Your Paper

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Abstract

We studied signals.

1 Introduction and Background

1.1 The Fourier Transform

Where $V(t)$ is voltage as a function of time and $V(\nu)$ is voltage as a function of frequency ν.

$V(\nu) = \int_{-T/2}^{T/2} V(t) \exp(2\pi i \nu t) dt$

Where T is the length of the time sample.

$V(t) = \frac{1}{\nu_s} \int_{-\nu_s/2}^{\nu_s/2} \tilde{V}(\nu) \exp(-2\pi i \nu t) d\nu$

where ν_s is the sampling frequency used.
What happens if you change the bounds in the manner (N / 2 - 1) / ν_s

1.2 Positive and Negative Frequencies

"complex inputs to a Fourier Transform break the positive/negative frequency degeneracy"
The sine function is odd:
$\sin(-\omega t) = -sin(\omega t)$
The cosine function is even:
$\cos(-\omega t) = \cos(\omega t)$
We can generalize under Euler's identity:
$A\exp(i\omega t) = A\cos(\omega t) + i \cdot A\sin(\omega t)$
$A\exp(-i\omega t) = A\cos(-\omega t) + i \cdot A\sin(-\omega t) = A\cos(\omega t) - i \cdot A\sin(\omega t)$
Show how such a formulation can explain a phase shift.

1.3 Nyquist's Theorem and Aliasing

$f_s > 2f_{max}$

1.4 The Convolution and Correlation Theorems

The convolution of two functions $f(t)$ and $g(t)$ is defined as:

$[f * g](\tau) = \int_{-T/2}^{T/2} f(t) g(\tau - t) dt$

The correlation of two functions $f(t)$ and $g(t)$ is defined as

$[f \star g](\tau) = \int_{-T/2}^{T/2} f(t) g(\tau + t) dt$

You may want to prove this by substituting the Fourier transform

$[f \tilde{*} g](\nu) \equiv \int_{-T/2}^{T/2} [f * g](\tau) \exp 2\pi i \tau \nu d\tau = \tilde{f}(\nu) \cdot \tilde{g}(\nu)$

$[f \tilde{\star} g](\nu) \equiv \int_{-T/2}^{T/2} [f \star g](\tau) \exp 2\pi i \tau \nu d\tau = \tilde{f}(\nu) \cdot \tilde{g^*}(\nu)$

Now: how is the ACF relevant to this paper

1.5 Sideband Theory

All that sweet, sweet trig.

2 Methods

This is the equipment I used. These are the libraries and functions I used. This is how I used them (*vague* hint at your code). Uncertainty? Technical errors?

To begin our investigation of the Nyquist criterion, we decided on a sampling frequency $\nu_s =$ 6.25 MHz at which the pico sampler (a PicoScope 2206A) remained for the duration of the data collection for all signals. We then used the N9310A RF Signal Generator to output signals at frequency ν_0: fractions of the sampling frequency. For example, one of our input signals was $\nu_0 = .4\nu_s = 2.5$ MHz. For all signals, we performed visual inspections of the pico sampler's input by using a T-joint and connecting the output to an oscilloscope (a Rigol DS1052E). We used the ugradio module to perform data collection (ugradio.pico.capture_data) and, later on, to perform Fourier and inverse Fourier transforms (ugradio.dft.dft and ugradio.dft.idft).

To consider the frequency resolution for two signals of similar frequency, we employed a second signal generator. The 83712B has a lower limit of 10 MHz output frequency. Consequently, we set our original signal generator to small increments above that frequency and needed to increase the sampling frequency to $\nu_s = 31.25$ MHz.

To investigate the impact of noise on the Fourier transform, we switched the pico sampler's input to an NOD 5250 noise generator (6 MHz bandpass) at zero attenuation. We collected 32 blocks of 16000 samples from the PicoScope, this time using the sampling frequency $\nu_s = 62.5$ MHz.

We explored mixers and sideband theory with a Mini-Circuits ZAD-1. We the 83712B as the local oscillator and the N9310A as the RF input. We sent the output to the oscilloscope and PicoScope, this time using a 50Ω terminator to eliminate bounce-back interference. The ZAD-1 works best for signals above 10 MHz, so we set the local oscillator to $\nu_{LO} = 11$ MHz and collected two sets of data: one for which $\nu_{RF} = 11.55$ MHz and one for which $\nu_{RF} = 10.45$ MHz (thus, $\Delta\nu = .05\nu_{LO} = .55$ MHz.

