Mixed-effects models

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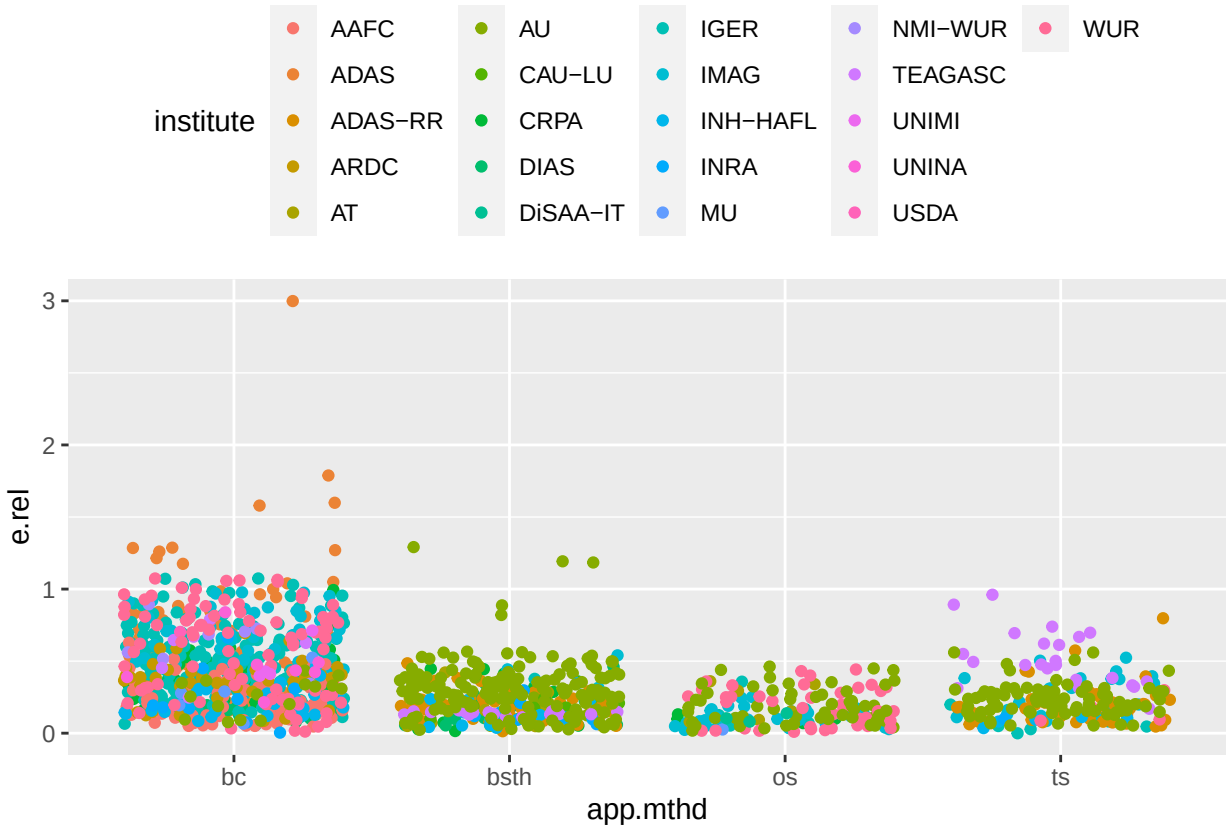
1. Take a look

