# Rules for Making Bode Plots

| Term | Magnitude | Phase |
| --- | --- | --- |
| **Constant:** K | $20\cdot\log_{10}(\lvert K\rvert)$ | K>0: 0°<br>K<0: ±180° |
| **Real Pole:** $\dfrac{1}{\frac{s}{\omega_0}+1}$ | • Low freq. asymptote at 0 dB<br>• High freq. asymptote at -20 dB/dec<br>• Connect asymptotic lines at $\omega_0$, | • Low freq. asymptote at 0°.<br>• High freq. asymptote at -90°.<br>• Connect with straight line from $0.1\cdot\omega_0$ to $10\cdot\omega_0$. |
| **Real Zero*:** $\dfrac{s}{\omega_0}+1$ | • Low freq. asymptote at 0 dB<br>• High freq. asymptote at +20 dB/dec.<br>• Connect asymptotic lines at $\omega_0$. | • Low freq. asymptote at 0°.<br>• High freq. asymptote at +90°.<br>• Connect with line from $0.1\cdot\omega_0$ to $10\cdot\omega_0$. |
| **Pole at Origin:** $\dfrac{1}{s}$ | • -20 dB/dec; through 0 dB at $\omega$=1. | • -90° for all $\omega$. |
| **Zero at Origin*:** s | • +20 dB/dec; through 0 dB at $\omega$=1. | • +90° for all $\omega$. |
| **Underdamped Poles:** $\dfrac{1}{\left(\frac{s}{\omega_0}\right)^2+2\zeta\left(\frac{s}{\omega_0}\right)+1}$ | • Low freq. asymptote at 0 dB.<br>• High freq. asymptote at -40 dB/dec.<br>• Connect asymptotic lines at $\omega_0$.<br>• Draw peak† at freq. $\omega_0$, with amplitude $H(j\omega_0)=-20\cdot\log_{10}(2\zeta)$ | • Low freq. asymptote at 0°.<br>• High freq. asymptote at -180°.<br>• Connect with straight line from $\omega=\omega_0\cdot 10^{-\zeta}$ to $\omega_0\cdot 10^{\zeta}$ |
| **Underdamped Zeros*:** $\left(\frac{s}{\omega_0}\right)^2+2\zeta\left(\frac{s}{\omega_0}\right)+1$ | • Low freq. asymptote at 0 dB.<br>• High freq. asymptote at +40 dB/dec.<br>• Connect asymptotic lines at $\omega_0$.<br>• Draw dip† at freq. $\omega_0$, with amplitude $H(j\omega_0)=+20\cdot\log_{10}(2\zeta)$ | • Low freq. asymptote at 0°.<br>• High freq. asymptote at +180°.<br>• Connect with straight line from $\omega=\omega_0\cdot 10^{-\zeta}$ to $\omega_0\cdot 10^{\zeta}$ |
| **Time Delay:** $e^{-sT}$ | • No change in magnitude | • Phase drops linearly. Phase = $-\omega T$ radians or $-\omega T\cdot 180/\pi$°.<br>On logarithmic plot phase appears to drop exponentially. |

**Notes:**

$\omega_0$ is assumed to be positive. If ω0 is negative, magnitude is unchanged, but phase is changed.

* Rules for drawing zeros create the mirror image (around 0 dB, or 0°) of those for a pole with the same $\omega_0$.

† We assume any peaks for $\zeta>0.5$ are too small to draw, and ignore them. However, for underdamped poles and zeros peaks exists for $0<\zeta<0.707=1/\sqrt{2}$ and peak freq. is not exactly at, $\omega_0$ (peak is at $\omega_{peak}=\omega_0\sqrt{1-2\zeta^2}$ ).

For $n^{th}$ order pole or zero make asymptotes, peaks and slopes n times higher than shown. For example, a double (i.e., repeated) pole has high frequency asymptote at -40 dB/dec, and phase goes from 0 to –180°). Don't change frequencies, only the plot values and slopes.


More detail at http://lpsa.swarthmore.edu/Bode/Bode.html

# Quick Reference for Making Bode Plots

If starting with a transfer function of the form (some of the coefficients $b_i$, $a_i$ may be zero).

$$H(s) = C\frac{s^n + \cdots + b_1 s + b_0}{s^m + \cdots + a_1 s + a_0}$$

Factor polynomial into real factors and complex conjugate pairs (p can be positive, negative, or zero; p is zero if $a_0$ and $b_0$ are both non-zero).

$$H(s) = C \cdot s^p \frac{(s+\omega_{z1})(s+\omega_{z2})\cdots(s^2+2\zeta_{z1}\omega_{0z1}s+\omega_{0z1}^2)(s^2+2\zeta_{z2}\omega_{0z2}s+\omega_{0z2}^2)\cdots}{(s+\omega_{p1})(s+\omega_{p2})\cdots(s^2+2\zeta_{p1}\omega_{0p1}s+\omega_{0p1}^2)(s^2+2\zeta_{p2}\omega_{0p2}s+\omega_{0p2}^2)\cdots}$$

