Autonomous Vehicle Simulation (AVS) Laboratory,
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Basilisk Technical Memorandum

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GUIDANCE MODULE FOR CELESTIAL TWO BODY POINT

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Scope/Contents
Generate the attitude reference to satisfy a primary attitude pointing constrain and the best as possible a secondary one.

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1.0	Initial copy	M. Cols	2018-04-23
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1 Model Description

1.1 Module Goal

This module computes a reference whose aim is to track the center of a primary target, e.g. pointing the communication antenna at the Earth, at the same time of trying to meet a secondary constraint as best as possible, e.g. a solar panel normal axis pointing the closest in the direction of the Sun. It is important to note that two pointing conditions in a three-dimensional space compose an overdetermined problem. Thus, the main constraint is always priorized over the secondary one so the former can always be met.
Figure **??** shows the case where Mars is the primary celestial body and the Sun is the secondary one. Note that the origin of the desired reference frame $\mathcal{R}$ is the position of the spacecraft.

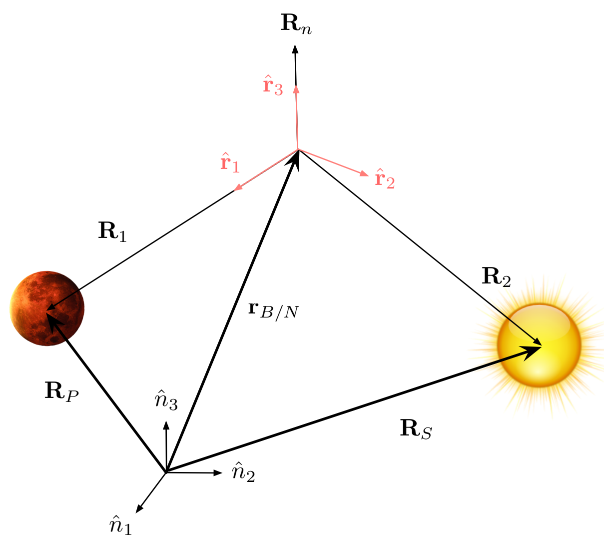

Fig. 1: Illustration of the restricted two-body pointing reference frame $\mathcal{R} : \{\hat{\boldsymbol{r}}_1, \hat{\boldsymbol{r}}_1, \hat{\boldsymbol{r}}_2\}$ and the inertial frame $\mathcal{N} : \{\hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}_2\}$.

Assuming knowledge of the position of the spacecraft $\boldsymbol{r}_{B/N}$ and the involved celestial bodies, $\boldsymbol{R}_{P1}$ and $\boldsymbol{R}_{P2}$ (all of them relative to the inertial frame $\mathcal{N}$ and expressed in inertial frame components), the relative position of the celestial bodies with respect to the spacecraft is obtained by simple subtraction:

$$\boldsymbol{R}_{P1} = \boldsymbol{R}_P - \boldsymbol{r}_{B/N} \tag{1a}$$
$$\boldsymbol{R}_{P2} = \boldsymbol{R}_S - \boldsymbol{r}_{B/N} \tag{1b}$$

In analogy, the inertial derivatives of these position vectors are obtained:

$$\boldsymbol{v}_{P1} = \boldsymbol{v}_P - \boldsymbol{v}_{B/N} \tag{2a}$$
$$\boldsymbol{v}_{P2} = \boldsymbol{v}_S - \boldsymbol{v}_{B/N} \tag{2b}$$
$$\boldsymbol{a}_{P1} = \boldsymbol{a}_P - \boldsymbol{a}_{B/N} \tag{2c}$$
$$\boldsymbol{a}_{P2} = \boldsymbol{a}_S - \boldsymbol{a}_{B/N} \tag{2d}$$