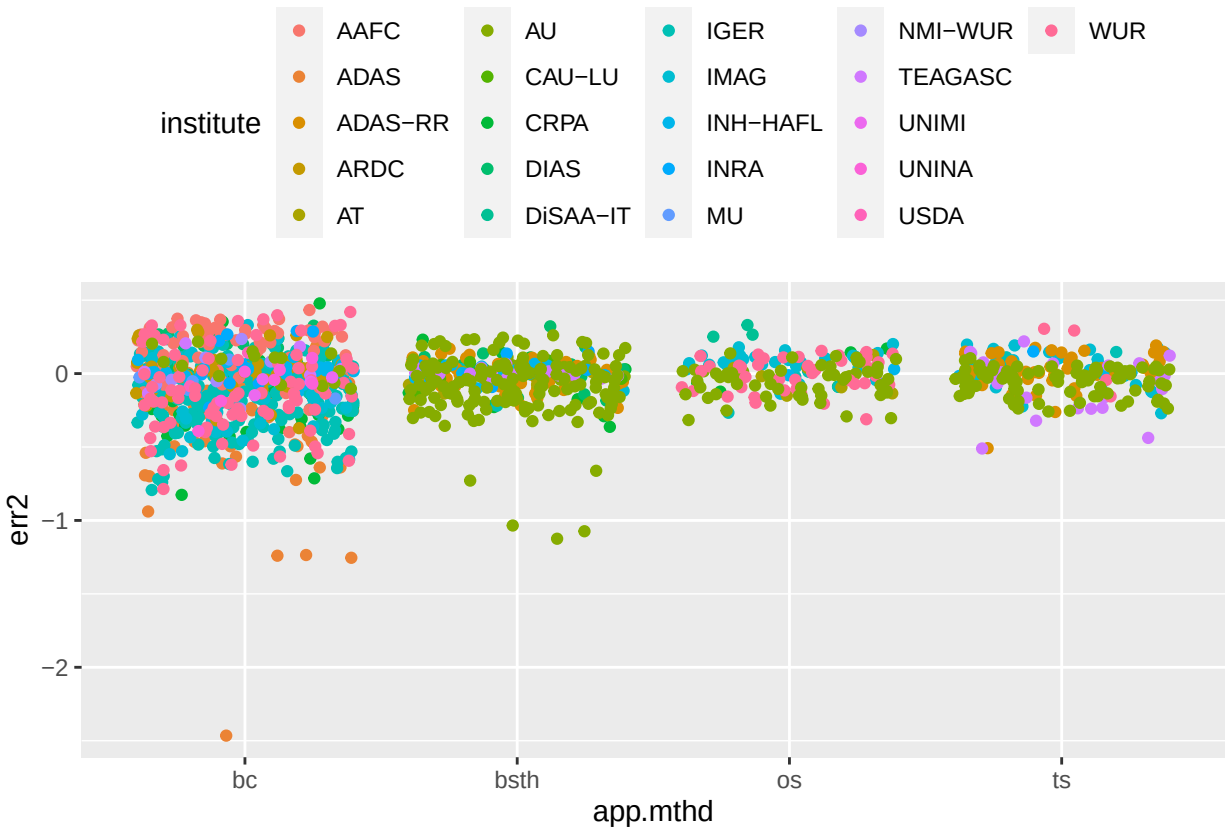
```
ggplot(dfinal, aes(app.mthd, e.rel, colour = institute)) +
  geom_jitter(height = 0) +
  theme(legend.position = 'top')
```

```
## Warning: Removed 33 rows containing missing values (`geom_point()`).
```




```
ggplot(dfinal, aes(app.mthd, err2, colour = institute)) +
  geom_jitter(height = 0) +
  theme(legend.position = 'top')
```

