Mixed-effects models

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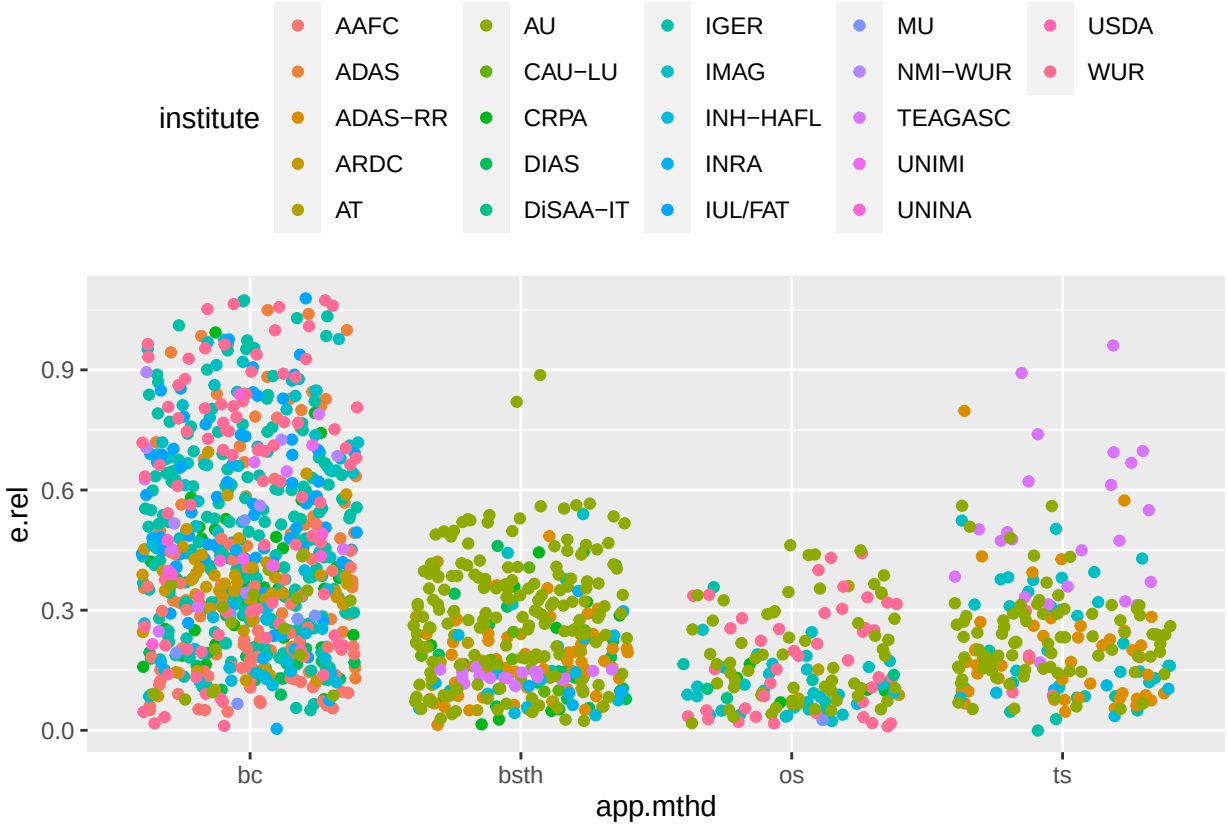
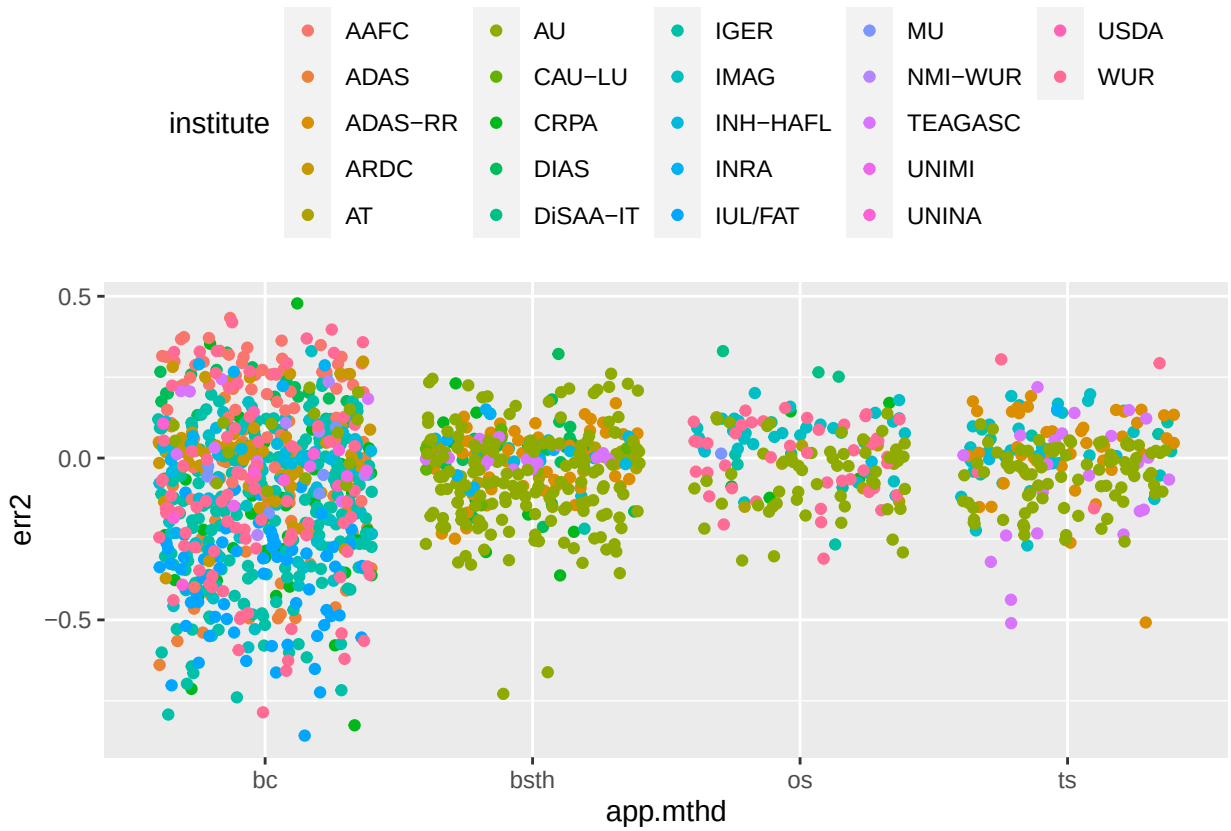
1. Take a look

```
ggplot(dfinal, aes(app.mthd, e.rel, colour = institute)) +
  geom_jitter(height = 0) +
  theme(legend.position = 'top')
```




```
ggplot(dfinal, aes(app.mthd, err2, colour = institute)) +
  geom_jitter(height = 0) +
  theme(legend.position = 'top')
```



