Autonomous Vehicle Simulation (AVS) Laboratory, University of Colorado

Basilisk Technical Memorandum

SPACECRAFT

Rev	**Change Description**	**By**	**Date**
1.0	Initial Draft	C. Allard	20170808
1.1	Added BOE calcs	C. Allard	20170814
1.2	Fixed typos and added tolerance discussion	C. Allard	20170919
1.3	Small updates to module description, fixed typos	C. Allard	20170919
1.4	Changed location of document to allow for multi-spacecraft document	C. Allard	20180613
1.5	Added the ability to prescribe a rotational reference motion	H. Schaub	2020-10-01
1.6	Renamed module `spacecraft`	H. Schaub	2021-02-15

Contents

1 Model Description

1.1 Introduction

SpacecraftPlus is an instantiation of the dynamicObject abstract class. This abstract class is representing systems that have equations of motion that need to be integrated and therefore the main goal of this dynamic object is finding the state derivatives and interfacing with the integrator to integrate the state forward in time. SpacecraftPlus is representing a spacecraft that can be simulating only the translational movement which would mean the spacecraft only has mass (for gravity only simulations, setting the mass is not necessary), it could be simulating only rotational dynamics which would result in the spacecraft only having inertia, and finally both translational and rotational dynamics can be simulated at a time which results in the spacecraft having mass, inertia and center of mass offset.

SpacecraftPlus is the module where the equations of motion of the spacecraft are computed including the interaction between the hubEffector, stateEffectors and dynamicEffectors. The hubEffector is where the translational and rotational state derivatives are computed, the stateEffectors give contributions to spacecraftPlus and computes their own derivatives and the dynamcEffectors provide force and torque contributions to spacecraftPlus.

To give more description on this complex relationship, Figure 1 shows an example of a spacecraft with a fuel slosh particle and two hinged rigid bodies connected to it. From a spacecraftPlus perspective there are some important variables and frame definitions needed to be defined. The body frame, $\mathcal{B} : \{\hat{\boldsymbol{b}}_1, \hat{\boldsymbol{b}}_2, \hat{\boldsymbol{b}}_3\}$, is a Cartesian reference frame that is fixed to the hub (the rigid body of the spacecraft) with no constraints on its orientation with respect to the hub. Additionally its origin, B, can be placed anywhere on the hub. The inertial frame, $\mathcal{N} : \{\hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}_2, \hat{\boldsymbol{n}}_3\}$, is the inertial reference frame used in the formulation of the dynamics. The hub has some remaining variables needed to be defined which are the mass of the hub, m_{hub}, center of mass of the hub location, B_c, and the inertia of the hub about its center of mass, $[I_{\text{hub},B_c}]$.

More variables needed to be defined for the spacecraftPlus, are the total mass of the spacecraft, m_{sc}, the total inertia of the spacecraft about point B, $[I_{\text{sc},B}]$, the center of mass location of the total spacecraft, point C, and the vector $\boldsymbol{c}$ which points from point B to point C. Additionally, the state variables for the hub are $\boldsymbol{r}_{B/N}$, $\dot{\boldsymbol{r}}_{B/N}$, $\boldsymbol{\sigma}_{\mathcal{B}/\mathcal{N}}$ and $\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}}$, which are the position and velocity of point B with respect to N which describes the translational motion of the spacecraft, the MRP set for the orientation of $\mathcal{B}$ with respect to $\mathcal{N}$, and the angular velocity of the $\mathcal{B}$ with respect to $\mathcal{N}$ with describes the rotation of the spacecraft, respectively.

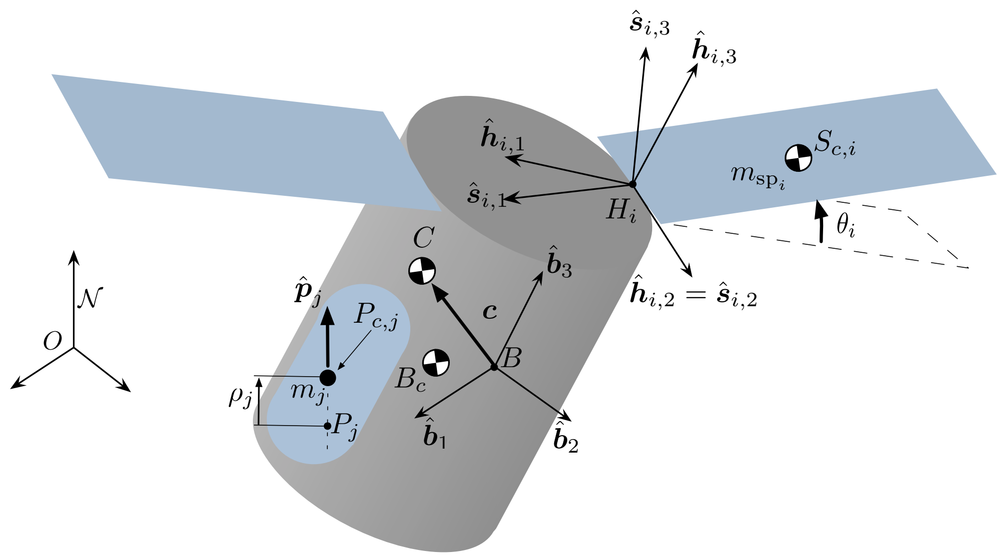

Fig. 1: Complex spacecraft that can be simulated using spacecraftPlus

1.2 Equations of Motion

The main description that is needed for spacecraftPlus are the equations of motion (EOMs). The following equation describes the translational EOM if only translation is enabled. This is Newton's

Second Law on the center of mass of the spacecraft but when translation only is enabled the center of mass of the spacecraft is coincident with point B.

$$m_{\text{sc}}\ddot{\boldsymbol{r}}_{B/N} = m_{\text{sc}}\ddot{\boldsymbol{r}}_{C/N} = \boldsymbol{F}_{\text{ext}} \tag{1}$$

The following equation describes the rotational EOM if only rotational is enabled. This is similar to what can be seen in Reference[2]

$$\Big[[I_{\text{sc},B}] + \sum_{i=1}^{N}[D_{\text{contr},i}]\Big]\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} = -[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][I_{\text{sc},B}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} + \boldsymbol{L}_B + \sum_{i=1}^{N}\boldsymbol{v}_{\text{rot,contr},i} \tag{2}$$

$[D_{\text{contr},i}]$ and $\boldsymbol{v}_{\text{rot},i}$ will be defined in the next set of equations. The $[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]$ matrix is the matrix representation of the cross product operation.

Finally, the two following equations describe the translational and rotational EOMs when the both rotational and translational equations are enabled.

$$\begin{aligned}\Big(m_{\text{sc}}[I_{3\times3}] + \sum_{i=1}^{N}[A_{\text{contr},i}]\Big)\ddot{\boldsymbol{r}}_{B/N} + \Big(-m_{\text{sc}}[\tilde{\boldsymbol{c}}] + \sum_{i=1}^{N}[B_{\text{contr},i}]\Big)\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}\\ = \boldsymbol{F}_{\text{ext}} - 2m_{\text{sc}}[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\boldsymbol{c}' - m_{\text{sc}}[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\boldsymbol{c} + \sum_{i=1}^{N}\boldsymbol{v}_{\text{trans,contr},i}\end{aligned} \tag{3}$$

$$\begin{aligned}\Big[m_{\text{sc}}[\tilde{\boldsymbol{c}}] + \sum_{i=1}^{N}[C_{\text{contr},i}]\Big]\ddot{\boldsymbol{r}}_{B/N} + \Big[[I_{\text{sc},B}] + \sum_{i=1}^{N}[D_{\text{contr},i}]\Big]\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}\\ = -[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][I_{\text{sc},B}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} - [I'_{\text{sc},B}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} + \boldsymbol{L}_B + \sum_{i=1}^{N}\boldsymbol{v}_{\text{rot,contr},i}\end{aligned} \tag{4}$$

Where $[A_{\text{contr},i}]$, $[B_{\text{contr},i}]$, $[C_{\text{contr},i}]$, $[D_{\text{contr},i}]$, $\boldsymbol{v}_{\text{trans,contr},i}$, and $\boldsymbol{v}_{\text{rot,contr},i}$, are the contributions from the stateEffectors using the back-substitution method seen in Reference[1] and also discussed more in detail in the hinged rigid body state effector document.

