answers

April 17, 2020

1 Question 1

1.

Let $H = 0$ if patient is healthy otherwise 1 which means affected. Let $A = 1$ if patient is affected else 0 which means not affected. $P(H = 0) = 0.03$. False negative rate is $P(A = 0|H = 1) = 0.03$. False positive rate is $P(A = 1|H = 0) = 0.03$.

Calculating false detection,

We first get $P(H = 0|A = 1) = \frac{P(H=0)P(A=1|H=0)}{P(H=0)P(A=1|H=0)+P(H=1)P(A=1|H=1)}$ from the Bayes Rule.

Plugging in the values, we get $\frac{(1-0.03)(0.05)}{(1-0.03)0.05+0.03(1-0.03)} = 0.625$.

Calculating false misdetection,

We first get $P(H = 1|A = 0) = \frac{P(H=1)P(A=0|H=1)}{P(H=1)P(A=0|H=1)+P(H=0)P(A=0|H=0)}$ from the Bayes Rule.

Plugging in the values, we get

$\frac{0.03*0.03}{0.03*0.03+(1-0.03)(1-0.05)} = 0.00097571552$.

2.

Using Bayes Rule to calculating the probability of both tests A_1 and A_1 for false detection,

$P(H = 0|A_1 = 1, A_2 = 1) = \frac{P(H=0)P(A_1=1,A_2=1|H=0)}{P(H=0)P(A_1=1,T_2=1|H=0)+P(H=1)P(A_1=1,A_2=1|H=1)} = \frac{P(H=0)P(A_1=1|H=0)P(A_2=1|H=0)}{P(H=0)P(A_1=1|H=0)P(A_2=1|H=0)+P(H=1)P(A_1=1|H=1)P(A_2=1|H=1)} = \frac{(1-0.03)(0.05*0.06)}{(1-0.03)(0.05*0.06+0.03(1-0.03)(1-0.04)} = 0.09697439875$.

Since the false discovery rate is lower, I agree with the company's claim.

3.

Separation: C independent of A, conditional on Y * We get equalized odds if both false and true positives are equal across groups * We get equalized opportunity if just true positives are equal across groups Sufficiency: Y independent of A, conditional on C

Seperation VS Sufficiency

Assume all events in the joint distribution of (A,C,Y) have positive probability. If A is dependent of Y, either Seperation holds or Sufficiency but not both.

Let A and C be independent given Y, and let A and Y given in dependent given C. If this is true, this implies A is independent of (C,Y), which then implies A is independent of Y.

4.

Assume P(X | Y, Z) = P(X | Z). From Bayes rule of conditonal independence, we get P(X | W, Y, Z) = P(X | Y, Z)

From this, we can get P(X | Z) = P(X | W, Y, Z), which is what we intend to show.

2 Question 2 & 3 attached at the end done by hand

3 Question 4

1.

Assume each variable is binary(that is, it can take two values). Then, we can deduce to the following:

A will need 1 parameter for p(a)

B will need 4 parameters for p(b|ac),p(b|āc),p(b|ač),p(a|āč)

C will need 2 parameters for p(c|a) and p(c|ā)

D will need 2 parameters (same as C)

E will need 1 parameter (same as A)

F will need 4 parameters (same as B)

G will need 4 parameters (same as B)

So in total we need 18 parameters.

2.
- (A, E)
- (B, E
- (C, E)

3.

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* C is independent from D given A
* B is independent from D given A
* F is independent of G given D
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4.

Any variable that has a conditional dependence with B, E, and F will be relevant. For instance, A, C, and G are relevant.

5.

If we were to calculate $P(B|G)$, we would need to first restrict the domain with the relevant variables. For instance, the restriction will be made. Using the min-fill heuristic which eliminates the next variable that causes the smallest size factor, the smallest to largest order from 4 will be eliminated.

4 Question 5

$ D = { Steel, Aluminium, Titanium}$

1.

Set D has three elements. L_{metal} has symbols o_1, o_2, o_3, *heavier_than*, and *expensive*.

We can denote *Steel* as o_1, *Aluminium* as o_2, and *Titanium* as o_3.

We can say $heavier\_than(o_1, o_2), heavier\_than(o_2, o_3), heavier\_than(o_1, o_2), heavier\_than(o_2, o_1), heavier\_tha$
$heavierthan(o_{1}, o_{3}).

We can say $expensive(o_1)$, $expensive(o_2)$, $expensive(o_3)$

Therefore, there are 12 L_{metal} structures.

2.

