

Autonomous Vehicle Simulation (AVS) Laboratory

AVS-Sim Technical Memorandum

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GUIDANCE MODULE TO PERFORM A CONSTANT SPINNING ABOUT AN ORBIT AXIS

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Status: Initial Version
Scope/Contents
Generate the attitude reference to achieve a spinning motion about a primary orbit frame axis. A chosen reference axis $\hat{b}_j = \hat{b}_{spin}$ is to line up with the orbit axis $\hat{o}_i = \hat{o}_{spin}$, and rotate a desired rate ω_{spin}

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1 Module Input and Output

Table 2 shows the input Configuration Data of the module Orbit Axis Spin.

Table 2: Input Configuration Data

Name	Type	Length	Description
o_{spin}	int []	1	Index of the orbit-frame axis about which to spin, i.e. 0, 1, 2.
b_{spin}	int []	1	Index of the body axis that is to line up with o_{spin} and start spinning, i.e. 0, 1, 2.
ω_{spin}	double []	1	Desired spin rate [rad/sec].

Table 3 shows the initial Attitude Reference input message from one of the orbit-frame pointing modules (either Hill Point or Velocity Point).

Table 3: Input Attitude Reference Message

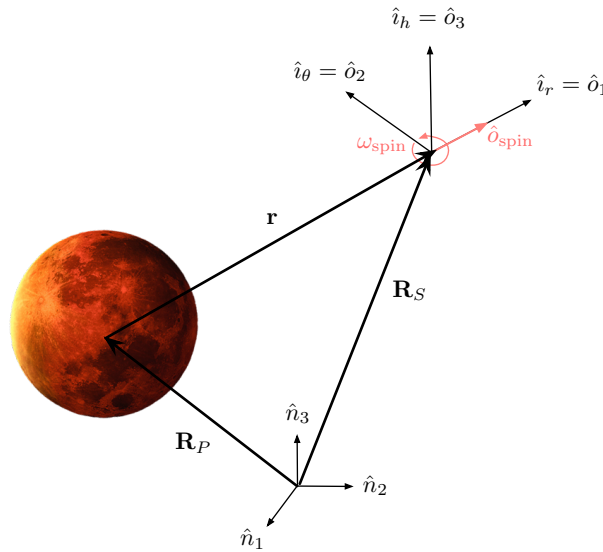
Name	Type	Length	Description
$\sigma_{R_0/N}$	double []	3	MRP attitude set of the orbit pointing frame with respect to the inertial frame.
$\omega_{R_0/N}$	double []	3	Angular velocity of the orbit pointing frame with respect to the inertial frame expressed in inertial frame components.
$\dot{\omega}_{R_0/N}$	double []	3	Angular acceleration of the orbit pointing frame with respect to the inertial frame expressed in inertial frame components.

Table 4 shows the Attitude Reference output message of the module Orbit Axis Spin.

Table 4: Output Attitude Reference Message

Name	Type	Length	Description
$\sigma_{R/N}$	double []	3	MRP attitude set of the reference frame with respect to the inertial frame.
${}^N\omega_{R/N}$	double []	3	Angular rate vector of the reference frame with respect to the inertial expressed in inertial frame components.
${}^N\dot{\omega}_{R/N}$	double []	3	Angular acceleration vector of the reference frame with respect to the inertial expressed in inertial frame components.

2 Introduction

**Fig. 1:** Illustration of spinning about the nadir orbit axis \hat{i}_r of the Hill orbit frame \mathcal{H} at a constant rate ω_{spin} .

In this note a method is discussed on how to compute the reference frame angular rate $\omega_{R/N}$ and acceleration $\dot{\omega}_{R/N}$ to achieve a particular family of stabilized spin motion. This module receives as input a constant pointing orbit-frame \mathcal{R}_0 . The goal is now to create a new reference \mathcal{R} that spins about any of the orbit axes that conform \mathcal{R}_0 at a constant rate ω_{spin} . Let us call the spinning orbit axis \hat{o}_{spin} . Note that the presented method is general enough to use any of the Hill $\mathcal{H} : \{\hat{i}_r, \hat{i}_\theta, \hat{i}_h\}$ or Velocity $\mathcal{V} : \{\hat{i}_n, \hat{i}_v, \hat{i}_h\}$ orbit frame orientations as the input reference. Figure 1 illustrates the case in which $\mathcal{R}_0 = \mathcal{H}$ and the spinning is about the nadir axis $\hat{o}_{\text{spin}} = \hat{i}_r$.

