Autonomous Vehicle Simulation (AVS) Laboratory, University of Colorado

Basilisk Technical Memorandum

Document ID: Basilisk-MRP_Steering

NONLINEAR RATE SERVO FEEDBACK CONTROL MODULE

Prepared by	H. Schaub

Status: Initial Documentation
Scope/Contents
This module uses the MRP Steering control logic to determine the ADCS control torque vector $\boldsymbol{L}_r$.

Rev:	**Change Description**	**By**
Draft	Initial Documentation Draft	H. Schaub

Contents

---

1 Overview

This module computes a commanded control torque vector $\boldsymbol{L}_r$ using a rate based steering law that drives a body frame $\mathcal{B}:\{\hat{\boldsymbol{b}}_1,\hat{\boldsymbol{b}}_2,\hat{\boldsymbol{b}}_3\}$ towards a time varying reference frame $\mathcal{R}:\{\hat{\boldsymbol{r}}_1,\hat{\boldsymbol{r}}_2,\hat{\boldsymbol{r}}_3\}$.

2 Initialization

Simply call the module reset function prior to using this control module. This will reset the prior function call time variable, and reset the rotational rate error integral measure. The control update period Δt is evaluated automatically.

3 Steering Law Goals

This technical note develops a rate based steering law that drives a body frame $\mathcal{B}:\{\hat{\boldsymbol{b}}_1,\hat{\boldsymbol{b}}_2,\hat{\boldsymbol{b}}_3\}$ towards a time varying reference frame $\mathcal{R}:\{\hat{\boldsymbol{r}}_1,\hat{\boldsymbol{r}}_2,\hat{\boldsymbol{r}}_3\}$. The inertial frame is given by $\mathcal{N}:\{\hat{\boldsymbol{n}}_1,\hat{\boldsymbol{n}}_2,\hat{\boldsymbol{n}}_3\}$. The RW coordinate frame is given by $\mathcal{W}_\rangle:\{\hat{\boldsymbol{g}}_{s_i},\hat{\boldsymbol{g}}_{t_i},\hat{\boldsymbol{g}}_{g_i}\}$. Using MRPs, the overall control goal is

$$\boldsymbol{\sigma}_{\mathcal{B}/\mathcal{R}}\rightarrow 0 \tag{1}$$

The reference frame orientation $\boldsymbol{\sigma}_{\mathcal{R}/\mathcal{N}}$, angular velocity $\boldsymbol{\omega}_{\mathcal{R}/\mathcal{N}}$ and inertial angular acceleration $\dot{\boldsymbol{\omega}}_{\mathcal{R}/\mathcal{N}}$ are assumed to be known.

The rotational equations of motion of a rigid spacecraft with N Reaction Wheels (RWs) attached are given by[1]

$$[I_{RW}]\dot{\boldsymbol{\omega}}=-[\tilde{\boldsymbol{\omega}}]\left([I_{RW}]\boldsymbol{\omega}+[G_s]\boldsymbol{h}_s\right)-[G_s]\boldsymbol{u}_s+\boldsymbol{L} \tag{2}$$

where the inertia tensor $[I_{RW}]$ is defined as

$$[I_{RW}]=[I_s]+\sum_{i=1}^{N}\left(J_{t_i}\hat{\boldsymbol{g}}_{t_i}\hat{\boldsymbol{g}}_{t_i}^T+J_{g_i}\hat{\boldsymbol{g}}_{g_i}\hat{\boldsymbol{g}}_{g_i}^T\right) \tag{3}$$

The spacecraft inertial without the N RWs is $[I_s]$, while J_{s_i}, J_{t_i} and J_{g_i} are the RW inertias about the body fixed RW axis $\hat{\boldsymbol{g}}_{s_i}$ (RW spin axis), $\hat{\boldsymbol{g}}_{t_i}$ and $\hat{\boldsymbol{g}}_{g_i}$. The $3\times N$ projection matrix $[G_s]$ is then defined as

$$[G_s]=\left[\cdots\ {}^{\mathcal{B}}\hat{\boldsymbol{g}}_{s_i}\cdots\right] \tag{4}$$

The RW inertial angular momentum vector $\boldsymbol{h}_s$ is defined as

$$h_{s_i}=J_{s_i}(\omega_{s_i}+\Omega_i) \tag{5}$$

Here Ω_i is the i^{th} RW spin relative to the spacecraft, and the body angular velocity is written in terms of body and RW frame components as

$$\boldsymbol{\omega}=\omega_1\hat{\boldsymbol{b}}_1+\omega_2\hat{\boldsymbol{b}}_2+\omega_3\hat{\boldsymbol{b}}_3=\omega_{s_i}\hat{\boldsymbol{g}}_{s_i}+\omega_{t_i}\hat{\boldsymbol{g}}_{t_i}+\omega_{g_i}\hat{\boldsymbol{g}}_{g_i} \tag{6}$$