The mixed signals were somewhat weak, so we used 1.5 dBm amplitudes on both signal generators. We again used a sampling frequency of $\nu_s = 62.5$ MHz, motivated by the following identities:

$\sin(\nu_{LO})\sin(\nu_{LO} + \Delta\nu) = \frac{1}{2}(\cos\Delta\nu - \cos(2\nu_{LO} + \Delta\nu))$ by evenness of the cosine function

and

$\sin(\nu_{LO})\sin(\nu_{LO} - \Delta\nu) = \frac{1}{2}(\cos\Delta\nu - \cos(2\nu_{LO} - \Delta\nu))$

Now, if we expect to see two signals, $2\nu_{LO} + \Delta\nu = 23.1$ 23.1 is the highest frequency that we would expect. Our PicoScope offers sampling rates by dividing the base 62.5 MHz. 31.25 MHz would have failed the Nyquist criterion, so we used the maximum sampling rate instead.

Finally, we constructed a single-sideband mixer by using two mixers. Both mixers used both the local oscillator and radio frequency signals, but one of the mixers' RF input was phase shifted by approximately $\pi/2$ by linking smaller cables together until we roughly satisfied the $\lambda/4$ criterion.

I don't remember the model of the other mixer

3 Notes to Self

What am I doing? Not ACF (5.3, 5.7) and procrastinating on 5.5 so that I can get some writing done.

I did ACF analysis for neither 5.3 nor 5.7

Major logic error: side-by-side captions are mutilated!

Idea: put signal and power spectrum plots side by side, not signals with signals

argue what the Nyquist criterion is, based on results.

* Include make and model of all equipment used.

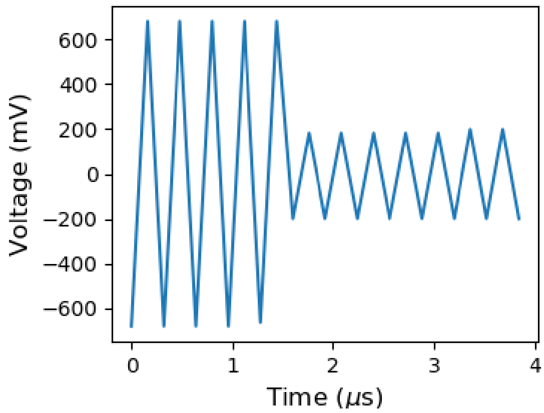

Figure 1: The oscilloscope displayed a constant signal throughout the period of data-taking. The data sampled from the pico sampler, however, reconstructs a signal with large aberrations in the first few microseconds.

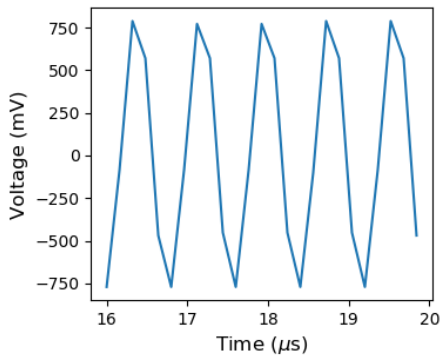

Figure 2: $\nu_0 = .2\nu_s = 1.25$ MHz.

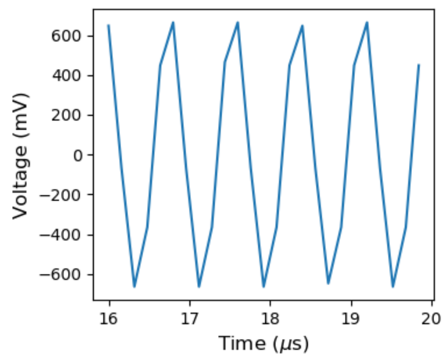

Figure 3: $\nu_0 = .8\nu_s = 5$ MHz.

* "Don't quote a number without the uncertainty and units."
Discussion on results for week 2, section 1

I need to include details about the equipment used, but how in-depth do I need to go? Current plan
* For most things, use model number and manufacturer
* When data analysis depends on a spec sheet, offer a brief summary of the specs to which you are referring to fine-tune your analysis

I need uncertainties on results but I do not yet know how to get these.
Closer to the end of writing, design a new subsection layout for the results section. You of course cannot make references like "5.2" and "7.3" in the submission.

4 Results

4.1 5.2

We took the default 16000 samples for each of our first signal samples. However, for the data analysis, we will be excluding the first 100 samples due to a peculiarity of the pico sampler which distorted these (see figure 1).

