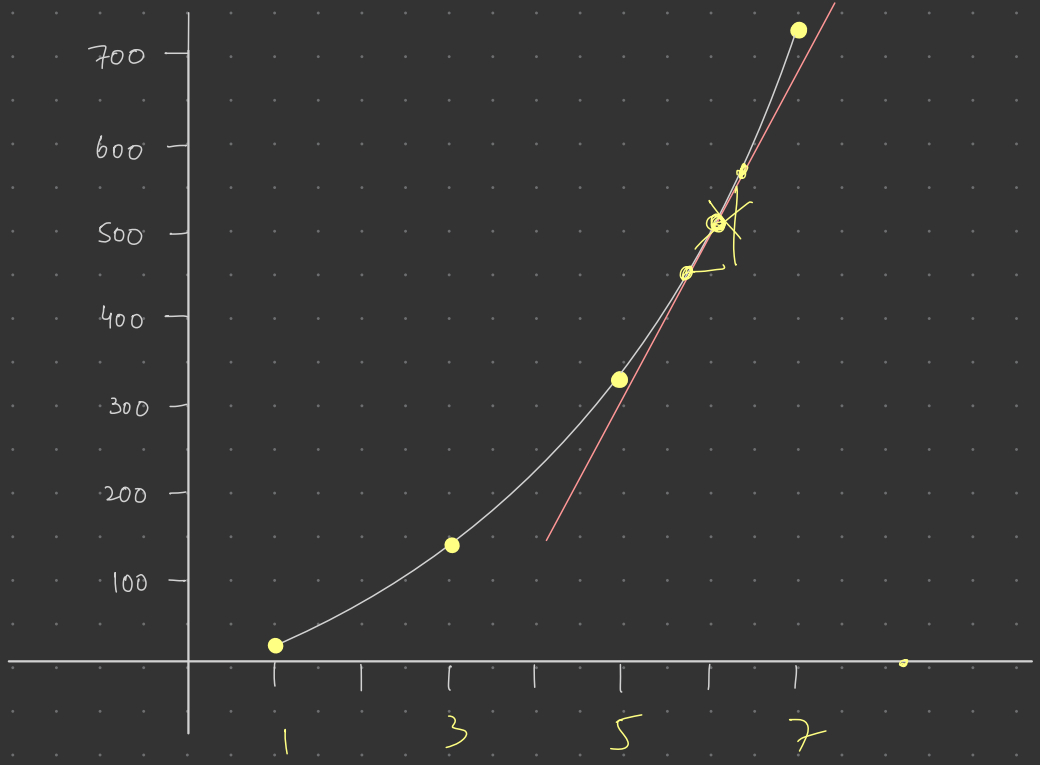
Numeric Differentiation



Given data points:

- No function definition given

- Find derivative "numerically" at a given point.

- Not the original polynomial!

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
x_0 1	y_0 24			
		Δy_0 96		
x_1 3	y_1 120		120	
		Δy_1 216		48
x_2 5	y_2 336		168	
		Δy_2 384		
x_3 7	y_3 720			

$h = 2, \quad x_0 = 1, \quad y_0 = 24$

$p = \dfrac{x - x_0}{h} = \dfrac{x-1}{2}$

$$y(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \cdots$$

$$= 24 + \left(\frac{x-1}{2}\right)(96) + \frac{\left(\frac{x-1}{2}\right)\left(\frac{x-1}{2}-1\right)}{2}(120) + \frac{\left(\frac{x-1}{2}\right)\left(\frac{x-1}{2}-1\right)\left(\frac{x-1}{2}-2\right)}{6}(48)$$

$$= 24 + (x-1)(48) + \left(\frac{x-1}{2}\right)\left(\frac{x-1-2}{2}\right)(60) + \left(\frac{x-1}{2}\right)\left(\frac{x-1-2}{2}\right)\left(\frac{x-1-4}{2}\right)(8)$$

$$= 24 + 48x - 48 + (x-1)(x-3)(15) + (x-1)(x-3)(x-5)$$

$$= -24 + 48x + (x^2-4x+3)(15) + (x^2-4x+3)(x-5)$$

$$= -24 + 48x + 15x^2 - 60x + 45 + x^3 - 4x^2 + 3x - 5x^2 + 20x - 15$$

$$= 6 + 11x + 6x^2 + x^3$$

$$y(x) = x^3 + 6x^2 + 11x + 6 \qquad y(8) = 990$$

To differentiate, we can do "perturbation".

$$\Delta x = 0.1$$

$$y'(x) = \frac{y(x+\Delta x) - y(x-\Delta x)}{2 \cdot \Delta x}$$

OR

calculate $y(x)$ closed form and then differentiate:

$$y(x) = x^3 + 6x^2 + 11x + 6$$

$$y'(x) = 3x^2 + 12x + 11$$

$$y'(8) = (3)(8)^2 + (12)(8) + 11$$

$$= 192 + 96 + 11$$

$$= \underline{299}$$