Drop values > 100% applied TAN.

```
dim(dfinal)
```

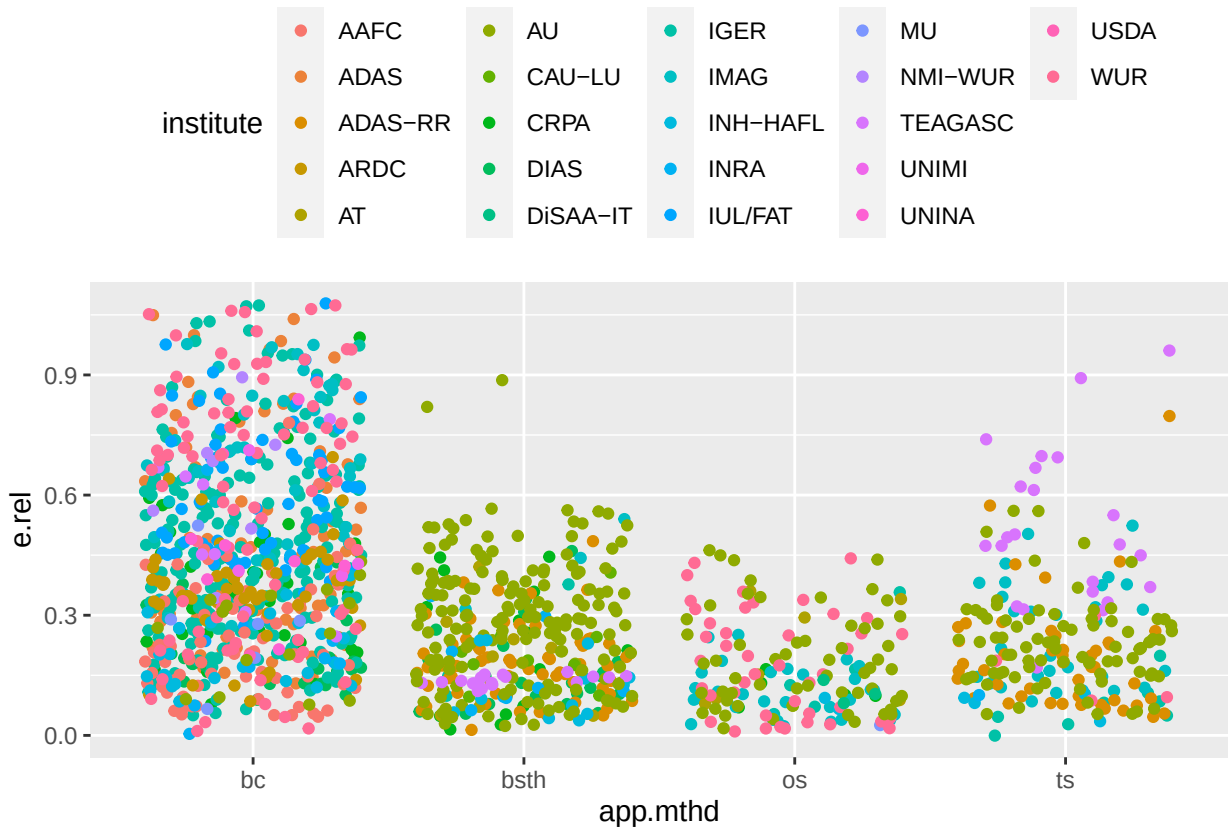
```
## [1] 1441  283
```

```
length(unique(dfinal$pmid))
```

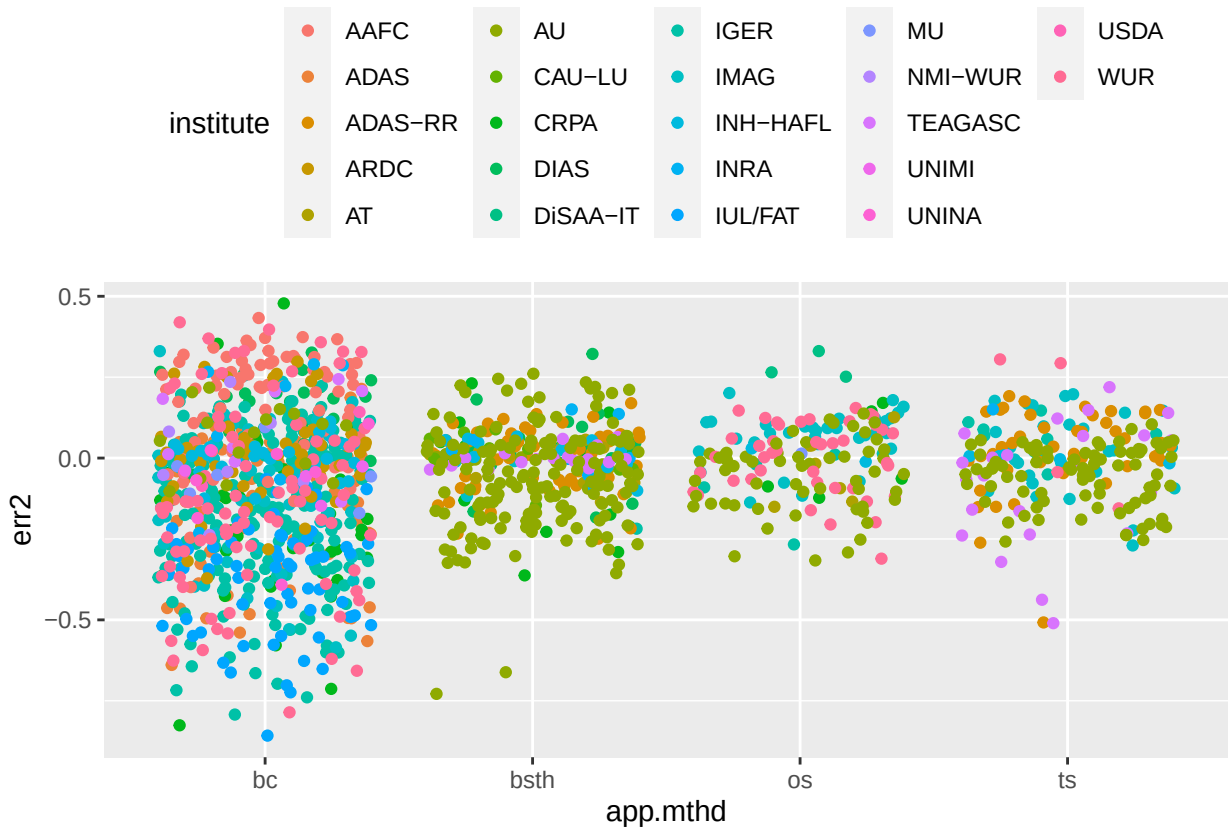
```
## [1] 1441
```

```
dfinal <- dfinal[e.rel < 1.1, ]
```

```
ggplot(dfinal, aes(app.mthd, e.rel, colour = institute)) +
  geom_jitter(height = 0) +
  theme(legend.position = 'top')
```




```
ggplot(dfinal, aes(app.mthd, err2, colour = institute)) +
  geom_jitter(height = 0) +
  theme(legend.position = 'top')
```