The equations can now be organized into the following matrix representation:

$$\begin{bmatrix}[A] & [B]\\ [C] & [D]\end{bmatrix}\begin{bmatrix}\ddot{\boldsymbol{r}}_{B/N}\\ \dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}\end{bmatrix} = \begin{bmatrix}\boldsymbol{v}_{\text{trans}}\\ \boldsymbol{v}_{\text{rot}}\end{bmatrix} \tag{5}$$

And are solved using the following equations:

$$\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} = \Big([D] - [C][A]^{-1}[B]\Big)^{-1}(\boldsymbol{v}_{\text{rot}} - [C][A]^{-1}\boldsymbol{v}_{\text{trans}}) \tag{6}$$

$$\ddot{\boldsymbol{r}}_{B/N} = [A]^{-1}(\boldsymbol{v}_{\text{trans}} - [B]\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}) \tag{7}$$

This shows that only two 3×3 matrices are needed to solve this system of equations. One remaining equation that should be included is the kinematic relationship between $\boldsymbol{\sigma}_{\mathcal{B}/\mathcal{N}}$ and $\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}}$. This relationship can be seen in the following equation:

$$\dot{\boldsymbol{\sigma}}_{\mathcal{B}/\mathcal{N}} = \frac{1}{4}[B(\boldsymbol{\sigma})]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \tag{8}$$

Where the full definition of $[B(\boldsymbol{\sigma})]$ can be seen in Reference.[2] The kinematic relationship between $\boldsymbol{r}_{B/N}$ and $\dot{\boldsymbol{r}}_{B/N}$ is trivial because the inertial time derivative of $\boldsymbol{r}_{B/N}$ is equal to $\dot{\boldsymbol{r}}_{B/N}$. This completes the necessary equations needed for discussion on the EOMs of the system.

1.3 Energy and Momentum Calculations

A key part of spacecraftPlus is finding the total energy and momentum of the spacecraft for validation purposes. This section describes how the energy and momentum is calculated in spacecraftPlus.

1.3.1 Total Orbital Kinetic Energy

The total orbital kinetic energy (i.e. kinetic energy of the center of mass) of the spacecraft is

$$T_{\text{orb}} = \frac{1}{2} m_{sc} \dot{\boldsymbol{r}}_{C/N} \cdot \dot{\boldsymbol{r}}_{C/N} \tag{9}$$

Expanding $\dot{\boldsymbol{r}}_{C/N}$ to be in terms of $\dot{\boldsymbol{r}}_{B/N}$ and $\dot{\boldsymbol{c}}$ results in

$$T_{\text{orb}} = \frac{1}{2} m_{sc} (\dot{\boldsymbol{r}}_{B/N} + \dot{\boldsymbol{c}}) \cdot (\dot{\boldsymbol{r}}_{B/N} + \dot{\boldsymbol{c}}) \tag{10}$$

Which simplifies to final desired equation

$$T_{\text{orb}} = \frac{1}{2} m_{sc} (\dot{\boldsymbol{r}}_{B/N} \cdot \dot{\boldsymbol{r}}_{B/N} + 2\dot{\boldsymbol{r}}_{B/N} \cdot \dot{\boldsymbol{c}} + \dot{\boldsymbol{c}} \cdot \dot{\boldsymbol{c}}) \tag{11}$$

1.3.2 Total Orbital Potential Energy

The total orbital potential energy depends on what type of gravity model you are using. For simplicity, the orbital potential energy due to point gravity is included here but would need to be changed for spherical harmonics, etc.

$$V_{\text{orb}} = \frac{\mu}{|\boldsymbol{r}_{C/N}|} \tag{12}$$

1.3.3 Total Orbital Energy

Since the total orbital energy of the spacecraft must be conserved when there are no non-conservative external forces and torques acting on the spacecraft, it convenient to combine the kinetic and potential energies into one term, E_{orb}. This can be seen in the following equation.

$$E_{\text{orb}} = T_{\text{orb}} - V_{\text{orb}} \tag{13}$$

1.3.4 Total Rotational and Deformational Kinetic Energy

The total rotational and deformational kinetic energy (i.e. kinetic energy about the center of mass) of the spacecraft is

$$\begin{aligned} T_{\text{rot}} = &\frac{1}{2} \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}}^T [I_{\text{hub},B_c}] \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} + \frac{1}{2} m_{\text{hub}} \dot{\boldsymbol{r}}_{B_c/C} \cdot \dot{\boldsymbol{r}}_{B_c/C} \\ &+ \sum_i^N \Big(\frac{1}{2} \boldsymbol{\omega}_{\mathcal{E}_i/\mathcal{N}}^T [I_{\text{eff},E_{c,i}}] \boldsymbol{\omega}_{\mathcal{E}_i/\mathcal{N}} + \frac{1}{2} m_{\text{eff}} \dot{\boldsymbol{r}}_{E_{c,i}/C} \cdot \dot{\boldsymbol{r}}_{E_{c,i}/C} \Big) \end{aligned} \tag{14}$$

Where N is the number of state effectors attached to the hub, "eff" is the current state effector which a frame specified as $\mathcal{E}_i$ and a center of mass location labeled as point $E_{c,i}$. Expanding these terms similar to orbital kinetic energy results in

$$\begin{aligned} T_{\text{rot}} = &\frac{1}{2} \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}}^T [I_{\text{hub},B_c}] \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} + \frac{1}{2} m_{\text{hub}} (\dot{\boldsymbol{r}}_{B_c/B} - \dot{\boldsymbol{c}}) \cdot (\dot{\boldsymbol{r}}_{B_c/B} - \dot{\boldsymbol{c}}) \\ &+ \sum_i^N \Big[\frac{1}{2} \boldsymbol{\omega}_{\mathcal{E}_i/\mathcal{N}}^T [I_{\text{eff},E_{c,i}}] \boldsymbol{\omega}_{\mathcal{E}_i/\mathcal{N}} + \frac{1}{2} m_{\text{eff}} (\dot{\boldsymbol{r}}_{E_{c,i}/B} - \dot{\boldsymbol{c}}) \cdot (\dot{\boldsymbol{r}}_{E_{c,i}/B} - \dot{\boldsymbol{c}}) \Big] \end{aligned} \tag{15}$$

Expanding further

$$T_{\text{rot}} = \frac{1}{2}\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}}^T[I_{\text{hub},B_c}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} + \frac{1}{2}m_{\text{hub}}(\dot{\boldsymbol{r}}_{B_c/B}\cdot\dot{\boldsymbol{r}}_{B_c/B} - 2\dot{\boldsymbol{r}}_{B_c/B}\cdot\dot{\boldsymbol{c}} + \dot{\boldsymbol{c}}\cdot\dot{\boldsymbol{c}})$$
$$+ \sum_i^N \Big[\frac{1}{2}\boldsymbol{\omega}_{\mathcal{E}_i/\mathcal{N}}^T[I_{\text{eff},E_{c,i}}]\boldsymbol{\omega}_{\mathcal{E}_i/\mathcal{N}} + \frac{1}{2}m_{\text{eff}}(\dot{\boldsymbol{r}}_{E_{c,i}/B}\cdot\dot{\boldsymbol{r}}_{E_{c,i}/B} - 2\dot{\boldsymbol{r}}_{E_{c,i}/B}\cdot\dot{\boldsymbol{c}} + \dot{\boldsymbol{c}}\cdot\dot{\boldsymbol{c}})\Big] \quad (16)$$

Combining like terms results in

$$T_{\text{rot}} = \frac{1}{2}\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}}^T[I_{\text{hub},B_c}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} + \frac{1}{2}m_{\text{hub}}\dot{\boldsymbol{r}}_{B_c/B}\cdot\dot{\boldsymbol{r}}_{B_c/B} + \sum_i^N\Big[\frac{1}{2}\boldsymbol{\omega}_{\mathcal{E}_i/\mathcal{N}}^T[I_{\text{eff},E_{c,i}}]\boldsymbol{\omega}_{\mathcal{E}_i/\mathcal{N}} + \frac{1}{2}m_{\text{eff}}\dot{\boldsymbol{r}}_{E_{c,i}/B}\cdot\dot{\boldsymbol{r}}_{E_{c,i}/B}\Big]$$
$$- \Big[m_{\text{hub}}\dot{\boldsymbol{r}}_{B_c/B} + \sum_i^N m_{\text{eff}}\dot{\boldsymbol{r}}_{E_{c,i}/B}\Big]\cdot\dot{\boldsymbol{c}} + \frac{1}{2}\Big[m_{\text{hub}} + \sum_i^N m_{\text{eff}}\Big]\dot{\boldsymbol{c}}\cdot\dot{\boldsymbol{c}} \quad (17)$$

Performing a final simplification yields

$$T_{\text{rot}} = \frac{1}{2}\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}}^T[I_{\text{hub},B_c}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} + \frac{1}{2}m_{\text{hub}}\dot{\boldsymbol{r}}_{B_c/B}\cdot\dot{\boldsymbol{r}}_{B_c/B}$$
$$+ \sum_i^N\Big[\frac{1}{2}\boldsymbol{\omega}_{\mathcal{E}_i/\mathcal{N}}^T[I_{\text{eff},E_{c,i}}]\boldsymbol{\omega}_{\mathcal{E}_i/\mathcal{N}} + \frac{1}{2}m_{\text{eff}}\dot{\boldsymbol{r}}_{E_{c,i}/B}\cdot\dot{\boldsymbol{r}}_{E_{c,i}/B}\Big] - \frac{1}{2}m_{sc}\dot{\boldsymbol{c}}\cdot\dot{\boldsymbol{c}} \quad (18)$$

1.3.5 Total Rotational Potential Energy

The total rotational potential energy is specific to each stateEffector. For example, the potential energy for hinged rigid bodies can be seen in the following equation.

$$V_{\text{rot}} = \frac{1}{2}k_\theta\theta^2 \quad (19)$$

Each stateEffector might not have a potential energy contribution, however each stateEffector will have the ability to add their contribution to the total potential energy.

