Autonomous Vehicle Simulation (AVS) Laboratory, University of Colorado

Basilisk Technical Memorandum

Document ID: Basilisk-hingedRigidBodyStateEffector

HINGED RIGID BODY DYNAMICS MODEL

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Status: Initial document draft
Scope/Contents
The hinged rigid body class is an instantiation of the state effector abstract class. The integrated test is validating the interaction between the hinged rigid body module and the rigid body hub that it is attached to. In this case, a hinged rigid body has a diagonal inertia tensor and is attached to the hub by a single degree of freedom torsional hinged with a linear spring constant and linear damping term. The integrated tests has three scenarios it is testing: one with gravity and damping, one without gravity and without damping, and one without gravity with damping . In the first two cases orbital energy, orbital momentum, rotational energy, and rotational angular momentum should all be conserved. In the last case only orbital momentum and rotational momentum should be conserved. This integrated test validates for both scenarios that all of these paramters are conserved.

Rev	Change Description	By	Date
1.0	Initial draft	C. Allard	20170705

1.1	Update to new format	C. Allard	20170714

Contents

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1 Model Description

1.1 Introduction

The hinged rigid body class is an instantiation of the state effector abstract class. The state effector abstract class is a base class for modules that have dynamic states or degrees of freedom with respect to the rigid body hub. Examples of these would be reaction wheels, variable speed control moment gyroscopes, fuel slosh particles, etc. Since the state effectors are attached to the hub, the state effectors are directly affecting the hub as well as the hub is back affecting the state effectors.

Specifically, a hinged rigid body state effector is a rigid body that has a diagonal inertia with respect to its $\mathcal{S}_i$ frame as seen in Figure 1. It is attached to the hub through a hinge with a linear torsional spring and linear damping term. The dynamics of this multi-body problem have been derived and can be seen in Reference [1]. The derivation is general for N number of panels attached to the hub but does not allow for multiple interconnected panels.

1.2 Equations of Motion

The following equations of motion (EOMs) are pulled from Reference [1] for convenience. Equation (1) is the spacecraft translational EOM, Equation (2) is the spacecraft rotational EOM, and Equation (3) is the hinged rigid body rotational EOM. These are the coupled nonlinear EOMs that need to be integrated in the simulation.

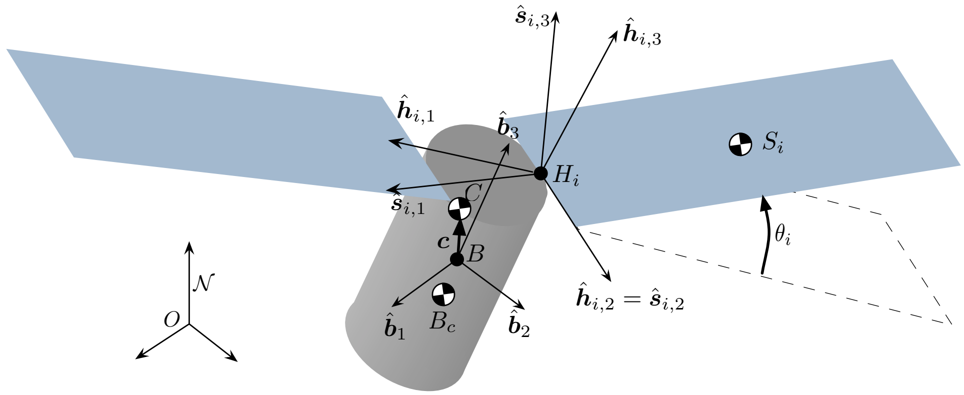

Fig. 1: Hinged rigid body frame and variable definitions

$$m_{\text{sc}}\ddot{\boldsymbol{r}}_{B/N} - m_{\text{sc}}[\tilde{\boldsymbol{c}}]\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} + \sum_{i}^{N} m_{\text{sp}_i} d_i \hat{\boldsymbol{s}}_{i,3}\ddot{\theta}_i = \boldsymbol{F}_{\text{ext}} - 2m_{\text{sc}}[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\boldsymbol{c}' - m_{\text{sc}}[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\boldsymbol{c} - \sum_{i}^{N} m_{\text{sp}_i} d_i \dot{\theta}_i^2 \hat{\boldsymbol{s}}_{i,1} \tag{1}$$

