# Physics Notes

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This compilation of notes are to be used as a reference for the GCE "A"-level Physics paper, as a refresher in definitions, theories as well as for general descriptions of presentation form. These notes are meant for free, public use, but at the reader's own risk.
Good luck with your exams.

# 1 Measurements

## 1.1 Units

Physics can be summarized as a collection of mathematical relationships between physical phenomena. Each and every physical quantity has a numerical magnitude and a unit. Note that it is nonsensical to compare a physical quantity to a unit (e.g. time cannot be compared to seconds).

$$\underbrace{F}_{\text{Physical Quantity}} = \underbrace{5}_{\text{Numerical Magnitude}} \underbrace{N}_{\text{Unit}}$$

**Definition 1.1: SI Base Units**

SI base units are a selection of fundamental physical quantities, from which all other physical quantities can be represented as a combination of SI Base Units. These quantities have been arbitrarily chosen for accessibility and reproducibility.

**Definition 1.2: Derived Units**

Derived Units are defined as products or quotients of base units and are obtained as products of base units

| Base Quantity | Base Unit | Symbol |
|---|---|---|
| Time | Second | s |
| Length | Meter | m |
| Mass | Kilogram | kg |
| Current | Ampere | A |
| Temperature | Kelvin | K |
| Amount of Substance | Mole | mol |

For a mathematical operation to be valid, addition and subtraction between physical quantities have to have the same unit and two sides of an equation must have the same unit.

**Definition 1.3: Homogeneous Equations**

An equation is homogeneous if both sides of an equation have the same resultant units. Also called Dimensionally Consistent.

The homogeneity of an equation can be used to determine the powers of physical quantities used to derive a value.

## 1.2 Numerical Magnitudes

Orders of magnitudes of a physical quantity can be used to represent decimal multiples of a number.

| Prefix | Symbol | Power of 10 |
|---|---|---|
| tera | T | 12 |
| giga | G | 9 |
| mega | M | 6 |
| kilo | k | 3 |
| deci | d | -1 |
| centi | c | -2 |
| milli | m | -3 |
| micro | $\mu$ | -6 |
| nano | n | -9 |
| pico | p | -12 |

**Definition 1.4: Standard Form**

Standard form is where the numerical magnitude of a physical quantity is written in the form $a \times 10^n$ where $1 \leq a < 10$ and $n$ is an integer.

Estimation of the order of magnitude of a physical quantity can be derived from estimating component values of a certain order of magnitude and then applying physical equations.

## 1.3 Error

Error in a reading is where there is uncertainty in the exact value of the numerical magnitude of a physical quantity.

**Definition 1.5: Systematic Error**

Systematic errors are caused by lapses in the measurement process, resulting in values consistently erroneous to give always smaller or always larger readings and can be eliminated if the source of error is known and accounted for.

**Definition 1.6: Random Error**

Random errors are caused by inherent inaccuracy and lack of precision in a reading, resulting in values scattered about a mean and can be mitigated by repeating measurements and finding lines of best fit but otherwise cannot be predicted.

**Definition 1.7: Accuracy**

Accurate readings are values which are close to the true value of a physical quantity and is influenced by systematic error.

**Definition 1.8: Precision**

Precise readings are values which agree which other and is influenced by random error.

### 1.3.1 Measuring Values

Precision of a measuring instrument is determined by its least count. Measurements of length and volume are read to their least count, or half their least count if the markings are larger than 1mm such as on a meter rule or a graph. Digital instruments are read and recorded to their displayed value except for tools which depend on other erroneous input such as human reaction time. Do note that the ruler is a special case, where since the error in reading is 0.5mm but two readings are made (one for the starting point of measurement, and one for the ending point, te result is obtained by $ending - starting$, though starting is usually at the zero mark) the total error is twice that error or 1mm. In questions which specify that the error accompanying each reading is one division, the absolute error is twice the least count (don't ask me why).

### 1.3.2 Error Propagation

**Equation 1.1: Error Propagation**

For a resultant value $Q$, two derivative values $X$ and $Y$ and their powers or coefficients $a$ and $b$

$$Q = aX + bY \quad \Delta Q = |a|\Delta X + |b|\Delta Y$$
$$Q = kX^aY^b \quad \frac{\Delta Q}{Q} = |a|\frac{\Delta X}{X} + |b|\frac{\Delta Y}{Y}$$

Absolute uncertainty is represented to 1 s.f. while fractional and percentage (fractional multipied by 100%) uncertainty is represented to 2 s.f. .