We may begin inspection of the samples with a qualitative approach. Figure 2 shows a signal which repeats about five times in the span of about 4 microseconds. This gives us 1.25 cycles per microsecond, or 1.25 MHz, as expected. Figure 3 still appoars as a sine wave, but visual inspection

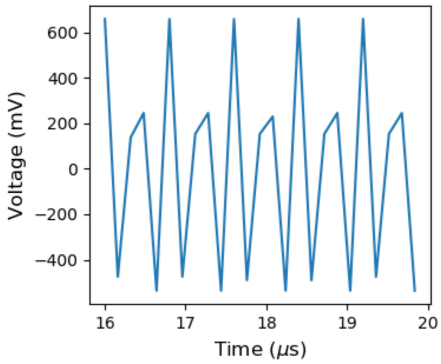

Figure 4: $\nu_0 = .4\nu_s = 2.5$ MHz.

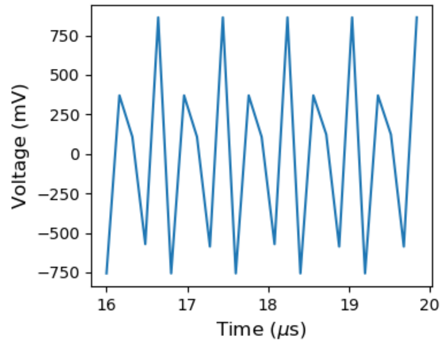

Figure 5: $\nu_0 = .6\nu_s = 3.75$ MHz.

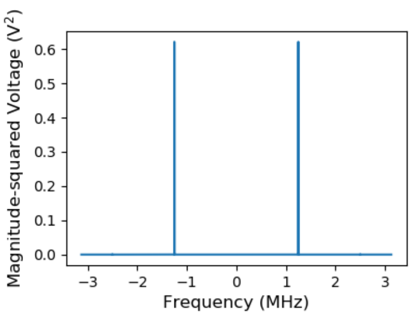

Figure 6: $\nu_0 = .2\nu_s = 1.25$ MHz. Peak amplitudes at $\pm$ 1.25 MHz

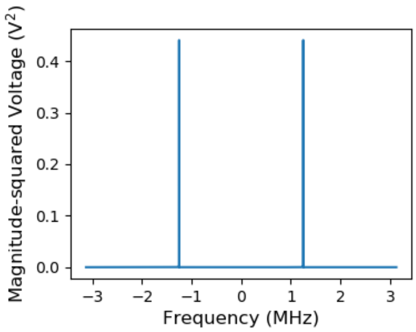

Figure 7: $\nu_0 = .8\nu_s = 5$ MHz. Peak amplitudes at $\pm$ 1.25 MHz

is no longer reliable. Again: we see about 5 samples in the span of about 4 microseconds. This would give 1.25 MHz, but because we know that our input signal was in fact $\nu_0 = 5$ MHz, we know that the waveform is aliased.

A more nuanced example comes from frequencies close to $.5\nu_s$: compare figures 4 and 9. Keeping in mind that only the frequency varied between trials and not the shape of the signal, a visual inspection is now confused: the sample looks like a sine wave in neither case. If take for granted that 4 is not a perfect sample, we may still visually estimate the frequency by noting 10 pairs of relative extrema (2.5 MHz).

The power spectra exactly confirm these estimations. Furthermore, there are no obvious aberrations in the power spectra to indicate whether the signal was aliased during sampling. Consider, for example, that figures 8 and 9 are identical. An argmax function yielded precisely the same values for the frequency peaks.

The power spectrum for our fifth trial ($\nu_0 = .5\nu_s = 3.125$ MHz) yielded an argmax with many digits (all other power spectra, even the aliased, returned frequencies to two decimal places). In other words, the calculation becomes more sensitive in a small region around this midpoint. All frequencies above this returned incorrect maxima in the power spectrum, so we have verified the Nyquist criterion.

?? No value and no uncertainty for this trial

4.2 5.3

The power spectra show two spikes symmetric about 0. We see them at the positive and negative of (for the non-aliased samples) the input frequency. We can consider the concept of a negative frequency in a broader context by returning to Euler's identity as recalled in the introduction.

Specifically, we calculated the voltage spectra of our samples. Since this is directly the Fourier transform, we now see real and imaginary components which are not visible in the power spectrum due to the magnitude-square operation applied to the outputs.

