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A simple condition monitoring model for a direct monitoring process

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Abstract

This paper addresses the problem of condition monitoring of a component which has available a measure of condition called wear. Wear accumulates over time and monitoring inspections are performed at chosen times to monitor and measure the cumulative wear. If past measurements of wear are available up to the present, and the component is still active, the decision problem is to choose an appropriate time for the next inspection based upon the condition information obtained to date. A simple model which minimizes the expected cost per unit time over the time interval between the current inspection and the next inspection time is derived, and numerical examples are given to demonstrate the solution method.

Keywords: Condition monitoring; Wear; Modelling; Maintenance

1. Introduction

The practice of applying some form of condition monitoring to plant as a basis for maintenance decision-making is growing in popularity. In order to assess the effectiveness of this monitoring process, it is necessary to model both the condition monitoring technique adopted and the context in which the monitoring process is utilized for maintenance decision making. In this way, the consequence of a particular choice of monitoring technique and of maintenance concept may be established and comparison made between alternatives. That is, the effectiveness and value of the monitoring process in the context of a maintenance problem may be assessed.

A monitoring inspection provides information z at the time of inspection. This information may or may not be precise, it may or may not be correct, and it may or may not be stochastic. However, information z is part of a wide set of information used as a basis for maintenance and subsequent

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monitoring decisions. The decision objective could be to prevent failure, to increase uptime, to reduce risk, or perhaps to rectify defects at less cost.

It is convenient here to classify information z into two classes, namely direct information and indirect information. Direct information is where z measures a variable which directly determines failure; for example the thickness of a brake pad, or the wear in a bearing. Indirect information z provides associated information which is influenced by the component condition, but is not a direct measure of the failure process; for example an oil analysis or a vibration frequency analysis. In both cases, the point of concern is to predict, given information z, the subsequent and conditional failure time distribution as an input to modelling maintenance practice. The distribution may be maintenance practice dependent.

This paper addresses the problem of condition monitoring of a component with direct measure information, which we will call wear. Throughout this paper, the generic term 'wear' denotes any type of degradation which can be measurable through inspections. Wear accumulates over time and monitoring inspections are performed at various times to check and measure the cumulative wear. The component is preventively replaced if the wear revealed at an inspection exceeds a certain defective level, and if the wear reaches or exceeds a wear limit, a failure occurs. Since we assume that the wear in the future could depend upon the entire wear history information available at past inspections, the underlying wear process is not Markovian and cannot be assumed to have the properties of stationary and independent increments over time. In this case, the traditional modelling objective of minimizing the asymptotic expected cost per unit time is evidently not a useful option. As new condition information becomes available at the time of an inspection, the inspection schedule is updated based upon the wear information obtained at the current and previous inspections. Here, a dynamic optimization procedure is needed to decide the subsequent inspection time based upon the available condition information to date.

Much of the previous work in modelling wear involves shock models in which the cumulative wear is a jump process (Esary et al., 1973; Mercer, 1961; Park, 1988b; Taylor, 1975). Christer and Wang (1992) present a special continuous wear model which assumes that the wear follows a linear pattern in which the rate of wear is a random variable. In their model, the condition information obtained at inspections is assumed to be (0, 1) type and the objective is to determine the inspection times over the envisaged life of the component to optimize the operating cost per unit time. Others have also studied continuous wear processes: Abdel-Hameed (1975), Park (1988a), and Giglmayr (1987). However, the modelling objectives adopted are typically minimizing long-run expected total cost or downtime per unit time. The key point is that previous models assume wear in the future depended only upon the current level of wear. Here we allow the future wear pattern to be dependent upon the history of wear, in other words we consider a non-stationary wear process with non-independent increments.

Thomas et al. (1991) classify the inspection models into categories according to the cost and quality of inspections, i.e. whether or not the inspection is associated with a cost and whether the inspection is perfect or not. The objective of the various classes of models is to maximize the expected time until a failure or an inspection replacement occurs. Here we assume that the monitoring inspection is costly and the wear measure z resulting from an inspection is perfect, and therefore our model belongs to the category of costly inspections and perfect information. The model differs from those considered by Thomas et al. in that it is concerned with determining the next inspection point only.

Assuming a history dependence of future wear is a natural step in the evolution of the modelling of condition monitoring for 'on condition' maintenance decision making. The long term objective is to model the general industrial case where condition information may not be precise, where it may not be accurate, and where it may not be a direct measure. This paper develops a prototype model of the problem of deciding at an inspection point the next inspection time based upon accurate direct condition information obtained at the current and past inspection times. Since this is an exploratory study, for sake of simplicity, care is taken to keep the model as simple as possible whilst attempting to capture the essence of the interactions being addressed.

2. The model

The following assumptions are made for the purpose of prototype model building.

