Autonomous Vehicle Simulation (AVS) Laboratory, University of Colorado

Basilisk Technical Memorandum

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REACTION WHEEL DYNAMICS MODEL

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Scope/Contents
The reaction wheel class is an instantiation of the state effector abstract class. The integrated test is validating the interaction between the reaction wheel module and the rigid body hub that it is attached to. More specifically, the reaction wheel module models three different cases: balanced wheels, simpled jitter, and fully coupled jitter. The details of each mode is described in detail in this document. This integrated test confirms that all three models are agreeing with physics and the tests use both energy and momentum checks and back of the envelope (BOE) calculations.

Rev	Change Description	By	Date
1.0	Initial Draft	C. Allard	20170816

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1 Model Description

1.1 Introduction

This module is modeling a reaction wheel connected to a rigid body hub. The reaction wheel model has three modes that can be ran: balanced wheels, simple jitter, and fully-coupled imbalanced wheels. The balanced wheels option is modeling the reaction wheels as having their principle inertia axes aligned with spin axis, $\hat{\boldsymbol{g}}_s$, and the center of mass of the wheel is coincident with $\hat{\boldsymbol{g}}_s$. This results in the reaction wheel not changing the mass properties of the spacecraft and results in simpler equations. The simple jitter option is approximating the jitter due to mass imbalances by applying an external force and torque to the spacecraft that is proportional to the wheel speeds squared. This is an approximation because in reality this is an internal force and torque. Finally, the fully-coupled mode is modeling reaction wheel imbalance dynamics by modeling the static and dynamic imbalances as internal forces and torques which is physically realistic and allows for energy and momentum conservation.

Figure 1 shows the frame and variable definitions used for this problem. The formulation involves a rigid hub with its center of mass location labeled as point B_c, and N_{rw} RWs with their center of mass locations labeled as W_{c_i}. The frames being used for this formulation are the body-fixed frame, $\mathcal{B} : \{\hat{\boldsymbol{b}}_1, \hat{\boldsymbol{b}}_2, \hat{\boldsymbol{b}}_3\}$, the motor frame of the i^{th} RW, $\mathcal{M}_i : \{\hat{\boldsymbol{m}}_{s_i}, \hat{\boldsymbol{m}}_{2_i}, \hat{\boldsymbol{m}}_{3_i}\}$ which is also body-fixed, and the wheel-fixed frame of the i^{th} RW, $\mathcal{W}_i : \{\hat{\boldsymbol{g}}_{s_i}, \hat{\boldsymbol{w}}_{2_i}, \hat{\boldsymbol{w}}_{3_i}\}$. The dynamics are modeled with respect to the $\mathcal{B}$ frame which can be generally oriented. The $\mathcal{W}_i$ frame is oriented such that the $\hat{\boldsymbol{g}}_{s_i}$ axis is aligned with the RW spin axis which is the same as the motor torque axis $\hat{\boldsymbol{m}}_{s_i}$, the $\hat{\boldsymbol{w}}_{2_i}$ axis is perpendicular to $\hat{\boldsymbol{g}}_{s_i}$ and points in the direction towards the RW center of mass W_{c_i}. The $\hat{\boldsymbol{w}}_{3_i}$ completes the right hand rule. The $\mathcal{M}_i$ frame is defined as being equal to the $\mathcal{W}_i$ frame at the beginning of the simulation and therefore the $\mathcal{W}_i$ and $\mathcal{M}_i$ frames are offset by an angle, θ_i, about the $\hat{\boldsymbol{m}}_{s_i} = \hat{\boldsymbol{g}}_{s_i}$ axes.

A few more key variables in Figure 1 need to be defined. The rigid spacecraft structure without the RWs is called the hub. Point B is the origin of the $\mathcal{B}$ frame and is a general body-fixed point that does not have to be identical to the total spacecraft center of mass, nor the rigid hub center of mass B_c. Point W_i is the origin of the $\mathcal{W}_i$ frame and can also have any location relative to point B. Point C is the center of mass of the total spacecraft system including the rigid hub and the RWs. Due to the RW imbalance, the vector $\boldsymbol{c}$, which points from point B to point C, will vary as seen by a body-fixed observer. The scalar variable d_i is the center of mass offset of the RW, or the distance from the spin axis, $\hat{\boldsymbol{g}}_{\mathsf{s}_i}$ to W_{c_i}. Finally, the inertial frame orientation is defined through $\mathcal{N} : \{\hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}_2, \hat{\boldsymbol{n}}_3\}$, while the origin of the inertial frame is labeled as N.

