



# SPARK 2014: Proof of Program Integrity

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- A runtime error is an error occurring at program execution (as opposed to at program compilation)
  - Among them are array access errors, range violations, overflows in computations...

```
type Nat_Array is array (Integer range <>) of Natural;
A : Nat_Array (1 .. 10);
I, J, P, Q : Integer;
A (I + J) := P / Q;
```

 Several runtime errors may occur when executing this program

$$A (I + J) := P / Q;$$

### The following errors could occur:

I+J might overflow

```
A (Integer'Last + 1) := P / Q;
```

I+J might be outside of the A's index range

```
A (A'Last + 1) := P / Q;
```

P/Q might overflow

```
A (I + J) := Integer'First / -1;
```

P/Q might be outside the element subtype

```
A (I + J) := 1 / -1;
```

Q might be zero

$$A (I + J) := P / 0;$$

- By default, the compiler will generate an executable that contains checks
  - So that an exception is raised if a runtime error occurs
  - For some runtime errors, an additional switch is required at compilation to get these checks

```
A (Integer'Last + 1) := P / Q;

raised CONSTRAINT_ERROR : overflow check failed

A (A'Last + 1) := P / Q;

raised CONSTRAINT_ERROR : index check failed

A (I + J) := Integer'First / (-1);

raised CONSTRAINT_ERROR : overflow check failed

A (I + J) := 1 / (-1);

raised CONSTRAINT_ERROR : range check failed

A (I + J) := P / O;

raised CONSTRAINT_ERROR : divide by zero
```

- GNATprove can statically verify absence of run-time errors
  - A logical formula, named Verification Condition, is generated for each possible runtime error
  - If the Verification Condition is true, the error cannot occur

```
A (Integer'Last + 1) := P / Q;
medium: overflow check might fail

A (A'Last + 1) := P / Q;
medium: array index check might fail

A (I + J) := Integer'First / (-1);
medium: overflow check might fail

A (I + J) := 1 / (-1);
medium: range check might fail

A (I + J) := P / 0;
medium: divide by zero might fail
```

#### Proof of Program Integrity – Modularity

- GNATprove uses contracts to perform proof of program modularly on a per subprogram basis
  - When verifying a subprogram's body, its precondition is all that is known about its inputs
  - When a subprogram is called, its post-condition is all that is known about its outputs

```
procedure Increment (X : in out Integer) with
    Pre => X < Integer'Last is

begin
    X := X + 1;
    -- info: overflow check proved
end;

X := Integer'Last - 2;
Increment (X);
    -- Here GNATprove does not know the value of X

X := X + 1;
    -- medium: overflow check might fail</pre>
```

#### Proof of Program Integrity – Modularity – Exceptions

Local subprograms without contracts can be inlined

```
procedure Increment (X : in out Integer) is
begin
    X := X + 1;
    -- info: overflow check proved, in call inlined at line 7
end Increment;

X := Integer'Last - 2;
Increment (X);

X := X + 1;
    -- info: overflow check proved
```

#### Values of expression functions are known

#### Proof of Program Integrity – Contracts

#### Ada 2012 contracts can be verified at runtime

- Runtime checks can be introduced for them during compilation
- If they do not hold at a subprogram call, an exception is raised

```
procedure Increment (X : in out Integer) with
  Pre => X < Integer'Last;</pre>
X := Integer'Last;
Increment (X);
-- raised ASSERT FAILURE : failed precondition
procedure Absolute (X : in out Integer) with
  Post \Rightarrow X \Rightarrow 0 is
begin
  if \times > 0 then
   X := -X;
  end if;
end Absolute;
X := 1;
Absolute (X);
-- raised ASSERT FAILURE : failed postcondition
```



#### Proof of Program Integrity – Contracts

#### GNATprove statically verifies pre and postconditions

- Preconditions are checked at each subprogram call
- Postconditions are verified once as part of the subprogram's body

```
procedure Increment (X : in out Integer) with
  Pre => X < Integer'Last;</pre>
X := Integer'Last;
Increment (X);
-- medium: precondition might fail
procedure Absolute (X : in out Integer) with
  Post => X >= 0 is
-- medium: postcondition might fail, requires X \ge 0
begin
  if X > 0 then
  X := -X;
  end if;
end Absolute;
X := 1;
Absolute (X);
```

