

# $\infty$ -former: Infinite Memory Transformer

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## Abstract

Transformers struggle when attending to long contexts, since the amount of computation grows with the context length, and therefore they cannot model long-term memories effectively. Several variations have been proposed to alleviate this problem, but they all have a finite memory capacity, being forced to drop old information. In this paper, we propose the  $\infty$ -former, which extends the vanilla transformer with an *unbounded* long-term memory. By making use of a continuous-space attention mechanism to attend over the long-term memory, the  $\infty$ -former’s attention complexity becomes independent of the context length. Thus, it is able to model arbitrarily long contexts and maintain “sticky memories” while keeping a fixed computation budget. Experiments on a synthetic sorting task demonstrate the ability of the  $\infty$ -former to retain information from long sequences. We also perform experiments on language modeling, by training a model from scratch and by fine-tuning a pre-trained language model, which show benefits of unbounded long-term memories.

## 1 Introduction

When reading or writing a document, it is important to keep in memory the information previously read or written. Humans have a remarkable ability to remember long-term context, keeping in memory the relevant details (Carroll, 2007; Kuhbandner, 2020). Recently, transformer-based language models have achieved impressive results by increasing the context size (Radford et al., 2018, 2019; Dai et al., 2019; Rae et al., 2019; Brown et al., 2020). However, as the computational burden of transformers grows with the length of the context, these models have computational limitations about how much information can fit into memory. For example, a regular transformer requires quadratic time to process an input sequence and linear time to attend to the context for generating every new word.

Several variations have been proposed to address this problem (Tay et al., 2020b). Some propose using sparse attention mechanisms, either with data-dependent patterns (Kitaev et al., 2020; Vyas et al., 2020; Tay et al., 2020a; Roy et al., 2021) or data-independent patterns (Child et al., 2019; Beltagy et al., 2020; Zaheer et al., 2020), reducing the self-attention complexity (Katharopoulos et al., 2020; Choromanski et al., 2021; Peng et al., 2021; Jaegle et al., 2021), and caching past representations in a memory (Dai et al., 2019; Rae et al., 2019). These models are able to reduce the attention complexity, and scale up to longer contexts. However, as their complexity still depends on the context length, they cannot deal with unbounded context.

In this paper, we propose the  $\infty$ -former (*infinite former*; Fig. 1): a transformer model extended with an unbounded long-term memory (LTM), which allows the model to attend to arbitrarily long contexts. In order for the LTM to be unbounded, we use a continuous-space attention framework (Martins et al., 2020) which trades off the number of information units that fit into memory (basis functions) with the granularity of their representations. In this framework, the input sequence is represented as a **continuous signal**, expressed as a linear combination of radial basis functions (RBFs). This representation comes with two significant advantages: (i) the context can be represented using a number of basis functions  $N$  smaller than the number of tokens in the context, reducing attention complexity; and (ii)  $N$  can be fixed, making it possible to represent unbounded context in memory, as described in §3.2 and Fig. 2, at the cost of losing resolution, without increasing its attention complexity,  $\mathcal{O}(L^2 + L \times N)$ , where  $L$  corresponds to the transformer sequence length (details in §3.1.1).

To mitigate the problem of losing resolution for old memories, we introduce the concept of “sticky memories” (§3.3), by attributing larger spaces in the LTM’s new signal to the relevant regions of the

previous memory’s signal. This creates a notion of permanence in the LTM, allowing the model to better capture long contexts without losing the relevant information, analogous to long-term potentiation and plasticity in the brain (Mills et al., 2014; Bamji, 2005).

To sum up, our contributions are the following:

- We propose the  $\infty$ -former, in which we extend the transformer model with a continuous long-term memory (§3.1). As the attention computational complexity is independent of the context length, the  $\infty$ -former is able to model long contexts.
- We propose a procedure that allows the model to keep unbounded context in memory (§3.2).
- We introduce sticky memories, a procedure that enforces the persistence of important information in the LTM (§3.3).
- We perform empirical comparisons in a synthetic task (§4.1), which considers increasingly long sequences, and in language modeling, by training a model from scratch (§4.2) and by fine-tuning a pre-trained language model (§4.3). These experiments show the benefits of using an unbounded memory.

## 2 Background

### 2.1 Transformer

A transformer (Vaswani et al., 2017) is composed of several layers, which encompass a multi-head self-attention layer (Bahdanau et al., 2015; Graves et al., 2014; Weston et al., 2014) followed by a feed-forward layer, along with residual connections (He et al., 2016) and layer normalization (Ba et al., 2016).

Let us denote the input sequence as  $X = [x_1, \dots, x_L] \in \mathbb{R}^{L \times e}$ , where  $L$  is the sequence length and  $e$  is the embedding size of the attention layer. Queries  $Q$ , keys  $K$ , and values  $V$  to be used in the self-attention computation are obtained by linearly projecting the input, or the output of the previous layer,  $X$ :

$$Q = XW^Q, \quad K = XW^K, \quad V = XW^V, \quad (1)$$

where  $W^Q, W^K, W^V \in \mathbb{R}^{e \times e}$  are learnable projection matrices. To perform multi-head self-attention,  $Q$ ,  $K$ , and  $V$  are split into heads  $Q_h, K_h, V_h \in \mathbb{R}^{L \times d}$  for  $h$  in  $\{1, \dots, H\}$  where  $H$

is the number of heads and  $d = e/H$ . Then, the context representation  $Z_h \in \mathbb{R}^{L \times d}$ , that corresponds to each attention head  $h$ , is obtained as:

$$Z_h = \text{softmax} \left( \frac{Q_h K_h^\top}{\sqrt{d}} \right) V_h, \quad (2)$$

where  $d$  is the hidden size dimension of each  $K_h$  and the softmax is performed row-wise. The head context representations are concatenated to obtain the final context representation  $Z \in \mathbb{R}^{L \times e}$ :

$$Z = [Z_1, \dots, Z_H] W^R, \quad (3)$$

where  $W^R \in \mathbb{R}^{e \times e}$  is another projection matrix that aggregates all head’s representations.