The normal vector $\boldsymbol{R}_n$ of the plane defined by $\boldsymbol{R}_{P1}$ and $\boldsymbol{R}_{P2}$ is computed through:

$$\boldsymbol{R}_n = \boldsymbol{R}_{P1} \times \boldsymbol{R}_{P2} \tag{3}$$

The inertial time derivative of $\boldsymbol{R}_n$ is found using the chain differentiation rule:

$$\boldsymbol{v}_n = \boldsymbol{v}_{P1} \times \boldsymbol{R}_{P2} + \boldsymbol{R}_{P1} \times \boldsymbol{v}_{P2} \tag{4}$$

And the second time derivative:

$$\boldsymbol{a}_n = \boldsymbol{a}_{P1} \times \boldsymbol{R}_{P2} + \boldsymbol{R}_{P1} \times \boldsymbol{a}_{P2} + 2\boldsymbol{v}_{P1} \times \boldsymbol{v}_{P2} \tag{5}$$

1.2 Special Case: No Secondary Constraint Applicable

If there is no incoming message with a secondary celestial body pointing condition or if the constrain is not valid, an artificial three-dimensional frame is defined instead. Note that a single condition pointing leaves one degree of freedom, hence standing for an underdetermined attitude problem. A secondary constrain is considered to be invalid if the angle between $\boldsymbol{R}_{P1}$ and $\boldsymbol{R}_{P2}$ is, in absolute value, minor than a set threshold. This could be the case where the primary and secondary celestial bodies are aligned as seen by the spacecraft. In such situation, the primary pointing axis would already satisfy both the primary and the secondary constraints.

Since the main algorithm of this module, which is developed in the following sections, assumes two conditions, the second one is arbitrarily set as following:

$$\boldsymbol{R}_{P2} = \boldsymbol{R}_{P1} \times \boldsymbol{v}_{P1} \equiv \boldsymbol{h}_{P1} \tag{6}$$

By setting the secondary constrain to have the direction of the angular momentum vector $\boldsymbol{h}_{P1}$, it is assured that it will always be valid ($\boldsymbol{R}_{P1}$ and $\boldsymbol{R}_{P2}$ are now perpendicular). The first and second time derivatives are steadily computed:

$$\boldsymbol{v}_{P2} = \boldsymbol{R}_{P1} \times \boldsymbol{a}_{P1} \tag{7}$$

$$\boldsymbol{a}_{P2} = \boldsymbol{v}_{P1} \times \boldsymbol{a}_{P1} \tag{8}$$

2 Reference Frame Definition

As illustrated in Figure **??**, the base vectors of the desired reference frame $\mathcal{R}$ are defined as following:

$$\hat{\boldsymbol{r}}_1 = \frac{\boldsymbol{R}_{P1}}{|\boldsymbol{R}_{P1}|} \tag{9a}$$

$$\hat{\boldsymbol{r}}_3 = \frac{\boldsymbol{R}_n}{|\boldsymbol{R}_n|} \tag{9b}$$

$$\hat{\boldsymbol{r}}_2 = \hat{\boldsymbol{r}}_3 \times \hat{\boldsymbol{r}}_1 \tag{9c}$$

Since the position vectors are given in terms of inertial $\mathcal{N}$-frame components, the DCM from the inertial frame $\mathcal{N}$ to the desired reference frame $\mathcal{R}$ is:

$$[RN] = \begin{bmatrix} {}^{\mathcal{N}}\hat{\boldsymbol{r}}_1^T \\ {}^{\mathcal{N}}\hat{\boldsymbol{r}}_2^T \\ {}^{\mathcal{N}}\hat{\boldsymbol{r}}_3^T \end{bmatrix} \tag{10}$$