#### Proof of Program Integrity – Contracts – Executable Semantics

#### The semantics in contracts are the usual Ada semantics

- Facilitate combination with testing on a per subprogram basis
- Facilitate debugging of assertions, as they can be executed

#### GNATprove also verifies integrity of assertions

- Proof of absence or runtime errors in preconditions is done once as part of the verification of the subprogram's body
- A switch can be used to disable overflow checks in assertions both for execution and static verification

```
function Add (X, Y : Integer) return Integer with
    Pre => X + Y in Integer;
    -- medium: overflow check might fail

X := Add (Integer'Last, 1);
    -- raised CONSTRAINT_ERROR : overflow check failed
```

#### Proof of Program Integrity – Contracts – Additional Contracts

- Assume generates an assumption for the proof tool
- Contract-Cases allows to annotate a subprogram by specifying a disjoint set of cases
  - A case is composed of a guard and a consequence
  - Guards of different cases must be disjoint and complete (there is always one and only one active case).
  - Only the consequence of the active case is verified

```
procedure Absolute (X : in out Integer) with
                    => X > Integer'First,
  Pre
  Contract Cases \Rightarrow (X < 0 \Rightarrow X = - X'Old,
                         X >= 0 => X = X' \text{ old});
-- info: disjoint contract cases proved
-- info: complete contract cases proved
-- info: contract case proved
pragma Assume (X < Integer'Last);</pre>
X := X + 1;
```

#### Proof of Program Integrity – Debug Failed Proof Attempts (1)

### Investigating Incorrect Code or Assertion

- [CODE] The check or assertion does not hold, because the code is wrong.
- [ASSERT] The assertion does not hold, because it is incorrect.

```
procedure Incr_Until (X : in out Natural) with
   Contract_Cases =>
      (Incremented => X > X'Old,
-- medium: contract case might fail
      others => X = X'Old) is
-- medium: contract case might fail
begin
   if X < 1000 then
      X := X + 1;
      Incremented := True;
   else
      Incremented := False;
   end if;
end Incr_Until;</pre>
```



#### Proof of Program Integrity – Debug Failed Proof Attempts (2)

#### Investigating Incorrect Code or Assertion

Test the program on representative inputs with the assertions enabled

```
procedure Incr Until (X : in out Natural) with
  Contract Cases =>
     (Incremented \Rightarrow X > X'Old,
     others => X = X'Old) is
begin
end Incr Until;
X := 0;
Incr Until (X);
X := 1000;
Incr Until (X);
-- raised ASSERT FAILURE : failed contract case at line 3
-- Incremented is True when evaluating the
-- Contract Cases' quards?
-- That is because they are evaluated before the call!
```

#### Proof of Program Integrity – Debug Failed Proof Attempts (3)

#### Investigating Unprovable Properties

- [SPEC] The check or assertion cannot be proved, because of some missing assertions about the behavior of the program
- [MODEL] The check or assertion is not proved because of current limitations in the model used by GNATprove

```
C : Natural := 100;
procedure Increase (X : in out Natural) with
  Post => (if X < C then X > X'Old else X = C) is
-- medium: postcondition might fail
begin
  if X < 90 then
    X := X + 10;
  elsif X >= C then
    X := C;
  else
    X := X + 1;
  end if;
end Increase;
```



#### Proof of Program Integrity – Debug Failed Proof Attempts (4)

#### Investigating Unprovable Properties

- GNATprove can precisely locate the part of large assertions that it cannot prove
- It also provides path information that might help the code review
- It may be useful to simplify the code during this investigation, for example by adding a simpler assertion and trying to prove it

```
C : Natural := 100; -- Requires C >= 90

procedure Increase (X : in out Natural) with
   Post => (if X < C then X > X'Old else X = C) is
-- medium: postcondition might fail, requires X = C

begin
   if X < 90 then
      X := X + 10;
   elsif X >= C then
      X := C;
...
```