### 2.2 Continuous Attention

Continuous attention mechanisms (Martins et al., 2020) have been proposed to handle arbitrary continuous signals, where the attention probability mass function over words is replaced by a probability *density* over a signal. This allows time intervals or compact segments to be selected.

To perform continuous attention, the first step is to transform the discrete text sequence represented by  $X \in \mathbb{R}^{L \times e}$  into a continuous signal. This is done by expressing it as a linear combination of basis functions. To do so, each  $x_i$ , with  $i \in \{1, \dots, L\}$ , is first associated with a position  $t_i \in [0, 1]$ , e.g., by setting  $t_i = i/L$ . Then, we obtain a continuous-space representation  $\bar{X}(t) \in \mathbb{R}^e$ , for any  $t \in [0, 1]$  as:

$$\bar{X}(t) = B^\top \psi(t), \quad (4)$$

where  $\psi(t) \in \mathbb{R}^N$  are  $N$  1D RBFs located in  $[0, 1]$ . The coefficient matrix  $B \in \mathbb{R}^{N \times e}$  in Eq. 4 is obtained with a multivariate ridge regression criterion (Brown et al., 1980) so that  $\bar{X}(t_i) \approx x_i$  for each  $i \in [L]$ , which leads to the closed form (see App. A for details):

$$B^\top = X^\top F^\top (F F^\top + \lambda I)^{-1} = X^\top G, \quad (5)$$

where  $F = [\psi(t_1), \dots, \psi(t_L)] \in \mathbb{R}^{N \times L}$  packs the basis vectors for the  $L$  locations. As  $G \in \mathbb{R}^{L \times N}$  only depends of  $F$ , it can be computed offline.

Having converted the input sequence into a continuous signal  $\bar{X}(t)$ , the second step is to attend over this signal. To do that, instead of having a discrete probability distribution over the input sequence as in standard attention mechanisms (like in Eq. 2), we have a probability density  $p$ , which



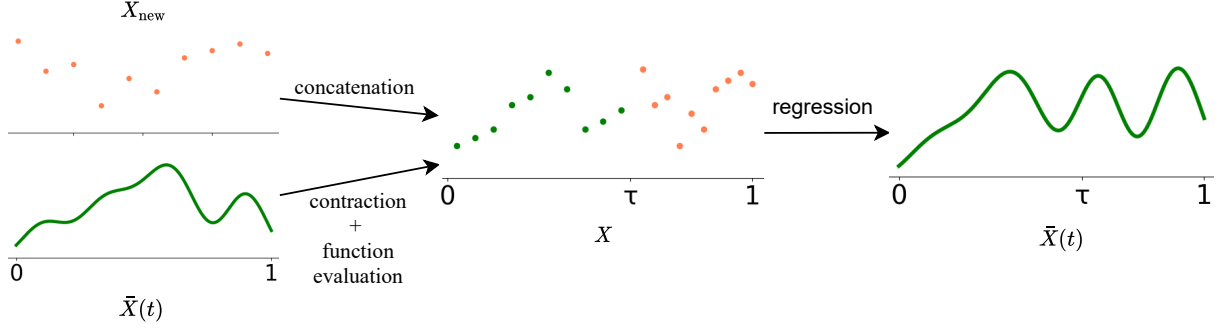


Figure 2: Diagram of the unbounded memory update procedure. This is performed in parallel for each embedding dimension, and repeated throughout the input sequence. We propose two alternatives to select the positions used for the function evaluation: linearly spaced or sticky memories.

$\mathcal{O}(L^2 + L \times L_{\text{STM}} + L \times N)$  when also using a short-term memory and  $\mathcal{O}(L^2 + L \times N)$  when only using the LTM, both  $\ll \mathcal{O}(L \times L_{\text{LTM}})$ , which would be the complexity of a vanilla transformer attending to the long-term memory. For this reason, the  $\infty$ -former can attend to arbitrarily long contexts without increasing the amount of computation.

### 3.2 Unbounded Memory

When representing the memory as a discrete sequence, to extend it, we need to store the new hidden states in memory. In a vanilla transformer, this is not feasible for long contexts due to the high memory requirements. However, the  $\infty$ -former can attend to unbounded context without increasing memory requirements by using continuous attention, as next described and shown in Fig. 2.

To be able to build an unbounded representation, we first sample  $M$  locations in  $[0, 1]$  and evaluate  $\tilde{X}(t)$  at those locations. These locations can simply be linearly spaced, or sampled according to the region importance, as described in §3.3.

