ECE570 Lecture 7: Game-Tree Search

Jeffrey Mark Siskind

School of Electrical and Computer Engineering

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PURDUE UNIVERSITY

Games

- Two-person
- Multiple-move
- Zero-sum
- Deterministic
- Complete information

A Formalization of Games

$p \in \{1, -1\}$	a player
b	a board
b^0	the initial board
$p(b)$	the player to move in b
m	a move
$m(b)$	the set of legal moves for $p(b)$ in b
$b'(m, b)$	the board that results when $p(b)$ takes move m in b
$w^0(b)$	who wins in b

$$w^0(b) = \begin{cases} p & p \text{ has won in } b \\ 0 & \text{otherwise} \end{cases}$$

Tic Tac Toe

1	X
-1	O
n	the board size
$b_{i,j} \in \{1, -1, 0\}$	the mark at $\langle i, j \rangle$
$b^0_{i,j} = 0$	the board is initially empty
$p(b^0) = 1$	X moves first
$p(b)$	standard definition
$m = \langle i, j \rangle$	a move
$m(b)$	standard definition
$b'(m, b)$	standard definition
$w^0(b)$	standard definition

Optimal Play—I

$$w^k(b) = \begin{cases} p & \text{player } p \text{ wins in } k \text{ or fewer moves given optimal play} \\ 0 & \text{otherwise} \end{cases}$$

Optimal Play—II

$$w^1(b) = \begin{cases} w^0(b) & w^0(b) \neq 0 \vee m(b) = \{\} \\ \max\limits_{m \in m(b)} w^0(b'(m, b)) & w^0(b) = 0 \wedge m(b) \neq \{\} \wedge p(b) = 1 \\ \min\limits_{m \in m(b)} w^0(b'(m, b)) & w^0(b) = 0 \wedge m(b) \neq \{\} \wedge p(b) = -1 \end{cases}$$

Optimal Play—III

$$w^1(b) = \begin{cases} w^0(b) & w^0(b) \neq 0 \vee m(b) = \{\} \\ p(b) \max\limits_{m \in m(b)} p(b) w^0(b'(m,b)) & \text{otherwise} \end{cases}$$

Optimal Play—IV

$$w^k(b) = \begin{cases} w^0(b) & w^0(b) \neq 0 \vee m(b) = \{\} \\ p(b) \max\limits_{m \in m(b)} p(b) w^{k-1}(b'(m, b)) & \text{otherwise} \end{cases}$$

Optimal Play—V

$$w^*(b) = \begin{cases} p & \text{player } p \text{ wins given optimal play} \\ 0 & \text{otherwise (draw or nontermination)} \end{cases}$$

Optimal Play—VI

$$w^*(b) = \begin{cases} w^0(b) & w^0(b) \neq 0 \vee m(b) = \{\} \\ p(b) \max\limits_{m \in m(b)} p(b) w^*(b'(m, b)) & \text{otherwise} \end{cases}$$

Optimal Play—VII

$\hat{m}(b) \subseteq m(b)$ the set of moves that lead to optimal play in b

$$\hat{m}(b) = \begin{cases} \{\} & w^0(b) \neq 0 \\ \{m \in m(b) | w^*(b'(m, b)) = w^*(b)\} & \text{otherwise} \end{cases}$$