

Machine Learning

Deciding what to try next

Debugging a learning algorithm:

Suppose you have implemented regularized linear regression to predict housing prices.

$$J(\theta) = \frac{1}{2m} \left[\sum_{i=1}^{m} (h_{\theta}(x^{(i)}) - y^{(i)})^2 + \lambda \sum_{j=1}^{m} \theta_j^2 \right]$$

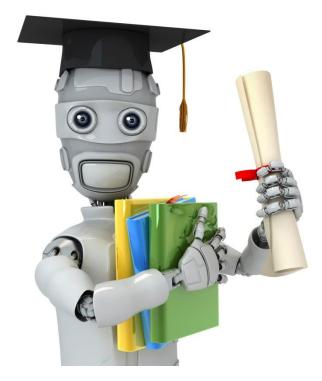
However, when you test your hypothesis on a new set of houses, you find that it makes unacceptably large errors in its predictions. What should you try next?

- Get more training examples
- Try smaller sets of features
- Try getting additional features
- Try adding polynomial features $(x_1^2, x_2^2, x_1x_2, \text{etc.})$
- Try decreasing λ
- Try increasing λ

Machine learning diagnostic:

Diagnostic: A test that you can run to gain insight what is/isn't working with a learning algorithm, and gain guidance as to how best to improve its performance.

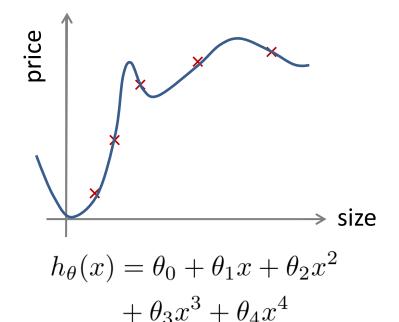
Diagnostics can take time to implement, but doing so can be a very good use of your time.



Machine Learning

Evaluating a hypothesis

Evaluating your hypothesis



Fails to generalize to new examples not in training set.

 $x_1 = \text{ size of house}$

 $x_2 = \text{ no. of bedrooms}$

 $x_3 = \text{ no. of floors}$

 $x_4 = age of house$

 x_5 = average income in neighborhood

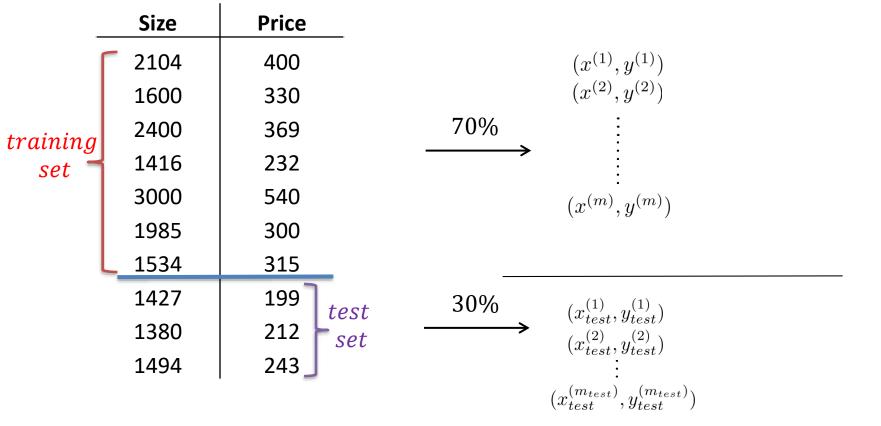
 $x_6 =$ kitchen size

•

 x_{100}

Evaluating your hypothesis

Dataset:



3

Training/testing procedure for linear regression

- Learn parameter θ from training data (minimizing training error $J(\theta)$)

Compute test set error:

$$J_{test}(\theta) = \frac{1}{2m_{test}} \sum_{i=1}^{m_{test}} (h_{\theta}(x_{test}^{(i)}) - y_{test}^{(i)})^2$$

Training/testing procedure for logistic regression

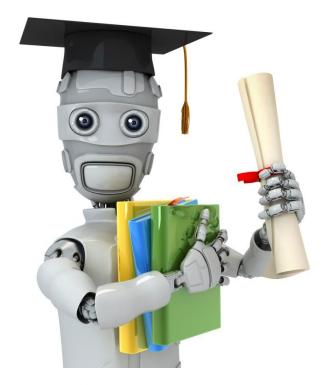
- Learn parameter heta from training data
- Compute test set error:

$$J_{test}(\theta) = -\frac{1}{m_{test}} \sum_{i=1}^{m_{test}} y_{test}^{(i)} \log h_{\theta}(x_{test}^{(i)}) + (1 - y_{test}^{(i)}) \log h_{\theta}(x_{test}^{(i)})$$

