# Mixed-effects models

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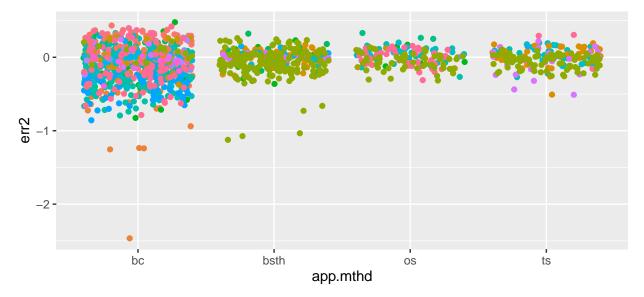
06 March, 2023 15:09

## 1. Take a look

```
ggplot(dfinal, aes(app.mthd, e.rel, colour = institute)) +
  geom_jitter(height = 0) +
  theme(legend.position = 'top')
## Warning: Removed 33 rows containing missing values (`geom_point()`).
                        AAFC
                                     ΑU
                                                   IGER
                                                                 MU
                                                                                USDA
                        ADAS
                                                                 NMI-WUR
                                                                                WUR
                                      CAU-LU
                                                   IMAG
           institute
                       ADAS-RR
                                      CRPA
                                                   INH-HAFL
                                                                 TEAGASC
                       ARDC
                                      DIAS
                                                   INRA
                                                                 UNIMI
                                                                 UNINA
                                      DiSAA-IT
                                                   IUL/FAT
  3 -
  2 -
e.rel
  1 ·
  0 -
                                   bsth
                                                         os
                                           app.mthd
ggplot(dfinal, aes(app.mthd, err2, colour = institute)) +
  geom_jitter(height = 0) +
  theme(legend.position = 'top')
```

