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Modelling condition monitoring intervals: A hybrid of simulation and analytical approaches

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This paper reports on a study of modelling condition monitoring intervals. The model is formulated based upon two important concepts. One is the failure delay time concept, which is used to divide the failure process of the item into two periods, namely a normal working period followed by a failure delay time period from a defect being first identified to the actual failure. The other is the conditional residual time concept, which assumes that the residual time also depends on the history condition information obtained. Stochastic filtering theory is used to predict the residual time distribution given all monitored information obtained to date over the failure delay time period. The solution procedure is carried out in two stages. We first propose a static model that is used to determine a fixed condition monitoring interval over the item life. Once the monitored information indicates a possible abnormality of the item concerned, that is the start of the failure delay time, a dynamic approach is employed to determine the next monitoring time at the current monitoring point given that the item is not scheduled for a preventive replacement before that time. This implies that the dynamic model overrides the static model over the failure delay time since more frequent monitoring might be needed to keep the item in close attention before an appropriate replacement is made prior to failure. Two key problems are addressed in the paper. The first is which criterion function we should use in determining the monitoring check interval, and the second is the optimization process for both models, which can be solved neither analytically nor numerically since they depend on two unknown quantities, namely, the available condition information and a decision of the time to replace the item over the failure delay time. For the first problem, we propose five appealingly good criterion functions, and test them using simulations to see which one performs best. The second problem was solved using a hybrid of simulation and analytical solution procedures. We finally present a numerical example to demonstrate the modelling methodology.

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Introduction

Condition monitoring is growing in popularity in industry, with considerable sums now being spent on condition monitoring hardware and software. Today, there are a large and growing variety of forms of condition monitoring techniques for machine condition monitoring and fault diagnosis. However, irrespective of the particular condition monitoring techniques used, the working principle of condition monitoring is the same, namely, condition data become available which need to be interpreted and appropriate decisions made accordingly. Among those decisions made in relation to condition monitoring, one such decision is to choose a condition monitoring interval. Obviously, more frequent monitoring checks cost money, particularly for those firms using external specialist condition monitoring services. On the other hand, a longer monitoring interval may save the monitoring cost, but may also increase the risk of a failure between the monitoring checks. Even for the case of in-house monitoring, there is still a cost of monitoring since the firm has to employ someone and use some staff time to do the condition monitoring. There is clearly an optimization problem here to balance the trade-off between more and fewer monitoring checks. It is noted however that the set-up of such a monitoring interval in industry is still largely governed by rule of thumb.

The literature on condition monitoring is extensive, which is evidenced by the proceedings of COMODEM.[1] It is noted however that very few studied the optimal determination of condition monitoring intervals. Christer and Wang[2] reported a model using binary information about wear to determine the monitoring interval. Wang[3] used random coefficient models to model the optimal wear checking interval. Both papers assumed that wear is observable by inspection. There is a rich source of literature on modelling inspection intervals using the delay time concept.[4–8] It is noted however that a common assumption used in these papers is that upon an inspection only binary information is available, which is not the case discussed here, where detailed condition information is obtained. There are also papers focused on specific-condition-based maintenance models, which can be

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developed further to model the condition intervals. Al-Najjar[9] reported the use of vibration signature to predict the remaining life of ball bearings in paper mills based upon the enveloped alarm level analysis. In his paper, the remaining life of a bearing is defined as the time when the vibration signal reaches a pre-set alarm level so that the prediction of the remaining life is achieved by the prediction of the future vibration signal. It is noted, however, that the set-up of such an alarm level itself is an open question that should be considered as a decision variable.[3] Other authors have considered proportional hazard modelling for modelling condition monitoring processes.[10,11] A Markovian model was developed by Gong and Tang[12] for monitoring machine operation using on-line sensors. However, both proportional hazard and Markovian models assume that the condition of the item monitored depends upon only the current condition monitoring reading. This is an approximation that lacks practical justification.

In this paper, we use the delay time concept to classify the deterioration process of a monitored item into a two-period process with the first period of a normal working condition up to u, and then a defect may present and lead to a failure at time $u+h$ if no maintenance intervention is involved. The second period, h, is called the failure delay time period.[4] The interest in condition-based maintenance is in the second period, that is, what to do based upon the condition information obtained, which has indicated something abnormal. Unlike the model of Christer and Waller,[4] where only binary condition information is provided, in our case here full condition monitoring information obtained is taken into account. We first propose a static model which is used to determine a fixed condition monitoring interval over the item life. Once the monitored information indicates a possible abnormality of the item concerned, that is, the start of the failure delay time, a dynamic approach is employed to determine the next monitoring time at the current monitoring point given that the item is not scheduled for a preventive replacement before that time. This implied that the dynamic model overrides the static model over the failure delay time since more frequent monitoring might be needed to keep the item in close attention before an appropriate replacement is made prior to failure. Both models were developed on the basis of the predicted residual life distribution of an item monitored over the failure delay time period conditional on available condition information to date. This condition information is assumed to be a continuous or discrete measure related to the true condition of the item monitored over time. The prediction is based upon the filtering theory using a recursive Bayesian updating mechanism.[13,14] Two key problems are addressed in the paper. The first is the criterion function that we should use in determining the monitoring interval for both the static and dynamic models. The second is the optimization process for both models given a criterion function. For the first problem we proposed five conventionally used criterion functions, and tested them using simulations. The second problem is solved by a combined procedure of simulation and numerical analysis. We also present a numerical example to demonstrate the modelling methodology.