Differences between voltage spectra of the same signal suggest that variation in imaginary and

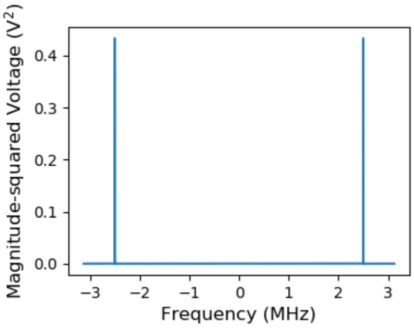

Figure 8: $\nu_0 = .4\nu_s = 2.5$ MHz. Peak amplitudes at $\pm$ 2.5 MHz

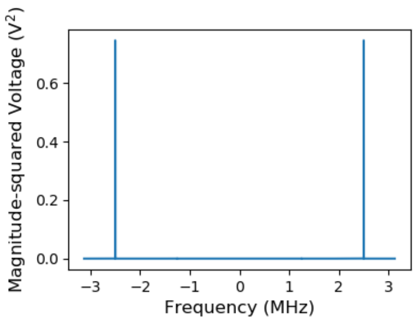

Figure 9: $\nu_0 = .6\nu_s = 3.75$ MHz. Peak amplitudes at $\pm$ 2.5 MHz

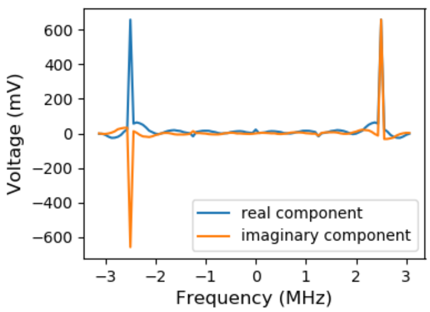

Figure 10: Voltage

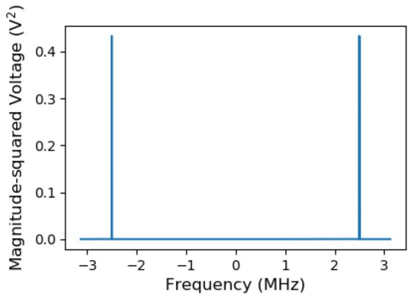

Figure 11: Powerish

real components are consequences of phase shifts. Taking multiple blocks of data one after the other (which loosely controls for environmental variation) allows the PicoScope to begin taking data points such that the signals do not overlap.

Power spectra are provided alongside the voltage spectra to demonstrate the power spectrum's destruction of information. The power spectrum does not change (figures 11, 13, and 15 are identical) and exhibits complete symmetry always. The voltage spectra, by contrast, are inconsistent. However, excepting noise, which seems to be influencing the patterns at the centers, the spectra are symmetric and antisymmetric in their real and imaginary components, respectively.

Phase is defined with respect to some reference point; although we can discuss the phase of a single sample through its voltage spectrum, phase only becomes physically meaningful when we have multiple signals to compare. Therefore, we would find voltage spectra to be physically insightful when we are investigating multiple signals at once. Power spectra are useful for both single and simultaneous signals but, as we have seen, combined signals lead to results about the combination, and information about the individual constituents is less accessible. However, power spectra immediately provide explicit characteristics of signals, so their utility in examining individual signals is demonstrated.

Come back to this section to discuss the ACF discrepancies

"What does it mean, that the voltage spectra are complex? What do the real and imaginary parts represent? Is the imaginary part less 'real' than the real part? What does it mean, for frequencies to be negative versus positive?"

"Why might we use power spectra instead of voltage spectra, and vice versa?"

"According to the correlation theorem, the Fourier transform of the power spectrum should equal the ACF. Does it? Explain any differences."

"you need to make sure dft.idft correctly infers the frequencies corresponding to each bins in your power spectrum array"

"When calculating a digital version of the correlation function, you have to worry about end effects. Suppose you are calculating an ACF for N samples with delays ΔN ranging up to N/2. Then the number of terms in the sum is always smaller than N because the delays spill over the edge of the available samples."

