Autonomous Vehicle Simulation (AVS) Laboratory

AVS-Sim Technical Memorandum
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MRP FEEDBACK ADCS CONTROL MODULE

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Scope/Contents
This module provides a general MRP feedback control law, applying to using N reaction wheels with general orientation.

Rev:	**Change Description**	**By**
Draft	Initial Draft document	H. Schaub

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1 Initialization

Simply call the module reset function prior to using this control module. This will reset the prior function call time variable, and reset the attitude error integral measure. The control update period Δt is evaluated automatically.

2 Algorithm

This module employed the MRP feedback algorithm of Example 8.16 of Reference1. This nonlinear attitude tracking control includes an integral measure of the attitude error. Further, we seek to avoid quadratic $\boldsymbol{\omega}$ terms to reduce the likelihood of control saturation during a detumbling phase. Let the new nonlinear feedback control be expressed as

$$[G_s]\boldsymbol{u}_s = K\boldsymbol{\sigma} + [P]\delta\boldsymbol{\omega} + [P][K_I]\boldsymbol{z} - [I_{\mathsf{RW}}](\dot{\boldsymbol{\omega}}_r - [\tilde{\boldsymbol{\omega}}]\boldsymbol{\omega}_r) + \boldsymbol{L}$$
$$- ([\tilde{\boldsymbol{\omega}}_r] + [\widetilde{K_I\boldsymbol{z}}])\left([I_{\mathsf{RW}}]\boldsymbol{\omega} + [G_s]\boldsymbol{h}_s\right) \quad (1)$$

The integral attitude error measure $\boldsymbol{z}$ is defined through

$$\boldsymbol{z} = K \int_{t_0}^{t} \boldsymbol{\sigma}\mathrm{d}t + [I_{\mathsf{RW}}](\delta\boldsymbol{\omega} - \delta\boldsymbol{\omega}_0)$$

The integral measure $\boldsymbol{z}$ must be computed to determine $[P][K_I]\boldsymbol{z}$, and the expression $[\widetilde{K_I\boldsymbol{z}}]$ is added to $[\widetilde{\boldsymbol{\omega}_r}]$ term.

To analyze the stability of this control, the following Lyapunov candidate function is used:

$$V(\delta\boldsymbol{\omega}, \boldsymbol{\sigma}, \boldsymbol{z}) = \frac{1}{2}\delta\boldsymbol{\omega}^T[I_{\mathsf{RW}}]\delta\boldsymbol{\omega} + 2K\ln(1 + \boldsymbol{\sigma}^T\boldsymbol{\sigma}) + \frac{1}{2}\boldsymbol{z}^T[K_I]\boldsymbol{z}$$

provides a convenient positive definite attitude error function. The attitude feedback gain K is positive, while the integral feedback gain $[K_I]$ is a symmetric positive definite matrix. The resulting Lyapunov rate expression, solved in Eq. (8.101), is given by

$$\dot{V} = (\delta\boldsymbol{\omega} + [K_I]\boldsymbol{z})^T \left([I_{\mathsf{RW}}]\frac{{}^{\mathcal{B}}\mathrm{d}}{\mathrm{d}t}(\delta\boldsymbol{\omega}) + K\boldsymbol{\sigma}\right)$$

Substituting the equations of motion of a spacecraft with N reaction wheels (see Eq. (8.160) in Reference 1), results in

$$\dot{V} = (\delta\boldsymbol{\omega} + [K_I]\boldsymbol{z})^T\left(-[\tilde{\boldsymbol{\omega}}]([I_{\mathsf{RW}}]\boldsymbol{\omega} + [G_s]\boldsymbol{h}_s) - [G_s]\boldsymbol{u}_s + \boldsymbol{L} - [I_{\mathsf{RW}}](\dot{\boldsymbol{\omega}}_r - [\tilde{\boldsymbol{\omega}}]\boldsymbol{\omega}_r) + K\boldsymbol{\sigma}\right)$$

Substituting the control expression in Eq. (1) and making use of $\boldsymbol{\alpha} = \boldsymbol{\omega}_r - [K_I]\boldsymbol{z}$ leads to

$$\dot{V} = (\delta\boldsymbol{\omega} + [K_I]\boldsymbol{z})^T\Big(-([\tilde{\boldsymbol{\omega}}] - [\tilde{\boldsymbol{\omega}}_r] + [\widetilde{K_I\boldsymbol{z}}])([I_{\mathsf{RW}}]\boldsymbol{\omega} + [G_s]\boldsymbol{h}_s) + (K\boldsymbol{\sigma} - K\boldsymbol{\sigma})$$

$$- [P]\delta\boldsymbol{\omega} - [P][K_I]\boldsymbol{z} + [I_{\mathsf{RW}}](\dot{\boldsymbol{\omega}}_r - [\tilde{\boldsymbol{\omega}}]\boldsymbol{\omega}_r) - [I_{\mathsf{RW}}](\dot{\boldsymbol{\omega}}_r - [\tilde{\boldsymbol{\omega}}]\boldsymbol{\omega}_r) + (\boldsymbol{L} - \boldsymbol{L})\Big)$$

$$= (\delta\boldsymbol{\omega} + [K_I]\boldsymbol{z})^T\Big(-([\widetilde{\delta\boldsymbol{\omega}}] + [\widetilde{K_I\boldsymbol{z}}])([I_{\mathsf{RW}}]\boldsymbol{\omega} + [G_s]\boldsymbol{h}_s) - [P](\delta\boldsymbol{\omega} + [K_I]\boldsymbol{z})\Big)$$

Because $(\delta\boldsymbol{\omega} + [K_I]\boldsymbol{z})^T([\widetilde{\delta\boldsymbol{\omega}}] + [\widetilde{K_I\boldsymbol{z}}]) = 0$, the Lyapunov rate reduces the negative semi-definite expression

$$\dot{V} = -(\delta\boldsymbol{\omega} + [K_I]\boldsymbol{z})^T[P](\delta\boldsymbol{\omega} + [K_I]\boldsymbol{z})$$

This proves the new control is globally stabilizing. Asymptotic stability is shown following the same steps as for the nonlinear integral feedback control in Eq. (8.104) in Reference 1.

One of the goals set forth at the beginning of the example was avoiding quadratic $\boldsymbol{\omega}$ feedback terms to reduce the odds of control saturation during periods with large $\boldsymbol{\omega}$ values. However, the control in Eq. (1) contains a product of $\boldsymbol{z}$ and $\boldsymbol{\omega}$. Let us study this term in more detail. The $\boldsymbol{\omega}$ expression with this product terms is found to be

$$[\widetilde{K_I\boldsymbol{z}}]([I_{\mathsf{RW}}]\boldsymbol{\omega}) \quad \Rightarrow \quad -([\widetilde{I_{\mathsf{RW}}\boldsymbol{\omega}}])([K_I][I_{\mathsf{RW}}]\boldsymbol{\omega} + \cdots)$$

If the integral feedback gain is a scalar K_I, rather than a symmetric positive definite matrix $[K_I]$, the quadratic $\boldsymbol{\omega}$ term vanishes. If the full 3×3 gain matrix is employed, then quadratic rate feedback terms are retained.

REFERENCES

[1] Hanspeter Schaub and John L. Junkins. *Analytical Mechanics of Space Systems*. AIAA Education Series, Reston, VA, 3rd edition, 2014.