Lab 2

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Abstract

We need more narrative drive. Consider using powerful transition phrases

We investigated the hyperfine hydrogen transition by calibrating our measurements to thermal noise and by averaging over many blocks of data taken in optimal time windows of the day. We investigated the speed of light using waveguides (How do they work?). Our Chi-Squared analysis is completely deficient because our data were taken too broadly to show noticeable inconsistencies.

This is a great place to have a to-do section, because I prefer to write abstracts at the end of the writing process.

* Ask Professor about the permissions thing (page 3), to make sure there are no technical difficulties.

Figure out how to make the section headers smaller. They are taking up far too much space.

1 Introduction and Background

In the first of two experiments, we seek to observe the 21-cm HI line, at 1420.4 MHz. However, we want to look at frequencies better suited to our lab equipment. For example, our PicoScope has a maximum sampling rate of 62.5 MHz, so anything above 125 MHz would be aliased.

The data that we sample from the Big Horn via the PicoScope will feature arbitrary units which depend on our setup. To calibrate the intensity of the spectrum, we first calculate the gain:

$$G = \frac{T_{\text{sys, cal}} - T_{\text{sys, cold}}}{\sum(s_{\text{cal}} - s_{\text{cold}})} \sum s_{\text{cold}} \approx \frac{300 \text{ K}}{\sum(s_{\text{cal}} - s_{\text{cold}})} \sum s_{\text{cold}} \tag{1}$$

Where $T_{\text{sys, cal}} \approx 98.6^\circ$ F ≈ 310 K represents the temperature of the human flesh that we use to cover most of the telescope. $T_{\text{sys, cold}}$ represents the temperature of the cold sky. We expect $T_{\text{sys, cold}} \ll T_{\text{sys, cal}}$, whereby we get the approximated form on the right. s_{cal} represents the spectrum for which we attempt to maximize thermal noise in the collector, and s_{cold} represents the spectrum for which we attempt to minimize.

To remove constant sources of interference in our data and to obtain the shape of the line, we divide an 'on' spectrum by an 'off' spectrum.

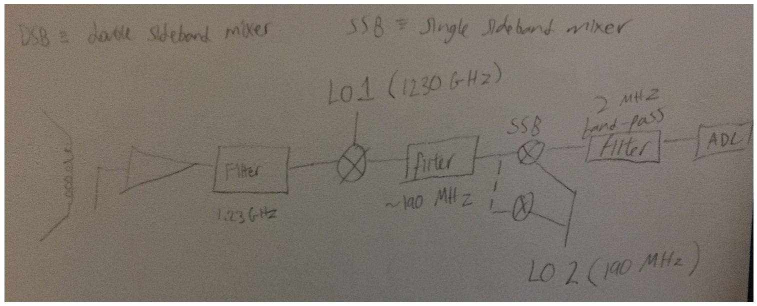

Figure 1: To reduce the frequencies of incoming signals to manageable levels, we employ a multi-stage combination of filters and mixers. The signal enters via transformer connected to the collector. The ADC represents the final step, when the PicoScope translates voltages over time into digital arrays.

$$s_{\text{line}} = \frac{s_{\text{on}}}{s_{\text{off}}} \tag{2}$$

The fully-calibrated spectrum represents a scaling of this s_{line} by the thermal gain from equation 1:

$$T_{\text{line}} = s_{line} \times G \tag{3}$$

To plot our calibrated spectrum against Doppler velocity, we use the following relation:

$$v = -c\frac{\Delta\nu}{\nu_0} \tag{4}$$

I should probably enumerate the rotation matrices that I plan to use.

For our second experiment, we seek to calculate to calculate the wavelength λ_{sl} of a signal inside the waveguide based on the positions of the minima in the squared voltage, which we call nulls. We expect a linear dependence of null spacings on λ_{sl}. We use the following equation for this first experiment:

$$x_m = A + m\frac{\lambda_{\text{sl}}}{2} \tag{5}$$

m is the index of the null, x_m is the position of null m, and A represents a constant offset which can relate whether the far end of the waveguide is open or shorted.

