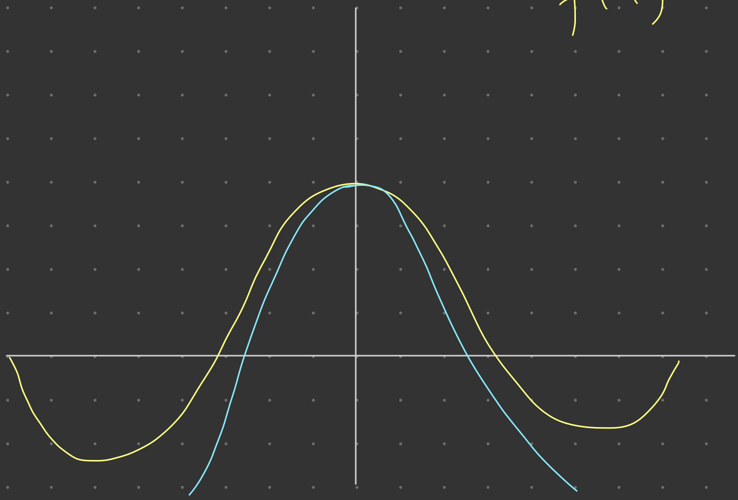
Taylor Series.

$\cos(x)$

$$f(x) = c_0 + c_1 x + c_2 x^2 \ldots$$

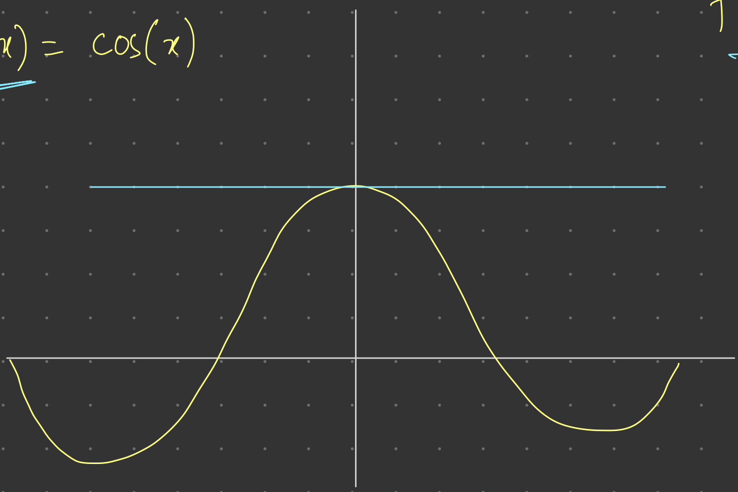
Let's focus on this first...



when x is 0,

we want $f(x) = \cos(x)$

$$f(x) = c_0 + \cancel{c_1 x + c_2 x^2}$$

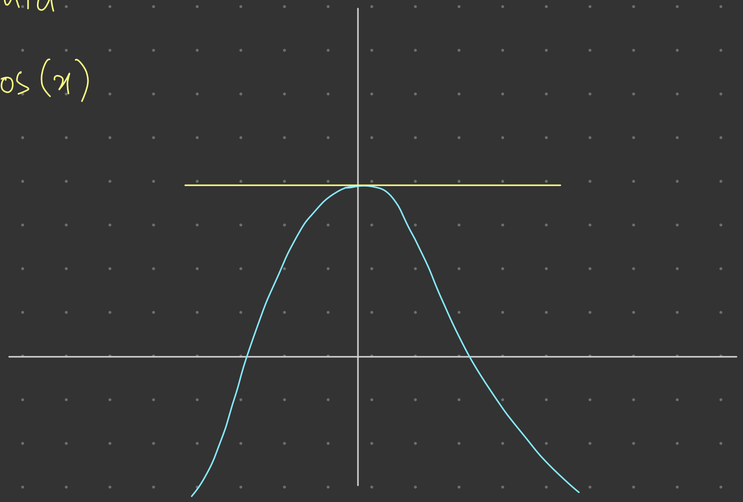


so, $\boxed{c_0} = \cos(0) \rightsquigarrow \boxed{1}$

(*) But more generally, just the value we already know for $\cos(x)$ at 0

Taking a step further ...