1. Condition information z on a component obtained at an inspection is a direct measure of wear which is assumed to be a random variable.
2. Wear increases monotonically over time starting from level zero. If past wear information is available, the future wear is assumed to be dependent upon all past condition information.
3. Failure of a component is dependent upon the level of wear.
4. There are two critical levels of wear, say z_d and z_f. If wear is below z_d, all is considered to be well, if $z_d \le z < z_f$, the component is classified as defective but working. If however, $z \ge z_f$, the component is in failure state, that is, a failure has occurred. It is assumed that a failed this state is immediately obvious.
5. If a defective state is identified at an inspection or a failure occurs, the component is immediately repaired or replaced.
6. A repair is equivalent to a replacement in the sense it restores the component to an as new condition. For convenience, the repair or replacement resulting from an inspection is called an inspection replacement, and the repair or replacement performed at a failure is called a failure replacement.
7. Condition monitoring inspections have been performed at times t_i, $i = 1, 2, \dots, n$, to observe the state of wear z, namely $Z(t_i)$, $i = 1, 2, \dots, n$. For convenience we define $Z(t_0 = 0) = 0$.
8. Condition information obtained in a monitoring check is assumed to be accurate in that it reveals the true wear.

The modelling objective is to decide at the current inspection t_n, assuming the component is still active at t_n, the next inspection time t_{n+1} based upon the condition information obtained at the past inspection times up to t_n. We select the inspection point t_{n+1} to optimize the expected cost per unit time over the period (t_n, t_{n+1}), that is, from the current inspection to the next scheduled inspection time. An on line computer facility is required here to solve an optimization problem at every inspection moment. This can be considered as a sub-optimisation in the overall inspection process. It is noted also that in the limiting case of deterministic wear, the objective function proposed here will choose the right decision, namely either to plan the next inspection just before the wear level will reach the critical level z_f, or at infinity.

It is important to note that as already indicated the wear process can not be modelled as a Markov process due to the possible dependency between the past and future wear level. This also implies that the operating, defective and failed state transition structure is not available unless we known all the possible wear paths in the past and future. Since the component is renewed at failure and inspection replacements, there is an implied regenerative process. In theory, if the expected cost and time over a regenerative cycle may be modelled with inspection times as decision variables, the traditional long-run average cost criterion can be used as an objective function. However, because of assumption 2, it is assumed that t_{n+1} is dependent upon the wear information up to t_n, which poses a highly complex problem in modelling both the expected cost, and the expected cycle length, of a regenerative cycle.

The current condition inspection takes place at time t_n, where n is the sequence number of inspections, and the next scheduled inspection will be at time t_{n+1}. This implies the wear reading at time t_n, $Z(t_n)$ satisfies $Z(t_n) < z_d$, otherwise a replacement is made at t_n. For convenience, the following terms are defined. Let:

c_b denote the average cost of a failure replacement.
c_m denote the average cost of an inspection with a replacement.
c_s denote the average cost of an inspection without a replacement.
$Z(t_i)$ denote the condition information (wear) measured at the i-th inspection time from new.

$\mathbf{Z}_n = Z(t_1), Z(t_2), \ldots, Z(t_n)$ denote the history vector of condition information measured at the past inspection times $t_1, t_2, \ldots, t_{n-1}$ and the current inspection time t_n. Clearly, $Z(t_i) < z_d$, $i = 1, 2, \ldots, n$, given that no inspection replacement is made at t_n, and if the process starts at time $t_0 = 0$, then $\mathbf{Z}_0 = Z(t_0) = 0$.

Δt denote the interval between t_n and t_{n+1}, i.e. $\Delta t = t_{n+1} - t_n$.

$C(\Delta t \mid \mathbf{Z}_n)$ denote the expected cost per unit time over (t_n, t_{n+1}) given the condition information history to t_n is $\mathbf{Z}_n$.

$N(\Delta t \mid \mathbf{Z}_n)$ denote the expected number of failures over (t_n, t_{n+1}) given the condition information history to t_n is $\mathbf{Z}_n$, and immediate replacement upon failures.

Consider now the cost criterion of our model. If over the interval (t_n, t_{n+1}) there is no failure and no defect is found at t_{n+1}, the only cost incurred over the interval (t_n, t_{n+1}) is the inspection cost c_s. Similarly, if there is no failure over (t_n, t_{n+1}) but a defective state is identified in the component at t_{n+1}, then the inspection replacement cost c_m occurs. If, however, a failure arises in (t_n, t_{n+1}), a failure replacement cost c_b will be incurred. Under the assumption (5), at the moment of failure the failed component is replaced or repaired to as new and the process resumes. There does, therefore, exist the possibility of another failure occurring before t_{n+1}, and we need to consider the possibility of more than one failure occurring in (t_n, t_{n+1}).