The rectangular waveguide, to which we switch for the X-band phase of the experiment, abides a nonlinear relationship between the waveguide's width and the wavelengths of the input in the waveguide and in free space.

$$\lambda_g = \frac{\lambda_{\text{fs}}}{[1 - (\frac{\lambda_{\text{fs}}}{2a})^2]^{1/2}} \tag{6}$$

λ_g is the guide wavelength, which we directly measure. λ_{fs} is the free-space wavelength of the signal, which we will calculate using the known input frequency and the relation $c = f\lambda_{\text{fs}}$. a is the width of the waveguide.

Finally, to analyze our results we define the reduced chi-squared function as follows:

$$\chi_r^2 = \frac{1}{N-M}\sum_i \frac{|y_i - \hat{y}_i|^2}{\sigma_i^2} \tag{7}$$

Where N is the number of data and M is the number of fit parameters. For each measurement i, y_i is the observed value, $\hat{y}_i$ is the value predicted by the fit, and σ_i is the expected error on that measurement.

The reduced chi-squared function will provide something clear to minimize when fitting the data to a model. Specifically, we want to calculate the width a based on our observations of null spacings, so we will calculate the value of a which minimizes the reduced chi-squared.

2 Methods

After setting the two local oscillators to 1230 and 190 MHz, as labeled in figure 1, we set the power levels to 8 and 0 dBm, respectively. These numbers are difficult to theoretically justify; through trial and error, and particularly through consultation with other groups, we found that these numbers led to the greatest HI resolution for the Big Horn pointed directly upward.

However, these numbers changed when taking data for Cassiopeia, by which point our signal-to-noise ratio was too low. We switched to 10 dBm for both and proceeded with the pico-sampling at 62.5 / 6 MHz, the rate we used for all captures.

Later on, we will see imperfections in the setup in the form of nearly symmetric 'bunny ear' spikes as well as an extreme spike at the center. We found the PicoScope to sometimes artifically attenuate and offset the signal going into the second port, because we would switch the cables going into A and B and we would see the same shape; if there were a problem in our mixer stages, we would expect the output to switch similarly.

We measure λ_g, the guide wavelength, as the distance between the nulls in the guide output. We measure a with a set of calipers and compare this with our results from a least-squares fit to the data based on equation 6.

3 Observations

Figure 2 shows a host of interfering signals, a consequence of the imperfections in our setup (such as low signal-to-noise ratio from low power on the local oscillators). How can I justify ignoring the 2 MHz spikes? Aren't those bad?

For the sake of brevity, we will plot the data for the 3GHz section later, in the analysis, along with the least-squares fit model.

Data from the XBand waveguide section, frequency and resulting null positions. All measurements come with an intrinsic reading error ($\pm .025$ cm) and, more importantly, poor resolution of the null's location. This poor resolution was partially due to variation in human judgment, but principally to the low signal-to-noise ratio. Near the null, the derivative of the magnitude-squared voltage with respect to waveguide slider position is at its most shallow. Consequently, there is a sort of range of positions over which the same null can be argued to

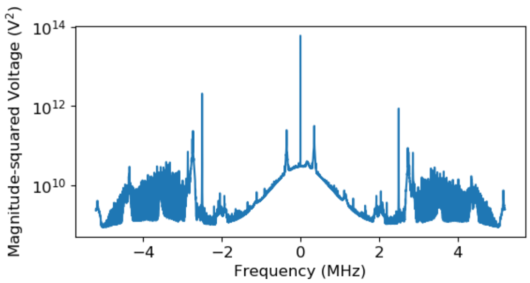

Figure 2: A semi-log plot over the range of frequencies sampled. We assume that our 2 MHz low-pass filter works, so we will dismiss the signals farther than 2 MHz from the center. Additionally, the large central spike and smaller 'bunny ear' spikes ($\pm$.5 MHz) appear on all data sets as persistent interference; we partially ignore these by limiting the y-axis. Maybe you should use an initial test sample, to discuss this.

