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An Inspection Policy for Deteriorating Processes Using Delay-Time Concept

SUSUMU OKUMURA
University of Shiga Prefecture, Japan

A method for determining the discrete time points of inspection for a deteriorating single-unit system under condition-based maintenance is developed. The system is in a normal state, a symptom state or a failed state. A delay-time model is utilized to describe the transition of the states. The transition time from a normal state to a symptom state and that from a symptom state to a failed state are derived assuming continuous deteriorating processes based on a reaction rate model which is categorized as a failure physics model. Failed-dangerous and failed-safe probabilities of an imperfect inspection are considered. A method for minimizing the long-run average cost per unit time is formulated. The characteristics and sensitivity of the proposed method are investigated. © 1997 IFORS. Published by Elsevier Science Ltd.

Key words: inspection, inspection policy, delay-time model, failure physics model, reaction rate model, failed-safe probability, failed-dangerous probability, condition-based maintenance

1. INTRODUCTION

It becomes increasingly important to prevent failure or deterioration of mechanical systems by performing effective maintenance as the complexity of the systems increases. Maintenance policies are classified into breakdown maintenance, time-based maintenance and condition-based maintenance. Breakdown maintenance is applied to a system in which economic losses when it fails are small since maintenance intervention is conducted after failure. Time-based maintenance is the policy in which maintenance action is performed at predetermined intervals or at failure, whichever occurs first. The intervals between preventive actions must be short in order to enhance system reliability, which leads to an increase in maintenance cost. Condition-based maintenance is the policy in which preventive intervention is conducted after identifying a symptom of impending failure at discrete time points or continuously with the aid of condition diagnosis techniques. This maintenance policy is effective for a system in which economic losses caused by failure or deterioration are large (Baldin, 1981).

Extensive surveys of preventive maintenance were done by McCall (1965), Pierskalla and Voelker (1976) and Valdez-Flores and Feldman (1989). A problem of determining discrete time points of inspection is dealt with in this paper. It is assumed that a maintained system is a single-component unit or that it can be modeled as a single entity. The following three states of a maintained system are considered; (1) a normal state: a state in which a system performs its desired functions completely, (2) a failed-state: a state in which a system deteriorates to a condition such that it is considered to have failed and (3) a symptom state: an intermediate state between normal and failed states, i.e., a state in which a symptom of failure due to deterioration can be observed by inspection.

Christer (1984, 1987) and Christer and Wang (1992) called the transition time from a symptom state to a failed state delay-time, and dealt with industrial plant maintenance problems. Although delay-time was assumed to have an arbitrary density function the transition time from a normal state to a symptom state was restricted to have a uniform distribution. A delay-time model is utilized to describe the transition of the states in this paper, and the time from a normal state to a symptom state, which is called symptom-time, is assumed to have an arbitrary density function.

When a distribution/density function for the time to failure is obtained, statistical characteristic values such as failure rate are conventionally used. Lu and Meeker (1993) developed a general path model and data analysis methods using degradation measures to estimate a time to failure distribution, and reviewed degradation models. However, it is expected that a significant interpretation can be given for the obtained distribution/density function itself if a failure mechanism can be

Correspondence: S. Okumura, Department of Mechanical Systems Engineering, University of Shiga Prefecture, 2500 Hassaka, Hikone, Shiga 522, Japan.

involved in the function. In this paper, we assume that the deterioration progresses according to a reaction rate model which is categorized as a failure physics model. Distribution functions of symptom-time and delay-time are derived when deterioration follows the model, and the relationship between progress of deterioration and a reaction rate model is discussed.

The true condition of a maintained system is not always correctly identified by an inspection. A system may be identified as being in a normal state by an inspection even if the system is actually in a symptom state (type I error). Moreover, a system may be identified as being in a symptom state by an inspection even if the system is actually in a normal state (type II error). Therefore, the following time-variant probabilities are considered with regard to an imperfect inspection: failed-dangerous probability (the probability of a type I error occurring) and failed-safe probability (the probability of a type II error occurring).

A new method for determining the discrete time points of inspection for a deteriorating delay-time modeled single-unit system is developed. The deterioration process conforms to a reaction rate model, and time-variant FD and FS probabilities of inspection are introduced. An inspection policy is formulated as a problem of minimizing the long-run average cost per unit time. The behavior of the model is investigated using numerical examples under varying FD and FS probabilities and coefficients of variation.

2. FORMULATION OF THE PROBLEM

2.1 Assumptions

(1) A maintained single-unit system deteriorates over time and undergoes three states; s_0, s_1, s_2, where s_0 is the state in which the system performs the desired functions completely (a normal state), s_1 is the state in which the system deteriorates and a symptom of impending failure appears (a symptom state) and s_2 is the state in which a system deteriorates to the condition that it is considered to have failed (a failed state).

(2) The times when a system enters from s_0 to s_1 and from s_1 to s_2 are denoted by x_{01} and x_{12}, respectively. The elapsed times from $t = 0$ to $t = x_{01}$ and from $t = x_{01}$ to x_{12} are random variables, and have probability density functions $f_1(t)$ and $f_2(t)$ $(t \geq 0)$, respectively. The times x_{01} and x_{12} are assumed to be independent for mathematical simplicity. The time to failure follows a probability density function $g(t) = \int_0^t f_1(t - x)f_2(x)\mathrm{d}x (t \geq 0)$. The cumulative distribution functions of $f_i(t)$ $(i = 1,2)$ and $g(t)$ are denoted by $F_i(t)$ and $G(t)$, respectively, and $\bar{F}_i(t) = 1 - F_i(t), \bar{G}(t) = 1 - G(t)$.

