Build Night 3

Baseline & Follow-the-Winner Strategies I

Team Meeting Agenda

- When2Meet for faculty advisor meeting (5min)
- Discuss homework (10min)
- Go over benchmark strategies (15min)
- Go over Follow-the-Winner strategies (15min)

Project Schedule

1. Welcome & Problem Definition
2. Power Outage – Python and Reading Review
3. Baseline + Follow-the-Winner Strategies I (you are here!)
4. Follow-the-Winner Strategies II
5. Follow-the-Loser Strategies I
6. Follow-the-Loser Strategies II
7. Pattern-Matching Strategies I
8. Pattern-Matching Strategies II
9. Backtesting, Experimentation, and Comparison
10. Poster Work & Presentation Practice

Homework Discussion

ALGORITHM 1: Online portfolio selection framework.

Input: $\mathbf{x}_1^n$: Historical market sequence
Output: S_n: Final cumulative wealth

Initialize $S_0 = 1, \mathbf{b}_1 = \left(\frac{1}{m}, \ldots, \frac{1}{m}\right)$
for $t = 1, 2, \ldots, n$ **do**
  Portfolio manager computes a portfolio $\mathbf{b}_t$;
  Market reveals the market price relative $\mathbf{x}_t$;
  Portfolio incurs period return $\mathbf{b}_t^\top \mathbf{x}_t$ and updates cumulative return $S_t = S_{t-1} \times \left(\mathbf{b}_t^\top \mathbf{x}_t\right)$;
  Portfolio manager updates his/her online portfolio selection rules ;
end

Benchmark Strategies

Buy-And-Hold

- Invest with an initial portfolio, long that portfolio with only adjustments based on market fluctuations
- Useful to produce a *market index*

$$\mathbf{b}_{t+1} = \frac{\mathbf{b}_t \odot \mathbf{x}_t}{\mathbf{b}_t^\top \mathbf{x}_t}$$

Buy-And-Hold: Example

- Excel time

Best Stock

- A special base of BAH where we put all capital with the best performance in hindsight (i.e. with the best cumulative performance)

$$\mathbf{b}^{\circ} = \arg\max_{\mathbf{b}\in\Delta_m} \mathbf{b} \cdot \left(\bigodot_{t=1}^{n} \mathbf{x}_t \right)$$

Constant Rebalanced Portfolios (BCRP)

- Rebalance the portfolio to some fixed proportion every period
- There is evidence to suggest that this strategy can produce an exponentially increasing return because it takes advantage of the underlying market volatility

Follow-the-Winner Strategies (Part I)

What is a FTW Strategy?

- We increase the weights of successful stocks based on the performance in the previous trading period

Exponential Gradient

- We track the stock with the best performance but keep our new portfolio close to the existing one
- Optimization problem:

$$\mathbf{b}_{t+1} = \underset{\mathbf{b} \in \Delta_m}{\arg\max} \quad \eta \log \mathbf{b} \cdot \mathbf{x}_t - R(\mathbf{b}, \mathbf{b}_t),$$

- η = learning rate , $R(b, b_t)$ is a regularization term

Exponential Gradient

- In English: “Find the value of *b* such that this expression is maximized”

$$\mathbf{b}_{t+1} = \arg\max_{\mathbf{b}\in\Delta_m} \quad \eta \log \mathbf{b}\cdot\mathbf{x}_t - R(\mathbf{b}, \mathbf{b}_t),$$

$$R(\mathbf{b}, \mathbf{b}_t) = \sum_{i=1}^{m} b_i \log \frac{b_i}{b_{t,i}}.$$

Exponential Gradient

- Using some Taylor series magic, the optimization problem yields the update rule:

$$b_{t+1,i} = b_{t,i} \exp\left(\eta \frac{x_{t,i}}{\mathbf{b}_t \cdot \mathbf{x}_t}\right)/Z, \quad i = 1, \ldots, m,$$

- *Z* makes sure the portfolio sums up to 1 (normalization term)

Exponential Gradient

- Regret = value of difference between a made decision and the optimal decision

Follow-the-Leader

- Tracks the BCRP strategy and uses it as the basis for an optimization problem

$$\mathbf{b}_{t+1} = \mathbf{b}_t^* = \arg\max_{\mathbf{b}\in\Delta_m} \sum_{j=1}^{t} \log\left(\mathbf{b}\cdot\mathbf{x}_j\right).$$

- We will have to feed this into a constrained optimization algorithm!
  - There may or may not be a closed-form solution for this

Follow the Regularized Leader

- Works similar to Follow the Leader but adds a regularization term

$$\mathbf{b}_{t+1} = \arg\max_{\mathbf{b}\in\Delta_m} \sum_{\tau=1}^{t} \log(\mathbf{b}\cdot\mathbf{x}_\tau) - \frac{\beta}{2} R(\mathbf{b}),$$

Follow the Regularized Leader

- There may or may not be a closed form solution? Math is fun :))

$$\mathbf{b}_1 = \left(\frac{1}{m}, \ldots, \frac{1}{m}\right), \quad \mathbf{b}_{t+1} = \Pi_{\Delta_m}^{\mathbf{A}_t}\left(\delta \mathbf{A}_t^{-1} \mathbf{p}_t\right),$$

with

$$\mathbf{A}_t = \sum_{\tau=1}^{t} \left(\frac{\mathbf{x}_\tau \mathbf{x}_\tau^\top}{(\mathbf{b}_\tau \cdot \mathbf{x}_\tau)^2}\right) + \mathbf{I}_m, \quad \mathbf{p}_t = \left(1 + \frac{1}{\beta}\right) \sum_{\tau=1}^{t} \frac{\mathbf{x}_\tau}{\mathbf{b}_\tau \cdot \mathbf{x}_\tau},$$

where β is the trade-off parameter, δ is a scale term, and $\Pi_{\Delta_m}^{\mathbf{A}_t}(\cdot)$ is an exact projection to the simplex domain.

Homework

This Week

- On Github
- Remember to schedule a meeting outside the Build Night this week!