Put polynomial into standard form for Bode Plots.

$$H(s) = C\frac{\omega_{z1}\omega_{z2}\cdots\omega_{0z1}^2\omega_{0z2}^2\cdots}{\omega_{p1}\omega_{p2}\cdots\omega_{0p1}^2\omega_{0p2}^2\cdots}\cdot s^p \frac{\left(\frac{s}{\omega_{z1}}+1\right)\left(\frac{s}{\omega_{z2}}+1\right)\cdots\left(\left(\frac{s}{\omega_{0z1}}\right)^2+2\zeta_{z1}\left(\frac{s}{\omega_{0z1}}\right)+1\right)\left(\left(\frac{s}{\omega_{0z2}}\right)^2+2\zeta_{z2}\left(\frac{s}{\omega_{0z2}}\right)+1\right)\cdots}{\left(\frac{s}{\omega_{p1}}+1\right)\left(\frac{s}{\omega_{p2}}+1\right)\cdots\left(\left(\frac{s}{\omega_{0p1}}\right)^2+2\zeta_{p1}\left(\frac{s}{\omega_{0p1}}\right)+1\right)\left(\left(\frac{s}{\omega_{0p2}}\right)^2+2\zeta_{p2}\left(\frac{s}{\omega_{0p2}}\right)+1\right)\cdots}$$

$$= K \cdot s^p \frac{\left(\frac{s}{\omega_{z1}}+1\right)\left(\frac{s}{\omega_{z2}}+\right)\cdots\left(\left(\frac{s}{\omega_{0z1}}\right)^2+2\zeta_{z1}\left(\frac{s}{\omega_{0z1}}\right)+1\right)\left(\left(\frac{s}{\omega_{0z2}}\right)^2+2\zeta_{z2}\left(\frac{s}{\omega_{0z2}}\right)+1\right)\cdots}{\left(\frac{s}{\omega_{p1}}+1\right)\left(\frac{s}{\omega_{p2}}+1\right)\cdots\left(\left(\frac{s}{\omega_{0p1}}\right)^2+2\zeta_{p1}\left(\frac{s}{\omega_{0p1}}\right)+1\right)\left(\left(\frac{s}{\omega_{0p2}}\right)^2+2\zeta_{p2}\left(\frac{s}{\omega_{0p2}}\right)+1\right)\cdots}$$

Take the terms (constant, real poles and zeros, origin poles and zeros, complex poles and zeros) one by one and plot magnitude and phase according to rules on previous page. Add up resulting plots.


More detail at http://lpsa.swarthmore.edu/Bode/Bode.html

# Matlab Tools for Bode Plots

```
>> n=[1 11 10];                       %A numerator polynomial (arbitrary)
>> d=[1 10 10000 0];                  %Denominator polynomial (arbitrary)
>> sys=tf(n,d)
Transfer function:
    s^2 + 11 s + 10
----------------------
s^3 + 10 s^2 + 10000 s

>> damp(d)                            %Find roots of den. If complex, show zeta, wn.
        Eigenvalue              Damping     Freq. (rad/s)
  0.00e+000                   -1.00e+000     0.00e+000
 -5.00e+000 + 9.99e+001i       5.00e-002     1.00e+002
 -5.00e+000 - 9.99e+001i       5.00e-002     1.00e+002

>> damp(n)                            %Repeat for numerator
Eigenvalue      Damping     Freq. (rad/s)
 -1.00e+000     1.00e+000      1.00e+000
 -1.00e+001     1.00e+000      1.00e+001

>> %Use Matlab to find frequency response (hard way).
>> w=logspace(-2,4);                  %omega goes from 0.01 to 10000;
>> fr=freqresp(sys,w);
>> subplot(211);  semilogx(w,20*log10(abs(fr(:))));  title('Mag response, dB')
>> subplot(212);  semilogx(w,angle(fr(:))*180/pi);   title('Phase resp, degrees')

>> %Let Matlab do all of the work
>> bode(sys)

>> %Find Freq Resp at one freq.     %Hard way
>> fr=polyval(n,j*10)./polyval(d,j*10)
fr =   0.0011 + 0.0010i

>> %Find Freq Resp at one freq.     %Easy way
>> fr=freqresp(sys,10)
fr =   0.0011 + 0.0009i

>> abs(fr)
ans =    0.0014

>> angle(fr)*180/pi   %Convert to degrees
ans =   38.7107


>> %You can even find impulse and step response from transfer function.
>> step(sys)
>> impulse(sys)
```

```
>> [n,d]=tfdata(sys,'v')          %Get numerator and denominator.
n =
     0     1    11    10
d =
           1          10       10000           0

>> [z,p,k]=zpkdata(sys,'v')       %Get poles and zeros
z =
   -10
    -1
p =
        0
  -5.0000 +99.8749i
  -5.0000 -99.8749i
k =
     1


>> %BodePlotGui - Matlab program shows individual terms of Bode Plot.  Code at:
>> %   http://lpsa.swarthmore.edu/NatSci/Bode/BodePlotGui.html
>>
>> BodePlotGui(sys)
```