3 Reference Frame Generation

3.1 Angular Velocity and Acceleration Descriptions

The reference frame \mathcal{R} is defined above, and the attitude reference frame tracking control requires the angular rate $\omega_{R/N}$ and acceleration $\dot{\omega}_{R/N}$. Let the MRP attitude set, angular velocity vector and angular acceleration vector associated with the constant pointing reference be $\sigma_{R_0/N}$, $\omega_{R_0/N}$ and $\dot{\omega}_{R_0/N}$ respectively. The angular velocity of the spinning reference frame is thus given by:

$$\omega_{R/N} = \omega_{\text{spin}} + \omega_{R_0/N} \quad (1a)$$

$$\omega_{R/N} = \omega_{\text{spin}} \hat{o}_{\text{spin}} + \omega_{R_0/N} \quad (1b)$$

Since \hat{o}_{spin} is aligned with one of the orbit axis defining \mathcal{R}_0 , taking the inertial derivative of (1):

$$\dot{\omega}_{R/N} = \dot{\omega}_{\text{spin}} + \dot{\omega}_{R_0/N} \quad (2a)$$

$$\dot{\omega}_{R/N} = \frac{\mathcal{R}_0 d}{dt}(\omega_{\text{spin}} \hat{o}_{\text{spin}}) + \omega_{R_0/N} \times (\omega_{\text{spin}} \hat{o}_{\text{spin}}) + \dot{\omega}_{R_0/N} \quad (2b)$$

3.2 MRP Attitude Set

Let ϕ_{spin} be the current spin angle that the reference frame has rotated about its spin axis \hat{o}_{spin} . The final reference frame orientation is eventually given by

$$[RN] = [M_{\hat{o}_{\text{spin}}}(\phi_{\text{spin}})][R_0N] \quad (3)$$

where $[R_0N]$ is the Direction Cosine Matrix associated with the orbit axis pointing attitude set, and $M_{\hat{o}_{\text{spin}}}$ is the principal axis rotation matrix about \hat{o}_{spin} . Assuming a constant spin rate, the spin angle is propagated using the simple Euler integration scheme

$$\phi_{\text{spin}, n+1} = \phi_{\text{spin}, n} + \omega_{\text{spin}} \Delta t \quad (4)$$

With $[RN]$ defined, the MRP attitude set $\sigma_{R/N}$ can readily be computed.

4 Spin Angle Initialization

The question remains on how to initialize the spin angle ϕ_{spin} . Since the initialization of the spin angle requires of knowing the actual body frame and doing it in a completely general way is not feasible, a separate algorithm is used. Being \mathcal{B} the body frame and \mathcal{R}_0 the orbit frame orientations, let us set \hat{b}_{spin} as the principal body axis that is to be aligned with the principal orbit axis \hat{o}_{spin} . The triodes of axes \hat{b}_i and \hat{o}_i are labeled so that the first index determines the spin axis, i.e. $\hat{b}_1 = \hat{b}_{\text{spin}}$ and $\hat{o}_1 = \hat{o}_{\text{spin}}$. The remaining two indices are set to yield a right-handed coordinate frame.

Let \mathcal{R} be the reference orientation in which \hat{o}_1 orbit axis and \hat{b}_1 body axis are lined up. Using the definition where $RN(i)$ is the i_{th} row of the $[RN]$ DCM matrix:

$$RN(\hat{b}_1) = R_0N(\hat{o}_1) \quad (5)$$

$$RN(\hat{b}_2) = R_0N(\hat{o}_2) \quad (6)$$

$$RN(\hat{b}_3) = RN(\hat{b}_1) \times RN(\hat{b}_2) \quad (7)$$

Now, the desired orientation for the body-frame $[RN]$ and the current one $[BN]$ are completely determined.

Let us designate the initial angle between \hat{b}_{spin} and \hat{o}_{spin} , before the alignment, as ϕ_{align} . Once a principal rotation of ϕ_{align} is applied to the body-frame for the purposes of this module, i.e. \hat{b}_{spin} becomes coincident with \hat{o}_{spin} , the other two body axes are only assured to be contained in the plane normal to the spinning axis. Let us eventually define ϕ_{spin} as the smallest angle to align the remaining axes with their corresponding desired ones. At this point, it is important to recall that any general alignment between reference frames can be achieved through a single rotation about a principal rotation axis and that this rotation can be either of ϕ or $(2\pi - \phi)$, which correspond to the short and long way respectively.

