1 Calculus II

1.1 Integration

1.1.1 Standard Integral Forms

$$\int f'(x)dx = f(x) + c$$
$$\int f'(x)(f(x))^n dx = \frac{1}{n+1} \quad f(x)^{n+1} + c$$
$$\int \frac{f'(x)}{f(x)} dx = \ln(|f(x)|) + c$$
$$\int f'(x)e^{f(x)} dx = e^{f(x)} + c$$

1.1.2 Special and Trigonometric Integrals

$$\int \frac{1}{x^2 + a^2} = \frac{1}{a} \tan^- 1\left(\frac{x}{a}\right) + c$$
$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^- 1\left(\frac{x}{a}\right) + c$$
$$\int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \ln\left(\frac{x-a}{x+a}\right) + c$$
$$\int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \ln\left(\frac{a+x}{a-x}\right) + c$$
$$\int \tan(x)dx = \ln(\sec(x))$$
$$\int \cot(x)dx = \ln(\sin(x))$$
$$\int \csc(x)dx = -\ln(\csc(x) + \cot(x))$$
$$\int \sec(x)dx = \ln(\sec(x) + \tan(x))$$

1.1.3 Integration by Substitution

Given a substitution $u(x)$ where $f(x) = g(u)$

$$\int f(x)dx = \int g(u)\frac{dx}{du} du$$

For definite integrals, substitute u back to receive the integral as a function of x. For definite integrals with limits $[a, b]$, change limits to $[u(a), u(b)]$ to evaluate the integral.

1.1.4 Integration by Parts

Recall the chain rule:

$$\frac{d}{dx} f(x)g(x) = f(x)g'(x) + f'(x)g(x)$$
$$f(x)g(x) = \int f(x)g'(x) + f'(x)g(x)dx$$
$$\int f(x)g'(x) = f(x)g(x) - \int f'(x)g(x)dx$$

To find the indefinite integral of a function involving multiplication, use integration by parts to eliminate one part of the integral.

1.2 Definite Integrals

1.2.1 Area Under Curve

Area bounded by x axis, $x = a$, $x = b$ and $f(x)$:

$$\int_a^b |f(x)| dx$$

Area bounded by y axis, $y = a$, $y = b$ and $f(y)$:

$$\int_a^b |f(y)| dy$$

1.2.2 Area of Parametric

Area bounded by x axis, $x = a$, $x = b$ and parametric curve $(x(t), y(t)$:

$$\int_a^b y dx = \int_c^d y(t) \frac{dx}{dt} dt$$

Where $x(c) = a, x(d) = b$

1.2.3 Volume of Rotation

Volume enclosed by curve rotated about x axis:

$$\pi \int_a^b (f(x))^2 dx$$