Autonomous Vehicle Simulation (AVS) Laboratory, University of Colorado

Basilisk Technical Memorandum

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HINGED RIGID BODY DYNAMICS MODEL

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Status: In Review
Scope/Contents
The hinged rigid body class is an instantiation of the state effector abstract class. The integrated test is validating the interaction between the hinged rigid body module and the rigid body hub that it is attached to. In this case, a hinged rigid body has a diagonal inertia tensor and is attached to the hub by a single degree of freedom torsional hinged with a linear spring constant and linear damping term. The integrated tests has three scenarios it is testing: one with gravity and no damping, one without gravity and without damping, and one without gravity with damping . In the first two cases orbital energy, orbital momentum, rotational energy, and rotational angular momentum should all be conserved. In the last case only orbital momentum, orbital energy, and rotational momentum should be conserved. This integrated test validates for all three scenarios that all of these paramters are conserved.

Rev	Change Description	By	Date
1.0	Initial Draft	C. Allard	20170703

1.1	Format Changes	C. Allard	20170712

Contents

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1 Model Description

1.1 Introduction

The hinged rigid body class is an instantiation of the state effector abstract class. The state effector abstract class is a base class for modules that have dynamic states or degrees of freedom with respect to the rigid body hub. Examples of these would be reaction wheels, variable speed control moment gyroscopes, fuel slosh particles, etc. Since the state effectors are attached to the hub, the state effectors are directly affecting the hub as well as the hub is back affecting the state effectors.

Specifically, a hinged rigid body state effector is a rigid body that has a diagonal inertia with respect to its $\mathcal{S}_i$ frame as seen in Figure 1. It is attached to the hub through a hinge with a linear torsional spring and linear damping term. The dynamics of this multi-body problem have been derived and can be seen in Reference [1]. The derivation is general for N number of panels attached to the hub but does not allow for multiple interconnected panels.

1.2 Equations of Motion

The following equations of motion (EOMs) are pulled from Reference [1] for convenience. Equation (1) is the spacecraft translational EOM, Equation (2) is the spacecraft rotational EOM, and Equation (3) is the hinged rigid body rotational EOM. These are the coupled nonlinear EOMs that need to be integrated in the simulation.

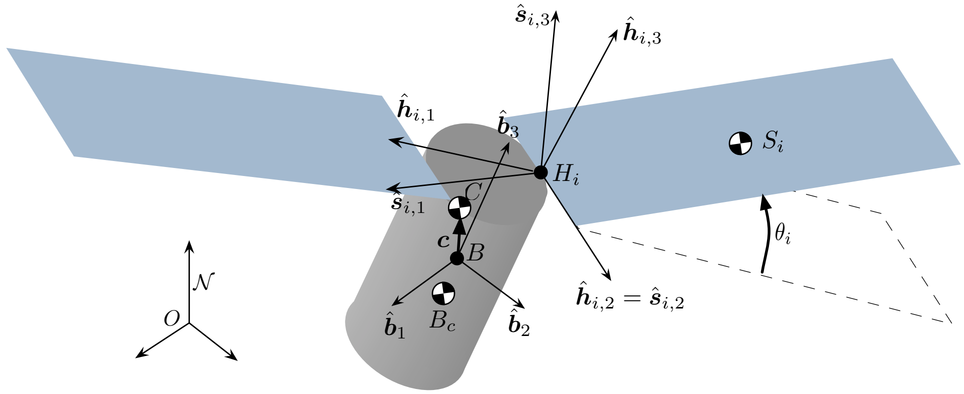

Fig. 1: Hinged rigid body frame and variable definitions

$$m_{\text{sc}}\ddot{\boldsymbol{r}}_{B/N} - m_{\text{sc}}[\tilde{\boldsymbol{c}}]\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} + \sum_{i}^{N} m_{\text{sp}_i} d_i \hat{\boldsymbol{s}}_{i,3}\ddot{\theta}_i = \boldsymbol{F}_{\text{ext}} - 2m_{\text{sc}}[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\boldsymbol{c}' - m_{\text{sc}}[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\boldsymbol{c} - \sum_{i}^{N} m_{\text{sp}_i} d_i \dot{\theta}_i^2 \hat{\boldsymbol{s}}_{i,1} \quad (1)$$

$$m_{\text{sc}}[\tilde{\boldsymbol{c}}]\ddot{\boldsymbol{r}}_{B/N} + [I_{\text{sc},B}]\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} + \sum_{i}^{N}\Big\{ I_{s_{i,2}}\hat{\boldsymbol{h}}_{i,2} + m_{\text{sp}_i} d_i [\tilde{\boldsymbol{r}}_{S_i/B}]\hat{\boldsymbol{s}}_{i,3}\Big\}\ddot{\theta}_i = - [\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][I_{\text{sc},B}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} - [I'_{\text{sc},B}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} - \sum_{i}^{N}\Big\{\dot{\theta}_i[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\left(I_{s_{i,2}}\hat{\boldsymbol{h}}_{i,2} + m_{\text{sp}_i} d_i [\tilde{\boldsymbol{r}}_{S_i/B}]\hat{\boldsymbol{s}}_{i,3}\right) + m_{\text{sp}_i} d_i \dot{\theta}_i^2 [\tilde{\boldsymbol{r}}_{S_i/B}]\hat{\boldsymbol{s}}_{i,1}\Big\} + \boldsymbol{L}_B \quad (2)$$