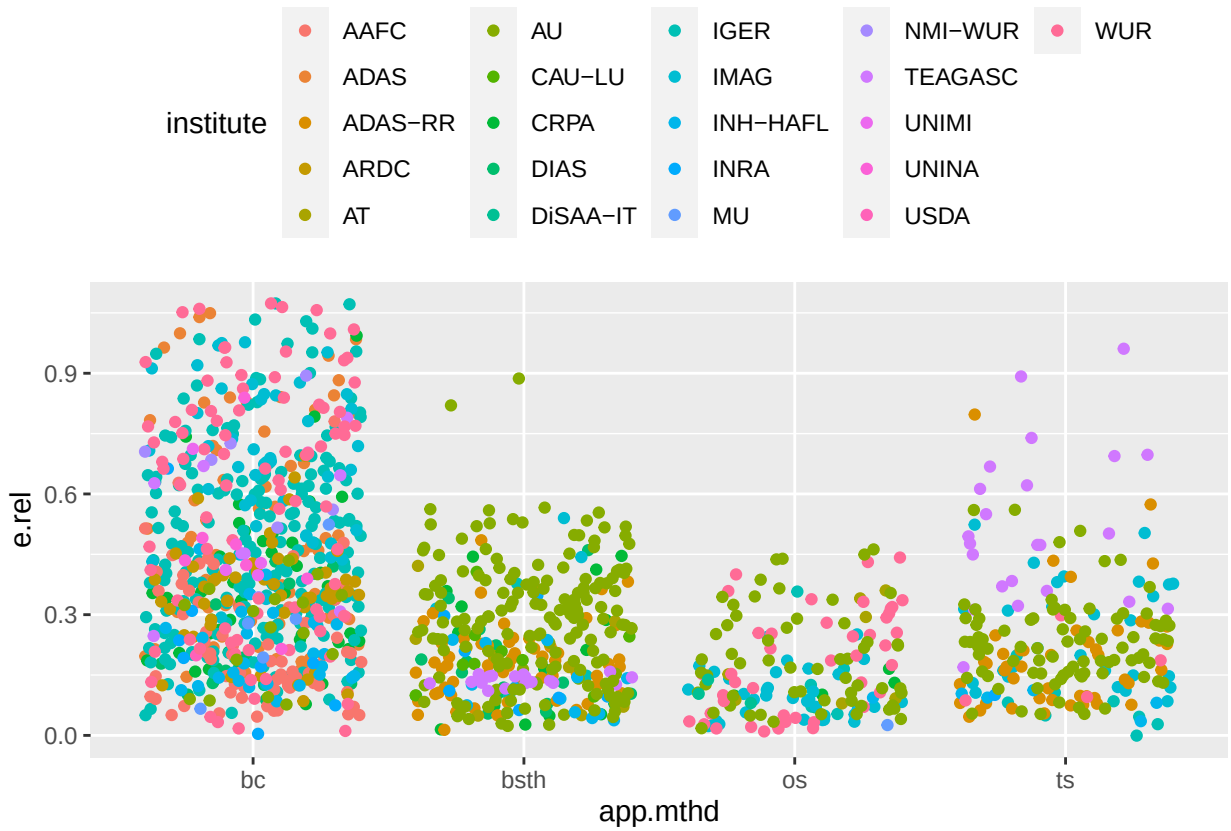
```
## Warning: Removed 33 rows containing missing values (`geom_point()`).
```



Drop values > 100% applied TAN.

```
dfinal <- dfinal[e.rel < 1.1, ]
```

```
ggplot(dfinal, aes(app.mthd, e.rel, colour = institute)) +
  geom_jitter(height = 0) +
  theme(legend.position = 'top')
```




```
ggplot(dfinal, aes(app.mthd, err2, colour = institute)) +
  geom_jitter(height = 0) +
  theme(legend.position = 'top')
```

2. Data prep

```
dfinal <- droplevels(dfinal[!is.na(e.rel), ])
dfinal$inst <- factor(dfinal$inst)
dfinal$inst.meas.tech <- interaction(dfinal$institute, dfinal$meas.tech)
dfinal$app.mthd <- factor(dfinal$app.mthd)
```

Get subset without crazy broadcast

```
dfinalb <- dfinal[app.mthd != 'bc', ]
```

3. Basic variability and comparison of simplest predictors

```
m0 <- lmer(e.rel ~ (1|inst.meas.tech), data = dfinal)
```

```
m1 <- lmer(e.rel ~ app.mthd + (1|inst.meas.tech), data = dfinal)
```

```
m2 <- lm(e.rel ~ app.mthd, data = dfinal)
```

```
AIC(m0, m1, m2)
```

```
##    df       AIC
## m0  3 -397.7495
## m1  6 -652.3573
## m2  5 -437.4887
```

```r
summary(m0)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: e.rel ~ (1 | inst.meas.tech)
##    Data: dfinal
##
## REML criterion at convergence: -403.7
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -2.1319 -0.6474 -0.1088  0.5276  3.3186
##
## Random effects:
##  Groups         Name        Variance Std.Dev.
##  inst.meas.tech (Intercept) 0.01526  0.1235
##  Residual                   0.04143  0.2035
## Number of obs: 1374, groups:  inst.meas.tech, 36
##
## Fixed effects:
##             Estimate Std. Error t value
## (Intercept)  0.31861    0.02449   13.01
```

```r
summary(m1)
```