(3) Failures of a system are known immediately since condition-based maintenance is applied to significant systems that may cause serious economic losses when they fail.

(4) Whether a system is in state s_0 or s_1 is known only by inspection. When a system is in s_0, the probability that the state of the system is correctly diagnosed as being in s_0 is given by $p_{00}(t)$, and the probability that the state is wrongly (FS) diagnosed as being in s_1 is given by $p_{01}(t)$: the two probabilities have the relation $p_{01}(t) = \bar{p}_{00}(t) = 1 - p_{00}(t)$. When a system is in s_1, the probability that the state is correctly diagnosed as being in s_1 is given by $p_{11}(t)$, and the probability that the state is wrongly (FD) diagnosed as being in s_0 is given by $p_{10}(t)$: the two probabilities have the relation $p_{10}(t) = \bar{p}_{11}(t) = 1 - p_{11}(t)$. We can regard failed-safe and failed-dangerous probabilities as a function of time elapsed. It is for this reason that the value of $p_{01}(t)$ for the neighborhood of 0 and that for the time just before the state of a maintained system enters from s_0 to s_1 are different as the deterioration of a system progresses as time proceeds. Probability $p_{01}(t)$ satisfies $p_{01}(0) = 0$, it becomes large as the state of a system approaches s_1 from s_0 and is non-decreasing. With regard to $p_{10}(t)$ the probability for the time just after the state of a system enters s_1 is large and becomes small as the state approaches s_2, $p_{10}(t)$ satisfies $p_{10}(\infty) = 0$ formally and is non-increasing.

(5) A system is replaced to an as new condition when the system is identified as being in state s_1 by an inspection or when it fails. If the true state of a maintained system is s_1 and that impending failure is identified by an inspection, then the system is replaced and it is restored to an as new condition or without deterioration. However, if the correct condition of a system is s_0 and the system is diagnosed to be in s_1 failed-safely by an inspection, then the system is replaced. If the intervals of inspection time are long, the system whose actual condition is s_1 can be left without inspection,

then the system fails and failure replacement is conducted. There is another case that a system is identified to be in s_0 failed-dangerously by an inspection when the true state of a system is s_1, then it is left without inspection intervention before its failure, and replaced.

(6) Inspections are carried out at discrete points in time t_i ($i = 1, 2, \ldots$) with negligible required time for an inspection. Discrete points $t_1, t_2, \ldots$ are denoted by $\mathbf{t}$, which is called inspection time vector. New inspection time begins with t_1 after a system is replaced and in operation again.

(7) M_P is the expected time required for a system to be replaced preventively and in operation again, and M_F is the expected time required for a failed system to be replaced and in operation again.

(8) C_P is the cost of a preventive replacement, C_F is the cost of a failure replacement, C_I is the cost of an inspection and C_C is the cost of a replaced system.

2.2. Description of deterioration processes on the basis of a reaction rate model

The deterioration of a system can be regarded microscopically as changes on the atomic or molecular level. Deleterious reaction proceeds due to stress applied to a system, and failures occur when the cumulative stress exceeds a certain limit. In a reaction rate model deterioration rate $K(t)$ at time t is defined as

$$K(t) = A\theta(t)\exp\left[-\frac{B}{k_B\theta(t)}\right]\exp\left[\left(C + \frac{D}{k_B\theta(t)}\right)S(t)\right], \tag{1}$$

where $\theta(t)$ is the absolute temperature at time t, $S(t)$ is stress except that due to temperature at time t, A, B, C and D are constants and k_B is Boltzmann's constant. This model can be applied to a wide variety of phenomena, e.g., not only chemical reactions in a narrow sense but also the rate of physical changes such as wear, deformation, growth of cracks and diffusion of heat, electric charge and mass (Shiomi, 1970). It is possible to incorporate the information concerning the degradation trend obtained by inspections in a reaction rate model. When $K(t)$ is integrable for $t \geq 0$, the cumulative degradation quantity

$$D(t) = \int_0^t K(t)\mathrm{d}t \tag{2}$$

is called a degradation trend function. Also, the degradation of a system quantitatively measured by an inspection is called the degradation trend value in this paper.

This paper deals with a decision making problem on the condition that the parameter vector involved in a degradation trend function is fixed at time 0. The measured degradation value at time t varies due to the variation of degradation progress and measurement errors, and it seems reasonable to suppose that both variations have the normality nature. Therefore, the degradation value measured at t is assumed to have a normal distribution which has $D(t)$ as the mean value. The variance of the normal distribution is expected to increase as time proceeds with the minimum variance at time 0. Therefore, a constant coefficient of variation is assumed. The reason for this is that the constant coefficient is a simple index and satisfies the condition that the variance of degradation grows as time proceeds:

$$N(D(t), \{c_v D(t)\}^2).$$

Let D_1 and $D_1 + D_2$ be the boundary values of $D(t)$ when the state of a system changes from s_0 to s_1 and when it changes from s_1 to s_2, respectively. Approximative cumulative functions $\hat{F}_i(t)$ ($i = 1, 2$) are obtained as

$$\hat{F}_i(t) = \int_{D_i}^{\infty} N_{\mathrm{pdf}}(x; D(t), \{c_v D(t)\}^2)\mathrm{d}x \tag{3}$$