Let *Steel* as o_1, *Aluminium* as o_2, and *Titanium* as o_3.

- From $heavierthan(o_2, o_1)$, we know that alumiunium is heavier than steel.
- From $heavierthan(o_1, o_3)$, we know that steel is heavier than titanium.
- From $expensive(o_1)$, steel is expensive.
- From $expensive(o_2)$, steel is expensive.

According to the above information, all structures of B are included in the domain, for the structures are analogou s to the structures of ϕ_1, but $< titatnium, aluminium >$ in $heavierthan^M$. From the domain of ϕ_1, we know that alumiunium is heavier than steel, and steel is heavier than titatnium. From this, we can infer that $o_2 > o_1$, and $o_1 > o_3$. There is no structure that implies titatinium is heavier than aluminium.

Therefore, other structures are included, despite $< titatnium, aluminium >$ in $heavierthan^M$.

3.

If we execute a universal generalisation on the statement to convert to clausal form, we get:

$heavierthan(x, y) \wedge heavierthan(y, z)) \rightarrow heavierthan(x, z)$.

From this we can infer that since $o_2 > o_1$, and $o_1 > o_3$, so $o_2 > o_1 > o_3$.

Examining $< titatnium, aluminium >$ in $heavierthan^M$, we see that this implies $o_3 > o_2$, which violates the above condition. Therefore, it is still not in ϕ_2

4.

If we execute a universal generalisation on the statement to convert to clausal form, we get:

$heavier\_than(x, y) \wedge heavier\_than(y, z)) \rightarrow heavier\_than(x, z)$.

This time, we are considering this as the $ONLY$ condition of ϕ_3.

This case, we are only interested in the structures that involve the structure *heavierthan*. From question 1, we know that there are 6 possible combinations of *heavier_than*. Enforcing the axiom, we only could get a subset of *heavier_than* structures.

For instance, $heavier\_than(o_2, o_1)$, $heavier\_than(o_1, o_3)$, $heavier\_than(o_2, o_3)$ satisfies the condition.

Therefore, 3 of the L_{metal} structures satisfy.

5 Question 6

1.

We will convert the sentences to clausal form.

- (1) $component(a, p_1) \wedge component(b, p_2) \wedge component(c, p_3) \wedge assemble\_before(a, b, p) \wedge assembly\_before(b, c, p)$

- (2) $component(c_1, p_1) \wedge component(c_2, p_2) \wedge assemble\_before(c_1, c_2, p) \rightarrow component(a, P_3) \wedge assemble\_before(a, c, p) \wedge assemble\_before(c, b, p)$
- (3) $component(c_1, a_1) \wedge component(c_2, a_2) \wedge component(c_1, c_2, p) \rightarrow precedes(a_1, a_2)$
- (4) $assemble\_before(c_1, c_2, a) \wedge assemblebefore(c_2, c_3, a)) \rightarrow assemblebefore(c_1, c_3, a)$
- (5) $assemble\_before(c_1, c_2, a) \wedge component(c_1, a_1) \rightarrow part(a_1, a)$

2.

- What is a part of p? (finding one answer is sufficient)
  - Assume $assemble\_before(c_1, c_2, a) \wedge component(c_1, a_1) \rightarrow part(a_1, a)$
  - One part of p is $component(c_1, p_1)$
- What does precede p3? (finding one answer is sufficient)
  - Assume $component(c_1, a_1) \wedge component(c_2, a_2) \wedge component(c_1, c_2, p) \rightarrow precedes(a_1, a_2)$
  - One part of p is $component(c_2, p_1)$

Q2. let F = Finches   C = Cardinals
K = Robins

~~let F/S → Finch sings~~
let F/S, C/S, R/S = Finch, cardinal, Robins sings
let F/D, C/D, R/D = Finch, cardinal, Robins don't sing.
let t>5 be longer than 5 mins.

1.