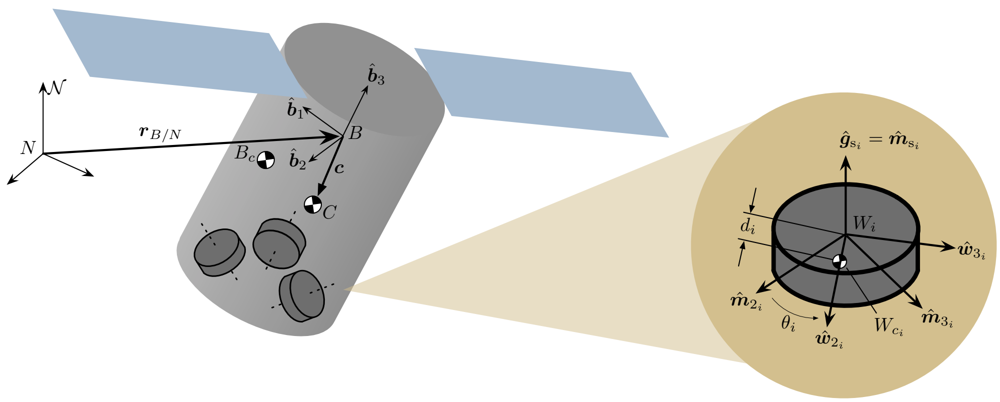

Fig. 1: Reaction wheel and spacecraft frame and variable definitions

1.2 Equations of Motion

The main introduction that is needed for this model is the equations of motion. Depending on the mode, the equations of motion are different. Each mode's equations of motion are discussed in the following sub sections.

1.2.1 Balanced Wheels

For balanced wheels, translational equation of motion is not coupled with $\dot{\Omega}$ as seen in the equation below.

$$m_{\text{sc}}[I_{3\times3}]\ddot{\boldsymbol{r}}_{B/N} - m_{\text{sc}}[\tilde{\boldsymbol{c}}]\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} = \boldsymbol{F}_{\text{ext}} - 2m_{\text{sc}}[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\boldsymbol{c}' - m_{\text{sc}}[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\boldsymbol{c} \tag{1}$$

The rotational equation of motion includes $\dot{\Omega}$ terms, and is thus coupled with wheel motion as seen below.

$$m_{\text{sc}}[\tilde{\boldsymbol{c}}]\ddot{\boldsymbol{r}}_{B/N} + [I_{\text{sc},B}]\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} + \sum_{i=1}^{N} J_{\mathsf{s}_i}\hat{\boldsymbol{g}}_{\mathsf{s}_i}\dot{\Omega}_i = -[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][I_{\text{sc},B}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} - \sum_{i=1}^{N}(\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times J_{\mathsf{s}_i}\Omega_i\hat{\boldsymbol{g}}_{\mathsf{s}_i}) + \boldsymbol{L}_B \tag{2}$$

The motor torque equation can be seen below.

$$\dot{\Omega}_i = \frac{u_{\mathsf{s}_i}}{J_{\mathsf{s}_i}} - \hat{\boldsymbol{g}}_{\mathsf{s}_i}^T\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} \tag{3}$$

Plugging Eq. (13) into Eq. (12)

$$m_{\text{sc}}[\tilde{\boldsymbol{c}}]\ddot{\boldsymbol{r}}_{B/N} + ([I_{\text{sc},B}] - \sum_{i=1}^{N} J_{\text{s}_i}\hat{\boldsymbol{g}}_{\text{s}_i}\hat{\boldsymbol{g}}_{\text{s}_i}^T)\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} = -[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][I_{\text{sc},B}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} - \sum_{i=1}^{N}(\hat{\boldsymbol{g}}_{\text{s}_i}u_{\text{s}_i} + \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times J_{\text{s}_i}\Omega_i\hat{\boldsymbol{g}}_{\text{s}_i}) - [I'_{\text{sc},B}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} + \boldsymbol{L}_B \quad (4)$$

