Physics Notes

Chang Si Yuan

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This compilation of notes are to be used as a reference for the GCE "A"-level Physics paper, as a refresher in definitions, theories as well as for general descriptions of presentation form. These notes are meant for free, public use, but at the reader's own risk.
Good luck with your exams.

1 Measurements

1.1 Units

Physics can be summarized as a collection of mathematical relationships between physical phenomena. Each and every physical quantity has a numerical magnitude and a unit. Note that it is nonsensical to compare a physical quantity to a unit (e.g. time cannot be compared to seconds).

$$\underbrace{F}_{\text{Physical Quantity}} = \underbrace{5}_{\text{Numerical Magnitude}} \underbrace{N}_{\text{Unit}}$$

Definition 1.1: SI Base Units

SI base units are a selection of fundamental physical quantities, from which all other physical quantities can be represented as a combination of SI Base Units. These quantities have been arbitrarily chosen for accessibility and reproducibility.

Definition 1.2: Derived Units

Derived Units are defined as products or quotients of base units and are obtained as products of base units

Base Quantity	Base Unit	Symbol
Time	Second	s
Length	Meter	m
Mass	Kilogram	kg
Current	Ampere	A
Temperature	Kelvin	K
Amount of Substance	Mole	mol

For a mathematical operation to be valid, addition and subtraction between physical quantities have to have the same unit and two sides of an equation must have the same unit.

Definition 1.3: Homogeneous Equations

An equation is homogeneous if both sides of an equation have the same resultant units. Also called Dimensionally Consistent.

The homogeneity of an equation can be used to determine the powers of physical quantities used to derive a value.

1.2 Numerical Magnitudes

Orders of magnitudes of a physical quantity can be used to represent decimal multiples of a number.

Prefix	Symbol	Power of 10
tera	T	12
giga	G	9
mega	M	6
kilo	k	3
deci	d	-1
centi	c	-2
milli	m	-3
micro	μ	-6
nano	n	-9
pico	p	-12

Definition 1.4: Standard Form

Standard form is where the numerical magnitude of a physical quantity is written in the form $a \times 10^n$ where $1 \leq a < 10$ and n is an integer.

Estimation of the order of magnitude of a physical quantity can be derived from estimating component values of a certain order of magnitude and then applying physical equations.

1.3 Error

Error in a reading is where there is uncertainty in the exact value of the numerical magnitude of a physical quantity.

Definition 1.5: Systematic Error

Systematic errors are caused by lapses in the measurement process, resulting in values consistently erroneous to give always smaller or always larger readings and can be eliminated if the source of error is known and accounted for.

Definition 1.6: Random Error

Random errors are caused by inherent inaccuracy and lack of precision in a reading, resulting in values scattered about a mean and can be mitigated by repeating measurements and finding lines of best fit but otherwise cannot be predicted.

Definition 1.7: Accuracy

Accurate readings are values which are close to the true value of a physical quantity and is influenced by systematic error.

Definition 1.8: Precision

Precise readings are values which agree which other and is influenced by random error.

1.3.1 Measuring Values

Precision of a measuring instrument is determined by its least count. Measurements of length and volume are read to their least count, or half their least count if the markings are larger than 1mm such as on a meter rule or a graph. Digital instruments are read and recorded to their displayed value except for tools which depend on other erroneous input such as human reaction time. Do note that the ruler is a special case, where since the error in reading is 0.5mm but two readings are made (one for the starting point of measurement, and one for the ending point, te result is obtained by subtracting starting value from ending value, though starting is usually at the zero mark) the total error is twice that error or 1mm. In questions which specify that the error accompanying each reading is one division, the absolute error is twice the least count.

1.3.2 Error Propagation

Equation 1.1: Error Propagation

For a resultant value Q, two derivative values X and Y and their powers or coefficients a and b

$$Q = aX + bY \quad \Delta Q = |a|\Delta X + |b|\Delta Y$$
$$Q = kX^aY^b \quad \frac{\Delta Q}{Q} = |a|\frac{\Delta X}{X} + |b|\frac{\Delta Y}{Y}$$

Absolute uncertainty is represented to 1 s.f. while fractional and percentage (fractional multipied by 100%) uncertainty is represented to 2 s.f. .

To find the situation where maximum fractional error occurs, adjust the values such that the value of Q is its smallest possible value.

1.3.3 DP and SF

Addition and subtraction operations in experimental situations require the result to follow the largest decimal place value of its derivatives. Multiplication and division operations in experimental situations require the result to follow the least significant figures of its derivatives. However, in exam settings seek to maintain all working in 5sf/dp and only reduce sf/dp when obtaining answers.

1.3.4 Scalars and Vectors

Definition 1.9: Scalar Quantity

A Scalar Quantity is a physical value with a numerical magnitude, and are represented by a magnitude and a unit.

Definition 1.10: Vector Quantity

A Vector Quantity is a physical value with a numerical magnitude as well as a direction, and are represented by a magnitude, unit and a direction.

Before solving questions involving vector quantities, a positive direction should be defined as whichever direction is most convenient.

2 Kinematics

Definition 2.1: Distance

Distance x is the length of a path followed by an object, measured in m.

Definition 2.2: Displacement

Displacement s is the distance moved in a specified direction from a reference point, measured in m. It is the vector equivalent of distance.

Definition 2.3: Speed

Speed v is the instantaneous speed of an object, defined as the rate of change of distance traveled with respect to time, measured in $\mathrm{m\,s^{-1}}$. Average speed refers to the distance traveled over a significantly large time taken.