$$= \Phi\left(\frac{D(t) - D_i}{c_v D(t)}\right), \tag{4}$$

where $N_{\mathrm{pdf}}(x; D(t), \{c_v D(t)\}^2)$ denotes the density function of $N(D(t), \{c_v D(t)\}^2)$, and $\Phi(\cdot)$ denotes the cumulative distribution function of the standard normal distribution $N(0,1)$. When $\theta(t) \equiv \theta$, $S(t) \equiv S$

$$K(t) = K \quad \text{(const.)} \tag{5}$$

$$D(t) = Kt. \tag{6}$$

When $\theta(t) \equiv \theta$

$$K(t) = K_1 \exp[K_2 S(t)] \quad (K_1, K_2\text{: const.}) \tag{7}$$

and when $S(t) \equiv S$

$$K(t) = K_1 \theta(t) \exp\left[\frac{K_2}{\theta(t)} + K_3\right] \quad (K_1, K_2, K_3\text{: const.}). \tag{8}$$

It is difficult to give $D(t)$ in explicit form when $\theta(t) \equiv \theta$ or $S(t) \equiv S$.

Approximative density functions $\hat{f}_i(t)$ are given by

$$\hat{f}_i(t) = \frac{\mathrm{d}}{\mathrm{d}t}\hat{F}_i(t) \tag{9}$$

$$= \frac{D_i \dfrac{\mathrm{d}D(t)}{\mathrm{d}t}}{\sqrt{2\pi c_v}\{D(t)\}^2} \exp\left[-\frac{(D(t) - D_i)^2}{2\{c_v D(t)\}^2}\right]. \tag{10}$$

In the following cases, $\hat{f}_i(t)$ can be obtained in a simple form as shown.

When $\theta(t) \equiv \theta, S(t) \equiv S$, since $D(t) = Kt$

$$\hat{f}_i(t) = \frac{D_i}{\sqrt{2\pi c_v} K t^2} \exp\left[-\frac{(Kt - D_i)^2}{2(c_v K t)^2}\right] \tag{11}$$

When $\theta(t) \equiv \theta, S(t) \equiv t$

$$D(t) = \frac{K_1}{K_2}(\exp[K_2 t] - 1) \tag{12}$$

$$\hat{f}_i(t) = \frac{D_i K_2^2 \exp[K_2 t]}{\sqrt{2\pi c_v} K_1 (\exp[K_2 t] - 1)^2} \exp\left[-\frac{\{K_1(\exp[K_2 t] - 1) - K_2 D_i\}^2}{2\{c_v K_1(\exp[K_2 t] - 1)\}^2}\right]. \tag{13}$$

When $\theta(t) \equiv \theta, S(t) \equiv t^2$

$$D(t) = \frac{K_1\sqrt{\pi}\,\mathrm{erfi}[\sqrt{K_2}t]}{2\sqrt{K_2}} \tag{14}$$

$$\hat{F}_i(t) = \frac{4D_i K_2 \exp[K_2 t^2]}{\sqrt{2\pi^3} c_v K_1 \{\mathrm{erfi}[\sqrt{K_2}t]\}^2} \exp\left[-\frac{\{\sqrt{\pi} K_1 \mathrm{erfi}[\sqrt{K_2}t] - 2\sqrt{K_2} D_i\}^2}{2\pi\{c_v K_1 \mathrm{erfi}[\sqrt{K_2}t]\}^2}\right], \tag{15}$$

where $\mathrm{erfi}(\cdot) = -i\mathrm{erf}(i\cdot) = i(1 - 2\Phi(i\sqrt{2}\cdot))$ (i: imaginary unit).

When $\theta(t) \equiv \theta, S(t) \equiv t^3$

$$D(t) = \frac{K_1 \Gamma(1/3, -K_2 t^3, 0)}{3\sqrt[3]{K_2}} \tag{16}$$

$$\hat{f}_i(t) = \frac{9D_i \sqrt[3]{K_2}^2 \exp[K_2 t^3]}{\sqrt{2\pi} c_v K_1 \{\Gamma(1/3, -K_2 t^3, 0)\}^2} \exp\left[-\frac{\{K_1 \Gamma(1/3, -K_2 t^3, 0) - 3\sqrt[3]{K_2} D_i\}^2 t^2}{2(c_v K_1)^2 \{\Gamma(1/3, -K_2 t^3, 0)\}^2}\right], \tag{17}$$

where $\Gamma(\cdot,\cdot,\cdot)$ is the generalized incomplete gamma function.

2.3. Long-run average cost per unit time

It is expected that a system which is suitable for condition-based maintenance causes huge economic losses. Therefore, it is reasonable to consider an appropriate criterion for the determination of inspection points in time as the total cost incurred which consists of preventive replacement cost, failure replacement cost and inspection cost as cost elements.

A cycle, which is the time between two successive replacements, is expected to repeat over an infinite time span. Therefore, the expected cost per unit time is considered to be the objective, and its minimization is tried. Typical possible operations when inspection is conducted i times in a cycle are shown in Fig. 1:

(1) A successful cycle: when a system is in state s_1 the system is correctly diagnosed as being in s_1, is replaced preventively and begins operation again.
(2) A failed-dangerous cycle: either no inspection is conducted after a system enters s_1; or when a system is in s_1 the system is failed-dangerously diagnosed as being in s_0, and fails before the time when the state is correctly identified as being in s_1. Failure replacement is conducted and the replaced system begins operation again.
(3) A failed-safe cycle: when a system is in state s_0 the system is failed-safely identified as being in s_1, is replaced preventively and begins operation again.