The following can be defined:

$$[A_{\text{contr}}] = [0_{3\times3}] \quad (5)$$

$$[B_{\text{contr}}] = [0_{3\times3}] \quad (6)$$

$$[C_{\text{contr}}] = [0_{3\times3}] \quad (7)$$

$$[D_{\text{contr}}] = -\sum_{i=1}^{N} J_{\text{s}_i}\hat{\boldsymbol{g}}_{\text{s}_i}\hat{\boldsymbol{g}}_{\text{s}_i}^T \quad (8)$$

$$\boldsymbol{v}_{\text{trans,contr}} = \boldsymbol{0} \quad (9)$$

$$\boldsymbol{v}_{\text{rot,contr}} = -\sum_{i=1}^{N}(\hat{\boldsymbol{g}}_{\text{s}_i}u_{\text{s}_i} + \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times J_{\text{s}_i}\Omega_i\hat{\boldsymbol{g}}_{\text{s}_i}) \quad (10)$$

These are the contributions needed for the back-substitution method used in spacecraft plus.

1.2.2 Simple Jitter

For simple jitter, like balanced wheels, the translational equation of motion is not coupled with $\dot{\Omega}$ as seen in the equation below, however the jitter does apply a force on the spacecraft.

$$m_{\text{sc}}[I_{3\times3}]\ddot{\boldsymbol{r}}_{B/N} - m_{\text{sc}}[\tilde{\boldsymbol{c}}]\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} = \boldsymbol{F}_{\text{ext}} - 2m_{\text{sc}}[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\boldsymbol{c}' - m_{\text{sc}}[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\boldsymbol{c} + U_{s_i}\Omega_i^2\hat{\boldsymbol{u}}_i \quad (11)$$

The rotational equation of motion is very similar to the balanced wheels EOM but has two additional torques due to the reaction wheel imbalance.

$$m_{\text{sc}}[\tilde{\boldsymbol{c}}]\ddot{\boldsymbol{r}}_{B/N} + [I_{\text{sc},B}]\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} + \sum_{i=1}^{N} J_{\text{s}_i}\hat{\boldsymbol{g}}_{\text{s}_i}\dot{\Omega}_i = -[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][I_{\text{sc},B}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} - \sum_{i=1}^{N}(\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times J_{\text{s}_i}\Omega_i\hat{\boldsymbol{g}}_{\text{s}_i}) + U_{s_i}\Omega_i^2[\tilde{\boldsymbol{r}}_{W_i/B}]\hat{\boldsymbol{u}}_i + U_{d_i}\Omega_i^2\hat{\boldsymbol{v}}_i + \boldsymbol{L}_B \quad (12)$$

The motor torque equation can be seen below:

$$\dot{\Omega}_i = \frac{u_{\text{s}_i}}{J_{\text{s}_i}} - \hat{\boldsymbol{g}}_{\text{s}_i}^T\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} \quad (13)$$

Plugging Eq. (13) into Eq. (12)

$$m_{\text{sc}}[\tilde{\boldsymbol{c}}]\ddot{\boldsymbol{r}}_{B/N} + ([I_{\text{sc},B}] - \sum_{i=1}^{N} J_{\text{s}_i}\hat{\boldsymbol{g}}_{\text{s}_i}\hat{\boldsymbol{g}}_{\text{s}_i}^T)\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} = -[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][I_{\text{sc},B}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} - \sum_{i=1}^{N}(\hat{\boldsymbol{g}}_{\text{s}_i}u_{\text{s}_i} + \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times J_{\text{s}_i}\Omega_i\hat{\boldsymbol{g}}_{\text{s}_i}) - [I'_{\text{sc},B}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} + U_{s_i}\Omega_i^2[\tilde{\boldsymbol{r}}_{W_i/B}]\hat{\boldsymbol{u}}_i + U_{d_i}\Omega_i^2\hat{\boldsymbol{v}} + \boldsymbol{L}_B \quad (14)$$

The following can be defined:

$$[A_{\textsf{contr}}] = [0_{3\times 3}] \tag{15}$$

$$[B_{\textsf{contr}}] = [0_{3\times 3}] \tag{16}$$

$$[C_{\textsf{contr}}] = [0_{3\times 3}] \tag{17}$$

$$[D_{\textsf{contr}}] = -\sum_{i=1}^{N} J_{\textsf{s}_i}\hat{\boldsymbol{g}}_{\textsf{s}_i}\hat{\boldsymbol{g}}_{\textsf{s}_i}^T \tag{18}$$

$$\boldsymbol{v}_{\textsf{trans,contr}} = U_{s_i}\Omega_i^2\hat{\boldsymbol{u}}_i \tag{19}$$

$$\boldsymbol{v}_{\textsf{rot,contr}} = U_{s_i}\Omega_i^2[\tilde{\boldsymbol{r}}_{W_i/B}]\hat{\boldsymbol{u}}_i + U_{d_i}\Omega_i^2\hat{\boldsymbol{v}} \tag{20}$$

These are the contributions needed for the back-substitution method used in spacecraft plus.

