# Build Night 3

Baseline & Follow-the-Winner Strategies I



#### Team Meeting Agenda

- When2Meet for faculty advisor meeting (5min)
- Discuss homework (10min)
- Go over benchmark strategies (15min)
- Go over Follow-the-Winner strategies (15min)

#### Project Schedule

- 1. Welcome & Problem Definition
- 2. Power Outage Python and Reading Review
- 3. Baseline + Follow-the-Winner Strategies I
- 4. Follow-the-Winner Strategies II
- Follow-the-Loser Strategies I
- 6. Follow-the-Loser Strategies II
- 7. Pattern-Matching Strategies I
- 8. Pattern-Matching Strategies II
- 9. Backtesting, Experimentation, and Comparison
- 10. Poster Work & Presentation Practice

(you are here!)

# Homework Discussion



#### **ALGORITHM 1:** Online portfolio selection framework.

**Input**:  $\mathbf{x}_1^n$ : Historical market sequence

**Output**:  $S_n$ : Final cumulative wealth

Initialize  $S_0=1,\, \mathbf{b}_1=\left(rac{1}{m},\, \ldots,\, rac{1}{m}
ight)$ 

**for** t = 1, 2, ..., n **do** 

Portfolio manager computes a portfolio  $\mathbf{b}_t$ ;

Market reveals the market price relative  $\mathbf{x}_t$ ;

Portfolio incurs period return  $\mathbf{b}_t^{\top}\mathbf{x}_t$  and updates cumulative return  $S_t = S_{t-1} \times (\mathbf{b}_t^{\top}\mathbf{x}_t)$ ;

Portfolio manager updates his/her online portfolio selection rules;

end

# Benchmark Strategies



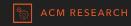
#### Buy-And-Hold

- Invest with an initial portfolio, long that portfolio with only adjustments based on market fluctuations
- Useful to produce a market index

$$\mathbf{b}_{t+1} = rac{\mathbf{b}_t \odot \mathbf{x}_t}{\mathbf{b}_t^{ op} \mathbf{x}_t}$$

# Buy-And-Hold: Example

Excel time



#### **Best Stock**

• A special base of BAH where we put all capital with the best performance in hindsight (i.e. with the best cumulative performance)

$$\mathbf{b}^{\circ} = \operatorname{arg\,max}_{\mathbf{b} \in \Delta_m} \mathbf{b} \cdot \left( \underbrace{\mathbf{o}}_{t=1}^n \mathbf{x}_t \right)$$

#### Constant Rebalanced Portfolios (BCRP)

- Rebalance the portfolio to some fixed proportion every period
- There is evidence to suggest that this strategy can produce an exponentially increasing return because it takes advantage of the underlying market volatility

Follow-the-Winner Strategies (Part I)

#### What is a FTW Strategy?

 We increase the weights of successful stocks based on the performance in the previous trading period

- We track the stock with the best performance but keep our new portfolio close to the existing one
- Optimization problem:

$$\mathbf{b}_{t+1} = \underset{\mathbf{b} \in \Delta_m}{\operatorname{arg\,max}} \quad \eta \log \mathbf{b} \cdot \mathbf{x}_t - R(\mathbf{b}, \mathbf{b}_t),$$

•  $\eta = \underline{\text{learning rate}}$ , R(b, b<sub>t</sub>) is a <u>regularization term</u>

In English: "Find the value of b such that this expression is maximized"

$$\mathbf{b}_{t+1} = \underset{\mathbf{b} \in \Delta_m}{\operatorname{arg\,max}} \quad \eta \log \mathbf{b} \cdot \mathbf{x}_t - R(\mathbf{b}, \mathbf{b}_t),$$

$$R(\mathbf{b},\mathbf{b}_t) = \sum_{i=1}^m b_i \log rac{b_i}{b_{t,i}}.$$

• Using some Taylor series magic, the optimization problem yields the update rule:

$$b_{t+1,i} = b_{t,i} \exp\left(\eta rac{\pmb{x}_{t,i}}{\pmb{\mathbf{b}}_t \cdot \pmb{\mathbf{x}}_t}
ight) / \pmb{Z}, \quad i = 1, \ldots, m,$$

ullet Z makes sure the portfolio sums up to 1 (normalization term)

 Regret = value of difference between a made decision and the optimal decision

#### Follow-the-Leader

Tracks the BCRP strategy and uses it as the basis for an optimization problem

$$\mathbf{b}_{t+1} = \mathbf{b}_t^* = rgmax_{\mathbf{b} \in \Delta_m} \sum_{j=1}^t \log \left( \mathbf{b} \cdot \mathbf{x}_j 
ight).$$

- We will have to feed this into a constrained optimization algorithm!
  - There may or may not be a closed-form solution for this

#### Follow the Regularized Leader

Works similar to Follow the Leader but adds a regularization term

$$\mathbf{b}_{t+1} = rgmax_{\mathbf{b} \in \Delta_m} \sum_{ au=1}^t \log(\mathbf{b} \cdot \mathbf{x}_{ au}) - rac{eta}{2} R(\mathbf{b}),$$

#### Follow the Regularized Leader

• There may or may not be a closed form solution? Math is fun :))

$$\mathbf{b}_1 = \left(rac{1}{m}, \dots, rac{1}{m}
ight), \quad \mathbf{b}_{t+1} = \Pi_{\Delta_m}^{\mathbf{A}_t} ig(\delta \mathbf{A}_t^{-1} \mathbf{p}_tig),$$

with

$$\mathbf{A}_t = \sum_{ au=1}^t \left( rac{\mathbf{x}_ au \mathbf{x}_ au^ op}{\left(\mathbf{b}_ au \cdot \mathbf{x}_ au
ight)^2} 
ight) + \mathbf{I}_m, \quad \mathbf{p}_t = \left(1 + rac{1}{eta}
ight) \sum_{ au=1}^t rac{\mathbf{x}_ au}{\mathbf{b}_ au \cdot \mathbf{x}_ au},$$

where  $\beta$  is the trade-off parameter,  $\delta$  is a scale term, and  $\Pi_{\Delta_m}^{\mathbf{A}_t}(\cdot)$  is an exact projection to the simplex domain.

# Homework



#### This Week

- On Github
- Remember to schedule a meeting outside the Build Night this week!