#### Proof of Program Integrity – Debug Failed Proof Attempts (5)

#### Investigating Prover Shortcomings

- [TIMEOUT] The check or assertion is not proved because the prover timeouts
- [PROVER] The check or assertion is not proved because the prover is not smart enough

```
3
```

```
function GCD (A, B : Positive) return Positive with
  Post => A mod GCD'Result = 0
            and B mod GCD'Result = 0 is
-- medium: postcondition might fail
begin
  if A > B then
    return GCD (A - B, B);
  elsif B > A then
    return GCD (A, B - A);
  else
    return A;
  end if;
end GCD;
```

#### Proof of Program Integrity – Debug Failed Proof Attempts (6)

#### Investigating Prover Shortcomings

- Test if the prover would find a proof given more time or if another prover can find a proof
- Non-linear arithmetic (multiplication, division, modulo...) is not well handled by provers in general







# Is this correct? 1/10



```
package Lists with SPARK Mode is
  function Goes To (I, J : Index) return Boolean;
 procedure Link (I, J : Index) with Post => Goes To (I, J);
private
  type Cell (Is Set : Boolean := True) is record ...
  type Cell Array is array (Index) of Cell;
 Memory : Cell Array;
end Lists;
package body Lists with SPARK Mode is
   function Goes To (I, J : Index) return Boolean is
   begin
     if Memory (I).Is Set then
       return Memory (I).Next = J;
     end if;
     return False;
   end Goes To;
  procedure Link (I, J : Index) is
   begin
      Memory (I) := (Is Set => True, Next => J);
   end Link;
end Lists;
```



### Is this correct? 1/10



```
package Lists with SPARK Mode is
  function Goes To (I, J : Index) return Boolean;
  procedure Link (I, J : Index) with Post => Goes To (I, J);
private
  type Cell (Is Set : Boolean := True) is record ...
  type Cell Array is array (Index) of Cell;
  Memory : Cell Array;
end Lists:
package body Lists with SPARK Mode is
   function Goes To (I, J : Index) return Boolean is
   begin
     if Memory (I).Is Set then
       return Memory (I).Next = J;
     end if;
     return False;
   end Goes To;
   procedure Link (I, J : Index) is
   begin
      Memory (I) := (Is Set \Rightarrow True, Next \Rightarrow J);
   end Link;
end Lists;
```

It is correct, but it cannot be verified with GNATprove. As Goes\_To has no postcondition, nothing is known about its result



### Is this correct? 2/10



```
package Lists with SPARK Mode is
  function Goes To (I, J : Index) return Boolean;
 procedure Link (I, J : Index) with Post => Goes To (I, J);
private
  type Cell (Is Set : Boolean := True) is record ...
  type Cell Array is array (Index) of Cell;
  Memory : Cell Array;
  function Goes To (I, J : Index) return Boolean is
      (Memory (I). Is Set and then Memory (I). Next = J);
end Lists;
package body Lists with SPARK Mode is
   procedure Link (I, J : Index) is
  begin
      Memory (I) := (Is Set => True, Next => J);
   end Link;
end Lists;
```



# Is this correct? 2/10



```
package Lists with SPARK Mode is
  function Goes To (I, J : Index) return Boolean;
 procedure Link (I, J : Index) with Post => Goes To (I, J);
private
  type Cell (Is Set : Boolean := True) is record ...
  type Cell Array is array (Index) of Cell;
  Memory : Cell Array;
  function Goes To (I, J : Index) return Boolean is
      (Memory (I). Is Set and then Memory (I). Next = J);
end Lists;
package body Lists with SPARK Mode is
   procedure Link (I, J : Index) is
  begin
      Memory (I) := (Is Set => True, Next => J);
   end Link;
end Lists;
```

Goes\_To is an expression function. As a consequence, its body is available for proof.



