Autonomous Vehicle Simulation (AVS) Laboratory

AVS-Sim Technical Memorandum

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PRV STEERING ADCS CONTROL MODULE

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Status: Initial Documentation
Scope/Contents
This module uses the PRV Steering control logic to determine the ADCS control torque vector $\boldsymbol{L}_r$.

Rev:	**Change Description**	**By**
Draft	Initial Documentation Draft	H. Schaub

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1 Initialization

Simply call the module reset function prior to using this control module. This will reset the prior function call time variable, and reset the attitude error integral measure. The control update period Δt is evaluated automatically.

2 Steering Law Goals

This technical note develops a new MRP based steering law that drives a body frame $\mathcal{B} : \{\hat{\boldsymbol{b}}_1, \hat{\boldsymbol{b}}_2, \hat{\boldsymbol{b}}_3\}$ towards a time varying reference frame $\mathcal{R} : \{\hat{\boldsymbol{r}}_1, \hat{\boldsymbol{r}}_2, \hat{\boldsymbol{r}}_3\}$. The inertial frame is given by $\mathcal{N} : \{\hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}_2, \hat{\boldsymbol{n}}_3\}$. The RW coordinate frame is given by $\mathcal{W}_\rangle : \{\hat{\boldsymbol{g}}_{s_i}, \hat{\boldsymbol{g}}_{t_i}, \hat{\boldsymbol{g}}_{g_i}\}$. The Using MRPs, the overall control goal is

$$\boldsymbol{\sigma}_{\mathcal{B}/\mathcal{R}} \rightarrow 0 \tag{1}$$

The reference frame orientation $\boldsymbol{\sigma}_{\mathcal{R}/\mathcal{N}}$, angular velocity $\boldsymbol{\omega}_{\mathcal{R}/\mathcal{N}}$ and inertial angular acceleration $\dot{\boldsymbol{\omega}}_{\mathcal{R}/\mathcal{N}}$ are assumed to be known.

The rotational equations of motion of a rigid spacecraft with N Reaction Wheels (RWs) attached are given by[1]

$$[I_{RW}]\dot{\boldsymbol{\omega}} = -[\tilde{\boldsymbol{\omega}}]\left([I_{RW}]\boldsymbol{\omega} + [G_s]\boldsymbol{h}_s\right) - [G_s]\boldsymbol{u}_s + \boldsymbol{L} \tag{2}$$

where the inertia tensor $[I_{RW}]$ is defined as

$$[I_{RW}] = [I_s] + \sum_{i=1}^{N}\left(J_{t_i}\hat{\boldsymbol{g}}_{t_i}\hat{\boldsymbol{g}}_{t_i}^T + J_{g_i}\hat{\boldsymbol{g}}_{g_i}\hat{\boldsymbol{g}}_{g_i}^T\right) \tag{3}$$

The spacecraft inertial without the N RWs is $[I_s]$, while J_{s_i}, J_{t_i} and J_{g_i} are the RW inertias about the body fixed RW axis $\hat{\boldsymbol{g}}_{s_i}$ (RW spin axis), $\hat{\boldsymbol{g}}_{t_i}$ and $\hat{\boldsymbol{g}}_{g_i}$. The $3 \times N$ projection matrix $[G_s]$ is then defined as

$$[G_s] = \left[\cdots {}^{\mathcal{B}}\hat{\boldsymbol{g}}_{s_i} \cdots\right] \tag{4}$$

The RW inertial angular momentum vector $\boldsymbol{h}_s$ is defined as

$$h_{s_i} = J_{s_i}(\omega_{s_i} + \Omega_i) \tag{5}$$

Here Ω_i is the i^{th} RW spin relative to the spacecraft, and the body angular velocity is written in terms of body and RW frame components as

$$\boldsymbol{\omega} = \omega_1\hat{\boldsymbol{b}}_1 + \omega_2\hat{\boldsymbol{b}}_2 + \omega_3\hat{\boldsymbol{b}}_3 = \omega_{s_i}\hat{\boldsymbol{g}}_{s_i} + \omega_{t_i}\hat{\boldsymbol{g}}_{t_i} + \omega_{g_i}\hat{\boldsymbol{g}}_{g_i} \tag{6}$$

