

Representations

Table of Contents:

A.B. Atanasov

1. Preambulum: On Abstraction
2. Stokes' Theorem and Modern Differential Geometry
 - (a) Generalizations of the Fundamental Theorem of Calculus
 - (b) The Notion of a Form
 - (c) The exterior derivative and the boundary
 - (d) The generalized stokes theorem
 - (e) To manifolds
 - (f) Vector Fields, Tensor Fields, and Differential Forms
 - (g) A Metric
 - (h) The Lie Derivative
 - (i) Exercises
3. Applications to Physics: Symplectic Geometry and Einstein Gravity
 - (a) Newton's Laws reformulated: Hamilton's Equations of Motion
 - (b) Time Evolution: Symplectomorphism
 - (c) The Riemann Curvature: Parallel Transport and its Violations
 - (d) Einstein's Field Equations of Gravitation
 - (e) Exercises
4. Lie Theory
 - (a) Vector Fields on a Homogenous Space
 - (b) Algebraic Background
 - (c) Cartan's Criterion
 - (d) The Example of \mathfrak{sl}_2 and its Generalizations
 - (e) The Euclidean Weight Space
 - (f) Reflection Groups
 - (g) Full Classification

- (h) Affine Lie Algebras and Kac Moody Algebras
 - (i) (You need to learn about the prior section)
 - (j) Exercises
5. Representation Theory
- (a) Vector Spaces: Direct Sum and Direct Product
 - (b) Representing Algebraic Structures as Operators on a Vector Space
 - (c) Some Examples in Lie Algebras
 - (d) Spinors and Triality
 - (e) Generalizations of the Fourier Transform
 - (f) Applications to Molecular Theory
 - (g) Representation theory of the Symmetric Group
 - (h) Weyl Character Formula