$$m_{\text{sp}_i} d_i \hat{\boldsymbol{s}}_{i,3}^T \ddot{\boldsymbol{r}}_{B/N} + \bigg[\left(I_{s_{i,2}} + m_{\text{sp}_i} d_i^2\right)\hat{\boldsymbol{s}}_{i,2}^T - m_{\text{sp}_i} d_i \hat{\boldsymbol{s}}_{i,3}^T[\tilde{\boldsymbol{r}}_{H_i/B}]\bigg]\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} + \left(I_{s_{i,2}} + m_{\text{sp}_i} d_i^2\right)\ddot{\theta}_i = -k_i\theta_i - c_i\dot{\theta}_i + \hat{\boldsymbol{s}}_{i,2}^T \boldsymbol{\tau}_{\text{ext},H_i} + \left(I_{s_{i,3}} - I_{s_{i,1}} + m_{\text{sp}_i} d_i^2\right)\omega_{s_{i,3}}\omega_{s_{i,1}} - m_{\text{sp}_i} d_i \hat{\boldsymbol{s}}_{i,3}^T[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\boldsymbol{r}_{H_i/B} \quad (3)$$

1.3 Back Substitution Method

In order to integrate the EOMs in a modular fashion, a back substitution method was developed and can be seen in Reference [1]. The hinged rigid body model must adhere to this analytical form, and the details are briefly summarized in the equations following. First the hinged rigid body EOM is substituted into the translational EOM and rearranged:

$$\Big(m_{\text{sc}}[I_{3\times3}] + \sum_{i=1}^{N} m_{\text{sp}_i} d_i \hat{\boldsymbol{s}}_{i,3}\boldsymbol{a}_{\theta_i}^T\Big)\ddot{\boldsymbol{r}}_{B/N} + \Big(-m_{\text{sc}}[\tilde{\boldsymbol{c}}] + \sum_{i=1}^{N} m_{\text{sp}_i} d_i \hat{\boldsymbol{s}}_{i,3}\boldsymbol{b}_{\theta_i}^T\Big)\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} = m_{\text{sc}}\ddot{\boldsymbol{r}}_{C/N} - 2m_{\text{sc}}[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\boldsymbol{c}' - m_{\text{sc}}[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\boldsymbol{c} - \sum_{i=1}^{N}\Big(m_{\text{sp}_i} d_i \dot{\theta}_i^2 \hat{\boldsymbol{s}}_{i,1} + m_{\text{sp}_i} d_i c_{\theta_i}\hat{\boldsymbol{s}}_{i,3}\Big) \quad (4)$$

Following the same pattern for the hub rotational EOM, Eq. (2), yields:

$$\Big[m_{\text{sc}}[\tilde{\boldsymbol{c}}] + \sum_{i=1}^{N}\big(I_{s_{i,2}}\hat{\boldsymbol{s}}_{i,2} + m_{\text{sp}_i}d_i[\tilde{\boldsymbol{r}}_{S_{c,i}/B}]\hat{\boldsymbol{s}}_{i,3}\big)\boldsymbol{a}_{\theta_i}^T\Big]\ddot{\boldsymbol{r}}_{B/N}$$
$$+ \Big[[I_{\text{sc},B}] + \sum_{i=1}^{N}\big(I_{s_{i,2}}\hat{\boldsymbol{s}}_{i,2} + m_{\text{sp}_i}d_i[\tilde{\boldsymbol{r}}_{S_{c,i}/B}]\hat{\boldsymbol{s}}_{i,3}\big)\boldsymbol{b}_{\theta_i}^T\Big]\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} = -[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][I_{\text{sc},B}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} - [I'_{\text{sc},B}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}}$$
$$- \sum_{i=1}^{N}\Big\{\big(\dot{\theta}_i[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}] + c_{\theta_i}[I_{3\times3}]\big)\Big(I_{s_{i,2}}\hat{\boldsymbol{s}}_{i,2} + m_{\text{sp}_i}d_i[\tilde{\boldsymbol{r}}_{S_{c,i}/B}]\hat{\boldsymbol{s}}_{i,3}\Big) + m_{\text{sp}_i}d_i\dot{\theta}_i^2[\tilde{\boldsymbol{r}}_{S_{c,i}/B}]\hat{\boldsymbol{s}}_{i,1}\Big\} + \boldsymbol{L}_B \tag{5}$$