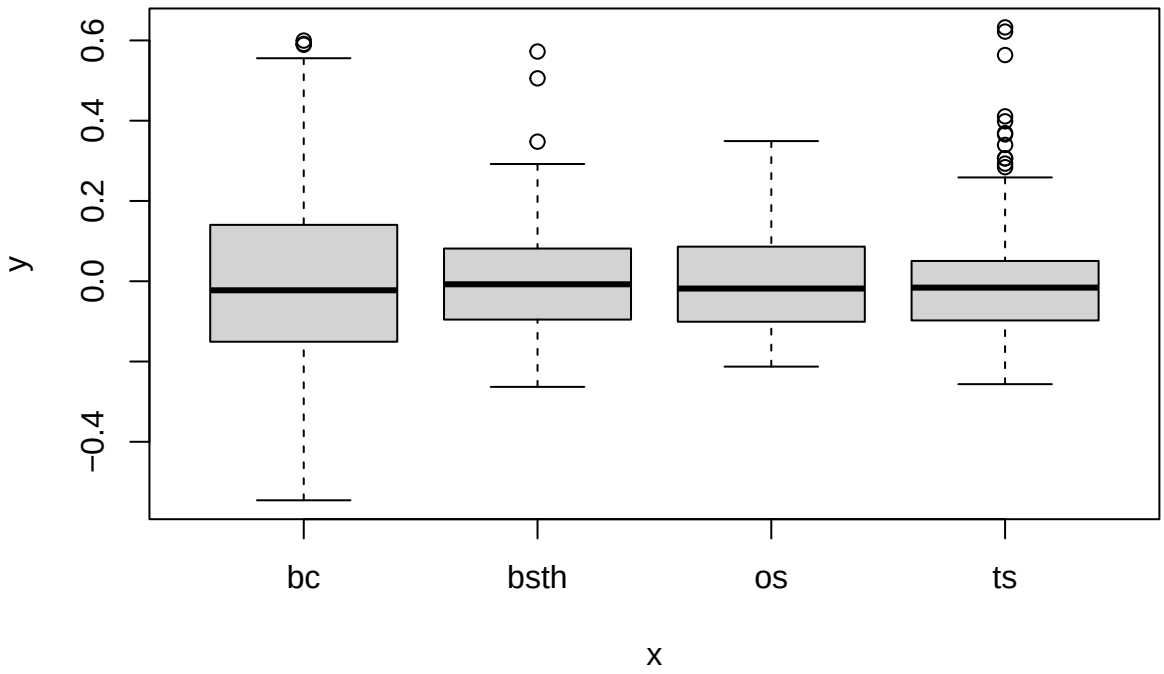
```
## Linear mixed model fit by REML ['lmerMod']
## Formula: e.rel ~ app.mthd + (1 | inst.meas.tech)
##    Data: dfinal
##
## REML criterion at convergence: -664.4
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -2.9610 -0.6694 -0.0821  0.5729  3.4315
##
## Random effects:
##  Groups         Name        Variance Std.Dev.
##  inst.meas.tech (Intercept) 0.01071  0.1035
##  Residual                   0.03397  0.1843
## Number of obs: 1374, groups:  inst.meas.tech, 36
##
## Fixed effects:
##              Estimate Std. Error t value
## (Intercept)   0.42453    0.02207   19.24
## app.mthdbsth -0.19357    0.01907  -10.15
## app.mthdos   -0.33425    0.01951  -17.13
## app.mthdts   -0.21337    0.02053  -10.39
##
## Correlation of Fixed Effects:
##             (Intr) app.mthdb app.mthds
## app.mthdbst -0.303
## app.mthdos  -0.236  0.437
## app.mthdts  -0.245  0.616     0.402
```

```r
summary(m2)
```

```
##
## Call:
## lm(formula = e.rel ~ app.mthd, data = dfinal)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.43101 -0.13592 -0.03796  0.11933  0.71341
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   0.43484    0.00785   55.39   <2e-16 ***
## app.mthdbsth -0.19986    0.01380  -14.48   <2e-16 ***
## app.mthdos   -0.27258    0.01821  -14.97   <2e-16 ***
## app.mthdts   -0.18736    0.01654  -11.33   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2059 on 1370 degrees of freedom
## Multiple R-squared:  0.2192, Adjusted R-squared:  0.2175
## F-statistic: 128.2 on 3 and 1370 DF,  p-value: < 2.2e-16
```

So, institute x measurement technique effect is around 12% of applied TAN (from model `m1`). Residuals are large, around 20% of applied TAN. Presumably residuals are smaller for injection.

```
res <- resid(m1)
plot(dfinal$app.mthd, res)
```



Perhaps, but could be worse.