1.3.6 Total Rotational Energy

Since the total rotational energy of the system is conserved when there are no non-conservative internal forces or torques acting on the system, it is convenient to combine the kinetic and potential energies into one term, E_{rot}. This can be seen in the following equation.

$$E_{\text{rot}} = T_{\text{rot}} + V_{\text{rot}} \quad (20)$$

1.4 Angular Momentum

1.4.1 Total Orbital Angular Momentum

The total orbital angular momentum of the spacecraft about point N is

$$\boldsymbol{H}_{\text{orb},N} = m_{sc}\boldsymbol{r}_{C/N}\times\dot{\boldsymbol{r}}_{C/N} \quad (21)$$

Expanding these terms yields

$$\boldsymbol{H}_{\text{orb},N} = m_{sc}(\boldsymbol{r}_{B/N} + \boldsymbol{c})\times(\dot{\boldsymbol{r}}_{B/N} + \dot{\boldsymbol{c}}) \quad (22)$$

The final form of this equation is

$$\boldsymbol{H}_{\text{orb},N} = m_{sc}\Big[\boldsymbol{r}_{B/N}\times\dot{\boldsymbol{r}}_{B/N} + \boldsymbol{r}_{B/N}\times\dot{\boldsymbol{c}} + \boldsymbol{c}\times\dot{\boldsymbol{r}}_{B/N} + \boldsymbol{c}\times\dot{\boldsymbol{c}}\Big] \quad (23)$$

1.4.2 Total Rotational Angular Momentum

The total rotational angular momentum of the spacecraft about point C is

$$\boldsymbol{H}_{\text{rot},C} = [I_{\text{hub},B_c}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} + m_{\text{hub}}\boldsymbol{r}_{B_c/C} \times \dot{\boldsymbol{r}}_{B_c/C} + \sum_{i}^{N}\Big[[I_{\text{eff},E_{c,i}}]\boldsymbol{\omega}_{\mathcal{E}_i/\mathcal{N}} + m_{\text{eff}}\boldsymbol{r}_{E_{c,i}/C} \times \dot{\boldsymbol{r}}_{E_{c,i}/C}\Big] \tag{24}$$

Expanding these terms yields

$$\boldsymbol{H}_{\text{rot},C} = [I_{\text{hub},B_c}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} + m_{\text{hub}}(\boldsymbol{r}_{B_c/B} - \boldsymbol{c}) \times (\dot{\boldsymbol{r}}_{B_c/B} - \dot{\boldsymbol{c}}) \\ + \sum_{i}^{N}\Big[[I_{\text{eff},E_{c,i}}]\boldsymbol{\omega}_{\mathcal{E}_i/\mathcal{N}} + m_{\text{eff}}(\boldsymbol{r}_{E_{c,i}/B} - \boldsymbol{c}) \times (\dot{\boldsymbol{r}}_{E_{c,i}/B} - \dot{\boldsymbol{c}})\Big] \tag{25}$$

Distributing this result

$$\boldsymbol{H}_{\text{rot},C} = [I_{\text{hub},B_c}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} + m_{\text{hub}}\Big(\boldsymbol{r}_{B_c/B} \times \dot{\boldsymbol{r}}_{B_c/B} - \boldsymbol{r}_{B_c/B} \times \dot{\boldsymbol{c}} - \boldsymbol{c} \times \dot{\boldsymbol{r}}_{B_c/B} + \boldsymbol{c} \times \dot{\boldsymbol{c}}\Big) \\ + \sum_{i}^{N}\Big[[I_{\text{eff},E_{c,i}}]\boldsymbol{\omega}_{\mathcal{E}_i/\mathcal{N}} + m_{\text{eff}}\Big(\boldsymbol{r}_{E_{c,i}/B} \times \dot{\boldsymbol{r}}_{E_{c,i}/B} - \boldsymbol{r}_{E_{c,i}/B} \times \dot{\boldsymbol{c}} - \boldsymbol{c} \times \dot{\boldsymbol{r}}_{E_{c,i}/B} + \boldsymbol{c} \times \dot{\boldsymbol{c}}\Big)\Big] \tag{26}$$

Simplifying this result yields the final equation

$$\boldsymbol{H}_{\text{rot},C} = [I_{\text{hub},B_c}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} + m_{\text{hub}}\boldsymbol{r}_{B_c/B} \times \dot{\boldsymbol{r}}_{B_c/B} \\ + \sum_{i}^{N}\Big[[I_{\text{eff},E_{c,i}}]\boldsymbol{\omega}_{\mathcal{E}_i/\mathcal{N}} + m_{\text{eff}}\boldsymbol{r}_{E_{c,i}/B} \times \dot{\boldsymbol{r}}_{E_{c,i}/B}\Big] - m_{sc}\boldsymbol{c} \times \dot{\boldsymbol{c}} \tag{27}$$

1.4.3 Contributions from Hub and State Effectors

During the integrate state method (after the integrator call), the spaceCraftPlus will ask both the hubEffector and stateEffectors for their contributions to $\boldsymbol{c}$, $\dot{\boldsymbol{c}}$, T_{orb}, E_{rot}, $\boldsymbol{H}_{\text{orb},N}$ and $\boldsymbol{H}_{\text{rot},C}$. The spaceCraftPlus will then manage the addition of these values and the modfying factors seen in the equations above.

2 Model Functions

This module is intended to be used as a model to represent a spacecraft that can be decomposed into a rigid body hub and has the ability to model state effectors such as reactions wheels, and flexing solar panels, etc attached to the hub.

- Updates the mass properties of the spacecraft by adding up all of the contributions to the mass properties from the hub and the state effectors
- Adds up all of the matrix contributions from the hub and state effectors for the back substitution method and gives this information to the hub effector
- Adds up all of the force and torque contributions from dynamicEffectors and gravityEffector
- Calls all of the *computeDerivatives* methods for the hub and all of the state effectors which is essentially doing $\dot{\boldsymbol{X}} = \boldsymbol{f}(\boldsymbol{X}, t)$
- Integrates the states forward one time step using the selected integrator
- Calculates the total energy and momentum of the spacecraft by adding up contributions from the hub and the state effectors
- Be able to have the attitude states and rates be prescribed through an optional input message

3 Model Assumptions and Limitations

Below is a summary of the assumptions/limitations:

- Translational only mode cannot have state effectors attached to it that change the mass properties of the spacecraft. However, a state effector like a battery could be used since this does not change the mass properties of the spacecraft.
- Rotational only mode can only have state effectors attached to it that do not change the mass properties of the spacecraft, i.e. balanced reaction wheels, and the $\mathcal{B}$ frame origin must coincident with the center of mass of the spacecraft.
- State effectors that are changing the mass properties of the spacecraft are considered new bodies that are added to the mass properties of the spacecraft. For example, adding flexing solar panels to the sim would require subtracting the mass and inertia of the solar panels from the total spacecraft which would then represent the rigid body hub, and then when the solar panels are added to the sim, their mass inertia are programatically added back to the spacecraft.
- The limitations of the sim are primarily based on what configuration you have set the spacecraft in (i.e. what state effectors and dynamic effectors you have attached to the spacecraft). Additionally you are limited to the current capability of Basilisk in regards to state effectors and dynamic effectors.
- The accuracy of the simulation is based upon the integrator and integrator step size
- As discussed in the description section, the body fixed frame $\mathcal{B}$ can be oriented generally and the origin of the $\mathcal{B}$ frame can be placed anywhere as long as it is fixed with respect to the body. This means that there no limitations from the rigid body hub perspective.

4 Test Description and Success Criteria

This test is located in `simulation/dynamics/spacecraftPlus/_UnitTest/test_spacecraftPlus.py`. Depending on the scenario, there are different success criteria. These are outlined in the following subsections:

4.1 Translation Only with Gravity Scenario

In this test the simulation is placed into orbit around Earth with point gravity and the spacecraft only has translational equations being evaluated. The following parameters are being tested.

- Conservation of orbital angular momentum
- Conservation of orbital energy
- Achieving the expected final position

4.2 Rotational Only Scenario

In this test, the spacecraft only has rotational equations being evaluated. The following parameters describe the success criteria.

- Conservation of rotational angular momentum
- Conservation of rotational energy

- Achieving the expected final attitude
- Switching of MRPs check
- Agreement with BOE calculation for rotational dynamics

The calculations for both the switching of the MRPs and the BOE for rotational dynamics need to be further discussed. The following sections outline these checks.