$$m_{\text{sc}}[\tilde{\boldsymbol{c}}]\ddot{\boldsymbol{r}}_{B/N} + [I_{\text{sc},B}]\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} + \sum_{i}^{N}\Big\{ I_{s_i,2}\hat{\boldsymbol{h}}_{i,2} + m_{\text{sp}_i} d_i [\tilde{\boldsymbol{r}}_{S_i/B}]\hat{\boldsymbol{s}}_{i,3}\Big\}\ddot{\theta}_i = - [\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][I_{\text{sc},B}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} - [I'_{\text{sc},B}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} - \sum_{i}^{N}\Big\{\dot{\theta}_i[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\Big(I_{s_i,2}\hat{\boldsymbol{h}}_{i,2} + m_{\text{sp}_i} d_i [\tilde{\boldsymbol{r}}_{S_i/B}]\hat{\boldsymbol{s}}_{i,3}\Big) + m_{\text{sp}_i} d_i \dot{\theta}_i^2 [\tilde{\boldsymbol{r}}_{S_i/B}]\hat{\boldsymbol{s}}_{i,1}\Big\} + \boldsymbol{L}_B \tag{2}$$

$$m_{\text{sp}_i} d_i \hat{\boldsymbol{s}}_{i,3}^T \ddot{\boldsymbol{r}}_{B/N} + \Big[\big(I_{s_{i,2}} + m_{\text{sp}_i} d_i^2\big)\hat{\boldsymbol{s}}_{i,2}^T - m_{\text{sp}_i} d_i \hat{\boldsymbol{s}}_{i,3}^T [\tilde{\boldsymbol{r}}_{H_i/B}]\Big]\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} + \big(I_{s_{i,2}} + m_{\text{sp}_i} d_i^2\big)\ddot{\theta}_i = -k_i\theta_i - c_i\dot{\theta}_i + \hat{\boldsymbol{s}}_{i,2}^T \boldsymbol{\tau}_{\text{ext},H_i} + \big(I_{s_{i,3}} - I_{s_{i,1}} + m_{\text{sp}_i} d_i^2\big)\omega_{s_{i,3}}\omega_{s_{i,1}} - m_{\text{sp}_i} d_i \hat{\boldsymbol{s}}_{i,3}^T [\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\boldsymbol{r}_{H_i/B} \tag{3}$$

1.3 Back Substitution Method

In order to integrate the EOMs in a modular fashion, a back substitution method was developed and can be seen in Reference [1]. The hinged rigid body model must adhere to this analytical form, and the details are briefly summarized in the equations following. First the hinged rigid body EOM is substituted into the translational EOM and rearranged:

$$\Big(m_{\text{sc}}[I_{3\times3}] + \sum_{i=1}^{N} m_{\text{sp}_i} d_i \hat{\boldsymbol{s}}_{i,3}\boldsymbol{a}_{\theta_i}^T\Big)\ddot{\boldsymbol{r}}_{B/N} + \Big(-m_{\text{sc}}[\tilde{\boldsymbol{c}}] + \sum_{i=1}^{N} m_{\text{sp}_i} d_i \hat{\boldsymbol{s}}_{i,3}\boldsymbol{b}_{\theta_i}^T\Big)\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} = m_{\text{sc}}\ddot{\boldsymbol{r}}_{C/N} - 2m_{\text{sc}}[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\boldsymbol{c}' - m_{\text{sc}}[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\boldsymbol{c} - \sum_{i=1}^{N}\Big(m_{\text{sp}_i} d_i \dot{\theta}_i^2 \hat{\boldsymbol{s}}_{i,1} + m_{\text{sp}_i} d_i c_{\theta_i}\hat{\boldsymbol{s}}_{i,3}\Big) \tag{4}$$

Following the same pattern for the hub rotational EOM, Eq. (2), yields:

$$\begin{aligned}
&\Big[m_{\text{sc}}[\tilde{\boldsymbol{c}}] + \sum_{i=1}^{N}\big(I_{s_{i,2}}\hat{\boldsymbol{s}}_{i,2} + m_{\text{sp}_i}d_i[\tilde{\boldsymbol{r}}_{S_{c,i}/B}]\hat{\boldsymbol{s}}_{i,3}\big)\boldsymbol{a}_{\theta_i}^T\Big]\ddot{\boldsymbol{r}}_{B/N} \\
&+\Big[[I_{\text{sc},B}] + \sum_{i=1}^{N}\big(I_{s_{i,2}}\hat{\boldsymbol{s}}_{i,2} + m_{\text{sp}_i}d_i[\tilde{\boldsymbol{r}}_{S_{c,i}/B}]\hat{\boldsymbol{s}}_{i,3}\big)\boldsymbol{b}_{\theta_i}^T\Big]\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} = -[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][I_{\text{sc},B}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} - [I'_{\text{sc},B}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \\
&-\sum_{i=1}^{N}\Big\{\big(\dot{\theta}_i[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}] + c_{\theta_i}[I_{3\times3}]\big)\Big(I_{s_{i,2}}\hat{\boldsymbol{s}}_{i,2} + m_{\text{sp}_i}d_i[\tilde{\boldsymbol{r}}_{S_{c,i}/B}]\hat{\boldsymbol{s}}_{i,3}\Big) + m_{\text{sp}_i}d_i\dot{\theta}_i^2[\tilde{\boldsymbol{r}}_{S_{c,i}/B}]\hat{\boldsymbol{s}}_{i,1}\Big\} + \boldsymbol{L}_B
\end{aligned} \tag{5}$$