Combining all the above three possible events, we have the expected cost per unit time measure

$$C(\Delta t \mid \mathbf{Z}_n) = \big(c_m P_r(\text{inspection replacement at time } t_{n+1} \mid \mathbf{Z}_n) \\ + c_s P_r(\text{no defect arises over } (t_n, t_{n+1}) \mid \mathbf{Z}_n) + c_b N(\Delta t \mid \mathbf{Z}_n)\big)/\Delta t. \tag{1}$$

This formulation assumes the inspection cost c_s at an inspection replacement is included in c_m, and that no inspection takes place at time t_{n+1} if there has been a failure replacement in (t_n, t_{n+1}).

According to assumption 4, Eq. (1) becomes

$$C(\Delta t \mid \mathbf{Z}_n) = \big(c_m P_r\{z_f > Z(t_{n+1}) \geq z_d \mid \mathbf{Z}_n\} + c_s P_r\{Z(t_{n+1}) < z_d \mid \mathbf{Z}_n\} + c_b N(\Delta t \mid \mathbf{Z}_n)\big)/\Delta t. \tag{2}$$

Since we know that

$$P_r\{z_f > Z(t_{n+1}) \geq z_d \mid \mathbf{Z}_n\} = P_r\{Z(t_{n+1}) < z_f \mid \mathbf{Z}_n\} - P_r\{Z(t_{n+1}) < z_d \mid \mathbf{Z}_n\},$$

it follows that

$$C(\Delta t \mid \mathbf{Z}_n) = \big(c_m P_r\{Z(t_{n+1}) < z_f \mid \mathbf{Z}_n\} + (c_s - c_m) P_r\{Z(t_{n+1}) < z_d \mid \mathbf{Z}_n\} + c_b N(\Delta t \mid \mathbf{Z}_n)\big)/\Delta t. \tag{3}$$

It is necessary to establish expressions for the component terms of Eq. (3). We first discuss the expression for the expected number of failures over Δt, i.e. $N(\Delta t \mid \mathbf{Z}_n)$.

3. Expected number of failures over Δt, $N(\Delta t \mid \mathbf{Z}_n)$

From the definition of $N(\Delta t \mid \mathbf{Z}_n)$, we have

$$N(\Delta t \mid \mathbf{Z}_n) = \sum_{m=1}^{\infty} m \cdot P_r\{m \text{ failures in } \Delta t \mid \mathbf{Z}_n\}. \tag{4}$$

Let $f(x; z_f \mid \mathbf{Z}_n)$ denote the p.d.f. of time x to failure measured from the inspection point t_n in the absence of further inspections, given the condition information $\mathbf{Z}_n$ and $Z(t_n) < Z_d$. Let $F(x; z_f \mid \mathbf{Z}_n)$ denote the c.d.f. of x given the same condition information. Since a failure occurs if and only if wear reaches or crosses level z_f, $f(x; z_f \mid \mathbf{Z}_n)$ is the p.d.f. of time to failure from t_n, where the wear increases continuously from level $Z(t_n)$ at time t_n to z_f at time $(t_n + x)$.

If x_1 denotes the time to the first failure from t_n, and x_j ($j=2, 3, \ldots, m$) denotes the time to the jth subsequent failure from the last renewal, then we have that the probability of m failures occurring in (t_n, t_{n+1}) given condition information $\mathbf{Z}_n$ is given by the convolution integral

$$P_r(m \text{ failures in } \Delta t \mid \mathbf{Z}_n)$$
$$= \int_0^{\Delta t} f(x_1; z_f \mid \mathbf{Z}_n) \int_0^{\Delta t - x_1} f(x_2; z_f \mid \mathbf{Z}_0) \int \cdots \int_0^{\Delta t - \Sigma_{j=1}^{m-1} x_j} f(x_m; z_f \mid \mathbf{Z}_0) \left\{1 - F\left(\Delta t - \Sigma_{j=1}^{m} x_j; z_f \mid \mathbf{Z}_0\right)\right\} \mathrm{d}x_m \, \mathrm{d}x_{m-1} \cdots \mathrm{d}x_1, \tag{5}$$

where, of course, it is necessary that $\Sigma_{j=1}^{m} x_j \le \Delta t$.