Total counts.

```
dim(dfinal)
```

```
## [1] 1441  283
```

```
length(unique(dfinal$pmid))
```

```
## [1] 1441
```

```
length(unique(dfinal$country))
```

```
## [1] 11
```

```
length(unique(dfinal$inst))
```

```
## [1] 22
```

```
unique(dfinal$country)
```

```
##  [1] "UK" "IT" "DK" "NL" "CH" "CA" "DE" "FR" "IE" "US" "SE"
```

```
unique(dfinal$institute)
```

```
##  [1] "ADAS"     "CRPA"     "DIAS"     "IGER"     "IMAG"     "IUL/FAT"
##  [7] "AAFC"     "ADAS-RR"  "ARDC"     "AT"       "AU"       "CAU-LU"
## [13] "INH-HAFL" "INRA"     "MU"       "NMI-WUR"  "TEAGASC"  "USDA"
## [19] "WUR"      "DiSAA-IT" "UNIMI"    "UNINA"
```

```
unique(dfinal$inst)
```

```
##  [1] 101 103 104 105 106 107 201 202 203 204 205 206 207 208 209 210 212 213
## [19] 214 303 304 305
```

```
## 22 Levels: 101 103 104 105 106 107 201 202 203 204 205 206 207 208 ... 305
```

2. Data prep

```
dfinal <- droplevels(dfinal[!is.na(e.rel), ])
dfinal$inst <- factor(dfinal$inst)
dfinal$inst.meas.tech <- interaction(dfinal$institute, dfinal$meas.tech)
dfinal$app.mthd <- factor(dfinal$app.mthd)
```

Get subset without crazy broadcast

```
dfinalb <- dfinal[app.mthd != 'bc', ]
```

3. Basic variability and comparison of simplest predictors

```
m0 <- lmer(e.rel ~ (1|inst.meas.tech), data = dfinal)
```

```
m1 <- lmer(e.rel ~ app.mthd + (1|inst.meas.tech), data = dfinal)
```

```
m2 <- lm(e.rel ~ app.mthd, data = dfinal)
```

```
AIC(m0, m1, m2)
```

```
##    df       AIC
## m0  3 -421.8459
## m1  6 -678.7292
## m2  5 -438.8121
```

```
summary(m0)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: e.rel ~ (1 | inst.meas.tech)
##    Data: dfinal
##
## REML criterion at convergence: -427.8
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -2.1406 -0.6467 -0.1080  0.5530  3.3263
##
## Random effects:
##  Groups         Name        Variance Std.Dev.
##  inst.meas.tech (Intercept) 0.01762  0.1327
##  Residual                   0.04119  0.2030
## Number of obs: 1441, groups:  inst.meas.tech, 37
##
## Fixed effects:
##             Estimate Std. Error t value
## (Intercept)  0.32878    0.02539   12.95
```

```
summary(m1)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: e.rel ~ app.mthd + (1 | inst.meas.tech)
##    Data: dfinal
```

