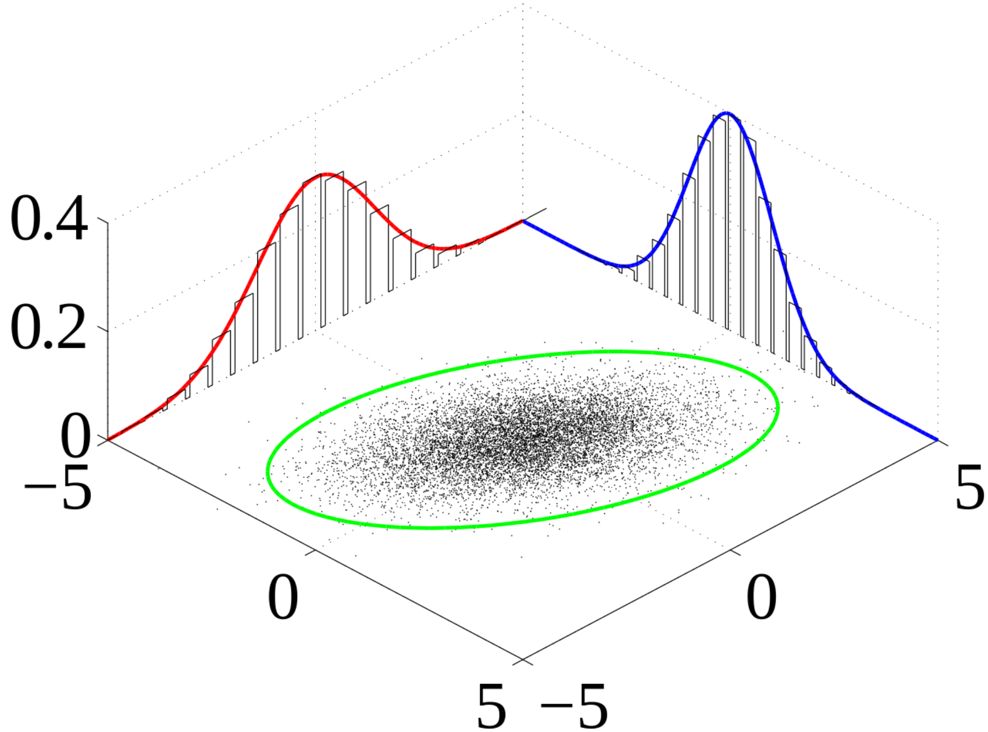

Simulation with Copulas in R

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2015-11-18

Problem

We want to calculate assets VAR on our risk drivers with incorporation of their dependence:

a) IR YR1 (PLN government bond IR at maturity of one year)
b) EQ index (WIG – index of stocks listed in Poland Stock Market)

What approach could be applied for such a problem?

Solve our problem with copula approach

- We need VAR of our assets
- To calculate it we need distibution of changes in value of assets.
- Since we are exposed only to those two risk drivers this distibution that we are seeking for will be joint distibution of changes in those risk drivers
- To construct joint distibution we need some framwork (in our case copulas) and marginal distibution for each risk driver
- To obtain marginal distibution of given risk driver we need to fit some teoretical distibution to our data

Distribution fitting - risk drivers as random variables

Since it is not core of the presentation topic we will use only one nice distibution. So called Normal Inverse Gaussian (NIG) distribution. It is a generalization of normal distribution. It is widely used within NN Internal Model modules.

NIG(μ, γ, σ,$\bar{\alpha}$) is a member of generalized hyperbolic distibutions and it is a mixture defined as:

$$NIG = \mu + W\gamma + \sqrt{W}\sigma Z,$$

where: W ~ GIG(λ=0.5,$\bar{\alpha}$), Z ~ N(0,1)

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Simulate from fitted distibutions

a) If we know that X has continuous cumulative density function F then $F(X) \sim U(0,1)$.
b) Moreover by the inversion method $X = F^{-1}(U)$. This function is also called quantile function.