Repeat without broadcast

```
m0b <- lmer(e.rel ~ (1|inst.meas.tech), data = dfinalb)
```

```
m1b <- lmer(e.rel ~ app.mthd + (1|inst.meas.tech), data = dfinalb)
```

```r
m2b <- lm(e.rel ~ app.mthd, data = dfinalb)
```

```r
AIC(m0b, m1b, m2b)
```

```
##     df       AIC
## m0b  3 -722.8347
## m1b  5 -733.1104
## m2b  4 -702.5938
```

```r
summary(m0b)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: e.rel ~ (1 | inst.meas.tech)
##    Data: dfinalb
##
## REML criterion at convergence: -728.8
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -1.7406 -0.7082 -0.1489  0.5329  4.5316
##
## Random effects:
##  Groups         Name        Variance Std.Dev.
##  inst.meas.tech (Intercept) 0.003508 0.05922
##  Residual                   0.019267 0.13881
## Number of obs: 686, groups:  inst.meas.tech, 19
##
## Fixed effects:
##             Estimate Std. Error t value
## (Intercept)  0.19900    0.01659      12
```

```r
summary(m1b)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: e.rel ~ app.mthd + (1 | inst.meas.tech)
##    Data: dfinalb
##
## REML criterion at convergence: -743.1
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -1.8037 -0.7134 -0.1433  0.5686  4.5847
##
## Random effects:
##  Groups         Name        Variance Std.Dev.
##  inst.meas.tech (Intercept) 0.003163 0.05624
##  Residual                   0.018580 0.13631
## Number of obs: 686, groups:  inst.meas.tech, 19
##
## Fixed effects:
##              Estimate Std. Error t value
## (Intercept)  0.221512   0.016987  13.040
## app.mthdos  -0.077669   0.016031  -4.845
## app.mthdts   0.004581   0.013043   0.351
##
```

```
## Correlation of Fixed Effects:
##             (Intr) app.mthds
## app.mthdos -0.330
## app.mthdts -0.240  0.347
```

```
summary(m2b)
```

```
##
## Call:
## lm(formula = e.rel ~ app.mthd, data = dfinalb)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.24808 -0.10570 -0.03087  0.07986  0.71341
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.234982   0.007965  29.502  < 2e-16 ***
## app.mthdos  -0.072725   0.014014  -5.190 2.78e-07 ***
## app.mthdts   0.012503   0.012954   0.965    0.335
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1445 on 683 degrees of freedom
## Multiple R-squared:  0.04971,    Adjusted R-squared:  0.04693
## F-statistic: 17.86 on 2 and 683 DF,  p-value: 2.741e-08
```

Less variability without broadcast.

4. ALFAM2 model residuals

```
m3 <- lmer(err2 ~ (1|inst.meas.tech), data = dfinal)
```

```
m4 <- lmer(err2 ~ app.mthd + (1|inst.meas.tech), data = dfinal)
```

```
summary(m0)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: e.rel ~ (1 | inst.meas.tech)
##    Data: dfinal
##
## REML criterion at convergence: -403.7
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -2.1319 -0.6474 -0.1088  0.5276  3.3186
##
## Random effects:
##  Groups         Name        Variance Std.Dev.
##  inst.meas.tech (Intercept) 0.01526  0.1235
##  Residual                   0.04143  0.2035
## Number of obs: 1374, groups:  inst.meas.tech, 36
##
## Fixed effects:
##             Estimate Std. Error t value
```

```
## (Intercept)  0.31861    0.02449   13.01
```

```
summary(m1)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: e.rel ~ app.mthd + (1 | inst.meas.tech)
##    Data: dfinal
##
## REML criterion at convergence: -664.4
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -2.9610 -0.6694 -0.0821  0.5729  3.4315
##
## Random effects:
##  Groups         Name        Variance Std.Dev.
##  inst.meas.tech (Intercept) 0.01071  0.1035
##  Residual                   0.03397  0.1843
## Number of obs: 1374, groups:  inst.meas.tech, 36
##
## Fixed effects:
##              Estimate Std. Error t value
## (Intercept)   0.42453    0.02207   19.24
## app.mthdbsth -0.19357    0.01907  -10.15
## app.mthdos   -0.33425    0.01951  -17.13
## app.mthdts   -0.21337    0.02053  -10.39
##
## Correlation of Fixed Effects:
##             (Intr) app.mthdb app.mthds
## app.mthdbst -0.303
## app.mthdos  -0.236  0.437
## app.mthdts  -0.245  0.616     0.402
```

```
summary(m3)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: err2 ~ (1 | inst.meas.tech)
##    Data: dfinal
##
## REML criterion at convergence: -866.7
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -4.1024 -0.5404  0.0910  0.6069  3.5164
##
## Random effects:
##  Groups         Name        Variance Std.Dev.
##  inst.meas.tech (Intercept) 0.007568 0.08699
##  Residual                   0.029763 0.17252
## Number of obs: 1374, groups:  inst.meas.tech, 36
##
## Fixed effects:
##             Estimate Std. Error t value
## (Intercept) -0.02310    0.01793  -1.288
```

```
summary(m4)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: err2 ~ app.mthd + (1 | inst.meas.tech)
##    Data: dfinal
##
## REML criterion at convergence: -867.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.9793 -0.5423  0.0830  0.5966  3.6275
##
## Random effects:
##  Groups         Name        Variance Std.Dev.
##  inst.meas.tech (Intercept) 0.00779  0.08826
##  Residual                   0.02938  0.17139
## Number of obs: 1374, groups:  inst.meas.tech, 36
##
## Fixed effects:
##              Estimate Std. Error t value
## (Intercept)  -0.04486    0.01933  -2.321
## app.mthdbsth  0.03217    0.01761   1.827
## app.mthdos    0.07936    0.01807   4.391
## app.mthdts    0.04612    0.01901   2.427
##
## Correlation of Fixed Effects:
##             (Intr) app.mthdb app.mthds
## app.mthdbst -0.323
## app.mthdos  -0.249  0.433
## app.mthdts  -0.261  0.612     0.399
```

```
AIC(m3, m4)
```

```
##    df       AIC
## m3  3 -860.6798
## m4  6 -855.4677
```