3 Steering Law

3.1 Steering Law Stability Requirement

As is commonly done in robotic applications where the steering laws are of the form $\dot{\boldsymbol{x}} = \boldsymbol{u}$, this section derives a kinematic based attitude steering law. Let us consider the simple Lyapunov candidate function[1,2]

$$V(\boldsymbol{\sigma}_{\mathcal{B}/\mathcal{R}}) = 2\ln\left(1 + \boldsymbol{\sigma}_{\mathcal{B}/\mathcal{R}}^T \boldsymbol{\sigma}_{\mathcal{B}/\mathcal{R}}\right) \tag{7}$$

in terms of the MRP attitude tracking error $\boldsymbol{\sigma}_{\mathcal{B}/\mathcal{R}}$. Using the MRP differential kinematic equations

$$\dot{\boldsymbol{\sigma}}_{\mathcal{B}/\mathcal{R}} = \frac{1}{4}[B(\boldsymbol{\sigma}_{\mathcal{B}/\mathcal{R}})]^{\mathcal{B}}\boldsymbol{\omega}_{\mathcal{B}/\mathcal{R}} = \frac{1}{4}\left[(1-\sigma_{\mathcal{B}/\mathcal{R}}^2)[I_{3\times3} + 2[\tilde{\boldsymbol{\sigma}}_{\mathcal{B}/\mathcal{R}}] + 2\boldsymbol{\sigma}_{\mathcal{B}/\mathcal{R}}\boldsymbol{\sigma}_{\mathcal{B}/\mathcal{R}}^T\right]{}^{\mathcal{B}}\boldsymbol{\omega}_{\mathcal{B}/\mathcal{R}} \tag{8}$$

where $\sigma_{\mathcal{B}/\mathcal{R}}^2 = \boldsymbol{\sigma}_{\mathcal{B}/\mathcal{R}}^T\boldsymbol{\sigma}_{\mathcal{B}/\mathcal{R}}$, the time derivative of V is

$$\dot{V} = \boldsymbol{\sigma}_{\mathcal{B}/\mathcal{R}}^T\left({}^{\mathcal{B}}\boldsymbol{\omega}_{\mathcal{B}/\mathcal{R}}\right) \tag{9}$$

To create a kinematic steering law, let $\mathcal{B}^*$ be the desired body orientation, and $\boldsymbol{\omega}_{\mathcal{B}^*/\mathcal{R}}$ be the desired angular velocity vector of this body orientation relative to the reference frame $\mathcal{R}$. The steering law requires an algorithm for the desired body rates $\boldsymbol{\omega}_{\mathcal{B}^*/\mathcal{R}}$ relative to the reference frame make $\dot{V}$ in Eq. (9) negative definite. For this purpose, let us select

$${}^{\mathcal{B}}\boldsymbol{\omega}_{\mathcal{B}^*/\mathcal{R}} = -\boldsymbol{f}(\boldsymbol{\sigma}_{\mathcal{B}/\mathcal{R}}) \tag{10}$$

where $\boldsymbol{f}(\boldsymbol{\sigma})$ is an even function such that

$$\boldsymbol{\sigma}^T\boldsymbol{f}(\boldsymbol{\sigma}) > 0 \tag{11}$$

The Lyapunov rate simplifies to the negative definite expression:

$$\dot{V} = -\boldsymbol{\sigma}_{\mathcal{B}/\mathcal{R}}^T\boldsymbol{f}(\boldsymbol{\sigma}_{\mathcal{B}/\mathcal{R}}) < 0 \tag{12}$$

3.2 Principal Angle Steering Law

Consider the following saturation function $\boldsymbol{f}(\boldsymbol{\sigma}_{\mathcal{B}/\mathcal{R}})$ which is colinear with the principal rotation axis $\hat{\boldsymbol{e}}_{\mathcal{B}/\mathcal{R}}$, and the magnitude scales uniformly with the principal rotation angle $\phi_{\mathcal{B}/\mathcal{R}}$:

$$\boldsymbol{f}(\boldsymbol{\sigma}_{\mathcal{B}/\mathcal{R}}) = \hat{\boldsymbol{e}}_{\mathcal{B}/\mathcal{R}} f(\phi_{\mathcal{B}/\mathcal{R}}) \tag{13}$$