Then, we concatenate the corresponding vectors with the new vectors coming from the short-term memory. For that, we first need to contract this function by a factor of  $\tau \in ]0, 1[$  to make room for the new vectors. We do this by defining:

$$X^{\text{contracted}}(t) = X(t/\tau) = B^\top \psi(t/\tau). \quad (13)$$

Then, we can evaluate  $\tilde{X}(t)$  at the  $M$  locations  $0 \leq t_1, t_2, \dots, t_M \leq \tau$  as:

$$x_m = B^\top \psi(t_m/\tau), \quad \text{for } m \in [M], \quad (14)$$

and define a matrix  $X_{\text{past}} = [x_1, x_2, \dots, x_M]^\top \in \mathbb{R}^{M \times e}$  with these vectors as rows. After that, we

concatenate this matrix with the new vectors  $X_{\text{new}}$ , obtaining:

$$X = [X_{\text{past}}^\top, X_{\text{new}}^\top]^\top \in \mathbb{R}^{(M+L) \times e}. \quad (15)$$

Finally, we perform multivariate ridge regression to compute the new coefficient matrix  $B \in \mathbb{R}^{N \times e}$ , via  $B^\top = X^\top G$ , as in Eq. 5. To do this, we need to associate the vectors in  $X_{\text{past}}$  with positions in  $[0, \tau]$  and in  $X_{\text{new}}$  with positions in  $]\tau, 1]$ . We consider the vectors to be linearly spaced.

### 3.3 Sticky Memories

When extending the LTM, we evaluate its current signal at  $M$  locations in  $[0, 1]$ , as shown in Eq. 14. These locations can be linearly spaced. However, some regions of the signal can be more relevant than others, and should consequently be given a larger “memory space” in the next step LTM’s signal. To take this into account, we propose sampling the  $M$  locations according to the signal’s relevance at each region (see Fig. 7 in App. B). To do so, we construct a histogram based on the attention given to each interval of the signal on the previous step. For that, we first divide the signal into  $D$  linearly spaced bins  $\{d_1, \dots, d_D\}$ . Then, we compute the probability given to each bin,  $p(d_j)$  for  $j \in \{1, \dots, D\}$ , as:

$$p(d_j) \propto \sum_{h=1}^H \sum_{i=1}^L \int_{d_j} \mathcal{N}(t; \mu_{h,i}, \sigma_{h,i}^2) dt, \quad (16)$$

where  $H$  is the number of attention heads and  $L$  is the sequence length. Note that Eq. 16’s integral can be evaluated efficiently using the erf function:

$$\int_a^b \mathcal{N}(t; \mu, \sigma^2) = \frac{1}{2} \left( \text{erf} \left( \frac{b}{\sqrt{2}} \right) - \text{erf} \left( \frac{a}{\sqrt{2}} \right) \right). \quad (17)$$



Then, we sample the  $M$  locations at which the LTM’s signal is evaluated at, according to  $p$ .

### 3.4 Implementation and Learning Details

Before applying multivariate ridge regression to convert the LTM’s input discrete sequence  $X$  into a continuous signal, we use a simple convolutional layer (with stride = 1 and width = 3) as a gate, to smooth the sequence:

$$\tilde{X} = \text{sigmoid}(\text{CNN}(X)) \odot X. \quad (18)$$

To train the model we use the cross entropy loss. Having a sequence of text  $X$  of length  $L$  as input, a language model outputs a probability distribution of the next word  $p(x_{t+1} | x_t, \dots, x_{t-L})$ . Given a corpus of  $T$  tokens, we train the model to minimize its negative log likelihood:

$$\mathcal{L}_{\text{NLL}} = - \sum_{t=0}^{T-1} \log p(x_{t+1} | x_t, \dots, x_{t-L}). \quad (19)$$

Additionally, in order to avoid having uniform distributions over the LTM, we regularize the continuous attention given to the LTM, by minimizing the Kullback-Leibler (KL) divergence,  $D_{\text{KL}}$ , between the attention probability density,  $\mathcal{N}(\mu_h, \sigma_h)$ , and a Gaussian prior,  $\mathcal{N}(\mu_0, \sigma_0)$ . As different heads should attend to different regions, we set  $\mu_0 = \mu_h$ , regularizing only the attention variance, and get:

$$\mathcal{L}_{\text{KL}} = \sum_{t=0}^{T-1} \sum_{h=1}^H D_{\text{KL}}(\mathcal{N}(\mu_h, \sigma_h) || \mathcal{N}(\mu_h, \sigma_0)) \quad (20)$$

$$= \sum_{t=0}^{T-1} \sum_{h=1}^H \frac{1}{2} \left( \frac{\sigma_h^2}{\sigma_0^2} - \log \left( \frac{\sigma_h}{\sigma_0} \right) - 1 \right). \quad (21)$$

Thus, the final loss that is minimized corresponds to:

$$\mathcal{L} = \mathcal{L}_{\text{NLL}} + \lambda_{\text{KL}} \mathcal{L}_{\text{KL}}, \quad (22)$$

where  $\lambda_{\text{KL}}$  is a hyperparameter that controls the amount of KL regularization.

## 4 Experiments

To understand if the  $\infty$ -former is able to model long contexts, we first performed experiments on a synthetic task, which consists of sorting tokens by their frequencies in a long sequence (§4.1). Then, we performed experiments on language modeling, by training a model from scratch (§4.2) and by fine-tuning a pre-trained language model (§4.3).

**Baselines.** We consider the transformer-XL<sup>4</sup> (Dai et al., 2019) and the compressive transformer<sup>5</sup> (Rae et al., 2019) as baselines for our experiments. The transformer-XL consists of a vanilla transformer (Vaswani et al., 2017) extended with a short-term memory which is composed of the hidden states of the previous steps. The compressive transformer is an extension of the transformer-XL: besides the short-term memory, it also has a long-term memory composed of the old vectors of the short-term memory, compressed using a CNN.

Both the transformer-XL and the compressive transformer require relative positional encodings. In contrast, there is no need for positional encodings in the memory in our approach because the memory vectors represent basis coefficients in a predefined continuous space.

