Performance Enhancements for Zero-Flow Simulation of Vapor Compression Cycles

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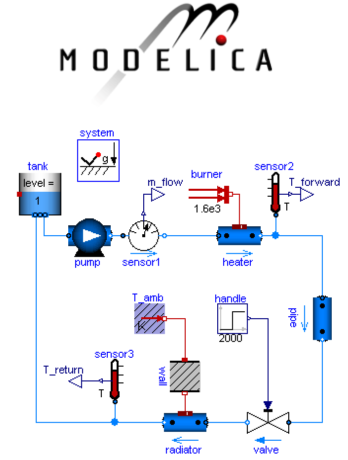
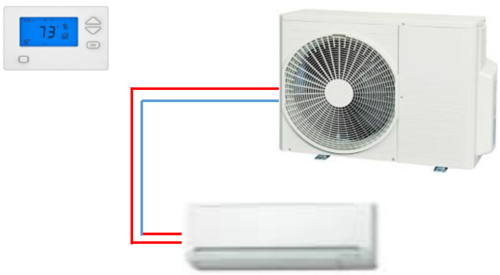
Motivation & Objective

Motivation:

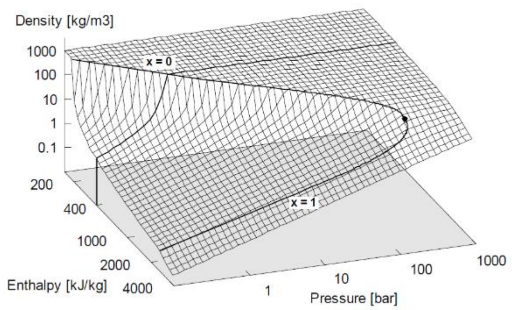
- Numerical simulations are widely employed in the HVAC&R industries
- Low and zero-flow phenomena are often encountered in the operation of vapor compression systems with large transients
- Simulation of system dynamics under low and zero-flow conditions presents numerical challenges
- Reported approaches cannot produce satisfactory results sometimes





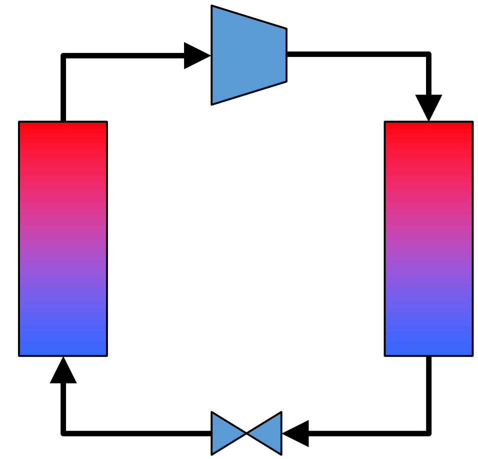
Objective:

- Explore effective techniques to improve the performance of zero-flow simulations, especially focusing on robustness and improvements in the simulation speed, with a goal of achieving faster than real-time dynamic simulation



Vapor Compression Cycle Model

- System dynamics are dominated by heat exchangers
- Refrigerant properties defined via Equation of State
  - Pressure and specific enthalpy
- Compressor and valve performance curves defined by the user



Dynamic HEX model

$$\frac{\partial(\rho A)}{\partial t}+\frac{\partial(\rho A v)}{\partial x}=0$$

$$\frac{\partial(\rho v A)}{\partial t}+\frac{\partial(\rho v^2 A)}{\partial x}=-A\frac{\partial p}{\partial x}-F_f$$

$$\frac{\partial(\rho u A)}{\partial t}+\frac{\partial(\rho v h A)}{\partial x}=vA\frac{\partial p}{\partial x}+vF_f+\frac{\partial Q}{\partial x}$$