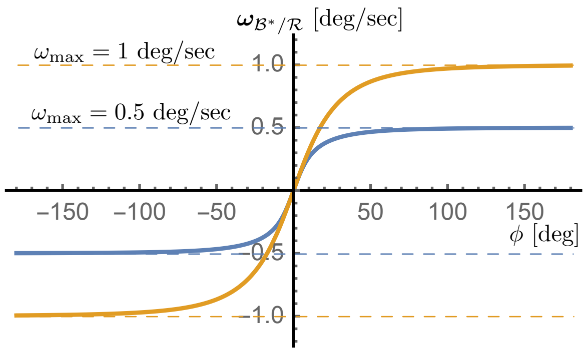
The scalar function $f(\phi_{\mathcal{B}/\mathcal{R}})$ is an even function where $f(\phi_{\mathcal{B}/\mathcal{R}})\phi_{\mathcal{B}/\mathcal{R}} \geqslant 0$. Note that $\hat{\boldsymbol{e}}_{\mathcal{B}/\mathcal{R}}$ is ill-defined for a zero principal rotation angle. This saturation function will need a check to avoid numerical issues right at the zero angle condition.

Using the MRP definition in terms of principal rotation angle and axis

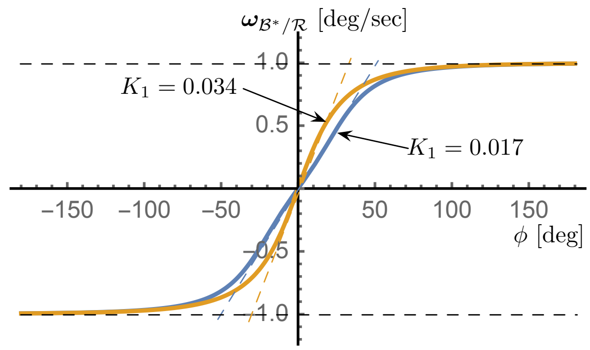
$$\boldsymbol{\sigma}_{\mathcal{B}/\mathcal{R}} = \hat{\boldsymbol{e}}_{\mathcal{B}/\mathcal{R}} \tan\left(\frac{\phi_{\mathcal{B}/\mathcal{R}}}{4}\right) \tag{14}$$

and substituting into Eq. (12), the Lyapunov rate is for this case

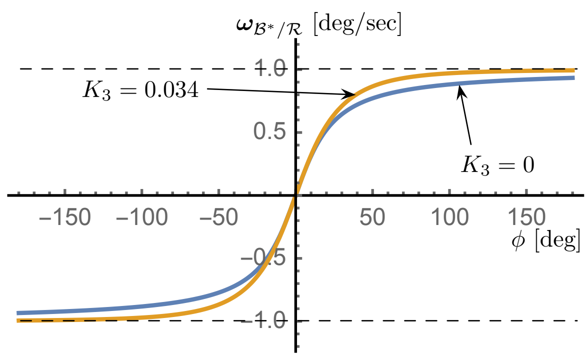
$$\dot{V} = -\boldsymbol{\sigma}_{\mathcal{B}/\mathcal{R}}^T\boldsymbol{f}(\boldsymbol{\sigma}_{\mathcal{B}/\mathcal{R}}) = -\tan\left(\frac{\phi_{\mathcal{B}/\mathcal{R}}}{4}\right)\hat{\boldsymbol{e}}_{\mathcal{B}/\mathcal{R}}^T\hat{\boldsymbol{e}}_{\mathcal{B}/\mathcal{R}} f(\phi_{\mathcal{B}/\mathcal{R}}) = -\tan\left(\frac{\phi_{\mathcal{B}/\mathcal{R}}}{4}\right) f(\phi_{\mathcal{B}/\mathcal{R}}) < 0 \tag{15}$$



(a) ω_{max} dependency with $K_1 = 0.034$, $K_3 = 0.034$



(b) K_1 dependency with $\omega_{\text{max}} = 1^\circ/s$, $K_3 = 0.034$



(c) K_3 dependency with $\omega_{\text{max}} = 1^\circ/s$, $K_1 = 0.034$

Fig. 1: Illustrations of Principal Angle Steering Parameters Influence.

This $\dot{V}$ is negative definite in terms of the attitude error, thus yielding asymptotic convergence. The saturation function in Eq. (13) has the convenient property that the resulting steering law employs an eigenaxis rotation towards the desired reference orientation. In contrast, the smoothing function in Eq. (**??**) saturates the MRP components individually, deviating fro man eigen-axis rotation. The benefit of the later is that small errors are reduced quickly, thus reducing the overall tracking error quicker. The benefit of the eigenaxis approach is that the closed-loop attitude path towards the reference is more predictable.