### 4.1 Sorting

In this task, the input consists of a sequence of tokens sampled according to a token probability distribution (which is not known to the system). The objective is to generate the tokens in the decreasing order of their frequencies in the sequence. One example can be:

$\underbrace{1 \ 2 \ 1 \ 3 \ 1 \ 0 \ 3 \ 1 \ 3 \ 2}_{\substack{1 \text{ occurs 4 times; } 3 \text{ occurs 3 times, etc.}}} <\text{SEP}> \ 1 \ 3 \ 2 \ 0$

To understand if the long-term memory is being effectively used and the transformer is not only performing sorting by modeling the most recent tokens, we design the token probability distribution to *change over time*: namely, we set it as a mixture of two distributions,  $p = \alpha p_0 + (1 - \alpha) p_1$ , where the mixture coefficient  $\alpha \in [0, 1]$  is progressively increased from 0 to 1 as the sequence is generated.

The vocabulary has 20 tokens and we experiment with sequences of length 4,000, 8,000, and 16,000. For all models we used a transformer with 3 layers and 6 attention heads and considered sequences of length 1,024 and a memory size 2,048. For the compressive transformer, both memories have a size of 1,024. For the  $\infty$ -former, we also consider a short-term memory of size 1,024 and a LTM with 1,024 basis functions. Further details and hyperparameters are described in App. C.1.

**Results.** As can be seen in Fig. 3, the transformer-XL achieves slightly higher accuracy than the compressive transformer and  $\infty$ -former for a short

<sup>4</sup>We use the authors’ implementation available at <https://github.com/kimiyoung/transformer-xl>.

<sup>5</sup>We use our implementation of the model.

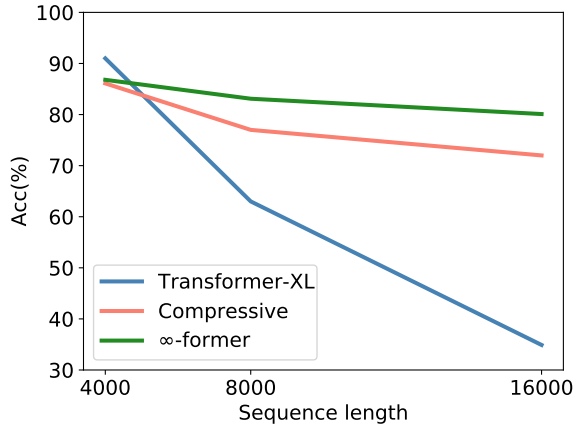


Figure 3: Sorting task accuracy for sequences of length 4,000, 8,000, and 16,000.

sequence length (4,000). This is because the transformer-XL is able to keep almost the entire sequence in memory. However, its accuracy degrades rapidly when the sequence length is increased. Both the compressive transformer and  $\infty$ -former also lead to smaller accuracies when increasing the sequence length. However, this decrease is not so significant for the  $\infty$ -former, which indicates that it is better at modeling long sequences.

## 4.2 Language Modeling

We also perform language modeling experiments on the Wikitext-103 dataset (Merity et al., 2017) which has a training set with 103 million tokens and validation and test sets with 217,646 and 245,569 tokens, respectively. For that, we follow the standard architecture of the transformer-XL (Dai et al., 2019), which consists of a transformer with 16 layers and 10 attention heads. For the transformer-XL, we experiment with a memory of size 150. For the compressive transformer, we consider that both memories have a size of 150 and a compression rate of 4. For the  $\infty$ -former we consider a short-term memory of size 150, a continuous long-term memory with 150 Gaussian RBFs, and a memory threshold of 900 tokens. Further details and hyperparameters are described in App. C.2.

**Results.** As can be seen in Table 1, extending the model with a long-term memory leads to a better perplexity, for both the compressive transformer and  $\infty$ -former. Moreover, the  $\infty$ -former slightly outperforms the compressive transformer. We can also see that using sticky memories leads to a somewhat lower perplexity, which shows that it helps the model to focus on the relevant memories.

	STM	LTM	Perplexity
Transformer-XL	150	—	24.52
Compressive	150	150	24.41
$\infty$ -former	150	150	24.29
$\infty$ -former (Sticky memories)	150	150	24.22

Table 1: Perplexity on Wikitext-103.

**Analysis.** To better understand whether  $\infty$ -former is paying more attention to the older memories in the LTM or to the most recent ones, we plotted a histogram of the attention given to each region of the long-term memory when predicting the tokens on the validation set. As can be seen in Fig. 4, in the first and middle layers, the  $\infty$ -former tends to focus more on the older memories, while in the last layer, the attention pattern is more uniform. In Figs. 5 and 8 (in App. D), we present examples of words for which the  $\infty$ -former has lower perplexity than the transformer-XL along with the attention given by the  $\infty$ -former to the last layer’s LTM. We can see that the word being predicted is present several times in the long-term memory and  $\infty$ -former gives higher attention to those regions.

To know whether the sticky memories approach attributes a larger space of the LTM’s signal to relevant phrases or words, we plotted the memory space given to each word<sup>6</sup> present in the long-term memory of the last layer when using and not using sticky memories. We present examples in Figs. 6 and 9 (in App. D) along with the phrases / words which receive the largest spaces when using sticky memories. We can see in these examples that this procedure does in fact attribute large spaces to old memories, creating memories that stick over time. We can also see that these memories appear to be relevant as shown by the words / phrases in the examples.