Static compressor model

$$\eta_v(f, P_r, \dots) = \frac{\dot{m}_{comp}}{\rho_{suc} V f}$$

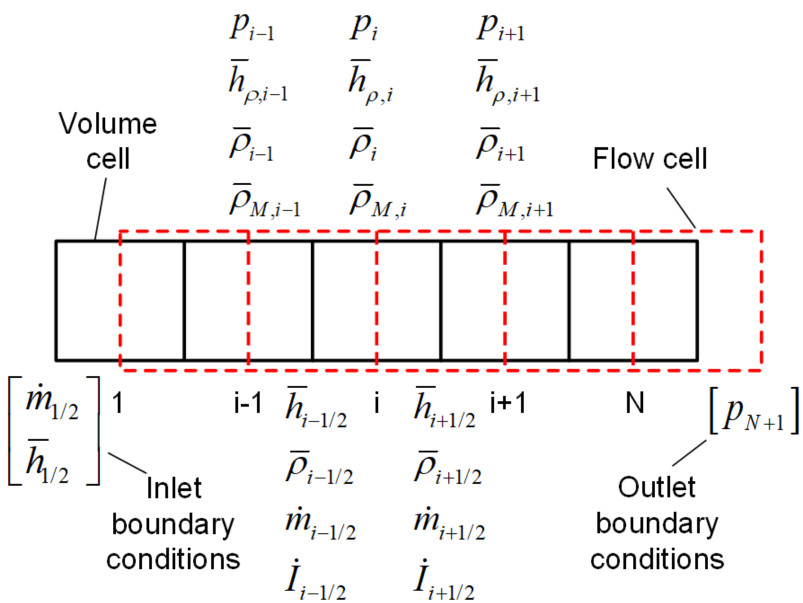
$$\eta_{is}(f, \dots) = \frac{h_{dis,isen} - h_{suc}}{h_{dis} - h_{suc}}$$

Static valve model

$$\dot{m} = C_v a_v \sqrt{\rho_{in} \Delta P}$$

Staggered Grid Scheme

- Use 1D separated flow model w/o interfacial exchange
- Shift the momentum balance half a cell compared to the mass- and energy balances to obtain a staggered grid
- The staggering tends to decouple the fast pressure from the slow thermal part of the equation system and thereby creates a more robust discretization scheme
- Use the upwind scheme to approximate interface variables



Mass balance

$$\frac{dM_i}{dt} = \dot{m}_{i-1/2} - \dot{m}_{i+1/2}$$

Energy balance

$$\frac{dU_i}{dt} = \dot{m}_{i-1/2}\bar{h}_{i-1/2} - \dot{m}_{i+1/2}\bar{h}_{i+1/2} + P\Delta z q_i''$$

Momentum balance

Ignored in our models

$$\Delta z \frac{d\dot{m}_{i+1/2}}{dt} = \left(\dot{I}_i - \dot{I}_{i+1}\right) - A\left(p_{i+1} - p_i\right) - \bar{\tau}_{w,i+1/2} P\Delta z - \boxed{\bar{\rho}_{i+1/2} g A \Delta z \sin\theta}$$

Momentum Balance Approximations

Uniform dp/dt:

$2N$+1 dynamic states

$$\frac{dp_1}{dt} = \dots = \frac{dp_i}{dt} = \dots = \frac{dp_{N+1}}{dt}$$

Linear Δp:

N+3 dynamic states

$$\frac{dp_i}{dt} = \left(1 - \frac{i-1}{N}\right)\frac{dp_1}{dt} + \frac{i-1}{N}\frac{dp_{N+1}}{dt}$$

$$N\Delta z \frac{d\bar{\dot{m}}}{dt} = \left(\dot{I}_1 - \dot{I}_{N+1}\right) - A\left(p_{N+1} - p_1\right) - P\Delta z \sum_{i=1}^{N} \bar{\tau}_{w,i+1/2}$$

Friction only:

$2N$ dynamic states

$$p_{i+1} = p_i - \frac{P}{A}\bar{\tau}_{w,i+1/2}\Delta z$$

✔

Pressure drop calculation

$$\frac{\Delta p}{\Delta p_0} = K\left(\frac{\dot{m}}{\dot{m}_0}\right)^{\lambda}$$

$$\Rightarrow \dot{m} = \dot{m}_0 \left(\frac{\Delta p}{K\Delta p_0}\right)^{1/\lambda}$$