Now, if we know the expression for $f(x_1; z_f \mid \mathbf{Z}_n)$ and $f(x_j; z_f \mid \mathbf{Z}_0)$, $j=2, 3, \ldots, m$, the probability of m renewals, Eq. (5), may be calculated analytically or numerically and, therefore, the expected number of failures, $N(\Delta t \mid \mathbf{Z}_n)$, may be determined from Eq. (4). However, it is not generally a simple task to derive the expressions for $f(x_1; z_f \mid \mathbf{Z}_n)$ and $f(x_j; z_f \mid \mathbf{Z}_0)$ from the wear process. Even when there is an expression for $f(x_1; z_f \mid \mathbf{Z}_n)$ and $f(x_j; z_f \mid \mathbf{Z}_0)$ and assuming independent wear increments (Cox, 1962; Prabhu, 1965), very few analytic solutions exist for the convolution expression (5) and a numerical technique will generally be necessary. The underlying modified renewal process may also be described in terms of integral equations (Christer and Jack, 1991). An alternative approximate procedure for determining $N(\Delta t \mid \mathbf{Z}_n)$ is given below by developing approximations for P_r (m failures in $\Delta t \mid \mathbf{Z}_n$) for use in Eq. (4).

4. Approximation to number of failures over Δt, $N(\Delta t \mid \mathbf{Z}_n)$

In order to use a numerical technique to establish the probability of m failures in Δt, we divide the interval Δt into M bins of length h. Here it is assumed that the bins are sufficiently small that more than one failure occurring in a bin is very unlikely, and that if a failure occurs it occurs at the end point of the bin, at time $k \cdot h$, where k is an integer satisfying $1 \le k \le M$. If the first failure occurs in the k_1-th bin, then the time to the first failure is $x_1 = h \cdot k_1$ ($1 \le k_1 \le M$). Let x_2 denote the time to the second failure from x_1. We have $x_2 = h \cdot k_2$ ($1 \le k_2 \le M - K_1$). Similarly, if x_j denotes the time to the j-th failure from the previous failure time x_{j-1}, then $x_j = h \cdot k_j$ ($1 \le k_j \le M - \Sigma_{i=1}^{j-1} k_i$). Here the k_j value denotes the number of bins from x_{j-1} to x_j. See Fig. 1.

For convenience we define the probability that wear is equal to or more than z_f given condition information $\mathbf{Z}_n$, that is a failure state at time $t_n + k_1 h$, as

$$P(z_f; x_1 \mid \mathbf{Z}_n) = P_r\{Z(t_n + x_1) \ge z_f \mid \mathbf{Z}_n\},$$

and similarly, for a subsequent failure, we define

$$P(z_f; x_j \mid \mathbf{Z}_0) = P_r\{Z(x_j) \ge z_f \mid \mathbf{Z}_0\}, \quad j = 2, 3, \ldots, m.$$

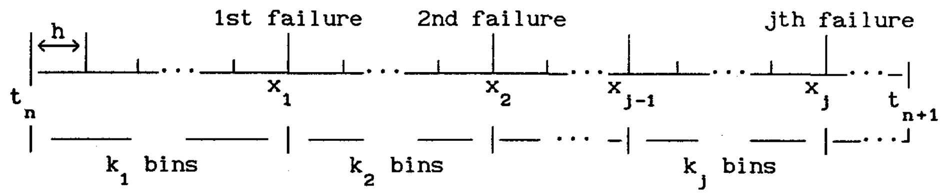

Fig. 1. Discrete approximation of a continuous process.

Since $Z(t_n + x_1) \geq z_f$ and $Z(x_j) \geq z_f$ occur if and only if the failure times are less than or equal to x_1 and x_j respectively, $j = 2, 3, \ldots, m$, we have

$$P(z_f;\ x_1 \mid Z_n) = F(x_1;\ z_f \mid Z_n) \ \text{ and } \ P(z_f;\ x_j \mid Z_0) = F(x_j;\ z_f \mid Z_0). \tag{6}$$

It is also convenient to define the failure interval probability

$$\Delta P(z_f;\ k_1 \mid Z_n) = P_r\{Z(t_n + h \cdot k_1) \geq z_f \mid Z_n\} - P_r\{Z[t_n + h \cdot (k_1 - 1)] \geq z_f \mid Z_n\},$$

and

$$\Delta P(z_f;\ k_j \mid Z_0) = P_r\{Z(h \cdot k_j) \geq z_f \mid Z_0\} - P_r\{Z[h \cdot (k_j - 1)] \geq z_f \mid Z_0\}, \quad j \geq 2.$$

After some manipulation and using Eq. (6), the discrete approximation form of Eq. (5) becomes

$$\begin{aligned} &P_r\ (m \text{ failures in } \Delta t \mid Z_n) \\ &= \sum_{k_1=1}^{M} \left(\Delta P(z_f;\ k_1 \mid Z_n) \left(\sum_{k_2=1}^{M-S_1} \Delta P(z_f;\ k_2 \mid Z_0) \left(\cdots \left(\sum_{k_j=1}^{M-S_1} \Delta P(z_f;\ k_j \mid Z_0) \left(\cdots \right.\right.\right.\right.\right. \\ &\quad \left.\left.\left.\left.\left. \times \left(\sum_{k_m=1}^{M-S_m} \Delta P(z_f;\ k_m \mid Z_0) P_r\left(z_f;\ \Delta t - h \cdot \sum_{j=1}^{m} k_j \mid Z_0 \right) \cdots \right) \right) \right) \right) \right), \end{aligned} \tag{7}$$

where $S_j = \sum_{i=1}^{j-1} k_i$ and $h \cdot \sum_{j=1}^{m} k_j \leq \Delta t$.