### Is this correct? 3/10



```
package Stacks with SPARK Mode is
  type Stack is private;
  function Peek (S : Stack) return Natural;
 procedure Push (S : in out Stack; E : Natural) with
    Post \Rightarrow Peek (S) = E;
private
  type Stack is record ...
  function Peek (S : Stack) return Natural is
     (if S.Top in S.Content'Range then S.Content (S.Top) else 0);
end Stacks;
package body Stacks with SPARK Mode is
 procedure Push (S : in out Stack; E : Natural) is
 begin
    if S.Top >= Max then
      return;
    end if;
    S.Top := S.Top + 1;
    S.Content (S.Top) := E;
  end Push:
end Stacks;
```



### Is this correct? 3/10



```
package Stacks with SPARK Mode is
  type Stack is private;
  function Peek (S : Stack) return Natural;
 procedure Push (S : in out Stack; E : Natural) with
    Post \Rightarrow Peek (S) = E;
private
  type Stack is record ...
  function Peek (S : Stack) return Natural is
     (if S.Top in S.Content'Range then S.Content (S.Top) else 0);
end Stacks:
package body Stacks with SPARK Mode is
 procedure Push (S : in out Stack; E : Natural) is
 begin
    if S.Top >= Max then
      return;
    end if:
    S.Top := S.Top + 1;
    S.Content (S.Top) := E_i
  end Push:
end Stacks;
```

The postcondition of Push is only true if the stack is not full when Push is called.



### Is this correct? 4/10



```
package Stacks with SPARK Mode is
  type Stack is private;
  function Peek (S : Stack) return Natural;
 procedure Push (S : in out Stack; E : Natural) with
    Post \Rightarrow Peek (S) = E;
private
  type Stack is record ...
  function Peek (S : Stack) return Natural is
     (if S.Top in S.Content'Range then S.Content (S.Top) else 0);
end Stacks;
package body Stacks with SPARK Mode is
 procedure Push (S : in out Stack; E : Natural) is
 begin
    if S.Top >= Max then
      raise Is Full E;
    end if;
    S.Top := S.Top + 1;
    S.Content (S.Top) := E;
  end Push:
end Stacks;
```



### Is this correct? 4/10



```
package Stacks with SPARK Mode is
  type Stack is private;
  function Peek (S : Stack) return Natural;
 procedure Push (S : in out Stack; E : Natural) with
    Post \Rightarrow Peek (S) = E;
private
  type Stack is record ...
  function Peek (S : Stack) return Natural is
     (if S.Top in S.Content'Range then S.Content (S.Top) else 0);
end Stacks:
package body Stacks with SPARK Mode is
 procedure Push (S : in out Stack; E : Natural) is
 begin
    if S.Top >= Max then
      raise Is Full E;
                              GNATprove can now verify Push's
    end if:
                              postcondition as it only considers paths
                              leading to normal termination. It will
    S.Top := S.Top + 1;
                              warn that Is Full E may be raised at
    S.Content (S.Top) := E;
                              runtime though, leading to an error.
  end Push;
end Stacks;
```



### Is this correct? 5/10



```
package Stacks with SPARK Mode is
  type Stack is private;
  function Peek (S : Stack) return Natural;
  function Is Full (S : Stack) return Natural;
  procedure Push (S : in out Stack; E : Natural) with
    Pre => not Is Full (S),
    Post \Rightarrow Peek (S) = E;
private
  type Stack is record ...
  function Peek (S : Stack) return Natural is
     (if S.Top in S.Content'Range then S.Content (S.Top) else 0);
  function Is Full (S : Stack) return Natural is (S.Top >= Max);
end Stacks;
package body Stacks with SPARK Mode is
 procedure Push (S : in out Stack; E : Natural) is
 begin
    if S.Top >= Max then
      raise Is Full E;
    end if:
    S.Top := S.Top + 1;
    S.Content (S.Top) := E;
  end Push:
end Stacks;
```



# Is this correct? 5/10



```
package Stacks with SPARK Mode is
  type Stack is private;
  function Peek (S : Stack) return Natural;
  function Is Full (S : Stack) return Natural;
  procedure Push (S : in out Stack; E : Natural) with
    Pre => not Is Full (S),
    Post \Rightarrow Peek (S) = E;
private
  type Stack is record ...
  function Peek (S : Stack) return Natural is
     (if S.Top in S.Content'Range then S.Content (S.Top) else 0);
  function Is Full (S : Stack) return Natural is (S.Top >= Max);
end Stacks;
package body Stacks with SPARK Mode is
  procedure Push (S : in out Stack; E : Natural) is
 begin
    if S.Top >= Max then
      raise Is Full E;
                                       In the context of the
    end if;
                                      precondition, GNATprove can
                                       now verify that Is Full E can
                                       never be raised at runtime.
```