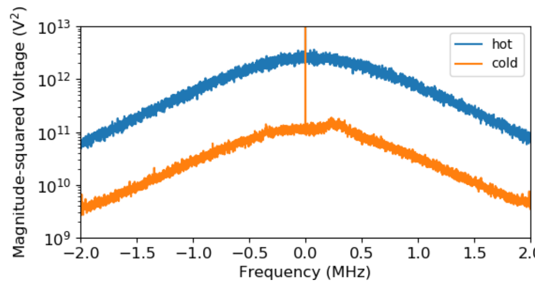

Figure 3: The 'hot' data correspond to three humans standing in front of the collector. The 'cold' data correspond to the collector pointing up at the cold sky. The noise is not an issue because of its regularity: observe the even discrepancy between the curves over the domain of interest.

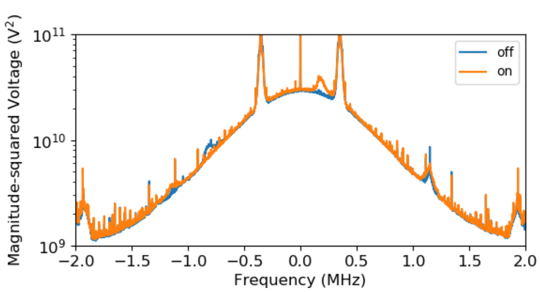

Figure 4: Combined plot of the 'on' and 'off' (LO1 at 1230 and 1231 MHz, respectively) power spectra. As we expect, the HI signal shifts by about 1 MHz between the two plots. This also supports our interpretation of the other patterns as interference: these patterns do not move between spectra.

Figure 5: This one is not helpful. Replace it with the final, calibrated spectrum, thermal power units.

appear. We decided that this uncertainty probably corresponds to a universal uncertainty of $\pm.2$ cm.

f (GHz)	Null Positions (cm)
7.0	8.8, 15.15
7.2	11.95, 17.15
7.5	10.05, 14.4
8.0	10.15, 13.35, 16.7
8.5	9.85, 12.7, 15.25
9.0	9.3, 12.3, 14.25, 16.7

f (GHz)	Null Positions (cm)
9.5	8.9, 11.15, 13.35, 15.45, 17.7
10.0	8.45, 10.45, 12.3, 14.35, 16.5
10.5	9.95, 11.75, 13.5, 15.45, 17.3
11.0	9.35, 11.4, 12.65, 14.45, 16.2, 17.85
11.5	8.95, 10.45, 12.0, 13.8, 15.15, 16.8
12.0	8.4, 9.85, 11.35, 12.85, 14.4, 15.9, 17.45

4 Analysis

Since the Horn was pointing straight up, we may say that the declination is equal to the latitude of the telescope. In our case, that is $37.873199° \approx 0.661012$ radians. The right ascension is equal to the local sidereal time (LST) of data-capture.

We labeled all of our samples with unix time. Included below is a table including examples of date conversions to Doppler corrections for the events of interest. For the sake of brevity, we omit the counterparts for the Cassiopeia measurements.

Label	Unix Time	Julian Date	LST
On-line start	1582937209.9447305	2458908.5325225084	0.8334479646673989
On-line stop	1582937704.4486437	2458908.5382459336	0.8695077621535888
Off-line start	1582937837.5090616	2458908.5397859844	0.8792106794915676
Off-line stop	1582938387.1761112	2458908.5461478718	0.9192930359644231

Label	PST	Doppler Correction (m / s)
On-line start	2/28/20 @ 16:46:49	-27648.19266498681
On-line stop	2/28/20 @ 16:55:04	-27897.548197380296
Off-line start	2/28/20 @ 16:57:17	-27960.009947759394
Off-line stop	2/28/20 @ 17:06:27	-28197.027228226034

The Doppler correction exhibits, at first glance, a critical time sensitivity. Observe that, just for one source (directly overhead), the final Doppler correction differed from the initial by about 550 m / s. However, as we shall soon see, the velocity axis will span many megameters, so taking the average of the Doppler corrections should suffice for a global correction.