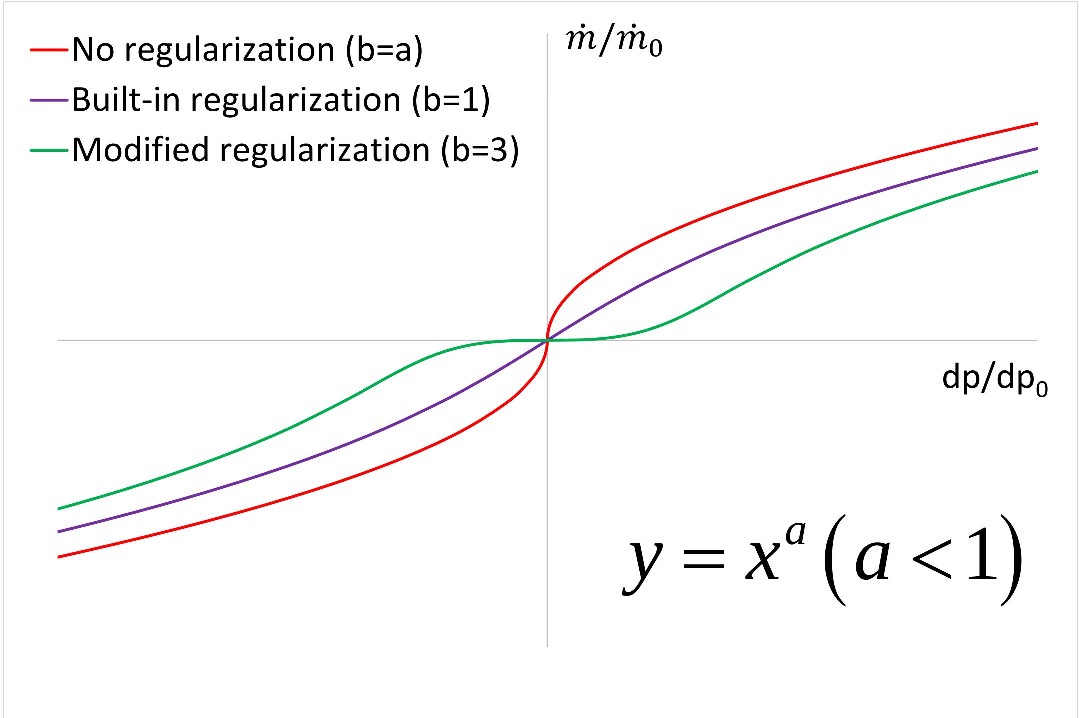
$$\lambda > 1$$

$$\frac{d\dot{m}}{d\left(\Delta p\right)} = \frac{\dot{m}_0^{\lambda}}{\lambda K \Delta p_0}\frac{1}{\dot{m}^{\lambda-1}}$$

