Autonomous Vehicle Simulation (AVS) Laboratory

AVS-Sim Technical Memorandum

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GUIDANCE MODULE FOR VELOCITY AXIS POINTING

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Scope/Contents
Generate the attitude reference to perform a constant pointing towards a Velocity orbit axis

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Contents

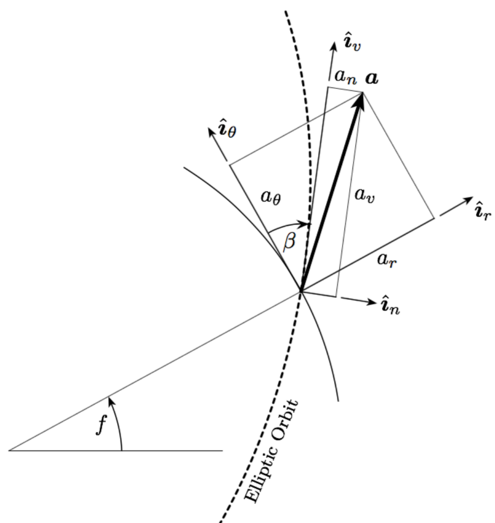

Fig. 1: Illustration of the planet orbit frames, Hill $\mathcal{H}$ and Velocity $\mathcal{V}$.

1 Reference Frame Definition

The velocity frame is defined as $\mathcal{V} : \{\hat{\boldsymbol{\imath}}_n, \hat{\boldsymbol{\imath}}_v, \hat{\boldsymbol{\imath}}_h\}$ as illustrated in Figure 1. Here $\hat{\boldsymbol{\imath}}_v$ is aligned with the orbit velocity direction, $\hat{\boldsymbol{\imath}}_h$ is aligned with the orbit normal direction and $\hat{\boldsymbol{\imath}}_n$ is the normal vector that completes a right-handed coordinate frame. On the other hand, the Hill orbit frame is defined as $\mathcal{O} : \{\hat{\boldsymbol{\imath}}_r, \hat{\boldsymbol{\imath}}_\theta, \hat{\boldsymbol{\imath}}_h\}$.

These two frames differ by a 3-axis rotation with the angle $-\beta$. In terms of β, the $[VH]$ DCM to map from $\mathcal{H}$ to $\mathcal{V}$ is given by

$$[VH] = [M_3(-\beta)] = \begin{bmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{1}$$

2 Velocity Frame Rate Development

Next the Velocity frame rate $\dot{\beta}$ relative to the orbit frame is determined. Note the following identities:

$$\tan\beta = \frac{e\sin f}{1 + e\cos f} \tag{2}$$

$$\cos^2\beta = \frac{(1 + e\cos f)^2}{1 + e^2 + 2e\cos f} \tag{3}$$

Taking the derivative of Eq. (2) yields

$$\dot{\beta} = \frac{e(e + \cos f)}{1 + e^2 + 2e\cos f}\dot{f} \tag{4}$$

To find the relative angular acceleration $\ddot{\beta}$, Eq. (4) is differentiated again.

$$\ddot{\beta} = \frac{e(e + \cos f)}{1 + e^2 + 2e\cos f}\ddot{f} + \frac{e(e^2 - 1)\sin f}{(1 + e^2 + 2e\cos f)^2}\dot{f}^2 \tag{5}$$

3 Angular Velocity Vectors

Next, let us evaluate the Velocity frame angular velocities. As both the $\mathcal{V}$ and $\mathcal{H}$ frame rotate about the common $\hat{\boldsymbol{\imath}}_h$ axis, note that

$$\boldsymbol{\omega}_{V/H} = -\dot{\beta}\hat{\boldsymbol{\imath}}_h \tag{6}$$

$$\boldsymbol{\omega}_{H/N} = \dot{f}\hat{\boldsymbol{\imath}}_h \tag{7}$$

This leads to

$$\boldsymbol{\omega}_{V/N} = \boldsymbol{\omega}_{V/H} + \boldsymbol{\omega}_{H/N} = \frac{1 + e\cos f}{1 + e^2 + 2e\cos f}\dot{f}\hat{\boldsymbol{\imath}}_h \tag{8}$$

Similarly, we find

$$\dot{\boldsymbol{\omega}}_{V/N} = \dot{\boldsymbol{\omega}}_{V/H} + \dot{\boldsymbol{\omega}}_{H/N} = \left(\frac{1 + e\cos f}{1 + e^2 + 2e\cos f}\ddot{f} - \frac{e(e^2 - 1)\sin f}{(1 + e^2 + 2e\cos f)^2}\dot{f}^2\right)\hat{\boldsymbol{\imath}}_h \tag{9}$$

The orbit frame angular rates and accelerations are determined through the standard astrodynamics relations:

$$\dot{f} = \frac{h}{r^2} \tag{10}$$

$$\ddot{f} = -\frac{\mu e \sin f}{r^3} \tag{11}$$