1.2.3 Fully-Coupled Jitter

The translational equation of motion is

$$\ddot{\boldsymbol{r}}_{B/N} - [\tilde{\boldsymbol{c}}]\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} + \frac{1}{m_{\textsf{sc}}}\sum_{i=1}^{N} m_{\textsf{rw}_i} d_i \hat{\boldsymbol{w}}_{3_i}\dot{\Omega}_i = \ddot{\boldsymbol{r}}_{C/N} - 2[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\boldsymbol{c}' - [\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\boldsymbol{c} + \frac{1}{m_{\textsf{sc}}}\sum_{i=1}^{N} m_{\textsf{rw}_i} d_i \Omega_i^2 \hat{\boldsymbol{w}}_{2_i} \tag{21}$$

The rotational equation of motion is

$$\begin{aligned} m_{\textsf{sc}}[\tilde{\boldsymbol{c}}]\ddot{\boldsymbol{r}}_{B/N} + [I_{\textsf{sc},B}]\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} + \sum_{i=1}^{N}\Big([I_{\textsf{rw}_i,W_{c_i}}]\hat{\boldsymbol{g}}_{s_i} + m_{\textsf{rw}_i} d_i [\tilde{\boldsymbol{r}}_{W_{c_i}/B}]\hat{\boldsymbol{w}}_{3_i}\Big)\dot{\Omega}_i \\ = \sum_{i=1}^{N}\Big[m_{\textsf{rw}_i}[\tilde{\boldsymbol{r}}_{W_{c_i}/B}] d_i \Omega_i^2 \hat{\boldsymbol{w}}_{2_i} - [I_{\textsf{rw}_i,W_{c_i}}]'\Omega_i\hat{\boldsymbol{g}}_{s_i} - [\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\Big([I_{\textsf{rw}_i,W_{c_i}}]\Omega_i\hat{\boldsymbol{g}}_{s_i} + m_{\textsf{rw}_i}[\tilde{\boldsymbol{r}}_{W_{c_i}/B}]\boldsymbol{r}'_{W_{c_i}/B}\Big)\Big] \\ - [\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][I_{\textsf{sc},B}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} - [I_{\textsf{sc},B}]'\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} + \boldsymbol{L}_B \end{aligned} \tag{22}$$

The motor torque equation is (note that $J_{12_i} = J_{23_i} = 0$)

$$\begin{aligned} \big[m_{\textsf{rw}_i} d_i \hat{\boldsymbol{w}}_{3_i}^T\big]\ddot{\boldsymbol{r}}_{B/N} + \big[(J_{11_i} + m_{\textsf{rw}_i} d_i^2)\hat{\boldsymbol{g}}_{\textsf{s}_i}^T + J_{13_i}\hat{\boldsymbol{w}}_{3_i}^T - m_{\textsf{rw}_i} d_i \hat{\boldsymbol{w}}_{3_i}^T[\tilde{\boldsymbol{r}}_{W_i/B}]\big]\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} + \big[J_{11_i} + m_{\textsf{rw}_i} d_i^2\big]\dot{\Omega}_i \\ = -J_{13_i}\omega_{w_{2_i}}\omega_{s_i} + \omega_{w_{2_i}}\omega_{w_{3_i}}(J_{22_i} - J_{33_i} - m_{\textsf{rw}_i} d_i^2) - m_{\textsf{rw}_i} d_i \hat{\boldsymbol{w}}_{3_i}^T[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\boldsymbol{r}_{W_i/B} + u_{s_i} \end{aligned} \tag{23}$$