4.2.1 MRP switching test

The MRP switching check needs to be discussed. In Basilisk the MRPs are switched to the shadow set in *hubEffector* after one step of the integration using the method *modifyStates()* which is available to all *stateEffectors* that need to change their states to a different but equivalent form. The MRP switching adheres to the following equation:[2]

$$\begin{aligned} &\textbf{if} \quad [s = |\boldsymbol{\sigma}(t + \mathrm{d}t)|] > 1 \quad \textbf{then} \\ &\quad \boldsymbol{\sigma}(t + \mathrm{d}t) = -\frac{\boldsymbol{\sigma}(t + \mathrm{d}t)}{s^2} \\ &\textbf{end if} \end{aligned} \tag{28}$$

To check that the switch in the simulation is behaving the way it should, the following check was developed. If the switch happened at time t_{s}, then there are two variables from the sim that will be used: $\boldsymbol{\sigma}(t_{\text{s-1}})$ and $\boldsymbol{\sigma}(t_{\mathsf{s}})$. The intermediate MRP that is switched in the sim is not an output of the simulation, but we will call this variable $\boldsymbol{\sigma}_0(t_{\mathsf{s}})$. To check the switching the following math occurs:

$$\boldsymbol{\sigma}_0(t_{\mathsf{s}}) \approx \boldsymbol{\sigma}(t_{\text{s-1}}) + \frac{\boldsymbol{\sigma}(t_{\text{s-1}}) - \boldsymbol{\sigma}(t_{\text{s-2}})}{\Delta t}\Delta t \tag{29}$$

Where this is an Euler approximation to the intermediate MRP before the switch occurs. Now using Eq. (28) the following definition is made:

$$\boldsymbol{\sigma}_{\mathsf{ch}}(t_{\mathsf{s}}) = -\frac{\boldsymbol{\sigma}_0(t_{\mathsf{s}})}{|\boldsymbol{\sigma}_0(t_{\mathsf{s}})|^2} \tag{30}$$

Where $\boldsymbol{\sigma}_{\mathsf{ch}}(t_{\mathsf{s}})$ is the MRP to check vs. the simulation MRP. Therefore, in the integrated test, the test is making sure that $\boldsymbol{\sigma}(t_{\mathsf{s}}) \approx \boldsymbol{\sigma}_{\mathsf{ch}}(t_{\mathsf{s}})$

4.2.2 Rotational Dynamics BOE description

To validate the rotational dynamics, a relationship that involves both the attitude and attitude rate was chosen. The following back of the envelope calculation is taken from[2] and repeated here for convenience. The angular momentum vector is chosen to be aligned with the inertial frame:

$${}^{\mathcal{N}}\boldsymbol{H} = -H\hat{\boldsymbol{n}}_3 = {}^{\mathcal{N}}\begin{bmatrix} 0 \\ 0 \\ -H \end{bmatrix} \tag{31}$$

The following relationship is written:

$${}^{\mathcal{B}}\boldsymbol{H} = [BN]^{\mathcal{N}}\boldsymbol{H} \tag{32}$$

Since MRPs are the attitude parameterization chosen for Basilisk, then the direction cosine matrix $[BN]$ is written in terms of the current MRPs. If there are no external torque's acting on the spacecraft, then the angular momentum vector will be conserved in the inertial frame. Therefore, using the definition of transformation from MRPs to the direction cosine matrix,[2] the following relationship will always hold:

$$ {}^{\mathcal{B}}\begin{bmatrix} H_1 \\ H_2 \\ H_3 \end{bmatrix} = -H \, {}^{\mathcal{B}}\begin{bmatrix} 8\sigma_1\sigma_3 - 4\sigma_2(1-\sigma^2) \\ 8\sigma_2\sigma_3 + 4\sigma_1(1-\sigma^2) \\ 4(-\sigma_1^2 - \sigma_2^2 + \sigma_3^2) + (1-\sigma^2)^2 \end{bmatrix} = {}^{\mathcal{B}}\begin{bmatrix} I_1\omega_1 \\ I_2\omega_2 \\ I_3\omega_3 \end{bmatrix} \tag{33} $$

Finally, the current angular velocity components in the body frame can be found from the current MRPs using the following relationship:

$$ \omega_1 = -\frac{H}{I_1}\Big[8\sigma_1\sigma_3 - 4\sigma_2(1-\sigma^2)\Big] \tag{34} $$

$$ \omega_2 = -\frac{H}{I_2}\Big[8\sigma_2\sigma_3 + 4\sigma_1(1-\sigma^2)\Big] \tag{35} $$

$$ \omega_3 = -\frac{H}{I_3}\Big[4(-\sigma_1^2 - \sigma_2^2 + \sigma_3^2) + (1-\sigma^2)^2\Big] \tag{36} $$

This gives a closed from solution between the current MRPs and the angular velocity of the spacecraft. The test picks 5 points during a simulation and verifies that this relationship holds true.

4.3 Translational and Rotational Scenario

In this test, the spacecraft is placed into an orbit with simple gravity and also has rotational states associated with the spacecraft. The following parameters describe the success criteria.

- Conservation of orbital angular momentum
- Conservation of orbital energy
- Achieving the expected final position (same as translational only test)
- Conservation of rotational angular momentum
- Conservation of rotational energy
- Achieving the expected final attitude (same as rotational only test)

4.4 Translational BOE Calculation Scenario

The translational BOE calculation can be seen in Figure 2. In this test a positive force is placed on the hub in the $\hat{\boldsymbol{b}}_1$ direction with no torque and no initial rotation of the spacecraft. This results in the 1 degree of freedom problem seen in Figure 2. The force is applied for some length of time, left off for a length of time, and then a negative force is applied ot the system for some length of time. The test is ensuring that Basilisk is giving the same results as the BOE calculation.

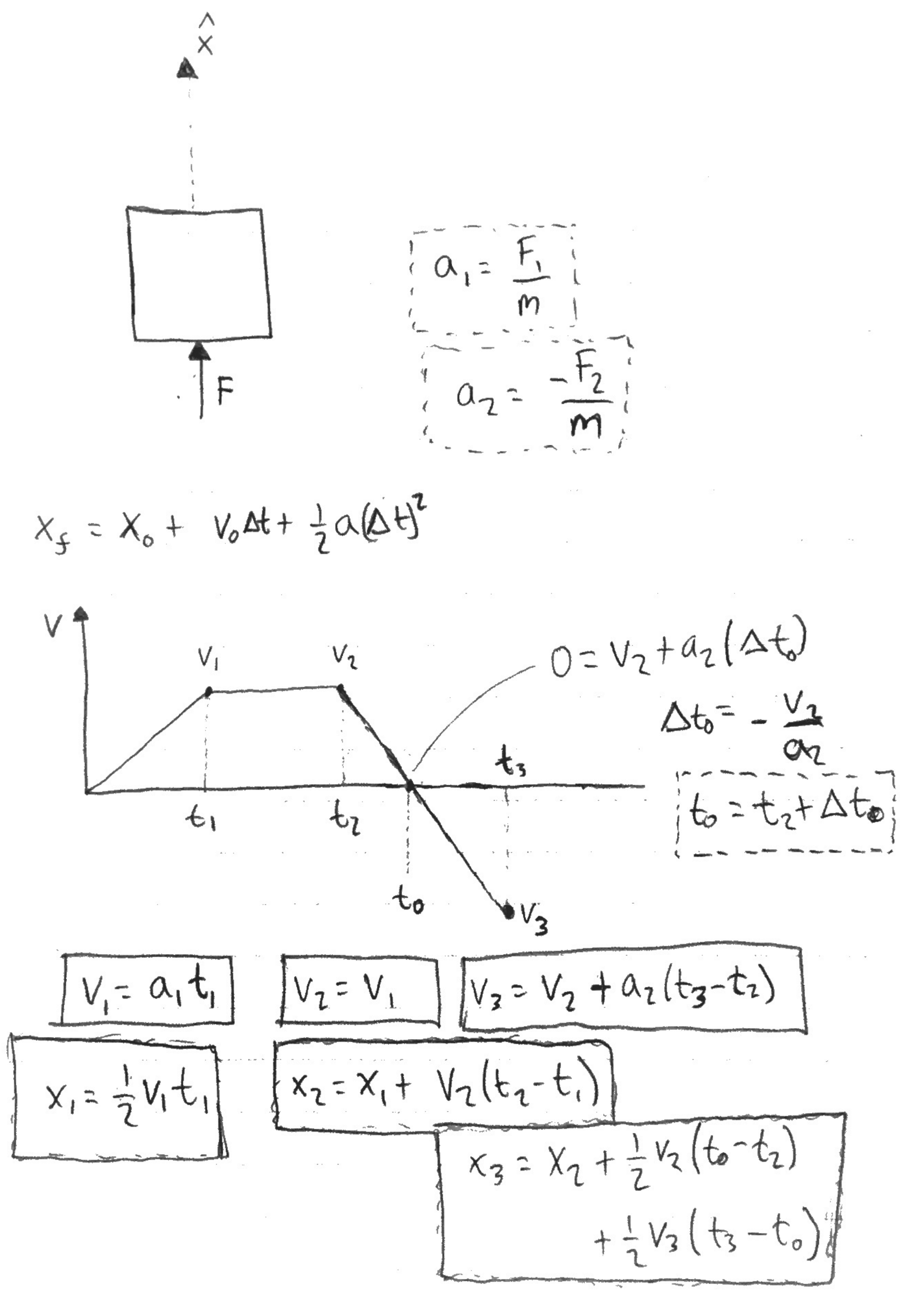

Fig. 2: Simple Translation BOE Calculation

4.5 Dynamics Calculated About Point B Test

The dynamics for the spacecraft have been derived in relation to the body frame origin, point B, which is not necessarily coincident with point C. Therefore the dynamicEffectors and stateEffectors define their torques about point B as oppose to about point C. This allows the simulation to have multi-body dynamics where the center of mass of the spacecraft can move with respect to the body frame. However, in rigid body dynamics it is very common to have the body frame origin coincident with the center of mass of the spacecraft. Therefore, to confirm that the dynamics has been developed correctly to handle this torque mapping a test has been created.