With the following definitions:

$$\boldsymbol{a}_{\theta_i} = -\frac{m_{\text{sp}_i}d_i}{\big(I_{s_{i,2}} + m_{\text{sp}_i}d_i^2\big)}\hat{\boldsymbol{s}}_{i,3} \tag{6a}$$

$$\boldsymbol{b}_{\theta_i} = -\frac{1}{\big(I_{s_{i,2}} + m_{\text{sp}_i}d_i^2\big)}\Big[\big(I_{s_{i,2}} + m_{\text{sp}_i}d_i^2\big)\,\hat{\boldsymbol{s}}_{i,2} + m_{\text{sp}_i}d_i[\tilde{\boldsymbol{r}}_{H_i/B}]\hat{\boldsymbol{s}}_{i,3}\Big] \tag{6b}$$

$$\begin{aligned}
c_{\theta_i} = &\frac{1}{\big(I_{s_{i,2}} + m_{\text{sp}_i}d_i^2\big)}\Big(-k_i\theta_i - c_i\dot{\theta}_i + \hat{\boldsymbol{s}}_{i,2}\cdot\boldsymbol{\tau}_{\text{ext},H_i} + \big(I_{s_{i,3}} - I_{s_{i,1}} + m_{\text{sp}_i}d_i^2\big)\,\omega_{s_{i,3}}\omega_{s_{i,1}} \\
&- m_{\text{sp}_i}d_i\hat{\boldsymbol{s}}_{i,3}^T[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\boldsymbol{r}_{H_i/B}\Big)
\end{aligned} \tag{6c}$$

The equations can now be organized into the following matrix respresentation:

$$\begin{bmatrix}[A] & [B]\\ [C] & [D]\end{bmatrix}\begin{bmatrix}\ddot{\boldsymbol{r}}_{B/N}\\ \dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}\end{bmatrix} = \begin{bmatrix}\boldsymbol{v}_{\text{trans}}\\ \boldsymbol{v}_{\text{rot}}\end{bmatrix} \tag{7}$$

Finally, the hinged rigid body model must make "contributions" to the matrices defined in Equations (7). These contributions are defined in the following equations:

$$[A_{\text{contr}}] = m_{\text{sp}_i}d_i\hat{\boldsymbol{s}}_{i,3}\boldsymbol{a}_{\theta_i}^T \tag{8}$$

$$[B_{\text{contr}}] = m_{\text{sp}_i}d_i\hat{\boldsymbol{s}}_{i,3}\boldsymbol{b}_{\theta_i}^T \tag{9}$$

$$[C_{\text{contr}}] = \big(I_{s_{i,2}}\hat{\boldsymbol{s}}_{i,2} + m_{\text{sp}_i}d_i[\tilde{\boldsymbol{r}}_{S_{c,i}/B}]\hat{\boldsymbol{s}}_{i,3}\big)\boldsymbol{a}_{\theta_i}^T \tag{10}$$

$$[D_{\text{contr}}] = \big(I_{s_{i,2}}\hat{\boldsymbol{s}}_{i,2} + m_{\text{sp}_i}d_i[\tilde{\boldsymbol{r}}_{S_{c,i}/B}]\hat{\boldsymbol{s}}_{i,3}\big)\boldsymbol{b}_{\theta_i}^T \tag{11}$$

$$\boldsymbol{v}_{\text{trans,contr}} = -\Big(m_{\text{sp}_i}d_i\dot{\theta}_i^2\hat{\boldsymbol{s}}_{i,1} + m_{\text{sp}_i}d_ic_{\theta_i}\hat{\boldsymbol{s}}_{i,3}\Big) \tag{12}$$

$$\boldsymbol{v}_{\text{rot,contr}} = -\Big\{\big(\dot{\theta}_i[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}] + c_{\theta_i}[I_{3\times3}]\big)\Big(I_{s_{i,2}}\hat{\boldsymbol{s}}_{i,2} + m_{\text{sp}_i}d_i[\tilde{\boldsymbol{r}}_{S_{c,i}/B}]\hat{\boldsymbol{s}}_{i,3}\Big) + m_{\text{sp}_i}d_i\dot{\theta}_i^2[\tilde{\boldsymbol{r}}_{S_{c,i}/B}]\hat{\boldsymbol{s}}_{i,1}\Big\} \tag{13}$$