## 4.3 Pre-trained language models

To understand if the LTM could also be used to extend a pre-trained language model, we fine-tune GPT-2 small (Radford et al., 2019) on Wikitext-103 (Merity et al., 2017) and a subset of PG-19 (Rae et al., 2019) containing the first 2,000 books ( $\approx 200$  million tokens) of the training set. To do so, we consider sequences of size 512, a long-term

<sup>6</sup>The (Voronoi) memory space attributed to each word is half the distance from the previous word plus half the distance to the next word in the LTM’s signal, being the word’s location computed based on the sampled positions from which we evaluate the signal when receiving new memory vectors.

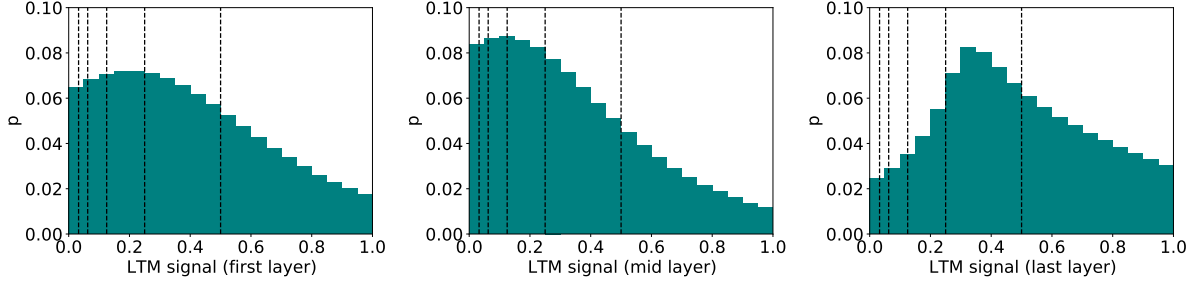


Figure 4: Histograms of attention given to the LTM by  $\infty$ -former, for the first (on the left), middle (on the middle), and last (on the right) layers. The dashed vertical lines represent the limits of the memory segments ( $\tau$ ) for the various memory updates.

GT: as the respective audio releases of the latter two concerts, Zoo TV Live and Hasta la Vista Baby! [U2](#)

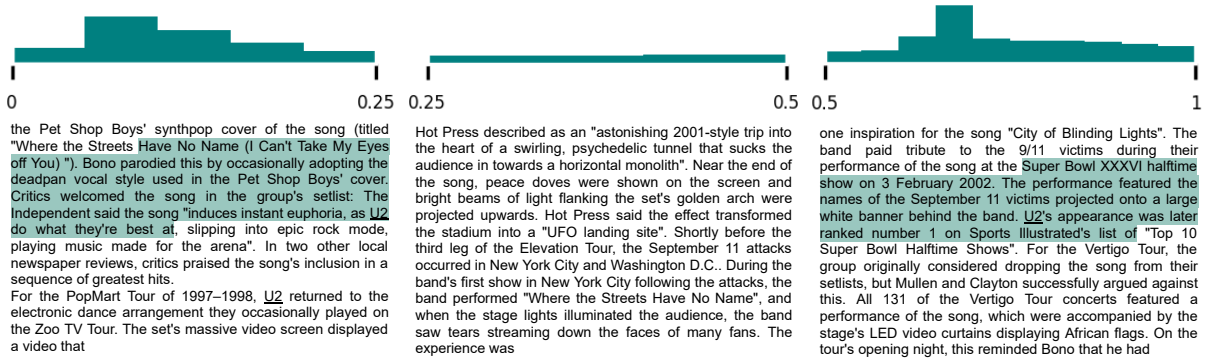


Figure 5: Example of attention given by  $\infty$ -former to the last layer’s long-term memory, when predicting the ground truth word “U2”. The words in the LTM which receive higher attention ( $> 0.05$ ) are shaded.

	Wikitext-103	PG19
GPT-2	16.85	33.44
$\infty$ -former	16.64	32.61
$\infty$ -former (Sticky memories)	16.61	32.48

Table 2: Perplexity of GPT-2 fine-tuned on Wikitext-103 and PG19.

memory with 512 Gaussian RBFs, and a memory threshold of 2,048 tokens. Further details and hyperparameters are described in App. C.3.

**Results.** The results reported in Table 2 show that by simply adding the long-term memory to GPT-2 and fine-tuning, we can improve perplexity on both Wikitext-103 and PG19. This shows the versatility of the  $\infty$ -former: it can be trained from scratch or used to improve a pre-trained model.

## 5 Related Work

**Continuous attention.** [Martins et al. \(2020\)](#) introduced 1D and 2D continuous attention, using Gaussians and truncated parabolas as densities. They applied it to RNN-based document classi-

fication, machine translation, and visual question answering. Several other works have also proposed the use of (discretized) Gaussian attention for natural language processing tasks: [Guo et al. \(2019\)](#) proposed a Gaussian prior to the self-attention mechanism to bias the model to give higher attention to nearby words, and applied it to natural language inference; [You et al. \(2020\)](#) proposed the use of hard-coded Gaussian attention as input-agnostic self-attention layer for machine translation; [Dubois et al. \(2020\)](#) proposed using Gaussian attention as a location attention mechanism to improve the model generalization to longer sequences. However, these approaches still consider discrete sequences and compute the attention by evaluating the Gaussian density at the token positions.