- Misclassification error (0/1 misclassification error):

$$\operatorname{err}(h_{\theta}(x), y) = \begin{cases} 1 & \text{if } h_{0}(x) \geq 0.5, y = 0 \\ & \text{or if } h_{\theta}(x) < 0.5, y = 1 \\ 0 & \text{otherwise} \end{cases}$$

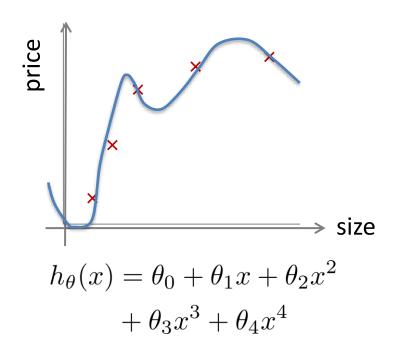
Test error =
$$\frac{1}{m_{\text{test}}} \sum_{i=1}^{m_{\text{test}}} \text{err} \left(h_{\theta} \left(x_{\text{test}}^{(i)} \right), y_{\text{test}}^{(i)} \right)$$



Machine Learning

Model selection and training/validation/test sets

Overfitting example



Once parameters $\theta_0, \theta_1, \dots, \theta_4$ were fit to some set of data (training set), the error of the parameters as measured on that data (the training error $J(\theta)$) is likely to be lower than the actual generalization error.

Model selection

- 1. $h_{\theta}(x) = \theta_0 + \theta_1 x$
- 2. $h_{\theta}(x) = \theta_0 + \theta_1 x + \theta_2 x^2$
- 3. $h_{\theta}(x) = \theta_0 + \theta_1 x + \dots + \theta_3 x^3$
- **10.** $h_{\theta}(x) = \theta_0 + \theta_1 x + \cdots + \theta_{10} x^{10}$

Choose $\theta_0 + \dots \theta_5 x^5$

How well does the model generalize? Report test set error $J_{test}(\theta^{(5)})$.

Problem: $J_{test}(\theta^{(5)})$ is likely to be an optimistic estimate of generalization error. I.e. our extra parameter (d = degree of polynomial) is fit to test set.

Evaluating your hypothesis

Dataset:

	Size	Price		$(x \cdot y \cdot y \cdot y)$
60%	2104	400		$(x^{(m)}, y^{(m)})$
	1600	330		$(x_{cv}^{(1)}, y_{cv}^{(1)})$
	2400	369	training	$(x_{cv}^{(2)},y_{cv}^{(2)}) \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ $
	1416	232	set	$(x_{\grave{c}v'},y_{\grave{c}v'})$
	3000	540	7	$(m_{\alpha n})$ $(m_{\alpha n})$
	1985	300		$(x_{cv}^{(m_{cv})}, y_{cv}^{(m_{cv})})$
20%	1534	315	Cross	${(x_{test}^{(1)}, y_{test}^{(1)})}$
	1427	199	validation set	$(x_{test}^{(2)},y_{test}^{(2)})$
20%	1380	212	test	(x_{test}, y_{test}) :
	1494	243	set	$(x_{test}^{(m_{test})}, y_{test}^{(m_{test})})$

 $(x^{(1)}, y^{(1)})$

 $(x^{(2)}, y^{(2)})$

Train/validation/test error

Training error:

$$J_{train}(\theta) = \frac{1}{2m} \sum_{i=1}^{m} (h_{\theta}(x^{(i)}) - y^{(i)})^2$$

Cross Validation error:

$$J_{cv}(\theta) = \frac{1}{2m_{cv}} \sum_{i=1}^{m_{cv}} (h_{\theta}(x_{cv}^{(i)}) - y_{cv}^{(i)})^2$$

Test error:

$$J_{test}(\theta) = \frac{1}{2m_{test}} \sum_{i=1}^{test} (h_{\theta}(x_{test}^{(i)}) - y_{test}^{(i)})^2$$

Model selection

1.
$$h_{\theta}(x) = \theta_0 + \theta_1 x$$

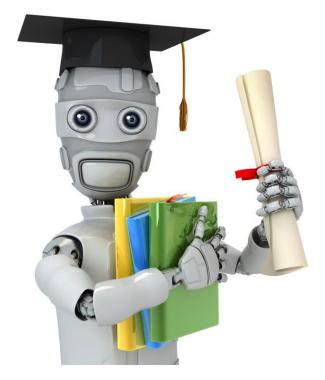
2.
$$h_{\theta}(x) = \theta_0 + \theta_1 x + \theta_2 x^2$$