Definition 2.4: Velocity

Velocity v is the instantaneous velocity of an object, defined as the rate of change of displacement with respect to time, measured in $\mathrm{m\,s^{-1}}$. Average velocity refers to the change in displacement over a significantly large time taken. It is the vector equivalent of speed.

Definition 2.5: Acceleration

Acceleration a is the instantaneous change in velocity of an object, defined as the rate of change of velocity with respect to time, measured in $\mathrm{m\,s^{-1}}$. Average acceleration refers to the change in velocity over a significantly large time taken.

Note that when faced with a kinematics graph (s/v/a against t), the gradient of the graph (differential) and the area under the graph (integral) obtain special meanings.

2.1 Equations of Motion

For a situation involving uniform acceleration and motion in a straight line, the following equations hold:

Final velocity from initial velocity and acceleration

$$v = u + at$$

Displacement from average velocity

$$s = \frac{1}{2}(u + v)t$$

Displacement from initial velocity and acceleration

$$s = ut + \frac{1}{2}at^2$$

Final velocity from displacement,
initial velocity and acceleration

$$v^2 = u^2 + 2as$$

For the condition of objects in freefall, acceleration is equal to which takes the value of 9.81 $\mathrm{m\,s^{-1}}$.For the conditions of objects in projectile motion with the assumption of no air resistance, acceleration in the vertical dimension behaves as if the object is in freefall, and acceleration in the horizontal direction is equal to zero.

2.2 Air Resistance

When objects move through air, it experiences viscous drag or air resistance. Air resistance acts opposite to the direction of velocity and is proportional to the velocity, or at higher velocities is proportional to the square of the velocity. The terminal velocity is the velocity at which the air resistance is equal to accelerative forces on an object, hence the resultant acceleration is equal to zero.

For an object projected upwards in freefall, the time of flight upwards will be smaller than the flight downwards.

On the way up, air resistance acts against upward motion and hence acts downwards in line with gravity, creating a larger resultant force downwards and a larger acceleration which retards its vertical motion, hence the velocity decreases at a faster rate and it takes less time to travel to the peak of the trajectory than if there had been no air resistance.

On the way down, air resistance acts against downward motion and hence acts upward and against gravity, reducing the resultant force downwards and a lower acceleration which accelerates the object downward, hence the velocity increases at a slower rate and it takes more time to travel the same distance downward had there been no air resistance.

3 Dynamics

Definition 3.1: Force

Force F is an action that causes a change in the physical shape or state of a body and is defined as the product of mass and acceleration with units N or $\mathrm{kg\,m\,s^{-1}}$

Multiple forces acting upon a body can be added together in a vector sum to find the resultant force.

3.1 Newton's Laws of Motion

Definition 3.2: Newton's First Law of Motion

Newton's First Law of Motion states that a body continues in its state of rest or motion in a straight line unless acted upon by an external force.

Definition 3.3: Newton's Second Law of Motion

Newton's Second Law of Motion states that the change in momentum of a body is proportional to the resultant force acting on it and occurs in the direction of said resultant force.

Definition 3.4: Newton's Third Law of Motion

Newton's Third Law of Motion states that for a force acting from a first body on a second body, there is an equal and opposite force acting from the second body on the first body.

3.2 Equilibrium

Definition 3.5: Inertia

Inertia is the tendency of a body to maintain its current motion or lack thereof unless acted upon a force.

Definition 3.6: Equilibrium

When a body experiences forces which do not change its state

For a object to be in equilibrium, the resultant force on the object must have zero magnitude and the resultant torque on the object about any axis must also have zero magnitude.

3.3 Momentum

Definition 3.7: Momentum

Momentum is defined as the product of the mass of an object and its velocity with units $\mathrm{kg\,m\,s^{-1}}$.

The total momentum of a system is equivalent to the vector sum of its component objects' momenta. Forces can be simplified into the change of momentum over time.

3.4 Action Reaction Pairs

Definition 3.8: Action Reaction Pairs

Action Reaction Paris are pairs of forces which arise due to Newton's Third Law of Motion which then are of the same type (Normal Contact with Normal Contact, Friction with Friction, Electric with Electric) and act upon different bodies, in addition to properties described in the law which states that the forces are equal in magnitude but opposite in direction.

3.5 Impulse

Definition 3.9: Impulse

Impulse is defined as the product of a force acting on an object and the time which the force is exerted, alternatively the amount of momentum that is transfered, with units $\mathrm{N\,s}$ or $\mathrm{kg\,m\,s^{-1}}$.

Change in momentum can also be measured by finding the area under a Force-Time graph.

3.6 Drawing Forces

Free body diagrams are rudimentary drawings which illustrate the location, direction and magnitude of multiple forces acting upon objects. When drawing diagrams, label with full names of forces unless the short forms are already defined (or define them yourself in a section of the question paper).

Weight W is drawn from the center of mass downward.

Normal Contact Force N is drawn from the point of contact between two bodies. For two contacting surfaces, the force is drawn perpendicular to the surface.

Frictional Force f is drawn on the surface which friction acts on.

Tension T / Compression is drawn along the wire, spring or strut. Tension is drawn inward while compression is drawn outward.

Upthrust U is drawn upwards from the center of mass which is below water level.

Viscous Force $\mathbf{F_v}$ is drawn from the center of the surface furthest from the direction of motion and is opposite to the direction of motion.

Lift L is drawn perpendicular to the axis of wings.

Resultant $\mathbf{F_{net}}$ is drawn disconnected from the body and is drawn with two arrows in the direction of motion.

