

3 Math

Lemma 3.1 (Brezis-Lieb) Let $\Omega \subset \mathbb{R}^N$ be an open set, and $\{u_n\} \subset L^p(\Omega)$, $1 \leq p \leq \infty$. If

1. $\{u_n\}$ is bounded in $L^p(\Omega)$, i.e., $\|u_n\|_{L^p(\Omega)} < \infty$ for some $C > 0$;
2. $u_n \rightarrow u$ a.e. in Ω , then

$$\lim_{n \rightarrow \infty} \left(\int_{\Omega} |u_n|^p dx - \int_{\Omega} |u_n - u|^p dx \right) = \int_{\Omega} |u|^p dx \quad (3.1)$$