$f'(x)$ should be $\frac{d}{dx}\cos(x)$



$$\frac{d}{dx}\cos(0) = -\sin(0)$$

$$f(x) = \overset{1}{\cancel{c_0}} + c_1 x$$

$$f'(x) = c_1$$

$$-\sin(x) = c_1$$

$$c_1 = -\sin(x)$$

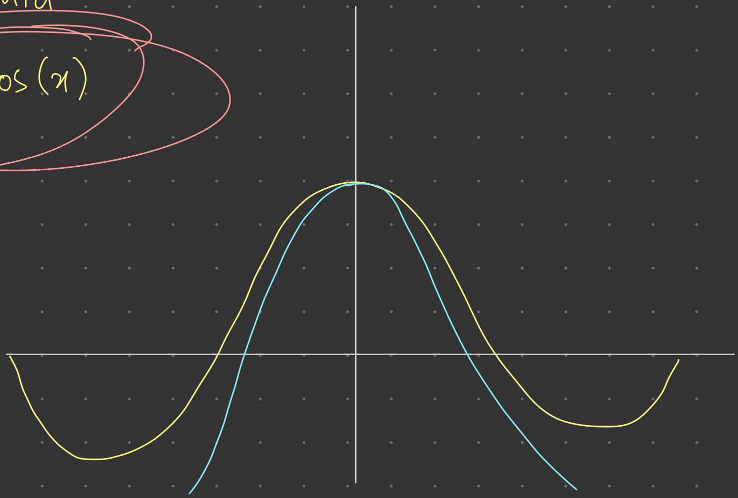
$$-\sin(0) = 0$$

$$0 = c_1 \rightsquigarrow c_1 = 0$$

$$c_0 = 1$$

Taking a step further...

$f''(x)$ should be $\frac{d^2}{dx^2}\cos(x)$



$$\frac{d^2}{dx^2}\cos(x) = \boxed{-\cos(x)}$$

$$f(x) = 1 + 0x + c_2 x^2$$

derivative → $f'(x) = 2c_2 x$

curvature → $f''(x) = 2c_2$

$$-\cos(x) = 2c_2$$

$$c_2 = \frac{-\cos(x)}{2}$$

where did this come from?

and this?

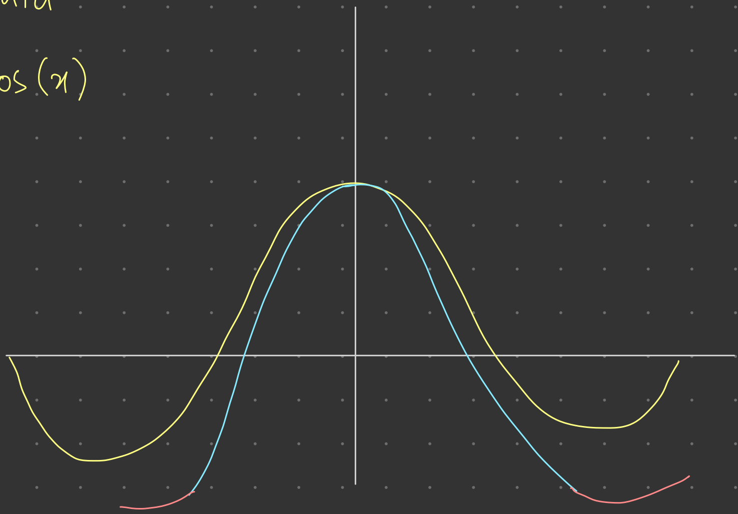
$$-\cos(x) = -1$$

$$c_2 = \frac{-1}{2}$$

$$\boxed{f(x) = 1 + 0x + \left(\frac{-1}{2}\right)x^2}$$

Taking a step further...

$f'''(x)$ should be $\frac{d^3}{dx^3}\cos(x)$



$$f(x) = 1 + 0x - \frac{1}{2}x^2 + c_3 x^3$$

$$f'(x) = \boxed{}x + 3c_3 x^2$$

$$f''(x) = 2 \cdot 3c_3 \cdot x$$

$$f'''(x) = \underbrace{1 \cdot 2 \cdot 3}\cdot c_3$$

$$f'''(x) = 3!\,c_3$$

$$\frac{d^3}{dx^3}\cos(x) = \sin(x)$$

$$\sin(x) = 3!\;c_3$$

$$c_3 = \frac{\sin(x)}{3!}$$

$$c_0 = 1 \rightsquigarrow \cos(x)$$

$$c_1 = -\sin(x) \rightsquigarrow \frac{d}{dx}\cos(x)$$

$$c_2 = \frac{-\cos(x)}{2!} \rightsquigarrow \frac{d^2}{dx^2}\cos(x) \quad /2!$$

$$c_3 = \frac{\sin(x)}{3!} \rightsquigarrow \frac{d^3}{dx^3}\cos(x) \quad /3!$$

Generalize: $\cancel{f}\,h(x) = \cos(x)$

$$f(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3 \ldots\ldots$$

$$= h(x) + h'(x)\cdot x^1 + \frac{h''(x)}{2}\cdot x^2 + \frac{h'''(x)}{3!}x^3$$

$$= h(x) + \frac{h'(x)\cdot x^1}{1!} + \frac{h''(x)}{2!}\cdot x^2 + \frac{h'''(x)}{3!}x^3$$

$$f(x) = \boxed{h(x)} + \sum_{i=1}^{n} \frac{h^i(x)\cdot x^i}{i!}$$

$$\frac{h^i(x)}{i!} \quad x^i \quad + \longrightarrow$$