A two-period failure process with additional condition information

First, we propose the following modelling assumptions.

(1) There are two periods in the plant life: the first period is the time length from new to the point when the item was first identified to be faulty, and the second period is the time interval from this point to failure if no maintenance intervention is carried out. The second period is often called the failure delay time.
(2) A threshold level is established to classify the item monitored to be in a potential faulty state if the condition information signal is above this level. Such a threshold level is usually determined by engineering experience or by a statistical analysis of past condition information.
(3) Plant items are monitored regularly at discrete time points of which a fixed interval is applied to the normal working period and a dynamic interval adopted over the failure delay time period. This dynamic interval shall be updated once new information becomes available at each checking point over the failure delay time period.
(4) The condition information obtained during the failure delay time is a random variable that follows a distribution function characterized by the current residual time among other parameters.

Assumptions (1)–(3) can often be observed in condition monitoring practice. Assumption (4) is first proposed in Wang and Christer,[13] which states that the rapid increase in observed condition information is partly due to the shortened residual time because of the fault. Generally speaking, there is a negative correlation between the observed condition information and the underlying residual time. However, this relationship is typically stochastic. Assumption (4) is the fundamental principle underpinning our model. For a detailed discussion on assumption (4), see Wang and Christer[13] and Wang[14].

The notation used in this paper is defined as follows:

t_i denotes the ith and the current monitoring time since the item was first suspected to be faulty but still operating (note that $i=1$ is the moment when the item was first identified to be faulty);

x_i denotes the residual time at time t_i;

y_i denotes the condition information obtained at time t_i;

$Y_i=\{y_i, y_{i-1}, \ldots, y_1\}$ denotes the history of cumulative condition monitoring information obtained up to t_i, the current monitoring decision point.

A typical condition monitoring process is depicted in Figure 1.

It is clear from Figure 1 that the condition information obtained before t_1 is not used in predicting the residual time since no such negative correlation between x_i and y_i exists. It is noted, however, that the time to t_1 is one of the important information sources to be used in determining the condition monitoring interval. This point will be addressed later.

Let $p_i\ (x_i|\,Y_i)$ denote the *pdf* of x_i conditional upon Y_i at time t_i and let $p(y_i|x_i)$ denote the *pdf* of y_i conditional upon x_i. Since the residual time at t_i is the residual time at t_{i-1} minus the interval between t_i and t_{i-1} provided the item has survived to t_i, and no maintenance action has been taken to alter the residual time, it follows that

$$x_i = \begin{cases} x_{i-1} - (t_i - t_{i-1}) & \text{if } x_{i-1} > t_i - t_{i-1} \\ \textit{not}\ \text{defined} & \text{else} \end{cases} \qquad (1)$$

The relationship between y_i and x_i is yet to be identified. From assumption (4), we know that it can be described by a distribution, say, $p(y_i|x_i)$. We will discuss this later when fitting the model to the data.

We wish to establish the expression of $p_i(x_i|\,Y_i)$ which is one of the key elements in subsequent decision modelling. It can be shown that[13] $p_i(x_i|\,Y_i)$ is given by

$$p_i(x_i|\,Y_i) = \frac{p(y_i|x_i)p_{i-1}(x_i + t_i - t_{i-1}|\,Y_{i-1})}{\int_0^{\infty} p(y_i|x_i)p_{i-1}(x_i + t_i - t_{i-1}|\,Y_{i-1})\mathrm{d}x_i} \qquad (2)$$

It can be shown that if $p(x_0|y_0) = p(x_0)$ and $p(y_i|x_i)$ are known, Equation (2) may be determined recursively.[13,14]

A static model using simulation and analytic analysis

Given the availability of a residual time model, Equation (2), we seek to determine a fixed optimal condition monitoring interval over the normal life period, and then proceed to determine a dynamic interval over the failure delay time period. If the item monitored will be replaced once it has been first identified to be defective, then delay time inspection modelling can be readily employed to recommend an optimal inspection/monitoring interval.[15] It is noted however in our case that the item monitored may not be replaced at the first moment it is identified to be abnormal depending on the severity of the suspected defect. This minor relaxation from the delay time modelling inspection assumption complicates our current modelling process since two more decisions enter into the decision process simultaneously, such as what is the time to the next check, and whether a preventive replacement should be made within the interval. It can be shown later that both decisions of the time to the next check and replacement involve the use of Equation (2) that cannot be evaluated if Y_i is unknown and cannot be described by a distribution function. In this case, we relate our solution procedure to a combination of simulation and analytical analysis. Figure 2 shows the flow chart of the solution process.