$\dot{m} \to 0$ → Infinity

We need to regularize the relation with locally non-singular substitute

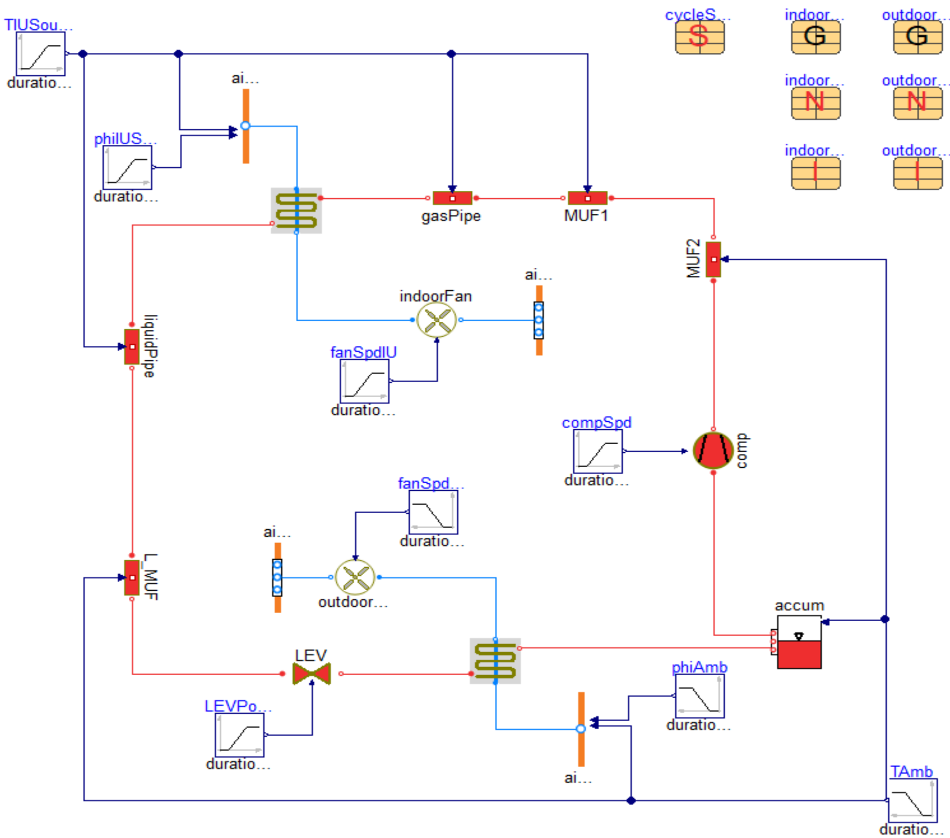
Pressure Drop Regularization

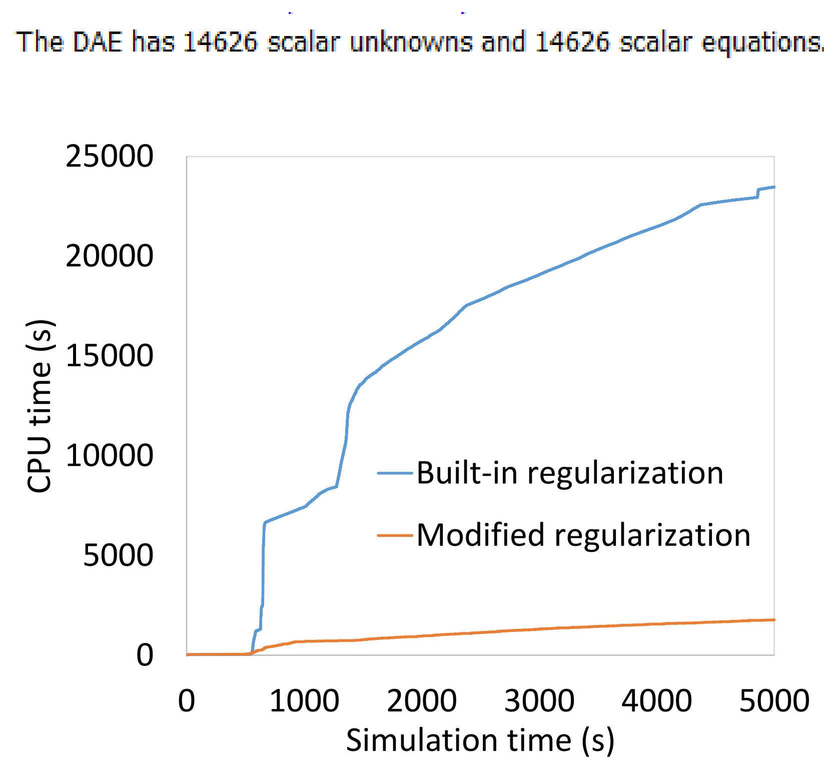



```
function regPowGen
  extends Modelica.Icons.Function;
  input Real x;
  input Real a;
  input Real delta=0.01;
  input Real b=1;
  output Real y;
algorithm
  y := x^b*(x*x+delta*delta)^((a-b)/2);
end regPowGen;
```