Let C be the cost incurred in a cycle, T the length of a cycle, I the number of inspections per cycle,

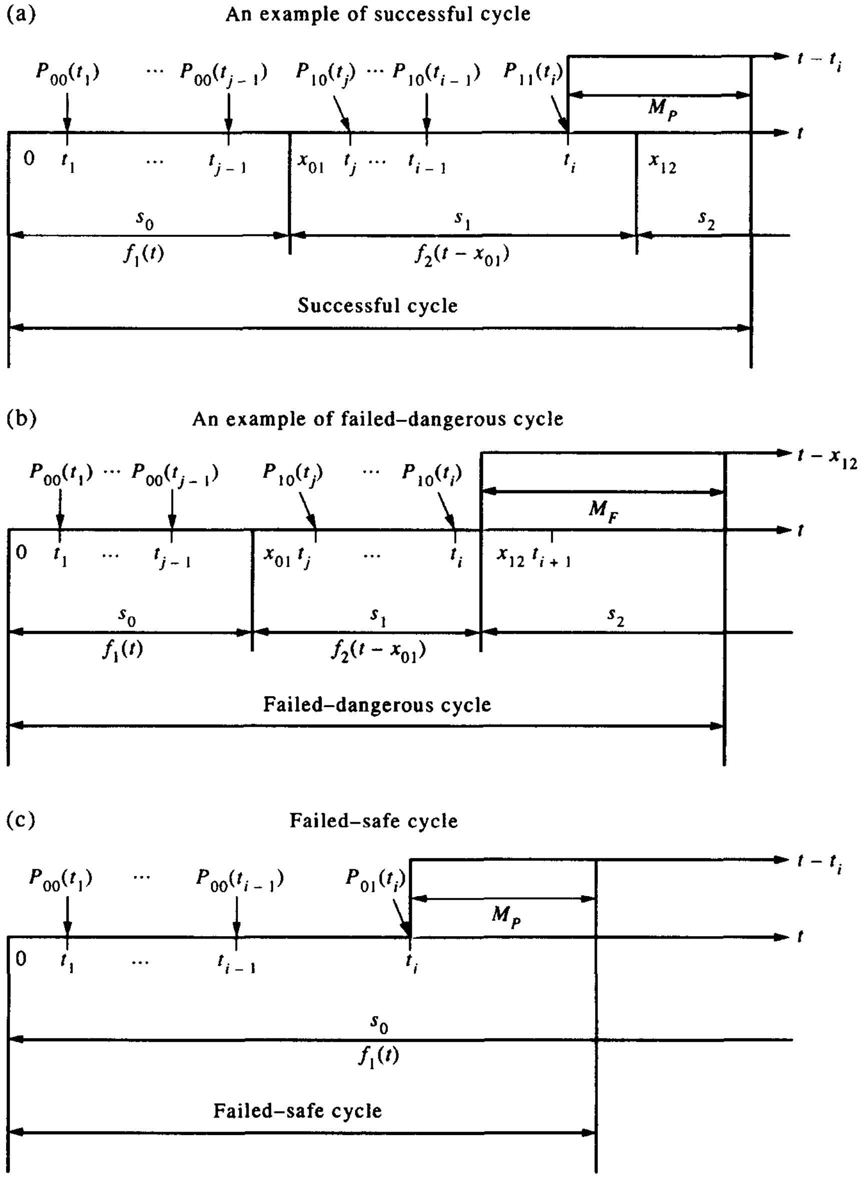

Fig. 1. Typical possible operations when inspection is conducted i times in a cycle.

and they are random variables. When their expected values are denoted by E_C, E_T, E_I, respectively, the long-run average cost per unit time is given by E_C/E_T. Let P_P be the probability that a cycle ends in a preventive replacement, and P_F be the probability that a cycle ends in a failure replacement. Expected value of the cost per cycle E_C is given by

$$E_C = C_I E_I + P_P C_P + P_F C_F + C_C. \tag{18}$$

$$E_I = \sum_{i=0}^{\infty} i p_i, \tag{19}$$

where p_i is the probability when inspection is conducted i times in a cycle.

Probability p_i is obtained in the following.

(1) A successful cycle: It is considered the case that the state of a system changes from s_0 to s_1 at time $x_{01} \in (t_{i-1}, t_i]$ $(i = 1, 2, \ldots; t_0 = 0)$. The probability that a system is correctly identified as being in s_1 at time t_i without failure from time x_{01} until t_i is given by

$$p_{11}(t_i) \prod_{k=1}^{i-1} p_{00}(t_k) \int_{t_{i-1}}^{t_i} f_1(x_{01}) \bar{F}_2(t_i - x_{01}) \mathrm{d}x_{01}, \tag{20}$$

where $\Pi^{y}_{\cdot = x} = 1$ for $y < x$. Then, the case that the state of a system changes at time $x_{01} \in (t_{j-1}, t_j]$ $(j = 1, 2, \ldots; t_0 = 0; j \leq i - 1)$ is considered. On the condition that the state of a system does not change from s_1 to s_2 from time x_{01} until time t_i, the probability that the state of the system is wrongly diagnosed at time t_k $(k = j, j + 1, \ldots, i - 1)$ as being in state s_0, and correctly diagnosed as being in state s_1 at time t_i is obtained by

$$p_{11}(t_i) \prod_{k=1}^{j-1} p_{00}(t_k) \prod_{k=j}^{i-1} p_{10}(t_k) \int_{t_{j-1}}^{t_j} f_1(x_{01}) \bar{F}_2(t_i - x_{01}) \mathrm{d}x_{01}, \tag{21}$$

