Random Passwords Aren't Good Enough

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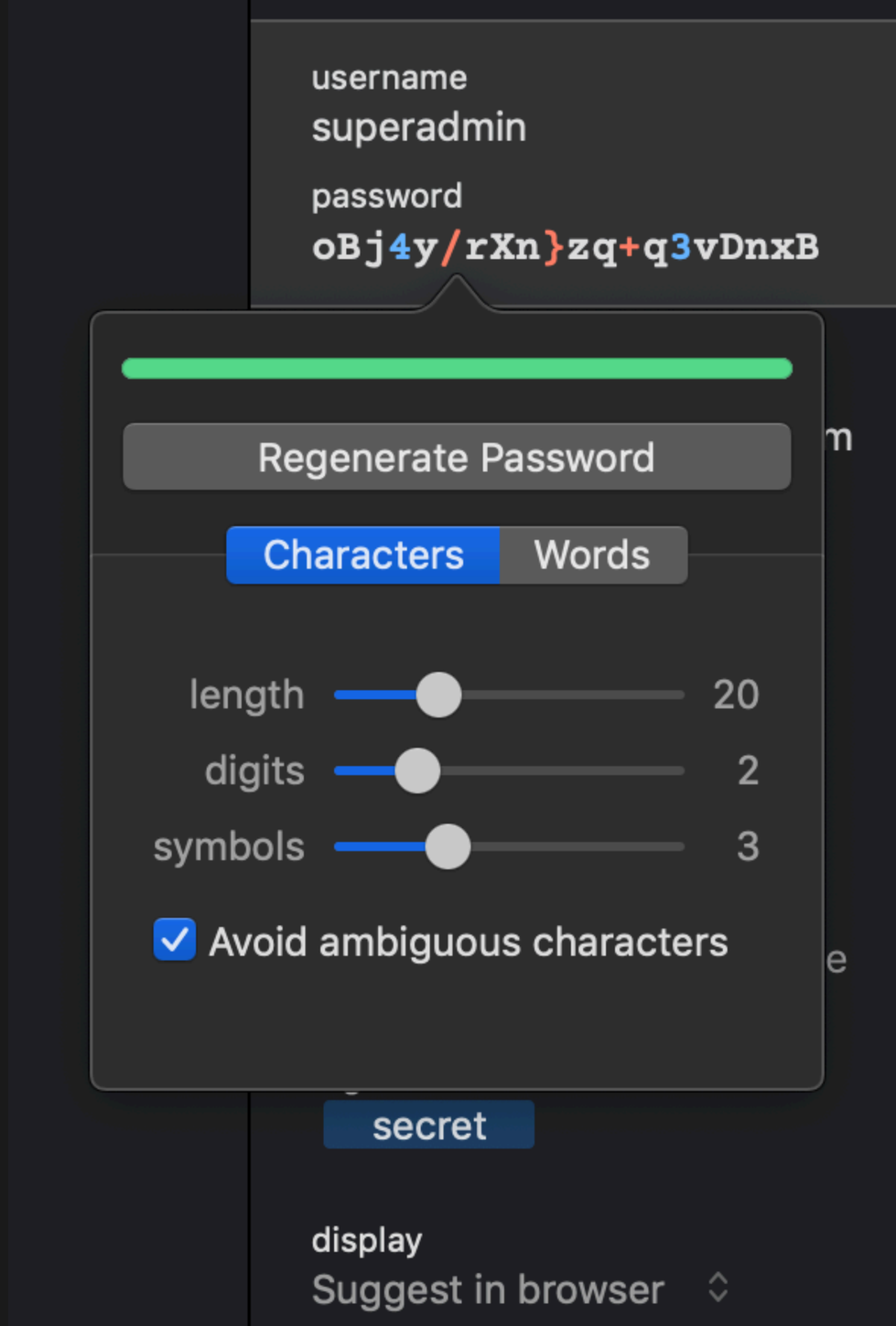
Weaver of Webs at 1Password