Given randomly generated times u and h from specified distributions for u and h, we generate observed condition monitoring data at each monitoring check point using $p(y_i|x_i)$ since x_i is known and equal to $u+h-t_i$; then we use an analytical criterion function to determine whether a replacement is required before the next monitoring point. This process continues until a failure or preventive replacement occurs. The process in Figure 2 is repeated for different intervals t and the best interval t that minimizes the total cost per unit time can be selected. Note that in Figure 2 the interval over the failure delay time is also fixed. It will be re-determined through our dynamic model as discussed later. It can be seen from Figure 2 that the process depends on the criterion function for the replacement decision, which we shall discuss in the next section.

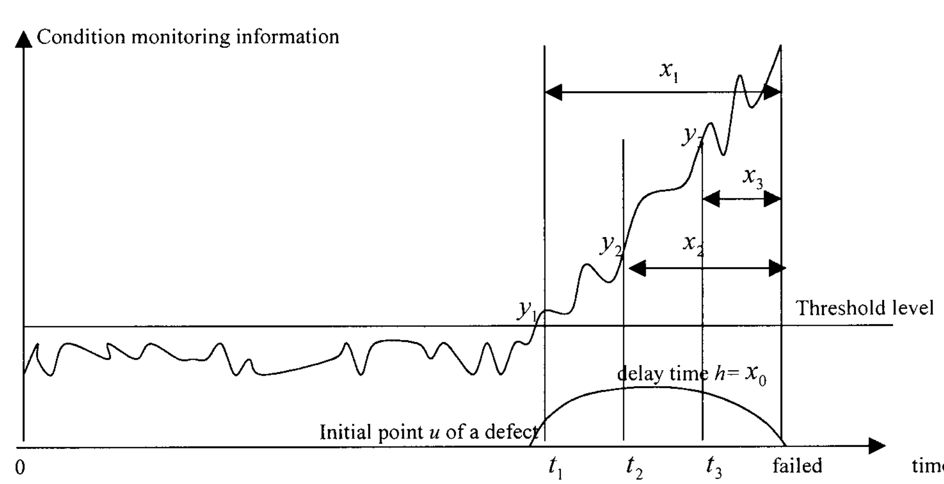

Figure 1 Condition monitoring practice.

