Autonomous Vehicle Simulation (AVS) Laboratory, University of Colorado

Basilisk Technical Memorandum

Document ID: Basilisk-MRP_Feedback

MRP FEEDBACK ADCS CONTROL MODULE

Prepared by	H. Schaub

Status: Initial Documentation Draft
Scope/Contents
This module provides a general MRP feedback control law, applying to using N reaction wheels with general orientation.

Rev:	**Change Description**	**By**
Draft	Initial Draft document	H. Schaub

Contents

---

1 Initialization

Simply call the module reset function prior to using this control module. This will reset the prior function call time variable, and reset the attitude error integral measure. The control update period Δt is evaluated automatically.

2 Algorithm

This module employs the MRP feedback algorithm of Example 8.14 of Reference 1. This nonlinear attitude tracking control includes an integral measure of the attitude error. Further, we seek to avoid quadratic $\boldsymbol{\omega}$ terms to reduce the likelihood of control saturation during a detumbling phase. Let the new nonlinear feedback control be expressed as

$$[G_s]\boldsymbol{u}_s = -\boldsymbol{L}_r \tag{1}$$

where

$$\begin{split}\boldsymbol{L}_r = -K\boldsymbol{\sigma} - [P]\delta\boldsymbol{\omega} - [P][K_I]\boldsymbol{z} + [I_{\text{RW}}](\dot{\boldsymbol{\omega}}_r - [\tilde{\boldsymbol{\omega}}]\boldsymbol{\omega}_r) - \boldsymbol{L} \\ + \left([\tilde{\boldsymbol{\omega}}_r] + [\widetilde{K_I\boldsymbol{z}}]\right)\left([I_{\text{RW}}]\boldsymbol{\omega} + [G_s]\boldsymbol{h}_s\right)\end{split} \tag{2}$$

and

$$h_{s_i} = I_{W_{s_i}}(\hat{\boldsymbol{g}}_{s_i}^T\boldsymbol{\omega}_{B/N} + \Omega_i) \tag{3}$$

with I_{W_s} being the RW spin axis inertia.

The integral attitude error measure $\boldsymbol{z}$ is defined through

$$\boldsymbol{z} = K\int_{t_0}^{t}\boldsymbol{\sigma}\,\text{d}t + [I_{\text{RW}}](\delta\boldsymbol{\omega} - \delta\boldsymbol{\omega}_0)$$

The integral measure $\boldsymbol{z}$ must be computed to determine $[P][K_I]\boldsymbol{z}$, and the expression $[\widetilde{K_I\boldsymbol{z}}]$ is added to $[\widetilde{\boldsymbol{\omega}_r}]$ term.

To analyze the stability of this control, the following Lyapunov candidate function is used:

$$V(\delta\boldsymbol{\omega}, \boldsymbol{\sigma}, \boldsymbol{z}) = \frac{1}{2}\delta\boldsymbol{\omega}^T[I_{\text{RW}}]\delta\boldsymbol{\omega} + 2K\ln(1 + \boldsymbol{\sigma}^T\boldsymbol{\sigma}) + \frac{1}{2}\boldsymbol{z}^T[K_I]\boldsymbol{z}$$

provides a convenient positive definite attitude error function. The attitude feedback gain K is positive, while the integral feedback gain $[K_I]$ is a symmetric positive definite matrix. The resulting Lyapunov rate expression, solved in Eq. (8.101), is given by

$$\dot{V} = (\delta\boldsymbol{\omega} + [K_I]\boldsymbol{z})^T\left([I_{\text{RW}}]\frac{{}^{\mathcal{B}}\text{d}}{\text{d}t}(\delta\boldsymbol{\omega}) + K\boldsymbol{\sigma}\right)$$

Substituting the equations of motion of a spacecraft with N reaction wheels (see Eq. (8.160) in Reference 1), results in

$$\dot{V} = (\delta\boldsymbol{\omega} + [K_I]\boldsymbol{z})^T \left(-[\tilde{\boldsymbol{\omega}}]([I_{\text{RW}}]\boldsymbol{\omega} + [G_s]\boldsymbol{h}_s) - [G_s]\boldsymbol{u}_s + \boldsymbol{L} - [I_{\text{RW}}](\dot{\boldsymbol{\omega}}_r - [\tilde{\boldsymbol{\omega}}]\boldsymbol{\omega}_r) + K\boldsymbol{\sigma}\right)$$

