Autonomous Vehicle Simulation (AVS) Laboratory, University of Colorado

Basilisk Technical Memorandum

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ECLIPSE C++ MODEL

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Status: Tested
Scope/Contents
The eclipse module is responsible for determining whether or not a spacecraft is experiencing a solar eclipse and how much of it is occulted. A unit test has been written and performed, comparing simulated and calculated shadow factors during full eclipse, partial eclipse, and no eclipse scenarios.

Rev:	**Change Description**	**By**	**Date**
1.0	First version - Mathematical formulation and implementation	G. Chapel	09/07/17

Contents

1 Model Description

The eclipse module is responsible for determining whether or not a spacecraft is within the shadow of a solar eclipse and if so, how much. The module finds the states of the sun, spacecraft and planets of interest, allowing each body's postition to be related and the construction of the conical shadow model. This provides the means for computing what percent of the spacecraft is illuminated where a shadow factor of 0.0 represents a total eclipse and 1.0 represents no eclipse.

To determine the states of the bodies in question, messages are passed into the code. For the spacecraft, Cartesian vectors provide the position and velocity of its center of mass. For the sun and planets, a time is chosen and input into the module along with SPK, LSK, and PCK files, indicating ephemeris data, leapsecond information, and reference frame orientation. The planets desired to be used in the module are specified through the Basilisk messaging system, where corresponding strings (e.g. "venus", "earth", or "mars barycenter") are input as Spice Objects. This allows the state data to be obtained at the given time, using Spice and the kernel files. The kernels used when testing are given in the Test Parameters section. Fig. 1 illustrates how the states are represented and will be identified in the mathematical model. Calculations in this model are taken from Montenbruck and Gill's *Satellite Orbits Models, Methods and Applications* text.[1]

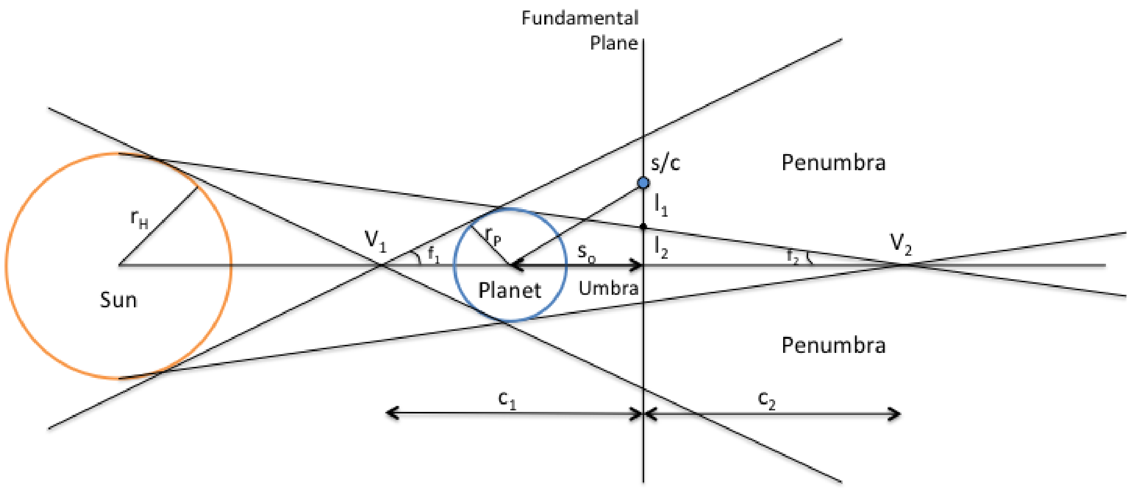

Fig. 1: Representation of a Conical Shadow

1.1 Mathematical model

1.1.1 Determining States

The initial step in the eclipse module is to obtain the celestial bodies' state data and transform them into usable terms. The relationships shown below remove the dependancy on relating position to the inertial frame *N* and instead utilize the planet *P*, spacecraft body *B*, and helio *H* frames.