To find the situation where maximum fractional error occurs, adjust the values such that the value of $Q$ is its smallest possible value.

### 1.3.3 DP and SF

Addition and subtraction operations in experimental situations require the result to follow the largest decimal place value of its derivatives. Multiplication and division operations in experimental situations require the result to follow the least significant figures of its derivatives. However, in exam settings seek to maintain all working in 5sf/dp and only reduce sf/dp when obtaining answers.

### 1.3.4 Scalars and Vectors

**Definition 1.9: Scalar Quantity**

A Scalar Quantity is a physical value with a numerical magnitude, and are represented by a magnitude and a unit.

**Definition 1.10: Vector Quantity**

A Vector Quantity is a physical value with a numerical magnitude as well as a direction, and are represented by a magnitude, unit and a direction.

Before solving questions involving vector quantities, a positive direction should be defined as whichever direction is most convenient.

# 2 Kinematics

**Definition 2.1: Distance**

Distance x is the length of a path followed by an object, measured in m.

**Definition 2.2: Displacement**

Displacement s is the distance moved in a specified direction from a reference point, measured in m. It is the vector equivalent of distance.

**Definition 2.3: Speed**

Speed v is the instantaneous speed of an object, defined as the rate of change of distance traveled with respect to time, measured in $\mathrm{m\,s^{-1}}$. Average speed refers to the distance traveled over a significantly large time taken.

**Definition 2.4: Velocity**

Velocity v is the instantaneous velocity of an object, defined as the rate of change of displacement with respect to time, measured in $\mathrm{m\,s^{-1}}$. Average velocity refers to the change in displacement over a significantly large time taken. It is the vector equivalent of speed.

**Definition 2.5: Acceleration**

Acceleration a is the instantaneous change in velocity of an object, defined as the rate of change of velocity with respect to time, measured in $\mathrm{m\,s^{-2}}$. Average acceleration refers to the change in velocity over a significantly large time taken.

Note that when faced with a kinematics graph (s/v/a against t), the gradient of the graph (differential) and the area under the graph (integral) obtain special meanings.

## 2.1 Equations of Motion

For a situation involving uniform acceleration and motion in a straight line, the following equations hold:

Final velocity from initial velocity and acceleration
$$v = u + at$$
Displacement from average velocity
$$s = \frac{1}{2}(u+v)t$$
Displacement from initial velocity and acceleration
$$s = ut + \frac{1/2}{a}t^2$$
Final velocity from displacement, initial velocity and acceleration
$$v^2 = u^2 + 2as$$

For the condition of objects in freefall, acceleration is equal to which takes the value of 9.81 $\mathrm{m\,s^{-2}}$.For the conditions of objects in projectile motion with the assumption of no air resistance, acceleration in the vertical dimension behaves as if the object is in freefall, and acceleration in the horizontal direction is equal to zero.

## 2.2 Air Resistance

When objects move through air, it experiences viscous drag or air resistance. Air resistance acts opposite to the direction of velocity and is proportional to the velocity, or at higher velocities is proportional to the square of the velocity. The terminal velocity is the velocity at which the air resistance is equal to accelerative forces on an object, hence the resultant acceleration is equal to zero.

For an object projected upwards in freefall, the time of flight upwards will be smaller than the flight downwards.

On the way up, air resistance acts against upward motion and hence acts downwards in line with gravity, creating a larger resultant force downwards and a larger acceleration which retards its vertical motion, hence the velocity decreases at a faster rate and it takes less time to travel to the peak of the trajectory than if there had been no air resistance.

On the way down, air resistance acts against downward motion and hence acts upward and against gravity, reducing the resultant force downwards and a lower acceleration which accelerates the object downward, hence the velocity increases at a slower rate and it takes more time to travel the same distance downward had there been no air resistance.

# 3 Dynamics

**Definition 3.1: Force**

Force F is an action that causes a change in the physical shape or state of a body and is defined as the product of mass and acceleration with units N or $\text{kgms}^{-2}$

Multiple forces acting upon a body can be added together in a vector sum to find the resultant force.

