Exercises for PMLS, Homework 6

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Discussion of Jansen-Rit Model Code: The provided code delves into the Jansen-Rit neural mass model and its application in understanding macroscopic electrophysiological activities within cortical columns. The model offers a structured framework, employing two key steps to capture the dynamics of interconnected neural populations: the first step involves transforming action potentials into post-synaptic membrane potentials using an impulse response, while the second step converts these potentials into rates of action potentials through a sigmoidal function. The resulting set of non-linear second-order differential equations encapsulates the coarse-grained activity of each population:

Piramidal cells:

$$\dot{y}_0(t) = y_3(t) \tag{1}$$
$$\dot{y}_3(t) = AaS[y_1(t) - y_2(t)] - 2ay_3(t) - a^2 y_0(t) \tag{2}$$

Excitatory interneuron:

$$\dot{y}_1(t) = y_4(t) \tag{3}$$
$$\dot{y}_4(t) = Aa(mu(t) + C_2 S[C_1 y_0(t)]) - 2ay_4(t) - a^2 y_1(t) \tag{4}$$
$$\tag{5}$$

Inhibitory interneurons:

$$\dot{y}_2(t) = y_5(t) \tag{6}$$
$$\dot{y}_5(t) = BbC_4 S[C_3 y_0] - 2by_5(t) - b^2 y_2(t) \tag{7}$$

Here, $C_{1,2,3,4}$ represent connectivity, A and B the max amplitude and a,b are rate constants.

Phase plane analysis can be used to show transition points (points where the system changes stability) linked to the change of different parameter configurations for a single node. The code demonstrates that variations in connectivity parameters and the external input (mu, assumed constant) contribute to these dynamic shifts. Specifically, when treating mu as a variable parameter, stability analysis reveals critical transition points, which are Hopf bifurcations (a critical point where, as a parameter changes, a system's stability switches, and a periodic solution arises). Following the suggestion of Fig.1, representing the eigenvalues obtained for mu constant in the range $mu \in [-20, 400]$, using parameters $A = 3.25, B = 22, C = 135, C_1 = 1 * C, C_2 = 0.8 * C, C_3 = 0.25 * C, C_4 = 0.25 * C$, some particular values of mu are chosen to show the different behavior of the stability solutions at different points of the bifurcation:

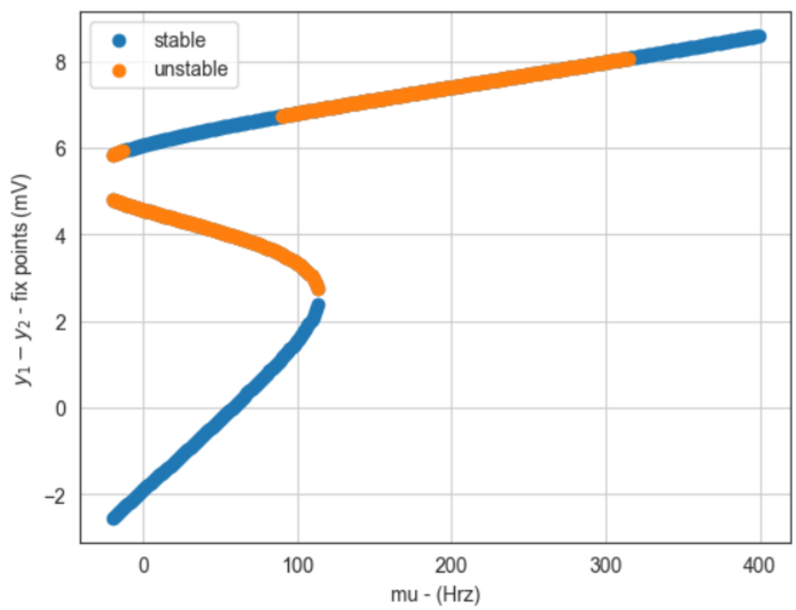
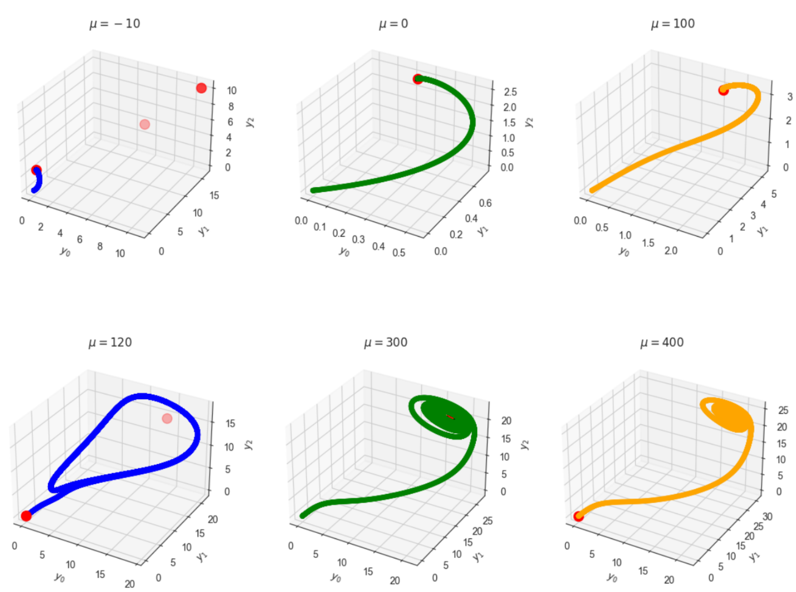


