



Autonomous Vehicle Simulation (AVS) Laboratory

Basilisk Technical Memorandum

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ALGORITHMS TO MAP DESIRED TORQUE VECTOR ONTO A SET OF THRUSTERS

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Scope/Contents
Include a short summary of what this system engineering report is about. Should be 300 words or less.

Rev:	Change Description	By
v0.1	Updated the thruster force evaluation to account for center of mass offsets	H. Schaub
v0.2	Updated the figure and the $[C]$ matrix notation	H. Schaub
v0.3	The thruster mapping logic has changed, and this documentation now reflects what the new algorithm does.	H. Schaub

Contents

1	Introduction	1
2	Simple Thruster Force Algorithm for a Thruster Configuration with Full Torque Control-	2
	ability	
3	Module Parameters	3
3.1	ϵ Parameter	3
3.2	$[C]$ matrix	4
3.3	thrForceSign Parameter	4

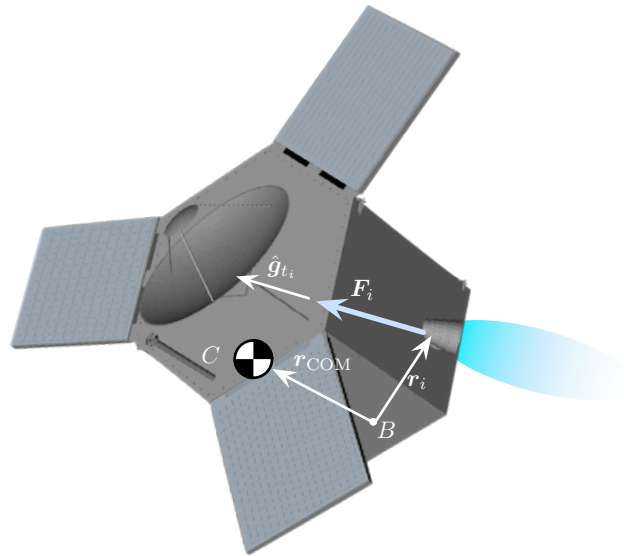


Fig. 1: Illustration of the Spacecraft Thruster Notation

1 Introduction

This technical note describes a general algorithm that maps a desired ADCS external control torque \mathbf{L}_r onto force commands for a cluster of thrusters. Let $\hat{\mathbf{c}}_j$ be the axis about which the thrusters are to produce the desired torque. The module can accept up to 3 orthogonal control axis $\hat{\mathbf{c}}_j$. The j^{th} component of \mathbf{L}_r is given by

$$L_{r,j} = \mathbf{L}_r \cdot \hat{\mathbf{c}}_j \quad (1)$$

The i^{th} thruster location relative to the spacecraft point B is given by \mathbf{r}_i as illustrated in Figure 1. The unit direction vector of the thruster force is $\hat{\mathbf{g}}_{t_i}$, while the thruster force is given by

$$\mathbf{F}_i = F_i \hat{\mathbf{g}}_{t_i} \quad (2)$$

The torque vector produced by each thruster about the body fixed point C is thus

$$\boldsymbol{\tau}_i = (\mathbf{r}_i - \mathbf{r}_{\text{COM}}) \times F_i \hat{\mathbf{g}}_{t_i} \quad (3)$$

The total torque onto the spacecraft about the body fixed axis $\hat{\mathbf{c}}_j$, due to a cluster of N thrusters, is

$$\tau_j = \sum_{i=1}^N \boldsymbol{\tau}_i \cdot \hat{\mathbf{c}}_j = \sum_{i=1}^N ((\mathbf{r}_i - \mathbf{r}_{\text{COM}}) \times \hat{\mathbf{g}}_{t_i}) \cdot \hat{\mathbf{c}}_j F_i = \sum_{i=1}^N d_i F_i \quad (4)$$

where

$$d_i = ((\mathbf{r}_i - \mathbf{r}_{\text{COM}}) \times \hat{\mathbf{g}}_{t_i}) \cdot \hat{\mathbf{c}}_j \quad (5)$$

In matrix form, the net spacecraft torque about the j^{th} axis is written compactly as

$$\tau_j = [d_1 \cdots d_N] \begin{bmatrix} F_1 \\ \vdots \\ F_N \end{bmatrix} = [D] \mathbf{F} \quad (6)$$

where $[D]$ is a $1 \times N$ matrix that maps the thruster forces F_i to the spacecraft torque τ .