Where we started

oBjyrXnzqqvDnxB

43

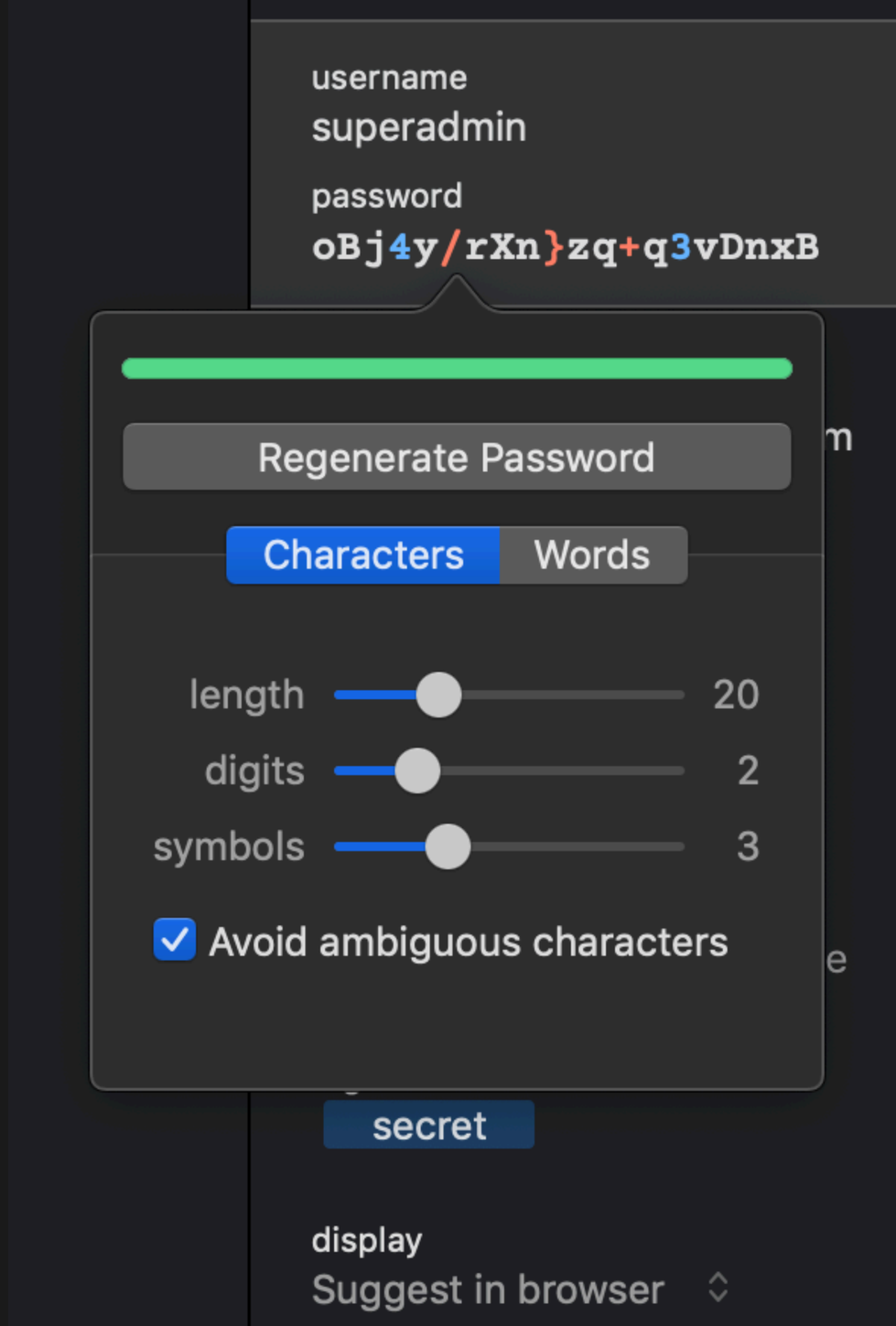
/}+



Where we started

`oBj4y/rXn}zq+q3vDnxB`

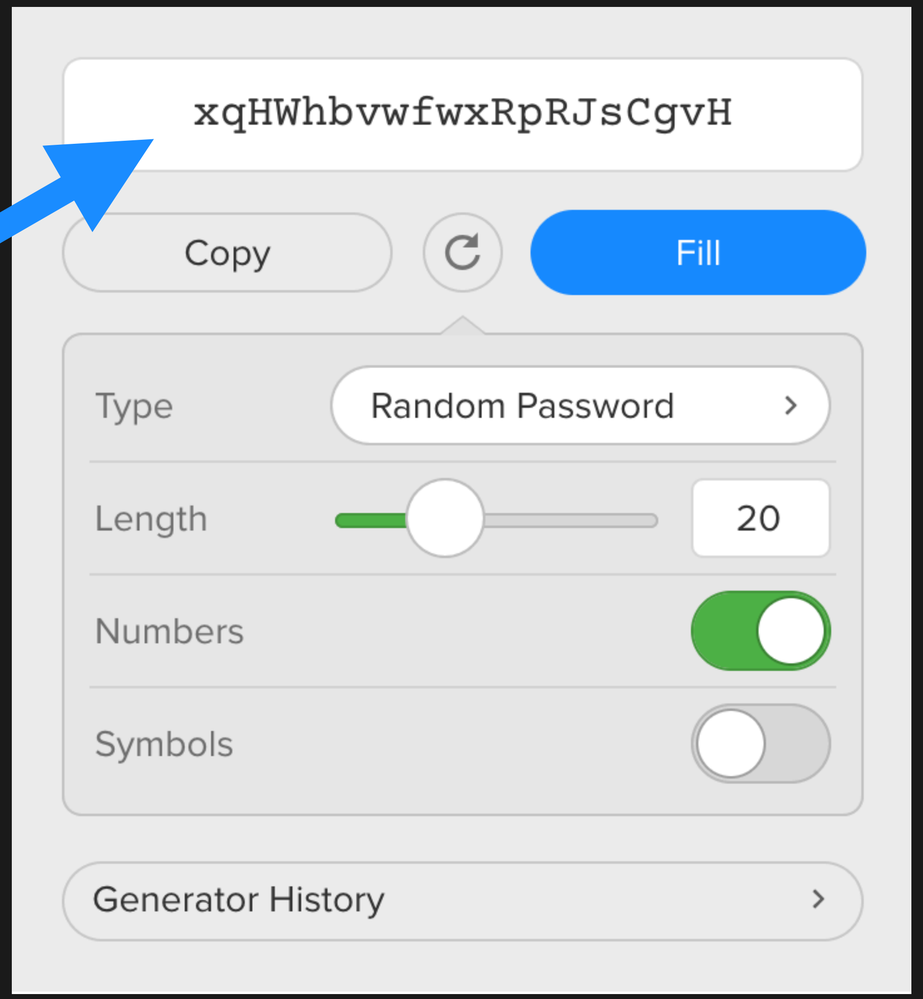
$$H(n) = l \lg |L| + d \lg |D| + s \lg |S| + \lg \binom{l+d}{d} + \lg \binom{l+d+s}{s}$$



Simplifying

– One slider
– Random mix
– Simpler entropy

$$H(n) = n \lg |A|$$



- 6 to 10 characters
- At least one lowercase let
- At least one number (0-9)
- No spaces, < or >

Password Must Contain
- ✖ at least 8 characters
- ✖ a number
- ✖ a lowercase letter
- ✖ an uppercase letter
- ✖ a special character (e.g. ,@$%*#?&)

ive and must contain:

assword

cters

umber and 1 letter

- ✖ Accepted symbols: _ ! . & @
- ✖ Password must be different from

Change Password

Please provide a secure password for your account. Password must be 7 - 10 characters in length, have at least one number, one special character and is case sensitive.

account

OK


ONE DOES NOT SIMPLY
PICK A RANDOM PASSWORD
imgflip.com

There are rules!

Password Length: 16

Include Numbers: ☑ (e.g. 123456)

Include Lowercase Characters: ☑ (e.g. abcdefgh)

Include Uppercase Characters: ☑ (e.g. ABCDEFGH)

Begin With A Letter: ☑ (don't begin with a number or symbol)

Include Symbols: ☑ !";#$%&'()*+,-./:;<=>?@[]^_`{|}~

No Similar Characters: ☑ (don't use characters like i, l, 1, L, o, 0, O, etc.)