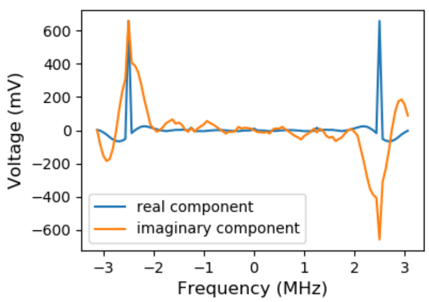

Figure 12: Voltage

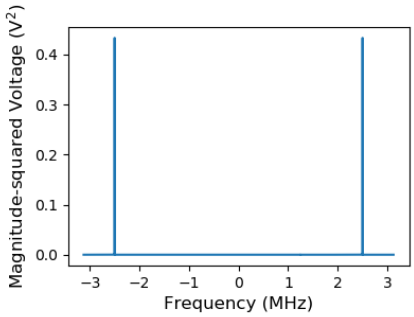

Figure 13: Powerish

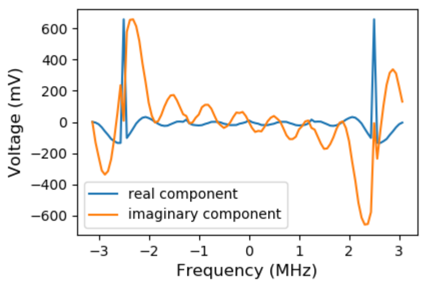

Figure 14: Voltage

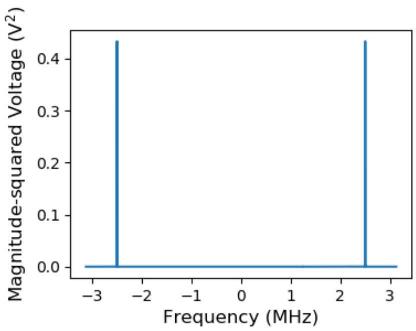

Figure 15: Powerishy Squishy

How am I supposed to account for this?
Differences? Probably because I did not zero out the middle

4.3 5.4

In physical application, all of our Fourier transforms must be discrete and have finite bounds. Finite bounds introduce spectral leakage, which in fact can be seens regardless of one's changing $\Delta\nu$. However, the effect is particularly pronounced when decreasing $\Delta\nu$.

Based on our results, we may suggest that spectral leakage becomes an increasingly impactful problem as one approaches the Nyquist criterion. To be sure, there is a variant pattern at the lower end of each leakage pattern, but this seems to stem more from noise than from the leakage pattern in particular. Consider figures 18 and 19. The as ν_0 increases in proportion to nu_s, we see the erroneous spikes increase in eminence. In fact, in figure 19, the leakage spikes overshadow the expected power spikes which we demonstrated earlier. By decreasing the size of the frequency interval, we are computationally introducing additional aliasing into data we have already collected, by treating irregular patterns as single cycles rather than combinations of the same sine wave