The final equation that is needed is:

$$\ddot{\theta}_i = \boldsymbol{a}_{\theta_i}^T\ddot{\boldsymbol{r}}_{B/N} + \boldsymbol{b}_{\theta_i}^T\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} + c_{\theta_i} \tag{14}$$

2 Model Functions

This module is intended to be used an approximation to a flexing body attached to the spacecraft. Examples include solar arrays, antennas, and other appended bodies that would exhibit flexing behavior. Below is a list of functions that this model performs:

- Should be a first-order approximation to a flexing body
- Is developed in such a way that does not require constraints to be met (or that could eventually diverge)
- Compute it's contributions to the mass properties of the spacecraft
- Adhere to the back substitution form and provide matrix contributions for the back substitution method
- Compute it's derivatives for θ and $\dot{\theta}$
- Add energy and momentum contributions to the spacecraft

3 Model Assumptions and Limitations

Below is a summary of the assumptions/limitations:

- The hinged rigid body must have a diagonal inertia tensor with respect the $\mathcal{S}_i$ frame as seen in Figure 1
- Only linear spring and damping terms
- Will only approximate one flexing mode at a time
- Cannot simulate multiple interconnected panels
- The hinged rigid will always stay attached to the hub (the hinge does not have torque limits)
- The hinge does not have travel limits, therefore if the spring is not stiff enough it will unrealistically travel through bounds such as running into the spacecraft hub
- The EOMs are nonlinear equations of motion, therefore there can be inaccuracies that result from integration. Having a time step of $<= 0.10$ sec is recommended.

4 Test Description and Success Criteria

This test is located in `SimCode/dynamics/HingedRigidBodies/UnitTest/test_hingedRigidBodyStateEffector.py`. `In this integrated energy and momentum are the primary methods for validation. Depending on the scenario, however, there are different success criteria. These are outlined in the following list:`

- `Gravity and no damping scenario:`

  `Conservation of orbital angular momentum`

  `Conservation of orbital energy`

  `Conservation of rotational angular momentum`

  `Conservation of rotational energy`

  `Achieving the expected final attitude`

- No gravity and no damping scenario:

  Conservation of orbital angular momentum

  Conservation of orbital energy

  Conservation of rotational angular momentum

  Conservation of rotational energy

  Achieving the expected final attitude

  Achieving the expected final position

  Conservation of velocity of center of mass

- No gravity with damping scenario:

  Conservation of orbital angular momentum

  Conservation of orbital energy

  Conservation of rotational angular momentum

  Conservation of velocity of center of mass

5 Test Parameters

Test parameters and inputs go here. I think that success criteria would work better here than in the test description section.

6 Test Results

The following figures show the conservation of the quantities described in the success criteria for each scenario.

