

# Linear Algebra

Midterm Review Question

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**Exercise** Consider the matrix:

$$A = \begin{bmatrix} 1 & 3 & 5 & 0 & 7 \\ 0 & 0 & 0 & 1 & 2 \\ 1 & 3 & 5 & 1 & 9 \end{bmatrix}$$

- Write down  $A = LU$  where  $L$  is a lower triangular matrix and  $U$  is a REF.
- Calculate the four fundamental subspaces

**Exercise 1.** all the possible rank of

$$\begin{bmatrix} 1 & 1 & a \\ 3 & 3 & a \\ a & a & a \end{bmatrix}$$

when  $a$  varies.

2. all the possible rank of

$$\begin{bmatrix} 1 & 3 & a \\ 3 & 1 & a \\ a & a & a \end{bmatrix}$$

when  $a$  varies.

**Exercise 1.** What is all the possible rank of

$$A = \begin{bmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \\ a & b & c & d \end{bmatrix}$$

when  $A, B, C, D$  varies 2. When is  $A$  invertible?

$$\begin{aligned} E_{21} &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow E_{21}A = \begin{bmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ a & b & c & c \\ a & b & c & d \end{bmatrix} \\ E_{31} &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow E_{31}E_{21}A = \begin{bmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & b-a & c-a & c-a \\ a & b & c & d \end{bmatrix} \\ E_{41} &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix} \rightarrow E_{41}E_{31}E_{21}A = \begin{bmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & b-a & c-a & c-a \\ 0 & b-a & c-a & d-a \end{bmatrix} \\ E_{32} &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow E_{32}E_{41}E_{31}E_{21}A = \begin{bmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & 0 & c-b & c-b \\ 0 & b-a & c-a & d-a \end{bmatrix} \\ E_{42} &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \rightarrow E_{42}E_{32}E_{41}E_{31}E_{21}A = \begin{bmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & 0 & c-b & c-b \\ 0 & 0 & c-b & d-b \end{bmatrix} \\ E_{43} &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} \rightarrow E_{43}E_{42}E_{32}E_{41}E_{31}E_{21}A = \begin{bmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & 0 & c-b & c-b \\ 0 & 0 & 0 & d-c \end{bmatrix} \end{aligned}$$

### Exercise

1. The complete solution of linear system  $Ax = b$  is  $\vec{x} = \begin{bmatrix} 0 \\ 2 \\ 3 \\ 1 \\ 2 \end{bmatrix} + x_1 \begin{bmatrix} 1 \\ 0 \\ 2 \\ -1 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ , then  $\dim(\text{col}(A)) = 3$

2. There exist a matrix  $A$  whose column space is spanned by  $(1, 2, 3)$  and  $(1, 0, 1)$  and whose nullspace is spanned by  $(1, 2, 3, 6)$

3.

- For a matrix  $A \in \mathbb{R}^{4 \times 5}$ , the largest possible rank of  $A$  is 5. **No**
- For a matrix  $A \in \mathbb{R}^{4 \times 5}$  there are possibility that linear system  $Ax = b$  have one and only have one solution. **No**
- For a matrix  $A \in \mathbb{R}^{4 \times 5}$ ,  $\text{rank}(A) = 4$ . There are possibility that linear system  $Ax = b$  have one and only have one solution. **No**
- For a matrix  $A \in \mathbb{R}^{4 \times 3}$ ,  $\text{rank}(A) = 3$ . There are possibility that linear system  $Ax = b$  have one and only have one solution. **Yes**
- For a matrix  $A \in \mathbb{R}^{5 \times 4}$ ,  $\text{rank}(A) = 4$ . There are possibility that linear system  $Ax = b$  have one and only have one solution. **Yes**
- For a matrix  $A \in \mathbb{R}^{4 \times 5}$ ,  $\text{rank}(A) = 4$ . There are possibility that linear system  $Ax = b$  have no solution. **No**
- For a matrix  $A \in \mathbb{R}^{5 \times 4}$ ,  $\text{rank}(A) = 4$ . There are possibility that linear system  $Ax = b$  have no solution. **Yes**