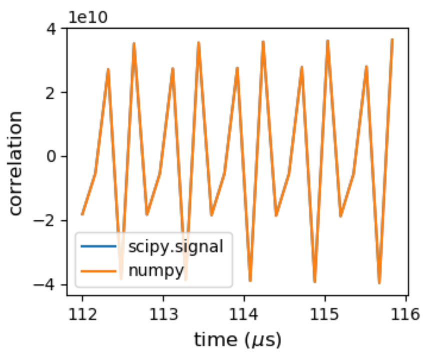

Figure 16: This is supposed to be the dft/idft version

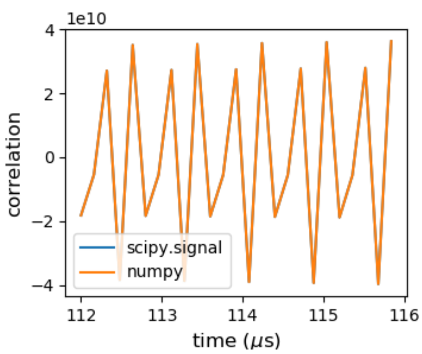

Figure 17: Powerish

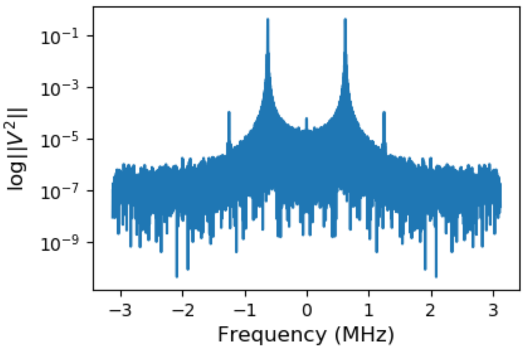

Figure 18: $\nu_0 = .1\nu_s = .625$ MHz

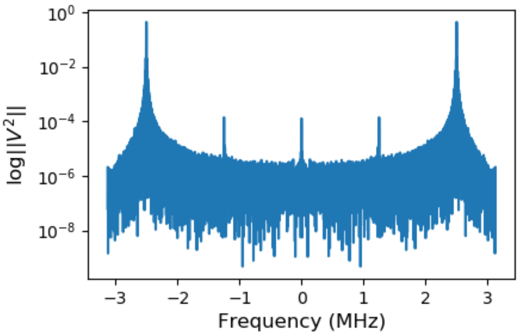

Figure 19: $\nu_0 = .4\nu_s = 2.5$ MHz

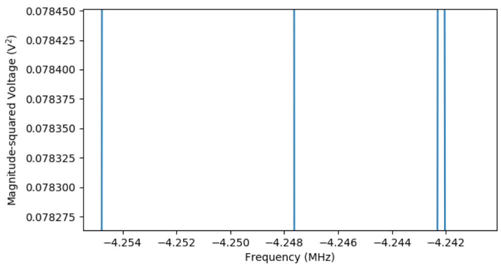

Figure 20

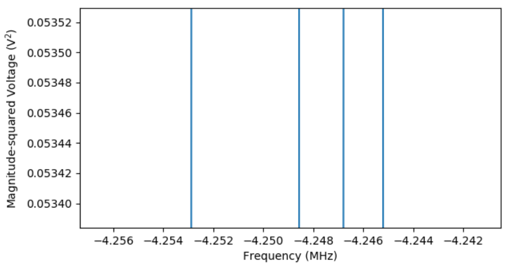

Figure 21

sampled with different offsets.

4.4 5.5

4.5 5.6

As we take the power spectrum out to increasing frequency ranges, we observe repetitions of the original signal which do not strictly increase in frequency, perhaps due to aliasing wrap-around.

Figures 22, 23, 24 all represent power spectra for the same data and therefore also for the same input signal frequency. We continue to see pairs of spikes symmetric about the zero frequency. While the number of spike pairs increases dramatically from the fourth (figure 22) to the eighth window (23), we see the number drop off steeply by the twelfth window.

4.6 5.7

First sample:

Mean = 3.899292452830189 mV Standard deviation = 20.079642416978363 mV Variance = 403.19203959371663 (mV)^2

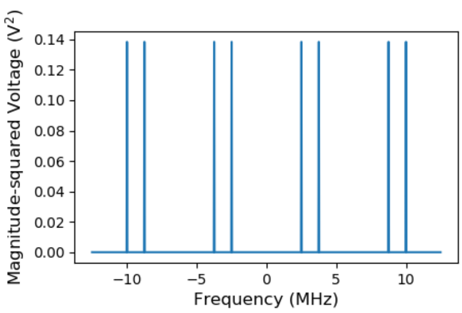

Figure 22: $\nu_0 = .4\nu_s = 2.5$ MHz. Fourth window.

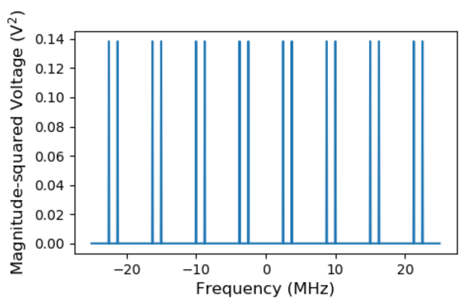

Figure 23: Same frequency. Eighth window.

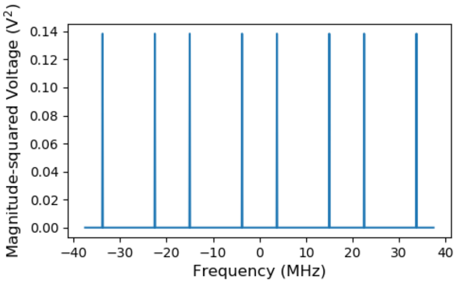

Figure 24: Same frequency. Twelfth window.

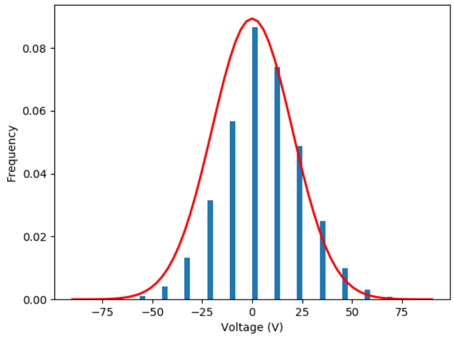

Figure 25

I am allowing the first 100 samples to contaminate. I do not know how much I should trod outside the 16000 recommendation.

4.7 7.1

Figure 31 demonstrates the principle from earlier that a sample can reconstruct a signal (e.g. via Fourier decomposition) without precise visual mimicry. In this case, each cycle of the original signal has produced a visually different sampled cycle.

These figures are missing captions!

Why do the power spectra look the way they do. Upper sideband and lower sideband.

