Autonomous Vehicle Simulation (AVS) Laboratory

Basilisk Technical Memorandum

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ALGORITHMS TO MAP DESIRED TORQUE VECTOR ONTO A SET OF THRUSTERS

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Scope/Contents
Include a short summary of what this system engineering report is about. Should be 300 words or less.

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Contents

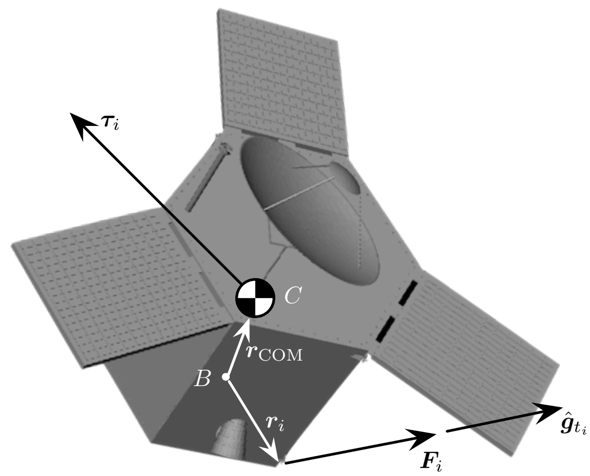

Fig. 1: Illustration of the Spacecraft Thruster Notation

1 Introduction

This technical note describes a general algorithm that maps a desired ADCS external control torque $\boldsymbol{L}_r$ onto force commands for a cluster of thrusters. The body fixed frame is given by $\mathcal{B} : \{\hat{\boldsymbol{b}}_1, \hat{\boldsymbol{b}}_2, \hat{\boldsymbol{b}}_3\}$. The j^{th} component of $\boldsymbol{L}_r$ is given by

$$L_{r,j} = \boldsymbol{L}_r \cdot \hat{\boldsymbol{b}}_j \tag{1}$$

The i^{th} thruster location relative to the spacecraft point B is given by $\boldsymbol{r}_i$ as illustrated in Figure 1. The unit direction vector of the thruster force is $\hat{\boldsymbol{g}}_{t_i}$, while the thruster force is given by

$$\boldsymbol{F}_i = F_i \hat{\boldsymbol{g}}_{t_i} \tag{2}$$

The toque vector produced by each thruster about the body fixed point B is thus

$$\boldsymbol{\tau}_i = (\boldsymbol{r}_i - \boldsymbol{r}_{\text{COM}}) \times F_i \hat{\boldsymbol{g}}_{t_i} \tag{3}$$

The total torque onto the spacecraft about the j^{th} body fixed axis, due to a cluster of N thrusters, is

$$\tau_j = \sum_{i=1}^{N} \boldsymbol{\tau}_i \cdot \hat{\boldsymbol{b}}_j = \sum_{i=1}^{N} ((\boldsymbol{r}_i - \boldsymbol{r}_{\text{COM}}) \times \hat{\boldsymbol{g}}_{t_i}) \cdot \hat{\boldsymbol{b}}_j F_i = \sum_{i=1}^{N} d_i F_i \tag{4}$$

where

$$d_i = ((\boldsymbol{r}_i - \boldsymbol{r}_{\text{COM}}) \times \hat{\boldsymbol{g}}_{t_i}) \cdot \hat{\boldsymbol{b}}_j \tag{5}$$

In matrix form, the net spacecraft torque about the j^{th} axis is written compactly as

$$\tau_j = \begin{bmatrix} d_1 \cdots d_N \end{bmatrix} \begin{bmatrix} F_1 \\ \vdots \\ F_N \end{bmatrix} = [D]\boldsymbol{F} \tag{6}$$

where $[D]$ is a $1 \times N$ matrix that maps the thruster forces F_i to the spacecraft torque τ.

The thruster control goal is to find a set of thruster forces $\boldsymbol{F}$ such that

$${}^{\mathcal{B}}\boldsymbol{L}_r = {}^{\mathcal{B}}\begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} \tag{7}$$

2 Simple Thruster Force Algorithm for a Thruster Configuration with Pure Couples

The goal of the thruster force algorithm is to determine a set of thruster forces $\boldsymbol{F}$ such that

$$\tau_j = \boldsymbol{L}_r \cdot \hat{\boldsymbol{b}}_j = [D]\boldsymbol{F} \tag{8}$$

The nest step to determine thruster forces F_i is to determine which thrusters are contributing to a positive torque. Using a minimum norm inverse of Eq. (8) yields

$$\boldsymbol{F}_j = [D]^T \left([D][D]^T\right)^{-1} \boldsymbol{L}_r \cdot \hat{\boldsymbol{b}}_j \tag{9}$$