6.1 Gravity with no damping scenario

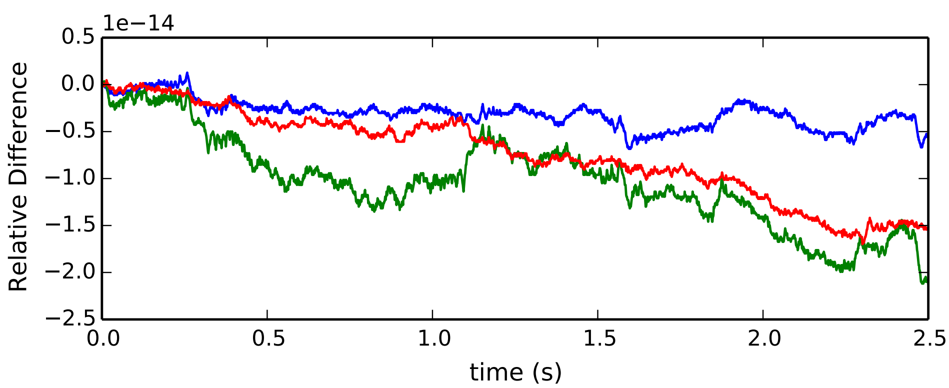

Fig. 2: Change in Orbital Angular Momentum with Gravity

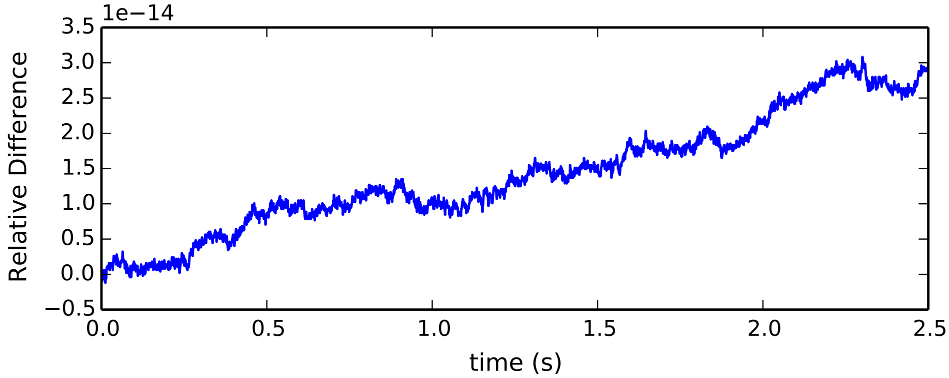

Fig. 3: Change in Orbital Energy with Gravity

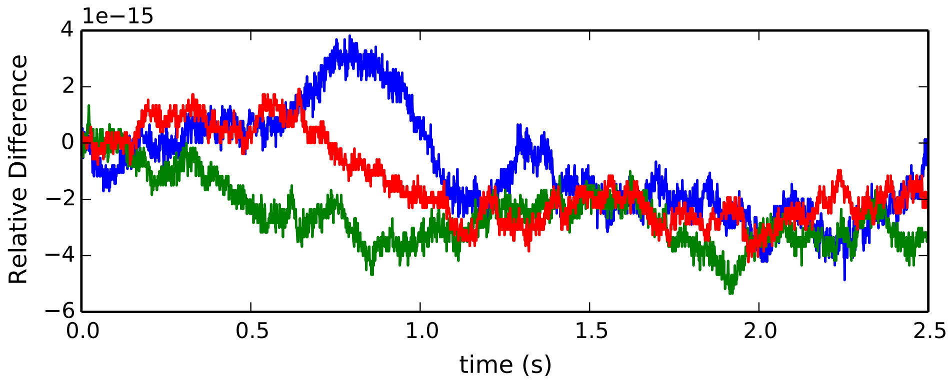

Fig. 4: Change In Rotational Angular Momentum with Gravity

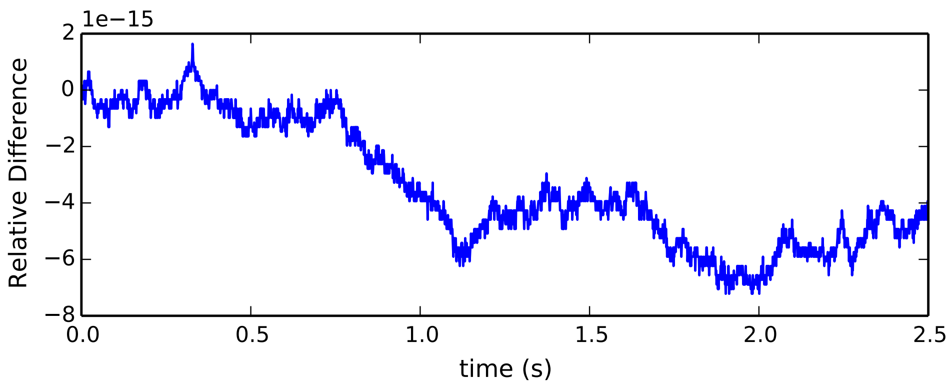

Fig. 5: Change In Rotational Energy with Gravity

6.2 No Gravity with no damping scenario