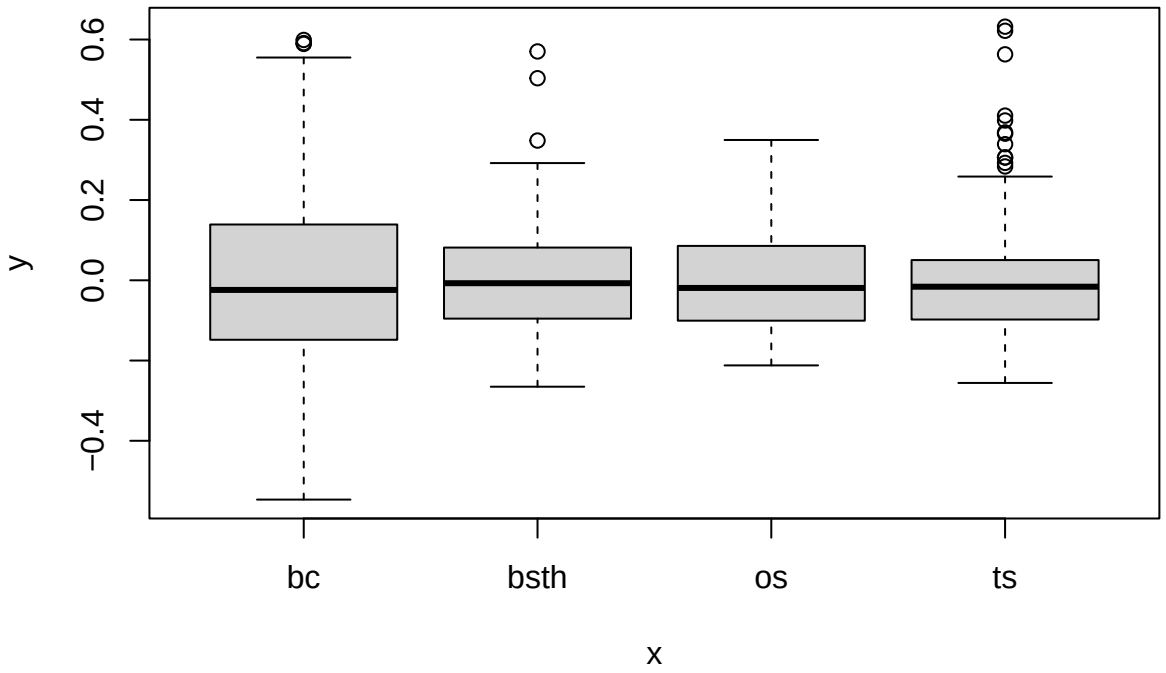
```
##
## REML criterion at convergence: -690.7
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -2.9593 -0.6684 -0.0838  0.5823  3.4226
##
## Random effects:
##  Groups         Name        Variance Std.Dev.
##  inst.meas.tech (Intercept) 0.01128  0.1062
##  Residual                   0.03410  0.1847
## Number of obs: 1441, groups:  inst.meas.tech, 37
##
## Fixed effects:
##              Estimate Std. Error t value
## (Intercept)   0.43034    0.02195   19.60
## app.mthdbsth -0.19508    0.01911  -10.21
## app.mthdos   -0.33559    0.01954  -17.17
## app.mthdts   -0.21475    0.02057  -10.44
##
## Correlation of Fixed Effects:
##             (Intr) app.mthdb app.mthds
## app.mthdbst -0.291
## app.mthdos  -0.227  0.436
## app.mthdts  -0.236  0.616     0.401
```

```
summary(m2)
```

```
##
## Call:
## lm(formula = e.rel ~ app.mthd, data = dfinal)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.44340 -0.13681 -0.03545  0.12378  0.71341
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)   0.447225   0.007547   59.26   <2e-16 ***
## app.mthdbsth -0.212243   0.013699  -15.49   <2e-16 ***
## app.mthdos   -0.284969   0.018189  -15.67   <2e-16 ***
## app.mthdts   -0.199741   0.016491  -12.11   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2074 on 1437 degrees of freedom
## Multiple R-squared:  0.235,  Adjusted R-squared:  0.2334
## F-statistic: 147.1 on 3 and 1437 DF,  p-value: < 2.2e-16
```

So, institute x measurement technique effect is around 12% of applied TAN (from model `m1`). Residuals are large, around 20% of applied TAN. Presumably residuals are smaller for injection.

```
res <- resid(m1)
plot(dfinal$app.mthd, res)
```



Perhaps, but could be worse.

