Your Paper

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February 10, 2020

Abstract

We studied signals.

1 Introduction and Background

1.1 The Fourier Transform

Where $V(t)$ is voltage as a function of time and $V(\nu)$ is voltage as a function of frequency ν.

$$V(\nu) = \int_{-T/2}^{T/2} V(t) \exp(2\pi i \nu t) dt$$

Where T is the length of the time sample.

$$V(t) = \frac{1}{\nu_s} \int_{-\nu_s/2}^{\nu_s/2} \tilde{V}(\nu) \exp(-2\pi i \nu t) d\nu$$

where ν_s is the sampling frequency used.

What happens if you change the bounds in the manner (N / 2 - 1) / ν_s

1.2 Positive and Negative Frequencies

"complex inputs to a Fourier Transform break the positive/negative frequency degeneracy"

The sine function is odd:

$\sin(-\omega t) = -sin(\omega t)$

The cosine function is even:

$\cos(-\omega t) = \cos(\omega t)$

We can generalize under Euler's identity:

$A\exp(i\omega t) = A\cos(\omega t) + i \cdot A\sin(\omega t)$

$A\exp(-i\omega t) = A\cos(-\omega t) + i \cdot A\sin(-\omega t) = A\cos(\omega t) - i \cdot A\sin(\omega t)$

1.3 Nyquist's Theorem and Aliasing

$f_s > 2f_{max}$

1.4 The Convolution and Correlation Theorems

The convolution of two functions $f(t)$ and $g(t)$ is defined as:

$$[f * g](\tau) = \int_{-T/2}^{T/2} f(t)g(\tau - t)dt$$

The correlation of two functions $f(t)$ and $g(t)$ is defined as

$$[f \star g](\tau) = \int_{-T/2}^{T/2} f(t)g(\tau + t)dt$$

You may want to prove this by substituting the Fourier transform

$$[f \, \tilde{*} \, g](\nu) \equiv \int_{-T/2}^{T/2} [f * g](\tau) \exp 2\pi i \tau \nu d\tau = \tilde{f}(\nu) \cdot \tilde{g}(\nu)$$

$[f \tilde{\star} g](\nu) \equiv \int_{-T/2}^{T/2} [f \star g](\tau) \exp 2\pi i \tau \nu d\tau = \tilde{f}(\nu) \cdot \tilde{g}^*(\nu)$

Now: how is the ACF relevant to this paper

1.5 Sideband Theory

All that sweet, sweet trig.

2 Methods

This is the equipment I used. These are the libraries and functions I used. This is how I used them (*vague* hint at your code). Uncertainty? Technical errors?

To begin our investigation of the Nyquist criterion, we decided on a sampling frequency $\nu_s = 6.25 MHz$ at which the pico sampler (a PicoScope 2206A) remained for the duration of the data collection for all signals. We then used the N9310A RF Signal Generator to output signals at frequency ν_0: fractions of the sampling frequency. For example, one of our input signals was $\nu_0 = .4\nu_s = 2.5 MHz$.

2.1 Sideband

We used one signal generator (N9310A) with a wide range of allowed frequencies, while the other (83712B) with a lower frequency limit of 10 MHz.

We set the 83712B to run at 11 MHz and 1.5 dBm (justify??). This gave $\Delta\nu = .05 \times \nu_{LO} = .55$ MHz. We set the N9310A to run at 1.5 dBm as well and, depending on the trial, either $\nu_{RF} = \nu_{LO} + \Delta\nu = 11.55$ MHz or $\nu_{RF} = \nu_{LO} - \Delta\nu = 10.45$ MHz.

When collecting data, we sampled at 32.5 MHz, which is more than double the Nyquist frequency. ?? why is this important and give a calculation of how far above the Nyquist this is

?? I didn't identify the sum and difference frequencies from observation!!
$\sin(\nu_{LO} + \Delta\nu) = \sin\nu_{LO}\cos\Delta\nu + \cos\nu_{LO}\sin\Delta\nu$
$\sin(\nu_{LO} - \Delta\nu) = \sin\nu_{LO}\cos\Delta\nu - \cos\nu_{LO}\sin\Delta\nu$

But these are not relevant, we want
$sin(a) + sin(a+b) =?$, right?
$\sin(\nu_{LO})\sin(\nu_{LO} + \Delta\nu) = \frac{1}{2}(\cos\Delta\nu - \cos(2\nu_{LO} + \Delta\nu))$ by evenness of the cosine function
and
$\sin(\nu_{LO})\sin(\nu_{LO} - \Delta\nu) = \frac{1}{2}(\cos\Delta\nu - \cos(2\nu_{LO} - \Delta\nu))$
Why do the power spectra look the way they do. Upper sideband and lower sideband.