Consider the $f(\phi_{\mathcal{B}/\mathcal{R}})$ function given by:

$$f(\phi_{\mathcal{B}/\mathcal{R}}) = \arctan\left((K_1\phi_{\mathcal{B}/\mathcal{R}} + K_3\phi^3_{\mathcal{B}/\mathcal{R}})\frac{\pi}{2\omega_{\text{max}}}\right)\frac{2\omega_{\text{max}}}{\pi} \tag{16}$$

The linear approximation of this function is

$$f(\phi_{\mathcal{B}/\mathcal{R}}) \approx K_1\phi_{\mathcal{B}/\mathcal{R}} + \text{ H.O.T} \tag{17}$$

The resulting attitude steering law is of the form:

$$\boldsymbol{\omega}_{\mathcal{B}*/\mathcal{R}} = -\hat{\boldsymbol{e}}_{\mathcal{B}/\mathcal{R}}\arctan\left((K_1\phi_{\mathcal{B}/\mathcal{R}} + K_3\phi^3_{\mathcal{B}/\mathcal{R}})\frac{\pi}{2\omega_{\text{max}}}\right)\frac{2\omega_{\text{max}}}{\pi} \tag{18}$$

The impacts of the steering law gains ω_{max}, K_1 and K_3 are illustrated in Figure 1. As the function $f(\phi)$ returns a value of $\pm\omega_{\text{max}}$ as $\phi \to \infty$, the gain ω_{max} determines the maximum rate limit that the steering law with request. This is illustrated in Figure 1(a) where reducing the ω_{max} value by a factor of 2 results in half of the asymptotic rate command.

The parameter K_1 determines the final pointing stiffness, and determines the final exponential convergence of the attitude pointing error as illustrated in Figure 1(b). Increasing this value results in faster final convergence once the principal rotation error has reduced past the saturated $f(\phi)$ function region.

Finally, the higher order ϕ polynomial is provided to cause the $f(\phi)$ function to saturate more quickly. Setting $K_3 = 0.034$ in Figure 1(c) doesn't change the initial slope of the rate command, but impacts how quickly the rates saturate on the maximum speed command.

Because $\hat{\boldsymbol{e}}'_{\mathcal{B}/\mathcal{R}} = 0$, the body relative time derivative of the steering control is

$$\frac{{}^{\mathcal{B}}\mathrm{d}({}^{\mathcal{B}}\boldsymbol{\omega}_{\mathcal{B}*/\mathcal{R}})}{\mathrm{d}t} = \boldsymbol{\omega}'_{\mathcal{B}*/\mathcal{R}} = -\hat{\boldsymbol{e}}_{\mathcal{B}/\mathcal{R}} \frac{\partial f(\phi_{\mathcal{B}/\mathcal{R}})}{\partial \phi_{\mathcal{B}/\mathcal{R}}} \dot{\phi}_{\mathcal{B}/\mathcal{R}} \tag{19}$$

The $f()$ function sensitivity is

$$\frac{\partial f}{\partial \phi_{\mathcal{B}/\mathcal{R}}} = \frac{(K_1 + 3K_3\phi^2_{\mathcal{B}/\mathcal{R}})}{1 + (K_1\phi_{\mathcal{B}/\mathcal{R}} + K_3\phi^3_{\mathcal{B}/\mathcal{R}})^2 \left(\frac{\pi}{2\omega_{\text{max}}}\right)^2} \tag{20}$$

The principal rotation angle time derivative is given by[3]

$$\dot{\phi}_{\mathcal{B}/\mathcal{R}} = \hat{\boldsymbol{e}}^T_{\mathcal{B}/\mathcal{R}} {}^{\mathcal{B}}\boldsymbol{\omega}_{\mathcal{B}*/\mathcal{R}} \tag{21}$$

Substituting Eqs. (10) and (21) into Eq. (19) yields

$$\begin{aligned}\boldsymbol{\omega}'_{\mathcal{B}*/\mathcal{R}} &= -\hat{\boldsymbol{e}}_{\mathcal{B}/\mathcal{R}} \frac{\partial f(\phi_{\mathcal{B}/\mathcal{R}})}{\partial \phi_{\mathcal{B}/\mathcal{R}}} \left(\hat{\boldsymbol{e}}^T_{\mathcal{B}/\mathcal{R}} {}^{\mathcal{B}}\boldsymbol{\omega}_{\mathcal{B}*/\mathcal{R}}\right) \\ &= -\hat{\boldsymbol{e}}_{\mathcal{B}/\mathcal{R}} \frac{\partial f(\phi_{\mathcal{B}/\mathcal{R}})}{\partial \phi_{\mathcal{B}/\mathcal{R}}} \left(\hat{\boldsymbol{e}}^T_{\mathcal{B}/\mathcal{R}}(-\hat{\boldsymbol{e}}_{\mathcal{B}/\mathcal{R}} f(\phi_{\mathcal{B}/\mathcal{R}}))\right) \\ &= \hat{\boldsymbol{e}}_{\mathcal{B}/\mathcal{R}} \frac{\partial f(\phi_{\mathcal{B}/\mathcal{R}})}{\partial \phi_{\mathcal{B}/\mathcal{R}}} f(\phi_{\mathcal{B}/\mathcal{R}}))\end{aligned} \tag{22}$$

