Autonomous Vehicle Simulation (AVS) Laboratory

Basilisk Technical Memorandum

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ALGORITHMS TO MAP DESIRED TORQUE VECTOR ONTO A SET OF THRUSTERS

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Scope/Contents
Include a short summary of what this system engineering report is about. Should be 300 words or less.

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v0.01	Updated the thruster force evaluation to account for center of mass offsets	H. Schaub

Contents

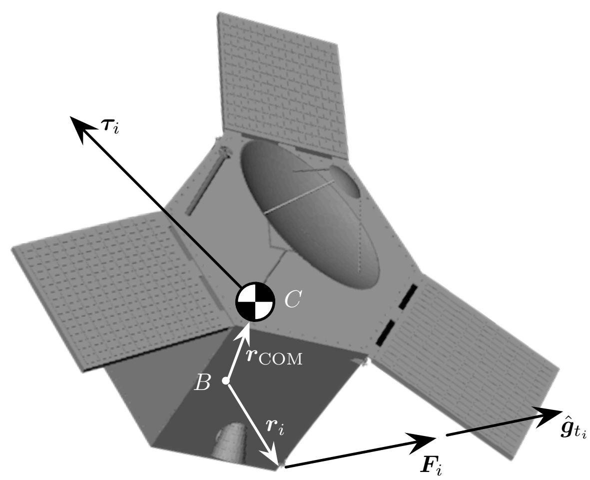

Fig. 1: Illustration of the Spacecraft Thruster Notation

1 Introduction

This technical note describes a general algorithm that maps a desired ADCS external control torque $\boldsymbol{L}_r$ onto force commands for a cluster of thrusters. Let $\hat{\boldsymbol{b}}_j$ be the axis about which the thrusters are to produce the desired torque. The module can accept up to 3 orthogonal control axis $\hat{\boldsymbol{b}}_j$. The j^{th} component of $\boldsymbol{L}_r$ is given by

$$L_{r,j} = \boldsymbol{L}_r \cdot \hat{\boldsymbol{b}}_j \tag{1}$$

The i^{th} thruster location relative to the spacecraft point B is given by $\boldsymbol{r}_i$ as illustrated in Figure 1. The unit direction vector of the thruster force is $\hat{\boldsymbol{g}}_{t_i}$, while the thruster force is given by

$$\boldsymbol{F}_i = F_i \hat{\boldsymbol{g}}_{t_i} \tag{2}$$

The toque vector produced by each thruster about the body fixed point B is thus

$$\boldsymbol{\tau}_i = (\boldsymbol{r}_i - \boldsymbol{r}_{\text{COM}}) \times F_i \hat{\boldsymbol{g}}_{t_i} \tag{3}$$

The total torque onto the spacecraft about the body fixed axis $\hat{\boldsymbol{b}}_j$, due to a cluster of N thrusters, is

$$\tau_j = \sum_{i=1}^{N} \boldsymbol{\tau}_i \cdot \hat{\boldsymbol{b}}_j = \sum_{i=1}^{N} ((\boldsymbol{r}_i - \boldsymbol{r}_{\text{COM}}) \times \hat{\boldsymbol{g}}_{t_i}) \cdot \hat{\boldsymbol{b}}_j F_i = \sum_{i=1}^{N} d_i F_i \tag{4}$$

where

$$d_i = ((\boldsymbol{r}_i - \boldsymbol{r}_{\text{COM}}) \times \hat{\boldsymbol{g}}_{t_i}) \cdot \hat{\boldsymbol{b}}_j \tag{5}$$

In matrix form, the net spacecraft torque about the j^{th} axis is written compactly as

$$\tau_j = \begin{bmatrix} d_1 \cdots d_N \end{bmatrix} \begin{bmatrix} F_1 \\ \vdots \\ F_N \end{bmatrix} = [D]\boldsymbol{F} \tag{6}$$

where $[D]$ is a $1 \times N$ matrix that maps the thruster forces F_i to the spacecraft torque τ.