For the upper sideband, we can see spikes at almost the difference frequency (.575 MHz ≈ .55 MHz). The other spikes are at 10.2 MHz? Why?

For the lower sideband, we see outer spikes at 9 MHz. The inner spikes are still at roughly the difference frequency...

3 Notes to Self

What am I doing? Not ACF (5.3, 5.7) and procrastinating on 5.5 so that I can get some writing done.

I did ACF analysis for neither 5.3 nor 5.7
Major logic error: side-by-side captions are mutilated!
Idea: put signal and power spectrum plots side by side, not signals with signals
argue what the Nyquist criterion is, based on results.
* Include make and model of all equipment used.
* "Don't quote a number without the uncertainty and units."
Discussion on results for week 2, section 1

I need to include details about the equipment used, but how in-depth do I need to go? Current plan
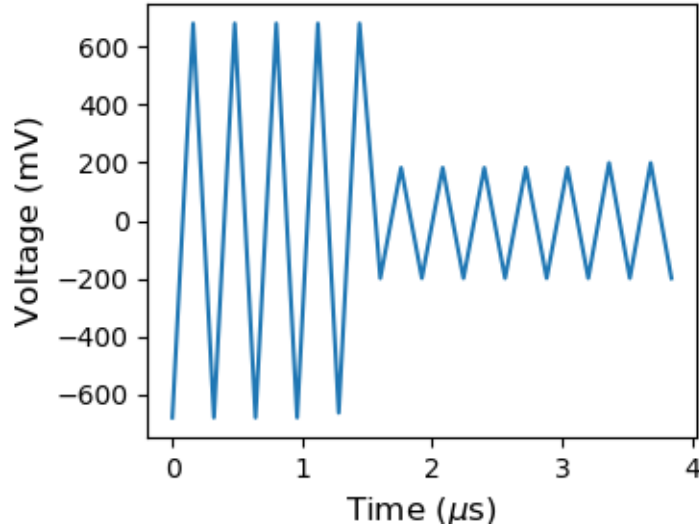
Figure 1: The oscilloscope displayed a constant signal throughout the period of data-taking. The data sampled from the pico sampler, however, reconstructs a signal with large aberrations in the first few microseconds.

    * For most things, use model number and manufacturer
    * When data analysis depends on a spec sheet, offer a brief summary of the specs to which you are referring to fine-tune your analysis

I need uncertainties on results but I do not yet know how to get these.
    Closer to the end of writing, design a new subsection layout for the results section. You of course cannot make references like "5.2" and "7.3" in the submission.

4 Results

4.1 5.2

Without taking the spectrum, we want to perform visual analysis of the plots and find periods
    Motivation
    * Pico sampler model, what sampling rate we used
    * We're using the ugradio pico sampler code
    * We're using the _ _ _ signal generator
    * Define terms ν_0 = input frequency. ν_s = sampling frequency.
    Finally, we took the default 16000 samples for each signal. However, for the data analysis, we will be excluding the first 100 samples due to a peculiarity of the pico sampler which distorted these (1).

      We may begin inspection of the samples with a qualitative approach. Figure 2 shows a signal which repeats about five times in the span of about 4 microseconds. This gives us 1.25 cycles per microsecond, or 1.25 MHz, as expected. Figure 5 is not as obviously derived from a sine wave (the shape is distorted by the shrinking gap between ν_0 and ν_s), but we may still say that there is a repeating signal with slightly under five repetitions in a span of 4 microseconds. This would give us slightly under 1.25 MHz, which is incorrect; thus we have qualitatively demonstrated an aliasing effect.