2 Simple Thruster Force Algorithm for a Thruster Configuration with Full Torque Controllability

The goal of the thruster force algorithm is to determine a set of thruster forces \mathbf{F} such that the net force onto the spacecraft is

$$\tau_j = \mathbf{L}_r \cdot \hat{\mathbf{c}}_j = [D] \mathbf{F}_j \quad (7)$$

without bleeding torque onto the un-controlled axes.

The following algorithm is applied individually to control the desired torque about each $\hat{\mathbf{c}}_j$ axis. The first step to determine which thruster forces F_i are contributing with the desired sign. If on-pulsing is used for attitude control, then only positive forces are sought. In contrast, if off-pulsing is used to achieve the required control torque, then negative thruster force solutions are sought. Each thruster can only produce a positive force. With off-pulsing, the nominal thrust force plus the negative correction must still yield a non-negative thrust force. The module parameter `thrForceSign` is either $+1$ or -1 to account for the desired force sign. The value of this parameter is represented through s_F .

Using a minimum norm inverse of Eq. (7) yields

$$\mathbf{F}_j = [D]^T ([D][D]^T)^{-1} \mathbf{L}_r \cdot \hat{\mathbf{c}}_j \quad (8)$$

This minimum norm inverse only requires inverting a 1×1 matrix. Using the SVD inverse technique, the value of this 1×1 matrix is the singular value. Thus, if this singular value is below a specified threshold ϵ , the thruster configuration is not contributing to a torque about the $\hat{\mathbf{c}}_j$ axis. In this case the inverse of this matrix is set to zero, and no thruster forces contribute to the desired torque about this axis.

Note that this force stack \mathbf{F} contains both positive and negative force values. Another step is required to ensure that the thrusters can only produce the desired force sign. Assume there are M force values in \mathbf{F}_j with a sign that matches s_F . The locations of these values is provided in the N -dimensional array \mathbf{t}_{used} which contains either 0 or 1 values. For example, consider $N = 8$ and only thrusters 2 and 6 produce forces of the desired sign. In this case we find

$$\mathbf{t}_{\text{used}} = [0 \ 1 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0] \quad (9)$$

This reduces the thruster force search to a subset of M thrusters. Let $\bar{\mathbf{F}}_j$ be a $M \times 1$ matrix of to be determined thruster forces. The corresponding $3 \times M$ mapping matrix $[\bar{D}]$ that projects $\bar{\mathbf{F}}_j$ onto a net body torque about point B is defined as:

$$[\bar{D}] = [\bar{\mathbf{d}}_1 \quad \cdots \quad \bar{\mathbf{d}}_M] \quad (10)$$

with

$$\bar{\mathbf{d}}_i = (\mathbf{r}_i - \mathbf{r}_{\text{COM}}) \times \hat{\mathbf{g}}_i \quad (11)$$

The net torque due to $\bar{\mathbf{F}}_j$ is

$$\bar{\boldsymbol{\tau}}_j = [\bar{D}]\bar{\mathbf{F}}_j \quad (12)$$

To enforce that $\bar{\mathbf{F}}_j$ only produces the desired torque about the $\hat{\mathbf{c}}_j$ axis, and not any torque about other axes, the following condition is established:

$$(\hat{\mathbf{c}}_j \cdot \mathbf{L}_r)\hat{\mathbf{c}}_j = [\bar{D}]\bar{\mathbf{F}}_j \quad (13)$$

If the mapping matrix $[\bar{D}]$ has rank 3, then a minimum norm inverse can be used to determine the smallest set of thruster forces that satisfy Eq. (13).

$$\bar{\mathbf{F}}_j = [\bar{D}]^T([\bar{D}][\bar{D}]^T)^{-1}\hat{\mathbf{c}}_j(\hat{\mathbf{c}}_j \cdot \mathbf{L}_r) \quad (14)$$

The rank condition can easily be checked by computing if the determinant of $[\bar{D}][\bar{D}]^T$ is greater than zero. If yes, a minimum norm inverse can be taken without numerical difficulties.

