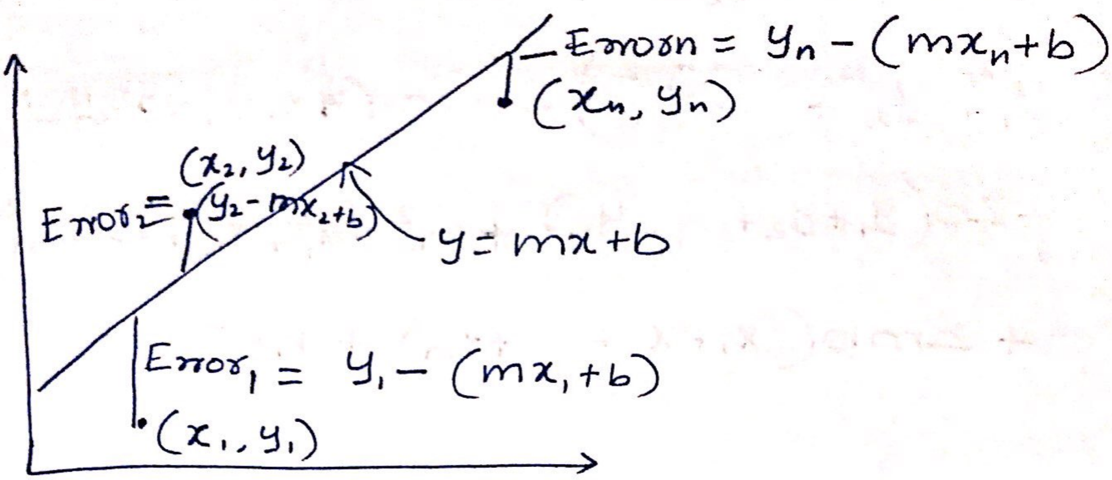


Least squares method.

$$SE_{line} = (y_1 - (mx_1+b))^2 + (y_2 - (mx_2+b))^2 + \cdots + (y_n - (mx_n+b))^2$$

find the m and b that minimizies SE_{line}

$$= y_1^2 - 2y_1(mx_1+b) + (mx_1+b)^2$$

$$+ y_2^2 - 2y_2(mx_2+b) + (mx_2+b)^2$$

$$\vdots$$

$$+ y_n^2 - 2y_n(mx_n+b) + (mx_n+b)^2$$

$$= y_1^2 - 2y_1mx_1 - 2y_1b + m^2x_1^2 + 2mx_1b + b^2$$

$$+ y_2^2 - 2y_2mx_2 - 2y_2b + m^2x_2^2 + 2mx_2b + b^2$$

$$\vdots$$

$$+ y_n^2 - 2y_nmx_n - 2y_nb + m^2x_n^2 + 2mx_nb + b^2$$

Add all the columns

$$SE_{line} = (y_1^2 + y_2^2 + \cdots y_n^2) - 2m(x_1y_1 + x_2y_2 + \cdots x_ny_n)$$
$$-2b(y_1 + y_2 + \cdots y_n) + m^2(x_1^2 + x_2^2 + \cdots x_n^2)$$
$$+ 2mb(x_1 + x_2 + \cdots + x_n) + nb^2$$

$$\frac{y_1^2 + y_2^2 + \cdots + y_n^2}{n} = \overline{y^2} \quad \left[y_1^2 + y_2^2 + \cdots y_n^2 = n\overline{y^2} \right]$$

$$\frac{x_1y_1 + x_2y_2 + \cdots + x_ny_n}{n} = \overline{xy} \quad \left[x_1y_1 + x_2y_2 + \cdots + x_ny_n = n\overline{xy} \right]$$

$$\frac{y_1 + y_2 + \cdots y_n}{n} = \overline{y}$$

$$\frac{x_1^2 + x_2^2 + \cdots + x_n^2}{n} = \overline{x^2}$$

$$\frac{x_1 + x_2 + \cdots + x_n}{n} = \overline{x}$$

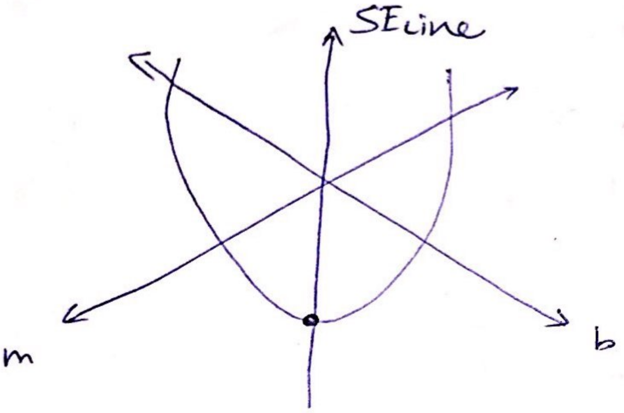
Scanned with CamScanner

Let's rewrite with new notation

$$SE_{LINE} = n\overline{y^2} - 2mn\,\overline{xy} - 2bn\overline{y} + m^2 n\,\overline{x^2} +$$

$$2mbn\overline{x} + nb^2$$

Let's optimize this.