The purpose of the algorithm presented in this section is to ensure that the spinning is always initialized to go the short way. Let us first compute the alignment angle between the prospective spinning axes:

$$\phi_{\text{align}} = \arccos(RN(\hat{b}_1) \cdot BN(\hat{b}_1)) \quad (8)$$

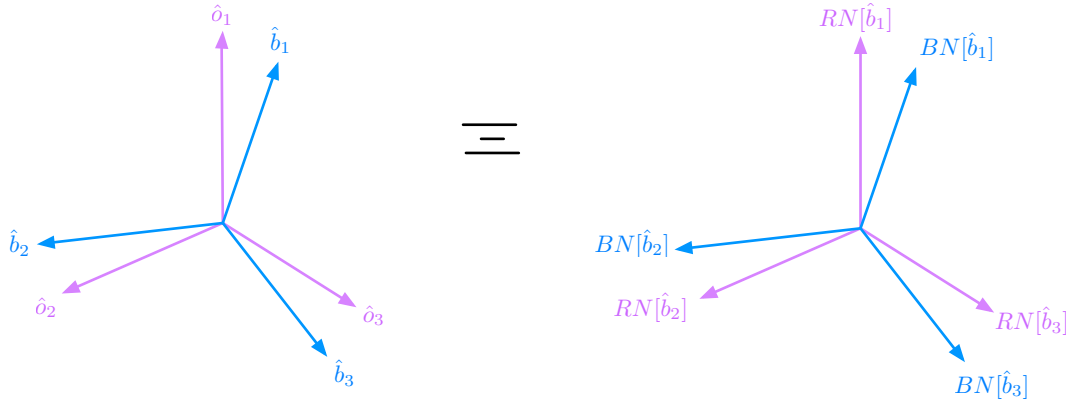


Fig. 2: Current body-frame $[BN]$ and goal reference frame $[RN]$

Note that $RN(\hat{b}_1)$ and $RN(\hat{b}_1)$ are vectors already normalized. And the principal rotation vector for the alignment is:

$$\hat{e}_{\text{align}} = RN(\hat{b}_1) \times BN(\hat{b}_1) \quad (9)$$

$$\mathbf{e}_{\text{align}} = \phi_{\text{align}} \hat{e}_{\text{align}} \quad (10)$$

Let us designate \mathcal{F} as the intermediate frame where \hat{b}_{spin} and \hat{o}_{spin} are aligned.

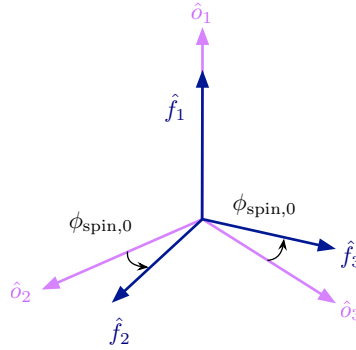


Fig. 3: Current body-frame \mathcal{B} and intermediate frame \mathcal{F}

Through the following function from the Rigid Body Kinematic library in Reference 1, the DCM corresponding to the previous principal rotation vector is computed:

$$[BF] = \text{PRV2C}(\mathbf{e}_{\text{align}}) \quad (11)$$

Now, we can align the spinning axes:

$$[FN] = [BF]^T [BN] \quad (12)$$

Next, let us compute the relative orientation between the intermediate frame \mathcal{F} and the reference \mathcal{R}

$$[RF] = [RN][FN]^T \quad (13)$$

Making use again of the same Rigid Body Kinematic Library, we compute the associated principal rotation vector (direction ang angle):

$$\mathbf{e}_{\text{spin}} = \text{C2PRV}([RF]) \quad (14)$$

Where $\phi_{\text{spin},0} = |e_{\text{spin}}|$ shall be checked if corresponds to the short or long rotation: if the angle between e_{spin} and $[RN(\hat{b}_{\text{spin}})]$ is 180 deg, then the computed ϕ_{spin} corresponds to the latter case and its sign needs to be switched in order to guarantee following the shortest path.

Mathematically, the check is accomplished through a simple if-statement:

If

$$e_{\text{spin}} \cdot RN[\hat{b}_{\text{spin}}] < 0$$

Then,

$$\phi_{\text{spin},0} = -\phi_{\text{spin},0}$$

Where, recall that $RN[\hat{b}_{\text{spin}}]$ is a base vector and the first expression is thus a dot product between vectors, directly related to the cosinus of the angle between them.

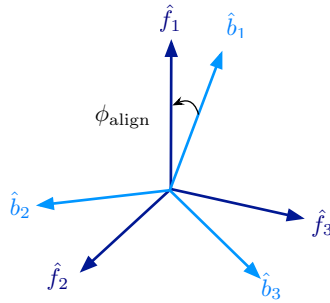


Fig. 4: Goal reference-frame \mathcal{R} and intermediate frame \mathcal{F}

REFERENCES

- [1] Hanspeter Schaub and John L. Junkins. *Analytical Mechanics of Space Systems*. AIAA Education Series, Reston, VA, 3rd edition, 2014.