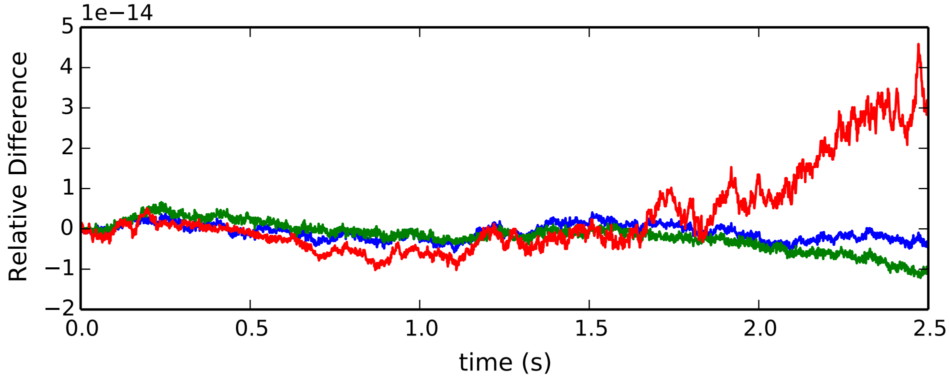

Fig. 6: Change in Orbital Angular Momentum No Gravity

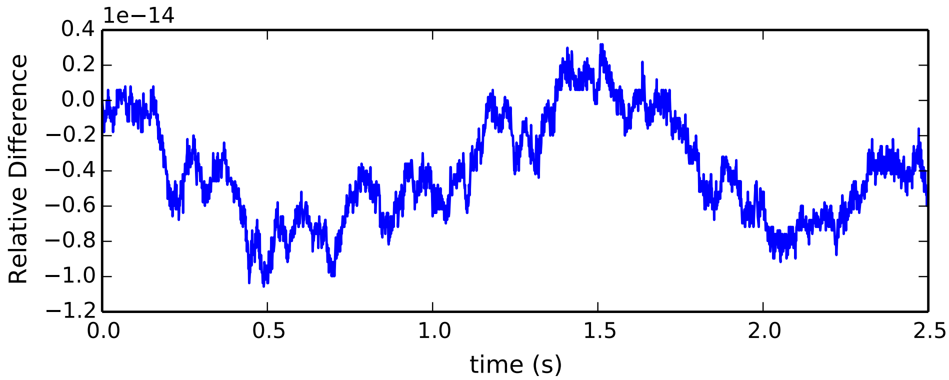

Fig. 7: Change in Orbital Energy No Gravity

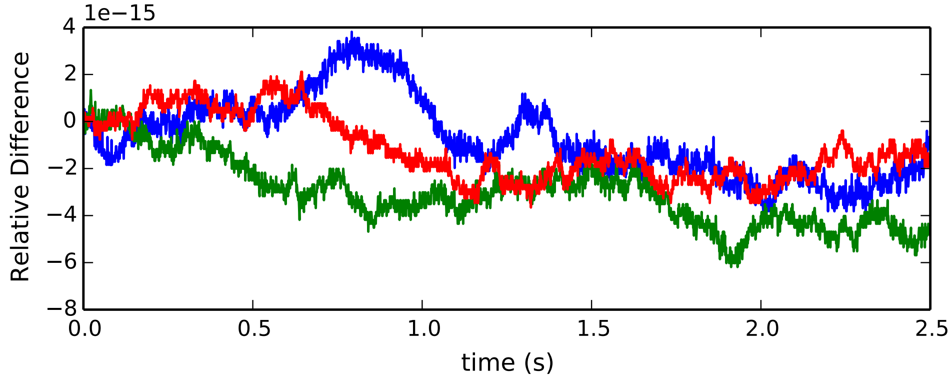

Fig. 8: Change In Rotational Angular Momentum No Gravity

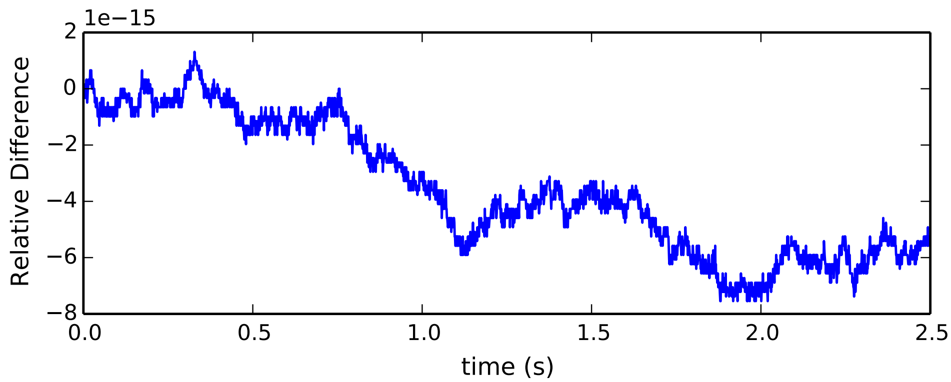

Fig. 9: Change In Rotational Energy No Gravity

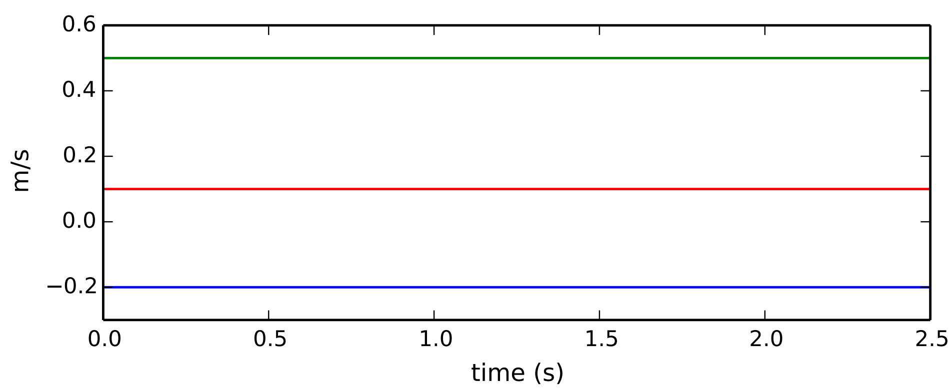

Fig. 10: Velocity Of Center Of Mass No Gravity