## 3.1 Newton's Laws of Motion

**Definition 3.2: Newton's First Law of Motion**

Newton's First Law of Motion states that a body continues in its state of rest or motion in a straight line unless acted upon by an external force.

**Definition 3.3: Newton's Second Law of Motion**

Newton's Second Law of Motion states that the change in momentum of a body is proportional to the resultant force acting on it and occurs in the direction of said resultant force.

**Definition 3.4: Newton's Third Law of Motion**

Newton's Third Law of Motion states that for a force acting from a first body on a second body, there is an equal and opposite force acting from the second body on the first body.

## 3.2 Equilibrium

**Definition 3.5: Inertia**

Inertia is the tendency of a body to maintain its current motion or lack thereof unless acted upon a force.

**Definition 3.6: Equilibrium**

When a body experiences forces which do not change its state

For a object to be in equilibrium, the resultant force on the object must have zero magnitude and the resultant torque on the object about any axis must also have zero magnitude.

## 3.3 Momentum

**Definition 3.7: Momentum**

Momentum is defined as the product of the mass of an object and its velocity with units Nm or $\text{kgm}^2\text{s}^{-2}$.

The total momentum of a system is equivalent to the vector sum of its component objects' momenta. Forces can be simplified into the change of momentum over time.

## 3.4 Action Reaction Pairs

**Definition 3.8: Action Reaction Pairs**

Action Reaction Paris are pairs of forces which arise due to Newton's Third Law of Motion which then are of the same type (Normal Contact with Normal Contact, Friction with Friction, Electric with Electric) and act upon different bodies, in addition to properties described in the law which states that the forces are equal in magnitude but opposite in direction.

## 3.5 Impulse

**Definition 3.9: Impulse**

Impulse is defined as the product of a force acting on an object and the time which the force is exerted, alternatively the amount of momentum that is transfered, with units Ns or $\text{kgms}^{-1}$.

Change in momentum can also be measured by finding the area under a Force-Time graph.

## 3.6 Drawing Forces

Free body diagrams are rudimentary drawings which illustrate the location, direction and magnitude of multiple forces acting upon objects. When drawing diagrams, label with full names of forces unless the short forms are already defined (or define them yourself in a section of the question paper).

**Weight W** is drawn from the center of mass downward.

**Normal Contact Force N** is drawn from the point of contact between two bodies. For two contacting surfaces, the force is drawn perpendicular to the surface.

**Frictional Force f** is drawn on the surface which friction acts on.

**Tension T / Compression** is drawn along the wire, spring or strut. Tension is drawn inward while compression is drawn outward.

**Upthrust U** is drawn upwards from the center of mass which is below water level.

**Viscous Force $\mathbf{F_v}$** is drawn from the center of the surface furthest from the direction of motion and is opposite to the direction of motion.

**Lift L** is drawn perpendicular to the axis of wings.

**Resultant $\mathbf{F_{net}}$** is drawn disconnected from the body and is drawn with two arrows in the direction of motion.

### 3.7 Collisions

Extremely quick collisions between objects involve high values of F such that assessment is more feasible by examining changes in momentum rather than forces exerted.

**Definition 3.10: Law of Conservation of Momentum**
The Law of Conservation of Momentum states that the total momentum of a system remains the same when no external force is applied.

Conservation of Momentum provides the equation:

**Equation 3.1: Conservation of Momentum**
For mass $m$ and initial and final velocities $u$ and $v$

$$m_1u_1 + m_2u_2 = m_1v_1m_2v_2$$

**Definition 3.11: Law of Conservation of Energy**
The Law of Conservation of Energy states that energy can neither be created nor destroyed, hence the total energy of a closed system remains the same.

Conservation of Energy provides the equation:

**Equation 3.2: Conservation of Kinetic Energy**
For mass $m$ and initial and final velocities $u$ and $v$

$$m_1u_1^2 + m_2u_2^2 = m_1v_1^2m_2v_2^2$$

**Definition 3.12: Elastic**
Elastic collisions maintain the property of conservation of momentum as well as conservation of kinetic energy.

Combining the equations of conservation of momentum and conservation of energy, we obtain this in the case of an elastic collision:

**Equation 3.3: Elastic Collision: Constant Relative Speed**
For initial and final velocities $u$ and $v$

$$u_1 - u_2 = v_2 - v_1$$

**Definition 3.13: Completely Inelastic**
Completely Inelastic collisions maintain the property of conservation of momentum but involve the conversion of kinetic energy to other forms of energy. Particles stick to each other after collision.