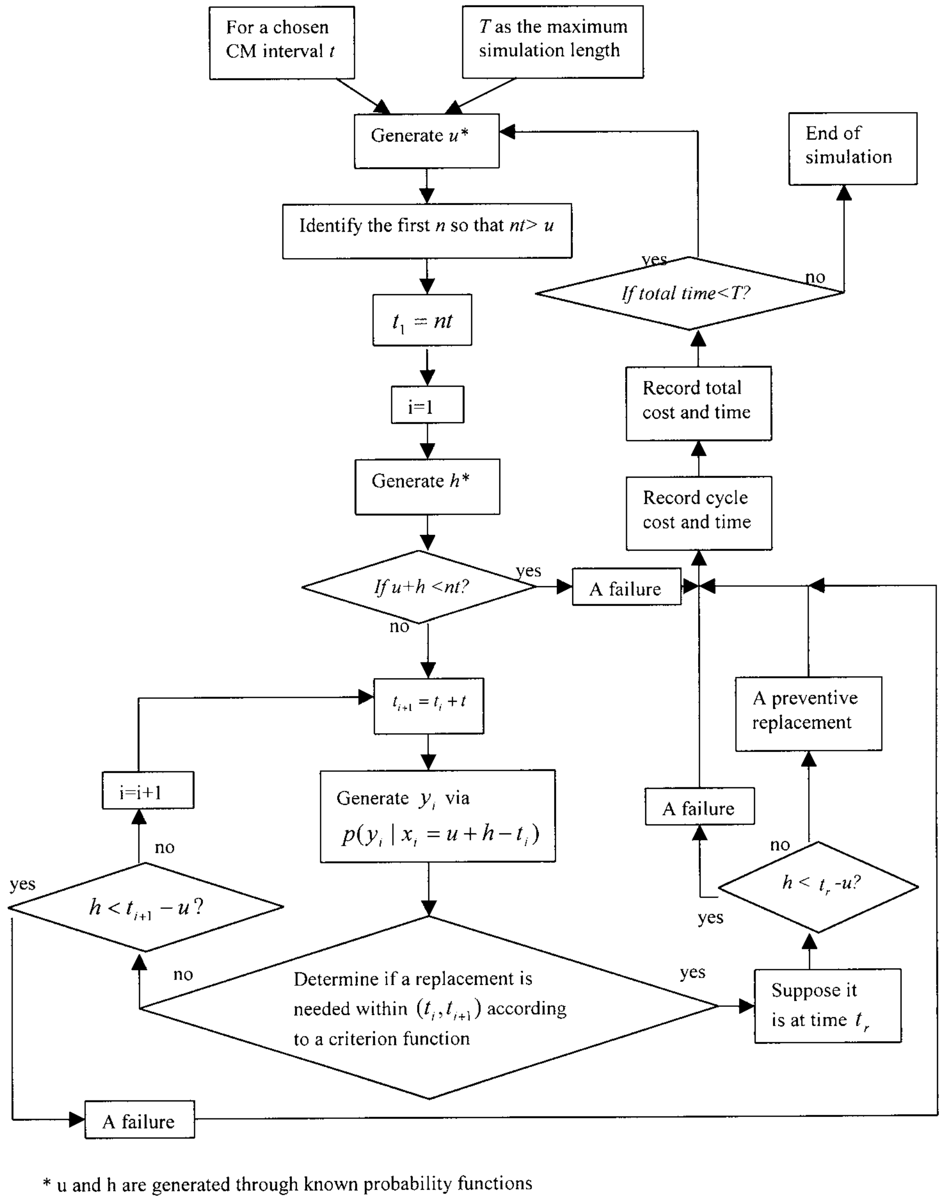

Figure 2 Solution process of the determination of a fixed condition monitoring interval.

Criterion functions

For a simplified condition monitoring process, there are two associated decisions to make, namely the time to the next monitoring check and the time to the next planned replacement if any. For both decisions, we need to choose an appropriate criterion function that will be minimized in terms of the decision variables. The traditional criterion function is the well-known renewal reward equation of the expected utility per unit time if the assumption of an independent and identical distribution (iid) of the time to failure is satisfied.[16] Note that in our case, at each checking point the residual life distribution (it is equivalent to the life distribution) depends on past condition monitoring history, which differs from item to item, and therefore the residual life distributions are not identical, although they may be independent. This implies that the use of the renewal reward equation may not guarantee an optimal solution. It is also noted that because of the availability of condition monitoring information, there is no point looking further than the next monitoring point since the residual life distribution will be updated at that point, and so will the associated decision. This means that the decision should be made over a finite time horizon from now to the next monitoring check which itself is a decision variable. For modelling purposes we only discuss the criterion function used in condition-based replacement decision-making since it will be in the same format for the model determining the optimal monitoring interval. We will discuss this point later. For simplicity, we also assume that the criterion function is cost related, say, to minimize a cost measure over a fixed horizon from t_i to t_{i+1} given a course of action. Since an optimal criterion function has not been established in the literature relating to the problem discussed here, we propose five options. Note that some of them have been used by others. Let C_{f} denote the mean cost per failure of the item, C_{p} the mean cost per preventive replacement, C_{m} the mean cost per condition monitoring and t_{r} the planned replacement time, $t_i<t_{\mathrm{r}}<t_{i+1}$.

Criterion function 1 (expected cost to a renewal/expected renewal length since new (time zero)). It is given by

$$\frac{(C_{\mathrm{f}}-C_{\mathrm{p}})P(t_{\mathrm{r}}-t_i|\,Y_i)+C_{\mathrm{p}}+\mathrm{i}C_{\mathrm{m}}}{t_i+(t_{\mathrm{r}}-t_i)(1-P(t_{\mathrm{r}}-t_i|\,Y_i))+\int_0^{t_{\mathrm{r}}-t_i}x_ip(x_i|\,Y_i)\mathrm{d}x_i} \tag{3}$$

where $P(t_{\mathrm{r}}-t_i|\,Y_i)=\int_0^{t_{\mathrm{r}}-t_i}p_i(x_i|\,Y_i)\mathrm{d}x_i$. Expression (3) is the commonly used criterion function in condition-based replacement modelling,[17,13,14] which is based upon the renewal reward equation. Note that there is an implicit assumption used in expression (3) which assumes that the probability of more than two renewals in (t_i, t_{i+1}) is almost zero. It is known that expression (3) is an approximation since the iid assumption is not satisfied here, and minimizing expression (3) in terms of t_{r} may not guarantee an optimal solution. Mathematically, we cannot evaluate how good this approximation is since we cannot quantify Y_i, but intuitively we argue that if the same monitoring process is repeated over a very long time period, there might be some subsets of the processes that may have similar histories so that these life distributions may be identical. However, one may ask why we need to include past cost and time such as iC_{m} and t_i in the model since they have already occurred. Before answering this question, we discuss a criterion function that only looks at the expected cost and time to a renewal from the current time t_i.

Criterion function 2 (expected renewal cost/expected renewal length since t_i). It is given by

$$\frac{(C_{\mathrm{f}}-C_{\mathrm{p}})P(t_{\mathrm{r}}-t_i|\,Y_i)+C_{\mathrm{p}}}{(t_{\mathrm{r}}-t_i)(1-P(t_{\mathrm{r}}-t_i|\,Y_i))+\int_0^{t_{\mathrm{r}}-t_i}x_ip(x_i|\,Y_i)\mathrm{d}x_i} \tag{4}$$

This criterion function looks logical as only the expected future cost and time are considered. However, it has a fundamental problem as $t_{\mathrm{r}}-t_i\to 0$, expression (4)$\to\infty$ so that no preventive replacement will be recommended to be made at time t_i, which might be conflicting in the case where the measured condition information signals have indicated a severe fault symptom and an immediate replacement may be required.