(2) A failed-dangerous cycle: The case that the state of a system enters s_1 at time $x_{01} \in [t_i, t_{i+1})$ $(t = 0, 1, \ldots; t_0 = 0)$ is considered. The probability that a system fails at time $x_{12} \in [x_{01}, t_{i+1})$ is given by

$$\prod_{k=1}^{i} p_{00}(t_k) \int_{t_i}^{t_{i+1}} f_1(x_{01}) F_2(t_{i+1} - x_{01}) \mathrm{d}x_{01}. \tag{22}$$

Then, the case that the state of a system enters s_1 at time $x_{01} \in [t_{j-1}, t_j)$ $(j = 1, 2, \ldots; t_0 = 0)$ is considered. The probability that a system is wrongly diagnosed as being in state s_0 at time t_k $(k = j, j + 1, \ldots, i)$, and the system fails at time $x_{12} \in [t_i, t_{i+1})$ is obtained by

$$\prod_{k=1}^{j-1} p_{00}(t_k) \prod_{k=j}^{i} p_{10}(t_k) \int_{t_{j-1}}^{t_j} f_1(x_{01})(F_2(t_{i+1} - x_{01}) - F_2(t_i - x_{01})) \mathrm{d}x_{01}. \tag{23}$$

(3) A failed-safe cycle: On the condition that a system does not enter s_1 from time $t = 0$ until time t_i, the probability that the system is wrongly diagnosed at time t_i as being in state s_1 when the system is actually in state s_0 is given by

$$p_{01}(t_i) \prod_{k=1}^{i-1} p_{00}(t_k) \bar{F}_1(t_i). \tag{24}$$

Combining Equations (20) and (21), Equations (22) and (23) introducing the following function:

$$\delta(x) = \begin{cases} 0 & (x = 0) \\ 1 & (x \neq 0), \end{cases} \tag{25}$$

and summing from $j = 1$ to i on j, probability p_i is represented by

$$\begin{aligned} p_i = {} & p_1(t_i) \sum_{j=1}^{i} \prod_{k=1}^{j-1} p_{00}(t_k) \prod_{k=j}^{i-1} p_{10}(t_k) \int_{t_{j-1}}^{t_j} f_1(x_{01}) \bar{F}_2(t_i - x_{01}) \mathrm{d}x_{01} \\ & + \sum_{j=1}^{i+1} \prod_{k=1}^{j-1} p_{00}(t_k) \prod_{k=j}^{i} p_{10}(t_k) \int_{t_{j-1}}^{t_j} f_1(x_{01})(F_2(\mathrm{t}_{i+1} - x_{01}) - \delta(i - j + 1) F_2(t_i - x_{01})) \mathrm{d}x_0 \\ & + p_{01}(t_i) \prod_{k=1}^{i-1} p_{00}(t_k) \bar{F}_1(t_i), \end{aligned} \tag{26}$$

where $t_0 = 0$.

Probability P_P when a cycle ends in a preventive replacement is given by

$$P_P = \sum_{j=1}^{\infty}\sum_{i=j}^{\infty} p_{11}(t_i)\prod_{k=1}^{j-1} p_{00}(t_k)\prod_{k=j}^{i-1} p_{10}(t_k)\int_{t_{j-1}}^{t_j} f_1(x_{01})\bar{F}_2(t_i - x_{01})\mathrm{d}x_{01} + \sum_{i=1}^{\infty} p_{01}(t_i)\prod_{k=1}^{i-1} p_{00}(t_k)\bar{F}_1(t_i) \tag{27}$$

which considers both a successful cycle and a failed-safe cycle.

Probability P_F is obtained by

$$P_F = \sum_{j=1}^{\infty}\sum_{i=j-1}^{\infty}\prod_{k=1}^{j-1} p_{00}(t_k)\prod_{k=j}^{i} p_{10}(t_k)\int_{t_{j-1}}^{t_j} f_1(x_{01})(F_2(t_{i+1} - x_{01}) - \delta(i-j+1)F_2(t_i - x_{01}))\mathrm{d}x_{01} \tag{28}$$

which reflects a failed-dangerous cycle, or simply by $P_F = 1 - P_P$.

Expected value of cycle length E_T is represented by the probabilistically weighted sum associated with the cycle length:

$$\begin{aligned} E_T = {} & (\text{Length of a successful cycle})\cdot(\text{Probability of a successful cycle}) \\ & + (\text{Length of a failed-dangerous cycle})\cdot(\text{Probability of a failed-dangerous cycle}) \\ & + (\text{Length of a failed-safe cycle})\cdot(\text{Probability of a failed-safe cycle}) \end{aligned} \tag{29}$$

Let T_1, T_2 and T_3 be the first term, the second term and the third term in the right side of Equation (29), respectively. The following equations are obtained:

$$T_1 = \sum_{j=1}^{\infty}\sum_{i=j}^{\infty}(t_i + M_P)p_{11}(t_i)\prod_{k=1}^{j-1} p_{00}(t_k)\prod_{k=j}^{i-1} p_{10}(t_k)\int_{t_{j-1}}^{t_j} f_1(x_{01})(\bar{F}_2(t_i - x_{01})\mathrm{d}x_{01} \tag{30}$$

$$T_2 = \sum_{j=1}^{\infty}\sum_{i=j-1}^{\infty}\prod_{k=1}^{j-1} p_{00}(t_k)\prod_{k=j}^{i} p_{10}(t_k)\int_{t_{j-1}}^{t_j} f_1(x_{01})\int_{(t_i - x_{01})\delta(i-j+1)}^{t_{i+1}-x_{01}} (t + x_{01} + M_F)f_2(t)\mathrm{d}t\mathrm{d}x_{01} \tag{31}$$

$$T_3 = \sum_{i=1}^{\infty}(t_i + M_P)p_{01}(t_i)\bar{F}_1(t_i)\prod_{k=1}^{i-1} p_{00}(t_k). \tag{32}$$