Figure 1: Fig.1

where $y_1 - y_2 = y = \frac{A}{a}mu + \frac{A}{a}C_2S(\frac{A}{a}C_1S(y)) - \frac{B}{b}C_4S(\frac{A}{a}C_3S(y))$.

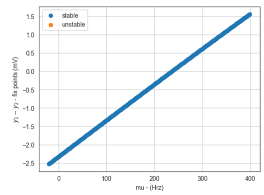
Whenever the system undergoes a stability change, a transition point emerges where the Jacobian matrix has eigenvalues with zero real parts. A closer examination of these eigenvalues reveals that, in the upper unstable section (between approximately 90 and 315), the fixed point experiences two complex conjugate eigenvalues crossing the imaginary axis. This phenomenon signifies two Hopf bifurcations, leading to oscillatory behavior in the system. For lower values of mu, the system demonstrates bistability with three fixed points. The ultimate stable fixed point reached depends on the system's initial conditions.

Some results of the trials are explicitly shown:

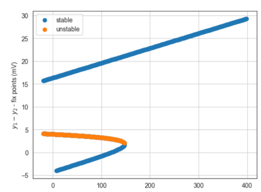


Additionaly, we have also investigated the influence of the other parameters in the generation of the bifurcation:

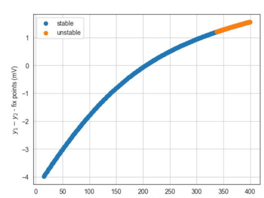
1. No bifurcation and no transitions if $A/a \sim 1/100$ or if $C_{3,4}$ and C_1 are very small because the equation becomes $y \sim \frac{A}{a} mu$. An example is given by: $A = 1, B = 22, C = 135, C_1 = 1 * C, C_2 = 0.8 * C, C_3 = 0.25 * C, C_4 = 0.25 * C, a = 100, b = 100$



2. if one between C_3, B/b or C_4 are small then there are unstable solutions but in correspondance of bistable points. For example, if: $A = 3.25, B = 22, C = 135, C_1 = C_2 = C_4 = C, C_3 = 0.01 * C, a = 100, b = 100$



3. if C_1 is small, instead, we have a transition for higher mu values. For example, if: $A = 3.25, B = 22, C = 135, C_3 = C_2 = C_4 = C, C_1 = 0.01 * C, a = 100, b = 100$



In the second section, the code extends the Jansen and Ritt model beyond single-node analysis to encompass a comprehensive whole-brain perspective, integrating anatomical connectivity. The brain undergoes segmentation into sub-regions employing the Schaefer et al. 2018 atlas, with the Jansen and Ritt Model applied to each distinct region. Computation of the distance between brain regions is executed based on their coordinates through Euclidean distance. The structural connectivity matrix is established using data from the Human Connectome Project (HCP) dataset and conduction delays between regions are determined considering distance and a conduction velocity of 5 ms.

The code illustrates that variations in alpha oscillations in specific brain regions, representing the maximum amplitude of the postsynaptic potential, can be ascribed to modifications in structural connectivity (assuming the model for the nodes remains consistent with identical parameters). Consequently, it is suggested that employing alternative connectivity matrices will yield distinct patterns in alpha oscillations.