The test runs two simulations: one with the body frame origin defined at the center of mass of the spacecraft, and one with the body frame origin not coincident with the center of mass. An external force and corresponding torque is applied to the spacecraft in both cases: in the first case the torque is being defined about point $B = C$, and in the second case the torque is being defined about point B. Both simulations are given identical initial conditions and the expectation is that the simulations should give identical results. The following parameters describe the success criteria.

- Agreement between both simulations for the translational states
- Agreement between both simulations for the attitude states

5 Test Parameters

Since this is an integrated test, the inputs to the test are the physical parameters of the spacecraft along with the initial conditions of the states. These parameters are outlined in Tables 2- 3. Additionally, the error tolerances can be seen in Table 4.

The energy/momentum conservation, rotational BOE, and translational BOE relative tolerance values were chosen to be 1e-10 to ensure cross-platform success. However, the expected relative tolerance for these three tests are $\approx$ 1e-15, which is about machine precision. This is because the integration time step was low enough that the integrator was introducing very small errors with respect to the exact solution. This gives great confidence in the simulations. The position and attitude relative tolerance values (including Point B Vs. Point C tests) were chosen because the values saved in python to compare to had 10 significant digits. Therefore, 1e-8 was chosen to ensure platform success. This agreement is very good and gives further confidence in the solution. Finally, the MRP switching relative tolerance was chosen to be 1e-5 because of the time step dependency of this test. Since the test involves a numerical difference the accuracy gets better the smaller the step size. For example with a step size of 0.001, the resulting relative accuracy was 1e-7 but if a step size of 0.001 is used, 1e-13 was the resulting relative accuracy. Therefore, 1e-5 was chosen to ensure cross platform success and to use a step size of 0.001 for speed. However, since as the step size goes down the simulation approaches the analytical solution, this tests gives very good confidence in the simulation.

Table 2: Spacecraft Hub Parameters

Name	Description	Value	Units
mHub	mass	100.0	kg
IHubPntBc_B	Inertia in $\mathcal{B}$ frame	$\begin{bmatrix} 500.0 & 0.0 & 0.0 \\ 0.0 & 200.0 & 0.0 \\ 0.0 & 0.0 & 300.0 \end{bmatrix}$	kg-m^2
r_BcB_B	CoM Location in $\mathcal{B}$ frame	$\begin{bmatrix} 0.0 & 0.0 & 0.0 \end{bmatrix}^T$	m

Table 3: Initial Conditions of the simulations

Name	Description	Value	Units
r_CN_NInit	Initial Position of S/C (gravity scenarios)	$[-4020339 \quad 7490567 \quad 5248299]^T$	m
v_CN_NInit	Initial Velocity of S/C (gravity scenarios)	$[-5199.78 \quad -3436.68 \quad 1041.58]^T$	m/s
r_CN_NInit	Initial Position of S/C (no gravity)	$[0.0 \quad 0.0 \quad 0.0]^T$	m
v_CN_NInit	Initial Velocity of S/C (no gravity)	$[0.0 \quad 0.0 \quad 0.0]^T$	m/s
sigma_BNInit	Initial MRP of $\mathcal{B}$ frame (All Except Rotation Only)	$[0.0 \quad 0.0 \quad 0.0]^T$	-
sigma_BNInit	Initial MRP of $\mathcal{B}$ frame (Rotation Only)	$[0.09734 \quad 0.62362 \quad 0.04679]^T$	-
omega_BN_BInit	Initial Angular Velocity (All Except Translation BOE)	$[0.5 \quad -0.4 \quad 0.7]^T$	rad/s
omega_BN_BInit	Initial Angular Velocity (Translation BOE)	$[0.0 \quad 0.0 \quad 0.0]^T$	rad/s

Table 4: Error Tolerance - Note: Relative Tolerance is abs($\frac{\text{truth}-\text{value}}{\text{truth}}$)

Test	Relative Tolerance
Energy and Momentum Conservation	1e-10
Position Check	1e-8
Attitude Check	1e-8
MRP Switching Check	1e-5
Rotational BOE	1e-10
Translational BOE	1e-10
Point B Vs. Point C	1e-8