Eq. (7) is still non-trivial to evaluate. However since the model objective is to determine the optimal time of the next inspection, the probability of having more than, say, 4 failures in an optimal interval (t_n, t_{n+1}) should be very small if $c_f > c_m$, otherwise it would not be the optimal interval. Since we know that

$$1 - P_r(Z(t_{n+1}) < z_f \mid Z_n) = \sum_{m=1}^{\infty} P(m \text{ failures in } \Delta t \mid Z_n), \tag{8}$$

and

$$P_r\ (\text{more than } r \text{ failures in } \Delta t \mid Z_n) = \sum_{m=r+1}^{\infty} P_r(m \text{ failures in } \Delta t \mid Z_n),$$

then since the RHS of Eq. (8) is rapidly convergent for finite Δt, relatively few terms need be calculated on the RHS equation (4) for any realistic problem. In fact, in the numerical example below, the probability of having 4 or more failures in Δt is very small when $\Delta t = 10$ ($\leq 10^{-4}$). The actual cut off point in the summation will, of course, also depend upon the relative values of c_m, c_s and c_b.

Now, if we can derive expressions to calculate values of $P_r\{Z(t_n + x) < z^* \mid Z_n\}$ and $P_r\{Z(x) < z^* \mid Z_0\}$, $0 < x \leq t_{n+1} - t_n$ for $z^* = z_f$ or z_d, the expected unit time cost, Eq. (3), can be determined using Eqs. (4), (6) and (7), and the decision problem of the next inspection point t_{n+1} solved. Procedures for evaluating these probabilities are given in the next section.

5. Statistical modelling of wear probability measures $P_r(Z(t_n + x) < z^* \mid Z_n)$ and $P_r(Z(x) < z^* \mid Z_0)$

In this section possible methods are considered for evaluating the probability measures $P_r(Z(t_n + x) < z^* \mid Z_n)$ and $P_r(Z(x) < z^* \mid Z_n)$, for $n \geq 0$. This is a relatively novel problem related to what Christer (1991) referred to as the prognosis issue. A two phase approach is suggested here. First, to model the condition history data to obtain a wear measure model, and secondly to utilize the wear measure as a covariate in a stochastic wear model.

First we consider the wear measure model. Having a measure of wear $Z(t_i)$ at an inspection point t_i, $i = 0, 1, \dots, n$, a wear measure model of wear at time t for $t > t_n$, $\hat{Z}(t)$, may be formed by extrapolating the time series $(t_i, Z(t_i))$, $i = 0, 1, \dots, n$, by fitting a regression model to the historical data using the least squares method. Previous histories can be utilized to suggest the appropriate mathematical form to be regressed. One such form the authors have observed to be used within industry to measure wear at time t based upon condition information is the power law function $\hat{Z}(t) = \lambda t^{\rho}$, for $t > t_n$. We will assume this function form of wear model for the purpose of demonstrating the modelling methodology. It should be noted, however, that although this wear measure can be found within industry, its use there as a deterministic model for future decision-making in what is actually a stochastic problem. Here the parameters of the function $\hat{Z}(t)$, i.e. λ and ρ, are determined by the historical data $(t_i, Z(t_i))$, and, therefore, the estimated increment in wear over (t_n, t), $\{\hat{Z}(t) - Z(t_n)\}$, is also dependent upon the wear history.

On the assumption that wear measures $Z(t_i)$ are accurate, and the process commences with a new component, the wear curve is constrained to pass through $(t_0, Z(t_0)$ and, in order to give weight to the last observation, $(t_n, Z(t_n))$. Other options do, of course, exist and would be explored in specific cases. Having a model for the measure of wear, a measure of the wear increment over time $(t_n, t_n + x)$ is given as

$$\Delta Z(x, t_n) = \hat{Z}(t_n + x) - Z(t_n). \tag{9}$$

$\hat{Z}(t_n + x)$ is one of many possible measures of wear at $t_n + x$ based upon the monitoring history of the component. It is, however, expected to be correlated to the actual wear which is a random variable, and the increment over $(t_n, t_n + x)$, $\Delta Z(x, t_n)$, is assumed, therefore, to be an appropriate covariate of wear.