3.7 Collisions

Extremely quick collisions between objects involve high values of F such that assessment is more feasible by examining changes in momentum rather than forces exerted.

Definition 3.10: Law of Conservation of Momentum

The Law of Conservation of Momentum states that the total momentum of a system remains the same when no external force is applied.

Conservation of Momentum provides the equation:

Equation 3.1: Conservation of Momentum

For mass m and initial and final velocities u and v

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

Definition 3.11: Law of Conservation of Energy

The Law of Conservation of Energy states that energy can neither be created nor destroyed, hence the total energy of a closed system remains the same.

Conservation of Energy provides the equation:

Equation 3.2: Conservation of Kinetic Energy

For mass m and initial and final velocities u and v

$$m_1u_1^2 + m_2u_2^2 = m_1v_1^2 + m_2v_2^2$$

Definition 3.12: Elastic

Elastic collisions maintain the property of conservation of momentum as well as conservation of kinetic energy.

Combining the equations of conservation of momentum and conservation of energy, we obtain this in the case of an elastic collision:

Equation 3.3: Elastic Collision: Constant Relative Speed

For initial and final velocities u and v

$$u_1 - u_2 = v_2 - v_1$$

Definition 3.13: Completely Inelastic

Completely Inelastic collisions maintain the property of conservation of momentum but involve the conversion of kinetic energy to other forms of energy. Particles stick to each other after collision.

4 Forces

4.1 Elastic Force

Definition 4.1: Hooke's Law

Hooke's Law states that the extension of a spring is proportional to the applied force if the limit of proportionality is not exceeded.

Equation 4.1: Elastic Force

For some distance of extension x, some proportionality constant k and some force F, the values are related by the equation

$$F = kx$$

Equation 4.2: Elastic Energy

For some distance of extension x and some proportionality constant k, the energy stored in a spring E is given by the equation

$$E = \frac{1}{2}kx^2$$

4.2 Frictional Force

Definition 4.2: Friction

Friction is a force which exists between two surfaces in contact with each other and resists motion between these two surfaces.

Friction is drawn along the line of contact between two objects. Note that friction between a wheel and a surface is in the direction of motion.

Equation 4.3: Frictional Force

For a given normal contact force N between two surfaces with a frictional constant μ the frictional force f is given by the equation

$$f = \mu N$$

4.3 Upthrust

Definition 4.3: Fluid

A Fluid is a substance which can flow, including most liquids and gases.

Definition 4.4: Density

Density ρ of a substance is its mass per unit volume, with units $\mathrm{kg\,m^{-3}}$

Definition 4.5: Pressure

Pressure p is the force per unit area exerted at right angles to a surface by some object, with units $\mathrm{N\,m^{-1}}$ or $\mathrm{kg\,m^{-1}\,s^{-1}}$

Note that when considering pressure in a liquid at sea level, the pressure due to atmosphere needs to be accounted for.

Equation 4.4: Pressure in Fluid

For the height $h$of a fluid above the level considered, its density ρ and the gravitational acceleration g at a point, pressure p is given by the equation

$$p = h\rho g$$

Definition 4.6: Upthrust

Upthrust is a vertically upward force exerted on a body by a fluid when it is fully or partially submerged in a fluid due to difference in fluid pressure at different heights.

Definition 4.7: Archimedes Principle

The Archimedes Principle states that the upthrust on a submerged object is equal to the weight of liquid displaced by said object.

Questions on upthrust usually involve a calculation of density, mass or volume of liquids or solids in a system. In the case of floating objects, utilize the equation $U = W$ to find the weight or upthrust experienced by an object.

4.4 Viscous Force

Definition 4.8: Viscous Force

Viscous Force is the force experienced by a body moving through a fluid when it receives normal contact force from the particles of the fluid after it imparts momentum onto fluid, written F_v.

The magnitude of the viscous force depends on the shape of the body and viscosity of the fluid, as well as the speed of the body which has a proportional relationship to the force at low velocity and a squared relationship with the force at higher velocities.

F_v is zero when a body is at rest. When a body affected by viscous force experiences a constant force or acceleration, the body speeds up at a decreasing rate as the resultant force is smaller due to the increasing viscous force. Terminal velocity is reached when the viscous force is equal to the applied force, resulting in equilibrium.

Definition 4.9: Terminal Velocity

Terminal Velocity is the speed at which the viscous force experienced by a body prevents further acceleration.

4.5 Calculating Equilibrium

A body in equilibrium must have both translational and rotational equilibrium. As such, it needs to have zero resultant force as well as zero torque about any axis.

Translational equilibrium is obtained when all forces acting on a body are added using vector addition and have zero resultant magnitude. Forces can be resolved into their dimensional components and summed together.

Definition 4.10: Principle of Moments

For any body in rotational equilibrium, the sum of all clockwise moments about an axis is equal to the sum of all anticlockwise moments.

Definition 4.11: Moment

A Moment is a physical value which involves the multiplication of a perpendicular distance from an arbitrary axis with another physical quantity existent at a point.

Definition 4.12: Torque

The Torque of a force about an arbitrary axis is defined as the product of the force and the perpendicular distance from the point to the line of action of the force, with units N m or kg m s^{-2}.

Definition 4.13: Couple

A Couple is a pair of equal and opposite parallel forces whose lines of action do not meet.