## Warning: Removed 33 rows containing missing values (`geom\_point()`).





Drop values > 100% applied TAN.

```
dim(dfinal)
```

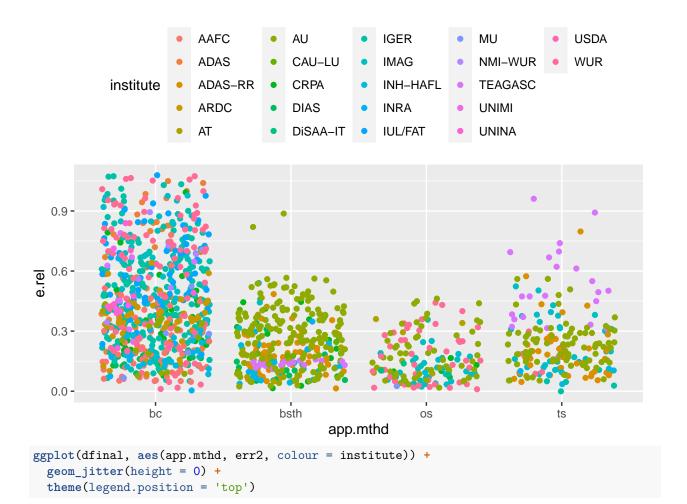
```
## [1] 1487 279
```

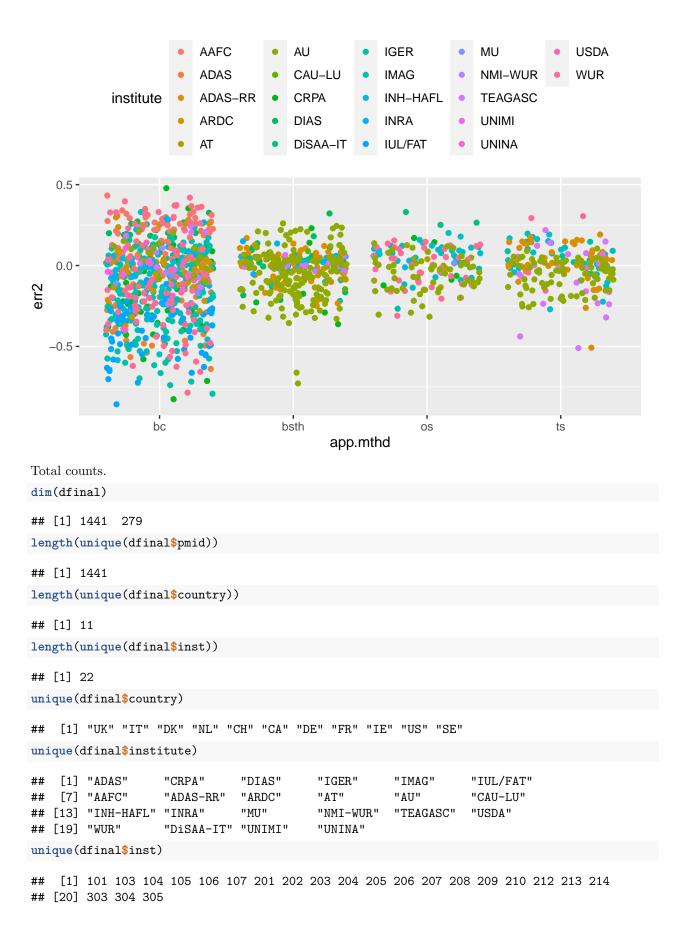
```
length(unique(dfinal$pmid))
```

```
## [1] 1487
```

```
dfinal <- dfinal[e.rel < 1.1, ]</pre>
```

```
ggplot(dfinal, aes(app.mthd, e.rel, colour = institute)) +
  geom_jitter(height = 0) +
  theme(legend.position = 'top')
```





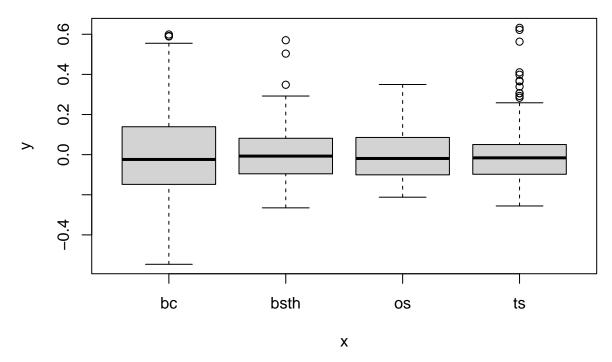
#### 2. Data prep

```
dfinal <- droplevels(dfinal[!is.na(e.rel), ])
dfinal$inst <- factor(dfinal$inst)
dfinal$inst.meas.tech <- interaction(dfinal$institute, dfinal$meas.tech)
dfinal$app.mthd <- factor(dfinal$app.mthd)</pre>
Get subset without crazy broadcast
dfinalb <- dfinal[app.mthd != 'bc', ]
```