4 Angular Velocity Servo Sub-System

To implement the kinematic steering control, a servo sub-system must be included which will produce the required torques to make the actual body rates track the desired body rates. The angular velocity tracking error vector is defined as

$$\delta\boldsymbol{\omega} = \boldsymbol{\omega}_{\mathcal{B}/\mathcal{B}*} = \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} - \boldsymbol{\omega}_{\mathcal{B}*/\mathcal{N}} \tag{23}$$

where the $\mathcal{B}^*$ frame is the desired body frame from the kinematic steering law. Note that

$$\boldsymbol{\omega}_{\mathcal{B}*/\mathcal{N}} = \boldsymbol{\omega}_{\mathcal{B}*/\mathcal{R}} + \boldsymbol{\omega}_{\mathcal{R}/\mathcal{N}} \tag{24}$$

where $\boldsymbol{\omega}_{\mathcal{R}/\mathcal{N}}$ is obtained from the attitude navigation solution, and $\boldsymbol{\omega}_{\mathcal{B}*/\mathcal{R}}$ is the kinematic steering rate command. To create a rate-servo system that is robust to unmodeld torque biases, the state $\boldsymbol{z}$ is defined as:

$$\boldsymbol{z} = \int_{t_0}^{t_f} {}^{\mathcal{B}}\delta\boldsymbol{\omega}\ \mathrm{d}t \tag{25}$$

The rate servo Lyapunov function is defined as

$$V_{\boldsymbol{\omega}}(\delta\boldsymbol{\omega}, \boldsymbol{z}) = \frac{1}{2}\delta\boldsymbol{\omega}^T [I_{\text{RW}}]\delta\boldsymbol{\omega} + \frac{1}{2}\boldsymbol{z}^T [K_I]\boldsymbol{z} \tag{26}$$

where the vector $\delta\boldsymbol{\omega}$ and tensor $[I_{\text{RW}}]$ are assumed to be given in body frame components, $[K_i]$ is a symmetric positive definite matrix. The time derivative of this Lyapunov function is

$$\dot{V}_{\boldsymbol{\omega}} = \delta\boldsymbol{\omega}^T \left([I_{\text{RW}}]\delta\boldsymbol{\omega}' + [K_I]\boldsymbol{z}\right) \tag{27}$$

Using the identities $\boldsymbol{\omega}'_{\mathcal{B}/\mathcal{N}} = \dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}$ and $\boldsymbol{\omega}'_{\mathcal{R}/\mathcal{N}} = \dot{\boldsymbol{\omega}}_{\mathcal{R}/\mathcal{N}} - \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \boldsymbol{\omega}_{\mathcal{R}/\mathcal{N}}$,[1] the body frame derivative of $\delta\boldsymbol{\omega}$ is

$$\delta\boldsymbol{\omega}' = \dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} - \boldsymbol{\omega}'_{\mathcal{B}*/\mathcal{R}} - \dot{\boldsymbol{\omega}}_{\mathcal{R}/\mathcal{N}} + \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \boldsymbol{\omega}_{\mathcal{R}/\mathcal{N}} \tag{28}$$

Substituting Eqs. (2) and (28) into the $\dot{V}_{\boldsymbol{\omega}}$ expression in Eq. (27) yields

$$\begin{aligned}\dot{V}_{\boldsymbol{\omega}} = \delta\boldsymbol{\omega}^T \Big(& - [\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}] \left([I_{RW}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} + [G_s]\boldsymbol{h}_s\right) - [G_s]\boldsymbol{u}_s + \boldsymbol{L} + [K_I]\boldsymbol{z} \\ & - [I_{\text{RW}}](\boldsymbol{\omega}'_{\mathcal{B}*/\mathcal{R}} + \dot{\boldsymbol{\omega}}_{\mathcal{R}/\mathcal{N}} - \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \boldsymbol{\omega}_{\mathcal{R}/\mathcal{N}})\Big)\end{aligned} \tag{29}$$