### Is this correct? 6/10



```
procedure Read Record (From : Integer) is
  function Read One (First : Integer; Offset : Integer)
    return Integer
  with
    Pre => Memory (First) + Offset in Memory'Range
  is
    Value : Integer := Memory (Memory (First) + Offset);
 begin
    if Is Too Coarse (Value) then
      Treat Value (Value);
    end if;
    return Value;
  end Read One;
begin
  Size := Read One (From, 0);
 pragma Assume (Size in 1 .. 10
               and then Memory (From) < Integer'Last - 2 * Size);</pre>
  Data1 := Read One (From, 1);
  Addr := Read One (From, Size + 1);
 pragma Assume (Memory (Addr) > Memory (From) + Size);
  Data2 := Read One (Addr, -Size);
end Read Record;
```



# Is this correct? 6/10



```
procedure Read Record (From : Integer) is
  function Read One (First : Integer; Offset : Integer)
    return Integer
 with
    Pre => Memory (First) + Offset in Memory'Range
  is
    Value : Integer := Memory (Memory (First) + Offset);
 begin
    if Is Too Coarse (Value) then
                                      It is correct, but it cannot be
      Treat Value (Value);
                                      verified with GNATprove. GNATprove
    end if;
                                      analyses Read One on its own and
    return Value;
                                      notices that an overflow may occur
  end Read One;
begin
                                      in its precondition in certain
  Size := Read One (From, 0);
                                      contexts.
 pragma Assume (Size in 1 .. 10
               and then Memory (From) < Integer'Last - 2 * Size);
  Data1 := Read One (From, 1);
  Addr := Read One (From, Size + 1);
 pragma Assume (Memory (Addr) > Memory (From) + Size);
  Data2 := Read One (Addr, -Size);
end Read Record;
```



### Is this correct? 7/10



```
procedure Read Record (From : Integer) is
  function Read One (First : Integer; Offset : Integer)
    return Integer
  with
    Pre => Memory (First) <= Memory'Last - Offset
  is
    Value : Integer := Memory (Memory (First) + Offset);
 begin
    if Is Too Coarse (Value) then
      Treat Value (Value);
    end if;
    return Value;
  end Read One;
begin
  Size := Read One (From, 0);
 pragma Assume (Size in 1 .. 10
               and then Memory (From) < Integer'Last - 2 * Size);</pre>
  Data1 := Read One (From, 1);
  Addr := Read One (From, Size + 1);
 pragma Assume (Memory (Addr) > Memory (From) + Size);
  Data2 := Read One (Addr, -Size);
end Read Record;
```



### Is this correct? 7/10



```
procedure Read Record (From : Integer) is
  function Read One (First : Integer; Offset : Integer)
    return Integer
 with
    Pre => Memory (First) <= Memory'Last - Offset
  is
    Value : Integer := Memory (Memory (First) + Offset);
 begin
    if Is Too Coarse (Value) then
                                      Unfortunately, our attempt to
      Treat Value (Value);
                                      correct Read One's precondition
    end if;
                                      failed. For example, an overflow
    return Value;
                                      will occur at runtime when Memory
  end Read One;
begin
                                       (First) is Integer'Last and Offset
  Size := Read One (From, 0);
                                      is negative.
 pragma Assume (Size in 1 .. 10
               and then Memory (From) < Integer'Last - 2 * Size);
  Data1 := Read One (From, 1);
  Addr := Read One (From, Size + 1);
 pragma Assume (Memory (Addr) > Memory (From) + Size);
  Data2 := Read One (Addr, -Size);
end Read Record;
```



### Is this correct? 8/10



```
procedure Read Record (From : Integer) is
  function Read One (First : Integer; Offset : Integer)
    return Integer
  is
    Value : Integer := Memory (Memory (First) + Offset);
 begin
    if Is Too Coarse (Value) then
      Treat Value (Value);
    end if;
    return Value;
  end Read One;
begin
  Size := Read One (From, 0);
 pragma Assume (Size in 1 .. 10
               and then Memory (From) < Integer'Last - 2 * Size);</pre>
  Data1 := Read One (From, 1);
  Addr := Read One (From, Size + 1);
 pragma Assume (Memory (Addr) > Memory (From) + Size);
  Data2 := Read One (Addr, -Size);
end Read Record;
```