With the following definitions:

$$\boldsymbol{a}_{\theta_i} = -\frac{m_{\text{sp}_i}d_i}{\big(I_{s_{i,2}} + m_{\text{sp}_i}d_i^2\big)}\hat{\boldsymbol{s}}_{i,3} \tag{6a}$$
$$\boldsymbol{b}_{\theta_i} = -\frac{1}{\big(I_{s_{i,2}} + m_{\text{sp}_i}d_i^2\big)}\Big[\big(I_{s_{i,2}} + m_{\text{sp}_i}d_i^2\big)\,\hat{\boldsymbol{s}}_{i,2} + m_{\text{sp}_i}d_i[\tilde{\boldsymbol{r}}_{H_i/B}]\hat{\boldsymbol{s}}_{i,3}\Big] \tag{6b}$$
$$c_{\theta_i} = \frac{1}{\big(I_{s_{i,2}} + m_{\text{sp}_i}d_i^2\big)}\Big(-k_i\theta_i - c_i\dot{\theta}_i + \hat{\boldsymbol{s}}_{i,2}\cdot\boldsymbol{\tau}_{\text{ext},H_i} + \big(I_{s_{i,3}} - I_{s_{i,1}} + m_{\text{sp}_i}d_i^2\big)\,\omega_{s_{i,3}}\omega_{s_{i,1}} - m_{\text{sp}_i}d_i\hat{\boldsymbol{s}}_{i,3}^T[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\boldsymbol{r}_{H_i/B}\Big) \tag{6c}$$

The equations can now be organized into the following matrix respresentation:

$$\begin{bmatrix}[A] & [B]\\ [C] & [D]\end{bmatrix}\begin{bmatrix}\ddot{\boldsymbol{r}}_{B/N}\\ \dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}\end{bmatrix} = \begin{bmatrix}\boldsymbol{v}_{\text{trans}}\\ \boldsymbol{v}_{\text{rot}}\end{bmatrix} \tag{7}$$

Finally, the hinged rigid body model must make "contributions" to the matrices defined in Equations (7). These contributions are defined in the following equations:

$$[A_{\text{contr}}] = m_{\text{sp}_i}d_i\hat{\boldsymbol{s}}_{i,3}\boldsymbol{a}_{\theta_i}^T \tag{8}$$
$$[B_{\text{contr}}] = m_{\text{sp}_i}d_i\hat{\boldsymbol{s}}_{i,3}\boldsymbol{b}_{\theta_i}^T \tag{9}$$
$$[C_{\text{contr}}] = \big(I_{s_{i,2}}\hat{\boldsymbol{s}}_{i,2} + m_{\text{sp}_i}d_i[\tilde{\boldsymbol{r}}_{S_{c,i}/B}]\hat{\boldsymbol{s}}_{i,3}\big)\boldsymbol{a}_{\theta_i}^T \tag{10}$$
$$[D_{\text{contr}}] = \big(I_{s_{i,2}}\hat{\boldsymbol{s}}_{i,2} + m_{\text{sp}_i}d_i[\tilde{\boldsymbol{r}}_{S_{c,i}/B}]\hat{\boldsymbol{s}}_{i,3}\big)\boldsymbol{b}_{\theta_i}^T \tag{11}$$
$$\boldsymbol{v}_{\text{trans,contr}} = -\Big(m_{\text{sp}_i}d_i\dot{\theta}_i^2\hat{\boldsymbol{s}}_{i,1} + m_{\text{sp}_i}d_ic_{\theta_i}\hat{\boldsymbol{s}}_{i,3}\Big) \tag{12}$$
$$\boldsymbol{v}_{\text{rot,contr}} = -\Big\{\big(\dot{\theta}_i[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}] + c_{\theta_i}[I_{3\times3}]\big)\Big(I_{s_{i,2}}\hat{\boldsymbol{s}}_{i,2} + m_{\text{sp}_i}d_i[\tilde{\boldsymbol{r}}_{S_{c,i}/B}]\hat{\boldsymbol{s}}_{i,3}\Big) + m_{\text{sp}_i}d_i\dot{\theta}_i^2[\tilde{\boldsymbol{r}}_{S_{c,i}/B}]\hat{\boldsymbol{s}}_{i,1}\Big\} \tag{13}$$

The final equation that is needed is:

$$\ddot{\theta}_i = \boldsymbol{a}_{\theta_i}^T\ddot{\boldsymbol{r}}_{B/N} + \boldsymbol{b}_{\theta_i}^T\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} + c_{\theta_i} \tag{14}$$

2 Model Functions

This module is intended to be used an approximation to a flexing body attached to the spacecraft. Examples include solar arrays, antennas, and other appended bodies that would exhibit flexing behavior. Below is a list of functions that this model performs:

- Compute it's contributions to the mass properties of the spacecraft
- Provides matrix contributions for the back substitution method
- Compute it's derivatives for θ and $\dot{\theta}$
- Adds energy and momentum contributions to the spacecraft

3 Model Assumptions and Limitations

Below is a summary of the assumptions/limitations:

- Is a first-order approximation to a flexing body
- Is developed in such a way that does not require constraints to be met
- The hinged rigid body must have a diagonal inertia tensor with respect the $\mathcal{S}_i$ frame as seen in Figure 1
- Only linear spring and damping terms
- Will only approximate one flexing mode at a time
- Cannot simulate multiple interconnected panels
- The hinged rigid will always stay attached to the hub (the hinge does not have torque limits)
- The hinge does not have travel limits, therefore if the spring is not stiff enough it will unrealistically travel through bounds such as running into the spacecraft hub
- The EOMs are nonlinear equations of motion, therefore there can be inaccuracies (and divergence) that result from integration. Having a time step of $<= 0.10$ sec is recommended.