The first step in the back-substitution method is to solve the motor torque equation for $\dot{\Omega}_i$ in terms of $\ddot{\boldsymbol{r}}_{B/N}$ and $\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}$

$$\begin{aligned} \dot{\Omega}_i = \frac{-m_{\textsf{rw}_i} d_i \hat{\boldsymbol{w}}_{3_i}^T}{J_{11_i} + m_{\textsf{rw}_i} d_i^2}\ddot{\boldsymbol{r}}_{B/N} + \frac{-\big[(J_{11_i} + m_{\textsf{rw}_i} d_i^2)\hat{\boldsymbol{g}}_{\textsf{s}_i}^T + J_{13_i}\hat{\boldsymbol{w}}_{3_i}^T - m_{\textsf{rw}_i} d_i \hat{\boldsymbol{w}}_{3_i}^T[\tilde{\boldsymbol{r}}_{W_i/B}]\big]}{J_{11_i} + m_{\textsf{rw}_i} d_i^2}\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} \\ + \frac{1}{J_{11_i} + m_{\textsf{rw}_i} d_i^2}\Big[\omega_{w_{2_i}}\omega_{w_{3_i}}(J_{22_i} - J_{33_i} - m_{\textsf{rw}_i} d_i^2) - J_{13_i}\omega_{w_{2_i}}\omega_{s_i} - m_{\textsf{rw}_i} d_i \hat{\boldsymbol{w}}_{3_i}^T[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\boldsymbol{r}_{W_i/B} + u_{s_i}\Big] \end{aligned} \tag{24}$$

$$\boldsymbol{a}_{\Omega_i} = -\frac{m_{\text{rw}_i} d_i \hat{\boldsymbol{w}}_{3_i}}{J_{11_i} + m_{\text{rw}_i} d_i^2} \tag{25}$$

$$\boldsymbol{b}_{\Omega_i} = -\frac{(J_{11_i} + m_{\text{rw}_i} d_i^2)\hat{\boldsymbol{g}}_{\text{s}_i} + J_{13_i}\hat{\boldsymbol{w}}_{3_i} + m_{\text{rw}_i} d_i [\tilde{\boldsymbol{r}}_{W_i/B}]\hat{\boldsymbol{w}}_{3_i}}{J_{11_i} + m_{\text{rw}_i} d_i^2} \tag{26}$$

$$c_{\Omega_i} = \frac{1}{J_{11_i} + m_{\text{rw}_i} d_i^2}\Big[\omega_{w_{2_i}}\omega_{w_{3_i}}(J_{22_i} - J_{33_i} - m_{\text{rw}_i} d_i^2) - J_{13_i}\omega_{w_{2_i}}\omega_{s_i} - m_{\text{rw}_i} d_i \hat{\boldsymbol{w}}_{3_i}^T [\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\boldsymbol{r}_{W_i/B} + u_{s_i}\Big] \tag{27}$$

$$\dot{\Omega}_i = \boldsymbol{a}_{\Omega_i}^T \ddot{\boldsymbol{r}}_{B/N} + \boldsymbol{b}_{\Omega_i}^T \dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} + c_{\Omega_i} \tag{28}$$

Plugging the equation above into Eq. (21) and multiplying both sides by m_{sc}, (plug $\dot{\Omega}_i$ into translation)

$$\begin{aligned}\Big[m_{\text{sc}}[I_{3\times3}] + \sum_{i=1}^{N} m_{\text{rw}_i} d_i \hat{\boldsymbol{w}}_{3_i}\boldsymbol{a}_{\Omega_i}^T\Big]\ddot{\boldsymbol{r}}_{B/N} + \Big[-m_{\text{sc}}[\tilde{\boldsymbol{c}}] + \sum_{i=1}^{N} m_{\text{rw}_i} d_i \hat{\boldsymbol{w}}_{3_i}\boldsymbol{b}_{\Omega_i}^T\Big]\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} \\ = m_{\text{sc}}\ddot{\boldsymbol{r}}_{C/N} - 2m_{\text{sc}}[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\boldsymbol{c}' - m_{\text{sc}}[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\boldsymbol{c} + \sum_{i=1}^{N} m_{\text{rw}_i} d_i \left(\Omega_i^2 \hat{\boldsymbol{w}}_{2_i} - c_{\Omega_i}\hat{\boldsymbol{w}}_{3_i}\right)\end{aligned} \tag{29}$$