- Original relation without regularization is not Lipschitz continuous
- Smaller Lipschitz constants around origin are very helpful

Case Study





- System was shut down at 500 sec and off-cycle lasted for 4500 sec
- CPU time: 23000 sec with built-in regularization vs. 1700 sec with modified regularization

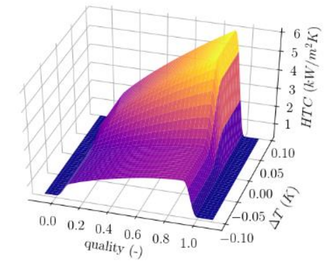
Heat Transfer Model: Static or Filtered?

- Multiple regimes are blended together to form a universal function
- Simplified closure relations can approximate full correlations

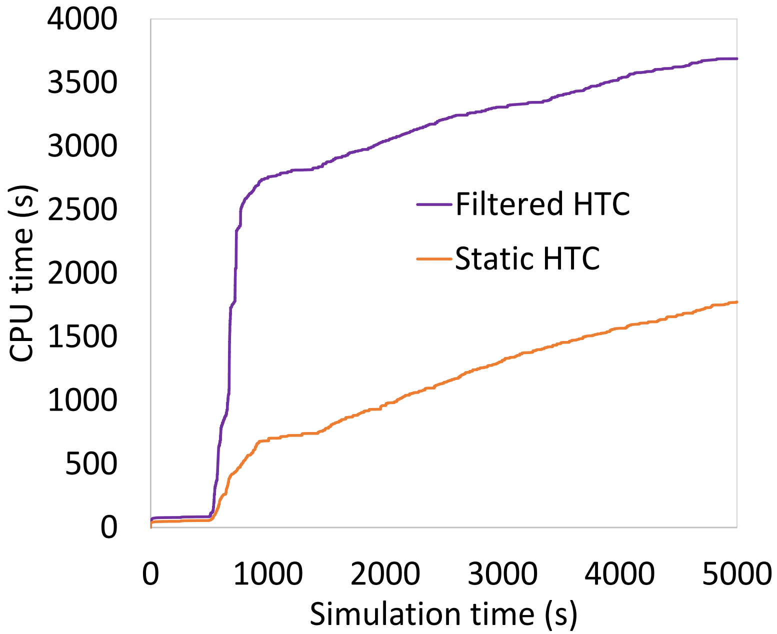
Static HTC $$\alpha = K\alpha_0\left(\frac{\dot{m}}{\dot{m}_0}\right)^c$$

Filtered HTC $$\frac{d\alpha}{dt} = \frac{1}{\tau}\left(\hat{\alpha} - \alpha\right)$$

- Filtered HTC model decouples the closure variable with other state variables, but increases the number of states
- Turning on 'log norm' showed filtered HTCs limited integration time steps
- CPU time: 3700 sec with filtered HTCs vs. 1700 sec with static HTCs




```
 6 197 1137 indoorCoil.circuits[1].heatTransfer.alphas[21] (#118)
17 357 1373 indoorCoil.circuits[1].heatTransfer.alphas[22] (#119)
28 315 1666 indoorCoil.circuits[1].heatTransfer.alphas[23] (#120)
32 501 2022 indoorCoil.circuits[1].heatTransfer.alphas[24] (#121)
51 544 2138 indoorCoil.circuits[1].heatTransfer.alphas[25] (#122)
32 333 1674 indoorCoil.circuits[1].heatTransfer.alphas[26] (#123)
19 193 1146 indoorCoil.circuits[1].heatTransfer.alphas[27] (#124)
 0   0    2 indoorCoil.header[1].mediums[1].p (#152)
 0   0    4 indoorCoil.header[1].mediums[2].p (#154)
13  21  120 indoorCoil.header[2].mediums[1].p (#156)
```