4 Test Description and Success Criteria

This test is located in `SimCode/dynamics/HingedRigidBodies/UnitTest/test_hingedRigidBodyStateEffector.py`. In this integrated test there are two hinged rigid bodies connected to the spacecraft hub. Energy and momentum are the primary methods for validation. Depending on the scenario, however, there are different success criteria. These are outlined in the following list:

- Gravity and no damping scenario:

    Conservation of orbital angular momentum

    Conservation of orbital energy

    Conservation of rotational angular momentum

    Conservation of rotational energy

    Achieving the expected final attitude

- No gravity and no damping scenario:

  Conservation of orbital angular momentum

  Conservation of orbital energy

  Conservation of rotational angular momentum

  Conservation of rotational energy

  Achieving the expected final attitude (same final attitude as the Gravity with no damping scenario)

  Achieving the expected final position

  Conservation of velocity of center of mass

- No gravity with damping scenario:

  Conservation of orbital angular momentum

  Conservation of orbital energy

  Conservation of rotational angular momentum

  Conservation of velocity of center of mass

5 Test Parameters

Since this is an integrated test, the inputs to the test are the physical parameters of the spacecraft along with the initial conditions of the states. These parameters are outlined in Tables 2- 5. Additionally, the error tolerance used for the relative accuracy checks is 1e-10.

Table 2: Spacecraft Hub Parameters

Name	Description	Value	Units
mHub	mass	750.0	kg
IHubPntBc_B	Inertia in $\mathcal{B}$ frame	$\begin{bmatrix} 900.0 & 0.0 & 0.0 \\ 0.0 & 600.0 & 0.0 \\ 0.0 & 0.0 & 600.0 \end{bmatrix}$	kg-m^2
r_BcB_B	CoM Location in $\mathcal{B}$ frame	$[0.0 \quad 0.0 \quad 1.0]^T$	m

Table 3: Hinged Rigid Body 1 Parameters

Name	Description	Value	Units
mass	mass	100.0	kg
IPntS_S	Inertia in $\mathcal{S}$ frame	$\begin{bmatrix} 100.0 & 0.0 & 0.0 \\ 0.0 & 50.0 & 0.0 \\ 0.0 & 0.0 & 50.0 \end{bmatrix}$	kg-m^2
d	CoM location	1.5	m
k	Spring Constant	100.0	N-m/rad
c	Damping Term	0.0 (6.0 - damping scenario)	N-m-s/rad
r_HB_B	Hinge Location in $\mathcal{B}$ frame	$[0.5 \quad 0.0 \quad 1.0]^T$	m
dcm_HB	$\mathcal{B}$ to $\mathcal{H}$ DCM	$\begin{bmatrix} -1.0 & 0.0 & 0.0 \\ 0.0 & -1.0 & 0.0 \\ 0.0 & 0.0 & 1.0 \end{bmatrix}$	-

Table 4: Hinged Rigid Body 2 Parameters

Name	Description	Value	Units
mass	mass	100.0	kg
IPntS_S	Inertia in $\mathcal{S}$ frame	$\begin{bmatrix} 100.0 & 0.0 & 0.0 \\ 0.0 & 50.0 & 0.0 \\ 0.0 & 0.0 & 50.0 \end{bmatrix}$	kg-m^2
d	CoM location	1.5	m
k	Spring Constant	100.0	N-m/rad
c	Damping Term	0.0 (7.0 - damping scenario)	N-m-s/rad
r_HB_B	Hinge Location in $\mathcal{B}$ frame	$[-0.5 \quad 0.0 \quad 1.0]^T$	m
dcm_HB	$\mathcal{B}$ to $\mathcal{H}$ DCM	$\begin{bmatrix} 1.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 1.0 \end{bmatrix}$	-

Table 5: Initial Conditions

Name	Description	Value	Units
(Panel 1) thetaInit	(Panel 1) Initial θ	5.0	deg
(Panel 1) thetaDotInit	(Panel 1) Initial $\dot{\theta}$	0.0	deg
(Panel 2) thetaInit	(Panel 2) Initial θ	0.0	deg
(Panel 2) thetaDotInit	(Panel 2) Initial $\dot{\theta}$	0.0	deg
r_CN_NInit	Initial Position of S/C (gravity scenarios)	$[-4020339 \quad 7490567 \quad 5248299]^T$	m
v_CN_NInit	Initial Velocity of S/C (gravity scenarios)	$[-5199.78 \quad -3436.68 \quad 1041.58]^T$	m/s
r_CN_NInit	Initial Position of S/C (no gravity)	$[0.1 \quad -0.4 \quad 0.3]^T$	m
v_CN_NInit	Initial Velocity of S/C (no gravity)	$[-0.2 \quad 0.5 \quad 0.1]^T$	m/s
sigma_BNInit	Initial MRP of $\mathcal{B}$ frame	$[0.0 \quad 0.0 \quad 0.0]^T$	-
omega_BN_BInit	Initial Angular Velocity of $\mathcal{B}$ frame	$[0.1 \quad -0.1 \quad 0.1]^T$	rad/s

6 Test Results

The following figures show the conservation of the quantities described in the success criteria for each scenario. The conservation plots are all relative difference plots. All conservation plots show integration error which is the desired result. In the python test these values are automatically checked therefore when the tests pass, these values have all been confirmed to be conserved.