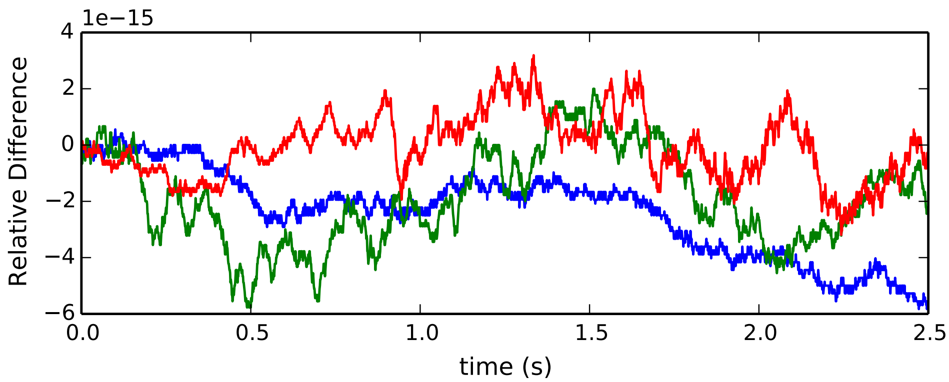

Fig. 11: Change In Velocity Of Center Of Mass No Gravity

6.3 No Gravity with damping scenario

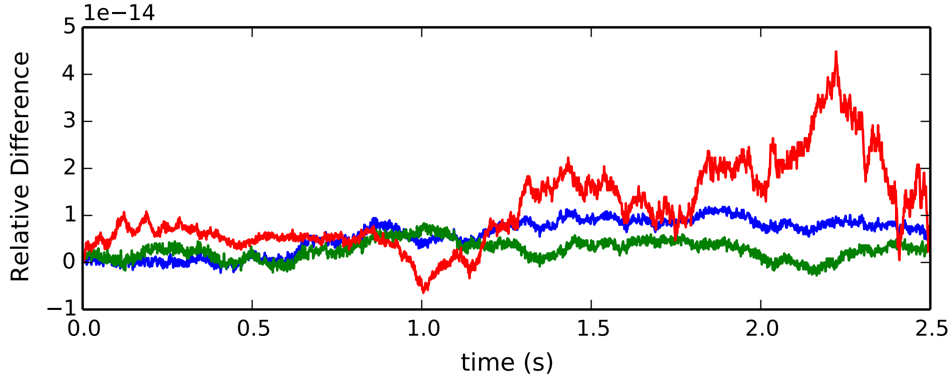

Fig. 12: Change in Orbital Angular Momentum No Gravity with Damping

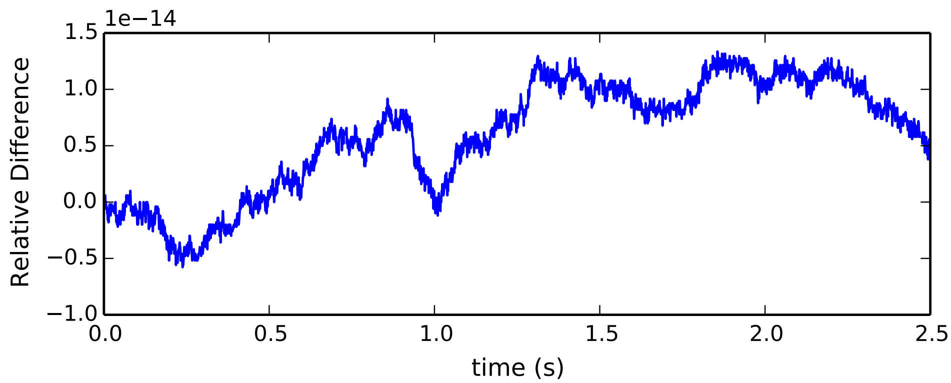

Fig. 13: Change in Orbital Energy No Gravity with Damping

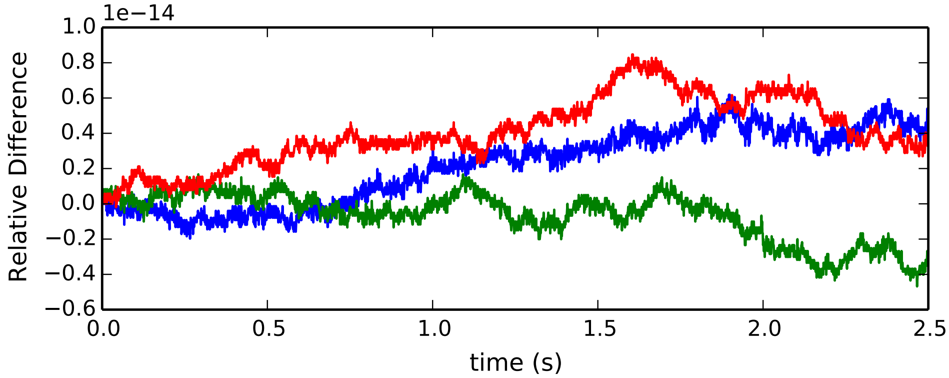

Fig. 14: Change In Rotational Angular Momentum No Gravity with Damping