Repeat without broadcast

```
m0b <- lmer(e.rel ~ (1|inst.meas.tech), data = dfinalb)
```

```
m1b <- lmer(e.rel ~ app.mthd + (1|inst.meas.tech), data = dfinalb)
```

```
m2b <- lm(e.rel ~ app.mthd, data = dfinalb)
```

```
AIC(m0b, m1b, m2b)
```

```
##     df       AIC
## m0b  3 -722.8347
## m1b  5 -733.1104
## m2b  4 -702.5938
```

```
summary(m0b)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: e.rel ~ (1 | inst.meas.tech)
##    Data: dfinalb
##
## REML criterion at convergence: -728.8
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -1.7406 -0.7082 -0.1489  0.5329  4.5316
##
## Random effects:
##  Groups         Name        Variance Std.Dev.
##  inst.meas.tech (Intercept) 0.003508 0.05922
##  Residual                   0.019267 0.13881
## Number of obs: 686, groups:  inst.meas.tech, 19
##
```

```
## Fixed effects:
##             Estimate Std. Error t value
## (Intercept)  0.19900    0.01659      12
```

```
summary(m1b)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: e.rel ~ app.mthd + (1 | inst.meas.tech)
##    Data: dfinalb
##
## REML criterion at convergence: -743.1
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -1.8037 -0.7134 -0.1433  0.5686  4.5847
##
## Random effects:
##  Groups         Name        Variance Std.Dev.
##  inst.meas.tech (Intercept) 0.003163 0.05624
##  Residual                   0.018580 0.13631
## Number of obs: 686, groups:  inst.meas.tech, 19
##
## Fixed effects:
##              Estimate Std. Error t value
## (Intercept)  0.221512   0.016987  13.040
## app.mthdos  -0.077669   0.016031  -4.845
## app.mthdts   0.004581   0.013043   0.351
##
## Correlation of Fixed Effects:
##             (Intr) app.mthds
## app.mthdos -0.330
## app.mthdts -0.240  0.347
```

```
summary(m2b)
```

```
##
## Call:
## lm(formula = e.rel ~ app.mthd, data = dfinalb)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.24808 -0.10570 -0.03087  0.07986  0.71341
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.234982   0.007965  29.502  < 2e-16 ***
## app.mthdos  -0.072725   0.014014  -5.190 2.78e-07 ***
## app.mthdts   0.012503   0.012954   0.965    0.335
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1445 on 683 degrees of freedom
## Multiple R-squared:  0.04971,    Adjusted R-squared:  0.04693
## F-statistic: 17.86 on 2 and 683 DF,  p-value: 2.741e-08
```

Less variability without broadcast.

4. ALFAM2 model residuals

```
m3 <- lmer(err2 ~ (1|inst.meas.tech), data = dfinal)
```

```
m4 <- lmer(err2 ~ app.mthd + (1|inst.meas.tech), data = dfinal)
```

```
summary(m0)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: e.rel ~ (1 | inst.meas.tech)
##    Data: dfinal
##
## REML criterion at convergence: -427.8
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -2.1406 -0.6467 -0.1080  0.5530  3.3263
##
## Random effects:
##  Groups         Name        Variance Std.Dev.
##  inst.meas.tech (Intercept) 0.01762  0.1327
##  Residual                   0.04119  0.2030
## Number of obs: 1441, groups:  inst.meas.tech, 37
##
## Fixed effects:
##             Estimate Std. Error t value
## (Intercept)  0.32878    0.02539   12.95
```

```
summary(m1)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: e.rel ~ app.mthd + (1 | inst.meas.tech)
##    Data: dfinal
##
## REML criterion at convergence: -690.7
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -2.9593 -0.6684 -0.0838  0.5823  3.4226
##
## Random effects:
##  Groups         Name        Variance Std.Dev.
##  inst.meas.tech (Intercept) 0.01128  0.1062
##  Residual                   0.03410  0.1847
## Number of obs: 1441, groups:  inst.meas.tech, 37
##
## Fixed effects:
##              Estimate Std. Error t value
## (Intercept)   0.43034    0.02195   19.60
## app.mthdbsth -0.19508    0.01911  -10.21
## app.mthdos   -0.33559    0.01954  -17.17
## app.mthdts   -0.21475    0.02057  -10.44
##
## Correlation of Fixed Effects:
##             (Intr) app.mthdb app.mthds
```

```
## app.mthdbst -0.291
## app.mthdos  -0.227  0.436
## app.mthdts  -0.236  0.616     0.401
```

```
summary(m3)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: err2 ~ (1 | inst.meas.tech)
##    Data: dfinal
##
## REML criterion at convergence: -902.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -4.1054 -0.5388  0.1000  0.6074  3.5407
##
## Random effects:
##  Groups         Name        Variance Std.Dev.
##  inst.meas.tech (Intercept) 0.01207  0.1098
##  Residual                   0.02965  0.1722
## Number of obs: 1441, groups:  inst.meas.tech, 37
##
## Fixed effects:
##             Estimate Std. Error t value
## (Intercept) -0.03566    0.02111  -1.689
```

```
summary(m4)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: err2 ~ app.mthd + (1 | inst.meas.tech)
##    Data: dfinal
##
## REML criterion at convergence: -904.7
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.9679 -0.5474  0.0938  0.5880  3.6548
##
## Random effects:
##  Groups         Name        Variance Std.Dev.
##  inst.meas.tech (Intercept) 0.01161  0.1077
##  Residual                   0.02929  0.1711
## Number of obs: 1441, groups:  inst.meas.tech, 37
##
## Fixed effects:
##              Estimate Std. Error t value
## (Intercept)  -0.05802    0.02173  -2.670
## app.mthdbsth  0.03633    0.01783   2.037
## app.mthdos    0.08261    0.01818   4.544
## app.mthdts    0.04962    0.01915   2.592
##
## Correlation of Fixed Effects:
##             (Intr) app.mthdb app.mthds
## app.mthdbst -0.272
## app.mthdos  -0.214  0.439
```

```
## app.mthdts  -0.220  0.619     0.404
```

```
AIC(m3, m4)
```

```
##    df       AIC
## m3  3 -896.5179
## m4  6 -892.7441
```