Moving on to rotation, (plug $\dot{\Omega}_i$ into rotation)

$$\begin{aligned}&\Big[m_{\text{sc}}[\tilde{\boldsymbol{c}}] + \sum_{i=1}^{N}\Big([I_{\text{rw}_i,W_{c_i}}]\hat{\boldsymbol{g}}_{s_i} + m_{\text{rw}_i} d_i [\tilde{\boldsymbol{r}}_{W_{c_i}/B}]\hat{\boldsymbol{w}}_{3_i}\Big)\boldsymbol{a}_{\Omega_i}^T\Big]\ddot{\boldsymbol{r}}_{B/N} \\ &\quad + \Big[[I_{\text{sc},B}] + \sum_{i=1}^{N}\Big([I_{\text{rw}_i,W_{c_i}}]\hat{\boldsymbol{g}}_{s_i} + m_{\text{rw}_i} d_i [\tilde{\boldsymbol{r}}_{W_{c_i}/B}]\hat{\boldsymbol{w}}_{3_i}\Big)\boldsymbol{b}_{\Omega_i}^T\Big]\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} \\ &= \sum_{i=1}^{N}\Big[m_{\text{rw}_i}[\tilde{\boldsymbol{r}}_{W_{c_i}/B}] d_i \Omega_i^2 \hat{\boldsymbol{w}}_{2_i} - [I_{\text{rw}_i,W_{c_i}}]'\Omega_i \hat{\boldsymbol{g}}_{s_i} - [\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\Big([I_{\text{rw}_i,W_{c_i}}]\Omega_i\hat{\boldsymbol{g}}_{s_i} + m_{\text{rw}_i}[\tilde{\boldsymbol{r}}_{W_{c_i}/B}]\boldsymbol{r}'_{W_{c_i}/B}\Big) \\ &\quad - \Big([I_{\text{rw}_i,W_{c_i}}]\hat{\boldsymbol{g}}_{s_i} + m_{\text{rw}_i} d_i [\tilde{\boldsymbol{r}}_{W_{c_i}/B}]\hat{\boldsymbol{w}}_{3_i}\Big)c_{\Omega_i}\Big] \\ &\quad - [\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][I_{\text{sc},B}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} - [I_{\text{sc},B}]'\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} + \boldsymbol{L}_B\end{aligned} \tag{30}$$

Now we have two equations containing $\ddot{\boldsymbol{r}}_{B/N}$ and $\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}$. Now the matrix contributions can be defined:

$$[A_{\text{contr}}] = \sum_{i=1}^{N} m_{\text{rw}_i} d_i \hat{\boldsymbol{w}}_{3_i}\boldsymbol{a}_{\Omega_i}^T \tag{31}$$

$$[B_{\text{contr}}] = \sum_{i=1}^{N} m_{\text{rw}_i} d_i \hat{\boldsymbol{w}}_{3_i}\boldsymbol{b}_{\Omega_i}^T \tag{32}$$

$$[C_{\text{contr}}] = \sum_{i=1}^{N}\Big([I_{\text{rw}_i,W_{c_i}}]\hat{\boldsymbol{g}}_{s_i} + m_{\text{rw}_i} d_i [\tilde{\boldsymbol{r}}_{W_{c_i}/B}]\hat{\boldsymbol{w}}_{3_i}\Big)\boldsymbol{a}_{\Omega_i}^T \tag{33}$$

$$[D_{\text{contr}}] = \sum_{i=1}^{N}\Big([I_{\text{rw}_i,W_{c_i}}]\hat{\boldsymbol{g}}_{s_i} + m_{\text{rw}_i} d_i [\tilde{\boldsymbol{r}}_{W_{c_i}/B}]\hat{\boldsymbol{w}}_{3_i}\Big)\boldsymbol{b}_{\Omega_i}^T \tag{34}$$

$$\boldsymbol{v}_{\text{trans,contr}} = \frac{1}{m_{\text{sc}}}\sum_{i=1}^{N} m_{\text{rw}_i} d_i \left(\Omega_i^2 \hat{\boldsymbol{w}}_{2_i} - c_{\Omega_i}\hat{\boldsymbol{w}}_{3_i}\right) \tag{35}$$