6 Test Results

6.1 Translation Only with Gravity Scenario

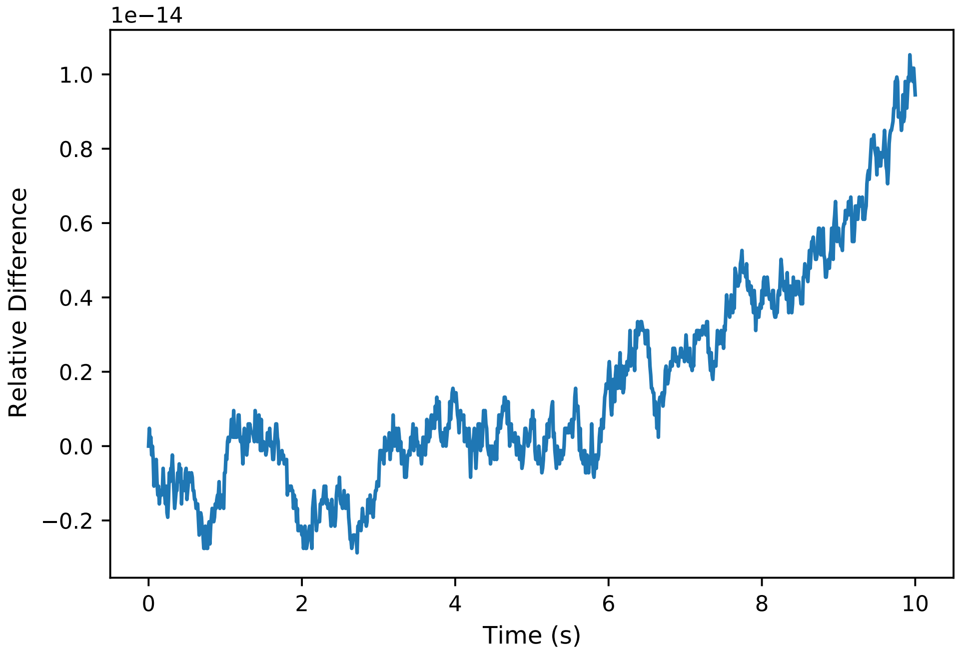

Fig. 3: Change in Orbital Energy Translation Only

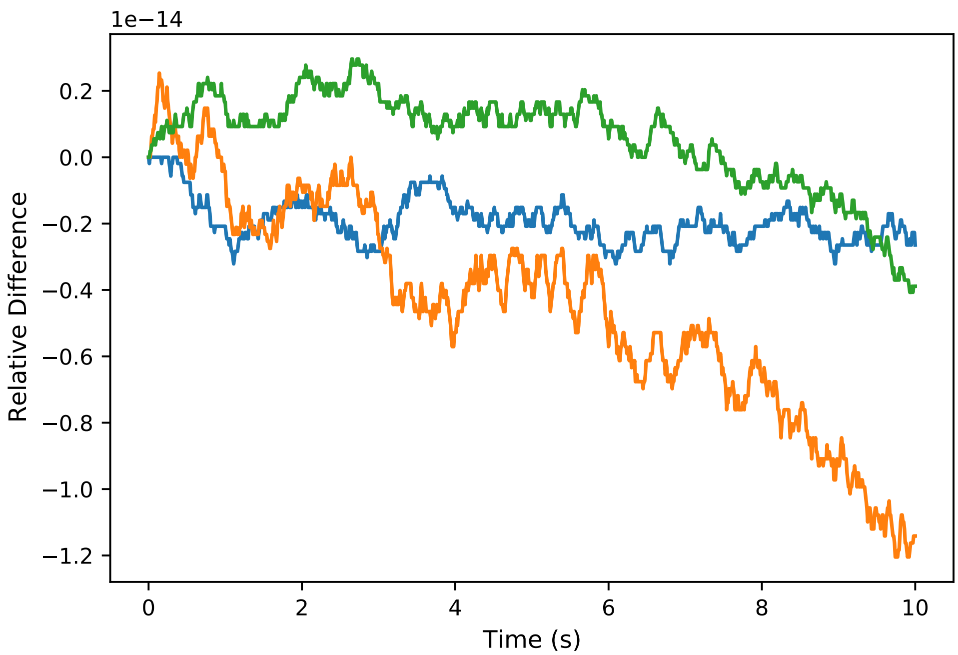

Fig. 4: Change in Orbital Angular Momentum Translation Only

6.2 Rotational Only Scenario

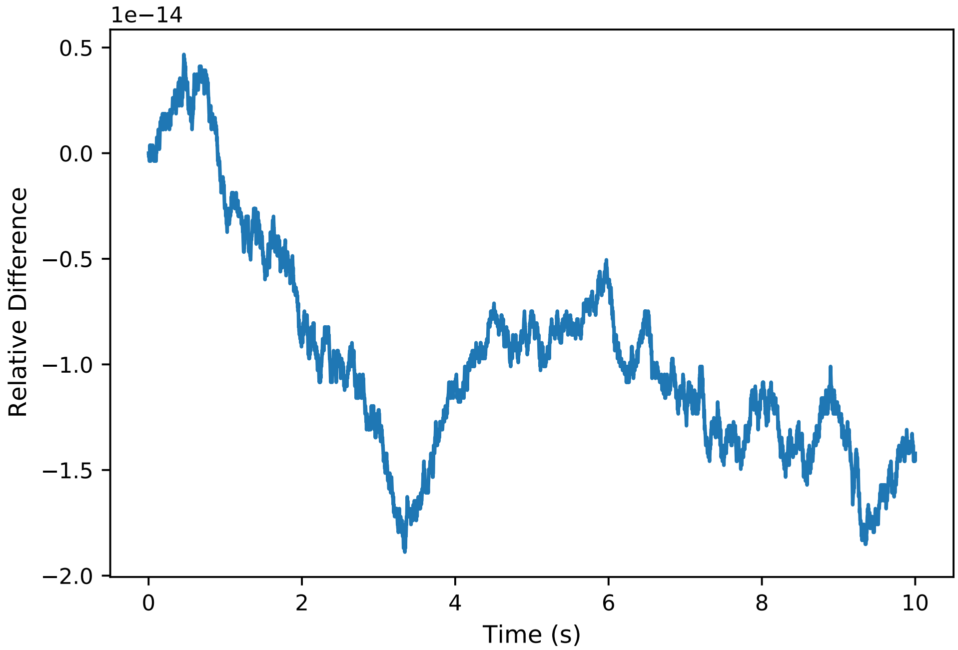

Fig. 5: Change in Rotational Energy Rotation Only

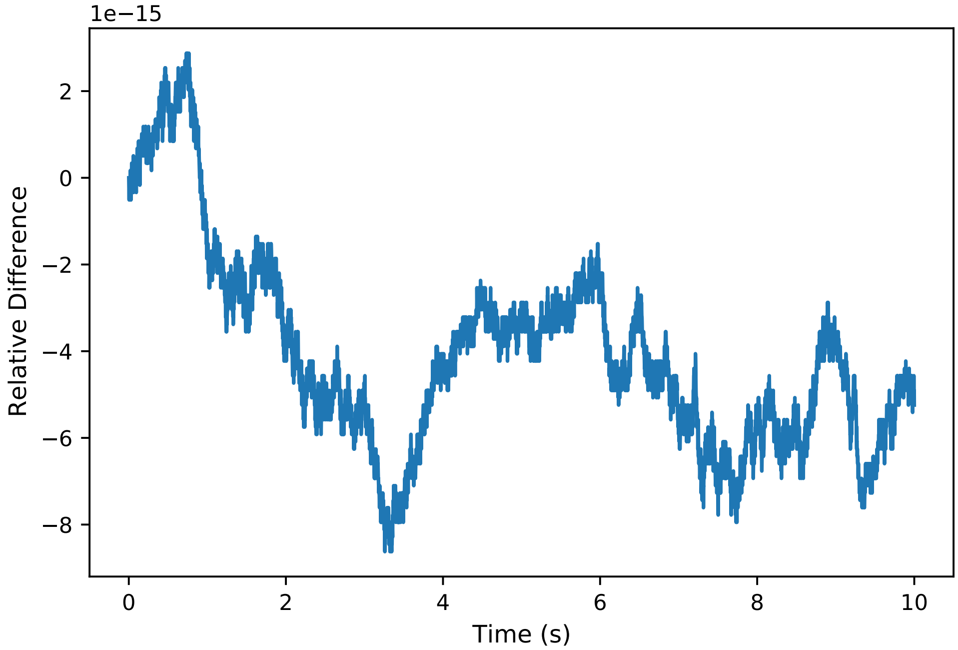

Fig. 6: Change in Rotational Angular Momentum Rotation Only

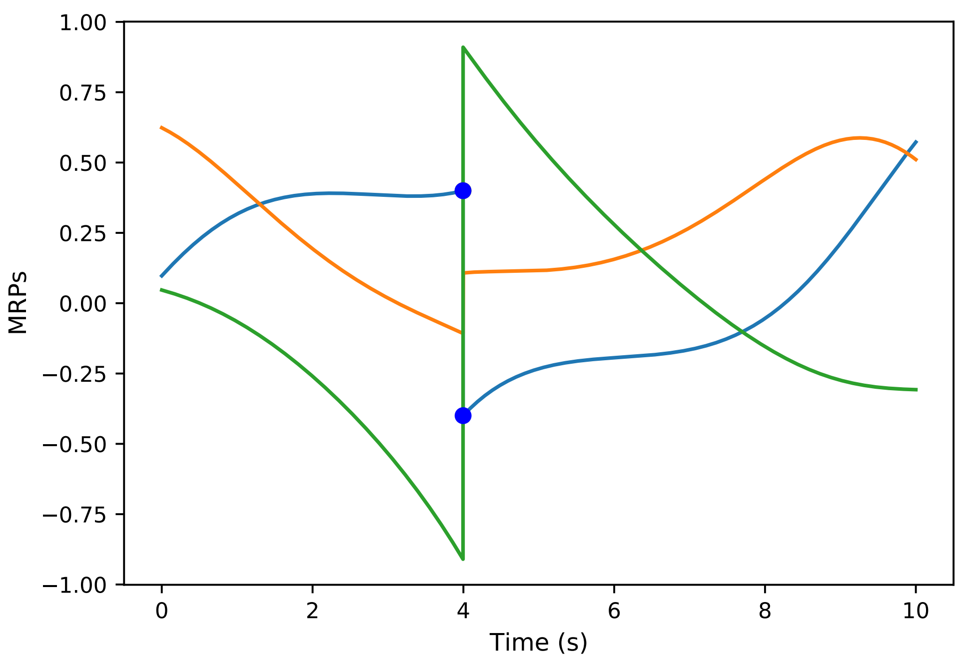

Fig. 7: Attitude of Spacecraft in MRPs

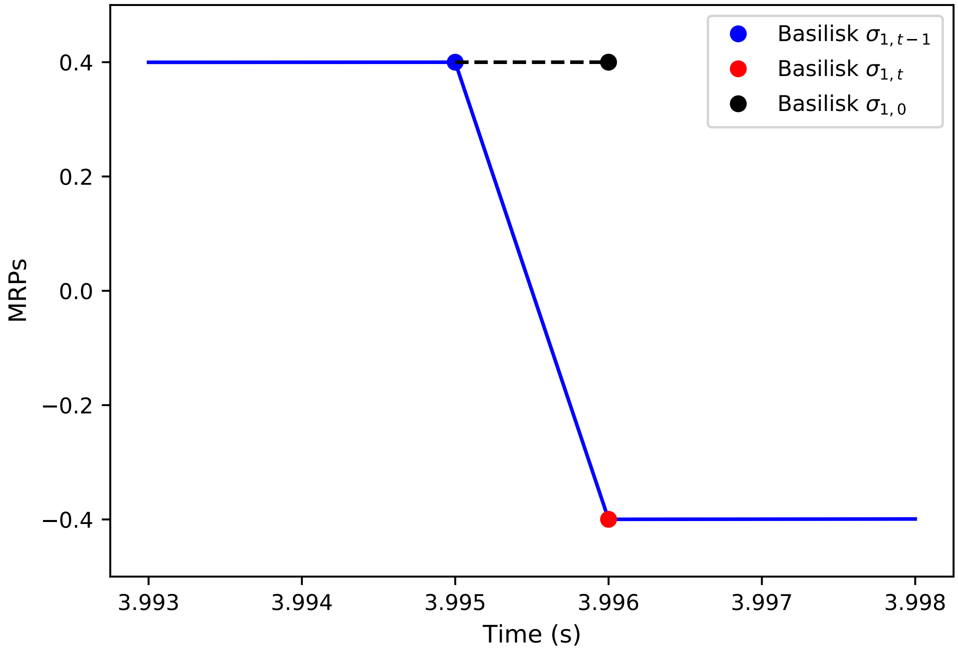