## 4 Forces

### 4.1 Elastic Force

**Definition 4.1: Hooke's Law**
Hooke's Law states that the extension of a spring is proportional to the applied force if the limit of proportionality is not exceeded.

**Equation 4.1: Elastic Force**
For some distance of extension $x$, some proportionality constant $k$ and some force $F$, the values are related by the equation

$$F = kx$$

And the energy stored in a spring is given by the equation:

**Equation 4.2: Elastic Energy**
For some distance of extension $x$ and some proportionality constant $k$, the energy stored in a spring $E$ is given by the equation

$$E = \frac{1}{2}kx^2$$

### 4.2 Frictional Force

**Definition 4.2: Friction**
Friction is a force which exists between two surfaces in contact with each other and resists motion between these two surfaces.

**Equation 4.3: Frictional Force**
For a given normal contact force $N$ between two surfaces with a frictional constant $\mu$ the frictional force $f$ is given by the equation

$$f = \mu N$$

### 4.3 Upthrust

**Definition 4.3: Fluid**
A Fluid is a substance which can flow, including most liquids and gases.

**Definition 4.4: Density**
Density $\rho$ a substance is its mass per unit volume, with units kgm$^{-3}$

**Definition 4.5: Pressure**
Pressure p is the force per unit area exerted at right angles to a surface by some object, with units Nm$^{-3}$ or kgm$^{-2}$s$^{-2}$

Note that when considering pressure in a liquid at sea level, the pressure due to atmosphere needs to be accounted for.

**Equation 4.4: Pressure in Fluid**
For the height $h$of a fluid above the level considered, its density $\rho$ and the gravitational acceleration $g$ at a point, pressure $p$ is given by the equation

$$p = h\rho g$$

**Definition 4.6: Upthrust**
Upthrust is a vertically upward force exerted on a body by a fluid when it is fully or partially submerged in a fluid due to difference in fluid pressure at different heights.

**Definition 4.7: Archimedes Principle**

The Archimedes Principle states that the upthrust on a submerged object is equal to the weight of liquid displaced by said object.

Questions on upthrust usually involve a calculation of density, mass or volume of liquids or solids in a system. In the case of floating objects, utilize the equation $U = W$ to find the weight or upthrust experienced by an object.

## 4.4 Viscous Force

**Definition 4.8: Viscous Force**

Viscous Force is the force experienced by a body moving through a fluid when it receives normal contact force from the particles of the fluid after it imparts momentum onto fluid, written $F_v$.

The magnitude of the viscous force depends on the shape of the body and viscosity of the fluid, as well as the speed of the body which has a proportional relationship to the force at low velocity and a squared relationship with the force at higher velocities.

$F_v$ is zero when a body is at rest. When a body affected by viscous force experiences a constant force or acceleration, the body speeds up at a decreasing rate as the resultant force is smaller due to the increasing viscous force. Terminal velocity is reached when the viscous force is equal to the applied force, resulting in equilibrium.

**Definition 4.9: Terminal Velocity**

Terminal Velocity is the speed at which a the viscous force experienced by a body causes further acceleration to be prevented.

## 4.5 Calculating Equilibrium

A body in equilibrium must have both translational and rotational equilibrium. As such, it needs to have zero resultant force as well as zero torque about any axis.

Translational equilibrium is obtained when all forces acting on a body are added using vector addition and have zero resultant magnitude. Forces can be resolved into their dimensional components and summed together.

**Definition 4.10: Principle of Moments**

For any body in rotational equilibrium, the sum of all clockwise moments about an axis is equal to the sum of all anticlockwise moments.

**Definition 4.11: Moment**

A Moment is a physical value which involves the multiplication of a perpendicular distance from an arbitrary axis with another physical quantity existent at a point.

**Definition 4.12: Torque**

The Torque of a force about an arbitrary axis is defined as the product of the force and the perpendicular distance from the point to the line of action of the force, with units Nm or $kgm^2s^{-2}$.

**Definition 4.13: Couple**

A Couple is a pair of equal and opposite parallel forces whose lines of action do not meet.