## 3. Basic variability and comparison of simplest predictors

```
m0 <- lmer(e.rel ~ (1|inst.meas.tech), data = dfinal)</pre>
m1 <- lmer(e.rel ~ app.mthd + (1 inst.meas.tech), data = dfinal)
m2 <- lm(e.rel ~ app.mthd, data = dfinal)
AIC(m0, m1, m2)
##
     df
               AIC
## m0 3 -421.8459
## m1 6 -678.7292
## m2 5 -438.8121
summary(m0)
## Linear mixed model fit by REML ['lmerMod']
## Formula: e.rel ~ (1 | inst.meas.tech)
##
     Data: dfinal
## REML criterion at convergence: -427.8
##
## Scaled residuals:
      Min 1Q Median
                                3Q
                                       Max
## -2.1406 -0.6467 -0.1080 0.5530 3.3263
##
## Random effects:
## Groups
                  Name
                               Variance Std.Dev.
## inst.meas.tech (Intercept) 0.01762 0.1327
                               0.04119 0.2030
## Residual
## Number of obs: 1441, groups: inst.meas.tech, 37
##
## Fixed effects:
               Estimate Std. Error t value
## (Intercept) 0.32878
                           0.02539
                                     12.95
summary(m1)
## Linear mixed model fit by REML ['lmerMod']
## Formula: e.rel ~ app.mthd + (1 | inst.meas.tech)
##
     Data: dfinal
## REML criterion at convergence: -690.7
##
```

```
## Scaled residuals:
##
               1Q Median
       Min
                                30
                                       Max
## -2.9593 -0.6684 -0.0838 0.5823
                                   3.4226
##
## Random effects:
                               Variance Std.Dev.
## Groups
                   Name
## inst.meas.tech (Intercept) 0.01128 0.1062
## Residual
                               0.03410 0.1847
## Number of obs: 1441, groups: inst.meas.tech, 37
##
## Fixed effects:
##
                Estimate Std. Error t value
## (Intercept)
                 0.43034
                            0.02195
                                      19.60
## app.mthdbsth -0.19508
                            0.01911 - 10.21
## app.mthdos
                -0.33559
                            0.01954
                                     -17.17
## app.mthdts
                -0.21475
                            0.02057 - 10.44
##
## Correlation of Fixed Effects:
##
               (Intr) app.mthdb app.mthds
## app.mthdbst -0.291
## app.mthdos -0.227 0.436
## app.mthdts -0.236 0.616
                                 0.401
summary(m2)
##
## Call:
## lm(formula = e.rel ~ app.mthd, data = dfinal)
##
## Residuals:
##
                       Median
        Min
                  1Q
                                     3Q
                                             Max
## -0.44340 -0.13681 -0.03545 0.12378 0.71341
##
## Coefficients:
##
                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                 0.447225
                            0.007547
                                       59.26
                                                <2e-16 ***
                                                <2e-16 ***
## app.mthdbsth -0.212243
                            0.013699
                                      -15.49
                -0.284969
                            0.018189 -15.67
## app.mthdos
                                                <2e-16 ***
## app.mthdts
                -0.199741
                            0.016491 -12.11
                                                <2e-16 ***
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.2074 on 1437 degrees of freedom
## Multiple R-squared: 0.235, Adjusted R-squared: 0.2334
## F-statistic: 147.1 on 3 and 1437 DF, p-value: < 2.2e-16
So, institute x measurement technique effect is around 12% of applied TAN (from model m1). Residuals are
large, around 20% of applied TAN. Presumably residuals are smaller for injection.
res <- resid(m1)
plot(dfinal$app.mthd, res)
```



Perhaps, but could be worse.

```
Repeat without broadcast
```

```
m0b <- lmer(e.rel ~ (1|inst.meas.tech), data = dfinalb)</pre>
m1b <- lmer(e.rel ~ app.mthd + (1|inst.meas.tech), data = dfinalb)</pre>
m2b <- lm(e.rel ~ app.mthd, data = dfinalb)</pre>
AIC(mOb, m1b, m2b)
##
       df
## m0b 3 -722.8347
## m1b 5 -733.1104
## m2b 4 -702.5938
summary(m0b)
## Linear mixed model fit by REML ['lmerMod']
## Formula: e.rel ~ (1 | inst.meas.tech)
##
      Data: dfinalb
##
## REML criterion at convergence: -728.8
##
## Scaled residuals:
                1Q Median
                                 3Q
##
       Min
                                        Max
## -1.7406 -0.7082 -0.1489 0.5329 4.5316
##
## Random effects:
## Groups
                   Name
                                Variance Std.Dev.
## inst.meas.tech (Intercept) 0.003508 0.05922
                                0.019267 0.13881
## Number of obs: 686, groups: inst.meas.tech, 19
```

```
## Fixed effects:
##
              Estimate Std. Error t value
                          0.01659
## (Intercept) 0.19900
summary(m1b)
## Linear mixed model fit by REML ['lmerMod']
## Formula: e.rel ~ app.mthd + (1 | inst.meas.tech)
     Data: dfinalb
##
##
## REML criterion at convergence: -743.1
##
## Scaled residuals:
##
      Min
              1Q Median
                               ЗQ
                                      Max
## -1.8037 -0.7134 -0.1433 0.5686 4.5847
##
## Random effects:
## Groups
                  Name
                              Variance Std.Dev.
## inst.meas.tech (Intercept) 0.003163 0.05624
## Residual
                              0.018580 0.13631
## Number of obs: 686, groups: inst.meas.tech, 19
##
## Fixed effects:
##
               Estimate Std. Error t value
## (Intercept) 0.221512
                          0.016987 13.040
## app.mthdos -0.077669
                          0.016031
                                    -4.845
## app.mthdts
              0.004581
                          0.013043
                                    0.351
##
## Correlation of Fixed Effects:
##
              (Intr) app.mthds
## app.mthdos -0.330
## app.mthdts -0.240 0.347
summary(m2b)
##
## Call:
## lm(formula = e.rel ~ app.mthd, data = dfinalb)
## Residuals:
##
       Min
                 1Q Median
                                   ЗQ
                                           Max
## -0.24808 -0.10570 -0.03087 0.07986 0.71341
##
## Coefficients:
##
               Estimate Std. Error t value Pr(>|t|)
## (Intercept) 0.234982 0.007965 29.502 < 2e-16 ***
## app.mthdos -0.072725
                          0.014014 -5.190 2.78e-07 ***
## app.mthdts
              0.012503
                         0.012954
                                    0.965
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1445 on 683 degrees of freedom
## Multiple R-squared: 0.04971,
                                   Adjusted R-squared: 0.04693
## F-statistic: 17.86 on 2 and 683 DF, p-value: 2.741e-08
Less variability without broadcast.
```