For the upper sideband, we can see spikes at almost the difference frequency (.575 MHz $\approx$ .55 MHz). The other spikes are at 10.2 MHz? Why?

For the lower sideband, we see outer spikes at 9 MHz. The inner spikes are still at roughly the difference frequency...

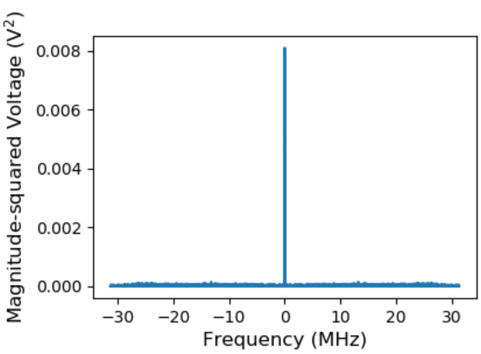

Figure 26

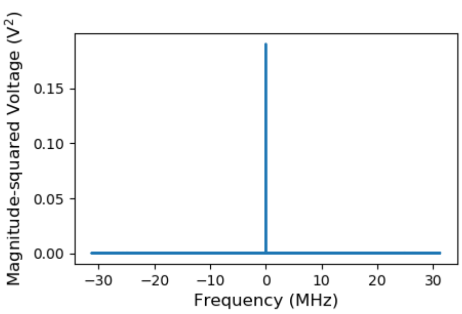

Figure 27

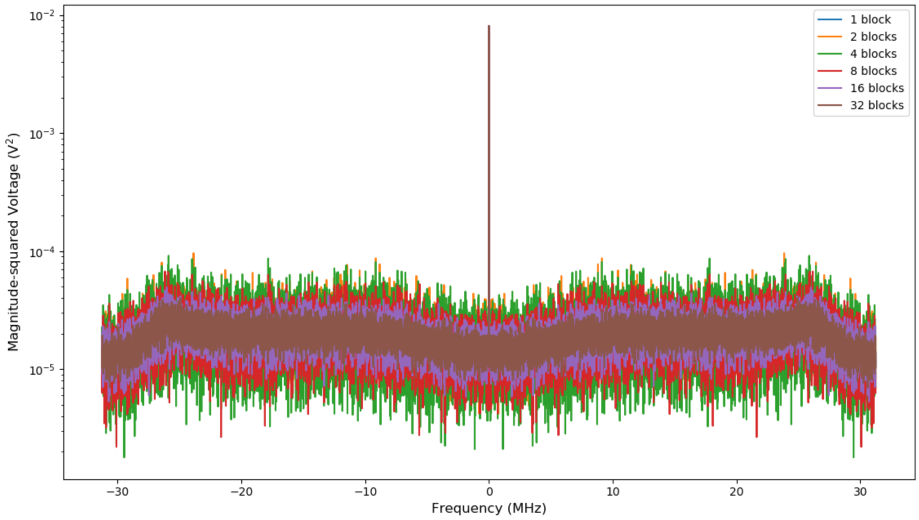

Figure 28

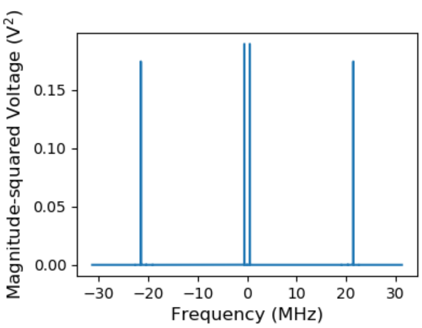

Figure 29

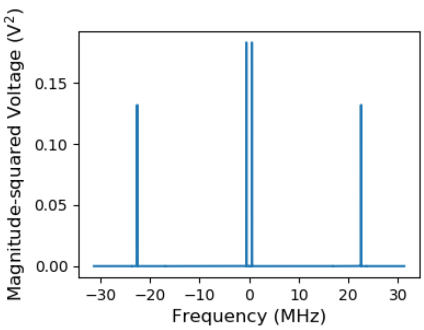

Figure 30

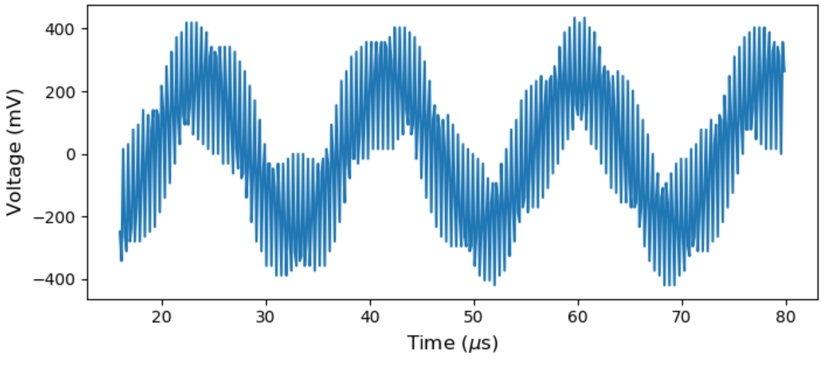