**Efficient transformers.** Several methods have recently been proposed that reduce the transformer’s attention complexity, and can, consequently, model longer contexts. Some of these do so by performing sparse attention, either by selecting pre-defined attention patterns ([Child et al., 2019](#); [Beltagy et al., 2020](#); [Zaheer et al., 2020](#)), or

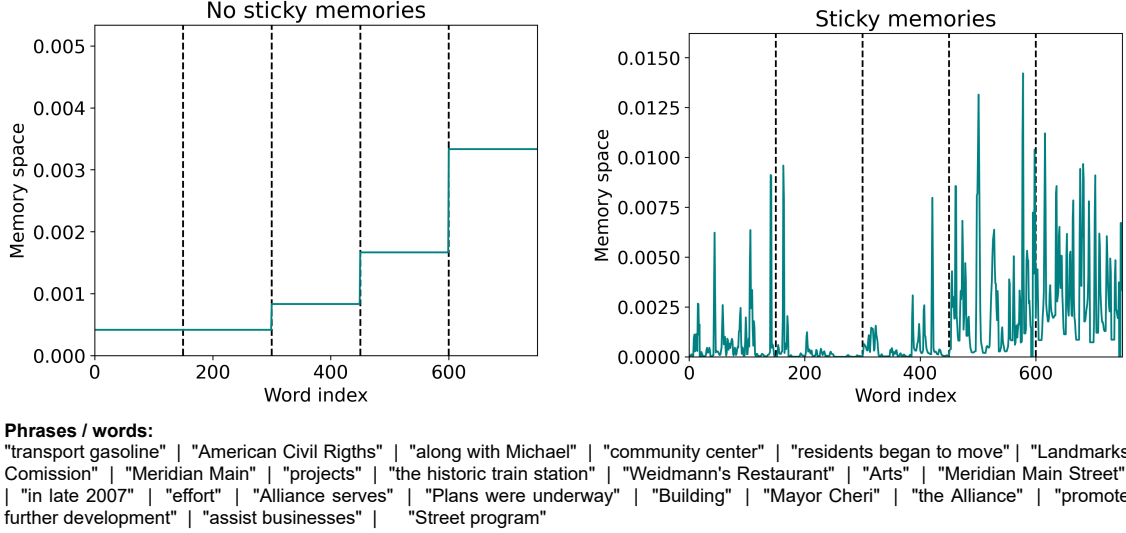


Figure 6: Example of the memory space attributed to each word in the last layer’s long-term memory (after 5 updates) without / with the sticky memories procedure, along with the words / phrases which have the largest memory spaces when using sticky memories (top peaks with space > .005). Excerpt of the sequence being generated in this example: “Given Meridian’s site as a railroad junction, its travelers have attracted the development of many hotels.” The dashed vertical lines represent the limits of the memory segments for the various memory updates.

by learning these patterns from data (Kitaev et al., 2020; Vyas et al., 2020; Tay et al., 2020a; Roy et al., 2021). Other works focus on directly reducing the attention complexity by applying the (reversed) kernel trick (Katharopoulos et al., 2020; Choromanski et al., 2021; Peng et al., 2021; Jaegle et al., 2021). Closer to our approach are the transformer-XL and compressive transformer models (Dai et al., 2019; Rae et al., 2019), which extend the vanilla transformer with a bounded memory. We use these models as baselines in our experiments (§4).

**Memory-augmented language models.** Several models that augment language models have been introduced. Graves et al. (2014) and Weston et al. (2014) proposed extending recurrent neural networks with an external memory, while Chandar et al. (2016) and Rae et al. (2016) proposed efficient procedures to read and write from these memories, using hierarchies and sparsity. Grave et al. (2016) and Merity et al. (2017) proposed the use of cache-based memories which store pairs of hidden states and output tokens from previous steps. The probability distribution over the words in the memory is then combined with the probability distribution over the words in the vocabulary given by the language model. More recently, Khandelwal et al. (2019) and Yogatama et al. (2021) proposed using nearest neighbors to retrieve words from a key-based memory constructed based on the

training data. Khandelwal et al. (2019) proposed interpolating the retrieved words probability distributions with the probability over the words in the vocabulary when generating a new word, while Yogatama et al. (2021) proposed combining the retrieved information at the architecture level. These models have the disadvantage of needing to perform a retrieval step when generating each token, which can be computationally expensive. These memories are orthogonal to  $\infty$ -former’s LTM and in future work the two can be combined.

## 6 Conclusions

In this paper, we proposed the  $\infty$ -former: a transformer extended with an unbounded long-term memory. By making use of a continuous-space attention framework, its attention complexity is independent of the length of the context being modeled, which allows the model to attend to arbitrarily long contexts while keeping a fixed computation budget. By updating the memory taking into account past usage, the model learns to keep “sticky memories”, enforcing the persistence of relevant information in memory. Experiments on a sorting synthetic task show that  $\infty$ -former scales up to long sequences, maintaining a high accuracy. Experiments on language modeling, by training models from scratch and by fine-tuning a pre-trained language model, have shown improvements in perplexity.



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## A Multivariate ridge regression

The coefficient matrix  $B \in \mathbb{R}^{N \times e}$  is obtained with multivariate ridge regression criterion so that  $\bar{X}(t_i) \approx x_i$  for each  $i \in [L]$ , which leads to the closed form:

$$\begin{aligned} B^\top &= \arg \min_{B^\top} \|B^\top F - X^\top\|_{\mathcal{F}}^2 + \lambda \|B\|_{\mathcal{F}}^2 \\ &= X^\top F^\top (F F^\top + \lambda I)^{-1} = X^\top G, \end{aligned} \quad (23)$$

where  $F = [\psi(t_1), \dots, \psi(t_L)]$  packs the basis vectors for  $L$  locations and  $\|\cdot\|_{\mathcal{F}}$  is the Frobenius norm. As  $G \in \mathbb{R}^{L \times N}$  only depends of  $F$ , it can be computed offline.