3 Base Vectors Time Derivatives

The first and second time derivatives of the base vectors that compound the reference frame $\mathcal{R}$ are needed in the following sections to compute the reference angular velocity and acceleration. Several lines of algebra lead to the following sets:

$$\dot{\hat{\boldsymbol{r}}}_1 = ([I_{3\times3}] - \hat{\boldsymbol{r}}_1\hat{\boldsymbol{r}}_1^T)\frac{\boldsymbol{R}_{P1}}{|\boldsymbol{R}_{P1}|} \tag{11a}$$

$$\dot{\hat{\boldsymbol{r}}}_3 = ([I_{3\times3}] - \hat{\boldsymbol{r}}_3\hat{\boldsymbol{r}}_3^T)\frac{\boldsymbol{R}_{n}}{|\boldsymbol{R}_{n}|} \tag{11b}$$

$$\dot{\hat{\boldsymbol{r}}}_2 = \dot{\hat{\boldsymbol{r}}}_3 \times \boldsymbol{r}_1 + \boldsymbol{r}_n \times \dot{\hat{\boldsymbol{r}}}_3 \tag{11c}$$

$$\ddot{\hat{\boldsymbol{r}}}_1 = \frac{1}{|\boldsymbol{R}_{P1}|}(([I_{3\times3}] - \hat{\boldsymbol{r}}_1\hat{\boldsymbol{r}}_1^T)\boldsymbol{a}_{P1} - 2\dot{\hat{\boldsymbol{r}}}_1(\hat{\boldsymbol{r}}_1 \cdot \boldsymbol{v}_{P1}) - \hat{\boldsymbol{r}}_1(\dot{\hat{\boldsymbol{r}}}_1 \cdot \boldsymbol{v}_{P1})) \tag{12a}$$

$$\ddot{\hat{\boldsymbol{r}}}_3 = \frac{1}{|\boldsymbol{R}_{n}|}(([I_{3\times3}] - \hat{\boldsymbol{r}}_3\hat{\boldsymbol{r}}_3^T)\boldsymbol{a}_{n} - 2\dot{\hat{\boldsymbol{r}}}_3(\hat{\boldsymbol{r}}_3 \cdot \boldsymbol{v}_{n}) - \hat{\boldsymbol{r}}_3(\dot{\hat{\boldsymbol{r}}}_3 \cdot \boldsymbol{v}_{n})) \tag{12b}$$

$$\ddot{\hat{\boldsymbol{r}}}_2 = \ddot{\hat{\boldsymbol{r}}}_3 \times \boldsymbol{r}_1 + \boldsymbol{r}_n \times \ddot{\hat{\boldsymbol{r}}}_3 + 2\dot{\hat{\boldsymbol{r}}}_3 \cdot \dot{\hat{\boldsymbol{r}}}_1 \tag{12c}$$

3.1 Angular Velocity and Acceleration Descriptions

Developing some more mathematics, the following elegant expressions of $\boldsymbol{\omega}_{R/N}$ and $\dot{\boldsymbol{\omega}}_{R/N}$ are found:

$$\boldsymbol{\omega}_{R/N} \cdot \hat{\boldsymbol{r}}_1 = \hat{\boldsymbol{r}}_3 \cdot \dot{\hat{\boldsymbol{r}}}_2 \tag{13a}$$

$$\boldsymbol{\omega}_{R/N} \cdot \hat{\boldsymbol{r}}_2 = \hat{\boldsymbol{r}}_1 \cdot \dot{\hat{\boldsymbol{r}}}_3 \tag{13b}$$

$$\boldsymbol{\omega}_{R/N} \cdot \hat{\boldsymbol{r}}_3 = \hat{\boldsymbol{r}}_2 \cdot \dot{\hat{\boldsymbol{r}}}_1 \tag{13c}$$

$$\dot{\boldsymbol{\omega}}_{R/N} \cdot \hat{\boldsymbol{r}}_1 = \dot{\hat{\boldsymbol{r}}}_3 \cdot \dot{\hat{\boldsymbol{r}}}_2 + \hat{\boldsymbol{r}}_3 \cdot \ddot{\hat{\boldsymbol{r}}}_2 - \boldsymbol{\omega}_{R/N} \cdot \dot{\hat{\boldsymbol{r}}}_1 \tag{14a}$$