Let $[P]^T = [P] >$ be a symmetric positive definite rate feedback gain matrix. The servo rate feedback control is defined as

$$\begin{aligned}[G_s]\boldsymbol{u}_s = [P]\delta\boldsymbol{\omega} + [K_I]\boldsymbol{z} - [\tilde{\boldsymbol{\omega}}_{\mathcal{B}*/\mathcal{N}}] \left([I_{\text{RW}}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} + [G_s]\boldsymbol{h}_s\right) \\ - [I_{\text{RW}}](\boldsymbol{\omega}'_{\mathcal{B}*/\mathcal{R}} + \dot{\boldsymbol{\omega}}_{\mathcal{R}/\mathcal{N}} - \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \boldsymbol{\omega}_{\mathcal{R}/\mathcal{N}}) + \boldsymbol{L}\end{aligned} \tag{30}$$

Defining the right-hand-side as $\boldsymbol{L}_r$, this is rewritten in compact form as

$$[G_s]\boldsymbol{u}_s = \boldsymbol{L}_r \tag{31}$$

The array of RW motor torques can be solved with the typical minimum norm inverse

$$\boldsymbol{u}_s = [G_s]^T \left([G_s][G_s]^T\right)^{-1} \boldsymbol{L}_r \tag{32}$$

To analyze the stability of this rate servo control, the $[G_s]\boldsymbol{u}_s$ expression in Eq. (30) is substituted into the Lyapunov rate expression in Eq. (29).

$$\begin{aligned}\dot{V}_{\omega} &= \delta\boldsymbol{\omega}^T \Big(- [P]\delta\boldsymbol{\omega} - [\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}] \left([I_{RW}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} + [G_s]\boldsymbol{h}_s\right) + [\tilde{\boldsymbol{\omega}}_{\mathcal{B}*/\mathcal{N}}] \left([I_{\text{RW}}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} + [G_s]\boldsymbol{h}_s\right) \Big) \\ &= \delta\boldsymbol{\omega}^T \Big(- [P]\delta\boldsymbol{\omega} - [\widetilde{\delta\boldsymbol{\omega}}] \left([I_{RW}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} + [G_s]\boldsymbol{h}_s\right) \Big) \\ &= -\delta\boldsymbol{\omega}^T[P]\delta\boldsymbol{\omega} < 0\end{aligned} \tag{33}$$

Thus, in the absence of unmodeled torques, the servo control in Eq. (30) is asymptotically stabilizing in rate tracking error $\delta\boldsymbol{\omega}$.

Next, the servo robustness to unmodeled external torques is investigated. Let us assume that the external torque vector $\boldsymbol{L}$ in Eq. (2) only approximates the true external torque, and the unmodeled component is given by $\Delta\boldsymbol{L}$. Substituting the true equations of motion and the same servo control in Eq. (30) into the Lyapunov rate expression in Eq. (27) leads to

$$\dot{V}_{\omega} = -\delta\boldsymbol{\omega}^T[P]\delta\boldsymbol{\omega} - \delta\boldsymbol{\omega}^T\Delta\boldsymbol{L} \tag{34}$$

This $\dot{V}_{\omega}$ is no longer negative definite due to the underdetermined sign of the $\delta\boldsymbol{\omega}^T\Delta\boldsymbol{L}$ components. Equating the Lyapunov rates in Eqs. (27) and (34) yields the following servo closed loop dynamics:

$$[I_{\text{RW}}]\delta\boldsymbol{\omega}' + [P]\delta\boldsymbol{\omega} + [K_I]\boldsymbol{z} = \Delta\boldsymbol{L} \tag{35}$$

Assuming that $\Delta\boldsymbol{L}$ is either constant as seen by the body frame, or at least varies slowly, then taking a body-frame time derivative of Eq. (35) is

$$[I_{\text{RW}}]\delta\boldsymbol{\omega}'' + [P]\delta\boldsymbol{\omega}' + [K_I]\delta\boldsymbol{\omega} = \Delta\boldsymbol{L}' \approx 0 \tag{36}$$

As $[I_{\text{RW}}]$, $[P]$ and $[K_I]$ are all symmetric positive definite matrices, these linear differential equations are stable, and $\delta\boldsymbol{\omega} \to 0$ given that assumption that $\Delta\boldsymbol{L}' \approx 0$.

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