Figure 31: Waveform in the case where $\nu_s = 62.5$ MHz and where $\nu_{RF} = \nu LO - \Delta\nu = 10.45$ MHz. The largest-frequency signal is that of the sum, $2\nu_{LO} + \Delta\nu = 23.1$ MHz.

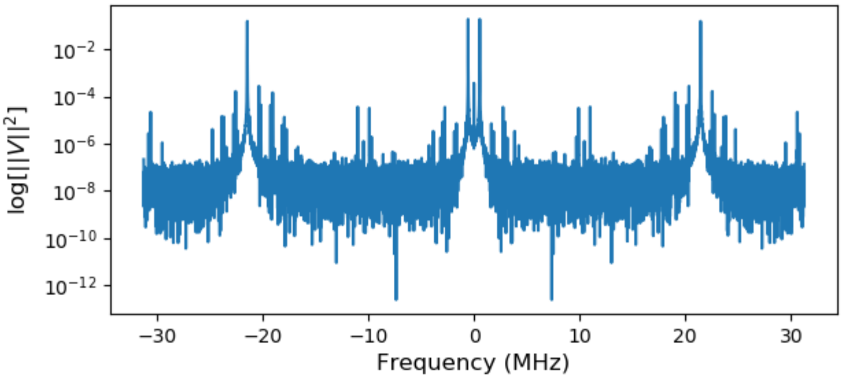

Figure 32

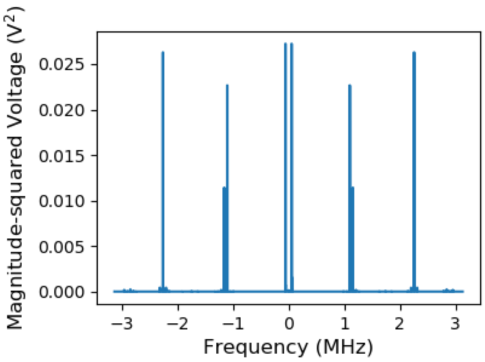

Figure 33

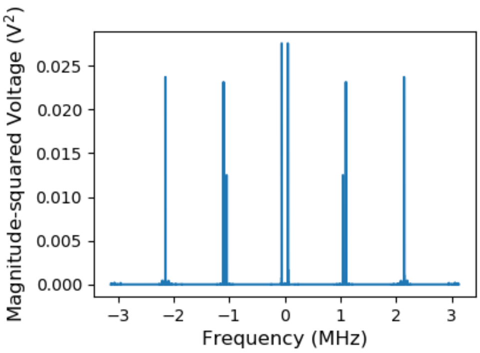

Figure 34

4.8 7.2

4.9 7.3

5 Conclusions

References

[Gre93] George D. Greenwade. The Comprehensive Tex Archive Network (CTAN). *TUGBoat*, 14(3):342–351, 1993.

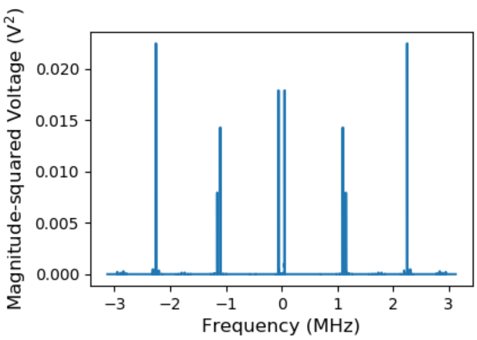

Figure 35

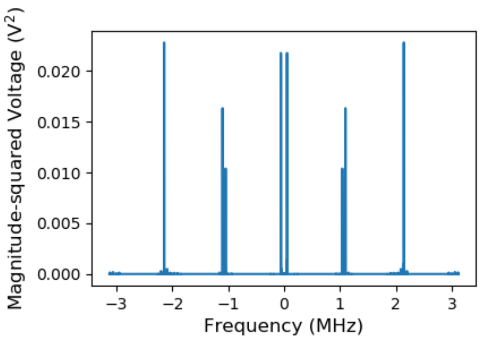

Figure 36