Let ϕ_i be the vector of parameters of $f_i(t)$ $(i = 1, 2)$. The objective is to determine inspection time vector $\mathbf{t}$ which minimizes E_C/E_T, i.e.,

$$\frac{C_I E_I(\mathbf{t}; \mathbf{a}_1) + (C_P - C_F)P_P(\mathbf{t}; \mathbf{a}_1) + (C_F + C_C)}{T_1(\mathbf{t}; \mathbf{a}_1) + T_2(\mathbf{t}; \mathbf{a}_2) + T_3(\mathbf{t}; \mathbf{a}_2)}, \tag{33}$$

where $\mathbf{a}_1 = (\phi_1, \phi_2, p_{01}, p_{10}(\mathbf{t})), \mathbf{a}_2 = (\phi_1, p_{01}(\mathbf{t}))$.

3. CHARACTERISTICS OF THE DETERMINATION METHOD

3.1. Relationship between the inspection time vector and preventive replacement probability

When obtaining inspection time vector which minimizes Equation (33) the following are needed: approximative density functions: $\hat{f}_i(t)$, cost parameters: C_I, C_P, C_F, mean time to repair: M_P, M_F, probabilities: $p_{ij}(t)$. With regard to $\hat{f}_i(t)$ we have two cases: (a) the density function is given in the explicit form, e.g., as shown in Equations (11), (13), (15), (17), (b) it is given in the implicit form. In the case we cannot have the function explicitly, we need to analyze it numerically based on Equations (1), (2) and (10) and obtain $\hat{f}_i(t)$ as discrete data.

When obtaining $\mathbf{t}$ which minimizes Equation (33) on these data, if the objective is as simple as Barlow and Proschan (1965) and Karpiński (1988) it is possible to make t_1 a decision variable on the condition that $\hat{f}_i(t)$ are the Pólya frequency functions of order two (PF_2) and the relation among (t_{i-1}, t_i, t_{i+1}) is derived. However, it is difficult to obtain the optimal solution of $\mathbf{t}$ under the consideration of $p_{ij}(t)$ and the difficulty in understanding the shape property of $\hat{f}_i(t)$ as in this research. Therefore, we obtain a nearly optimal solution of the inspection time vector by adding practical constraint on t. Equidistant inspection points in time which are the same inspection intervals are considered to be a practical condition in industry:

$$t_i = iu; i = 1, 2 \ldots, \tag{34}$$

where u is a positive constant, and a decision variable. Then, u corresponds to inspection interval.

It is difficult for the developed model to be investigated analytically; therefore, a numerical method is used. We investigate how coefficients of variation of $\hat{f}_i(t)$ and probabilities $p_{01}(t)$ and $p_{10}(t)$ influence preventive replacement probability P_P before examining the relationship among the inspection time vector, the cost parameters and parameters of $\hat{f}_i(t)$.

When obtaining u^* which minimizes Equation (33) we introduce the minimum increasing width $\Delta u > 0$. The cost ratio E_C/E_T is denoted by $r(u)$. Let T be the minimum value which satisfies $G(T) = 1$ and α be the maximum value ($\alpha \in \mathbb{N}$) which satisfies $\alpha\Delta u \leq T$. Under this notation u^* is obtained by solving the following:

$$r(u) \to \min$$

$$\text{s.t.} u \in \{\Delta u, \ldots, \alpha\Delta u\}.$$

The minimum width Δu is set to 1, which is supposed as being 1 day, in the numerical examples.

We consider the cases that $p_{01}(t)$ and $p_{10}(t)$ are dependent on time and constant over time for numerical experiments as typical examples. It may safely be assumed that $p_{01}(0) = 0$. Failed-safe probability $p_{01}(t)$ can be considered to have a positive correlation with the distribution $F_1(t)$ of the transition time from s_0 to s_1 since failed-safe probability becomes large as the state of a unit approaches s_1. Therefore, it is supposed that $p_{01}(t) = p_{01}^M F_1(t)$ where p_{01}^M is the maximum value of $p_{01}(t)$.

Failed-dangerous probability $p_{10}(t)$ can be considered to have a negative correlation with the distribution $F_2(t - x_{01})$ which satisfies $F_2(t) = 0$ if $t \leq 0$ because the probability is large just after the state of a maintained system enters s_1 and it becomes small as the state approaches s_2. From this reason, $p_{10}(t)$ is supposed to be given by $p_{10}^M \bar{F}_2(t - x_{01})$, where p_{10}^M is the maximum value of $p_{10}(t)$. Supposing $p_{01}^M = p_{10}^M$ to be 0.2 for convenience, $(p_{01}(t), p_{10}(t)) = (0, 0), (0.2, 0.2), (0.2\hat{F}_1(t), 0.2\hat{\bar{F}}_2(t - x_{01}))$ with approximative distribution functions are considered for numerical examples.

The following are supposed:

(1) $\theta(t)$: constant
(2) $S(t)$: constant
    Then, $D(t) = Kt$ is monotonically increasing, where K is a positive constant.
(3) Variance of measured degradation is monotonically increasing since coefficient c_v is constant over time.