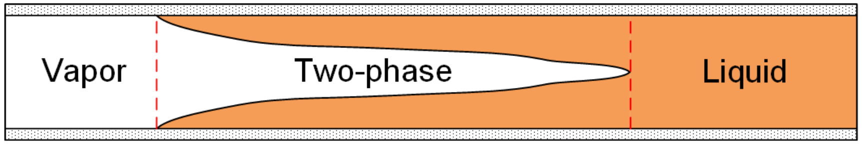
Discretization Methods

Continuity $$\frac{\partial(\rho A)}{\partial t}+\frac{\partial(\rho A v)}{\partial z}=0$$

Momentum $$\frac{\partial(\rho v A)}{\partial t}+\frac{\partial(\rho v^2 A)}{\partial z}=-A\frac{\partial p}{\partial z}-F_f$$

Energy $$\frac{\partial(\rho u A)}{\partial t}+\frac{\partial(\rho v h A)}{\partial z}=vA\frac{\partial p}{\partial z}+vF_f+\frac{\partial Q}{\partial z}$$

Moving Boundary Method

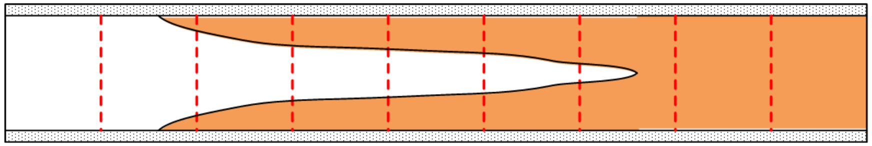


Reynolds transport theorem

$$\frac{d}{dt}\int_0^L\frac{\partial(\rho A)}{\partial t}dz+\frac{d}{dt}\int_0^L\frac{\partial \dot m}{\partial z}dz=0$$

$$\frac{d}{dt}\int_0^L\frac{\partial\left[A(\rho h-p)\right]}{\partial t}+\frac{d}{dt}\int_0^L\frac{\partial(\dot m h)}{\partial z}dz=q'$$

Finite Volume Method



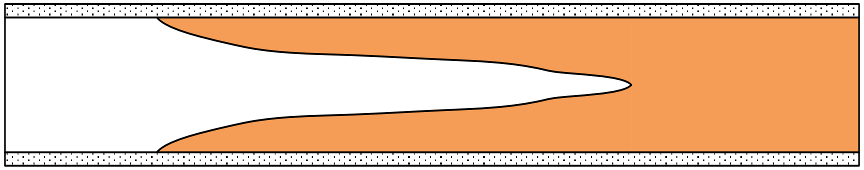
Staggered grid scheme with upwind difference

$$A\Delta z\frac{d\rho_i}{dt}=\dot m_{i-1/2}-\dot m_{i+1/2}$$

$$\Delta z\frac{d\dot m_{i+1/2}}{dt}=-\Delta\dot I_i-A\Delta p_i-\overline{\tau}_{w,i+1/2}P\Delta z$$

$$A\Delta z\frac{d(\rho_i u_i)}{dt}=\dot m_{i-1/2}h_{i-1/2}-\dot m_{i+1/2}h_{i+1/2}+P\Delta z q_i''$$

Lumped Parameter Method



Single control volume with mean properties

$$AL\frac{d\overline{\rho}}{dt}=\dot m_{in}-\dot m_{out}$$

$$AL\frac{d\left(\overline{\rho h}-p\right)}{dt}=\dot m_{in}h_{in}-\dot m_{out}h_{out}+PLq''$$

