2.12

$X_t = Z \mathbb{1}_{\{t \in A\}}$

$\mathbb{P}\{X_{t_1} \le a_1, X_{t_2} \le a_2, \dots, X_{t_n} \le a_n\}$

$= \mathbb{P}\{Z \mathbb{1}_{\{t_1 \in A\}} \le a_1, \dots, Z \mathbb{1}_{\{t_n \in A\}} \le a_n\}$

$= \mathbb{P}\{Z \le \min \{a_i \text{ s.t } t_i \in A\} \cup \{0\}\}$ if one is not in A and one is

$\mathbb{P}\{Z \le 0\}$ if all not in A

$\mathbb{P}\{Z \le \min \{a_i, \forall i \in \{1, \dots, n\}\}\}$ if all in A

(2)

$f_i : \mathbb{R}^d \to \mathbb{R}$

$$X_t = \sum^{n} Z_i f_i(t)$$

deine covariance funct

$$E[X_t] = \sum f_i(t) E[Z_i]$$
$$= \sum^{n} f_i(t) m_i$$

$$E[X_t X_s] = \sum_i^{n} \sum_j^{n} f_i(t) f_j(s) E[Z_i Z_j]$$

$$Cov(X_t, X_s) = E[(X_t - E[X_t])(X_s - E[X_s])]$$
$$= E[X_t X_s] - E[X_t] E[X_s]$$
$$= \sum_i^{n} \sum_j^{n} f_i(t) f_j(s) E[Z_i Z_j]$$
$$- \sum_i^{n} \sum_j^{n} f_i(t) f_j(t) E[Z_i] E[Z_i]$$

$$= \sum_i^{n} \sum_j^{n} f_i(t) \left\{ f_j(s) E[Z_i Z_j] - f_j(t) E[Z_i] E[Z_j] \right\}$$

(6)

Spectral measure of 1-dim
Ornstein - Uhlenbeck process w/ covariance

$$p(t) = \exp\{-\beta|t|\}/2\beta\}$$

$$f(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \exp(-i\omega t)\, p(t)\, dt$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \exp\{-i\omega t - \beta|t|\}/2\beta\}\, dt$$

$$= \frac{1}{4\pi\beta}\int_{-\infty}^{\infty} \exp\}-i\omega t - \beta|t|\}\, dt$$

$$= \frac{1}{4\pi\beta}\left(\int_{0}^{\infty} \exp\}-i\omega t - \beta t\}\, dt + \int_{-\infty}^{0} \exp\}-i\omega t + \beta t\}\, dt\right\}$$

$$= \frac{1}{4\pi\beta}\left\}\int_{0}^{\infty} \exp\}-i\omega t - \beta t\}\, dt + \int_{0}^{\infty} \exp\}i\omega t - \beta t\}\, dt\right)$$

$$= \frac{1}{4\pi\beta}\int_{0}^{\infty} \exp\}-2\beta t\}\, dt$$

$$= \frac{1}{4\pi\beta}\left(\frac{1}{-2\beta}\exp\}-2\beta t\}\Big|_{0}^{\infty}\right)$$

$$= \frac{1}{4\pi\beta}\left(\frac{1}{2\beta}\right\} = \frac{1}{8\pi\beta^2}$$

$$= \frac{1}{4\pi\beta} \left\{ \int_0^\infty \exp\{-\beta t\} \exp\{i\omega t\} + \exp\{-i\omega t\} \, dt \right\}$$

$$= \frac{2}{4\pi\beta} \left\{ \int_0^\infty \exp\{-\beta t\} \cos(\omega t) \, dt \right\}$$

$$= \frac{\cancel{2}\, 1}{2\pi\beta} \left\{ \int_0^\infty \exp\{-\beta t\} \cos(\omega t) \, dt \right\}$$

$$= \frac{1}{2\pi(\beta^2 + \omega^2)}$$