Reassuring that `m3` is actually a better model than `m4`, meaning adding application method on top of ALFAM2 predictions doesn't help.

Again, exclude broadcast.

```
m3b <- lmer(err2 ~ (1|inst.meas.tech), data = dfinalb)
```

```
m4b <- lmer(err2 ~ app.mthd + (1|inst.meas.tech), data = dfinalb)
```

```
summary(m0b)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: e.rel ~ (1 | inst.meas.tech)
##    Data: dfinalb
##
## REML criterion at convergence: -728.8
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -1.7406 -0.7082 -0.1489  0.5329  4.5316
```

```
##
## Random effects:
##  Groups         Name        Variance Std.Dev.
##  inst.meas.tech (Intercept) 0.003508 0.05922
##  Residual                   0.019267 0.13881
## Number of obs: 686, groups:  inst.meas.tech, 19
##
## Fixed effects:
##             Estimate Std. Error t value
## (Intercept)  0.19900    0.01659      12
```

```
summary(m1b)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: e.rel ~ app.mthd + (1 | inst.meas.tech)
##    Data: dfinalb
##
## REML criterion at convergence: -743.1
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -1.8037 -0.7134 -0.1433  0.5686  4.5847
##
## Random effects:
##  Groups         Name        Variance Std.Dev.
##  inst.meas.tech (Intercept) 0.003163 0.05624
##  Residual                   0.018580 0.13631
## Number of obs: 686, groups:  inst.meas.tech, 19
##
## Fixed effects:
##              Estimate Std. Error t value
## (Intercept)  0.221512   0.016987  13.040
## app.mthdos  -0.077669   0.016031  -4.845
## app.mthdts   0.004581   0.013043   0.351
##
## Correlation of Fixed Effects:
##             (Intr) app.mthds
## app.mthdos -0.330
## app.mthdts -0.240  0.347
```

```
summary(m3b)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: err2 ~ (1 | inst.meas.tech)
##    Data: dfinalb
##
## REML criterion at convergence: -900.2
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -4.8657 -0.5597  0.1550  0.6170  2.5881
##
## Random effects:
##  Groups         Name        Variance Std.Dev.
##  inst.meas.tech (Intercept) 0.00298  0.05459
```

```
##  Residual                   0.01498  0.12239
## Number of obs: 686, groups:  inst.meas.tech, 19
##
## Fixed effects:
##              Estimate Std. Error t value
## (Intercept) -0.008732   0.015141  -0.577
```

```
summary(m4b)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: err2 ~ app.mthd + (1 | inst.meas.tech)
##    Data: dfinalb
##
## REML criterion at convergence: -886.8
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -4.8666 -0.5765  0.1516  0.6033  2.6082
##
## Random effects:
##  Groups         Name        Variance Std.Dev.
##  inst.meas.tech (Intercept) 0.002823 0.05313
##  Residual                   0.015029 0.12259
## Number of obs: 686, groups:  inst.meas.tech, 19
##
## Fixed effects:
##              Estimate Std. Error t value
## (Intercept) -0.012550   0.015805  -0.794
## app.mthdos   0.009106   0.014461   0.630
## app.mthdts   0.005060   0.011744   0.431
##
## Correlation of Fixed Effects:
##            (Intr) app.mthds
## app.mthdos -0.321
## app.mthdts -0.232  0.348
```

```
AIC(m3b, m4b)
```

```
##     df       AIC
## m3b  3 -894.2066
## m4b  5 -876.8290
```