$$\boldsymbol{s}_{P/H} = \boldsymbol{r}_{N/H} - \boldsymbol{r}_{N/P} \tag{1}$$

$$\boldsymbol{r}_{B/H} = \boldsymbol{r}_{N/H} - \boldsymbol{r}_{N/B} \tag{2}$$

$$\boldsymbol{s}_{P/B} = \boldsymbol{r}_{N/B} - \boldsymbol{r}_{N/P} \tag{3}$$

The previous three equations provide coordinates for the sun with respect to both the occulting planet and occulted spacecraft as well as the spacecraft's position with respect to the planet, respectively. The parameters on the right side of these equations come from the input state data where $\boldsymbol{r}_{N/H}$, $\boldsymbol{r}_{N/P}$, and $\boldsymbol{r}_{N/B}$ are the sun, planet, and spacecraft positions in the inertial frame.

This module supports the use of multiple occulting bodies, so it is important to analyze only the planet with the highest potential to cause an eclipse. Thus, the closest planet is determined by comparing the magnitude of each planet's distance to the spacecraft, $|\boldsymbol{s}_{P/B}|$. Note that if the spacecraft is closer to the sun than the planet, i.e. $|\boldsymbol{r}_{B/H}| < |\boldsymbol{s}_{P/H}|$, an eclipse is not possible and the shadow fraction is immediately set to 1.0.

1.2 Eclipse Conditions

When analyzing the conical shadow model, there are critical distances and conical dimensions that must be considered. These parameters are determined by first knowing the planet's equatorial radius r_P, which is used to solve for the angles of the shadow cones. Angles f_1 and f_2 are computed as shown below, where the subscript 1 relates to the cone of the penumbra and 2 relates to the umbra.

$$f_1 = \frac{\arcsin(r_H + r_P)}{|\boldsymbol{s}_{P/H}|} \tag{4}$$

$$f_2 = \frac{\arcsin(r_H - r_P)}{|\boldsymbol{s}_{P/H}|} \tag{5}$$

Here r_H indicates the equatorial radius of the sun, which is 695000 km. Both the sun and planet radii must be input in terms of meters.

As shown by Fig. 1, the fundamental plane is perpendicular to the shadow axis and coincident with the spacecraft body. The distance between the plane-axis intersection and the center of the planet is given by s_0 as shown by Eq. 6.

$$s_0 = \frac{-\boldsymbol{s}_{P/B} \cdot \boldsymbol{s}_{P/H}}{|\boldsymbol{s}_{P/H}|} \tag{6}$$

This distance and the shadow cone angles can now be used to determine the distances, c_1 and c_2, between the fundamental plane and the cones' vertices V_1 and V_2. These are calculated as follows:

$$c_1 = s_0 + \frac{r_P}{\sin(f_1)} \tag{7}$$

$$c_2 = s_0 - \frac{r_P}{\sin(f_2)} \tag{8}$$

As shown in Eq. 9 and 10, these are then used to find the radii, l_1 and l_2, of the shadow cones in the fundamental plane.

$$l_1 = c_1 \tan(f_1) \tag{9}$$

$$l_2 = c_2 \tan(f_2) \tag{10}$$

Finding these parameters provides insight into the type of eclipse that the spacecraft is experiencing. To determine the type, it is useful to compare the cone radii to the distance between the spacecraft and the shadow axis, which is given by l.

$$l = \sqrt{|\boldsymbol{s}_{P/B}|^2 - s_0^2} \tag{11}$$

Total and annular eclipses both require the spacecraft to be relatively close to the shadow axis, where $|l| < |l_2|$. The difference between these two types is that the planet is closer to the spacecraft for a total eclipse ($c_2 < 0$) than during an annular eclipse ($c_2 > 0$). If the spacecraft is further from the shadow axis but still within a cone radius ($|l| < |l_1|$), it is experiencing a partial eclipse.