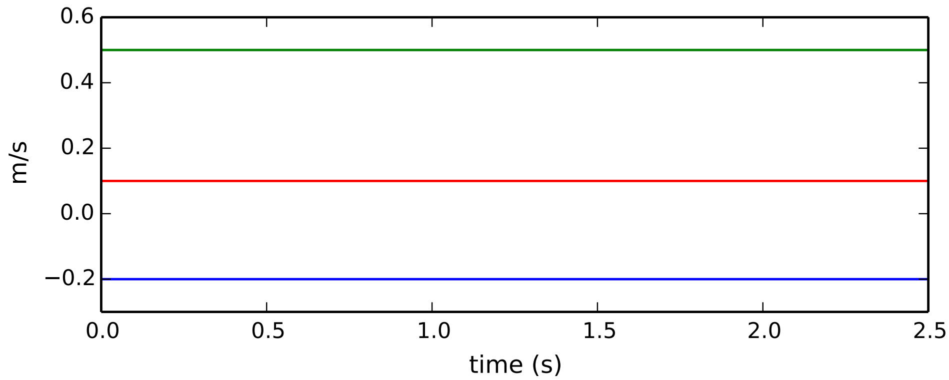

Fig. 15: Velocity Of Center Of Mass No Gravity with Damping

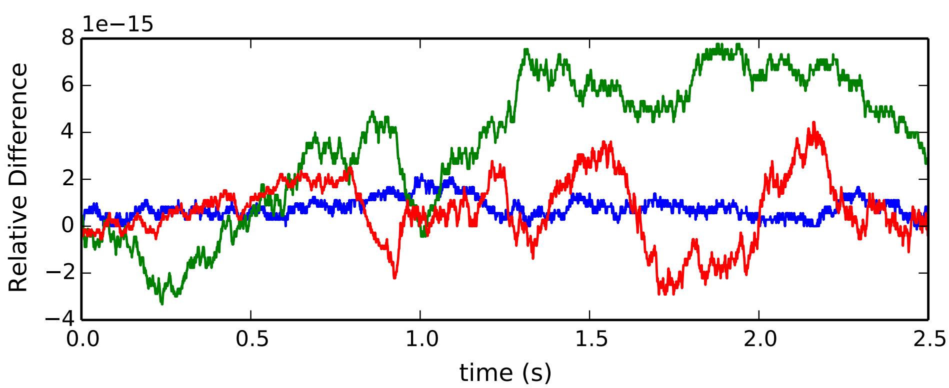

Fig. 16: Change In Velocity Of Center Of Mass No Gravity with Damping

7 User Guide

REFERENCES

[1] C. Allard, Hanspeter Schaub, and Scott Piggott. General hinged solar panel dynamics approximating first-order spacecraft flexing. In *AAS Guidance and Control Conference*, Breckenridge, CO, Feb. 5--10 2016. Paper No. AAS-16-156.