Reassuring that `m3` is actually a better model than `m4`, meaning adding application method on top of ALFAM2 predictions doesn't help.

Again, exclude broadcast.

```
m3b <- lmer(err2 ~ (1|inst.meas.tech), data = dfinalb)
```

```
m4b <- lmer(err2 ~ app.mthd + (1|inst.meas.tech), data = dfinalb)
```

```
summary(m0b)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: e.rel ~ (1 | inst.meas.tech)
##    Data: dfinalb
##
## REML criterion at convergence: -728.8
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -1.7406 -0.7082 -0.1489  0.5329  4.5316
##
## Random effects:
##  Groups         Name        Variance Std.Dev.
##  inst.meas.tech (Intercept) 0.003508 0.05922
##  Residual                   0.019267 0.13881
## Number of obs: 686, groups:  inst.meas.tech, 19
##
## Fixed effects:
##             Estimate Std. Error t value
## (Intercept)  0.19900    0.01659      12
```

```
summary(m1b)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: e.rel ~ app.mthd + (1 | inst.meas.tech)
##    Data: dfinalb
##
## REML criterion at convergence: -743.1
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -1.8037 -0.7134 -0.1433  0.5686  4.5847
##
## Random effects:
##  Groups         Name        Variance Std.Dev.
##  inst.meas.tech (Intercept) 0.003163 0.05624
##  Residual                   0.018580 0.13631
## Number of obs: 686, groups:  inst.meas.tech, 19
##
## Fixed effects:
```

```
##              Estimate Std. Error t value
## (Intercept)  0.221512   0.016987  13.040
## app.mthdos  -0.077669   0.016031  -4.845
## app.mthdts   0.004581   0.013043   0.351
##
## Correlation of Fixed Effects:
##            (Intr) app.mthds
## app.mthdos -0.330
## app.mthdts -0.240  0.347
```

```
summary(m3b)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: err2 ~ (1 | inst.meas.tech)
##    Data: dfinalb
##
## REML criterion at convergence: -900.2
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -4.8657 -0.5597  0.1550  0.6170  2.5881
##
## Random effects:
##  Groups         Name        Variance Std.Dev.
##  inst.meas.tech (Intercept) 0.00298  0.05459
##  Residual                   0.01498  0.12239
## Number of obs: 686, groups:  inst.meas.tech, 19
##
## Fixed effects:
##              Estimate Std. Error t value
## (Intercept) -0.008732   0.015141  -0.577
```

```
summary(m4b)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: err2 ~ app.mthd + (1 | inst.meas.tech)
##    Data: dfinalb
##
## REML criterion at convergence: -886.8
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -4.8666 -0.5765  0.1516  0.6033  2.6082
##
## Random effects:
##  Groups         Name        Variance Std.Dev.
##  inst.meas.tech (Intercept) 0.002823 0.05313
##  Residual                   0.015029 0.12259
## Number of obs: 686, groups:  inst.meas.tech, 19
##
## Fixed effects:
##              Estimate Std. Error t value
## (Intercept) -0.012550   0.015805  -0.794
## app.mthdos   0.009106   0.014461   0.630
## app.mthdts   0.005060   0.011744   0.431
```

```
##
## Correlation of Fixed Effects:
##             (Intr) app.mthds
## app.mthdos -0.321
## app.mthdts -0.232  0.348
```

```
AIC(m3b, m4b)
```

```
##     df       AIC
## m3b  3 -894.2066
## m4b  5 -876.8290
```