$$\boldsymbol{v}_{\text{rot,contr}} = \sum_{i=1}^{N}\Big[m_{\text{rw}_i}[\tilde{\boldsymbol{r}}_{W_{c_i}/B}]d_i\Omega_i^2\hat{\boldsymbol{w}}_{2_i} - [I_{\text{rw}_i,W_{c_i}}]'\Omega_i\hat{\boldsymbol{g}}_{s_i} - [\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\Big([I_{\text{rw}_i,W_{c_i}}]\Omega_i\hat{\boldsymbol{g}}_{s_i} + m_{\text{rw}_i}[\tilde{\boldsymbol{r}}_{W_{c_i}/B}]\boldsymbol{r}'_{W_{c_i}/B}\Big) \\ - \Big([I_{\text{rw}_i,W_{c_i}}]\hat{\boldsymbol{g}}_{s_i} + m_{\text{rw}_i}d_i[\tilde{\boldsymbol{r}}_{W_{c_i}/B}]\hat{\boldsymbol{w}}_{3_i}\Big)c_{\Omega_i}\Big] \tag{36}$$

This concludes the equations that are necessary to define the three different modes of the reaction wheel. Reference [1] explains in further detail the EOMs for the simple-jitter and fully-coupled modes. Reference [2] gives more details on the derivation for balanced reaction wheels.

2 Model Functions

This model is used to approximate the behavior of a reaction wheel. Below is a list of functions that this model performs:

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3 Model Assumptions and Limitations

Below is a summary of the assumptions/limitations:

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4 Test Description and Success Criteria

The tests are located in `SimCode/dynamics/reactionWheels/_UnitTest/test_reactionWheelStateEffector_integrated.py` **and** `SimCode/dynamics/reactionWheels/_UnitTest/ test_reactionWheelStateEffector_ConfigureRWRequests.py`. Depending on the test, there are different success criteria. These are outlined in the following subsections:

4.1 Balanced Wheels Scenario - Integrated Test

In this test the simulation is placed into orbit around Earth with point gravity, has 3 reaction wheels attached to the spacecraft, and the wheels are in “Balanced” mode. Each wheel is given a commanded torque for half the simulation and the rest of the simulation the torques are set to zero. The following parameters are being tested:

- Conservation of orbital angular momentum
- Conservation of orbital energy
- Conservation of rotational angular momentum
- Conservation of rotational energy (second half of the simulation)
- Achieving the expected final attitude
- Achieving the expected final position

4.2 Simple Jitter Scenario - Integrated Test

In this test the simulation is placed into orbit around Earth with point gravity, has 3 reaction wheels attached to the spacecraft, and the wheels are in "Simple Jitter" mode. Each wheel is given a commanded torque for half the simulation and the rest of the simulation the torques are set to zero. The following parameters are being tested:

- Achieving the expected final attitude
- Achieving the expected final position

4.3 Fully Coupled Jitter Scenario - Integrated Test

In this test the simulation is placed into orbit around Earth with point gravity, has 3 reaction wheels attached to the spacecraft, and the wheels are in "Fully Coupled Jitter" mode. Each wheel is given a commanded torque for half the simulation and the rest of the simulation the torques are set to zero. The following parameters are being tested:

- Conservation of orbital angular momentum
- Conservation of orbital energy
- Conservation of rotational angular momentum
- Conservation of rotational energy (second half of the simulation)
- Achieving the expected final attitude
- Achieving the expected final position

4.4 BOE Calculation Scenario - Integrated Test

For this

4.5 Friction Scenario - Integrated Test

In the test the goal is to validate that the friction model is matching the desired

4.6 Saturation - Unit Test

4.7 Minimum Torque - Unit Test

5 Test Parameters

Since this is an integrated test, the inputs to the test are the physical parameters of the spacecraft along with the initial conditions of the states. These parameters are outlined in Tables 2- 5. Additionally, the error tolerances can be seen in Table 6.

Table 2: Spacecraft Hub Parameters

Name	Description	Value	Units
mHub	mass	750.0	kg
IHubPntBc_B	Inertia in $\mathcal{B}$ frame	$\begin{bmatrix} 900.0 & 0.0 & 0.0 \\ 0.0 & 600.0 & 0.0 \\ 0.0 & 0.0 & 600.0 \end{bmatrix}$	kg-m^2
r_BcB_B	CoM Location in $\mathcal{B}$ frame	$[0.0 \quad 0.0 \quad 1.0]^T$	m