4.2 5.3

"What does it mean, that the voltage spectra are complex? What do the real and imaginary parts represent? Is the imaginary part less 'real' than the real part? What does it mean, for frequencies to be negative versus positive?"
    "Why might we use power spectra instead of voltage spectra, and vice versa?"
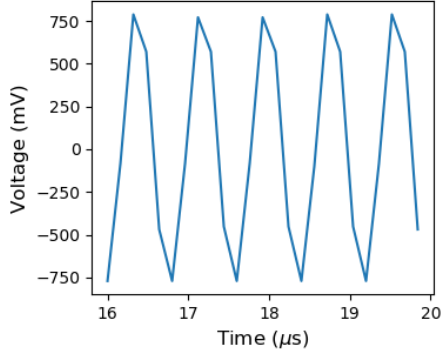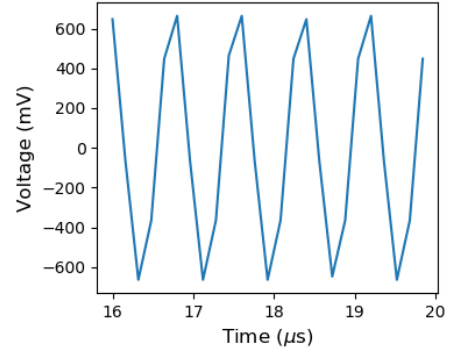
Figure 2: $\nu_0 = .2\nu_s = 1.25$ MHz.
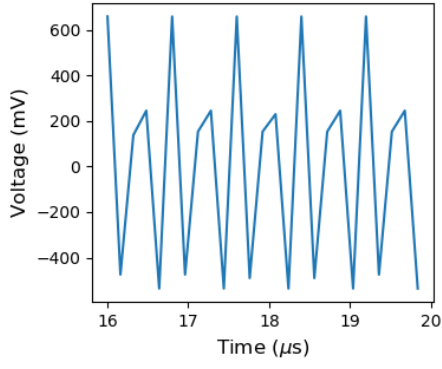

Figure 3: $\nu_0 = .8\nu_s = 5$ MHz.

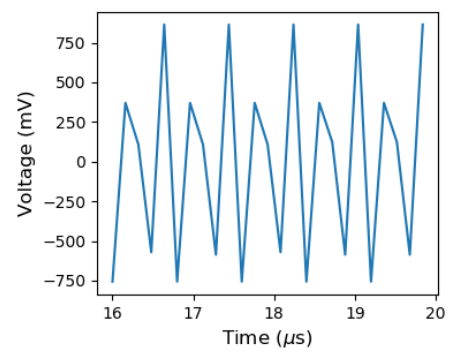Figure 4: $\nu_0 = .4\nu_s = 2.5$ MHz.
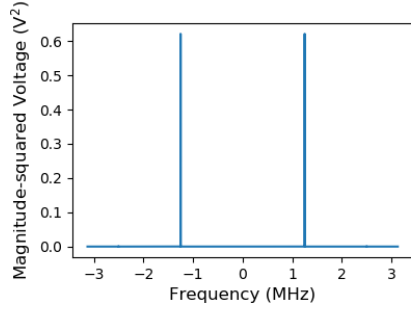

Figure 5: $\nu_0 = .6\nu_s = 3.75$ MHz.

Figure 6: $\nu_0 = .2\nu_s = 1.25$ MHz. Peak amplitudes at $\pm$ 1.25 MHz
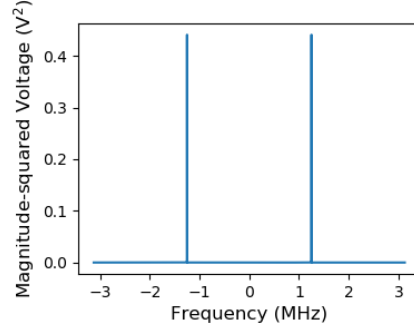
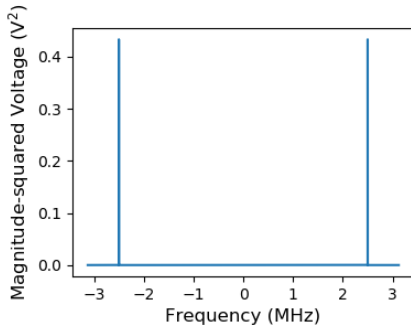

Figure 7: $\nu_0 = .8\nu_s = 5$ MHz. Peak amplitudes at $\pm$ 1.25 MHz

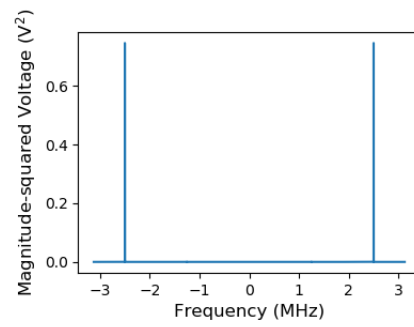Figure 8: $\nu_0 = .4\nu_s = 2.5$ MHz. Peak amplitudes at $\pm$ 2.5 MHz