1.3 Percent Shadow

With the eclipse type determined, the shadow fraction can now be found. To find the shadow fraction, the apparent radii of the sun and planet and the apparent seperation of both bodies are needed. These are given, respectively, by a, b, and c in the equations below.

$$a = \arcsin(\frac{r_H}{|\boldsymbol{r}_{B/H}|}) \tag{12}$$

$$b = \arcsin(\frac{r_P}{|\boldsymbol{s}_{P/B}|}) \tag{13}$$

$$c = \arccos(\frac{-\boldsymbol{s}_{P/B} \cdot \boldsymbol{r}_{B/H}}{|\boldsymbol{s}_{P/B}||\boldsymbol{r}_{B/H}|}) \tag{14}$$

Fig. 2 below illustrates the overlapping disk model that represents the occultation, where the solid orange line indicates the sun and the dotted blue line indicates the planet.

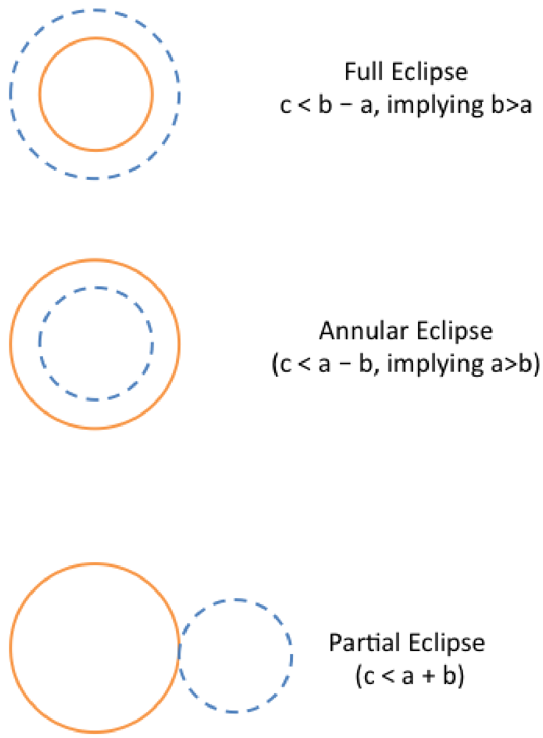

Fig. 2: Occultation Disk Model

1.3.1 Total Eclipse ($c < b - a$)

This type assumes that the apparent radius of the planet is larger than that of the sun ($b > a$). A total eclipse produces a total shadow, so the shadow fraction is 0.0.

1.3.2 Annular Eclipse ($c < a - b$)

This type assumes the apparent radius of the sun is larger than that of the planet ($a > b$). Use the equation for a circular area, $A = \pi r^2$, to find the area of the sun and planet faces, replacing r with the corresponding apparent radius. The shadow fraction is then just the ratio of the planet's area to the sun's area.

$$ShadowFraction = \frac{A_P}{A_H} \tag{15}$$

1.3.3 Partial Eclipse ($c < a + b$)

For a partial eclipse, the occulted area is given by Eq. 16.

$$A = a^2 \arccos(\frac{x}{a}) + b^2 \arccos(\frac{c - x}{b}) - cy \tag{16}$$

Parameters a, b, and c are those calculated previously in Eq. 12, 13, and 14. The values x and y are given by the following equations.

$$x = \frac{c^2 + a^2 - b^2}{2c} \tag{17}$$

$$y = \sqrt{a^2 - x^2} \tag{18}$$

Like with the annular partial eclipse, the shadow factor for this type is the ratio between the occulted area and the sun's apparent area. This is given by the equation below.

$$ShadowFraction = 1 - \frac{A}{\pi a^2} \tag{19}$$

2 Model Functions

This module contains several functions in order to obtain state data and describe eclipse characteristics. These functions are explained below.