No Duplicate Characters: ☑ (don't use the same character more than once)

No Sequential Characters: ☑ (don't use sequential characters, e.g. abc, 789)

Auto Generate On The First Call: ☑ (generate passwords automatically when you open this page)

Quantity: 1

Save My Preference: ☐ (save all the settings above in cookies)

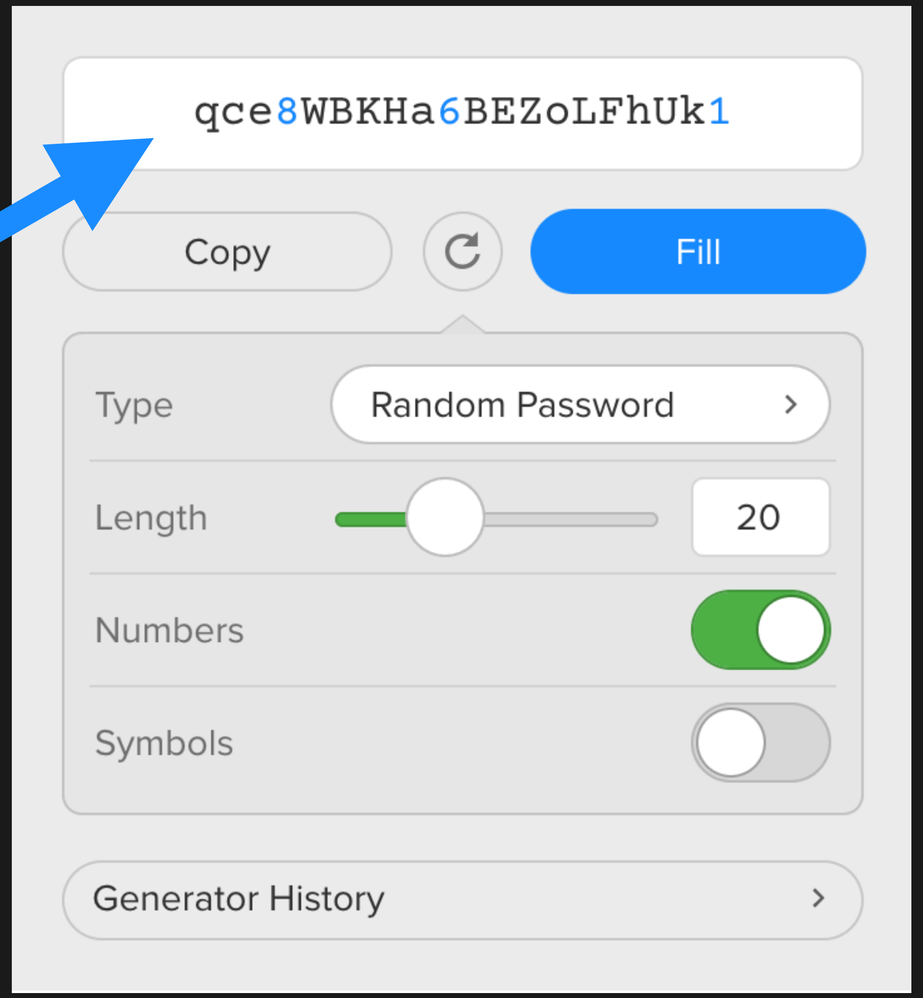
Generate Passwords

Your New Passwords:

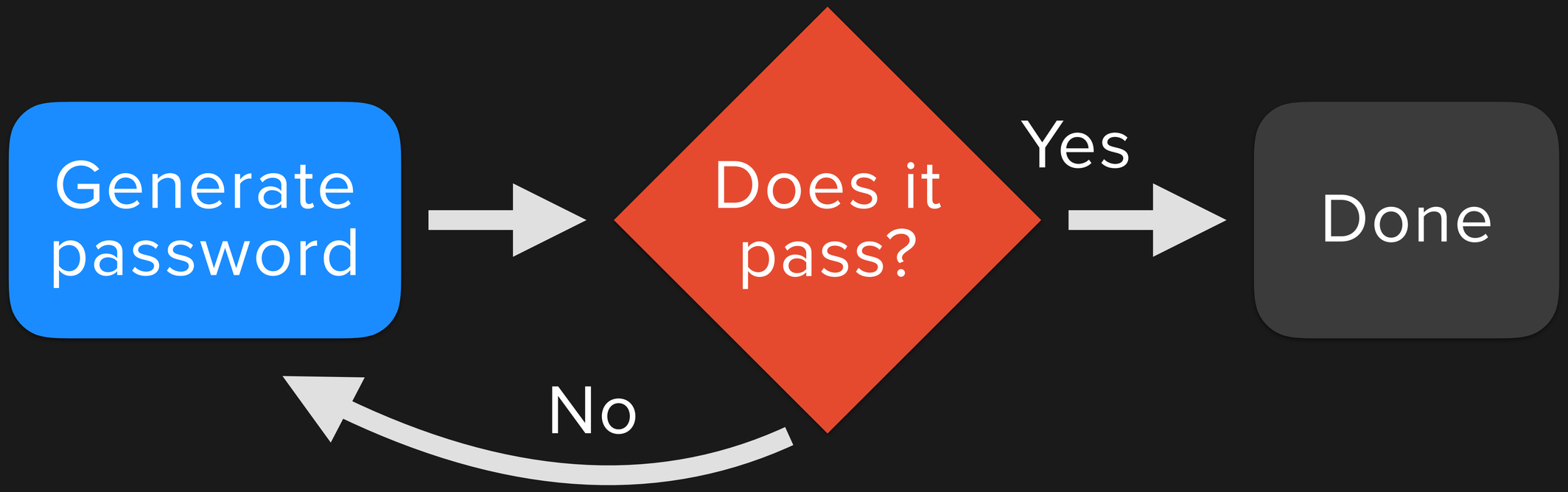
S:5gz>MZ4H&t%,VA

What we want

– Required characters
– Uniform distribution
– Accurate entropy
– Simple interface



The obvious dumb approach



The obvious dumb approach

An equation for entropy is easily derived:

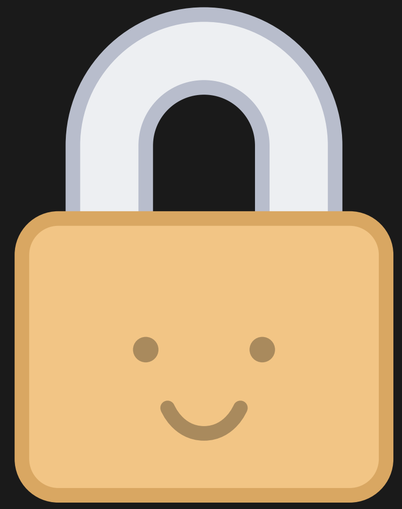
$$H(n) = \lg(|L \cup D|^n - |L|^n - |D|^n)$$

Two drawbacks:

– Generator not guaranteed to terminate

– Entropy function operates on giant integers

I can do better.