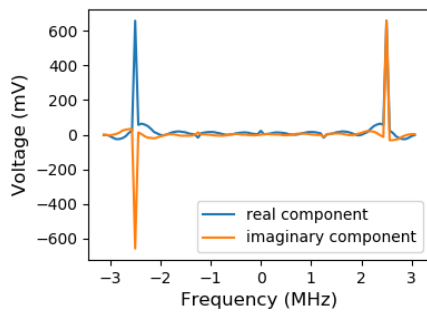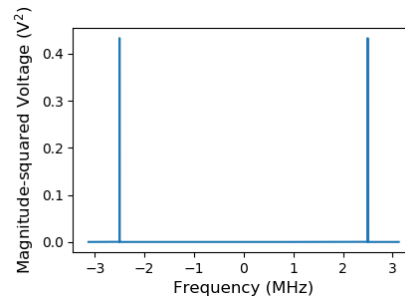Figure 9: $\nu_0 = .6\nu_s = 3.75$ MHz. Peak amplitudes at $\pm$ 2.5 MHz

4

Figure 10: Voltage

Figure 11: Powerish
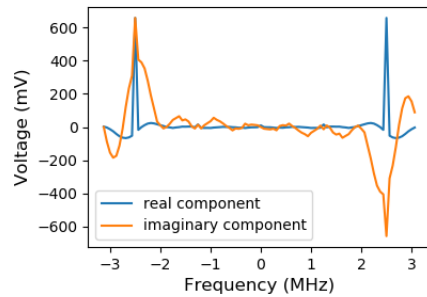
Figure 12: Voltage
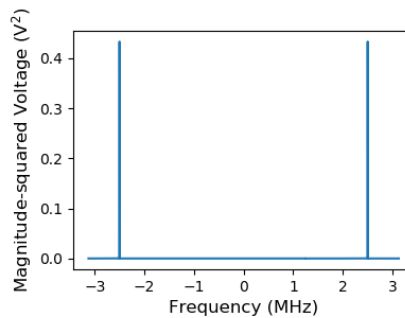
Figure 13: Powerish

"According to the correlation theorem, the Fourier transform of the power spectrum should equal the ACF. Does it? Explain any differences."

"you need to make sure dft.idft correctly infers the frequencies corresponding to each bins in your power spectrum array"

"When calculating a digital version of the correlation function, you have to worry about end effects. Suppose you are calculating an ACF for N samples with delays ΔN ranging up to N/2. Then the number of terms in the sum is always smaller than N because the delays spill over the edge of the available samples."

How am I supposed to account for this?

Differences? Probably because I did not zero out the middle

4.3 5.4

The y-axis is labeled incorrectly. We want log of square of voltage.

Recall that spectral leakage is introduced by the finite bounds on our Fourier transforms. Why does that math correspond to these results?
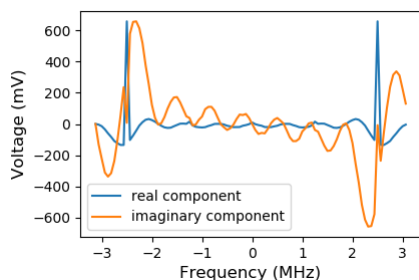
Figure 14: Voltage
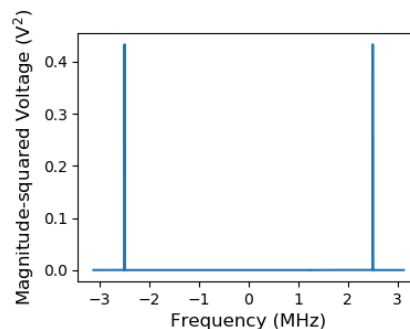
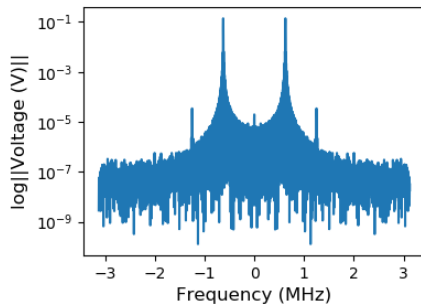Figure 15: Powerish
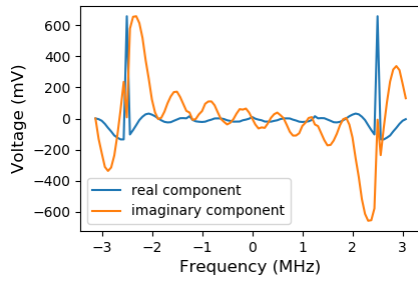
Figure 16: Voltage
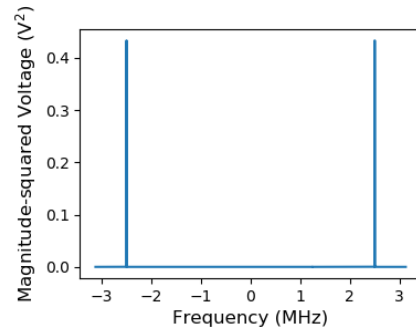
Figure 17: Powerish