Equation 4.5: Torque of A Couple

For the magnitude of one force in the couple F and the perpendicular distance between the two forces d, total torque τ is given by the equation

$$\tau = Fd$$

A couple has the special quality that it has zero resultant force but still has a torque. A couple will continue to rotate until the lines of action of the two forces coincide and have zero perpendicular distance. Multiple forces which all pass through one single point have no net torque.

4.6 Calculating Center of Mass

Definition 4.14: Center of Gravity

The Center of Gravity of an object is the point where gravitational attraction on the body appears to act.

CG is calculated by finding the point where when used as a pivot results in rotational equilibrium. This means that the point is vertically in line with the center of gravity. CG of an irregularly shaped object can be obtained by pivoting the body and drawing a line vertically down from the pivot multiple times, where the point where the lines intersect would be the center of gravity.

5 Work Energy and Power

5.1 Work

Definition 5.1: Work

Work WD is defined as the product of a force and the displacement in the direction of the force, measured in Joules J or kg m s^{-2}

Negative work done is a sign of a dissipative force.

Equation 5.1: Work Done on a System

For a force F, displacement s and angle between Force and Displacement θ, work done W is given by the equation

$$W = Fs\cos(\theta)$$

Equation 5.2: Work Done by a Gas

For a contained gas of changing volume V and external pressure p, work done $W$is given by the equation

$$W = p\Delta V$$

Amount of work done can be measured as the integral of the Force-Distance graph for normal motion, the integral of the Pressure-Volume graph for work done by a gas and the integral of the Force-Extension graph for work done by a spring.

5.2 Energy

Definition 5.2: Energy

Energy is the quantification of an object's capacity to do work,measured in J or kg m s^{-2}

Equation 5.3: Kinetic Energy

For a object of mass m and speed v, the amount of Kinetic Energy E_k contained is given by the equation

$$E_k = \frac{1}{2}mv^2$$

Note that an object whose velocity changes has an energy change of $\frac{1}{2}m(v^2 - u^2)$ rather than $\frac{1}{2}m(v - u)^2$.

Equation 5.4: Gravitational Potential Energy

For a object of mass m, gravitational acceleration g and (relative) height h, the amount of Gravitational Potential Energy E_p contained is given by the equation

$$E_p = mgh$$

Equation 5.5: Elastic Potential Energy

For a spring of proportionality constant k and extension x, the amount of Elastic Potential Energy U_E contained is given by the equation

$$U_E = \frac{1}{2}kx^2$$

Given the Energy-Distance graph of a object experiencing a field force, the gradient of the graph gives the force at a certain distance.

Definition 5.3: Principle of Conservation of Energy

The Principle of Conservation of Energy states that energy cannot be destroyed or created, only converted and transferred.

The sum of all kinetic and potential energy at any point in time is constant, even in the presence of a dissipative force so long as no work is done on a system. Dissipative forces are also lessened in a system with more uniform motion.

Equation 5.6: Efficiency

To obtain the efficiency η of a system, use the equation

$$\eta = \frac{\text{useful energy output}}{\text{total energy input}} \times 100\%$$

Definition 5.4: Power

Power is the quantification of work done with respect to time, defined as the rate which work is done with respect to time or the amount of energy transferred with respect to time, measured in W or kg m s^{-2}

For questions involving mass flow rates, calculate values in terms of their algebraic quantities and then cancel out the time quantity at the end to find the rate.

6 Circular Motion

6.1 Kinematics of Circular Motion

Definition 6.1: Angular Displacement

Angular Displacement θ is the angle an object makes with reference to a line, measured in radians.

Definition 6.2: Radian

A Radian is the angle subtended by an arc of length equal to its radius.

Definition 6.3: Angular Velocity

Angular Velocity ω is defined as the rate of change of angular displacement with respect to time, measured in rad s^{-1}

Definition 6.4: Period

A period of a system T is the time taken for a system to complete one cycle of motion, measured in s

Period, with regard to circular motion, is the time taken for one complete revolution to finish.

Equation 6.1: Linear Velocity

For some angular velocity ω and some radius r, the linear velocity of an object in uniform circular motion is given by the equation

$$v = r\omega$$

6.2 Dynamics of Circular Motion

A object in uniform circular motion orbits an object at a constant radius, linear velocity and angular velocity, but with a changing direction.

Velocity is constantly changing since direction is changing despite linear velocity remaining the same, hence there is a force acting on the body.

Linear velocity is constant, hence the force acting on the body is perpendicular to the direction of motion in order to keep the linear velocity unchanged. As such, no work is done on the force as well.

Equation 6.2: Centripetal Acceleration

For a radius r, linear velocity v and angular velocity ω, the centripetal acceleration a is given by the equation

$$a = r\omega^2 = v\omega = \frac{v^2}{r}$$

Definition 6.5: Centripetal Force

Centripetal Force $\mathrm{F_c}$ is a name given to any force which allows a body to undergo circular motion, typically calculated as the product of centripetal acceleration and the mass of an object, measured in N.

6.3 Special Cases

6.3.1 Racecar on Inclined Track

Horizontal component of normal force of ground on the car provides centripetal force. When the car goes above or below the speed needed to maintain this centripetal force, friction between the car tires and the track contributes additional force towards maintaining circular motion. Alternatively, the car will experience horizontal acceleration in the event it is no longer able to maintain circular motion and "slide" up and down the incline.