Take 1

```
oBjy/rXnzq+q3vDnxB
4
}
```

Take 1

```
oBj4y/rXn}zq+q3vDnxB
```

Take 1

```
oBj4y/rXn}zq+q3vDnxB
```

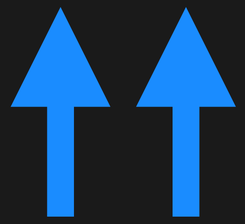
```
a1
```

```
1
```

Take 1

```
oBj4y/rXn}zq+q3vDnxB
```

```
  a  1
_ _ _ _
```

1 

Take 1

```
oBj4y/rXn}zq+q3vDnxB
```

```
a11
```

Take 1

```
oBj4y/rXn}zq+q3vDnxB
```

```
a11
```

❌ Uniform distribution

Take 2

```
oBjyr3X}nzqqvD+nxB
```

Take 2

```
oBjyrXnzqqvDnxB
3
}+
```

Take 2

```
oBjyrXnzqqvDnxB
43
/}+
```

Take 2

```
oBj4y/rXn}zq+q3vDnxB
```

Take 2

```
oBj4y/rXn}zq+q3vDnxB
```

```
a1
```

Take 2

```
oBj4y/rXn}zq+q3vDnxB
```

```
a
```

```
1
```

Take 2

```
oBj4y/rXn}zq+q3vDnxB
```

```
a
```

```
11
```

Take 2

```
oBj4y/rXn}zq+q3vDnxB
```

```
a11   1a1   11a
```

Take 2

```
oBj4y/rXn}zq+q3vDnxB

a11   1a1   11a

aa   a1   1a   11
```

Take 2

aa	a1	1a	11
aa1	a11	a11	111
a1a	1a1	1a1	
1aa	11a	11a	

Take 2

aa	a1	1a	11
aa1	a11		111
a1a	1a1		
1aa	11a		
1/12	1/6		1/4

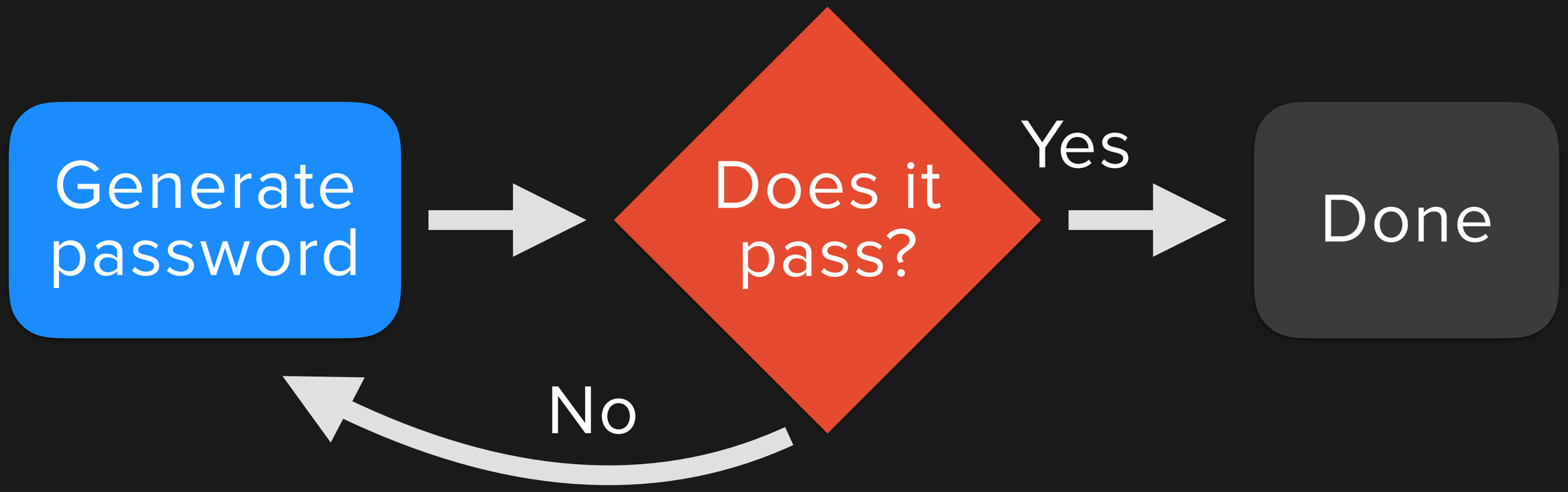
Take 2

```
oBj4y/rXn}zq+q3vDnxB

a11   1a1   11a

aa   a1   1a   11
```

❌ Uniform distribution

Ok, I give up.

The obvious dumb approach



The obvious dumb approach

Recall our entropy equation:

$$H(n) = \lg(|L \cup D|^n - |L|^n - |D|^n)$$

And the drawbacks:

– Generator not guaranteed to terminate

– Entropy function operates on giant integers

Notation update

Entropy in bits

$$H(n) = \lg(|L \cup D|^n - |L|^n - |D|^n)$$

Number of possible results

$$N(n) = |L \cup D|^n - |L|^n - |D|^n$$

$$H(n) = \lg N(n)$$

What about symbols?

Two required sets:

$$N(n) = |L \cup D|^n - |L|^n - |D|^n$$

Three required sets:

$$N(n) = |L \cup D \cup S|^n - |L|^n - |D|^n - |S|^n - |L \cup D|^n - |L \cup S|^n - |D \cup S|^n$$

What's wrong?

We said...

$$N(n) = |L \cup D \cup S|^n - |L|^n - |D|^n - |S|^n - |L \cup D|^n - |L \cup S|^n - |D \cup S|^n$$

But...

$$L \subseteq L \qquad L \subseteq (L \cup D) \qquad L \subseteq (L \cup S)$$

We need...

$$N(n) = |L \cup D \cup S|^n - |L|^n - |D|^n - |S|^n - N_{\{L,D\}}(n) - N_{\{L,S\}}(n) - N_{\{D,S\}}(n)$$