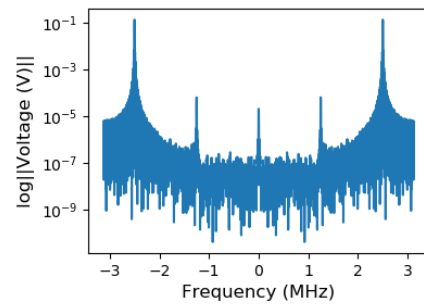Figure 18: $\nu_0 = .1\nu_s = .625$ MHz

Figure 19: $\nu_0 = .4\nu_s = 2.5$ MHz

4.4 5.5

The separation looks pretty good so far.

4.5 5.6

4.6 5.7

First sample:

Mean = 3.899292452830189 mV Standard deviation = 20.079642416978363 mV Variance = 403.19203959371663 (mV)^2

I am allowing the first 100 samples to contaminate. I do not know how much I should trod outside the 16000 recommendation.
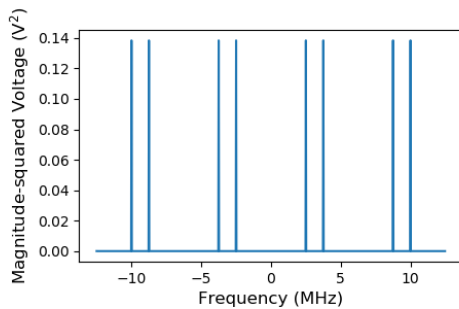
4.7 7.1

These figures are missing captions!