6.3.2 Vertical Circular Motion

Apparent weight is given by the normal contact force acting on an object. Since centripetal force is the vector sum of gravity and normal force, at the bottom of a loop where centripetal force acts upwards but gravity acts downwards, the normal contact force is largest so as to act against gravity to obtain the necessary centripetal force, hence apparent weight is highest at the bottom.
Objects falling at the top of circular motion / objects (not) in contact with other objects can be explained by saying that "contact" is caused by having a normal force between two objects, and at the top of a loop and at low linear velocity the weight of an object is higher or lower than its necessary centripetal force, hence there is (no) normal force between object and another object.

Alternatively, argue that objects in circular motion tend to move tangentially to the path of circular motion and at sufficient speeds press against other objects rather than fall due to gravity.

6.3.3 Vertical Circular Motion with Uniform Speed

Note that due to the presence of gravitational acceleration, any additional force required to act upon an object to keep it in circular motion changes at different stages of the rotation in order to keep total centripetal acceleration constant.

Also note that work is done in the process of maintaining uniform speed as $\mathrm{E_k}$ is constant but $\mathrm{E_p}$ changes.

6.3.4 Vertical Circular Motion with Non-uniform Speed

In order to maintain circular motion throughout a vertical loop without any external work being done, the object must have enough energy at the bottom of the loop such that it can reach the top of the loop and still have the necessary linear velocity given the centripetal acceleration provided by gravity. Velocity at the bottom is typically in the form $v_{\text{bottom}} = \sqrt{5gr}$ while velocity at the top is in the form $v_{\text{top}} = \sqrt{gr}$, but its derivation is required to be shown.

7 Gravitation

Definition 7.1: Gravitation

Gravitation is defined as the attractive force between to masses.

Equation 7.1: Gravitational Equations

For the mass of two planetary objects m_1 and m_2, the radius between them r, and the gravitational constant G, the force experienced by each of the planets F, the strength of gravitational field or acceleration due to gravity g, the potential energy of a each planet U and the potential gradient of a planet ϕ are given by the equations

$$F = G\frac{m_1 m_2}{r^2}$$
$$g = G\frac{m_1}{r^2}$$
$$U = -G\frac{m_1 m_2}{r}$$
$$\phi = -G\frac{m_1}{r}$$

7.1 Gravitational Force

Definition 7.2: Newton's Law of Gravitation

Newton's Law of Gravitation states that every point mass attracts every single other point mass along a line intersecting both points, which is proportional to the product of the two masses and inversely proportional to the square of the distance between the two masses.

Gravitational force is a field force, which does not require contact between two objects to have effect. Gravitational force is also a case of an inverse-square law.

Gravitational Force is the basis which other gravitational quantities are calculated on.

7.2 Gravitational Field

Gravitational Fields are drawn as a set of field lines which demonstrate the direction and magnitude of acceleration at a certain point. Lines are drawn pointing towards masses and more dense field lines indicate stronger fields.

Definition 7.3: Gravitational Field Strength

The Gravitational Field Strength $g$at a point is the gravitational force experienced per unit mass at a point, with units $\mathrm{m\,s^{-1}}$.

Gravitational field varies in the case of hollow or solid masses as well as whether the object is inside or outside the mass. Hollow masses have zero gravitational field inside a mass. Solid masses have fields which vary linearly inside of them and fields which follow inverse square law outside of them.

When calculating gravitational field strength which involves some element of circular motion (planetary rotation, object in orbit), do consider that observed acceleration is a composite of acceleration due to gravity and centripetal acceleration.

7.3 Gravitational Potential

Definition 7.4: Gravitational Potential Energy

The Gravitational Potential Energy U of a mass at a point is the work done by an external force in bringing a test mass from infinity to that point, with units J.

Negative total energy indicates that a mass is bounded to the gravitational field of said mass.

Definition 7.5: Gravitational Potential

Gravitational Potential ϕ at a point is the work done per unit mass by a external force in bringing a test mass from infinity to a point, with units $\mathrm{J\,kg^{-1}}$.

Gravitational Potential (Energy) is negative because the direction of displacement is opposite that of the external force.

7.4 Escape Velocity

The escape velocity of a body is the velocity required such that said body is able to travel to infinity away from a planetary mass. Escape velocity is independent of the mass of the escaping object. Escape from Earth requires a speed of about $11.2 \times 10^3\,\mathrm{m\,s^{-1}}$.

Equation 7.2: Escape Velocity

For gravitational acceleration g and radius from the original point R, the escape velocity v is given by the equation

$$v = \sqrt{2gR}$$

7.5 Planetary Orbit

Definition 7.6: Kepler's Third Law

Kepler's Third Law states that the square of the period of a planetary orbit is proportional to the cube of its radius. Mathematically, it is presented as

$$T^2 \propto R^3$$

Given the radius and period of one planet's orbit, you can then infer mathematically the period or radius of another planet when given one of the two variables.

7.5.1 Geostationary Orbit

A geostationary satellite maintains the same position relative to a point on a planet's surface.

- Orbital period is same as the planet's rotation period (24 hours for Earth)
- Plane of orbit is the same as the planet's equator
- Direction of orbit is same as the planet's direction of rotation (eastward for Earth)

Geostationary orbits allow for uninterrupted surveillance of one point on a planet and is easier to communicate with, and has a high field of view due to its large orbital radius. However, geostationary orbits face a significant loss in signal strength due to the large radius, as well as poorer quality in terms of imaging satellites as well as significant latency in signal.

8 Thermodynamics

Definition 8.1: Internal Kinetic Energy

Internal Kinetic Energy $\mathrm{E_K}$ is the amount of energy stored in an object's molecule's translational and rotational energy.

Definition 8.2: Temperature

Temperature is a quantization of the internal kinetic energy of an object.