3.
$$h_{\theta}(x) = \theta_0 + \theta_1 x + \dots + \theta_3 x^3$$

10.
$$h_{\theta}(x) = \theta_0 + \theta_1 x + \dots + \theta_{10} x^{10}$$

Pick
$$\theta_0 + \theta_1 x_1 + \dots + \theta_4 x^4$$

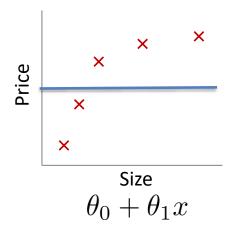
Estimate generalization error for test set $J_{test}(\theta^{(4)})$



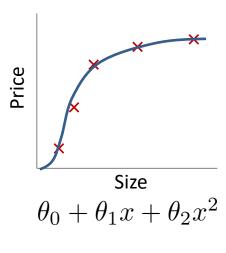
Machine Learning

Diagnosing bias vs. variance

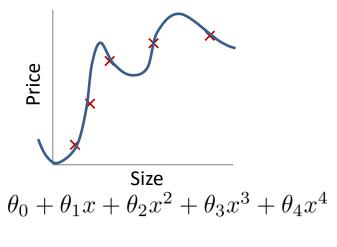
Bias/variance



High bias (underfit)



"Just right"

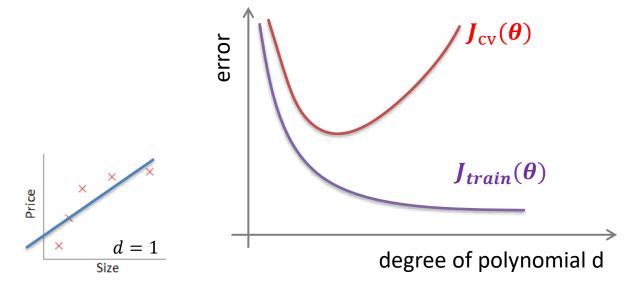


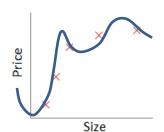
High variance (overfit)

Bias/variance

Training error:
$$J_{train}(\theta) = \frac{1}{2m} \sum_{i=1}^{n} (h_{\theta}(x^{(i)}) - y^{(i)})^2$$

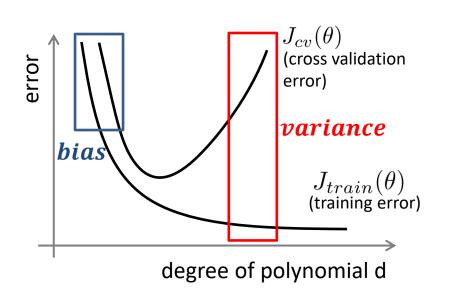
Cross validation error: $J_{cv}(\theta) = \frac{1}{2m_{cv}} \sum_{i=1}^{m_{cv}} (h_{\theta}(x_{cv}^{(i)}) - y_{cv}^{(i)})^2$





Diagnosing bias vs. variance

Suppose your learning algorithm is performing less well than you were hoping. ($J_{cv}(\theta)$ or $J_{test}(\theta)$ is high.) Is it a bias problem or a variance problem?



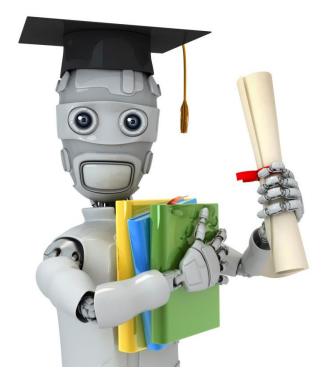
Bias (underfit): $J_{train}(\theta)$ will be high

 $J_{\rm cv}(\boldsymbol{\theta}) \approx J_{train}(\boldsymbol{\theta})$

Variance (overfit):

 $J_{train}(\theta)$ will be low

$$J_{\rm cv}(\theta) \gg J_{train}(\theta)$$



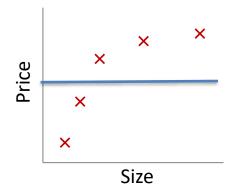
Machine Learning

Regularization and bias/variance

Linear regression with regularization

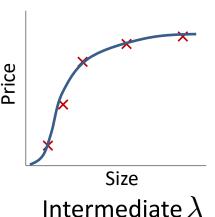
Model:
$$h_{\theta}(x) = \theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \theta_4 x^4$$

$$J(\theta) = \frac{1}{2m} \sum_{i=1}^{m} (h_{\theta}(x^{(i)}) - y^{(i)})^2 + \frac{\lambda}{2m} \sum_{j=1}^{m} \theta_j^2$$

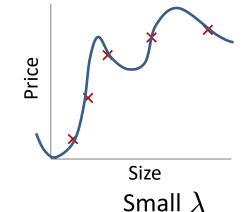


Large λ High bias (underfit)