Figure 20: $\nu_0 = .4\nu_s = 2.5$ MHz. Fourth window.
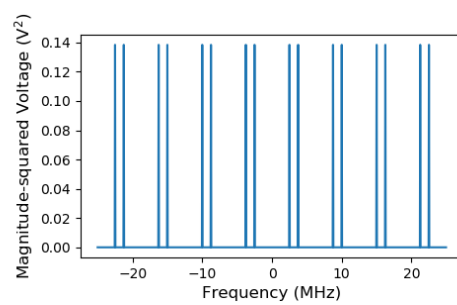
Figure 21: Same frequency. Eighth window.
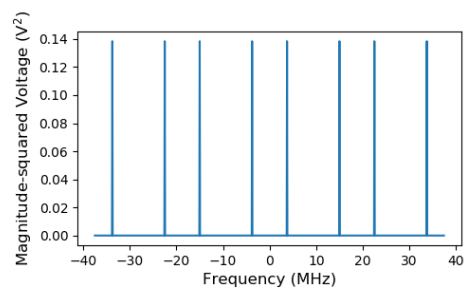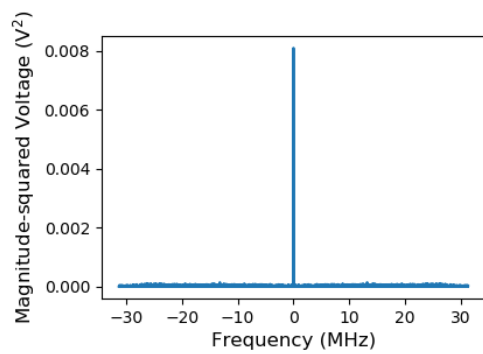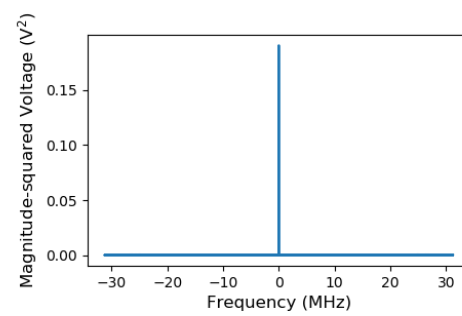
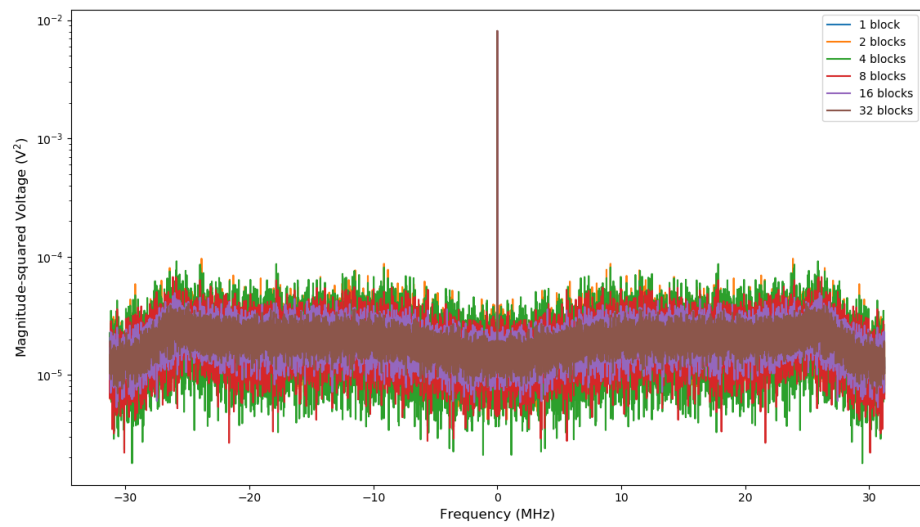Figure 22: Same frequency. Twelth window.
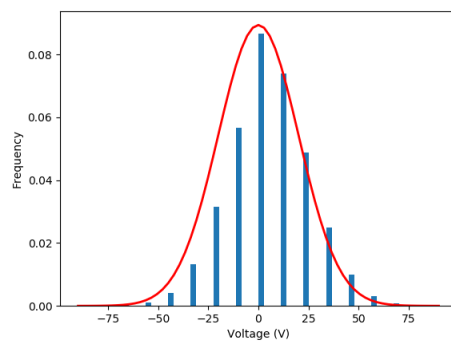
Figure 23

Figure 24

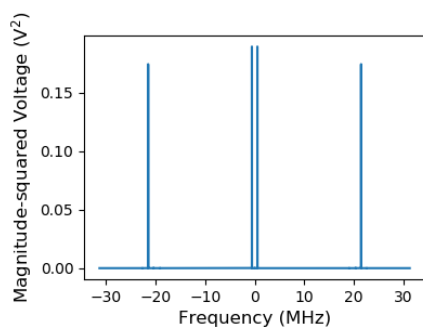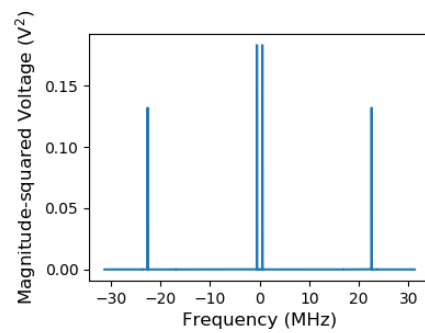

Figure 25

Figure 26
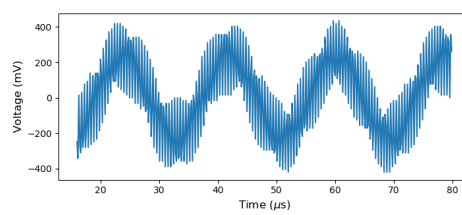

Figure 27

Figure 28

Figure 29

4.8   7.2

4.9   7.3

Single side band mixer is more difficult to perform.

5   Conclusions

References

[Gre93] George D. Greenwade. The Comprehensive Tex Archive Network (CTAN). *TUGBoat*, 14(3):342–351, 1993.