## B Sticky memories

We present in Fig. 7 a scheme of the sticky memories procedure. First we sample  $M$  locations from the previous step LTM attention histogram (Eq. 16). Then, we evaluate the LTM’s signal at the sampled locations (Eq. 14). Finally, we consider that the sampled vectors,  $X_{\text{past}}$ , are linearly spaced in  $[0, \tau]$ . This way, the model is able to attribute larger spaces of its memory to the relevant words.

## C Experimental details

### C.1 Sorting

For the compressive transformer, we consider compression rates of size 2 for sequences of length 4,000, from 2 to 6 for sequences of length 8,000, and from 2 to 12 for sequences of length 16,000. We also experiment training the compressive transformer with and without the attention reconstruction auxiliary loss. For the  $\infty$ -former we consider 1,024 Gaussian RBFs  $\mathcal{N}(t; \tilde{\mu}, \tilde{\sigma}^2)$  with  $\tilde{\mu}$  linearly spaced in  $[0, 1]$  and  $\tilde{\sigma} \in \{.01, .05\}$ . We set  $\tau = 0.75$  and for the KL regularization we used  $\lambda_{\text{KL}} = 1 \times 10^{-5}$  and  $\sigma_0 = 0.05$ .

For this task, for each sequence length, we created a training set with 8,000 sequences and validation and test sets with 800 sequences. We trained all models with batches of size 8 for 20 epochs on 1 Nvidia GeForce RTX 2080 Ti or 1 Nvidia GeForce GTX 1080 Ti GPU with  $\approx 11$  Gb of memory, using the Adam optimizer (Kingma and Ba, 2015). For the sequences of length 4,000 and 8,000 we used a learning rate of  $2.5 \times 10^{-4}$  while for sequences of length 16,000 we used a learning rate of  $2 \times 10^{-4}$ . The learning rate was decayed to 0 until the end of training with a cosine schedule.

### C.2 Language Modeling

For the language modeling experiments on the English dataset Wikitext-103<sup>7</sup>, we use a transformer with 16 layers, 10 heads, embeddings of size 410, and a feed-forward hidden size of 2100. For the compressive transformer, we follow Rae et al. (2019) and use a compression rate of 4 and the attention reconstruction auxiliary loss. For the  $\infty$ -former we consider 150 Gaussian RBFs  $\mathcal{N}(t; \tilde{\mu}, \tilde{\sigma}^2)$  with  $\tilde{\mu}$  linearly spaced in  $[0, 1]$  and  $\tilde{\sigma} \in \{.01, .05\}$ . We set  $\tau = 0.5$  and for the KL regularization we used  $\lambda_{\text{KL}} = 1 \times 10^{-5}$  and  $\sigma_0 = 0.1$ .

We trained all models with batches of size 40 for 250,000 steps on 1 Nvidia Titan RTX or 1 Nvidia Quadro RTX 6000 with  $\approx 24$  GPU Gb of memory using the Adam optimizer (Kingma and Ba, 2015) with a learning rate of  $2.5 \times 10^{-4}$ . The learning rate was decayed to 0 until the end of training with a cosine schedule.

### C.3 Pre-trained Language Models

In these experiments, we fine-tune the GPT-2 small, which is composed of 12 layers with 12 attention heads, on the Wikitext-103 and on a subset of the English dataset PG19<sup>8</sup> containing the first 2,000 books. We consider sequences of length 512 and a long-term memory with 512 Gaussian RBFs  $\mathcal{N}(t; \tilde{\mu}, \tilde{\sigma}^2)$  with  $\tilde{\mu}$  linearly spaced in  $[0, 1]$  and  $\tilde{\sigma} \in \{.005, .01\}$  and for the KL regularization we use  $\lambda_{\text{KL}} = 1 \times 10^{-6}$  and  $\sigma_0 = 0.05$ . We set  $\tau = 0.5$ .

We fine-tuned GPT-2 small with a batch size of 1 on 1 Nvidia GeForce RTX 2080 Ti or 1 Nvidia GeForce GTX 1080 Ti GPU with  $\approx 11$  Gb of memory, using the Adam optimizer (Kingma and Ba, 2015) for 1 epoch with a learning rate of  $5 \times 10^{-5}$  for the GPT-2 parameters and a learning rate of  $2.5 \times 10^{-4}$  for the LTM parameters.

## D Examples

We present in Fig 8 an additional example of the attention given to the last layer’s LTM when predicting a word. In Fig. 9 we present an example of the memory space attributed to each word when using / not using sticky memories, along with the words or phrases which have the largest spaces in the memory.

<sup>7</sup>Dataset available at <https://blog.einstein.ai/the-wikitext-long-term-dependency-language-modeling-dataset/>.

<sup>8</sup>Dataset available at <https://github.com/deepmind/pg19>.