Criterion function 3 (expected cost to a renewal since new divided by t_{r}). It is given by

$$\frac{(C_{\mathrm{f}}-C_{\mathrm{p}})P(t_{\mathrm{r}}-t_i|\,Y_i)+C_{\mathrm{p}}+\mathrm{i}C_{\mathrm{m}}}{t_{\mathrm{r}}} \tag{5}$$

This criterion function is similar to expression (3) except that the denominator is t_{r} rather than the expected time to a renewal. The rationale is to minimize the expected unit time cost over a finite horizon t_{r}.

Criterion function 4 (expected cost since t_i divided by $(t_{\mathrm{r}}-t_i)$). It is given by

$$\frac{(C_{\mathrm{f}}-C_{\mathrm{p}})P(t_{\mathrm{r}}-t_i|\,Y_i)+C_{\mathrm{p}}}{t_{\mathrm{r}}-t_i} \tag{6}$$

This is a simplified version of criterion function 2, which shares the same problem as discussed.

To determine whether a preventive replacement should be scheduled within t_i and t_{i+1} using one of the criterion functions 1–4, we can simply enumerate the expression over the range of t_{r} from t_i to $t_{i+1}+\Delta$, where Δ is an arbitrary time unit. If the t_{r} that minimizes the chosen criterion function is located inside (t_i, t_{i+1}), then this minimum point is the recommended preventive replacement time. The reason why we compute one more unit beyond t_{i+1} is if the minimum is at $t_{i+1}+\Delta$, where the recommendation would be to wait until new information becomes available at time t_{i+1}.

Criterion function 5: This criterion function differs from the other four as it evaluates the following inequality to determine whether a preventive replacement should be scheduled within (t_i, t_{i+1}):

$$\frac{C_{\mathrm{p}}}{C_{\mathrm{f}}+C_{\mathrm{p}}}\leq P(t_{\mathrm{r}}-t_i|\,Y_i) \tag{7}$$

The rationale behind inequality (7) is as follows.

If a preventive replacement is scheduled at time t_{r} where $t_i<t_{\mathrm{r}}<t_{i+1}$, then there would be two expected costs exclusively, namely a preventive replacement cost if the item has not failed before t_{r}, and a failure replacement cost otherwise. If the ratio between these two is less than 1 for the first time for a given t_{r}, then this t_{r} is the recommended replacement time, that is,

$$\inf\left(t_{\mathrm{r}}:\frac{C_{\mathrm{p}}(1-P(t_{\mathrm{r}}-t_i|\,Y_i))}{C_{\mathrm{f}}P(t_{\mathrm{r}}-t_i|\,Y_i)}\leq 1,\ t_i\leq t_{\mathrm{r}}<t_{i+1}\right)$$

and after some manipulation, inequality (7) follows. This implies that if the expected failure replacement cost is equal to or larger than the expected preventive replacement cost over (t_i, t_{i+1}), a preventive replacement is favourable, or otherwise wait until new information becomes available at time t_{i+1}. This criterion function was used by Wang *et al*[18] and Wang and Gu.[19]

Now we have a question as to which of the above would give the best cost saving over a very long time period, namely the expected unit time cost measure over T, $T\to\infty$. Because of the updating nature of the criterion function which depends on Y_i, the analytic form of the long-term expected cost per unit time is not available; instead, we use the approach described in Figure 2 with simulation and analytic decision-making at each monitoring point using one of the above criterion functions to investigate which criterion function would give the lowest long-term cost per unit time. For illustration, we give an example.

Numerical example

Table 1 shows the parameter values and the distributions used for describing u, the initial point, $h = x_0$, the delay time, and $y_i|x_i$, which are taken from Wang[14] based on real data.

Note that in Table 1 the relationship between y_i and x_i is described by letting $1/\alpha = A + Be^{-Cx_i}$, which establishes a negative correlation as expected. The cost parameters are $C_f = £6000$, $C_p = £2000$ and $C_m = £30$, respectively, which are taken from Wang *et al.*[20]

For a detailed discussion as to why to choose a Weibull distribution for these random variables, and how these parameters were estimated, see Wang.[14] For our purpose here we only use them to demonstrate the comparison and solution procedures for determining the optimal monitoring interval.

The timescale for each simulation run is set as $T = 60\,000$, sufficiently large for comparison purposes. Figure 3 shows the result of the simulation test.

It can be seen from Figure 3 that criterion functions 1 and 3 produce the best results. Criterion function 4 is the worst among these five, and criterion function 5 is marginally better than criterion functions 2 and 4.

In order to observe the performance of these criterion functions in a variety of situations, we carried out three more simulation tests. The first two were conducted by changing the failure cost from £6000 to £3000 and £9000, respectively, while the other model parameters are kept unchanged. The third one was to change α_h from 0.011 to 0.051, a short mean delay time, while all other parameters are the same. The results are shown in Figures 4-6.