2 Simple Thruster Force Algorithm for a Thruster Configuration with Full Torque Controllability

The goal of the thruster force algorithm is to determine a set of thruster forces $\boldsymbol{F}$ such that the net force onto the spacecraft is

$$\tau_j = \boldsymbol{L}_r \cdot \hat{\boldsymbol{b}}_j = [D]\boldsymbol{F}_j \tag{7}$$

without bleeding torque onto the un-controlled axes.

The following algorithm is applied individually to control the desired torque about each $\hat{\boldsymbol{b}}_j$ axis. The first step to determine which thruster forces F_i are contributing with a positive force value. Each thruster can only produce a positive force. Using a minimum norm inverse of Eq. (7) yields

$$\boldsymbol{F}_j = [D]^T \left([D][D]^T\right)^{-1} \boldsymbol{L}_r \cdot \hat{\boldsymbol{b}}_j \tag{8}$$

This minimum norm inverse only requires inverting a 1×1 matrix. Using the SVD inverse technique, the value of this 1×1 matrix is the singular value. Thus, if this singular value is below a specified threshold ϵ, the thruster configuration is not contributing to a torque about the $\hat{\boldsymbol{b}}_j$ axis. In this case the inverse of this matrix is set to zero, and not thruster forces contribute to the desired torque about this axis.

Note that this force stack $\boldsymbol{F}$ contains both positive and negative force values. Another step is required to ensure that the thrusters can only produce positive forces. Assume there are M positive force values in $\boldsymbol{F}_j$. The locations of these values is provided in the N-dimensional array $\boldsymbol{t}_{\text{used}}$ which contains either 0 or 1 values. For example, consider $N = 8$ and only thrusters 2 and 6 produce positive forces. In this case we find

$$\boldsymbol{t}_{\text{used}} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \tag{9}$$

This reduces the thruster force search to a subset of M thrusters. Let $\bar{\boldsymbol{F}}_j$ be a $M \times 1$ matrix of to be determined thruster forces. The corresponding $3 \times M$ mapping matrix $[\bar{D}]$ that projects $\bar{\boldsymbol{F}}_j$ onto a net body torque about point B is defined as:

$$[\bar{D}] = \begin{bmatrix} \bar{\boldsymbol{d}}_1 & \cdots & \bar{\boldsymbol{d}}_M \end{bmatrix} \tag{10}$$

with

$$\bar{\boldsymbol{d}}_i = (\boldsymbol{r}_i - \boldsymbol{r}_{\text{COM}}) \times \hat{\boldsymbol{g}}_i \tag{11}$$

The net torque due to $\bar{\boldsymbol{F}}_j$ is

$$\bar{\boldsymbol{\tau}}_j = [\bar{D}]\bar{\boldsymbol{F}}_j \tag{12}$$

To enforce that $\bar{\boldsymbol{F}}_j$ only produces the desired torque about the $\hat{\boldsymbol{b}}_j$ axis, and not any torque about other axes, the following condition is established:

$$(\hat{\boldsymbol{b}}_j \cdot \boldsymbol{L}_r)\hat{\boldsymbol{b}}_j = [\bar{D}]\bar{\boldsymbol{F}}_j \tag{13}$$

If the mapping matrix $[\bar{D}]$ has rank 3, then a minimum norm inverse can be used to determine the smallest set of thruster forces that satisfy Eq. (13).

$$\bar{\boldsymbol{F}}_j = [\bar{D}]^T([\bar{D}][\bar{D}]^T)^{-1}\hat{\boldsymbol{b}}_j(\hat{\boldsymbol{b}}_j \cdot \boldsymbol{L}_r) \tag{14}$$

The rank condition can easily be checked by computing if the determinant of $[\bar{D}][\bar{D}]^T$ is greater than zero. If yes, a minimum norm inverse can be taken without numerical difficulties.