Notation update

The set of distinct required character sets

$$\mathscr{R} = \{R_1, R_2, R_3, \ldots, R_k\}$$

e.g. $\mathscr{R} = \{\text{"abc"}, \text{"123"}, \text{"!@#"}\}$

Possible passwords given $\mathscr{R}$ and length n

$$N_{\mathscr{R}}(n)$$

Finding a pattern

$$N_{\emptyset}(n) = 0$$

$$N_{\{R_1\}}(n) = \left|R_1\right|^n - \left(N_{\emptyset}(n)\right)$$

$$N_{\{R_1,R_2\}}(n) = \left|R_1 \cup R_2\right|^n - \left(N_{\emptyset}(n) + N_{\{R_1\}}(n) + N_{\{R_2\}}(n)\right)$$

$$N_{\{R_1,R_2,R_3\}}(n) = \left|R_1 \cup R_2 \cup R_3\right|^n - \Big(N_{\emptyset}(n) + N_{\{R_1\}}(n) + N_{\{R_2\}}(n) + N_{\{R_3\}}(n) + N_{\{R_1,R_2\}}(n) + N_{\{R_1,R_3\}}(n) + N_{\{R_2,R_3\}}(n)\Big)$$

The power set

The set of all subsets of a set

$$\mathscr{P}(\mathscr{R}) = \{x : x \subseteq \mathscr{R}\}$$

$$\mathscr{P}(\{R_1, R_2, R_3\}) = \{\emptyset, \{R_1\}, \{R_2\}, \{R_3\}, \{R_1, R_2\}, \{R_1, R_3\}, \{R_2, R_3\}, \{R_1, R_2, R_3\}\}$$

The set of all proper subsets of a set

$$\{x : x \subset \mathscr{R}\} \rightarrow \mathscr{P}(\mathscr{R}) \backslash \{\mathscr{R}\}$$

To recurse, divine

$$N_{\{R_1,R_2,R_3\}}(n) = \left|R_1 \cup R_2 \cup R_3\right|^n - \Big(N_{\emptyset}(n) + N_{\{R_1\}}(n) + N_{\{R_2\}}(n) + N_{\{R_3\}}(n) + N_{\{R_1,R_2\}}(n) + N_{\{R_1,R_3\}}(n) + N_{\{R_2,R_3\}}(n)\Big)$$

$$N_{\mathscr{R}}(n) = \left|\bigcup_{X\in\mathscr{R}} X\right|^n - \sum_{Y\in\mathscr{P}(\mathscr{R})\setminus\{\mathscr{R}\}} N_Y(n)$$

A final touch

Let A be the set of other allowed characters

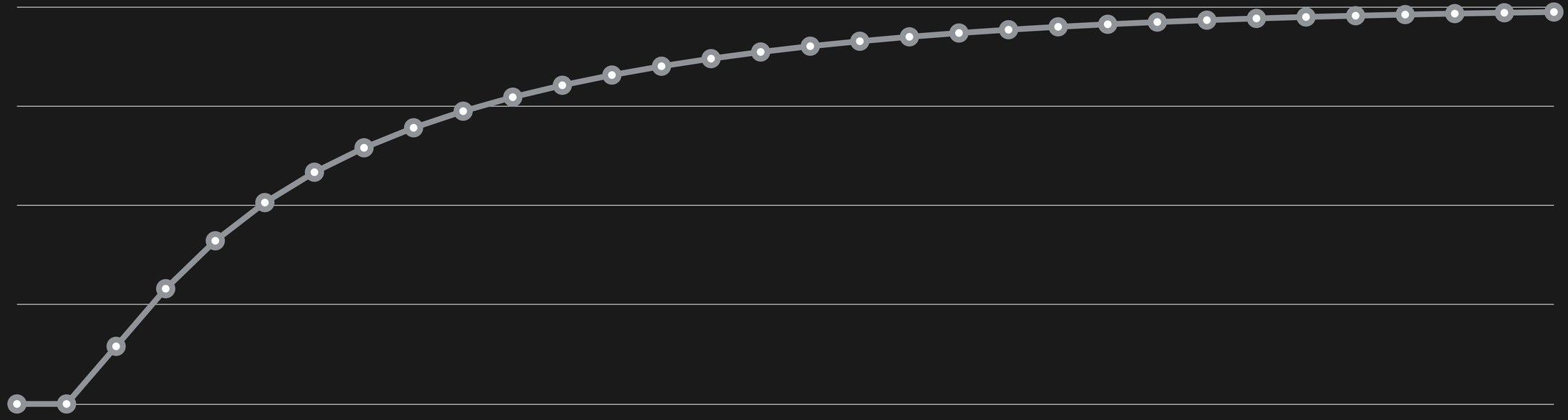
$$N^A_{\mathscr{R}}(n) = \left| \bigcup_{X \in \mathscr{R} \cup \{A\}} X \right|^n - \sum_{Y \in \mathscr{P}(\mathscr{R}) \setminus \{\mathscr{R}\}} N^A_Y(n)$$