Here too, `m3b` is better.

5. "Institution effect"

Our best estimate of an "institution effect" is from `m3`, where we have corrected for different application methods, manure DM, pH, and weather using the ALFAM2 model.

```
VarCorr(m1)
```

```
##  Groups         Name        Std.Dev.
##  inst.meas.tech (Intercept) 0.10621
##  Residual                   0.18467
```

```
VarCorr(m1b)
```

```
##  Groups         Name        Std.Dev.
##  inst.meas.tech (Intercept) 0.056243
##  Residual                   0.136309
```

```
VarCorr(m3)
```

```
##  Groups         Name        Std.Dev.
##  inst.meas.tech (Intercept) 0.10984
##  Residual                   0.17220
```

```
VarCorr(m3b)
```

```
##  Groups         Name        Std.Dev.
##  inst.meas.tech (Intercept) 0.054594
##  Residual                   0.122390
```

```
summary(m1)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: e.rel ~ app.mthd + (1 | inst.meas.tech)
##    Data: dfinal
##
## REML criterion at convergence: -690.7
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -2.9593 -0.6684 -0.0838  0.5823  3.4226
##
## Random effects:
##  Groups         Name        Variance Std.Dev.
##  inst.meas.tech (Intercept) 0.01128  0.1062
##  Residual                   0.03410  0.1847
```

```
## Number of obs: 1441, groups:  inst.meas.tech, 37
##
## Fixed effects:
##              Estimate Std. Error t value
## (Intercept)   0.43034    0.02195   19.60
## app.mthdbsth -0.19508    0.01911  -10.21
## app.mthdos   -0.33559    0.01954  -17.17
## app.mthdts   -0.21475    0.02057  -10.44
##
## Correlation of Fixed Effects:
##             (Intr) app.mthdb app.mthds
## app.mthdbst -0.291
## app.mthdos  -0.227  0.436
## app.mthdts  -0.236  0.616     0.401
```

```
VarCorr(m1b)
```

```
##  Groups         Name        Std.Dev.
##  inst.meas.tech (Intercept) 0.056243
##  Residual                   0.136309
```

```
summary(m3)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: err2 ~ (1 | inst.meas.tech)
##    Data: dfinal
##
## REML criterion at convergence: -902.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -4.1054 -0.5388  0.1000  0.6074  3.5407
##
## Random effects:
##  Groups         Name        Variance Std.Dev.
##  inst.meas.tech (Intercept) 0.01207  0.1098
##  Residual                   0.02965  0.1722
## Number of obs: 1441, groups:  inst.meas.tech, 37
##
## Fixed effects:
##             Estimate Std. Error t value
## (Intercept) -0.03566    0.02111  -1.689
```

```
VarCorr(m3b)
```

```
##  Groups         Name        Std.Dev.
##  inst.meas.tech (Intercept) 0.054594
##  Residual                   0.122390
```

```
summary(m3b)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: err2 ~ (1 | inst.meas.tech)
##    Data: dfinalb
##
## REML criterion at convergence: -900.2
##
```

```
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -4.8657 -0.5597  0.1550  0.6170  2.5881
##
## Random effects:
##  Groups         Name        Variance Std.Dev.
##  inst.meas.tech (Intercept) 0.00298  0.05459
##  Residual                   0.01498  0.12239
## Number of obs: 686, groups:  inst.meas.tech, 19
##
## Fixed effects:
##              Estimate Std. Error t value
## (Intercept) -0.008732   0.015141  -0.577
```