	F	R	C
~~Bird~~ Appear	7/12	4/12	1/12

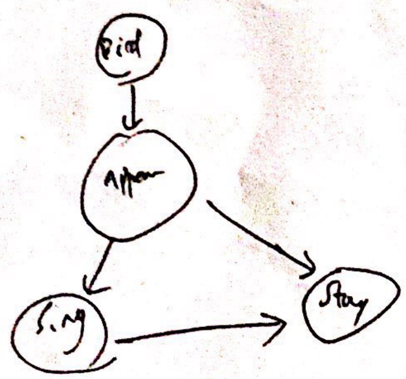
	F/S	F/D
H	0.7	0.3
E	0	1.0

	C/S	C/D
H	0	1.0
E	0.1	0.9

	R/S	R/D
H	0.2	0.8
E	0.2	0.8

~~t>5 1/4~~
~~t<5 3/4~~

	F	R	C
t>5	1/4	1/10	1/10
t<5	3/4	9/10	9/10



2. A total of 12 parameters.

3. $P(t>5 \mid appear) = \underline{0.62}$

4. ______

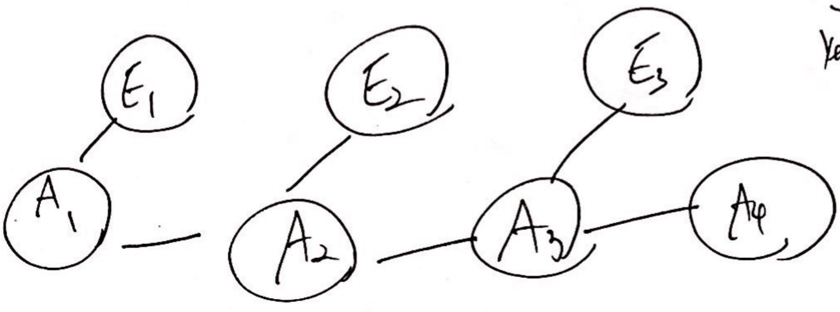
Q3.

~~CPT~~	P	Y
Cart	0.1	0.9

1. Let A_n = Cart colour

Let E_n = letter origin

	Dom	internati
Blue	0.7	0.3
Yellow	0.4	0.6



	B	Y
B	0.7	0.3
Y	0.5	0.5

let d = domestic, i = international.

2: ~~$P(E_1 = dom$~~

$P(E_1=d, E_2=i, E_3=d) = \sum_{A_1}\sum_{A_2}\sum_{A_3} P(A_3, A_2, A_1, E_1=d, E_2=i, E_3=d)$

$\sum_{A_1}\sum_{A_2}\sum_{A_3} = P(A_3|A_2)P(A_2|A_1)P(A_1)P(E_1=d|A_1)P(E_2=i|A_2)P(E_3=d|A_3) =$

$\sum_{A_3} P(E_3=d|A_3)\underbrace{\sum_{A_2} P(A_3|A_2)P(E_2=i|A_2)}_{\cancel{f_2(A_3)}}\underbrace{\sum_{A_1} P(A_1)P(A_2|A_1)P(E_1=d|A_1)}_{f_1(A_2)}$

$= \cancel{\sum_{A_3} P(E_3=d|A_3)}\ \cancel{P(E_3=d|A_3)}\ \sum_{A_3} P(E_3=d|A_3)f_2(A_3)$.

3. we want to know $P(A=blue | E_1 = international | E_2 = international | E_3 = Domestic)$.

$\Rightarrow \sum_{A_1}\sum_{A_2}\sum_{A_3} P(A_4=Blue), A_3, A_2, A_1, E_1=i, E_2=i, E_3=d) / \sum_{A_1}\sum_{A_2}\sum_{A_3}\sum_{A_4} P(A_4, A_3, A_2, A_1, E_1=i, E_2=i, E_3=d)$

* Restrict CPT, Eliminate A_1, A_2, A_3. Normalize $\Rightarrow f_3(A_4=Blue)/f_3(A_4=Blue)+f_3(A_4=Yellow)$.

$A_1 \Rightarrow f_1(A_2=Blue) = P(A_1=Blue)P(E_1=i|A_1=Blue)P(A_2=Blue|A_1=Blue) + P(A_1=Yellow)P(E_1=i|A_1=Yellow)P(A_2=Blue|A_1=Yellow)$.

~~$f_2(A_2=Yellow)$~~ Doing the same for A_1, A_2, A_3 we get

$f_3(A_4)$ Blue $\Rightarrow P(E_3=d|A_3=Blue)P(A_4=Blue|A_3=Blue)f_2(A_3=Blue) + P(E_3=d|A_3=Yellow)P(A_4=Blue|A_3=Yellow)f_2(A_3=Yellow) \Rightarrow A$.

Yellow $\Rightarrow P(E_3=d|A_3=Blue)P(A_4=Yellow|A_3=True)f_2(A_3=Blue) + P(E_3=d|A_3=Yellow)P(A_4=Yellow|A_3=Yellow)f_2(A_3=yellow) \Rightarrow B$.

we normalize such that $\dfrac{A}{(A+B)}$.

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