$$N^A_{\emptyset}(n) = \left|A\right|^n$$

$$H^A_{\mathscr{R}}(n) = \lg N^A_{\mathscr{R}}(n)$$

Considering drawbacks

Probability of keeping a candidate



(lower + upper + digits – ambiguous)

Considering drawbacks

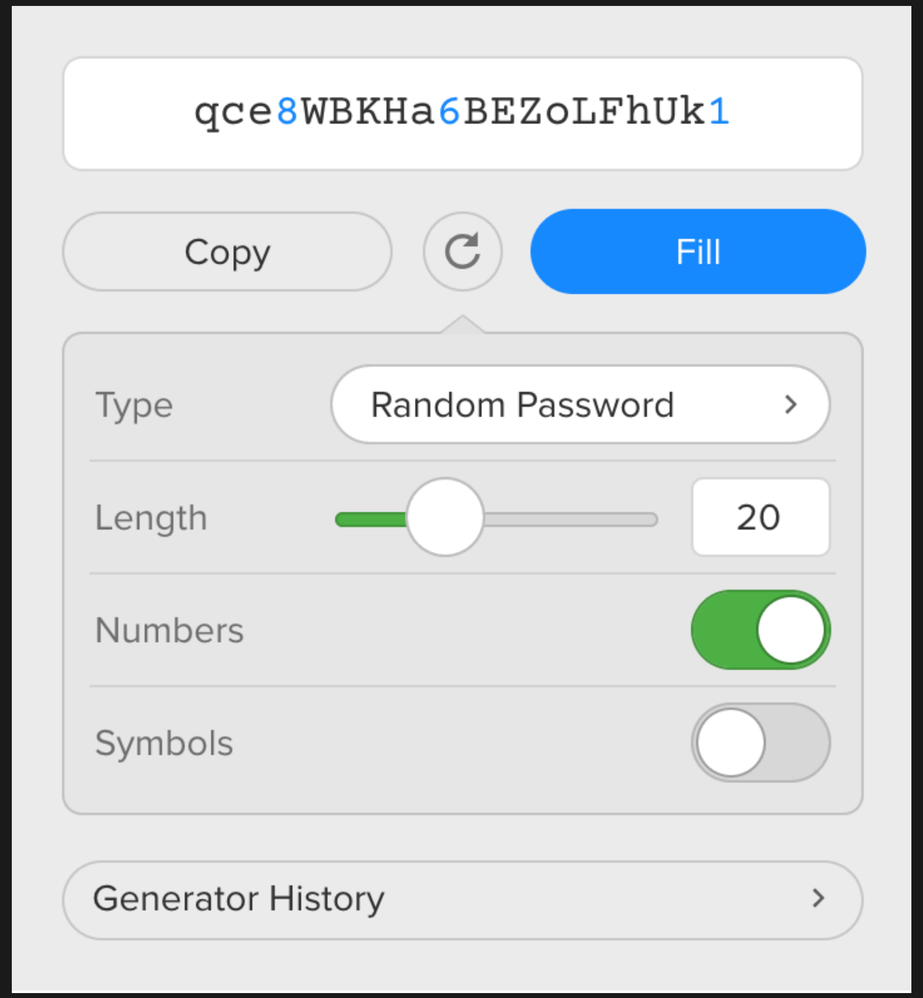
Probability of keeping a candidate

Size of values in memory (golang)

- `big.Int`
- `float64`
- `math.MaxFloat64` (1024 bits)

What we want

- Required characters ✔
- Uniform distribution ✔
- Accurate entropy ✔
- Simple interface ✔



Good enough.

Get it

Code, slides, and demo CLI:

github.com/1password/spg

```
import "go.1password.io/spg"
```

In action:

“1Password X” on the Chrome App Store

1password.com/password-generator