Linear Algebra Cheat Sheet Inverse Matrix

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Properties of Inverse Matrix

- $AA^{-1} = I$, $A^{-1}A = I$
- $(A^T)^{-1} = (A^{-1})^T$
- $\bullet (AB)^{-1} = B^{-1}A^{-1}$

Question 1 If A and M have inverse matrix A^{-1} and M^{-1} and

X. Y. 2 are matrix!

•
$$AX = B$$

$$A^{-1}(Ax) = A^{-1}B \Rightarrow x = Ix = A^{-1}B$$

$$YM = C$$
 $(YM)M^{\dagger} = CM$

$$\bullet \overrightarrow{A} \overrightarrow{Z} M^{\top} = D$$

$$M^T$$
 is invertible. $(M^T)^{-1} = (M^{-1})^T$

• YM = C $(YM)M^{-1} = CM^{-1}$ \Rightarrow $Y = YI = CM^{-1}$ • $AZM^{T} = D$ M^{T} is invertible. $(M^{T})^{-1} = (M^{-1})^{T}$ A-1 (A-2 NT) (NT) = A-10 MT) => 2= I. 2. I what is X, Y, Z?

2 Elimination

= A-1 D . (MT)-1 - A-1 D (M-1)T

Elimination as Matrix Operation We can write the operations to change equivalent linear system by $[A|b] \rightarrow [E_{ij}A|E_{ij}b]$ and $[P_{ij}A|P_{ij}b]$.

- Elimination matrix E_{ij} :
 - Replace row (i) by **row(i) + row (j) ← Fij A Fij b
 - Identity matrix except $a_{ij} = *$



- Permutation matrix P_{ij} :
 - Swtich Row (i) with Row (j) ← [PijA|Pijb]
 - Identity matrix except $a_{ij} = a_{ji} = 1, a_{ii} = a_{jj} = 0$

Question 2 What is the matrix after the following operations

- Change Row 2 of A to Row 2 + 2* Row 1
- Switch Row 3 and Row 4 of the new matrix
- Change Row 4 of the new matrix to Row 4 + 2* Row 2



3 Inverse Matrix

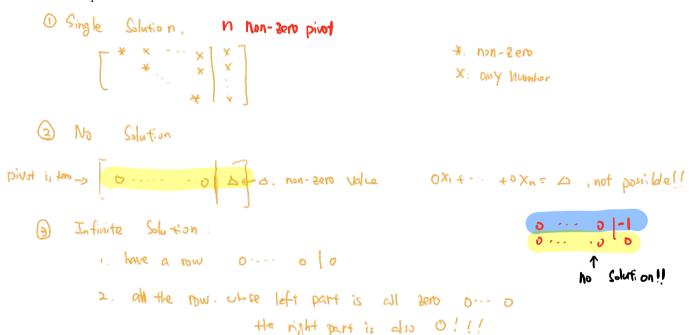
- The inverse of a matrix exists if and only if the matrix is a square matrix and all column vectors are linear independent.
- The inverse of a matrix exists if and only if elimination produced n non-zero pivots.

Questions (answer is in the slide) Can you describe how the upper triangular form and their pivots look like for the following three cases

- The linear system have a single solution
- The linear system have no solution
- The linear system have infinite solutions

Questions Please ensure you know the answer of the following questions

- How to calculate the inverse of a matrix?
- What is the inverse of the elimination matrix? What is the inverse of the permutation matrix?



Lu Decomposition!
lower tradular
$$A = L \cdot u \in \text{upper tradular}$$

Let's assume
$$A = L_1 U_1 = L_2 \cdot U_2 \cdot L_1 = U_3 \cdot U_1^{-1}$$

L.T.

$$L_3^{-1} L_1 = D = u_3 \cdot u_1^{-1}$$

diagnal

if A is symmetric

A= L· D· LT

(LDL desoportion)

1. LDM. CDL. are Unique!!

2. LU is not unique.

A= L· U = Lr U = died

Then L = Lr U = (!)

> 15 - Ly or Uz Ut are both L.T. and U.T. means. They are dia !!!

of diag of Lz. and L1 are 1. the my D is identify hint!! LI=Lz. D =) (L1) = (L2) 11 · d11 => d11=1 => D is identity!