

# Linear Algebra

Midterm Sample Question

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**Exercise** True or False? In both cases, explain clearly.

- Every Diagonal matrix is an invertible matrix. **No**
- An upper triangular matrix times an upper triangular matrix is a upper triangular matrix. **Yes**
- The inverse of a permutation matrix is also a permutation matrix. **Yes**
- The transpose of an elimination matrix is also a an elimination matrix. **Yes**
- Only symmetric matrix have a LDL decomposition. **Yes**
- The LU decomposition of a matrix is unique **No**
- The inverse of upper triangular matrix is lower triangular matrix **No**
- An Elimination matrix times an Elimination matrix matrix is stil an Elimination matrix. **No**
- Every invertible matrix is a square matrix. **Yes**
- $E_{21}E_{32}A$  means change Row 2 of matrix A by linear combination of Row 2 and Row 1 and then change Row 3 by linear combination of Row 3 and Row 2. **No**
- $\left\{ \begin{bmatrix} x \\ x+2y \end{bmatrix} \mid 3x+2y=0 \right\}$  is a vector space. **Yes**
- $\left\{ \begin{bmatrix} x \\ x+2y+1 \end{bmatrix} \mid 3x+2y=0 \right\}$  is a vector space. **No**
- The set of all polynomials of degree less than 3 forms a vector space. This means any polynomial that can be written in the form  $ax^2+bx+c$ , where  $a, b$  and  $c$  are constants (which can include zero), belongs to this vector space. **Yes**

- $A$  is an invertible matrix then  $A^{-1}A^{\top}A^2$  is also invertible. **Yes, the inverse matrix is  $A^{-2}(A^{-1})^{-\top}A$**
- For a matrix  $A \in \mathbb{R}^{4 \times 5}$ , the largest possible rank of  $A$  is 5. **No**
- For a matrix  $A \in \mathbb{R}^{4 \times 5}$  there are possibility that linear system  $Ax = b$  have one and only have one solution. **No**
- For a matrix  $A \in \mathbb{R}^{4 \times 5}$ ,  $\text{rank}(A) = 4$ . There are possibility that linear system  $Ax = b$  have one and only have one solution. **No**
- For a matrix  $A \in \mathbb{R}^{4 \times 3}$ ,  $\text{rank}(A) = 3$ . There are possibility that linear system  $Ax = b$  have one and only have one solution. **Yes**
- For a matrix  $A \in \mathbb{R}^{5 \times 4}$ ,  $\text{rank}(A) = 4$ . There are possibility that linear system  $Ax = b$  have one and only have one solution. **Yes**
- For a matrix  $A \in \mathbb{R}^{4 \times 5}$ ,  $\text{rank}(A) = 4$ . There are possibility that linear system  $Ax = b$  have no solution. **No**
- For a matrix  $A \in \mathbb{R}^{5 \times 4}$ ,  $\text{rank}(A) = 4$ . There are possibility that linear system  $Ax = b$  have no solution. **Yes**
- The exist matrixs  $A$  and matrix  $B$ ,  $\text{rank}(A) = 4$  and  $\text{rank}(AB) = 3$ . **No**
- $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right\}$  is a basis of  $\mathbb{R}^3$ . **No**
- The column vectors of a full column rank  $m \times n$  matrix is a basis of  $\mathbb{R}^m$ . **No**
- The column vectors of a full row rank  $m \times n$  matrix is a basis of  $\mathbb{R}^m$ . **No**

### Questions

- Compute Angle, matrix product, inverse matrix, LU decomposition, LDU decomposition, complete solution
- Compute the rank of the four subspaces