Fig. 8: MRP Switching

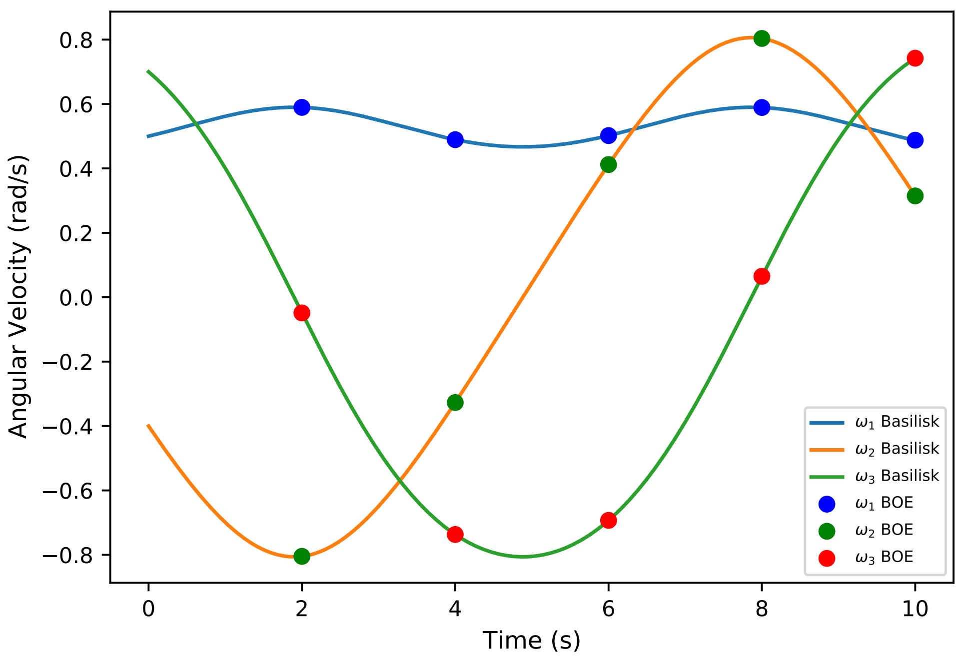

Fig. 9: Basilisk Vs BOE Calc For Rotation

6.3 Translational and Rotational Scenario

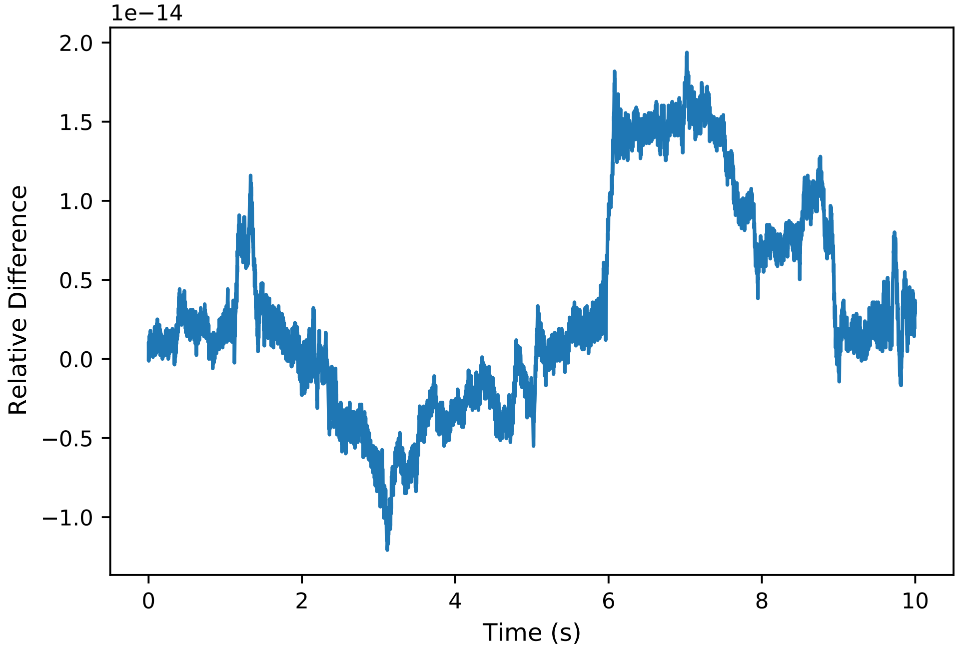

Fig. 10: Change in Orbital Energy Translation And Rotation

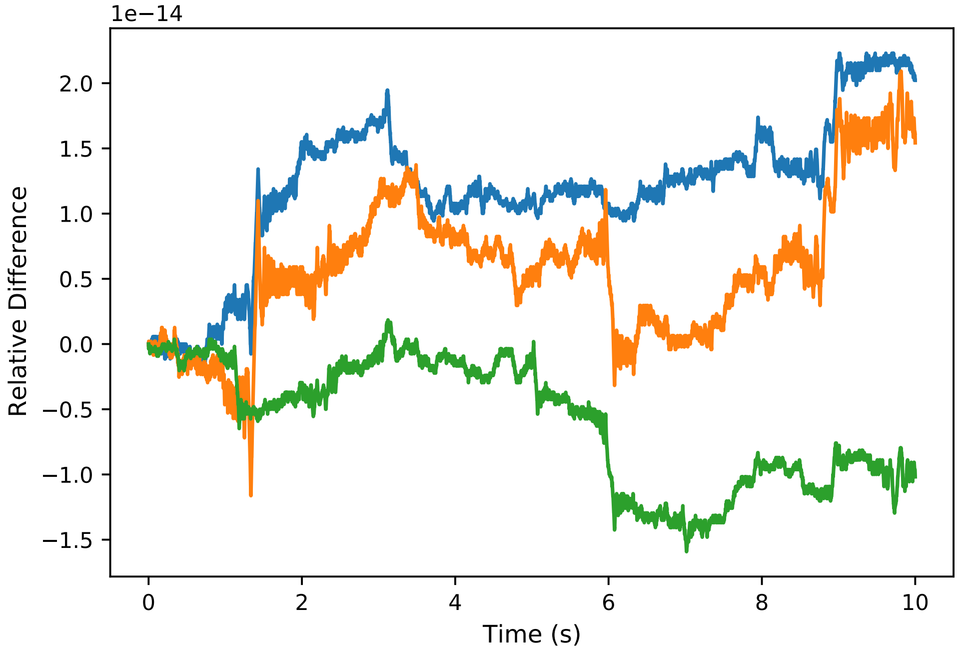

Fig. 11: Change in Orbital Angular Momentum Translation And Rotation

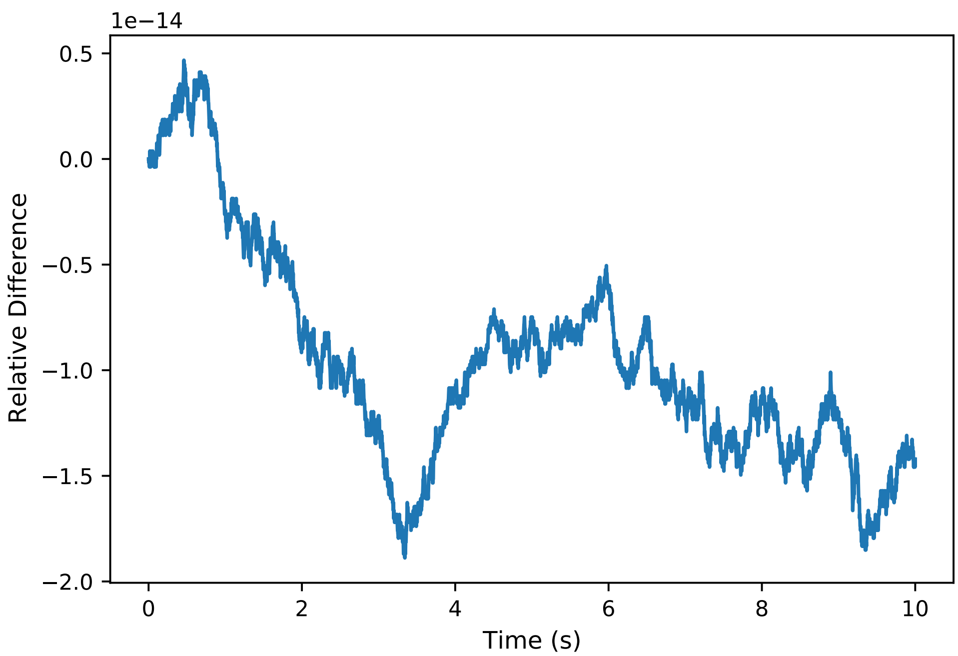

Fig. 12: Change in Rotational Energy Translation And Rotation

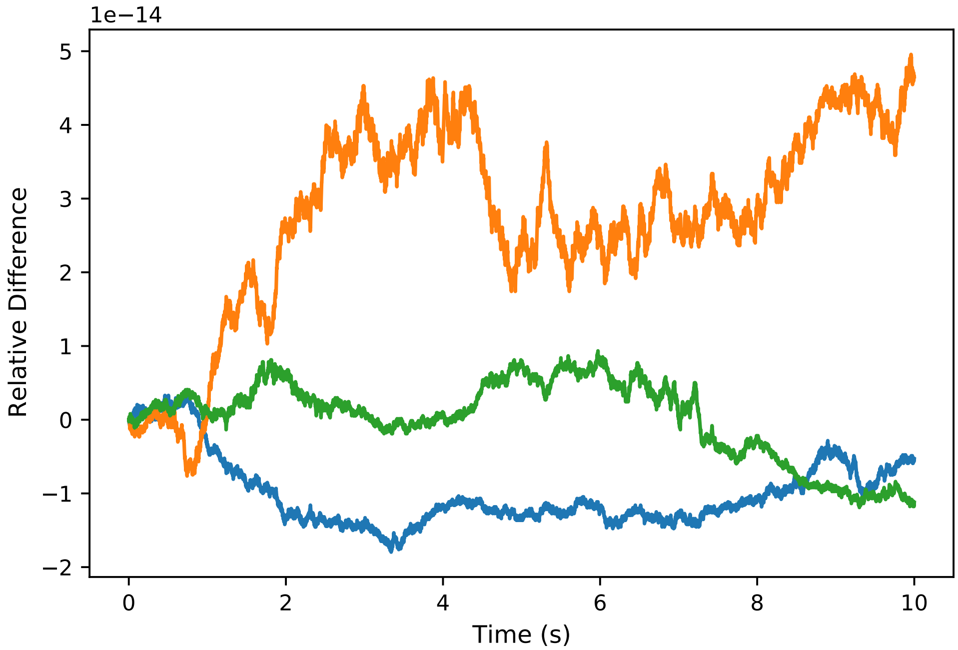

Fig. 13: Change in Rotational Angular Momentum Translation And Rotation