The second phase is to utilize this wear measure, Eq. (9), as a covariate in the stochastic wear model. Past history data could suggest an appropriate distribution of wear, but for demonstration purposes, we suppose the distribution of wear increment ΔZ over time period $(t_n, t_n + x)$ is Weibull with shape parameter β and scale parameter α, namely

$$f(\Delta Z) = \alpha\beta(\alpha\, \Delta Z)^{\beta - 1} \exp\{-(\alpha\, \Delta Z)^{\beta}\}.$$

If the scale parameter α is assumed to be proportional to $\{\Delta Z(x, t_n)\}^{-1}$, i.e.

$$\alpha(x, t_n) = \alpha_0\{\Delta Z(x, t_n)\}^{-1}, \tag{10}$$

then, from the definition of the Weibull distribution, we have that the mean wear increment over $(t_n, t_n + x)$ will take the form of

$$\mu(x, t_n) = \Gamma(1 + 1/\beta)\Delta Z(x, t_n)/\alpha_0 = \mu_0 \Delta Z(x, t_n) \tag{11}$$

where μ_0 is a constant. That is, on the assumption of relation (10), the mean wear increment over $(t_n, t_n + x)$ is proportional to the wear measure obtained from the regression model (9). Clearly, the mean wear increment $\mu(x, t_n)$ is dependent upon both t_n and x as well as the wear history $\mathbf{Z}_n$.

For any wear level z^*, we have, assuming Eq. (10) for α, that the probability the wear at $t_n + x$ does not reach level z^* given $z^* \geq Z(t_n)$, is

$$P_r(Z(t_n + x) < z^* \mid \mathbf{Z}_n) = 1 - \exp\left\{-\{\Delta z^* \alpha(x, t_n)\}^{\beta}\right\}, \tag{12}$$

where $\Delta z^* = z^* - Z(t_n)$, α is given by Eq. (10), and $\Delta Z(x, t_n)$ is given by Eq. (9) and is assumed to take the form of $\Delta Z(x, t_n) = \lambda(t_n + x)^{\rho} - Z(t_n)$. A similar parameterization form was used to model the growth of 'pit depth' by Cottis et al. (1990) and the growth of cracks at welds by Scarf et al. (1993).

It is not immediately possible to parameterize $P_r(Z(x) < z^* \mid \mathbf{Z}_0)$ in the same way as $P_r(Z(t_n + x) < z^* \mid \mathbf{Z}_n)$ because of the absence of wear history $\mathbf{Z}_n$ and, therefore, of a wear model $\Delta Z(x)$. It is proposed in the $\mathbf{Z}_0$ case that a prior model of wear, obtained perhaps from an analysis of a similar component, be utilized initially and updated as inspection data of the component becomes available.

There could be a considerable amount of data of similar components for use in determining the appropriate distribution and parameters for a stochastic wear model, and appropriate statistical tests are required to direct model selection (Baker and Wang, 1991).

The following numerical example illustrates the idea.

6. Numerical example

Suppose the cumulative wear limit for a breakdown z_f is 20, breakdown cost $c_b = £200$, the defective level z_d is 18 with inspection repair cost $c_m = £100$, and the inspection cost $c_s = £20$. The Weibull distribution is chosen as our underlying distribution of wear increment over $(t_n, t_n + x)$ with the scale parameter $\alpha = \alpha_0 \Delta Z(x, t_n)^{-1}$ and shape parameter β. Suppose further that we have two cases in hand. For case 1, there were two previous inspections at $t_1 = 4$ and $t_2 = 8$ with $Z(t_1) = 3$ and $Z(t_2) = 8$. For case 2, we also have two previous inspections with $t_1 = 4$, $t_2 = 6$ and $Z(t_1) = 6$, $Z(t_2) = 8$. In both cases, the limited data uniquely define wear curves which satisfy all data points, namely,

$$\hat{Z}(t) = \begin{cases} 0.422 \cdot t^{1.415}, & \text{case 1,} \quad \text{for } t > t_2 = 8, \\ 2.245 \cdot t^{0.709}, & \text{case 2,} \quad \text{for } t > t_2 = 6. \end{cases}$$

See Fig. 2.