6.1 Gravity with no damping scenario

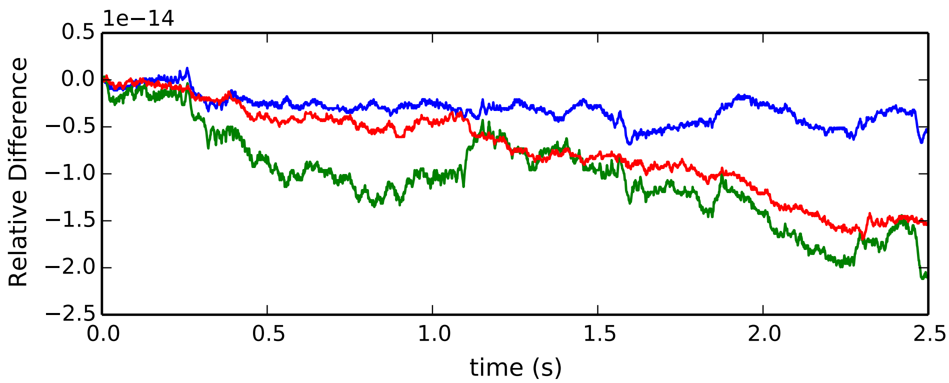

Fig. 2: Change in Orbital Angular Momentum with Gravity

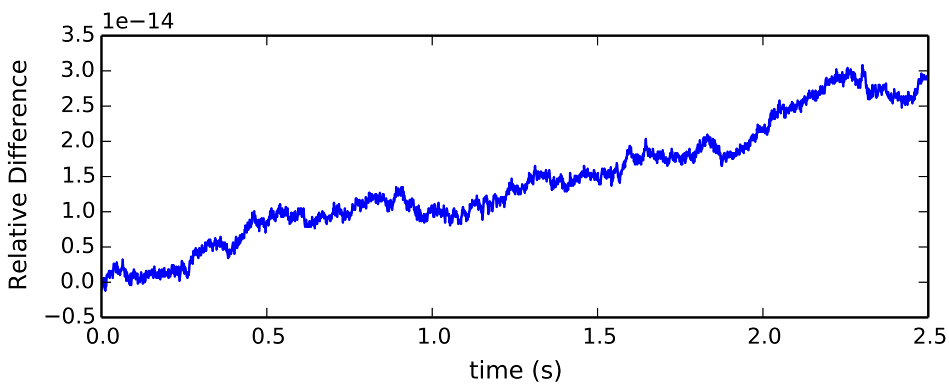

Fig. 3: Change in Orbital Energy with Gravity

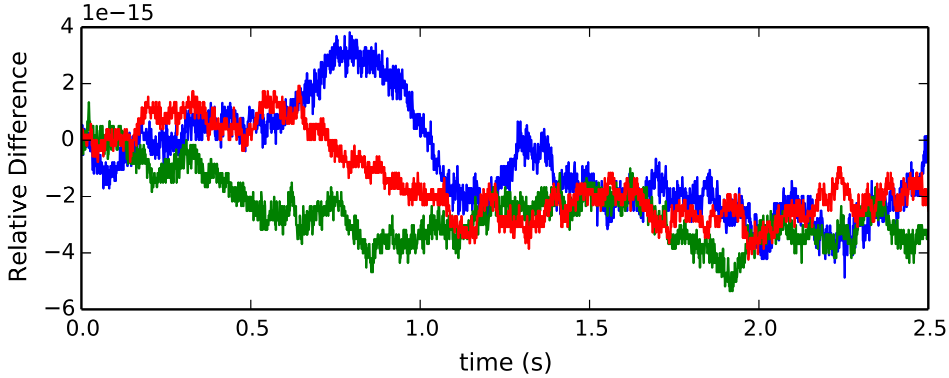

Fig. 4: Change In Rotational Angular Momentum with Gravity

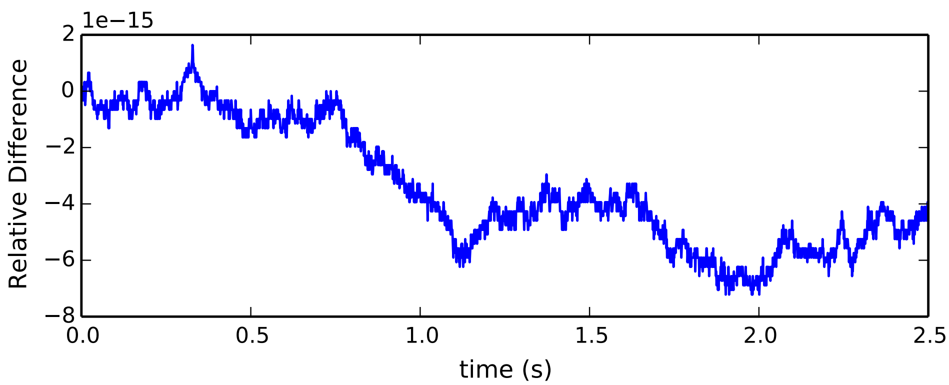

Fig. 5: Change In Rotational Energy with Gravity

6.2 No Gravity with no damping scenario

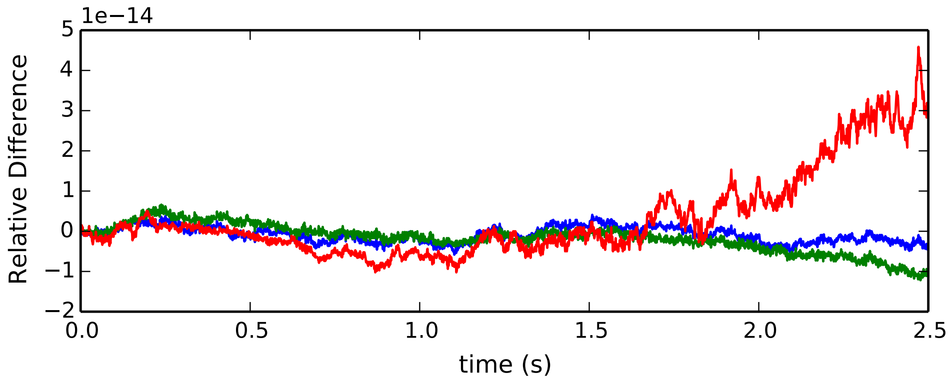

Fig. 6: Change in Orbital Angular Momentum No Gravity

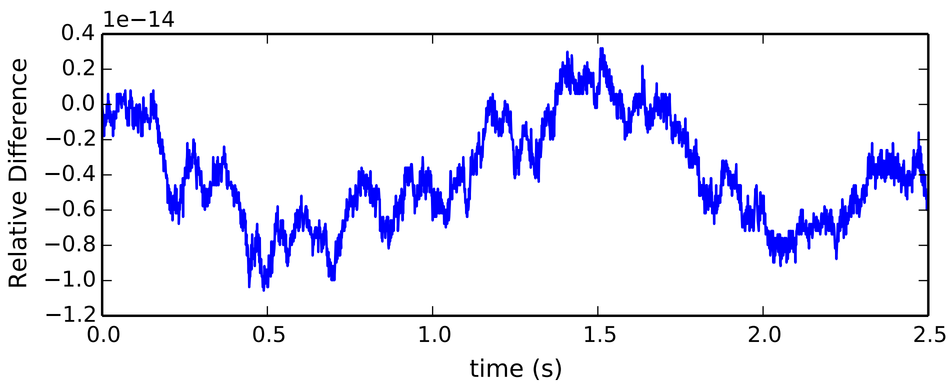

Fig. 7: Change in Orbital Energy No Gravity

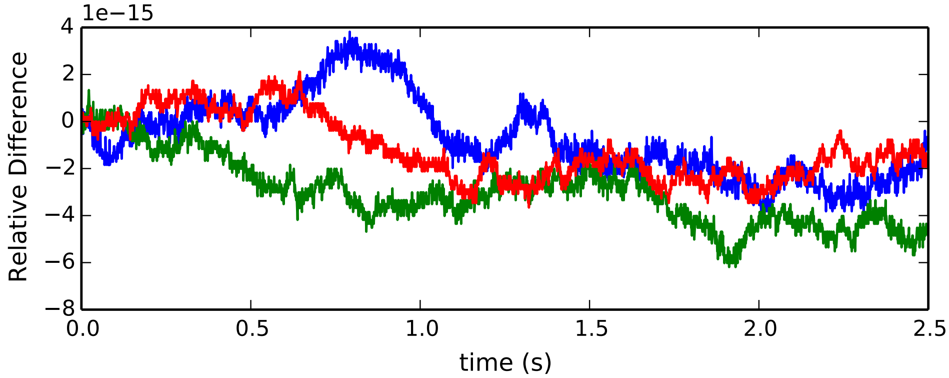

Fig. 8: Change In Rotational Angular Momentum No Gravity

