Autonomous Vehicle Simulation (AVS) Laboratory

AVS-Sim Technical Memorandum

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GUIDANCE MODULE TO PERFORM AN INERTIALLY CONSTANT SPIN

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Scope/Contents
Generate the reference attitude trajectory for a general 3D inertial spin with a constant inertial angular velocity vector. A corrected body frame will align with the desired reference frame.

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1 Introduction

This technical note discusses the guidance mathematics to perform a constant spin about an inertially fixed axis. Instead of aligning the body frame $\mathcal{B}$ with a desired reference frame $\mathcal{R}$, rather an arbitrary body-fixed frame can be driven towards the desired reference frame.

2 Reference Frame Definitions

Let the primary body-fixed coordinate frame be $\mathcal{B} : \{\hat{\boldsymbol{b}}_1, \hat{\boldsymbol{b}}_2, \hat{\boldsymbol{b}}_3\}$. However, instead of aligning this frame with a reference, a corrected body frame $\mathcal{B}_c$ is to be aligned with a reference frame. Let the uncorrected reference orientation be given by $\mathcal{R}_0$. Thus, the guidance goal is to drive $\mathcal{B}_c \to \mathcal{R}_0$, which yields

$$[R_0N] = [B_cB][BN] \tag{1}$$

where $\mathcal{N}$ is an inertial reference frame. Rearranging this relationship, with perfect attitude tracking the inertial body frame orientation should be

$$[BN] = [B_cB]^T[R_0N] = [RN] \tag{2}$$

where $\mathcal{R}$ is a corrected reference frame. Note that $[B_cB] = [R_0R]$. Thus, the corrected reference orientation is computed using

$$[RN] = [R_0R]^T[R_0N] \tag{3}$$

where the body-frame correction is subtracted from the original reference orientation.

The benefit of of driving $\mathcal{B} \to \mathcal{R}$ instead of $\mathcal{B}_c \to \mathcal{R}_0$ is that the body frame, along with the many device position and orientation vectors expressed in body-frame components, don't have to be rotated for each control evaluation. In simple terms, if the corrected body frame is a 60°rotation from the body frame, then the 60°is subtracted from the original reference orientation. This allows all body inertia tensor and reaction wheel heading vector descriptions to remain in the primary body frame $\mathcal{B}$.

Assume the initial uncorrected reference frame $\mathcal{R}_\prime$ is given through the MRP set $\boldsymbol{\sigma}_{R_0/N}$

$$[R_0N(\boldsymbol{\sigma}_{R_0/N})] \tag{4}$$

The relative orientation of the corrected body frame relative to the primary body frame is a constant MRP set

$$[B_cB(\boldsymbol{\sigma}_{B_c/B})] = [R_0R(\boldsymbol{\sigma}_{R_0/R})] \tag{5}$$

To apply this correction to the original reference frame, using the Direction Cosine Matrix (DCM) description, this is determined through

$$[RN(\boldsymbol{\sigma}_{R/N})] = [R_0R(\boldsymbol{\sigma}_{R_0/R})]^T[R_0N(\boldsymbol{\sigma}_{R_0/N})] = [R_0R(-\boldsymbol{\sigma}_{R_0/R})][R_0N(\boldsymbol{\sigma}_{R_0/N})] \tag{6}$$

where the convenient MRP identity

$$[R_0R(\boldsymbol{\sigma}_{R_0/R})]^T = [R_0R(-\boldsymbol{\sigma}_{R_0/R})] \tag{7}$$

Note the following MRP addition property developed in Reference 1. If

$$[BN(\boldsymbol{\sigma})] = [FB(\boldsymbol{\sigma}'')][BN(\boldsymbol{\sigma}')] \tag{8}$$

then

$$\boldsymbol{\sigma} = \frac{(1-|\boldsymbol{\sigma}'|^2)\boldsymbol{\sigma}'' + (1-|\boldsymbol{\sigma}''|^2)\boldsymbol{\sigma}' - 2\boldsymbol{\sigma}'' \times \boldsymbol{\sigma}'}{1+|\boldsymbol{\sigma}'|^2|\boldsymbol{\sigma}''|^2 - 2\boldsymbol{\sigma}' \cdot \boldsymbol{\sigma}''} \tag{9}$$

In the RigidBodyKinematics software library of Reference 1, this MRP evaluation is achieved with

$$\boldsymbol{\sigma} = \texttt{addMRP}(\boldsymbol{\sigma}', \boldsymbol{\sigma}'')$$

Thus, to properly apply the body frame orientation correction to the original reference frame, this function should be used with

$$\boldsymbol{\sigma}_{R/N} = \texttt{addMRP}(\boldsymbol{\sigma}_{R_0/N}, -\boldsymbol{\sigma}_{R_0/R})$$

3 Reference Frame Angular Velocity Vector

The angular velocity of the original reference frame $\mathcal{R}_0$ is

$$\boldsymbol{\omega}_{R_0/N} \tag{10}$$

The angular velocity tracking error is defined as

$$\delta\boldsymbol{\omega} = \boldsymbol{\omega}_{B/N} - \boldsymbol{\omega}_{R/N} \tag{11}$$

The correct reference frame angular velocity is

$$\boldsymbol{\omega}_{R/N} = \boldsymbol{\omega}_{R/R_0} + \boldsymbol{\omega}_{R_0/N} = \boldsymbol{\omega}_{R_0/N} \tag{12}$$

because the body frame correction $[B_cB] = [R_0R]$ is a constant angular offset.

4 Reference Frame Propagation

The MRP differential kinematic equations are

$$\boldsymbol{\sigma}_{R/N} = \frac{1}{4}[B(\boldsymbol{\sigma}_{R/N})]^{\mathcal{R}}\boldsymbol{\omega}_{R/N} \tag{13}$$

where

$$[B(\boldsymbol{\sigma}_{R/N})] = (1-\sigma_{R/N}^2)[I_{3\times3}] + 2[\tilde{\boldsymbol{\sigma}}_{R/N}] + 2\boldsymbol{\sigma}_{R/N}\boldsymbol{\sigma}_{R/N}^T \tag{14}$$

If there inertially fixed reference angular velocity vector is given in inertial frame components, it must be rotated using the appropriate DCM:

$$^{\mathcal{R}}\boldsymbol{\omega}_{R/N} = [RN]^{\mathcal{N}}\boldsymbol{\omega}_{R/N} \tag{15}$$

REFERENCES

[1] Hanspeter Schaub and John L. Junkins. *Analytical Mechanics of Space Systems*. AIAA Education Series, Reston, VA, 3rd edition, 2014.