

## Exercises from Lecture 2

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1. Generalize the example in class to  $S_n$  acting on  $\mathbb{C}$ , and preserves  $\mathbb{C}\langle e_1 + \dots e_n \rangle = \mathbb{C}$ . Then write  $\mathbb{C}^n = \mathbb{C} \oplus V$  and show that  $V$  is irreducible.
  2. A representation  $V$  of  $G$  is called self-dual iff there is a  $G$ -isomorphism  $V \rightarrow V^*$ .
    - (a)  $V$  is self dual iff  $\mathbb{C} \subseteq V \otimes V$  is a direct summand.
    - (b) Use this to show that all irreps of the symmetric group  $S_n$  are self-dual
- (Hint: A representation is self-dual iff its characters take real values)