Table 3: Hinged Rigid Body 1 Parameters

Name	Description	Value	Units
mass	mass	100.0	kg
IPntS_S	Inertia in $\mathcal{S}$ frame	$\begin{bmatrix} 100.0 & 0.0 & 0.0 \\ 0.0 & 50.0 & 0.0 \\ 0.0 & 0.0 & 50.0 \end{bmatrix}$	kg-m^2
d	CoM location	1.5	m
k	Spring Constant	100.0	N-m/rad
c	Damping Term	0.0 (6.0 - damping scenario)	N-m-s/rad
r_HB_B	Hinge Location in $\mathcal{B}$ frame	$[0.5 \quad 0.0 \quad 1.0]^T$	m
dcm_HB	$\mathcal{B}$ to $\mathcal{H}$ DCM	$\begin{bmatrix} -1.0 & 0.0 & 0.0 \\ 0.0 & -1.0 & 0.0 \\ 0.0 & 0.0 & 1.0 \end{bmatrix}$	-

Table 4: Hinged Rigid Body 2 Parameters

Name	Description	Value	Units
mass	mass	100.0	kg
IPntS_S	Inertia in $\mathcal{S}$ frame	$\begin{bmatrix} 100.0 & 0.0 & 0.0 \\ 0.0 & 50.0 & 0.0 \\ 0.0 & 0.0 & 50.0 \end{bmatrix}$	kg-m^2
d	CoM location	1.5	m
k	Spring Constant	100.0	N-m/rad
c	Damping Term	0.0 (7.0 - damping scenario)	N-m-s/rad
r_HB_B	Hinge Location in $\mathcal{B}$ frame	$[-0.5 \quad 0.0 \quad 1.0]^T$	m
dcm_HB	$\mathcal{B}$ to $\mathcal{H}$ DCM	$\begin{bmatrix} 1.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 1.0 \end{bmatrix}$	-

Table 5: Initial Conditions for Energy Momentum Conservation Scenarios

Name	Description	Value	Units
(Panel 1) thetaInit	(Panel 1) Initial θ	5.0	deg
(Panel 1) thetaDotInit	(Panel 1) Initial $\dot{\theta}$	0.0	deg
(Panel 2) thetaInit	(Panel 2) Initial θ	0.0	deg
(Panel 2) thetaDotInit	(Panel 2) Initial $\dot{\theta}$	0.0	deg
r_CN_NInit	Initial Position of S/C (gravity scenarios)	$[-4020339 \quad 7490567 \quad 5248299]^T$	m
v_CN_NInit	Initial Velocity of S/C (gravity scenarios)	$[-5199.78 \quad -3436.68 \quad 1041.58]^T$	m/s
r_CN_NInit	Initial Position of S/C (no gravity)	$[0.1 \quad -0.4 \quad 0.3]^T$	m
v_CN_NInit	Initial Velocity of S/C (no gravity)	$[-0.2 \quad 0.5 \quad 0.1]^T$	m/s
sigma_BNInit	Initial MRP of $\mathcal{B}$ frame	$[0.0 \quad 0.0 \quad 0.0]^T$	-
omega_BN_BInit	Initial Angular Velocity of $\mathcal{B}$ frame	$[0.1 \quad -0.1 \quad 0.1]^T$	rad/s

Table 6: Error Tolerance - Note: Relative Tolerance is abs($\frac{\text{truth}-\text{value}}{\text{truth}}$)

Test	Relative Tolerance
Energy and Momentum Conservation	1e-10
Steady State Deflection	1e-6
Frequency verification	5e-3
Max deflection with force on	5e-3
Max deflection with force off	5e-3
Lagrangian vs Basilisk comparison	1e-10

6 Test Results

7 User Guide

This section is to outline the steps needed to setup a reaction wheel state effector in python using Basilisk.

1. Import the reactionWheelStateEffector class:
   import reactionWheelStateEffector

2. Create an instantiation of a reaction wheel state effector:
   rws = reactionWheelStateEffector.ReactionWheelStateEffector()

3. Finally, add the reaction wheel object to your spacecraftPlus:
   scObject.addStateEffector(rws). See spacecraftPlus documentation on how to set up a spacecraftPlus object.

REFERENCES

[1] John Alcorn, Cody Allard, and Hanspeter Schaub. Fully-coupled dynamical modeling of a rigid spacecraft with imbalanced reaction wheels. In *AIAA/AAS Astrodynamics Specialist Conference*, Long Beach, CA, Sept. 12–15 2016.

[2] Hanspeter Schaub and John L. Junkins. *Analytical Mechanics of Space Systems*. AIAA Education Series, Reston, VA, 3rd edition, 2014.