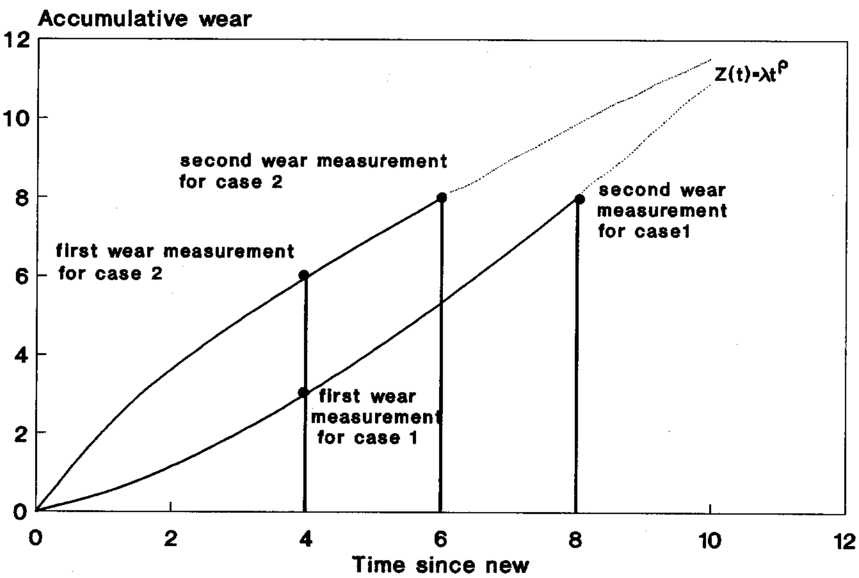

Fig. 2. Wear paths for case 1 and case 2.

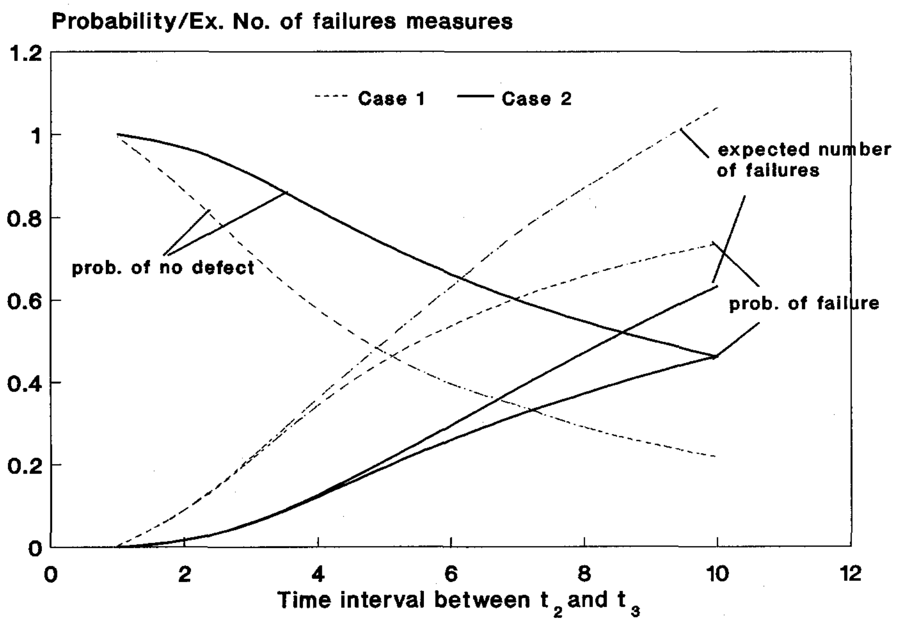

Fig. 3. Probabilities of at least one failure, of no defects arising, and the expected number of failures over (t_2, t_3) for cases 1 and 2.

The imposed constraints $\hat{Z}(t_2) = Z(t_2)$ and $\hat{Z}(0) = 0$ are satisfied automatically here.

If we assume for demonstration purposes that the parameter values for α_0 and β are 0.55 and 1.2 respectively, Eqs. (9), (12), (7) and (4) provide measures of expected numbers of failures over time from the current inspection, and the respective probabilities of no defect arising and of failures (≥ 1) over time, all of which are conditional upon the previous wear information. For example in case 1, the probabilities of no defect arising and of failures are given respectively by

$$P_r\{Z(t_3) < z_d \mid Z_2\} = 1 - \exp\left\{-\left\{[z_d - Z(t_2)]\alpha_0\left[\hat{Z}(t_3) - Z(t_2)\right]\right\}^{\beta}\right\},$$

and

$$P_r\{Z(t_3) \geq z_f \mid Z_2\} = \exp\left\{-\left\{[z_f - Z(t_2)]\alpha_0\left[\hat{Z}(t_3) - Z(t_2)\right]\right\}^{\beta}\right\},$$

where $z_d = 18$, $z_f = 20$, $\alpha_0 = 0.55$, $\beta = 1.2$, $Z(t_2) = 8$ and $\hat{Z}(t_3) = 0.422 \cdot t_3^{1.415}$.

These probability measures together with the expected number of failures from Eqs. (4) and (7) for both case 1 and case 2 are shown graphically in Fig. 3. The cost consequence of objective function (3) can now be calculated and is shown in Fig. 4.

As we expected, as the time to the next inspection increases, the probability of failures increases and the probability of no defect arising decreases. It is also noted from Fig. 4 that for the two wear patterns, the optimal time to the next inspection for case 1 is shorter than that for case 2. This is again as expected, since the former has a trend of greater rate of wear over Δt compared to the latter.