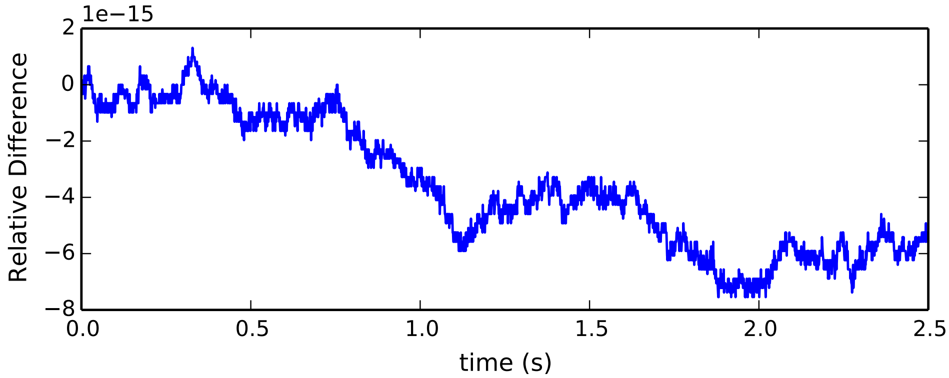

Fig. 9: Change In Rotational Energy No Gravity

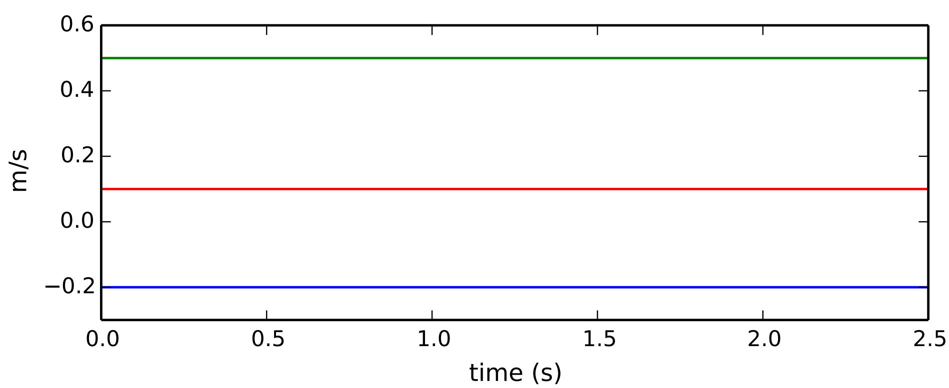

Fig. 10: Velocity Of Center Of Mass No Gravity

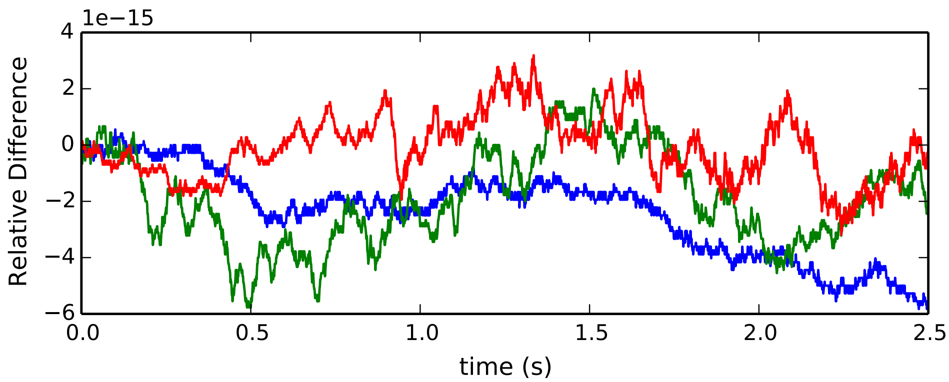

Fig. 11: Change In Velocity Of Center Of Mass No Gravity

6.3 No Gravity with damping scenario

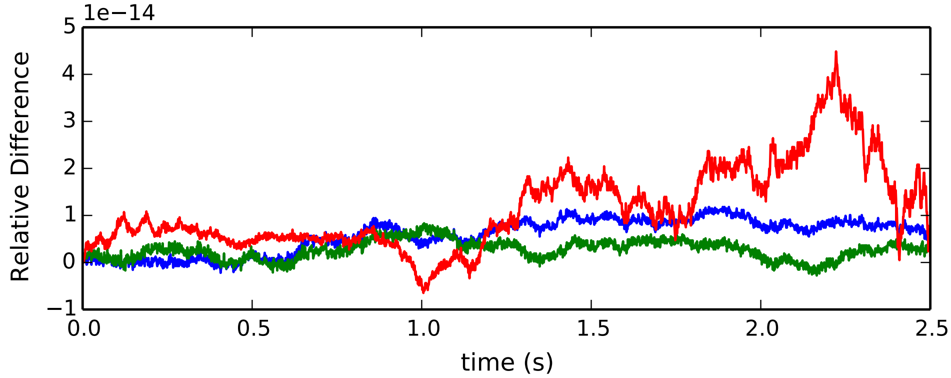

Fig. 12: Change in Orbital Angular Momentum No Gravity with Damping

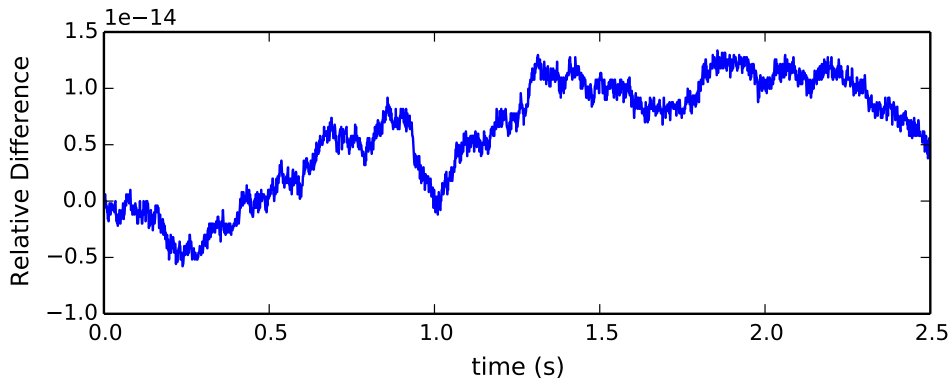

Fig. 13: Change in Orbital Energy No Gravity with Damping

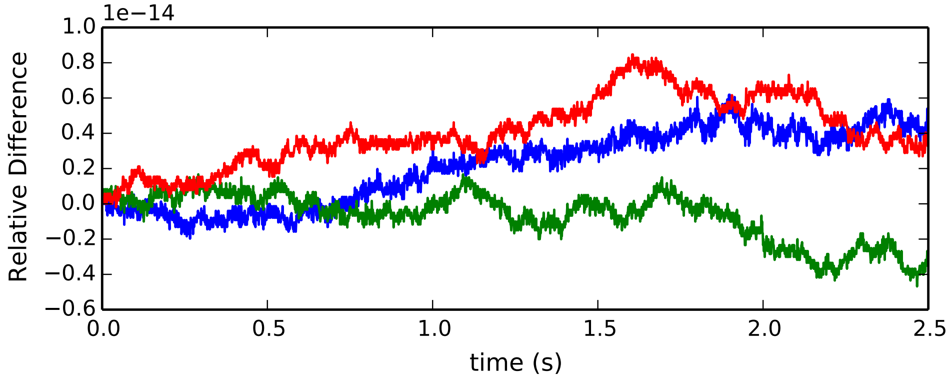

Fig. 14: Change In Rotational Angular Momentum No Gravity with Damping

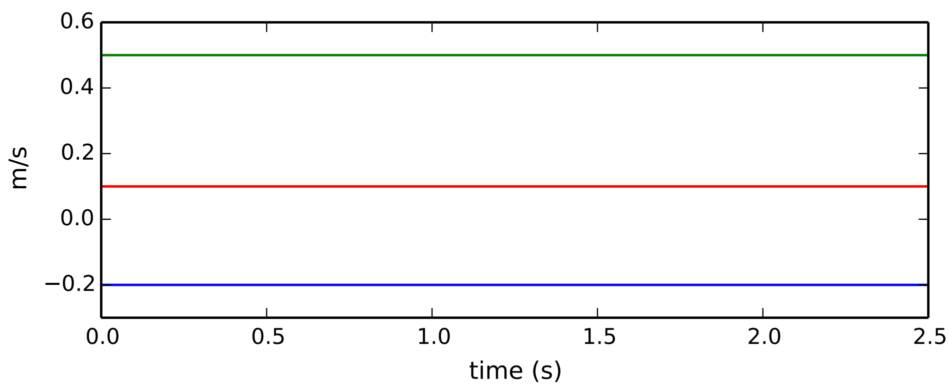