Equation 8.1: Thermodynamic Temperature

For an object with temperature T_C in °C, its thermodynamic temperature $T_K$in K is given by the equation:

$$T_K = T_C + 273.15$$

Definition 8.3: Kelvin

The Kelvin K is a measurement of temperature. 1 K is $\frac{1}{273.16}$ of the temperature difference between absolute zero and the triple point of water (0.01 °C).

Definition 8.4: Heat

Heat is the flow of internal kinetic energy due to the difference in temperature between them.

Definition 8.5: Thermal Equilibrium

Objects that are in Thermal Equilibrium have no net flow of heat, and occurs if and only if these objects have the same temperature.

8.1 Laws of Thermodynamics

Definition 8.6: 0th Law of Thermodynamics

If an A and B are in thermal equilibrium and B and C are in thermal equilibrium, A and C are in thermal equilibrium.

Definition 8.7: 1st Law of Thermodynamics

The 1 1st Law of Thermodynamics states that the total energy in a closed system, including internal kinetic energy, is constant.

Definition 8.8: 2nd Law of Thermodynamics

The 2nd Law of Thermodynamics states that the entropy of a closed system increases.

Definition 8.9: 3rd Law of Thermodynamics

The 3rd Law of Thermodynamics states that no object can achieve 0 K.

8.2 Ideal Gas Law

Definition 8.10: Ideal Gas

An Ideal Gas satisifes the relationship of $pV = nRT$.

Equation 8.2: Ideal Gas Law

For an ideal gas with pressure p, volume V, amount of gas in mol n and thermodynamic temperature T

$$pV = nRT$$

8.3 Internal Kinetic Energy of a Gas

Equation 8.3: Internal Kinetic Energy

The average Internal Kinetic Energy $\langle E_K \rangle$ of a gas molecule with mass m and mean square speed c or thermodynamic temperature T using the Boltzmann constant k is given by the equation:

$$\langle E_K \rangle = \frac{1}{3} m \left\langle c^2 \right\rangle = \frac{3}{2} kT$$

The derivation of the expression for $\langle E_K \rangle$ is as follows:

1. Pressure in a container arises when gas molecules collide against a container.
2. Model the collision of gas molecules with container as a elastic collision, where a gas molecule has initial momentum mc. Due to conservation of kinetic energy, the final momentum of the molecule is $-mc$. The total change in momentum is $\Delta p = -2mc$.
3. Model the container as a cube of side length 2d. A collision occurs every time a molecule moves twice distance between two walls of the container, therefore it occurs once every $\Delta t = \frac{2d}{c}$.
4. Since force is defined as change of momentum per unit time, the force exerted of wall on the molecule is $F = \frac{\Delta p}{\Delta t} = -\frac{2mc^2}{2d} = -\frac{mc^2}{d}$.
5. By Newton's third law, the force of molecule on the wall is $F = \frac{mc^2}{d}$.
6. Considering the total force on the wall as a summation of the forces by each molecule, $F = \sum \frac{mc^2}{d} = \frac{Nm\langle c^2 \rangle}{d}$
7. Since pressure is force per unit area and the "unit area" is modeled as a square side of a container, $P = \frac{F}{d} = \frac{Nm\langle c^2 \rangle}{d^3} = \frac{Nm\langle c^2 \rangle}{V}$
8. Accounting for the fact that each of the three dimensions were arbitrarily chosen, pressure has to be divided by 3. $P = \frac{1}{3}\frac{Nm\langle c^2 \rangle}{V}$
9. Rearranging,

$$\frac{1}{3} Nm \left\langle c^2 \right\rangle = pV = NkT$$
$$m \left\langle c^2 \right\rangle = 3kT$$
$$\langle E_K \rangle = \frac{1}{2} m \left\langle c^2 \right\rangle = \frac{3}{2} kT$$

9 Oscillations

9.1 Simple Harmonic Motion

Definition 9.1: Simple Harmonic Motion

Simple Harmonic Motion (SHM) occurs when a body oscillates about a point where its acceleration is proportional to its displacement from said point and directed towards said point.

9.2 SHM Equations

Equation 9.1: SHM Equation

For

$$\begin{aligned} a &= -kx \\ x &= x_o \cos \omega t \\ v &= \frac{dx}{dt} = -x_o \omega \sin \omega t \\ a &= \frac{dv}{dt} = -x_o \omega^2 \cos \omega t \\ &= -\omega^2 x \\ k &= \omega^2 \end{aligned}$$

9.2.1 Graphs of SHM

Bodies undergoing SHM have sinusoidal graphs of displacement, velocity and acceleration due to the trigonometric functions in their equations. Graphs of energy of SHM hold the property of conservation of energy, where the sum of potential energy and kinetic energy, usually in the form of square trigonometric function graphs, always add up to a constant.

9.3 Cases of SHM

9.3.1 Horizontal Oscillations

Horizontal Spring Systems involve a mass connected to a spring. The spring creates a horizontal restoring force.

Pendulum Systems involves a mass connected to a inextensible string. The net force of gravity and tension in the string creates a horizontal restoring force which is assumed to be proportional to horizontal displacement at small angles $\theta < 6°$.

9.3.2 Vertical Oscillations

Vertical Spring Systems involve a mass connected to a vertical spring. The net force of the spring and gravity provides the restoring force which acts on the oscillating mass. The center of oscillations is assumed to be the same position as when the mass is in stationary equilibrium and any SHM acts around this position. Energy is also conserved, where the summation of spring potential and gravitational potential stores energy to be converted into kinetic energy.