$$\dot{\boldsymbol{\omega}}_{R/N} \cdot \hat{\boldsymbol{r}}_2 = \dot{\hat{\boldsymbol{r}}}_1 \cdot \dot{\hat{\boldsymbol{r}}}_3 + \hat{\boldsymbol{r}}_1 \cdot \ddot{\hat{\boldsymbol{r}}}_3 - \boldsymbol{\omega}_{R/N} \cdot \dot{\hat{\boldsymbol{r}}}_2 \tag{14b}$$

$$\dot{\boldsymbol{\omega}}_{R/N} \cdot \hat{\boldsymbol{r}}_3 = \dot{\hat{\boldsymbol{r}}}_2 \cdot \dot{\hat{\boldsymbol{r}}}_1 + \hat{\boldsymbol{r}}_2 \cdot \ddot{\hat{\boldsymbol{r}}}_1 - \boldsymbol{\omega}_{R/N} \cdot \dot{\hat{\boldsymbol{r}}}_3 \tag{14c}$$

Note that $\boldsymbol{\omega}_{R/N} \cdot \hat{\boldsymbol{r}}_1$ is the first component of the angular velocity of the reference with respect to the inertial expressed in reference frame components. Hence,

$$\boldsymbol{\omega}_{R/N} = {}^{\mathcal{R}}\begin{bmatrix} \boldsymbol{\omega}_{R/N} \cdot \hat{\boldsymbol{r}}_1 \\ \boldsymbol{\omega}_{R/N} \cdot \hat{\boldsymbol{r}}_2 \\ \boldsymbol{\omega}_{R/N} \cdot \hat{\boldsymbol{r}}_3 \end{bmatrix} \tag{15}$$

Similarly for the angular acceleration:

$$\dot{\boldsymbol{\omega}}_{R/N} = {}^{\mathcal{R}}\begin{bmatrix} \dot{\boldsymbol{\omega}}_{R/N} \cdot \hat{\boldsymbol{r}}_1 \\ \dot{\boldsymbol{\omega}}_{R/N} \cdot \hat{\boldsymbol{r}}_2 \\ \dot{\boldsymbol{\omega}}_{R/N} \cdot \hat{\boldsymbol{r}}_3 \end{bmatrix} \tag{16}$$

Eventually, in inertial frame components:

$${}^{\mathcal{N}}\boldsymbol{\omega}_{R/N} = [RN]\ {}^{\mathcal{R}}\boldsymbol{\omega}_{R/N} \tag{17a}$$

$${}^{\mathcal{N}}\dot{\boldsymbol{\omega}}_{R/N} = [RN]\ {}^{\mathcal{R}}\dot{\boldsymbol{\omega}}_{R/N} \tag{17b}$$

4 Module Functions

- **parseInputMessages**: This method takes the navigation translational info as well as the spice data of the primary celestial body and, if applicable, the second one, and computes the relative state vectors necessary to create the restricted 2-body pointing reference frame.
- **computeCelestialTwoBodyPoint**: This method takes the spacecraft and points a specified axis at a named celestial body specified in the configuration data. It generates the commanded attitude and assumes that the control errors are computed downstream.

5 Module Assumptions and Limitations

This module does not add any assumptions to the guidance scheme.

6 Test Description and Success Criteria

The mathematics in this module are straight forward and can be tested in a series of input and output evaluation tests.

6.1 Check 1

Here a check is performed where the sun vector measurement $\boldsymbol{s}$ has a non-zero length and is not aligned with $\hat{\boldsymbol{s}}_c$.

6.2 Check 2

The sun direction vector $\boldsymbol{s}$ is given a norm value that is less than `minUnitMag`. In this case the attitude tracking $\boldsymbol{\sigma}_{B/R}$ should be set to zero. Further, the body rate errors are now evaluated relative to a fixed $\boldsymbol{\omega}_{R/N}$ vector.

