Linear Algebra

Midterm Review Question

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January 2024

Exercise Consider the matrix:

$$A = \begin{bmatrix} 1 & 3 & 5 & 0 & 7 \\ 0 & 0 & 0 & 1 & 2 \\ 1 & 3 & 5 & 1 & 9 \end{bmatrix}$$

- Write down $A = LU$ where L is an lower traingular matrix and U is a REF.
- Calculate the four fundamental subspaces

Exercise 1. all the possible rank of

$$\begin{bmatrix} 1 & 1 & a \\ 3 & 3 & a \\ a & a & a \end{bmatrix}$$

when a varies.

2. all the possible rank of

$$\begin{bmatrix} 1 & 3 & a \\ 3 & 1 & a \\ a & a & a \end{bmatrix}$$

when a varies.

Exercise 1. What is all the possible rank of

$$A = \begin{bmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \\ a & b & c & d \end{bmatrix}$$

when A, B, C, D varies 2. When is A invertible?

$$E_{21} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \to \quad E_{21}A = \begin{bmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ a & b & c & c \\ a & b & c & d \end{bmatrix}$$

$$E_{31} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \to \quad E_{31}E_{21}A = \begin{bmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & b-a & c-a & c-a \\ a & b & c & d \end{bmatrix}$$

$$E_{41} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix} \quad \to \quad E_{41}E_{31}E_{21}A = \begin{bmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & b-a & c-a & c-a \\ 0 & b-a & c-a & d-a \end{bmatrix}$$

$$E_{32} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \to \quad E_{32}E_{41}E_{31}E_{21}A = \begin{bmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & 0 & c-b & c-b \\ 0 & b-a & c-a & d-a \end{bmatrix}$$

$$E_{42} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \quad \to \quad E_{42}E_{32}E_{41}E_{31}E_{21}A = \begin{bmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & 0 & c-b & c-b \\ 0 & 0 & c-b & d-b \end{bmatrix}$$

$$E_{43} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} \quad \to \quad E_{43}E_{42}E_{32}E_{41}E_{31}E_{21}A = \begin{bmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & 0 & c-b & c-b \\ 0 & 0 & 0 & d-c \end{bmatrix}$$

Exercise

1. The complete solution of linear system $Ax = b$ is $\vec{x} = \begin{bmatrix} 0 \\ 2 \\ 3 \\ 1 \\ 2 \end{bmatrix} + x_1 \begin{bmatrix} 1 \\ 0 \\ 2 \\ -1 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$, then $\dim(\text{col}(A)) = 3$

2. There exist a matrix A whose column space is spanned by $(1, 2, 3)$ and $(1, 0, 1)$and whose nullspace is spanned by $(1, 2, 3, 6)$

3.

- For a matrix $A \in \mathbb{R}^{4\times 5}$, the largest possible rank of A is 5. No
- For a matrix $A \in \mathbb{R}^{4\times 5}$ there are possibility that linear system $Ax = b$ have one and only have one solution. No
- For a matrix $A \in \mathbb{R}^{4\times 5}$, $\text{rank}(A) = 4$. There are possibility that linear system $Ax = b$ have one and only have one solution. No
- For a matrix $A \in \mathbb{R}^{4\times 3}$, $\text{rank}(A) = 3$. There are possibility that linear system $Ax = b$ have one and only have one solution. Yes
- For a matrix $A \in \mathbb{R}^{5\times 4}$, $\text{rank}(A) = 4$. There are possibility that linear system $Ax = b$ have one and only have one solution. Yes
- For a matrix $A \in \mathbb{R}^{4\times 5}$, $\text{rank}(A) = 4$. There are possibility that linear system $Ax = b$ have no solution. No
- For a matrix $A \in \mathbb{R}^{5\times 4}$, $\text{rank}(A) = 4$. There are possibility that linear system $Ax = b$ have no solution. Yes
- $Y = AX$ and A is an invertible matrix, then $\text{rank}(Y) = \text{rank}(X)$. Yes