4 Angular Velocity Servo Sub-System

To implement the kinematic steering control, a servo sub-system must be included which will produce the required torques to make the actual body rates track the desired body rates. The angular velocity tracking error vector is defined as

$$\delta\boldsymbol{\omega} = \boldsymbol{\omega}_{\mathcal{B}/\mathcal{B}*} = \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} - \boldsymbol{\omega}_{\mathcal{B}*/\mathcal{N}} \tag{7}$$

where the $\mathcal{B}^*$ frame is the desired body frame from the kinematic steering law. Note that

$$\boldsymbol{\omega}_{\mathcal{B}*/\mathcal{N}} = \boldsymbol{\omega}_{\mathcal{B}*/\mathcal{R}} + \boldsymbol{\omega}_{\mathcal{R}/\mathcal{N}} \tag{8}$$

where $\boldsymbol{\omega}_{\mathcal{R}/\mathcal{N}}$ is obtained from the attitude navigation solution, and $\boldsymbol{\omega}_{\mathcal{B}*/\mathcal{R}}$ is the kinematic steering rate command. To create a rate-servo system that is robust to unmodeld torque biases, the state $\boldsymbol{z}$ is defined as:

$$\boldsymbol{z} = \int_{t_0}^{t_f} {}^{\mathcal{B}}\delta\boldsymbol{\omega}\ \mathrm{d}t \tag{9}$$

The rate servo Lyapunov function is defined as

$$V_{\boldsymbol{\omega}}(\delta\boldsymbol{\omega}, \boldsymbol{z}) = \frac{1}{2}\delta\boldsymbol{\omega}^T[I_{\text{RW}}]\delta\boldsymbol{\omega} + \frac{1}{2}\boldsymbol{z}^T[K_I]\boldsymbol{z} \tag{10}$$

where the vector $\delta\boldsymbol{\omega}$ and tensor $[I_{\text{RW}}]$ are assumed to be given in body frame components, $[K_i]$ is a symmetric positive definite matrix. The time derivative of this Lyapunov function is

$$\dot{V}_{\boldsymbol{\omega}} = \delta\boldsymbol{\omega}^T\left([I_{\text{RW}}]\delta\boldsymbol{\omega}' + [K_I]\boldsymbol{z}\right) \tag{11}$$

Using the identities $\boldsymbol{\omega}'_{\mathcal{B}/\mathcal{N}} = \dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}$ and $\boldsymbol{\omega}'_{\mathcal{R}/\mathcal{N}} = \dot{\boldsymbol{\omega}}_{\mathcal{R}/\mathcal{N}} - \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \boldsymbol{\omega}_{\mathcal{R}/\mathcal{N}}$,[1] the body frame derivative of $\delta\boldsymbol{\omega}$ is

$$\delta\boldsymbol{\omega}' = \dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} - \boldsymbol{\omega}'_{\mathcal{B}*/\mathcal{R}} - \dot{\boldsymbol{\omega}}_{\mathcal{R}/\mathcal{N}} + \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \boldsymbol{\omega}_{\mathcal{R}/\mathcal{N}} \tag{12}$$

Substituting Eqs. (2) and (12) into the $\dot{V}_{\boldsymbol{\omega}}$ expression in Eq. (11) yields

$$\begin{aligned}\dot{V}_{\boldsymbol{\omega}} = \delta\boldsymbol{\omega}^T\Big(&- [\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\left([I_{RW}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} + [G_s]\boldsymbol{h}_s\right) - [G_s]\boldsymbol{u}_s + \boldsymbol{L} + [K_I]\boldsymbol{z} \\ &- [I_{\text{RW}}](\boldsymbol{\omega}'_{\mathcal{B}*/\mathcal{R}} + \dot{\boldsymbol{\omega}}_{\mathcal{R}/\mathcal{N}} - \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \boldsymbol{\omega}_{\mathcal{R}/\mathcal{N}})\Big)\end{aligned} \tag{13}$$