Here too, `m3b` is better.

5. "Institution effect"

Our best estimate of an "institution effect" is from `m3`, where we have corrected for different application methods, manure DM, pH, and weather using the ALFAM2 model.

```
VarCorr(m1)
```

```
##  Groups         Name        Std.Dev.
##  inst.meas.tech (Intercept) 0.10348
##  Residual                   0.18431
```

```
VarCorr(m1b)
```

```
##  Groups         Name        Std.Dev.
##  inst.meas.tech (Intercept) 0.056243
##  Residual                   0.136309
```

```
VarCorr(m3)
```

```
##  Groups         Name        Std.Dev.
##  inst.meas.tech (Intercept) 0.086993
##  Residual                   0.172521
```

```
VarCorr(m3b)
```

```
##  Groups         Name        Std.Dev.
##  inst.meas.tech (Intercept) 0.054594
##  Residual                   0.122390
```

```
summary(m1)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: e.rel ~ app.mthd + (1 | inst.meas.tech)
##    Data: dfinal
##
## REML criterion at convergence: -664.4
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -2.9610 -0.6694 -0.0821  0.5729  3.4315
##
## Random effects:
##  Groups         Name        Variance Std.Dev.
##  inst.meas.tech (Intercept) 0.01071  0.1035
##  Residual                   0.03397  0.1843
## Number of obs: 1374, groups:  inst.meas.tech, 36
##
## Fixed effects:
##              Estimate Std. Error t value
## (Intercept)   0.42453    0.02207   19.24
## app.mthdbsth -0.19357    0.01907  -10.15
## app.mthdos   -0.33425    0.01951  -17.13
## app.mthdts   -0.21337    0.02053  -10.39
##
## Correlation of Fixed Effects:
##             (Intr) app.mthdb app.mthds
## app.mthdbst -0.303
## app.mthdos  -0.236  0.437
## app.mthdts  -0.245  0.616     0.402
```

```
VarCorr(m1b)
```

```
##  Groups         Name        Std.Dev.
##  inst.meas.tech (Intercept) 0.056243
##  Residual                   0.136309
```

```
summary(m3)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: err2 ~ (1 | inst.meas.tech)
##    Data: dfinal
```

```
##
## REML criterion at convergence: -866.7
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -4.1024 -0.5404  0.0910  0.6069  3.5164
##
## Random effects:
##  Groups         Name        Variance Std.Dev.
##  inst.meas.tech (Intercept) 0.007568 0.08699
##  Residual                   0.029763 0.17252
## Number of obs: 1374, groups:  inst.meas.tech, 36
##
## Fixed effects:
##             Estimate Std. Error t value
## (Intercept) -0.02310    0.01793  -1.288
```

```
VarCorr(m3b)
```

```
##  Groups         Name        Std.Dev.
##  inst.meas.tech (Intercept) 0.054594
##  Residual                   0.122390
```

```
summary(m3b)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: err2 ~ (1 | inst.meas.tech)
##    Data: dfinalb
##
## REML criterion at convergence: -900.2
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -4.8657 -0.5597  0.1550  0.6170  2.5881
##
## Random effects:
##  Groups         Name        Variance Std.Dev.
##  inst.meas.tech (Intercept) 0.00298  0.05459
##  Residual                   0.01498  0.12239
## Number of obs: 686, groups:  inst.meas.tech, 19
##
## Fixed effects:
##              Estimate Std. Error t value
## (Intercept) -0.008732   0.015141  -0.577
```