6.3 Check 3

The sun direction vector $\boldsymbol{s}$ aligned with $\hat{\boldsymbol{s}}_c$. In this case the attitude tracking $\boldsymbol{\sigma}_{B/R}$ should be set to zero. Further, the body rate errors are simply the inertial body angular rates.

6.4 Check 4

The sun direction vector $\boldsymbol{s} \approx -\hat{\boldsymbol{s}}_c$. In this case the attitude tracking $\boldsymbol{\sigma}_{B/R}$ should be set to $\hat{\boldsymbol{e}}_{180}$. Further, the body rate errors are simply the inertial body angular rates.

6.5 Check 4

The sun direction vector $\boldsymbol{s} \approx -\hat{\boldsymbol{s}}_c$, but $\hat{\boldsymbol{s}}_c = \hat{\boldsymbol{b}}_1$. In this case the attitude tracking $\boldsymbol{\sigma}_{B/R}$ should be set to $\hat{\boldsymbol{e}}_{180}$ that is evaluated with the cross product with $\hat{\boldsymbol{b}}_2$. Further, the body rate errors are simply the inertial body angular rates.

7 Test Parameters

The unit test verify that the module output guidance message vectors match expected values.

The nominal module test input values are $\hat{\boldsymbol{s}}_c = (0, 0, 1)$, $\boldsymbol{s} = (1, 0, 0)$ and ${}^{\mathcal{B}}\boldsymbol{\omega}_{B/N} = (0.01, 0.50, -0.20)$ rad/sec. The nominal body-fixed search rate is set to ${}^{\mathcal{B}}\boldsymbol{\omega}_{R/N} = (0.0, 0.0, 0.1)$ rad/sec. This rate is only used if no sun direction vector is available. The small angle is set to $\epsilon = 0.01$ degrees.

Table 2: Error tolerance for each test.

Output Value Tested	Tolerated Error
$\boldsymbol{\sigma}_{B/R}$	
$\boldsymbol{\omega}_{B/R}$	
$\boldsymbol{\omega}_{R/N}$	
$\dot{\boldsymbol{\omega}}_{R/N}$	

8 Test Results

All of the tests passed:

Table 3: Test results

Check	Pass/Fail
1	
2	
3	
4	
5	

9 User Guide

9.1 Input/Output Messages

The module has 2 required input messages, and 1 output message:

- `attGuidanceOutMsgName` – This output message, of type `AttGuidFswMsg`, provide the attitude tracking errors and the reference frame states.
- `sunDirectionInMsgName` – This input message, of type `NavAttIntMsg`, receives the sun heading vector $\boldsymbol{s}$
- `imuInMsgName` – This input message, of type `IMUSensorBodyFswMsg`, receives the inertial angular body rates $\boldsymbol{\omega}_{B/N}$

9.2 Module Parameters and States

The module has the following parameter that can be configured:

- `sHatBdyCmd` – [REQUIRED] This 3x1 array contains the commanded body-relative vector $\hat{\boldsymbol{s}}_c$ that is to be aligned with the sun heading $\boldsymbol{s}$
- `minUnitMag` – This double contains the minimum norm value of $\boldsymbol{s}$ such that a tracking error attitude solution $\boldsymbol{\sigma}_{B/R}$ is still computed. If the norm is less than this, then $\boldsymbol{\sigma}_{B/R}$ is set to zero. The default `minUnitMag` value is zero.
- `omega_RN_B` – This vectors specifies the body-fixed search rate to rotate and search for the sun if no good sun direction vector is visible. Default value is a zero vector.
- `smallAngle` – This double specifies what is considered close for $\boldsymbol{s}$ and $\hat{\boldsymbol{s}}_c$ to be collinear. Default value is zero.
- `sunAxisSpinRate` – Specifies the nominal spin rate about the sun heading vector. This is only used if a sun heading solution is available. Default value is zero bring the spacecraft to rest.