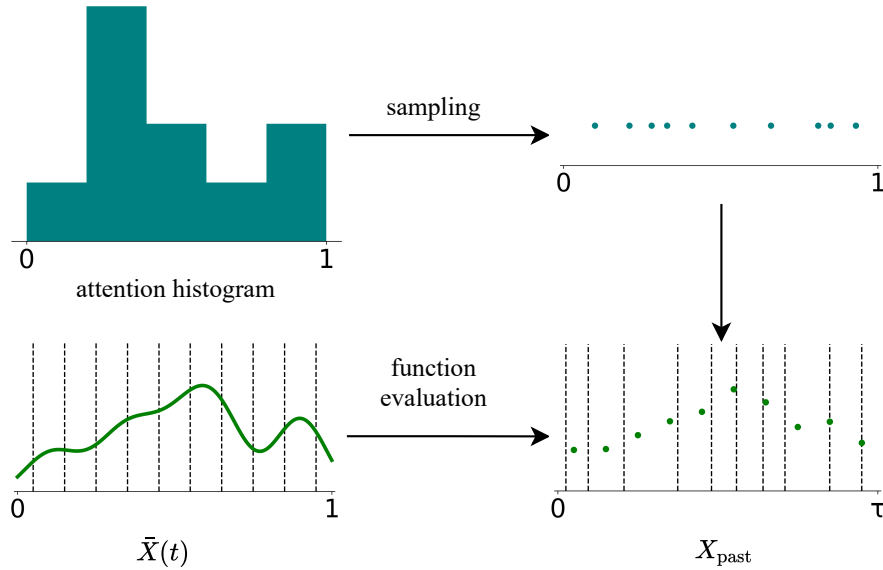


Figure 7: Sticky memories procedure diagram. The dashed vertical lines correspond to the position of the words in the LTM signal.

GT: for the first time on 26 January 1942, and attacked regularly thereafter, damaging some aircraft. The intact Hudsons were withdrawn to Darwin but Headlam

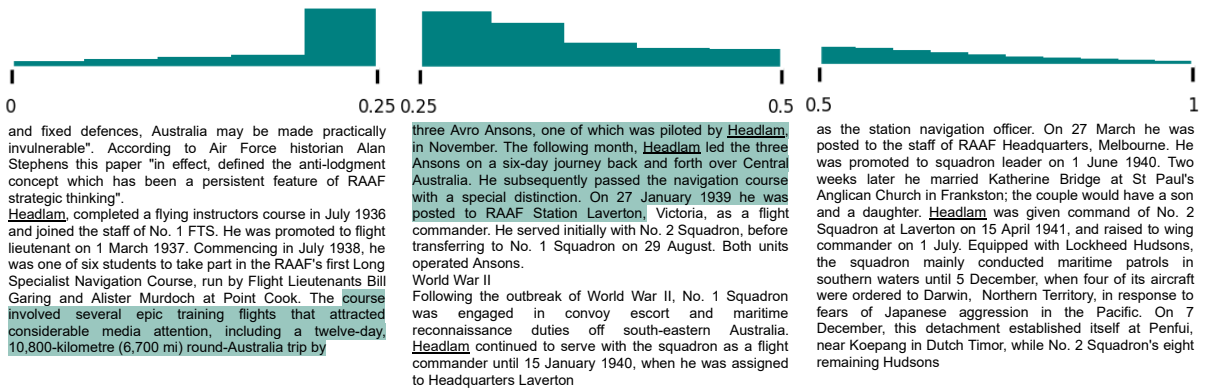
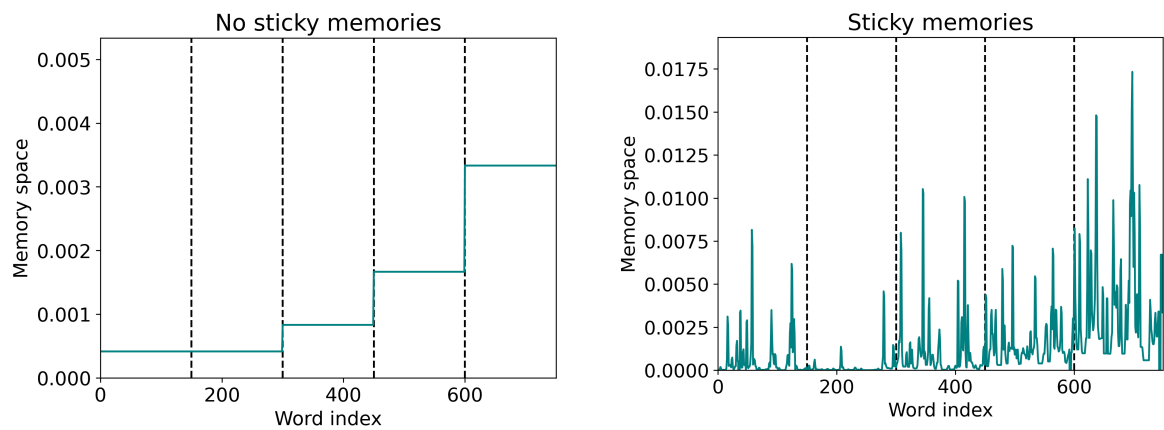


Figure 8: Example of attention given by  $\infty$ -former to the last layer's long-term memory, when predicting the ground truth word "Headlam". The words in the long-term memory which receive higher attention (bigger than 0.05) are shaded.





**Phrases / words:**

"July 1936" | "Headlam continued to serve" | "27 March" | "in Frankston" | "daughter" | "four of its aircraft" | "in response to fears of Japanese" | "stationed at Darwin" | "attacked the Japanese" | "forced it aground" | "dispersed at Penfui" | "three Japanese floatplanes" | "attacked regularly" | "withdrawn to Darwin" | "his staff remained at Penfui" | "ordered to evacuate" | "assistance from Sparrow Force" | "Four of No. 2 Squadron's Hudsons were destroyed" | "relocated to Daly Waters"

Figure 9: Example of the memory space attributed to each word in the last layer's long-term memory (after 5 updates) without / with the sticky memories procedure, along with the words / phrases which have the largest memory spaces when using sticky memories (top peaks with space > .005) Excerpt of the sequence being generated in this example: "Headlam became Officer Commanding North-Western Area in January 1946. Posted to Britain at the end of the year, he attended the Royal Air Force Staff College, Andover, and served with RAAF Overseas Headquarters, London." The dashed vertical lines represent the limits of the memory segments for the various memory updates.