### 4. ALFAM2 model residuals

```
m3 <- lmer(err2 ~ (1|inst.meas.tech), data = dfinal)
m4 <- lmer(err2 ~ app.mthd + (1 inst.meas.tech), data = dfinal)
summary(m0)
## Linear mixed model fit by REML ['lmerMod']
## Formula: e.rel ~ (1 | inst.meas.tech)
     Data: dfinal
##
## REML criterion at convergence: -427.8
##
## Scaled residuals:
##
      Min
               1Q Median
                               3Q
## -2.1406 -0.6467 -0.1080 0.5530 3.3263
##
## Random effects:
## Groups
                  Name
                              Variance Std.Dev.
## inst.meas.tech (Intercept) 0.01762 0.1327
                              0.04119 0.2030
## Number of obs: 1441, groups: inst.meas.tech, 37
## Fixed effects:
              Estimate Std. Error t value
## (Intercept) 0.32878
                          0.02539
                                    12.95
summary(m1)
## Linear mixed model fit by REML ['lmerMod']
## Formula: e.rel ~ app.mthd + (1 | inst.meas.tech)
##
     Data: dfinal
##
## REML criterion at convergence: -690.7
##
## Scaled residuals:
              1Q Median
                               3Q
      Min
## -2.9593 -0.6684 -0.0838 0.5823 3.4226
##
## Random effects:
## Groups
                  Name
                              Variance Std.Dev.
## inst.meas.tech (Intercept) 0.01128 0.1062
## Residual
                              0.03410 0.1847
## Number of obs: 1441, groups: inst.meas.tech, 37
## Fixed effects:
##
               Estimate Std. Error t value
## (Intercept)
                0.43034
                           0.02195
                                    19.60
## app.mthdbsth -0.19508
                           0.01911 -10.21
## app.mthdos
              -0.33559
                           0.01954 - 17.17
## app.mthdts
              -0.21475
                           0.02057 -10.44
## Correlation of Fixed Effects:
               (Intr) app.mthdb app.mthds
```

```
## app.mthdbst -0.291
## app.mthdos -0.227 0.436
## app.mthdts -0.236 0.616
                                0.401
summary(m3)
## Linear mixed model fit by REML ['lmerMod']
## Formula: err2 ~ (1 | inst.meas.tech)
##
     Data: dfinal
##
## REML criterion at convergence: -902.5
## Scaled residuals:
##
      Min
               1Q Median
                               ЗQ
                                      Max
## -4.1054 -0.5388 0.1000 0.6074 3.5407
##
## Random effects:
## Groups
                  Name
                              Variance Std.Dev.
## inst.meas.tech (Intercept) 0.01207 0.1098
## Residual
                              0.02965 0.1722
## Number of obs: 1441, groups: inst.meas.tech, 37
##
## Fixed effects:
##
              Estimate Std. Error t value
## (Intercept) -0.03566
                          0.02111 -1.689
summary(m4)
## Linear mixed model fit by REML ['lmerMod']
## Formula: err2 ~ app.mthd + (1 | inst.meas.tech)
     Data: dfinal
##
## REML criterion at convergence: -904.7
##
## Scaled residuals:
##
      Min
               1Q Median
                               3Q
                                      Max
## -3.9679 -0.5474 0.0938 0.5880 3.6548
##
## Random effects:
## Groups
                  Name
                              Variance Std.Dev.
## inst.meas.tech (Intercept) 0.01161 0.1077
                              0.02929 0.1711
## Residual
## Number of obs: 1441, groups: inst.meas.tech, 37
##
## Fixed effects:
               Estimate Std. Error t value
## (Intercept) -0.05802
                           0.02173 -2.670
## app.mthdbsth 0.03633
                           0.01783
                                    2.037
## app.mthdos
                0.08261
                           0.01818
                                   4.544
## app.mthdts
                0.04962
                           0.01915
##
## Correlation of Fixed Effects:
               (Intr) app.mthdb app.mthds
## app.mthdbst -0.272
## app.mthdos -0.214 0.439
```

```