Those transformations are denoted in R by a) *pnorm* and b) *qnorm* functions (example for normal distribution). We will use those extensively.

U denotes uniform distribution on (0,1) interval

Simulate from fitted distributions

a) If we know that X has continuous cumulative density function F then F(X) ~ U(0,1).

b) Moreover by the inversion method X = $F^{-1}(U)$. This function is also called quantile function.

Simple example of inversion method for exponential distribution

a) $F(x) = 1 - e^{-\lambda x}$

b) $u = 1 - e^{-\lambda x}$

$e^{-\lambda x} = 1 - u$

$\ln(e^{-\lambda x}) = \ln(1-u)$

$-\lambda x = \ln(1-u)$

$x = \frac{\ln(1-u)}{-\lambda}$

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Simulate from fitted distribution

Other function that has to be mentioned in contexts of simulation is function to simulate observation from given distribution.

It is denoted as:

- *rnorm* (for normal distribution)
- *runif* (for uniform distribution)
- *rghyp* (for generalized hyperbolic distributions)

Obtain joint distribution with Copulas

Copula is an function to capture complex dependency between variables. It could be perceived an extension of the concept of multivariate distribution. Why copulas approach is often better than multivariate distribution one?

Modelling with use of copula allows us to break down modelling process into two parts:

a) Choice of arbitrary marginal distribution for all variables
b) Choice of arbitrary copula function

Estimation of marginal distribution and copula can be done separately. This provide mass flexibility.

Elliptical copulas

a) Gaussian copula
- Multivariate normal distribution is an Gaussian copula with normal margins. As copula dependency is margin independent the Gaussian copula with any margins provide same dependency as multivariate normal distribution.
- Its parameter θ is defined as linear correlation
- It assume zero tail dependence

b) Student t copula
- It allows for both lower and upper tail dependence (symmetric)

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Parametric copulas applied in presentation

a) Archimedean copulas:

Clayton - with parameter θ directly related to kendall tau:

$\tau = \frac{\theta}{\theta+2}$ It allows for **lower** tail dependence

Gumbel - with parameter θ directly related to kendall tau:

$\tau = 1 - \theta^{-1}$ It allows for **upper** tail dependence

b) BB7 – complex copula (one of mixtures of max-id copulas) that allows for **both** upper and lower tail dependence

c) Rotated Gumbel copula (270 degrees)

Algorithm for simulation of Clayton copula [6]

a) Sample $V \sim F = \mathcal{LS}^{-1}[\psi]$
b) Sample $R_j \sim Exp(1), j \in \{1, \dots, d\}$
c) Set $U_j = \psi\left(\frac{R_j}{V}\right), j \in \{1, \dots, d\}$

Family	Parameter	$\psi(t)$	$V \sim F = \mathcal{LS}^{-1}[\psi]$
Clayton	$\theta \in (0, \infty)$	$(1+t)^{-1/\theta}$	$\Gamma(1/\theta, 1)$
Frank	$\theta \in (0, \infty)$	$-\log(1-(1-e^{-\theta})\exp(-t))/\theta$	$\mathrm{Log}(1-e^{-\theta})$
Gumbel	$\theta \in [1, \infty)$	$\exp(-t^{1/\theta})$	$S(1/\theta, 1, \cos^{\theta}(\pi/(2\theta)), \mathbf{1}_{\{\theta=1\}}; 1)$

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References

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3. H. Joe, *Dependence Modeling with Copulas,* 2014 (not yet published on CRAN package *CopulaModel - copula.stat.ubc.ca*)
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5. H. Joe, H. Li, A. K. Nikoloulopoulos, *Tail dependence functions and vine copulas*, 2010
6. M. Hofer, M.Machler, Nested Archimedean Copulas Meet R – Vignette, https://cran.r-project.org/web/packages/copula/vignettes/nacopula-pkg.pdf

Questions & Answers

Exercises – instructions in code

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