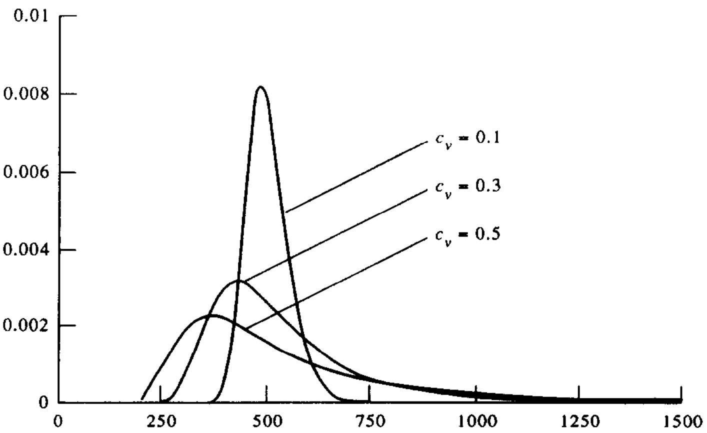

Fig. 2. Probability density functions ($K = 1.0$, $D_1 = 500$).

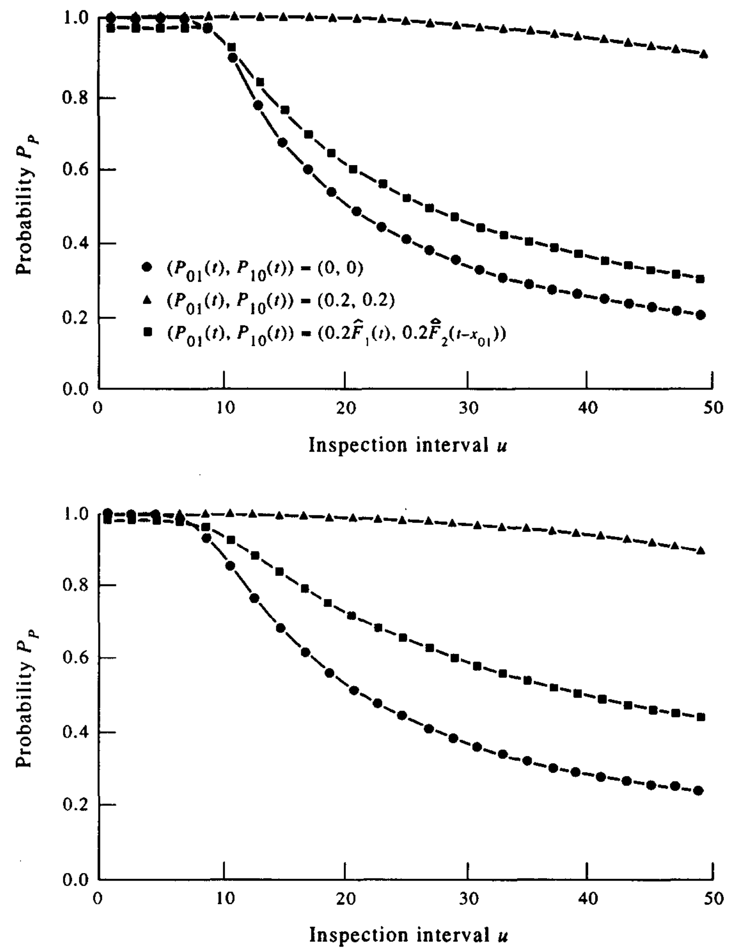

Fig. 3. Relationship between inspection interval and preventive replacement probability.

Parameters for $\hat{f}_i(t)$ are $\phi_1 = (500, \{0.1, 0.3\}, 1.0)$, $\phi_2 = (10, \{0.1, 0.3\}, 1.0)$ where $\phi_i = (D_i, c_v, K)$. Expected values M_P and M_F are 1 and 5, respectively. Fig. 2 shows some examples of the density functions for $\hat{f}_1(t)$ when $D_1 = 500$, $K = 1.0$.

Fig. 3 shows the relationship between inspection interval and preventive replacement probability. We can confirm the following from Fig. 3: (1) preventive replacement probability P_P is a monotonic decreasing curve that approaches 1 when $u \to 0$ and approaches 0 when $u \to \infty$, (2) P_P keeps the value near 1 from $u = 0$ to the neighborhood of the expected value of $\hat{f}_2(t)$ for whichever c_v, $p_{01}(t)$ and $p_{10}(t)$, (3) when $(p_{01}(t), p_{10}(t)) = (0,0)$, P_P is stable for whichever c_v, (4) when $(p_{01}(t), p_{10}(t)) = (0.2, 0.2)$ and $(0.2\hat{F}_1(t), 0.2\hat{\hat{F}}_2(t - x_{01}))$, P_P is a large value for $c_v = 0.3$, which is notable when $(0.2\hat{F}_1(t), 0.2\hat{\hat{F}}_2(t - x_{01}))$.

3.2. Relationship among the inspection time vector, the cost parameters and deterioration distribution

Inspection time vector denoted by $\mathbf{t}^*$ which minimizes Equation (33) with the constraint Equation (34) is obtained under varying probabilities $p_{01}(t)$ and $p_{10}(t)$, cost ratios C_F/C_P, C_I/C_P and C_C/C_P, and coefficient of variation c_v. Density functions $\hat{f}_i(t)$ are the same as those of the previous subsection. Cost ratio C_F/C_P varies from 1 to 20, C_I/C_P is assumed to vary from 0.05 to 0.5 and C_C/C_P is supposed to be 0.5 for convenience.