## app.mthdts -0.220 0.619
AIC(m3, m4)
##
      df
               AIC
## m3 3 -896.5179
## m4 6 -892.7441
Reassuring that m3 is actually a better model than m4, meaning adding application method on top of ALFAM2
predictions doesn't help.
Again, exclude broadcast.
m3b <- lmer(err2 ~ (1|inst.meas.tech), data = dfinalb)
m4b <- lmer(err2 ~ app.mthd + (1 inst.meas.tech), data = dfinalb)
summary(m0b)
## Linear mixed model fit by REML ['lmerMod']
## Formula: e.rel ~ (1 | inst.meas.tech)
      Data: dfinalb
##
## REML criterion at convergence: -728.8
##
## Scaled residuals:
       Min
##
                1Q Median
                                3Q
                                        Max
## -1.7406 -0.7082 -0.1489 0.5329
                                   4.5316
##
## Random effects:
## Groups
                   Name
                               Variance Std.Dev.
## inst.meas.tech (Intercept) 0.003508 0.05922
                               0.019267 0.13881
## Residual
## Number of obs: 686, groups: inst.meas.tech, 19
##
## Fixed effects:
               Estimate Std. Error t value
## (Intercept) 0.19900
                           0.01659
                                         12
summary(m1b)
## Linear mixed model fit by REML ['lmerMod']
## Formula: e.rel ~ app.mthd + (1 | inst.meas.tech)
##
      Data: dfinalb
##
## REML criterion at convergence: -743.1
##
## Scaled residuals:
                10 Median
                                3Q
## -1.8037 -0.7134 -0.1433 0.5686 4.5847
##
## Random effects:
                   Name
                               Variance Std.Dev.
## Groups
## inst.meas.tech (Intercept) 0.003163 0.05624
                               0.018580 0.13631
## Residual
## Number of obs: 686, groups: inst.meas.tech, 19
## Fixed effects:
```

```
Estimate Std. Error t value
##
## (Intercept) 0.221512 0.016987 13.040
## app.mthdos -0.077669 0.016031 -4.845
## app.mthdts
               0.004581
                          0.013043
                                   0.351
## Correlation of Fixed Effects:
             (Intr) app.mthds
## app.mthdos -0.330
## app.mthdts -0.240 0.347
summary(m3b)
## Linear mixed model fit by REML ['lmerMod']
## Formula: err2 ~ (1 | inst.meas.tech)
     Data: dfinalb
##
##
## REML criterion at convergence: -900.2
## Scaled residuals:
      Min
              1Q Median
                               3Q
                                      Max
## -4.8657 -0.5597 0.1550 0.6170 2.5881
##
## Random effects:
## Groups
                              Variance Std.Dev.
                  Name
## inst.meas.tech (Intercept) 0.00298 0.05459
## Residual
                              0.01498 0.12239
## Number of obs: 686, groups: inst.meas.tech, 19
## Fixed effects:
               Estimate Std. Error t value
##
## (Intercept) -0.008732 0.015141 -0.577
summary(m4b)
## Linear mixed model fit by REML ['lmerMod']
## Formula: err2 ~ app.mthd + (1 | inst.meas.tech)
##
     Data: dfinalb
##
## REML criterion at convergence: -886.8
##
## Scaled residuals:
      Min 1Q Median
                               3Q
                                      Max
## -4.8666 -0.5765 0.1516 0.6033 2.6082
##
## Random effects:
## Groups
                 Name
                              Variance Std.Dev.
## inst.meas.tech (Intercept) 0.002823 0.05313
## Residual
                              0.015029 0.12259
## Number of obs: 686, groups: inst.meas.tech, 19
## Fixed effects:
               Estimate Std. Error t value
## (Intercept) -0.012550 0.015805 -0.794
## app.mthdos
              0.009106 0.014461
                                   0.630
## app.mthdts
              0.005060
                                   0.431
                        0.011744
```

```
##
## Correlation of Fixed Effects:
## (Intr) app.mthds
## app.mthdos -0.321
## app.mthdts -0.232  0.348

AIC(m3b, m4b)

## df    AIC
## m3b  3 -894.2066
## m4b  5 -876.8290

Here too, m3b is better.
```

