1 Complex Numbers

1.1 Imaginary Numbers

$$i = sqrt(-1)$$
$$i^2 = -1$$
$$i^4 = i^2 \times i^2 = 1$$

1.2 Cartesian Representation

1.2.1 Complex Numbers

Complex numbers are written in the form:

$$z = a + ib \quad a, b \in \mathbb{R}$$
$$\text{Where} \quad Re(z) = a \quad Im(z) = b$$

And populate the set $\mathbb{C}$

1.2.2 Conjugates

$$\text{For} \quad w = a + ib \quad z = c + id$$
$$w^* = a - ib$$
$$ww^* = a^2 + b^2 = |w|^2$$
$$(w + z)^* = w^* + z^*$$
$$(wz)^* = w^* z^*$$

1.2.3 Algebraic Manipulation

$$\text{For} \quad w = a + ib \quad z = c + id$$
$$w = z \implies a = c, b = d \quad \textbf{IMPT}$$
$$w + z = (a + c) + i(b + d)$$
$$w - z = (a - c) + i(b - d)$$
$$w * z = (ac - bd) + i(ad + bc)$$
$$|w| = sqrt(a^2 + b^2)$$
$$sqrt(w) \quad \text{occurs with a} \pm \text{sign}$$

For division, remove i from denominator by multiplying numerator and denominator by conjugate and then solve:

$$\begin{aligned} \frac{w}{z} &= \frac{w}{z} \times \frac{z^*}{z^*} \\ &= \frac{wz^*}{zz^*} \\ &= \frac{wz^*}{c^2 + d^2} \end{aligned}$$

1.3 Complex Polynomial Roots

1.3.1 Theorem of Algebra

A polynomial of degree n has n real or complex roots

1.3.2 Finding Complex Roots

If a polynomial has all real coefficients, complex roots occur in conjugate pairs. Write as such in exams:

$$f(x) \quad \text{has real coefficients}$$

$$a + ib \quad \text{is a root} \implies a - ib \quad \text{is a root}$$

For a polynomial with complex coefficients, use quadratic general formula. Note that $\pm$ will still be present somewhere

1.4 Polar Representation

1.4.1 Polar Representation

$$z = re^{i\theta} \quad r \in \mathbb{R}_0^+ \quad -\pi < \theta \leq \pi$$

$$|z| = r \quad arg(z) = \theta = tan^{-1}(\frac{a}{b})$$

$$Re(z) = rcos(\theta)$$

$$Im(z) = rsin(\theta)$$

$$re^{i\pi} = -1 \quad re^{i0} = 1$$

$$re^{i\frac{\pi}{2}} = i \quad re^{i\frac{-\pi}{2}} = -i$$

1.4.2 Algebraic Manipulation

$$\text{For} \quad z = r_1 e^{i\theta_1} \quad z = r_2 e^{i\theta_2}$$

$$z_1 z_2 = r_1 r_2 e^{i(\theta_1 + \theta_2)}$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} e^{i(\theta_1 - \theta_2)}$$

$$z_1^n = r_1^n e^{in\theta_1}$$

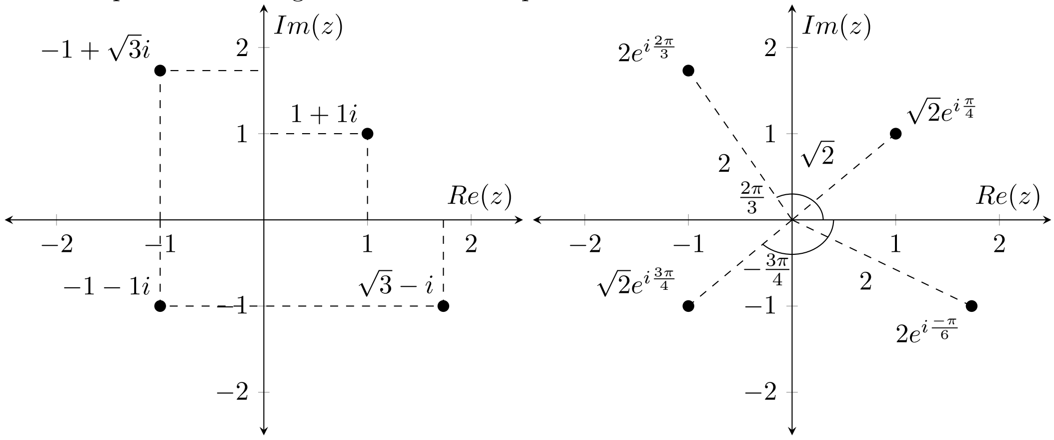
$$\sqrt{z_1} = \sqrt{r_1} e^{i(\frac{\theta_1}{2})}, \sqrt{r_1} e^{i(\frac{\theta_1}{2} + \pi)}$$

1.5 Geometric Representation

1.5.1 Argand Diagrams

Cartesian form describes the x and y coordinate on the real and imaginary axis
Polar form describes the distance between the point and the origin as well as the angle a line

from the point to the origin makes with the positive real axis



1.5.2 Geometric Manipulation

Multiplying by $re^{i\theta}$ scales the number by a factor of r and rotates anticlockwise by an angle of θ about the origin
Multiplying by i rotates a complex number by $\frac{\pi}{2}$, or 90° anticlockwise
The conjugate of a complex number is a reflection of the complex number on the x axis