- **Interface: Planet States** The code receives planet state data through Spice when a time and planet name are input as Spice objects through the Basilisk messaging system.
- **Interface: Sun States** Like the previous function, the code receives sun state data through Spice when given a time, input as a Spice object through the Basilisk messaging system.
- **Interface: Spacecraft States** The code receives spacecraft state data through Basilisk's spacecraftPlus messaging system.
- **Find Closest Planet** If given multiple planets, the code iterates through the planet list and determines which is the closest to the spacecraft.
- **Planet Radius** The code specifies the equatorial radius of a planet when a planet name is input as a Spice object.
- **Eclipse Type** The code uses calculations from Montenbruck and Gill's *Satellite Orbits Models, Methods and Applications* text.[1] to define shadow cone dimensions. This provides the means of determining the type of eclipse.
- **Percent Shadow** The code calculates and outputs the shadow factor of the eclipse, where 0.0 indicates a total eclipse and 1.0 represents no eclipse.

3 Model Assumptions and Limitations

- **Occultation Model:** Since the apparent radius of the sun is relatively small, the occultation can be modeled as overlapping disks.
- **No Eclipse:** If the spacecraft is closer to the sun than the planet, an eclipse is not possible.
- **Planets:** The allowed planets for use as occulting bodies are Mercury, Venus, Earth, and Mars.
- **Sun and Planet States:** The data defining the sun and planet states is obtained through an external Spice package. Errors may be derived from this package but will be small.
- **Spacecraft States:** Spacecraft states must be input as Cartesian vectors. In the test, a conversion from orbital elements is performed.
- **Apparent Radii:** When determining the type of eclipse, assume that the apparent separation $c \geqslant 0$.
  - Total Eclipse ($c < b - a$): Assume the apparent radius of the planet is greater than that of the sun ($b > a$).
  - Annular Eclipse ($c < a - b$): Assume the apparent radius of the sun is greater than that of the planet ($a > b$).

4 Test Description and Success Criteria

The unit test, test_eclipse.py, validates the internal aspects of the Basilisk eclipse module by comparing simulated output with expected output. It validates the computation of a shadow factor for total eclipse, partial eclipse,annular eclipse, and no eclipse scenarios. The test is designed to analyze one type at a time for both Earth and Mars and is then repeated for all three.

4.1 Sun States

This test begins by specifying a UTC time through the use of a Spice object. This, along with the implementation of Spice kernels, allows the states of the sun and any desired planets to be determined. One time is provided, fixing the sun and planet states so the spacecraft states may be varied. After specifying these, the spacecraft states are set and the three eclipse types are considered.

4.2 Spacecraft States

Earth is set as the zero base for all eclipse types to test it as the occulting body. For full, partial, and no eclipse cases, orbital elements describing the spacecraft states are then converted to Cartesian vectors. These orbital elements vary for each eclipse type since the Sun and planet states are fixed. The conversion is made using the orbitalMotion *elem2rv* function, where the inputs are six orbital elements (a, e, i, Ω, ω, f) and the outputs are Cartesian position and velocity vectors. For the annular eclipse case, the conversion is avoided and a Cartesian position vector is initially provided instead. The vectors are then passed into spacecraftPlus and, subsequently, the eclipse module through the Basilisk messaging system.

Testing the no eclipse case with Mars as the occulting body is the same as the Earth no eclipse test, except Mars is set as the zero base. The Mars full, partial, and annular eclipse cases, however, are like the Earth annular case where Cartesian vectors are, instead, the initial inputs. Since the test is performed as a single step process, the velocity is not necessarily needed as an input, so only a position vector is provided for these cases. These inputs are more clearly illustrated in the Test Parameters section.

4.3 Planet States

Once the spacecraft states are defined, the planet names are provided as Spice objects. Since the module determines the closest planet to the spacecraft, multiple names may be input and the chosen one depends purely on the position of the spacecraft. For this test, only Earth and Mars were used as occulting bodies, since no appreciable difference in the algorithm presents itself when testing various planets.