This minimum norm inverse only requires inverting a 1×1 matrix. Using the SVD inverse technique, the value of this 1×1 matrix is the singular value. Thus, if this singular value is below a specified threshold, the thruster configuration is not contributing to a torque about the $\hat{\boldsymbol{b}}_j$ axis. In this case the inverse of this matrix is set to zero, and not thruster forces contribute to the desired torque about this axis. An common example of such a scenario is a cluster of Δv thruster that are all pointing in the same direction. Here off-pulsing can be used to impart differential forces, and thus torques, to rotate the spacecraft. However, such a configuration is not capable of producing torques about the Δv thrust axis.

Note that this force stack $\boldsymbol{F}$ contains both positive and negative force values. As the thruster can only produce positive forces, another step is required that computes the thruster forces subject to $F_i > 0$. The inverse in Eq. (9) determines which thrusters require a positive force to achieve $\boldsymbol{L}_r$. Because the thruster configuration is such that pure couples are produced, the following simple logic in Algoritm 1 enforces this $F_i > 0$ constraint. Here the individual thruster values are either doubled, or set to zero, depending on their sign. This process is then repeated for all three body axes $\hat{\boldsymbol{b}}_j$, and the net set of thruster forces is

$$\boldsymbol{F} = \sum_{j=1}^{3} \boldsymbol{F}_j \tag{10}$$

If the thruster cluster configuration is symmetric and aligned such that it produces pure torques, then the minimum norm solution to produce the desired $\boldsymbol{L}_r$ will also result in a thruster solution that produces a net 0 force onto the spacecraft. Using the super-particle theorem,[1] the total thruster force is given by

$$\boldsymbol{F}_{T,j} = [G_t]\boldsymbol{F}_j = [G_t][D]^T([D][D])^{-1}\boldsymbol{L}_r \cdot \hat{\boldsymbol{b}}_j = \boldsymbol{0} \tag{11}$$

With a pure-couple thruster configuration the expression satisfies $[G_t][D]^T = \boldsymbol{0}$.

$i = 1$
while $i \leqslant 3$ **do**
  if $F_i > 0$ **then**
    $F_i * = 2$
  else
    $F_i = 0$
  end if
  $i + = 1$
end while

Algorithm 1: Logic to Enforce $F_i > 0$ with the Simple Thruster Force Algorithm

3 Module Parameters

3.1 ϵ Parameter

The minimum norm inverse in Eq. (9) requires a non-zero value of $[D][D]^T$. For this setup, this matrix is a scalar value

$$D_2 = [D][D]^T \tag{12}$$

The d_i matrix components are given in Eq. (5). Using the robust SVD inverse technique, $D_2 > \epsilon$, then the $1/D_2$ math is evaluated as normal. However, if $D_2 < \epsilon$, then the inverse $1/D_2$ is set to zero. In the latter case there is no control authority about the current axis of interest. To set this epsilon parameter, not the definition of the $[D]$ matrix components $d_i = (\boldsymbol{r}_i \times \hat{\boldsymbol{g}}_{t_i}) \cdot \hat{\boldsymbol{b}}_j$. Note that $\boldsymbol{r}_i \times \hat{\boldsymbol{g}}_{t_i}$ is a scaled axis along which the i^{th} thruster can produce a torque. The value d_i will be near zero if the dot product of this axis with the current control axis $\hat{\boldsymbol{b}}_j$ is small.

To determine an appropriate ϵ value, let α be the minimum desired angle to avoid the control axis $\hat{\boldsymbol{b}}_j$ and the scaled thruster torque axis $\boldsymbol{r}_i \times \hat{\boldsymbol{g}}_{t_i}$ being orthogonal. If $\bar{r}$ is a mean distance of the thrusters to the spacecraft center of mass, then the d_i values must satisfy

$$\frac{d_i}{\bar{r}} > \cos(90^\circ - \alpha) = \sin\alpha \tag{13}$$

Thus, to estimate a good value of ϵ, the following formula can be used

$$\epsilon \approx d_i^2 = \sin^2\alpha \ \bar{r}^2 \tag{14}$$

For example, if $\bar{r} = 1.3$ meters, and we want α to be at least 1°, then we would set $\epsilon = 0.000515$.

REFERENCES

[1] Hanspeter Schaub and John L. Junkins. *Analytical Mechanics of Space Systems*. AIAA Education Series, Reston, VA, 3rd edition, 2014.