Let $[P]^T = [P] >$ be a symmetric positive definite rate feedback gain matrix. The servo rate feedback control is defined as

$$\begin{aligned}[G_s]\boldsymbol{u}_s = [P]\delta\boldsymbol{\omega} + [K_I]\boldsymbol{z} - [\tilde{\boldsymbol{\omega}}_{\mathcal{B}*/\mathcal{N}}]\left([I_{\text{RW}}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} + [G_s]\boldsymbol{h}_s\right) \\ - [I_{\text{RW}}](\boldsymbol{\omega}'_{\mathcal{B}*/\mathcal{R}} + \dot{\boldsymbol{\omega}}_{\mathcal{R}/\mathcal{N}} - \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \boldsymbol{\omega}_{\mathcal{R}/\mathcal{N}}) + \boldsymbol{L}\end{aligned} \tag{14}$$

Defining the right-hand-side as $\boldsymbol{L}_r$, this is rewritten in compact form as

$$[G_s]\boldsymbol{u}_s = -\boldsymbol{L}_r \tag{15}$$

The array of RW motor torques can be solved with the typical minimum norm inverse

$$\boldsymbol{u}_s = [G_s]^T\left([G_s][G_s]^T\right)^{-1}(-\boldsymbol{L}_r) \tag{16}$$

To analyze the stability of this rate servo control, the $[G_s]\boldsymbol{u}_s$ expression in Eq. (14) is substituted into the Lyapunov rate expression in Eq. (13).

$$\begin{aligned}\dot{V}_\omega &= \delta\boldsymbol{\omega}^T\Big(-[P]\delta\boldsymbol{\omega} - [\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\left([I_{RW}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} + [G_s]\boldsymbol{h}_s\right) + [\tilde{\boldsymbol{\omega}}_{\mathcal{B}*/\mathcal{N}}]\left([I_{\text{RW}}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} + [G_s]\boldsymbol{h}_s\right)\Big) \\ &= \delta\boldsymbol{\omega}^T\Big(-[P]\delta\boldsymbol{\omega} - [\widetilde{\delta\boldsymbol{\omega}}]\left([I_{RW}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} + [G_s]\boldsymbol{h}_s\right)\Big) \\ &= -\delta\boldsymbol{\omega}^T[P]\delta\boldsymbol{\omega} < 0 \end{aligned} \tag{17}$$

Thus, in the absence of unmodeled torques, the servo control in Eq. (14) is asymptotically stabilizing in rate tracking error $\delta\boldsymbol{\omega}$.

Next, the servo robustness to unmodeled external torques is investigated. Let us assume that the external torque vector $\boldsymbol{L}$ in Eq. (2) only approximates the true external torque, and the unmodeled component is given by $\Delta\boldsymbol{L}$. Substituting the true equations of motion and the same servo control in Eq. (14) into the Lyapunov rate expression in Eq. (11) leads to

$$\dot{V}_\omega = -\delta\boldsymbol{\omega}^T[P]\delta\boldsymbol{\omega} - \delta\boldsymbol{\omega}^T\Delta\boldsymbol{L} \tag{18}$$

This $\dot{V}_\omega$ is no longer negative definite due to the underdetermined sign of the $\delta\boldsymbol{\omega}^T\Delta\boldsymbol{L}$ components. Equating the Lyapunov rates in Eqs. (11) and (18) yields the following servo closed loop dynamics:

$$[I_{\text{RW}}]\delta\boldsymbol{\omega}' + [P]\delta\boldsymbol{\omega} + [K_I]\boldsymbol{z} = \Delta\boldsymbol{L} \tag{19}$$

Assuming that $\Delta\boldsymbol{L}$ is either constant as seen by the body frame, or at least varies slowly, then taking a body-frame time derivative of Eq. (19) is

$$[I_{\text{RW}}]\delta\boldsymbol{\omega}'' + [P]\delta\boldsymbol{\omega}' + [K_I]\delta\boldsymbol{\omega} = \Delta\boldsymbol{L}' \approx 0 \tag{20}$$

As $[I_{\text{RW}}]$, $[P]$ and $[K_I]$ are all symmetric positive definite matrices, these linear differential equations are stable, and $\delta\boldsymbol{\omega} \to 0$ given that assumption that $\Delta\boldsymbol{L}' \approx 0$.

REFERENCES

[1] Hanspeter Schaub and John L. Junkins. *Analytical Mechanics of Space Systems*. AIAA Education Series, Reston, VA, 3rd edition, 2014.

[2] Panagiotis Tsiotras. New control laws for the attitude stabilization of rigid bodies. In *13th IFAC Symposium on Automatic Control in Aerospace*, pages 316–321, Palo Alto, CA, Sept. 12–16 1994.

[3] Peter Carlisle Hughes. *Spacecraft Attitude Dynamics*. John Wiley & Sons, Inc., New York, 1986.