Substituting the control expression in Eq. (1) and making use of $\boldsymbol{\alpha} = \boldsymbol{\omega}_r - [K_I]\boldsymbol{z}$ leads to

$$\begin{aligned}\dot{V} &= (\delta\boldsymbol{\omega} + [K_I]\boldsymbol{z})^T \Big(- ([\tilde{\boldsymbol{\omega}}] - [\tilde{\boldsymbol{\omega}}_r] + [\widetilde{K_I\boldsymbol{z}}])([I_{\text{RW}}]\boldsymbol{\omega} + [G_s]\boldsymbol{h}_s) + (K\boldsymbol{\sigma} - K\boldsymbol{\sigma}) \\ &\quad - [P]\delta\boldsymbol{\omega} - [P][K_I]\boldsymbol{z} + [I_{\text{RW}}](\dot{\boldsymbol{\omega}}_r - [\tilde{\boldsymbol{\omega}}]\boldsymbol{\omega}_r) - [I_{\text{RW}}](\dot{\boldsymbol{\omega}}_r - [\tilde{\boldsymbol{\omega}}]\boldsymbol{\omega}_r) + (\boldsymbol{L} - \boldsymbol{L})\Big) \\ &= (\delta\boldsymbol{\omega} + [K_I]\boldsymbol{z})^T \Big(- ([\widetilde{\delta\boldsymbol{\omega}}] + [\widetilde{K_I\boldsymbol{z}}])([I_{\text{RW}}]\boldsymbol{\omega} + [G_s]\boldsymbol{h}_s) - [P](\delta\boldsymbol{\omega} + [K_I]\boldsymbol{z})\Big)\end{aligned}$$

Because $(\delta\boldsymbol{\omega} + [K_I]\boldsymbol{z})^T([\widetilde{\delta\boldsymbol{\omega}}] + [\widetilde{K_I\boldsymbol{z}}]) = 0$, the Lyapunov rate reduces the negative semi-definite expression

$$\dot{V} = -(\delta\boldsymbol{\omega} + [K_I]\boldsymbol{z})^T[P](\delta\boldsymbol{\omega} + [K_I]\boldsymbol{z})$$

This proves the new control is globally stabilizing. Asymptotic stability is shown following the same steps as for the nonlinear integral feedback control in Eq. (8.104) in Reference 1.

One of the goals set forth at the beginning of the example was avoiding quadratic $\boldsymbol{\omega}$ feedback terms to reduce the odds of control saturation during periods with large $\boldsymbol{\omega}$ values. However, the control in Eq. (1) contains a product of $\boldsymbol{z}$ and $\boldsymbol{\omega}$. Let us study this term in more detail. The $\boldsymbol{\omega}$ expression with this product terms is found to be

$$[\widetilde{K_I\boldsymbol{z}}]([I_{\text{RW}}]\boldsymbol{\omega}) \quad \Rightarrow \quad -([\widetilde{I_{\text{RW}}\boldsymbol{\omega}}])([K_I][I_{\text{RW}}]\boldsymbol{\omega} + \cdots)$$

If the integral feedback gain is a scalar K_I, rather than a symmetric positive definite matrix $[K_I]$, the quadratic $\boldsymbol{\omega}$ term vanishes. If the full 3×3 gain matrix is employed, then quadratic rate feedback terms are retained.

3 Unit Test

The unit test for this module `test_MRP_feedback` tests a set of gains K, K_i, P on a rigid body with no external torques, and with a fixed input reference attitude message. The torque requested by the controller is evaluated against python computed torques at 0s, 0.5s, 1s, 1.5s and 2s to within a tolerance of 10^{-8}. After 1s the simulation is stopped and the Reset() function is called to check that integral feedback related variables are properly reset. The following permutations are run:

- The test is run for a case with error integration feedback ($k_i = 0.01$) and one case where k_i is set to a negative value, resulting in a case with no integrator.
- The RW array number is configured either to 4 or 0
- The integral limit term is set to either 0 or 20
- The RW availability message is tested in 3 manners. Either the availability message is not written where all wheels should default to being available. If the availability message is written, then the RWs are either zero to available or not available.
- The control parameter $\delta\omega_0$ is set to either a zero or non-zero vector

All permutations of these test cases are expected to pass.

4 User Guide

This module requires the following variables:

- σ_{BN} as `guidCmdData.sigma_BR`
- ${}^{B}\omega_{BR}$ as `guidCmdData.omega_BR_B`
- ${}^{B}\omega_{RN}$ as `guidCmdData.omega_RN_B`
- ${}^{B}\dot{\omega}_{RN}$ as `guidCmdData.domega_RN_B`
- $[I]$, the inertia matrix of the body as `vehicleConfigOut.ISCPntB_B`
- Ω_i, speed of each reaction wheel in `rwSpeedMessage.wheelSpeeds`
- Gains k, P in `moduleConfig`.
- The integral gain K_i in `moduleConfig`. Setting this variable to a negative number disables the error integration for the controller, leaving just PI terms. Zero is not supported as a value for k_i. This variable is used to compute the integralLimit, used to limit the degree of integrator windup and reduce the chance of controller saturation. The integrator is required to maintain asymptotic tracking in the presence of an external disturbing torque.

REFERENCES

[1] Hanspeter Schaub and John L. Junkins. *Analytical Mechanics of Space Systems*. AIAA Education Series, Reston, VA, 3rd edition, 2014.