From these figures, we can see that criterion functions 1 and 3 produce the best results. For all cases the use of criterion functions 1 and 3 resulted in similar results to those expected, since the denominators in expressions (3) and (5) are largely dominated by t_i. In conclusion, we recommend using criterion function 1 since it is the closest to the renewal reward equation in format, and its performance in our simulation tests is good across all situations tested. This is, we believe, because the iid assumption is approximately satisfied if the sample size is sufficiently large as in our simulation. This also demonstrates that we need to include the past cost and time in order to formulate the complete renewal cycle.

Table 1 Distribution and parameter values used in the example

	Distribution		*Parameter values*			
u	$p(u) = \alpha_u\beta_u(\alpha_u u)^{\beta_u-1}e^{-(\alpha_u u)^{\beta_u}}$	$\alpha_u = 0.004$	$\beta_u = 1.67$			
$h = x_0$	$p(x_0) = \alpha_h\beta_h(\alpha_h x_0)^{\beta_h-1}e^{-(\alpha_h x_0)\beta_h}$	$\alpha_h = 0.011$	$\beta_u = 1.67$			
$y_i\|x_i$	$p(y_i\|x_i) = \alpha\beta(\alpha y_i)^{\beta-1}e^{-(\alpha y_i)^{\beta}}$*		$\beta = 4.559$	$A = 7.069$	$B = 27.09$	$C = 0.052$

$^*1/\alpha = A + Be^{-Cx_i}$.

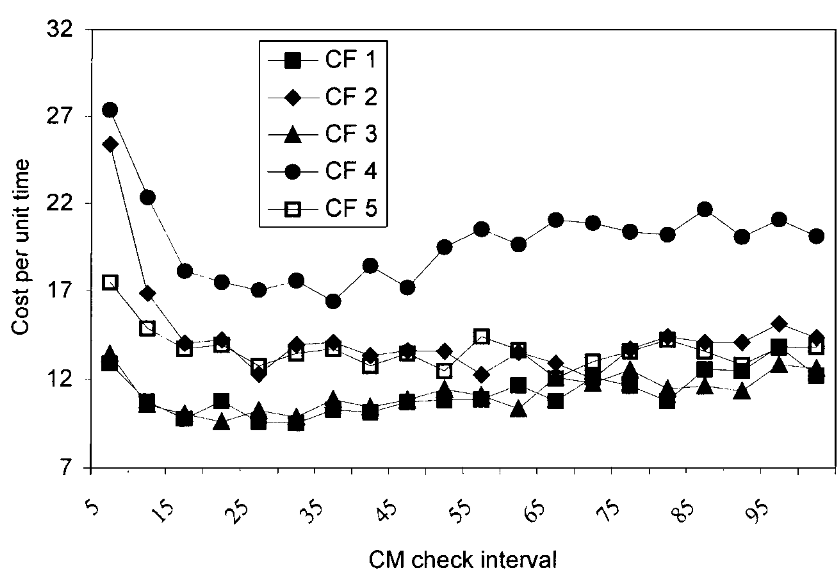

Figure 3 Comparison of different criterion functions in terms of unit time cost over T.

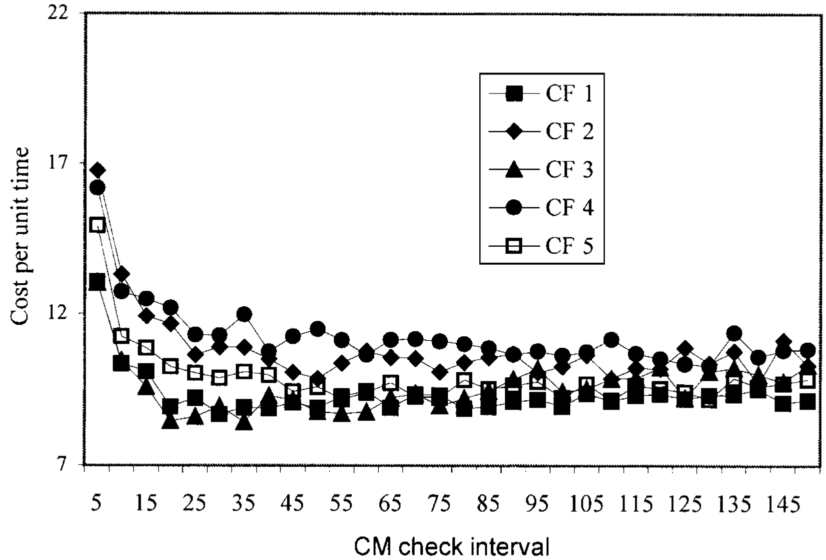

Figure 4 Comparison of different criterion functions in terms of unit time cost over T when $C_f = 3000$.