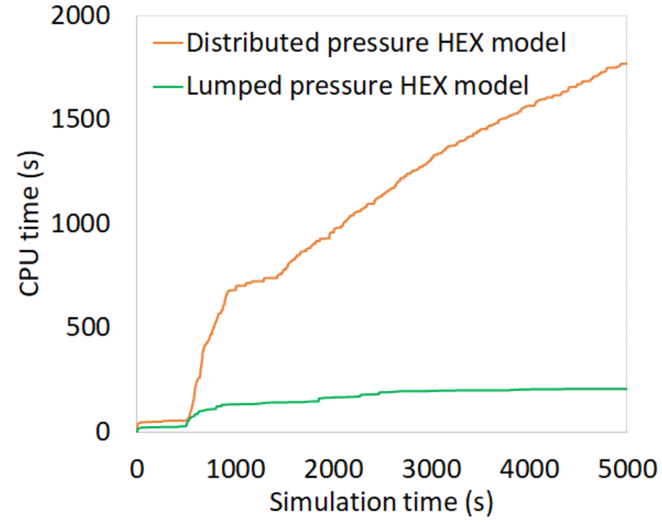
Single-Pressure HEX Model

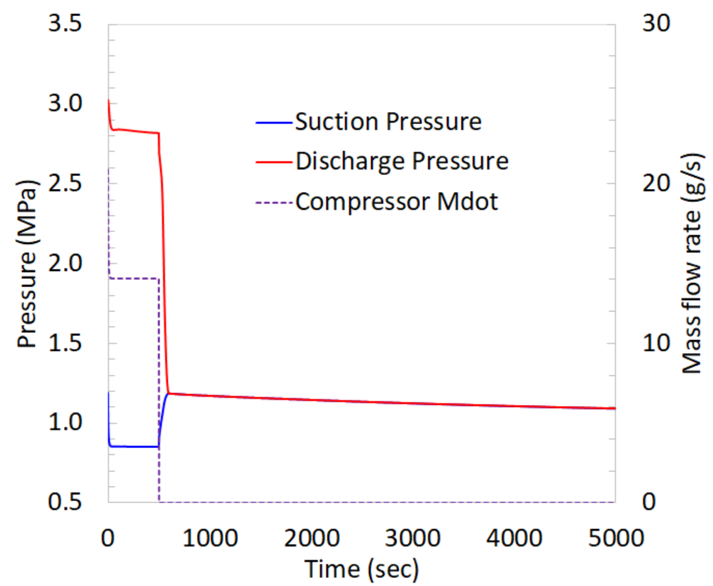
$$A\Delta z\left(\frac{\partial\bar{\rho}_i}{\partial p}\frac{dp}{dt}+\frac{\partial\bar{\rho}_i}{\partial\bar{h}_{\rho,i}}\frac{d\bar{h}_{\rho,i}}{dt}\right)=\dot{m}_{i-1/2}-\dot{m}_{i+1/2}$$

$$A\Delta z\left(\bar{\rho}_i\frac{d\bar{h}_{\rho,i}}{dt}-\frac{dp}{dt}\right)=$$

