Linear Algebra Cheat Sheet Inverse Matrix

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Properties of Inverse Matrix

- $AA^{-1} = I$, $A^{-1}A = I$
- \bullet $(A^T)^{-1} = (A^{-1})^T$
- $(AB)^{-1} = B^{-1}A^{-1}$

Question 1 If A and M have inverse matrix A^{-1} and M^{-1} and

X. Y. 2 are matrix!

$$\bullet \ AX = B$$

•
$$AX = B$$
 $A^{-1}(AX) = A^{-1}B$ $\Rightarrow X = IX = A^{-1}B$

•
$$YM = C$$

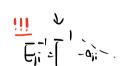
$$\bullet AZM^{\mathsf{T}} = D$$

$$M^T$$
 is invertible. $(M^T)^{-1} = (M^{-1})^T$

•
$$YM = C$$
 $(YM)M^{\dagger} = CM^{-1} \Rightarrow Y = YI = CM^{-1}$
• $AZM^{\dagger} = D$ M^{\dagger} is invertible. $(M^{\dagger})^{-1} = (M^{-1})^{\dagger}$
what is X, Y, Z ?

$$A^{\dagger} (A 2 M^{\dagger}) (M^{\dagger})^{-1} = A^{\dagger} D M^{\dagger})^{-1} \Rightarrow 2 = I \cdot 2 \cdot I$$

= A-1 D · (MT)- - A-1 D (M-1)T Elimination as Matrix Operation We can write the operations to change



equivalent linear system by $[A|b] \rightarrow [E_{ij}A|E_{ij}b]$ and $[P_{ij}A|P_{ij}b]$. • Elimination matrix E_{ij} :

- Replace row (j) by **row(i) + row (j)
$$\leftarrow$$
 [Fig Al Fig b]
- Identity matrix except $a_{ij} = *$

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$$a_{ij} = *$$



• Permutation matrix P_{ij} :

- Swtich Row (i) with Row (j)
$$\leftarrow$$
 P: A P: b - Identity matrix except $a_{ij} = a_{ji} = 1$, $a_{ii} = a_{jj} = 0$

- Identity matrix except
$$a_{ij} = a_{ji} = 1$$
, $a_{ij} = a_{jj} = 0$

Question 2 What is the matrix after the following operations



3 Inverse Matrix

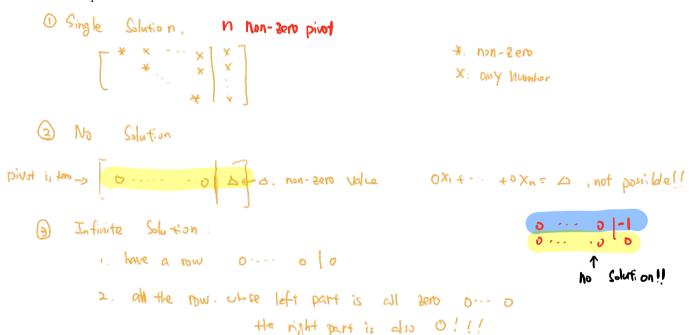
- The inverse of a matrix exists if and only if the matrix is a square matrix and all column vectors are linear independent.
- The inverse of a matrix exists if and only if elimination produced n non-zero pivots.

Questions (answer is in the slide) Can you describe how the upper triangular form and their pivots look like for the following three cases

- The linear system have a single solution
- The linear system have no solution
- The linear system have infinite solutions

Questions Please ensure you know the answer of the following questions

- How to calculate the inverse of a matrix?
- What is the inverse of the elimination matrix? What is the inverse of the permutation matrix?



Lu Decomposition!
lower tradular
$$A = L \cdot u \in \text{upper tradular}$$

Let's assume
$$A = L_1 U_1 = L_2 \cdot U_2 \cdot L_1 = U_3 \cdot U_1^{-1}$$

L.T.

$$L_3^{-1} L_1 = D = u_3 \cdot u_1^{-1}$$

diagnal

if A is symmetric

A= L· D· LT

(LDL desoportion)

1. LDM. CDL. are Unique!!

2. LU is not unique.

A= L· U = Lr U = died

Then L = Lr U = (!)

> 15 - Ly or Uz Ut are both L.T. and U.T. means. They are dia !!!

of diag of Lz. and L1 are 1. the my D is identify hint!! LI=Lz. D =) (L1) = (L2) 11 · d11 => d11=1 => D is identity!