9.4 Damping

Definition 9.2: Damping

Damping occurs when there is an external force acting on the object undergoing SHM, usually proportional to velocity (such as air resistance).

Definition 9.3: Light Damping

Light Damping occurs when a damping force causes the amplitude of SHM to decrease exponentially over time.

Definition 9.4: Critical Damping

Critical Damping occurs when a damping force causes a body undergoing SHM to return to equilibrium position and stop oscillating within the shortest possible time.

Definition 9.5: Heavy Damping

Heavy Damping occurs when a damping force causes a body undergoing SHM to return to equilibrium position and stop oscillating over a period of time longer than if the system were critically damped.

9.4.1 Forced Oscillations

Definition 9.6: Forced Oscillations

Forced Oscillations arise when a system has to receive external force in order to undergo SHM.

9.5 Resonance

Definition 9.7: Resonance

Resonance occurs when a system responds to a driving force with a maximum amplitude. This implies the maximal transfer of energy between driving and driven systems, therefore implying that the driving frequency is equal to the natural frequency of the driven system.

Definition 9.8: Natural Frequency

The Natural Frequency, also known as the Resonant Frequency or Resonance Frequency, is the frequency at which a system is able to receive energy at a maximum.

RI's tutorial solutions have used the terms "Resonance Frequency", "System at Resonance" and "Resonance Occurs". Use these if phrasing is vague and if you worry about presentation / phrasing marks.

10 Wave Motion

Definition 10.1: Wave

Waves are displacements in a system where oscillations in a medium are propagated and therefore energy is propagated without physically relocating the medium.

10.1 Transverse and Longitudinal Waves

Definition 10.2: Transverse Wave

Transverse Waves involve particles oscillating in a direction perpendicular to the direction of energy transfer.

Definition 10.3: Longitudinal Wave

Longitudinal Waves involve particles oscillating in a direction parallel to the direction of motion of energy transfer.

10.2 Intensity of Waves

Definition 10.4: Intensity

Intensity is the quantization of how much energy is transferred to a surface area by a wave per unit time, measured in $\mathrm{W\,m^{-2}}$

Equation 10.1: Intensity

For power P spread across surface area S, or a proportionality constant k, frequency f and amplitude of wave A, intensity I is defined as

$$I = \frac{P}{S} = kf^2A^2$$

As wave energy dissipates across a larger surface area and its total power remains the same, its intensity decreases. Cylindrically radiating waves have $S = 2\pi r$ and follow a inverse rule while spherically radiating waves have $S = 4\pi r^2$ and follow a inverse square rule.

10.3 Polarization

Equation 10.2: Polarization

For a angle θ between the original and new planes of polarized light, the amplitude of wave A and intensity of wave I are changed as:

$$A\prime = A\cos\theta$$
$$I\prime = I\cos^2\theta$$

11 Superposition

11.1 Principle of Superposition

Definition 11.1: Principle of Superposition

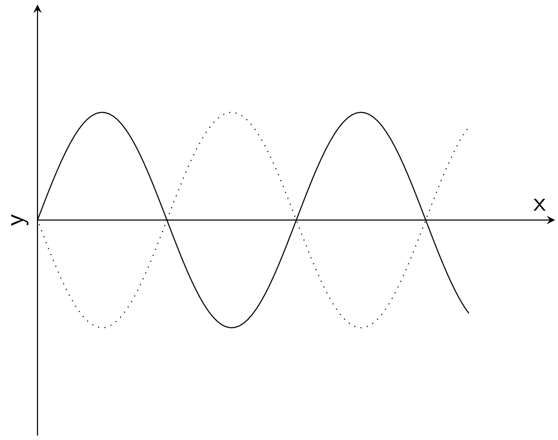
The Principle of Superposition states that the effect of two stimuli at a point is the sum of the two responses.

11.2 Stationary Waves

Definition 11.2: Stationary Waves

Stationary Waves result from the superposition of two progressive waves of same frequency, amplitude and speed traveling along the same line but in opposite directions.

The resultant stationary wave has zero net energy transfer, and is drawn as the envelope of the displaced particles, with one waveform drawn with a solid line and one waveform with a dotted line.



Definition 11.3: Node

The Node is a location in a stationary wave where the particles no longer oscillate, typically when two waves reach the point π radians out of phase.

Definition 11.4: Antinode

The Antinode is a location in a stationary wave where the particles oscillate with the maximum displacement, typically when two waves reach the point in phase.

Depending on the environment in which a stationary wave is produced determines the distribution of nodes and antinodes. Since the distribution of nodes and antinodes determine the wavelength of a stationary wave and the speed of the wave in the propagation medium is typically constant, the frequencies at which a stationary wave is produced is now determined.

Definition 11.5: Hard Boundary

A Hard Boundary is able to supply a restoring force which then causes a reflected wave to undergo a π radian phase shift (due to conservation of energy). A Hard Boundary in a stationary wave creates a node.

Definition 11.6: Soft Boundary

A Soft Boundary is unable to supply a restoring force and hence causes a wave to reflect with the same polarity back. A Soft Boundar in a stationary wave creates an antinode.

Definition 11.7: Fundamental Frequency

The Fundamental Frequency of a stationary wave is the lowest frequency at which a stationary wave can be produced.

Definition 11.8: Harmonic

The n^{th} Harmonic of a wave is the frequency where a stationary wave can be produced where $n = \frac{f_{\text{hamonic}}}{f_{\text{fundamental}}}$

Definition 11.9: Overtone

The n^{th} Overtone of a wave is the nth next highest frequency than the fundamental frequency that a stationary wave can be produced.