### Is this correct? 8/10



```
procedure Read Record (From : Integer) is
  function Read One (First : Integer; Offset : Integer)
    return Integer
  is
    Value : Integer := Memory (Memory (First) + Offset);
 begin
                                       We could have fixed the contract
    if Is Too Coarse (Value) then
                                       on Read One to handle correctly
      Treat Value (Value);
                                       positive and negative values of
    end if:
    return Value;
                                       Offset. However, we found it
  end Read One;
                                       simpler to let the function be
begin
                                       inlined for proof by removing its
  Size := Read One (From, 0);
                                       precondition.
 pragma Assume (Size in 1 .. 10
               and then Memory (From) < Integer'Last - 2 * Size);</pre>
  Data1 := Read One (From, 1);
  Addr := Read One (From, Size + 1);
 pragma Assume (Memory (Addr) > Memory (From) + Size);
  Data2 := Read One (Addr, -Size);
end Read Record;
```



### Is this correct? 9/10



```
procedure Compute (X : in out Integer) with
  Contract Cases => ((X in -100 .. 100) => X = X'Old * 2,
                    (X in 0 .. 199) => X = X'Old + 1,
                     (X in -199 ... 0) => X = X'Old - 1,
                     X >= 200 => X = 200,
                                     => X = -200)
                     others
is
begin
  if X in -100 .. 100 then
   X := X * 2;
 elsif X in 0 .. 199 then
   X := X + 1;
  elsif X in -199 .. 0 then
   X := X - 1;
  elsif X \ge 200 then
   X := 200;
 else
  X := -200;
  end if;
end Compute;
```



### Is this correct? 9/10





```
procedure Compute (X : in out Integer) with
  Contract Cases \Rightarrow ((X in -100 .. 100) \Rightarrow X = X'Old * 2,
                     (X in 0 .. 199) => X = X'Old + 1,
                     (X in -199 ... 0) => X = X'Old - 1,
                      X >= 200 => X = 200,
                      others
                                       => X = -200)
is
begin
  if X in -100 .. 100 then
    X := X * 2;
                                     We duplicated in Compute's
  elsif X in 0 .. 199 then
                                     contract the content of its body.
   X := X + 1;
  elsif X in -199 .. 0 then
                                     This is not correct with respect
    X := X - 1;
                                     to the semantics of Contract Cases
  elsif X >= 200 then
                                     which expects disjoint cases, like
   X := 200;
                                     a case statement.
  else
   X := -200;
  end if;
end Compute;
```



### Is this correct? 10/10



```
procedure Compute (X : in out Integer) with
  Contract Cases => ((X in 1 .. 199) => X >= X'Old,
                    (X in -199 ... -1) => X <= X'Old,
                     X >= 200 => X = 200,
                     X \le -200 = X = -200
is
begin
  if X in -100 ... 100 then
   X := X * 2;
 elsif X in 0 .. 199 then
   X := X + 1;
  elsif X in -199 .. 0 then
   X := X - 1;
  elsif X >= 200 then
   X := 200;
 else
  X := -200;
 end if;
end Compute;
```



### Is this correct? 10/10





```
procedure Compute (X : in out Integer) with
  Contract Cases \Rightarrow ((X in 1 .. 199) \Rightarrow X >= X'Old,
                     (X in -199 ... -1) => X <= X'Old,
                      X >= 200 => X = 200
                      X <= -200 => X = -200
is
begin
  if X in -100 ... 100 then
   X := X * 2;
                                     Here, GNATprove can successfully
  elsif X in 0 .. 199 then
                                     check that the different cases are
    X := X + 1;
  elsif X in -199 .. 0 then
                                     disjoint. It can also successfully
   X := X - 1;
                                     verify each case on its own. This
  elsif X >= 200 then
                                     is not enough though, as a
   X := 200;
                                     Contract Cases must also be total.
  else
                                     Here, we forgot the value 0.
   X := -200;
  end if;
end Compute;
```





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