If the determinant of $[\bar{D}][\bar{D}]^T$ is near zero, then $\bar{\boldsymbol{F}}_j$ cannot generate a general 3D torque vector. As the spacecraft is setup with pairs of thrusters to produce the control torques, in this case the rank of $[\bar{D}]$ is 2, and not all body axis are influenced by $\bar{\boldsymbol{F}}_j$. In this case the thruster forces are determined through a least-squares inverse that selects $\bar{\boldsymbol{F}}_j$ such that the controllable axes satisfy the condition in Eq. (13).

$$\bar{\boldsymbol{F}}_j = ([\bar{D}]^T[\bar{D}])^{-1}[\bar{D}]^T\hat{\boldsymbol{b}}_j(\hat{\boldsymbol{b}}_j \cdot \boldsymbol{L}_r) \tag{15}$$

The final step is to sum the individual $\bar{\boldsymbol{F}}_j$ thruster solutions to the yield the net set of thruster forces required to produce $\boldsymbol{L}_r$. This is done using the $\boldsymbol{t}_{\text{used}}$ matrix to determine which thrusters have non-zero contributions.

If the thruster cluster configuration is such that pairs of thrusters produce full controllabilty, then the minimum norm solution to produce the desired $\boldsymbol{L}_r$ will also result in a thruster solution that produces a net 0 force onto the spacecraft. Using the super-particle theorem,[1] the total thruster force is given by

$$\boldsymbol{F}_{T,j} = [G_t]\boldsymbol{F}_j = [G_t][D]^T([D][D])^{-1}\boldsymbol{L}_r \cdot \hat{\boldsymbol{b}}_j = \boldsymbol{0} \tag{16}$$

With a pure-couple thruster configuration the expression satisfies $[G_t][D]^T = \boldsymbol{0}$.

3 Module Parameters

3.1 ϵ Parameter

The minimum norm inverse in Eq. (8) requires a non-zero value of $[D][D]^T$. For this setup, this matrix is a scalar value

$$D_2 = [D][D]^T \tag{17}$$

The d_i matrix components are given in Eq. (5). Using the robust SVD inverse technique, $D_2 > \epsilon$, then the $1/D_2$ math is evaluated as normal. However, if $D_2 < \epsilon$, then the inverse $1/D_2$ is set to zero. In the latter case there is no control authority about the current axis of interest. To set this epsilon parameter, not the definition of the $[D]$ matrix components $d_i = (\boldsymbol{r}_i \times \hat{\boldsymbol{g}}_{t_i}) \cdot \hat{\boldsymbol{b}}_j$. Note that $\boldsymbol{r}_i \times \hat{\boldsymbol{g}}_{t_i}$ is a scaled axis along which the i^{th} thruster can produce a torque. The value d_i will be near zero if the dot product of this axis with the current control axis $\hat{\boldsymbol{b}}_j$ is small.

To determine an appropriate ϵ value, let α be the minimum desired angle to avoid the control axis $\hat{\boldsymbol{b}}_j$ and the scaled thruster torque axis $\boldsymbol{r}_i \times \hat{\boldsymbol{g}}_{t_i}$ being orthogonal. If $\bar{r}$ is a mean distance of the thrusters to the spacecraft center of mass, then the d_i values must satisfy

$$\frac{d_i}{\bar{r}} > \cos(90^\circ - \alpha) = \sin\alpha \tag{18}$$

Thus, to estimate a good value of ϵ, the following formula can be used

$$\epsilon \approx d_i^2 = \sin^2\alpha\ \bar{r}^2 \tag{19}$$

For example, if $\bar{r} = 1.3$ meters, and we want α to be at least 1°, then we would set $\epsilon = 0.000515$.

3.2 $[B]$ matrix

The module requires control control axis matrix $[B]$ to be defined. Up to 3 orthogonal control axes can be selected. Not that in python the matrix is given in a 1D form by defining `controlAxes_B`. Thus, the $\hat{\boldsymbol{b}}_j$ axes are concatenated to produce the input matrix $[B]$.

REFERENCES

[1] Hanspeter Schaub and John L. Junkins. *Analytical Mechanics of Space Systems*. AIAA Education Series, Reston, VA, 3rd edition, 2014.