The relationship among C_F/C_P, C_I/C_P and u^* which minimizes E_C/E_T when $c_v = 0.1$ is shown in Fig. 4. The notation ∞in Fig. 4 corresponds to $u^* = \infty$, i.e., no inspection gives the minimum cost ratio. From this figure, it is confirmed that when C_I/C_P is a large value and C_F/C_P is a small value no

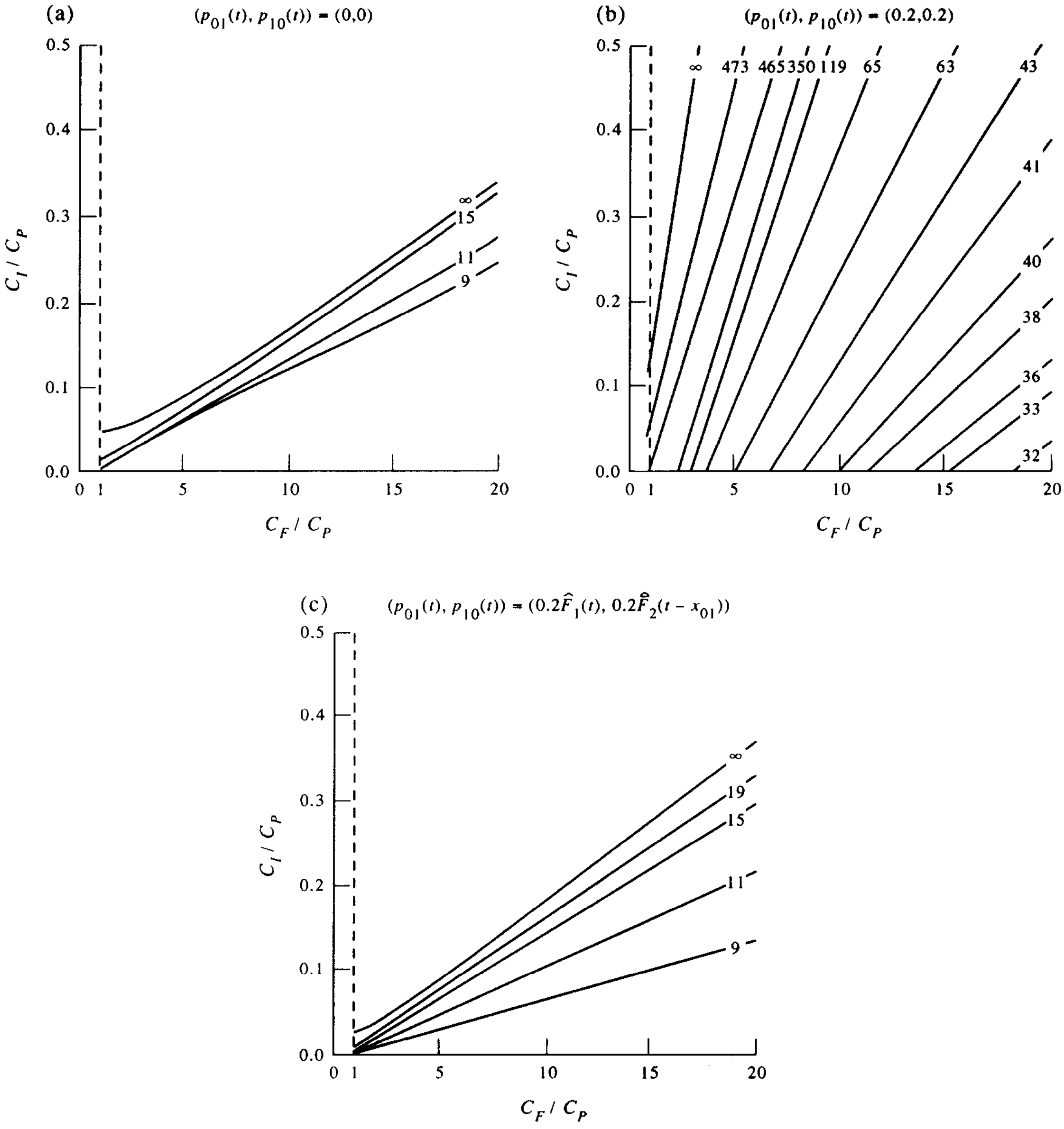

Fig. 4. Relationship among C_F/C_P, C_I/C_P and u^* ($c_v = 0.1$).

inspection makes E_C/E_T minimize. When C_F/C_P is a large value an appropriate inspection interval makes E_C/E_T minimize.

4. CONCLUDING REMARKS

In this paper, we have developed a method for determining the discrete time points of inspection for a deteriorating single-unit system which has a normal state, a symptom state and a failed state. The transitions of the states are described using a delay-time model in which the transition time from a normal state to a symptom state (symptom-time) and that from a symptom state to a failed-state (delay-time) are assumed to be independent and to have arbitrary probability density functions. The transition times are derived assuming continuous deterioration processes based on a reaction rate model which is categorized as a failure physics model. A method for determining the inspection time vector which minimizes the long-run average cost per unit time is developed considering FD and FS probabilities with regard to an imperfect inspection. Then, the relationship between the inspection time vector and the probability that a system is replaced preventively is investigated with varying FD and FS probabilities and coefficients of variation. The relationships between the inspection time vector and inspection cost, preventive maintenance cost and breakdown maintenance cost are investigated in addition to FD and FS probabilities and coefficients of variation.

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