Fig. 15: Velocity Of Center Of Mass No Gravity with Damping

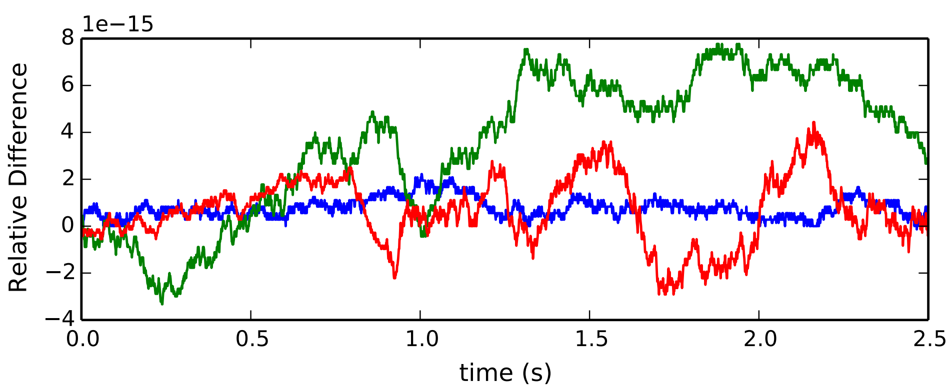

Fig. 16: Change In Velocity Of Center Of Mass No Gravity with Damping

7 User Guide

This section is to outline the steps needed to setup a Hinged Rigid Body State Effector in python using Basilisk.

1. Import the hingedRigidBodyStateEffector class:
   import hingedRigidBodyStateEffector
2. Create an instantiation of a Hinged Rigid body:
   panel1 = hingedRigidBodyStateEffector.HingedRigidBodyStateEffector()
3. Define all physical parameters for a Hinged Rigid Body. For example:
   IPntS_S = [[100.0, 0.0, 0.0], [0.0, 50.0, 0.0], [0.0, 0.0, 50.0]] Do this for all of the parameters for a Hinged Rigid Body seen in the Hinged Rigid Body 1 Parameters Table.
4. Define the initial conditions of the states:
   *panel1.thetaInit = 5*numpy.pi/180.0   panel1.thetaDotInit = 0.0*
5. Define a unique name for each state:
   panel1.nameOfThetaState = "hingedRigidBodyTheta1"   panel1.nameOfThetaDotState = "hingedRigidBodyThetaDot1"
6. Finally, add the panel to your spacecraftPlus:
   scObject.addStateEffector(unitTestSim.panel1). See spacecraftPlus documentation on how to set up a spacecraftPlus object.

REFERENCES

[1] C. Allard, Hanspeter Schaub, and Scott Piggott. General hinged solar panel dynamics approximating first-order spacecraft flexing. In *AAS Guidance and Control Conference*, Breckenridge, CO, Feb. 5--10 2016. Paper No. AAS-16-156.