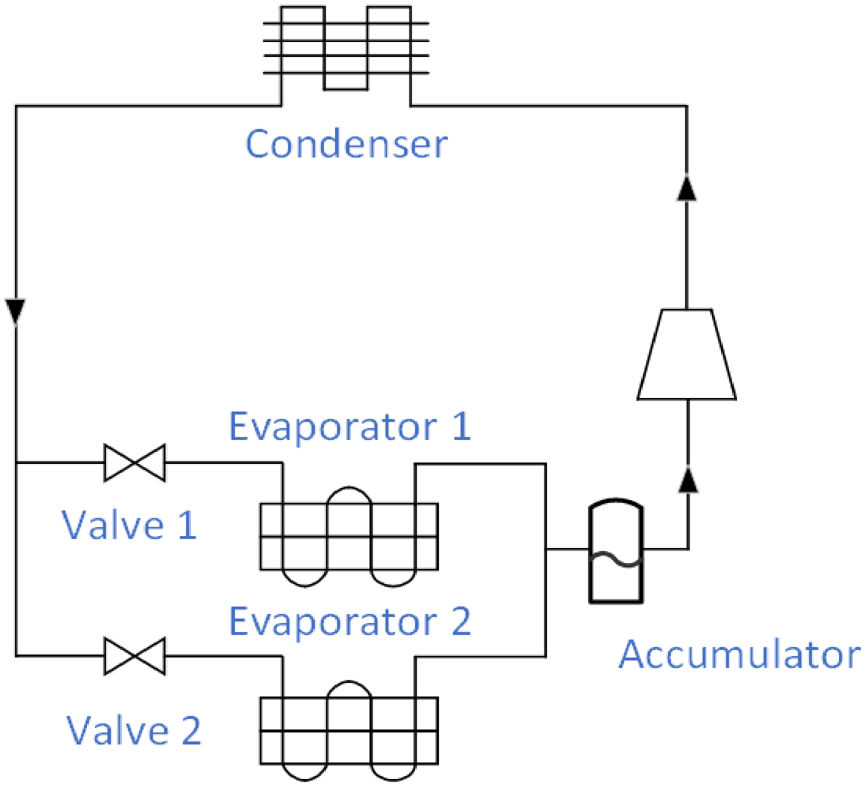
$$\dot{m}_{i-1/2}\left(\bar{h}_{i-1/2}-\bar{h}_{\rho,i}\right)\bar{h}_{i-1/2}-\dot{m}_{i+1/2}\left(\bar{h}_{i+1/2}-\bar{h}_{\rho,i}\right)+P\Delta z q_i''$$

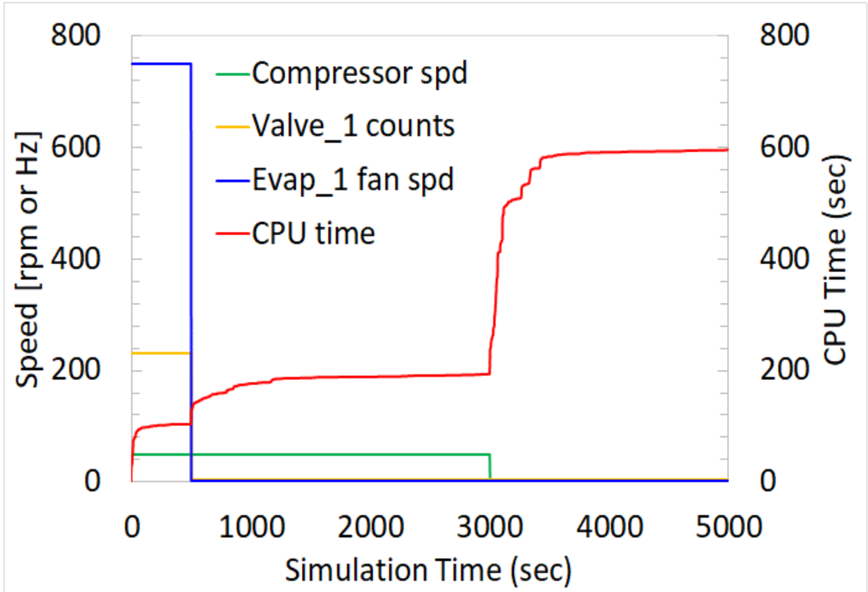
- Dependence of mass flow rates upon pressure difference can be removed
- N+1 states and pressure drop is lumped together and calculated at the inlet or outlet depending on model structure
- CPU time: 1700 sec with distributed HEX model vs. 200 sec with lumped pressure drop model
- 100x speedup with all the enhancements





Another Case Study





- System operated for the first 500 sec with two active evaporator branches
- The first branch was off (the associated fan and valve were closed) since 500 sec
- Entire system was turned off at 3000 sec
- 600 sec CPU time with all the enhancements

Conclusions & Future Work

- Reducing the sensitivity of mass flow to pressure differences is a key to accelerating zero-flow simulation
- Static heat transfer model seems more efficient, if no spurious oscillations appear in the simulation
- Lumping pressure drops at the inlet or outlet of HEXs or pipes is helpful for further speeding up simulation
- We would like to evaluate these enhancements with models that can generate Analytical Jacobians

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