6.4 Translational BOE Calculation Scenario

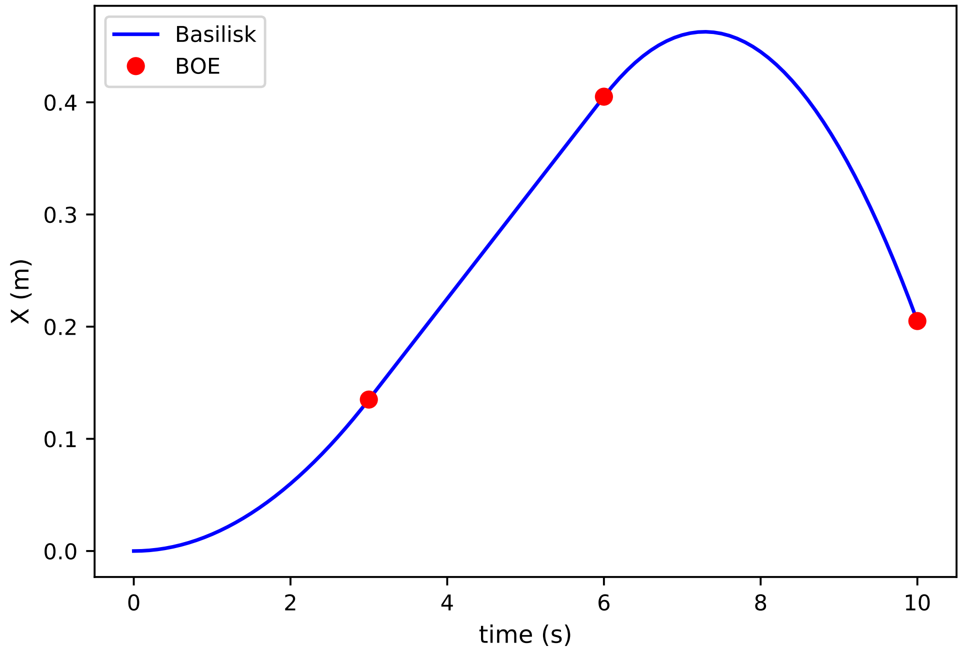

Fig. 14: Translation Position BOE

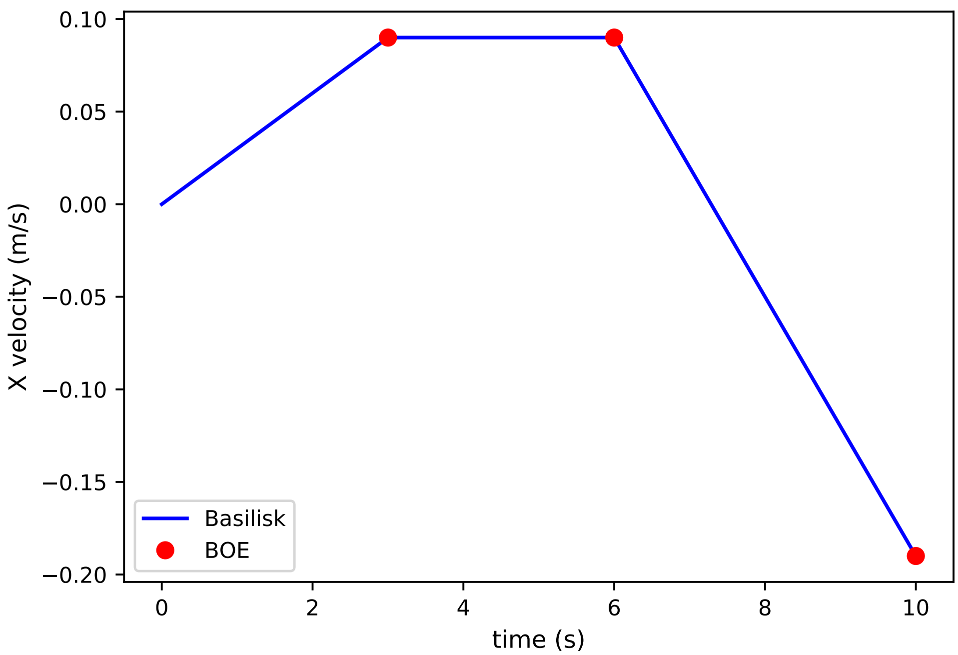

Fig. 15: Translation Velocity BOE

6.5 Dynamics Calculated About Point B Test

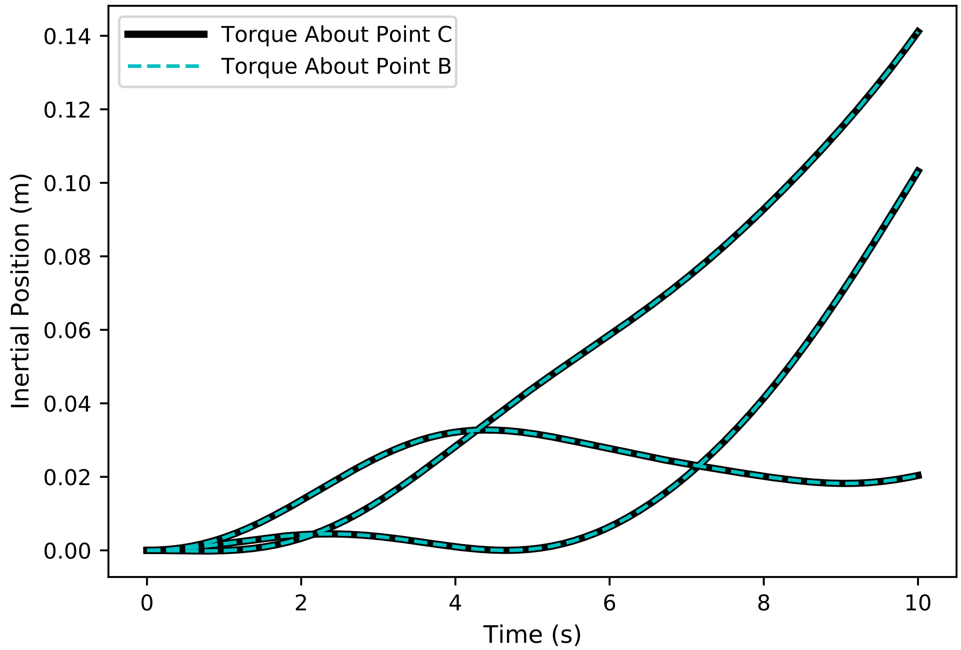

Fig. 16: PointB Vs PointC Translation

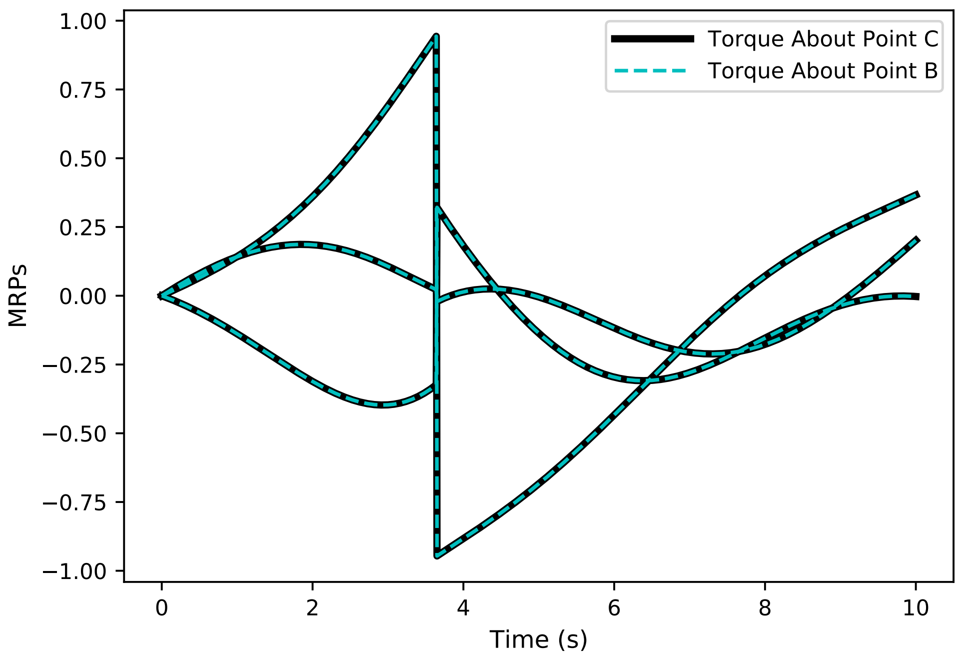

Fig. 17: PointB Vs PointC Attitude

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[2] Hanspeter Schaub and John L. Junkins. *Analytical Mechanics of Space Systems*. AIAA Education Series, Reston, VA, 3rd edition, 2014.