

# Linear Algebra

Midterm Sample Question

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**Exercise** True or False? In both cases, explain clearly.

- Every Diagonal matrix is an invertible matrix. **No**
- An upper triangular matrix times an upper triangular matrix is a upper triangular matrix. **Yes**
- The inverse of a permutation matrix is also a permutation matrix. **Yes**
- The transpose of an elimination matrix is also an elimination matrix. **Yes**
- If  $A$  and  $B$  are elimination matrix, then  $AB = BA$ . **No**
- Only symmetric matrix have a LDL decomposition. **Yes**
- The LU decomposition of a matrix is unique **No**
- The inverse of upper triangular matrix is lower triangular matrix **No**
- An Elimination matrix times an Elimination matrix is still an Elimination matrix. **No**
- Every invertible matrix is a square matrix. **Yes**
- $E_{21}E_{32}A$  means change Row 2 of matrix  $A$  by linear combination of Row 2 and Row 1 and then change Row 3 by linear combination of Row 3 and Row 2. **No**
- $\left\{ \begin{bmatrix} x \\ x+2y \end{bmatrix} \mid 3x+2y=0 \right\}$  is a vector space. **Yes**
- $\left\{ \begin{bmatrix} x \\ x+2y+1 \end{bmatrix} \mid 3x+2y=0 \right\}$  is a vector space. **No**
- All 3 by 3 matrices with the column vector  $(1, 1, 1)$  in their column space forms a vector space. **No**

- The set of all polynomials of degree less than 3 forms a vector space. This means any polynomial that can be written in the form  $ax^2 + bx + c$ , where  $a, b$  and  $c$  are constants (which can include zero), belongs to this vector space. **Yes**
- $A$  is an invertible matrix then  $A^{-1}A^{\top}A^2$  is also invertible. **Yes, the inverse matrix is  $A^{-2}(A^{-1})^{-\top}A$**
- For a matrix  $A \in \mathbb{R}^{4 \times 5}$ , the largest possible rank of  $A$  is 5. **No**
- For a matrix  $A \in \mathbb{R}^{4 \times 5}$  there are possibility that linear system  $Ax = b$  have one and only have one solution. **No**
- For a matrix  $A \in \mathbb{R}^{4 \times 5}$ ,  $\text{rank}(A) = 4$ . There are possibility that linear system  $Ax = b$  have one and only have one solution. **No**
- For a matrix  $A \in \mathbb{R}^{4 \times 3}$ ,  $\text{rank}(A) = 3$ . There are possibility that linear system  $Ax = b$  have one and only have one solution. **Yes**
- For a matrix  $A \in \mathbb{R}^{5 \times 4}$ ,  $\text{rank}(A) = 4$ . There are possibility that linear system  $Ax = b$  have one and only have one solution. **Yes**
- For a matrix  $A \in \mathbb{R}^{4 \times 5}$ ,  $\text{rank}(A) = 4$ . There are possibility that linear system  $Ax = b$  have no solution. **No**
- For a matrix  $A \in \mathbb{R}^{5 \times 4}$ ,  $\text{rank}(A) = 4$ . There are possibility that linear system  $Ax = b$  have no solution. **Yes**
- The exist matrixs  $A$  and matrix  $B$ ,  $\text{rank}(A) = 4$  and  $\text{rank}(AB) = 3$ . **No**
- $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right\}$  is a basis of  $\mathbb{R}^3$ . **No**
- The column vectors of a full column rank  $m \times n$  matrix is a basis of  $\mathbb{R}^m$ . **No**
- The column vectors of a full row rank  $m \times n$  matrix is a basis of  $\mathbb{R}^m$ . **No**
- The complete solution of linear system  $Ax = b$  is  $\vec{x} = \begin{bmatrix} 0 \\ 2 \\ 3 \\ 1 \\ 2 \end{bmatrix} + x_1 \begin{bmatrix} 1 \\ 0 \\ 2 \\ -1 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ , then  $\dim(\text{col}(A)) = 3$  **Yes**
- All symmetric matrix  $A \in \mathbb{R}^{3 \times 3}$  forms a vector space whose dimension is 6. **Yes**

- Suppose the matrices  $A$  and  $B$  have the same column space, then  $A$  and  $B$  have the same nullspaces. **No**
- Matrices  $A$  and its row echelon form always have the same column space. **No**
- If two  $m \times n$  matrices  $A$  and  $B$  have the same 4 fundamental spaces, then  $A = B$ . **No**
- Suppose  $A$  and  $B$  have the same column space, then  $A$  and  $B$  have the same rank. **Yes**
- $\text{rank}(A) = \text{rank}(A^\top)$  **Yes**
- There exist a matrix  $A$  whose column space is spanned by  $(1, 2, 3)$  and  $(1, 0, 1)$  and whose nullspace is spanned by  $(1, 2, 3, 6)$  **No, The dimensions of such a matrix must be 3 by 4 ( $m = 3$  and  $n = 4$ ). The dimension of the column space is 2, because the given vectors are independent. That means the dimension of the nullspace must be  $4 - 2 = 2$ . The null space cannot be spanned by 1 vector.**

#### Questions

- Compute Angle, matrix product, inverse matrix, LU decomposition, LDU decomposition, complete solution
- Compute the rank of the four subspaces