To produce a stationary wave, the relationships between need to be maintained.

Equation 11.1: Frequency for two-node or two-antinode Systems

Where v is the speed of wave in a system, L is the length of system and n is the overtone number, the fundamental frequency for stationary wave f is

$$f = \frac{(n+1)v}{2L}$$

Equation 11.2: Frequency for one-node one-antinode Systems

Where v is the speed of wave in a system, L is the length of system and n is the overtone number, the fundamental frequency for stationary wave f is

$$f = \frac{(2n+1)v}{2L}$$

11.2.1 Stationary Waves in String

Ends of string fixed to a stationary point behave like hard boundaries and create nodes. Ends of string which can oscillate perpendicularly to the direction of the wave (by means of a guide rail or some similar mechanism) behave like soft surfaces and create antinodes.

The speed of the wave in string v depends on the material of the string and is proportional to the tension on the string.

11.2.2 Stationary Waves in Pipes

Definition 11.10: End Correction

The End Correction e of a stationary wave in a pipe is an extra effective length added to open ends, which arises because a antinode occurs slightly outside the end of a pipe.

Closed ends of pipes behave like hard boundaries and create nodes. Open ends of pipes behave like soft surfaces because pressure inside a pipe is partially reflected when it comes into contact with an external system, creating antinodes which are one end correction away from the edge of the pipe. Pipes with two open ends have to account for two end corrections.
The speed of the wave in a pipe v depends on the speed of wave in the medium inside the pipe, such as the speed of sound in air $334\,\mathrm{m\,s^{-1}}$ for pipes in air.

11.2.3 Diffraction

Definition 11.11: Diffraction

Diffraction is where waves bend when passed through an aperture of comparable length to its wavelength or when passing around an obstacle.

For the case of an aperture (also known as Fraunhofer diffraction), waves radiate in a radial fashion.

Definition 11.12: Huygen's Diffraction

Diffraction occurs as points on a wavefront can be treated as secondary sources of wavelets, where the envelope of the secondary wavelets form the next wavefront of the original wave.

11.3 Interference

Definition 11.13: Coherence

Two waves or sources are coherent if they have a constant phase difference, implying equal frequency, speed and wavelength.

Definition 11.14: Interference

Interference between two waves occurs when both reach a point in space, where their resultant effect is obtained through the principle of superposition, hence the effective displacement is the vector sum of the displacement due to each waves.

Definition 11.15: Constructive Interference

Destructive Interference occurs when two coherent waves reach a point in phase to form a resultant maximum displacement.

Definition 11.16: Destructive Interference

Destructive Interference occurs when two coherent waves reach a point with a phase difference of π radians to form a resultant minimum displacement.

Definition 11.17: Fringe

A Fringe is a location at which constructive (bright fringe) or destructive (dark fringe) interference can be observed.

11.3.1 Double Source Interference

Definition 11.18: Path Difference

The Path Difference is the difference in the distance that each wave travels fro its source to the point where two waves meet.

Definition 11.19: Order

The Order of a fringe is the rounded-up absolute path difference divided by wavelength between two waves which cause the formation of a fringe.

Two coherent sources of radial waves produce a interference pattern of dark and bright fringes on a plane far away from the two sources.

In the case of a double-slit experiment where the distance between source and observation screen is sufficiently large and the angle between 0^{th} order maximum and other fringes are small enough to use small angle approximation, the distance between two successive bright or dark fringes is assumed to be constant.

Equation 11.3: Two-Slit Experiment Fringe Septation

For some wavelength λ, distance between source and screen D and separation of sources a, the separation between two fringes x is

$$x = \frac{\lambda D}{a}$$

11.3.2 Diffraction Grating

Definition 11.20: Diffraction Grating

A Diffraction Grating is a sheet of material with multiple apertures on its surface. In optics, typical diffraction gratings have around 100 to 1000 apertures per mm.

Light passed through a Diffraction Grating creates sharp bright fringes.

Equation 11.4: Condition for Constructive Interference

For some slit separation d, wavelength λ, non-zero integer n and angular displacement θ_n, bright fringes form where the following equation is satisfied

$$d \sin\theta_n = n\lambda, \quad n < \frac{d}{\lambda}$$

11.3.3 Single Source Interference

A single source of waves when observed on a screen forms a unique interference pattern. The center of the pattern is a diffuse section due to diffraction. However, when considering the aperature source as multiple coherent sources of waves, there arises a difference in path difference which then causes minima to form.

Equation 11.5: Condition for Single-Source Minima

For some opening of length b, wavelength λ and angular displacement from source θ, single-source minima form where

$$\sin\theta = \frac{\lambda}{b}$$

11.4 Rayleigh Condition

Definition 11.21: Resolution of Images

Two images are clearly resolved when they are distinguishable from each other.

Equation 11.6: Rayleigh Condition

For a image to just be resolved when observed through an aperture, angular separation (crossing both sides of the medial line) θ, wavelength λ and size of aperture b

$$\sin\theta = \frac{\lambda}{b}$$
$$\theta \approx \frac{\lambda}{b}$$

For a circular aperture, $\theta \approx 1.22\frac{\lambda}{b}$.

The Rayleigh Condition was arbitrarily set such that the peak intensity from one image intersects with the minimum of another image. Should the images be any closer the combination of the two graphs will form one single peak (and hence the images are indistinguishable) and should they be any further away the peaks will be more resolved.