4.4 Comparison

The shadow factor obtained through the module is compared to the expected result, which is either trivial or calculated, depending on the eclipse type. Full eclipse and no eclipse shadow factors are compared without the need for computation, since they are just 0.0 and 1.0, respectively. The partial and annular eclipse shadow factors, however, vary between 0.0 and 1.0, based on the cone dimensions, and are calculated using MATLAB and Spice data.

5 Test Parameters

The previous description only briefly mentions the input parameters required to perform the tests on this module, so this section further details the parameters set by the user and built into the unit test.

1. Eclipse Condition Input:
   One of the two user-defined inputs is the eclipse identifier, appropriately named *eclipseCondition*. This specifies the eclipse type and can be set, via string inputs, to either *full*, *partial*, *annular*, or *none* for the full, partial, annular, and no eclipse cases. This input changes the spacecraft state parameters to position the spacecraft in the desired eclipse.

2. Planet Input:
   The second user input is *planet*, which allows the user to test either Earth or Mars as the occulting body. Only these two planets are tested since there is no appreciable difference in the eclipse algorithm when varying the planet. This input changes the zero base to reference the chosen planet and the spacecraft state parameters to position the spacecraft in the desired eclipse.

3. Zero Base:
   The zero base references either Earth or Mars depending on the planet input.

4. Orbital Elements:
   Orbital elements describe the characteristics of the orbit plane and are used in this module to define the states of the spacecraft. They consist of the semimajor axis a in units of kilometers, eccentricity e, inclination i, ascending node Ω, argument of pariapses ω, and the true anomaly f. The Euler angles and true anomaly are all in units of degrees but are converted to radians in the test. Using orbitalMotion, all of the elements are converted to Cartesian position and velocity vectors that are then translated from kilometers to meters. The table below shows the values used when testing full, partial, and no eclipse cases for Earth as well as the no eclipse case for Mars.

Table 2: Orbital Element Values for Each Eclipse Condition

Element	Full Eclipse	Partial Eclipse	No Eclipse
a	6878.1366 km	6878.1366 km	9959991.68982 km
e	0.00001	0.00001	0.00001
i	5°	5°	5°
Ω	48.2°	48.2°	48.2°
ω	0	0	0
f	173°	107.5°	107.5°

The orbital elements remain the same when testing the no eclipse case for Earth and Mars, but the zero base is changed to reference the appropriate planet. The parameters used when testing full, partial, and annular eclipse cases for Mars as well as the annular case for Earth are shown under Cartesian Vectors.

5. Cartesian Vectors:
   Cartesian position vectors are used as inputs when testing the full, partial, and annular eclipse cases for Mars as well as the annular case for Earth. Velocity vectors are not needed since this test is performed as a single step process and eclipses at a single point in time depend only on the position of the celestial bodies at that time. The vectors are shown in the Table 3 below. The parameters for the no eclipse case were given previously under Orbital Elements.

Table 3: Position Vectors for Each Eclipse Condition

Eclipse Type	**X**	**Y**	**Z**
Full Eclipse	-2930233.559 m	2567609.100 m	41384.233 m
Partial Eclipse	-6050166.455 m	2813822.447 m	571725.565 m
Annular Eclipse (Mars)	-427424601.171 km	541312532.797 km	259820030.623 km
Annular Eclipse (Earth)	-326716535.629 km	-287302983.139 km	-124542549.301 km

6. Standard Gravitational Parameter:
   The gravitational parameter μ is necessary for converting between orbital elements and Cartesian vectors. It is the product of a body's gravitational constant and mass, specifying the attracting body in units of $\frac{km^3}{s^2}$. This test only uses the conversion when considering Earth as the occulting body, so only Earth's gravitational parameter is given below. The value is obtained through orbitalMotion.

$$\mu_{Earth} = GM_{Earth} = 398600.436 \quad \frac{km^3}{s^2} \tag{20}$$

7. Planet Names
   The eclipse module accepts and will analyze multiple planets at a time, so to indicate which planets are of interest, the names must be input. These are input as Spice objects of type string and can either be *venus*, *earth*, or *mars barycenter*. Spice uses this information and the time input to provide state data, which is then used to determine the closest planet. The closest planet is the only one further evaluated for potential eclipse conditions.

