Autonomous Vehicle Simulation (AVS) Laboratory

AVS-Sim Technical Memorandum

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GUIDANCE MODULE TO PERFORM A CONSTANT SPINNING ABOUT AN ORBIT AXIS

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Scope/Contents	
Generate the attitude reference to achieve a spinning motion about a primary orbit frame axis. A chosen reference axis $\hat{r}_j = \hat{r}_{spin}$ is to line up with the orbit axis $\hat{o}_i = \hat{o}_{spin}$, and rotate a desired rate ω_{spin}	

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Contents

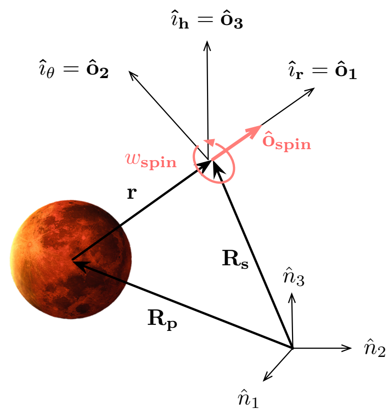

Fig. 1: Illustration of spinning about the nadir orbit axis $\hat{\imath}_r$ of the Hill orbit frame $\mathcal{H}$ at a constant rate ω_{spin}.

1 Introduction

In this note a method is discussed on how to compute the reference frame angular rate $\boldsymbol{\omega}_{R/N}$ and acceleration $\dot{\boldsymbol{\omega}}_{R/N}$ to achieve a particular family of stabilized spin motion. This module receives as input the generated reference for a constaint pointing towards an orbit reference frame. Let us call the input reference $\mathcal{R}_0$. The goal is now to create a new reference $\mathcal{R}$ that spins about any of the orbit axes, $\hat{o}_{\text{spin}}$, at a constant rate ω_{spin}. Note that the presented method is general enough to use any of the Hill $\mathcal{H} : \{\hat{\boldsymbol{\imath}}_r, \hat{\boldsymbol{\imath}}_\theta, \hat{\boldsymbol{\imath}}_h\}$ or Velocity $\mathcal{V} : \{\hat{\boldsymbol{\imath}}_n, \hat{\boldsymbol{\imath}}_v, \hat{\boldsymbol{\imath}}_h\}$ orbit frame orientations as the input reference. Figure 1 illustrates the case in which $\mathcal{R}_0$ is aligned with $\mathcal{H}$ and the spinning is about the nadir axis $\hat{\boldsymbol{\imath}}_r$.

2 Angular Velocity and Acceleration Descriptions

The reference frame $\mathcal{R}$ is defined above, and the attitude reference frame tracking control requires the angular rate $\boldsymbol{\omega}_{R/N}$ and acceleration $\dot{\boldsymbol{\omega}}_{R/N}$. Let the MRP attitude set, angular velocity vector and angular acceleration vector associated with the constant pointing reference be $\boldsymbol{\sigma}_{R_0/N}$, $\boldsymbol{\omega}_{R_0/N}$ and $\dot{\omega}_{R_0/N}$ respectively. The angular velocity of the spinning reference frame is thus given by:

$$\boldsymbol{\omega}_{R/N} = \boldsymbol{\omega}_{\text{spin}} + \boldsymbol{\omega}_{R_0/N} \tag{1a}$$

$$\boldsymbol{\omega}_{R/N} = \omega_{\text{spin}} \hat{o}_{\text{spin}} + \boldsymbol{\omega}_{R_0/N} \tag{1b}$$

Since $\hat{o}_{\text{spin}}$ is aligned with one of the orbit axis defining $\mathcal{R}_0$, taking the inertial derivative of (1):

$$\dot{\boldsymbol{\omega}}_{R/N} = \dot{\boldsymbol{\omega}}_{\text{spin}} + \dot{\boldsymbol{\omega}}_{R_0/N} \tag{2a}$$

$$\dot{\boldsymbol{\omega}}_{R/N} = \frac{^{\mathcal{R}_0}\mathrm{d}}{\mathrm{dt}}(\omega_{\text{spin}}\hat{o}_{\text{spin}}) + \boldsymbol{\omega}_{R_0/N} \times (\omega_{\text{spin}}\hat{o}_{\text{spin}}) + \dot{\boldsymbol{\omega}}_{R_0/N} \tag{2b}$$

3 MRP Attitude Set

Let be ϕ_{spin} be the current spin angle that the reference frame has rotated about its spin axis $\hat{o}_{\text{spin}}$. The final reference frame orientation is eventually given by

$$[RN] = [M_{\hat{o}_{\text{spin}}}(\phi_{\text{spin}})][R_0N] \tag{3}$$

where $[R_0N]$ is the Direction Cosine Matrix associated with the orbit axis pointing attitude set, and $M_{\hat{o}_{\text{spin}}}$ is the principal axis rotation matrix about $\hat{o}_{\text{spin}}$. Assuming a constant spin rate, the spin angle is propagated using the simple Euler integration scheme

$$\phi_{\text{spin, n+1}} = \phi_{\text{spin, n}} + \omega_{\text{spin}}\Delta t \tag{4}$$

With $[RN]$ defined, the MRP attitude set $\sigma_{R/N}$ can readily be computed.

4 Spin Angle Initialization Process

Any alignment between reference frames, say $\mathcal{B}$ and $\mathcal{R}$, can be achieved through a single rotation about a principal rotation axis. If ϕ_{align} is the principal rotation angle, then the alignment can be achieved through a rotation of whether ϕ_{align} or $(2\pi - \phi_{\text{align}})$. In order to determine the initial spin angle and assure that the chosen path is always the shortest rotation, the original $\mathcal{B}$ and final $\mathcal{R}$ frame orientations must be known.

4.1 Algorithm Outlines

The question remains on how to initialize the spin angle ϕ_{spin}. Since the initialization of the spin angle does not allow further generality in the generatiton of reference attitudes, a separate algorithm is used. Being $\mathcal{B}$ the body frame and $\mathcal{R}_0$ the orbit frame orientations, let us set $\hat{b}_{\text{spin}}$ as the principal body axis that is to be aligned with the principal orbit axis $\hat{o}_{\text{spin}}$. The triodes of axes $\hat{b}_i$ and $\hat{o}_i$ are labeled so that the first index determines the spin axis, i.e. $\hat{b}_1 = \hat{b}_{\text{spin}}$ and $\hat{o}_1 = \hat{o}_{\text{spin}}$. The remaining two indices are set to yield a right-handed coordinate frame. Note that $\mathcal{R}_0$ could reference to either Hill orbit frame $\mathcal{H}$ or Velocity $\mathcal{V}$.

Let $\mathcal{R}$ be the reference orientation in which $\hat{o}_1$ orbit axis and $\hat{b}_1$ body axis are lined up. Using the definition where $RN(i)$ is the i_{th} row of the $[RN]$ DCM matrix:

$$RN(\hat{b}_1) = R_0N(\hat{o}_1) \tag{5}$$

$$RN(\hat{b}_2) = R_0N(\hat{o}_2) \tag{6}$$

$$RN(\hat{b}_3) = RN(\hat{b}_1) \times RN(\hat{b}_2) \tag{7}$$

Having $[RN]$ and $[BN]$ defined, the spin axis alignment angle ϕ_{align} is computed as the relative orientation between $\hat{b}_{\text{spin}}$ and $\hat{o}_{\text{spin}}$. Next, the body frame is rotated about a principal rotation angle to align these two axes. Eventually, the initial spin angle ϕ_{spin} is defined as the smallest angle to align the remaining two.