If I have extra space, put the results from the Cass. Doppler correction calculation

What was the point of fitting anything to a Gaussian? Furthermore, include the parameters of the two Gaussians that you have here put forward.

A quick calculation: take the average of all null spacings.

Plot: data (position versus null), fit line, error bars...

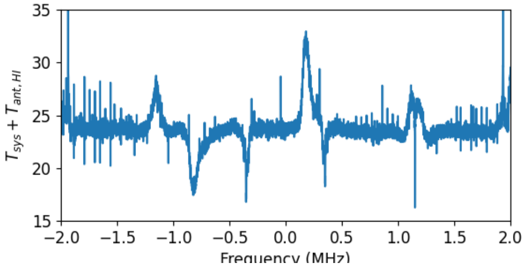

Figure 6: Here is a fully calibrated line. We see HI for sure!

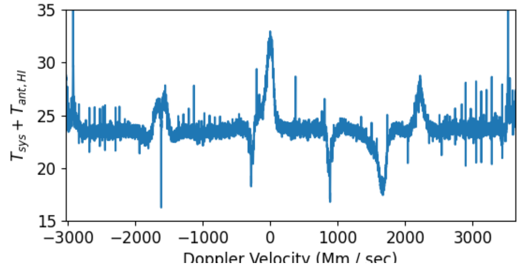

Figure 7: Here is that same line, now with a velocity axis. What is the point I am trying to make with this plot?

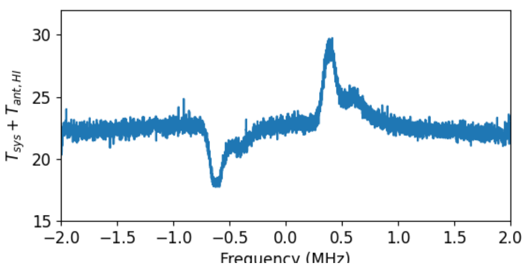

Figure 8: ...

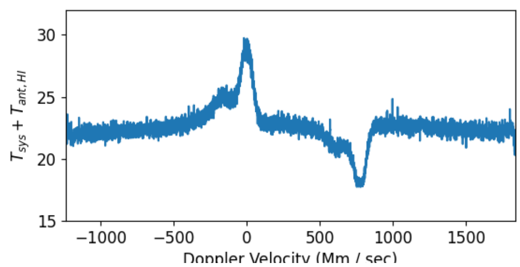

Figure 9: ...

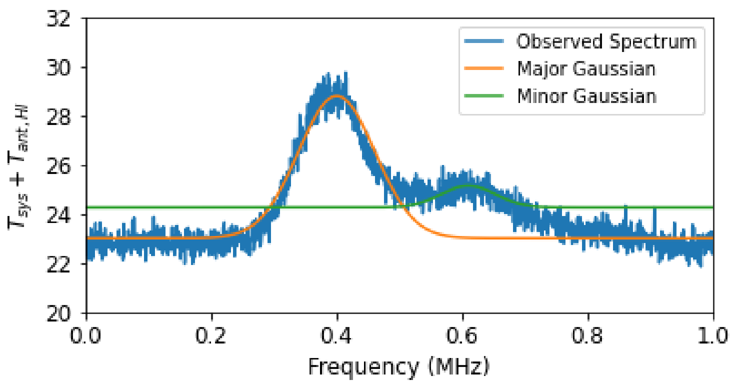

Figure 10: The minor Gaussian, I cannot say for sure it is there. The major one, by contrast, looks pretty neat.

5 Conclusions

On the bright side, if you cannot reach ten pages, that means that you can expand all graphs until the font size is no longer a problem!

6 Acknowledgments

You have to remedy your complete ignorance of BibTex