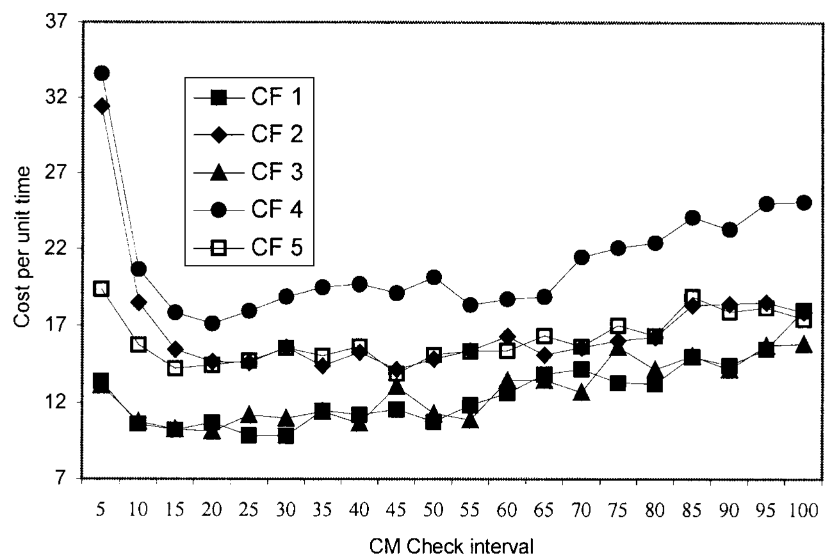

Figure 5 Comparison of different criterion functions in terms of unit time cost over T when $C_j = 9000$.

A dynamic model to determine the next monitoring point

Once the component has been identified to be possibly defective, it requires close attention, and therefore, in most cases, a reduced monitoring interval is proposed if the item is not planned to be replaced in the near future. Suppose now we are at time t_i, and we seek to determine such an interval $t = t_{i+1} - t_i$ in terms of, for example, a minimum expected cost measure. Similarly, the decision of the time to the next check involves the use of Equation (2) that cannot be evaluated if Y_{i+1} is unknown and cannot be described by a distribution. As in the determination of the static monitoring interval, we use a procedure combining simulation and analytical analysis. Figure 7 shows the flow chart of the procedure.

For simplicity, we assume from t_i that the subsequent monitoring interval is constant in the procedure, but in fact it will be nonconstant since a new decision will be made at time t_{i+1}once new information becomes available. The above process in Figure 7 is repeated for different intervals t, and we can simply pick up the one with the lowest average total cost per unit time as the time interval to the next

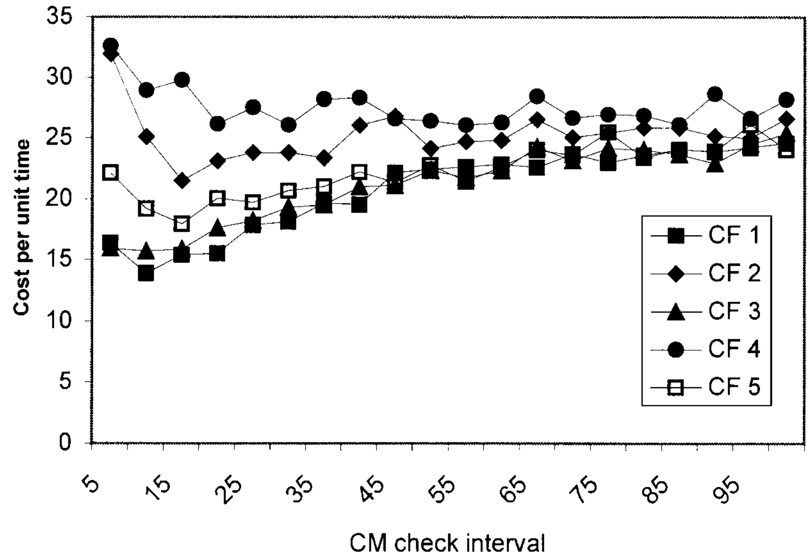

Figure 6 Comparison of different criterion functions in terms of unit time cost over T when $\alpha_h = 0.051$.