If the determinant of $[\bar{D}][\bar{D}]^T$ is near zero, then $\bar{\mathbf{F}}_j$ cannot generate a general 3D torque vector. As the spacecraft is setup with pairs of thrusters to produce the control torques, in this case the rank of $[\bar{D}]$ is 2, and not all body axis are influenced by $\bar{\mathbf{F}}_j$. In this case the thruster forces are determined through a least-squares inverse that selects $\bar{\mathbf{F}}_j$ such that the controllable axes satisfy the condition in Eq. (13).

$$\bar{\mathbf{F}}_j = ([\bar{D}]^T[\bar{D}])^{-1}[\bar{D}]^T\hat{\mathbf{c}}_j(\hat{\mathbf{c}}_j \cdot \mathbf{L}_r) \quad (15)$$

The final step is to sum the individual $\bar{\mathbf{F}}_j$ thruster solutions to the yield the net set of thruster forces required to produce \mathbf{L}_r . This is done using the \mathbf{t}_{used} matrix to determine which thrusters have non-zero contributions.

If the thruster cluster configuration is such that pairs of thrusters produce full controllability, then the minimum norm solution to produce the desired \mathbf{L}_r will also result in a thruster solution that produces a net 0 force onto the spacecraft. Using the super-particle theorem,¹ the total thruster force is given by

$$\mathbf{F}_{T,j} = [G_t]\mathbf{F}_j = [G_t][D]^T([D][D])^{-1}\mathbf{L}_r \cdot \hat{\mathbf{c}}_j = \mathbf{0} \quad (16)$$

With a pure-couple thruster configuration the expression satisfies $[G_t][D]^T = \mathbf{0}$.

3 Module Parameters

3.1 ϵ Parameter

The minimum norm inverse in Eq. (8) requires a non-zero value of $[D][D]^T$. For this setup, this matrix is a scalar value

$$D_2 = [D][D]^T \quad (17)$$

The d_i matrix components are given in Eq. (5). Using the robust SVD inverse technique, $D_2 > \epsilon$, then the $1/D_2$ math is evaluated as normal. However, if $D_2 < \epsilon$, then the inverse $1/D_2$ is set to zero. In the latter case there is no control authority about the current axis of interest. To set this epsilon parameter,

not the definition of the $[D]$ matrix components $d_i = (\mathbf{r}_i \times \hat{\mathbf{g}}_{t_i}) \cdot \hat{\mathbf{c}}_j$. Note that $\mathbf{r}_i \times \hat{\mathbf{g}}_{t_i}$ is a scaled axis along which the i^{th} thruster can produce a torque. The value d_i will be near zero if the dot product of this axis with the current control axis $\hat{\mathbf{c}}_j$ is small.

To determine an appropriate ϵ value, let α be the minimum desired angle to avoid the control axis $\hat{\mathbf{c}}_j$ and the scaled thruster torque axis $\mathbf{r}_i \times \hat{\mathbf{g}}_{t_i}$ being orthogonal. If \bar{r} is a mean distance of the thrusters to the spacecraft center of mass, then the d_i values must satisfy

$$\frac{d_i}{\bar{r}} > \cos(90^\circ - \alpha) = \sin \alpha \quad (18)$$

Thus, to estimate a good value of ϵ , the following formula can be used

$$\epsilon \approx d_i^2 = \sin^2 \alpha \bar{r}^2 \quad (19)$$

For example, if $\bar{r} = 1.3$ meters, and we want α to be at least 1° , then we would set $\epsilon = 0.000515$.

3.2 $[C]$ matrix

The module requires control control axis matrix $[C]$ to be defined. Up to 3 orthogonal control axes can be selected. Let M be the number of control axes. The $M \times 3$ $[C]$ matrix is then defined as

$$[C] = \begin{bmatrix} \hat{\mathbf{c}}_1 \\ \vdots \end{bmatrix} \quad (20)$$

Not that in python the matrix is given in a 1D form by defining `controlAxes_B`. Thus, the $\hat{\mathbf{c}}_j$ axes are concatenated to produce the input matrix $[C]$.

3.3 thrForceSign Parameter

Before this module can be run, the parameter `thrForceSign` must be set to either $+1$ (on-pulsing) or -1 (off-pulsing).

REFERENCES

- [1] Hanspeter Schaub and John L. Junkins. *Analytical Mechanics of Space Systems*. AIAA Education Series, Reston, VA, 3rd edition, 2014.