The method of the parameterization of $P_r(Z(t_n + x) < z^* \mid Z_n)$ is not unique. Since the covariate path has been modelled as $\Delta Z(x)$, it becomes the time dependent covariate and the traditional PHM or accelerated life model (Cox and Oakes, 1984) can also be used here to model $P_r(Z(t_n + x) < z^* \mid Z_n)$.

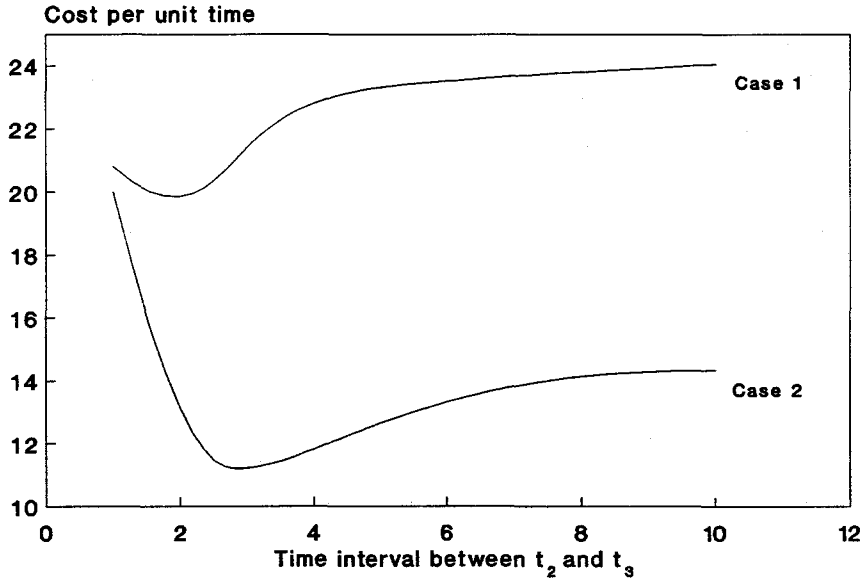

Fig. 4. Expected cost per unit time over (t_2, t_3) for cases 1 and 2.

However, as already indicated the final selection of the parameterization of $P_r(Z(t_n + x) < z^* \mid Z_n)$ is determined by how good this model fits the wear data, which requires suitable statistical tests. We leave this to a future study when real life data is available.

7. Conclusions

For a direct wear monitoring process, this paper has established a model for condition monitoring inspection which may be used to minimize the expected total cost per unit time over the time interval from the current inspection to the next inspection time. A statistical model has been proposed to model the key probability measures in the formulation of the average cost. Equally well, a model could have been formulated in terms of downtime or risk. Numerical examples are given to demonstrate the model's computation and operation. In truncating the summation of probability of m and more failures, the process does inevitably underestimate the expected number of failures over Δt. However, even for relative large Δt the error is expected to be small, and in the numerical example, the probability of more than 4 failures in Δt, where $\Delta t = 10$, is of order 10^{-4} with negligible influence upon the objective function curve (Fig. 4). This is particularly true in the neighborhood of the optimal value.

The existence of a finite optimal solution for Δt depends upon the relative values of parameters c_m, c_s and c_b as well as the underlying wear process. It can be shown that if $c_b > c_m > c_s$ and the wear is monotonically increasing, which is relative general for a wear process, an unique and finite optimal solution to the model exists for the perfect inspection case considered here.

The probability of a specific wear level at time $t_n + x$ conditional on previous wear information is modelled taking the wear increment measure $\Delta Z(x, t_n)$ as a covariate obtained by fitting a regression model to the data of wear history. Basically, there could be some loss in information since it is not guaranteed that the fitted curve passes through all the previous measuring points. But, since we have assumed that wear increases monotonically, a power law function is expected to give a good fit. The specific wear model based upon Weibull distribution is given in Section 4 for the purpose of demonstration. The model developed in this paper is aimed at exploring how condition monitoring information might be used to influence subsequent on condition maintenance decision making. More specific studies on the statistical modelling of the wear process and the development of appropriate statistical tests are required, but await real life data. It is hoped this prototype modelling will motivate the necessary data collection.

There remains the problem of modelling the indirect condition information case, that is, where condition information obtained is an indirect measure of the failure characteristics, i.e. failure may be mainly determined by age but is influenced by the condition information obtained in the past. A model under periodic inspections for this case of indirect condition information dependent failure rate would be a very useful extension to the current research. The assumption of perfect testing also requires to be relaxed.

Finally, it is noted that as the number of inspections increases, the cost per unit time over the horizon to the next optimal inspection may have increased to the point where a replacement is appropriate, that is the problem no longer remains a simple inspection problem but now becomes an inspection/replacement decision. This relates to the issue of 'one period look ahead rules' (Dekker and Dijkstra, 1992).

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