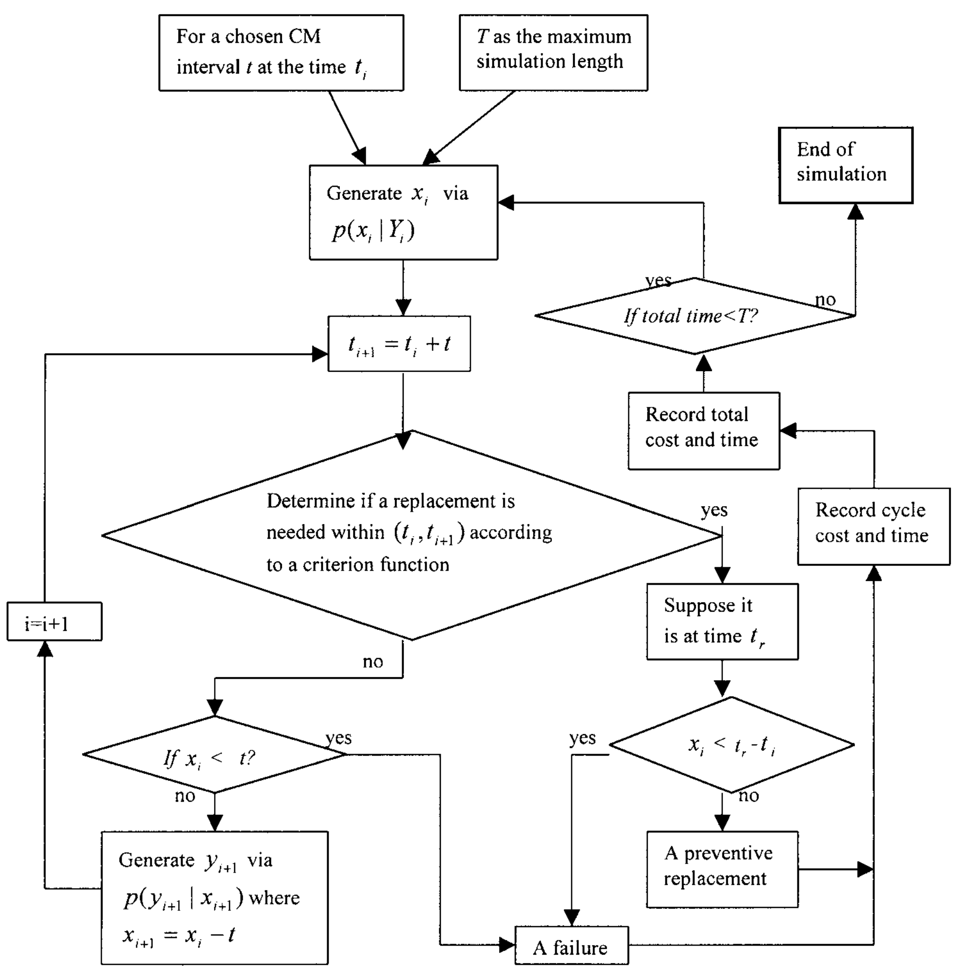

Figure 7 Solution process of the determination of a dynamic condition monitoring interval.

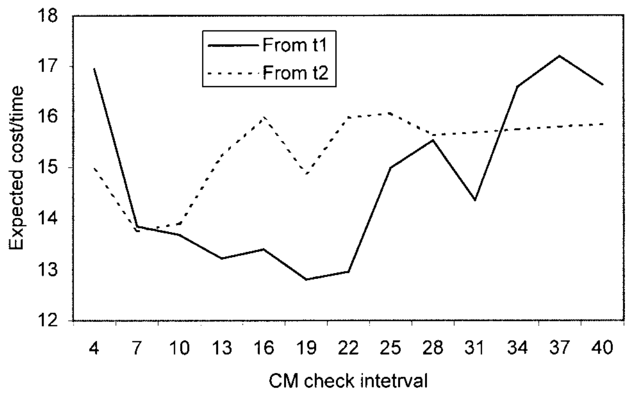

Figure 8 Optimal CM check interval when $t_1 = 100$ with $y_1 = 7.1$ and $t_2 = 120$ with $y_2 = 10.2$.

monitoring check. The same process will be repeated at each new monitoring point and therefore a series of intervals can be obtained, which are dynamic in that they are dependent on the history of condition monitoring to t_i. Criterion function 1 is chosen for the replacement decision, which has been discussed earlier.

Now we consider an example using the same parameter values in Figure 3. If we adopt the recommended normal CM check interval, 25, see Figure 3, and $t_1 = 100$ is the first point that the reading is above threshold level 5,[14] with $y_1 = 7.1$, then in Figure 8 the solid line shows the expected unit time cost in terms of various condition monitoring intervals using the procedure illustrated in Figure 7. It can be seen that the optimal time to the next check is about 20. If a preventive replacement is not recommended before the next check and a failure does not occur, then we have $t_2 = 120$. Supposing $y_2 = 10.2$. and applying the simulation procedure of Figure 7, the dotted line shows the expected unit time cost, and the recommended time to the next check, in this case, is about 7. Interestingly, both curves in Figure 8 show some local minima which are as expected due to the stochastic nature of the problem. This point has been discussed by Newby and Dagg.[21]

Conclusions

The purpose of this paper is to present a simulation and analytical approach for determining the condition monitoring interval both at normal and abnormal working periods. The solution procedure is carried out in two stages. We first propose a static model that is used to determine a fixed condition monitoring interval over the item life. Once the monitored information indicates a possible abnormality of the item concerned, that is, the start of the failure delay time, a dynamic approach is employed to determine the next monitoring time at the current monitoring point given that the item is not scheduled for a preventive replacement before that time. Two key problems are addressed in the paper. The first is which criterion function we should use for determining the time of the next monitoring check, and the second is the optimisation process for both the static and dynamic models that can be solved neither analytically nor numerically since the models depend on two unknown quantities, namely, the available condition information and a decision of the time to replace the item over the failure delay time. For the first problem, we proposed five conventionally used criterion functions, and tested them using simulations to see which one performs best. It is concluded that the conventional renewal reward equation measured from new is the best one from our simulation tests. The second problem was solved using an adaptive numerical and simulation solution procedure. This procedure is demonstrated through a numerical example that produced results as expected.

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