**Equation 4.5: Torque of A Couple**

For the magnitude of one force in the couple $F$ and the perpendicular distance between the two forces $d$, total torque $\tau$ is given by the equation

$$\tau = Fd$$

A couple has the special quality that it has zero resultant force but still has a torque. A couple will continue to rotate until the lines of action of the two forces coincide and have zero perpendicular distance.

## 4.6 Calculating Center of Mass

**Definition 4.14: Center of Gravity**

The Center of Gravity of an object is the point where gravitational attraction on the body appears to act.

CG is calculated by finding the point where when used as a pivot results in rotational equilibrium. This means that the point is vertically in line with the center of gravity. CG of an irregularly shaped object can be obtained by twice pivoting the body and drawing a line vertically down from the pivot, where the point where the lines intersect would be the center of gravity.

# 5 Work Energy and Power

## 5.1 Work

**Definition 5.1: Work**

Work is defined as the product of a force and the displacement in the direction of the force, measured in Joules J or $kgm^2s^{-2}$

**Equation 5.1: Work Done on a System**

For a force $F$, displacement $s$ and angle between Force and Displacement $\theta$, work done $W$ is given by the equation

$$W = Fscos(\theta)$$

**Equation 5.2: Work Done by a Gas**

For a contained gas of volume $V$ and external pressure $p$, work done $W$is given by the equation

$$W = p\Delta V$$

Amount of work done can be measured as the integral of the Force-Distance graph for normal motion, the integral of the Pressure-Volume graph for work done by a gas and the integral of the Force-Extension graph for work done by a spring.

## 5.2 Energy

**Definition 5.2: Energy**

Energy is the quantification of an object's capacity to do work,measured in J or kgm$^2$s$^{-2}$

**Equation 5.3: Kinetic Energy**

For a object of mass $m$ and speed $v$, the amount of Kinetic Energy $E_k$ contained is given by the equation

$$E_k = \frac{1}{2}mv^2$$

Note that an object whose velocity changes has an energy change of $\frac{1}{2}m(v^2 - u^2)$ rather than $\frac{1}{2}m(v-u)^2$.

**Equation 5.4: Gravitational Potential Energy**

For a object of mass $m$, gravitational acceleration $g$ and (relative) height $h$, the amount of Gravitational Potential Energy $E_p$ contained is given by the equation

$$E_p = mgh$$

**Equation 5.5: Elastic Potential Energy**

For a spring of proportionality constant $k$ and extension $x$, the amount of Elastic Potential Energy $U_E$ contained is given by the equation

$$U_E = \frac{1}{2}kx^2$$

Given the Energy-Distance graph of a object experiencing a field force, the gradient of the graph gives the force at a certain distance.

**Definition 5.3: Principle of Conservation of Energy**

The Principle of Conservation of Energy states that the total energy of a system is constant while no work is done on the system.

The sum of all kinetic and potential energy at any point in time is constant, even in the presence of a dissipative force. Dissipative forces are also lessened in a system with more uniform motion.

**Equation 5.6: Efficiency**

To obtain the efficiency $\eta$ of a system, use the equation

$$\eta = \frac{\text{useful energy output}}{\text{total energy input}} \times 100\%$$

**Definition 5.4: Power**

Power is the quantification of work done with respect to time, defined as the rate which work is done with respect to time or the amount of energy transferred with respect to time, measured in W or kgm$^2$s$^{-3}$

For questions involving mass flow rates, calculate values in terms of their algebraic quantities and then cancel out the time quantity at the end to find the rate.

# 6 Circular Motion

# 7 Gravitation

**Definition 7.1: Gravitation**

Gravitation is defined as the attractive force between to masses.

**Definition 7.2: Newton's Law of Gravitation**

Newton's Law of Gravitation states that every point mass attracts every single other point mass along a line intersecting both points, which is proportional to the product of the two masses and inversely proportional to the square of the distance between the two masses.

**Equation 7.1: Gravitational Equations**

For the mass of two planetary objects $m_1$ and $m_2$, the radius between them $r$, and the gravitational constant $G$, the force experienced by each of the planets $F$, the acceleration of a planet $g$, the potential energy of a each planet $U$ and the potential gradient of a planet $\phi$ are given by the equations

$$F = G\frac{m_1 m_2}{r^2}$$
$$g = G\frac{m_1}{r}$$
$$U = -G\frac{m_1 m_2}{r}$$
$$\phi = -G\frac{m_1}{r}$$