#### 5. "Institution effect"

Our best estimate of an "institution effect" is from m3, where we have corrected for different application methods, manure DM, pH, and weather using the ALFAM2 model.

```
VarCorr(m1)
## Groups
                   Name
                               Std.Dev.
## inst.meas.tech (Intercept) 0.10621
## Residual
                               0.18467
VarCorr(m1b)
## Groups
                   Name
                               Std.Dev.
## inst.meas.tech (Intercept) 0.056243
## Residual
                               0.136309
VarCorr(m3)
                               Std.Dev.
## Groups
                   Name
## inst.meas.tech (Intercept) 0.10984
## Residual
                               0.17220
VarCorr(m3b)
                               Std.Dev.
## Groups
                   Name
## inst.meas.tech (Intercept) 0.054594
## Residual
                               0.122390
summary(m1)
## Linear mixed model fit by REML ['lmerMod']
## Formula: e.rel ~ app.mthd + (1 | inst.meas.tech)
     Data: dfinal
##
## REML criterion at convergence: -690.7
##
## Scaled residuals:
##
      Min
                1Q Median
                                ЗQ
                                       Max
## -2.9593 -0.6684 -0.0838 0.5823 3.4226
##
## Random effects:
## Groups
                               Variance Std.Dev.
                   Name
## inst.meas.tech (Intercept) 0.01128 0.1062
## Residual
                               0.03410 0.1847
```

```
## Number of obs: 1441, groups: inst.meas.tech, 37
##
## Fixed effects:
              Estimate Std. Error t value
##
## (Intercept) 0.43034
                           0.02195
## app.mthdbsth -0.19508 0.01911 -10.21
## app.mthdos -0.33559
                           0.01954 - 17.17
## app.mthdts -0.21475
                           0.02057 -10.44
##
## Correlation of Fixed Effects:
              (Intr) app.mthdb app.mthds
## app.mthdbst -0.291
## app.mthdos -0.227 0.436
## app.mthdts -0.236 0.616
                                0.401
VarCorr(m1b)
## Groups
                  Name
                              Std.Dev.
## inst.meas.tech (Intercept) 0.056243
## Residual
                              0.136309
summary(m3)
## Linear mixed model fit by REML ['lmerMod']
## Formula: err2 ~ (1 | inst.meas.tech)
     Data: dfinal
##
##
## REML criterion at convergence: -902.5
##
## Scaled residuals:
##
      Min
              1Q Median
                               3Q
## -4.1054 -0.5388 0.1000 0.6074 3.5407
##
## Random effects:
## Groups
                              Variance Std.Dev.
                  Name
## inst.meas.tech (Intercept) 0.01207 0.1098
                              0.02965 0.1722
## Residual
## Number of obs: 1441, groups: inst.meas.tech, 37
##
## Fixed effects:
              Estimate Std. Error t value
## (Intercept) -0.03566
                          0.02111 -1.689
VarCorr(m3b)
## Groups
                  Name
                              Std.Dev.
## inst.meas.tech (Intercept) 0.054594
## Residual
                              0.122390
summary(m3b)
## Linear mixed model fit by REML ['lmerMod']
## Formula: err2 ~ (1 | inst.meas.tech)
##
     Data: dfinalb
## REML criterion at convergence: -900.2
##
```

```
## Scaled residuals:
## Min 1Q Median 3Q Max
## -4.8657 -0.5597 0.1550 0.6170 2.5881
##
## Random effects:
## Groups Name Variance Std.Dev.
## inst.meas.tech (Intercept) 0.00298 0.05459
## Residual 0.01498 0.12239
## Number of obs: 686, groups: inst.meas.tech, 19
##
## Fixed effects:
## Estimate Std. Error t value
## (Intercept) -0.008732 0.015141 -0.577
```