8. Time:
   In order to specify the states of the planets and sun, time is a neccessary input. Only one time is used for every test, fixing the sun and planet states and varying the states of the spacecraft. For this test, the input *2021 MAY 04 07:47:49.965 (UTC)* is used, where the time is represented in UTC form.

9. Spice Kernels
   Spice information is gathered from a collection of kernels, specfically SPK, LSK, and PCK files. The binary SPK used is *de430.bsp*, which provides ephemeris data. The LSK file is *naif0011.tls*, which offers leapsecond information. For reference frame orientation, two PCK files are given. These are *de-403-masses.tpc* and *pck00010.tpc*.

Table 4: Error Tolerance of Shadow Factor Comparison-Note: Absolute Tolerance is abs(truth-value)

Test	Tolerated Error
Full Eclipse	1e-12
Partial Eclipse	1e-12
Annular Eclipse	1e-12
No Eclipse	1e-12

6 Test Results

All checks within test_eclipse.py passed as expected. Table 5 shows the results below.

Table 5: Test Results

Test	Earth Results	Mars Results	Notes
Full Eclipse	PASSED	PASSED	
Partial Eclipse	PASSED	PASSED	
Annular Eclipse	PASSED	PASSED	
No Eclipse	PASSED	PASSED	

7 User Guide

When using this model, the user should follow the setup procedure corresponding to his or her desired conversion described below. The procedures outline the required inputs and some recommended parameter values.

- *eclipseCondition*
  - *full* for full eclipse
  - *partial* for partial eclipse
  - *annular* for annular eclipse
  - *none* for no eclipse
- μ is recommended to be $3.86\text{e}{+}14\ \frac{\text{m}^3}{\text{s}^2}$ for Earth.
- *UTCCalInit* is used in this test as *2021 MAY 04 07:47:49.965 (UTC)* but can be any UTC time.
- Keplerian orbital elements are given in Table 2 previously. These are just suggested values that are used for the test and can be varied. The module only accepts Cartesian vectors.
  - To convert from orbital elements to Cartesian vectors, use the *elem2rv* method from orbitalMotion
- Planet Names
  - *venus*
  - *earth*
  - *mars barycenter*

7.1 Variable Definition and Code Description

The variables in Table 6 are available for user input. Variables used by the module but not available to the user are not mentioned here. Variables with default settings do not necessarily need to be changed by the user, but may be.

Table 6: Definition and Explanation of Variables Used.

Variable	LaTeX Equivalent	Variable Type	Notes
r$_B N_N$	$\boldsymbol{r}$	double	[m]Default setting: 0.0. Spacecract position vector used as an input.
μ	μ	double	[m3/s2] Required Input. This is the gravitational parameter of the body. Replaces the product of Earth's gravitational force and mass for this test..
a	a	double	[km] Required Input. The semimajor axis of the body's orbit.
e	e	double	Required Input. The eccentricity of the body's orbit.
i	i	double	[rad] Required Input. The inclination of the body's orbit
Omega	Ω	double	[rad] Required Input. The ascending node of the body's orbit.
omega	ω	double	[rad] Required Input. The argument of periapses of the body's orbit.
f	f	double	[rad] Required Input. The true anomaly of the body's orbit
spiceObject.UTCCalInit	*UTCCalInit*	str	Required Input. A UTC time that provides the means of obtaining planet and sun state data.
Planet Names	N/A	str	Required Input. Identifies the planets desired to be evaluated.

REFERENCES

[1] Montenbruck, O., and Gill, E., *Satellite Orbits Models, Methods and Applications*, 1st ed. Berlin: Springer Berlin, 2000