$$\frac{\partial SE}{\partial m} = 0 \qquad \frac{\partial SE}{\partial b} = 0$$

$$\frac{\partial SE}{\partial m} = -2n\,\overline{xy} + 2mn\overline{x^2} + 2bn\overline{x} = 0 \quad \text{— (1)}$$

$$\frac{\partial SE}{\partial b} = -2n\overline{y} + 2mn\overline{x} + 2nb = 0 \quad \text{— (2)}$$

divide eq (1) and eq (2) by $2n$

$$-\overline{xy} + m\overline{x^2} + b\overline{x} = 0$$

$$-\overline{y} + 2m\overline{x} + b = 0$$

~~Subtrac~~
Add $\overline{xy}$ at both sides in eq(1)

$$\cancel{\overline{xy}} - \cancel{\overline{xy}} + m\overline{x^2} + b\overline{x} = 0 + \overline{xy}$$

Add $\overline{y}$ at both sides for eq(2)

$$\cancel{\overline{y}} - \cancel{\overline{y}} + m\overline{x} + b = 0 + \overline{y}$$

$$m\overline{x^2} + b\overline{x} = \overline{xy} \quad \text{—— (1)}$$
$$m\overline{x} + b = \overline{y} \quad \text{—— (2)}$$

divide eq(1) by $\overline{x}$ $\quad (y = mx + b)$

$$\frac{m\overline{x^2}}{\overline{x}} + \frac{b\cancel{\overline{x}}}{\cancel{\overline{x}}} = \frac{\overline{xy}}{\overline{x}}$$

$$\frac{m\overline{x^2}}{\overline{x}} + b = \frac{\overline{xy}}{\overline{x}} \quad \text{—— (1)}$$
$$\underset{(-)}{m\overline{x}} \underset{(-)}{+ b} \underset{(-)}{=} \overline{y} \quad \text{—— (2)}$$

eq(1) − eq(2)

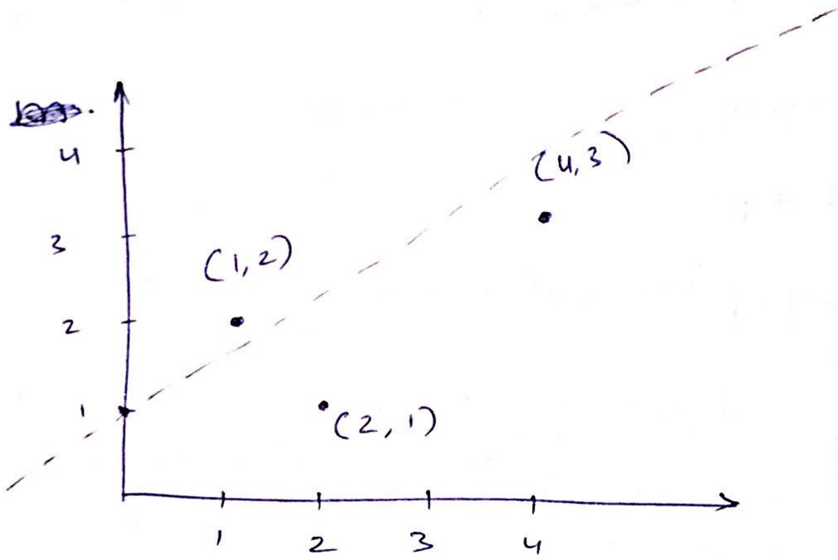
$$m\left(\overline{x} - \frac{\overline{x^2}}{\overline{x}}\right) = \overline{y} - \frac{\overline{xy}}{\overline{x}}$$

$$m = \frac{\overline{y} - \frac{\overline{xy}}{\overline{x}}}{\overline{x} - \frac{\overline{x^2}}{\overline{x}}} \quad \text{—— (1)}$$

To simply eq 1 divide it by $\bar{x}$

$$m = \frac{\bar{y} - \frac{\overline{xy}}{\cancel{\bar{x}}}}{\bar{x} - \frac{\overline{x^2}}{\bar{x}}} \times \frac{\cancel{\bar{x}}}{\bar{x}} = \frac{\bar{x}\,\bar{y} - \overline{xy}}{(\bar{x})^2 - \overline{x^2}}$$

$$b = \bar{y} - m\bar{x}$$



x	y	$\bar{x}$	$\bar{y}$	$\bar{x}\,\bar{y}$	xy	$\overline{xy}$	x^2
1	2						
2	1						
4	3						

$\bar{x} = \frac{1+2+4}{3} = \frac{7}{3}$ $\quad \bar{y} = \frac{2+1+3}{3} = 2$ $\quad \overline{xy} = \frac{2+2+12}{3} = \frac{16}{3}$

$\overline{x^2} = \frac{1^2+2^2+4^2}{3} = \frac{21}{3} = 7$ $\quad b = 2 - \frac{\cancel{3}}{\cancel{7}} \times \frac{\cancel{7}}{\cancel{3}} = 1$

$$m = \frac{\frac{7}{3} \times 2 - \frac{16}{3}}{\left(\frac{7}{3}\right)^2 - 7} = \frac{\frac{14}{3} - \frac{16}{3}}{\frac{49}{9} - \frac{63}{9}} = \frac{\frac{-2}{3}}{\frac{-14}{9}} = \frac{+\cancel{2}}{\cancel{3}} \times \frac{+\cancel{9}^{3}}{\cancel{14}_{7}} = \frac{3}{7}$$