 $\lambda = 10000. \ \theta_1 \approx 0, \theta_2 \approx 0, \dots$ $h_{\theta}(x) \approx \theta_0$



"Just right"



High variance (overfit)

Choosing the regularization parameter λ

$$h_{\theta}(x) = \theta_{0} + \theta_{1}x + \theta_{2}x^{2} + \theta_{3}x^{3} + \theta_{4}x^{4}$$

$$J(\theta) = \frac{1}{2m} \sum_{i=1}^{m} (h_{\theta}(x^{(i)}) - y^{(i)})^{2} + \frac{\lambda}{2m} \sum_{j=1}^{m} \theta_{j}^{2}$$

$$J_{train}(\theta) = \frac{1}{2m} \sum_{i=1}^{m} (h_{\theta}(x^{(i)}) - y^{(i)})^{2}$$

$$J_{cv}(\theta) = \frac{1}{2m_{cv}} \sum_{i=1}^{m_{cv}} (h_{\theta}(x^{(i)}_{cv}) - y^{(i)}_{cv})^{2}$$

$$J_{test}(\theta) = \frac{1}{2m_{test}} \sum_{i=1}^{m_{test}} (h_{\theta}(x^{(i)}_{test}) - y^{(i)}_{test})^{2}$$

Choosing the regularization parameter λ

Model:
$$h_{\theta}(x) = \theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \theta_4 x^4$$

$$J(\theta) = \frac{1}{2m} \sum_{i=1}^{m} (h_{\theta}(x^{(i)}) - y^{(i)})^2 + \frac{\lambda}{2m} \sum_{i=1}^{m} \theta_j^2$$

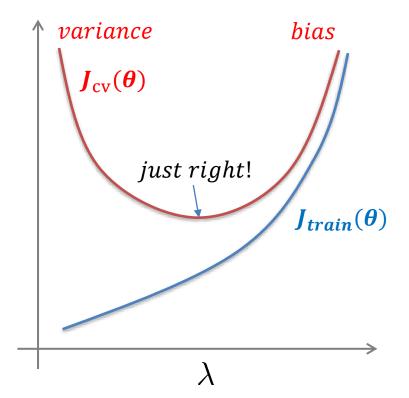
- 1. Try $\lambda = 0$
- 2. Try $\lambda = 0.01$
- 3. Try $\lambda = 0.02$
- 4. Try $\lambda = 0.04$
- 5. Try $\lambda = 0.08$
 - :
- **12.** Try $\lambda = 10$

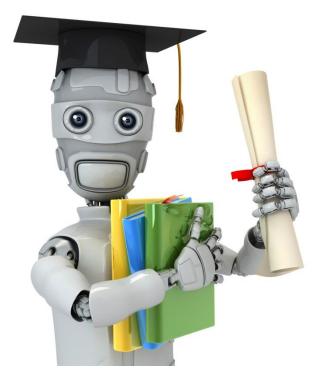
Bias/variance as a function of the regularization parameter λ

$$J(\theta) = \frac{1}{2m} \sum_{i=1}^{m} (h_{\theta}(x^{(i)}) - y^{(i)})^{2} + \frac{\lambda}{2m} \sum_{j=1}^{m} \theta_{j}^{2}$$

$$J_{train}(\theta) = \frac{1}{2m} \sum_{\substack{i=1 \ p_{cv}}}^{m} (h_{\theta}(x^{(i)}) - y^{(i)})^{2}$$

$$J_{cv}(\theta) = \frac{1}{2m_{cv}} \sum_{i=1}^{m} (h_{\theta}(x_{cv}^{(i)}) - y_{cv}^{(i)})^{2}$$



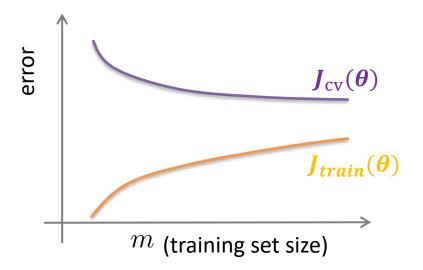


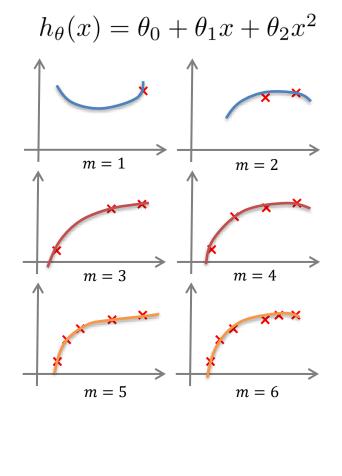
Machine Learning

Learning curves

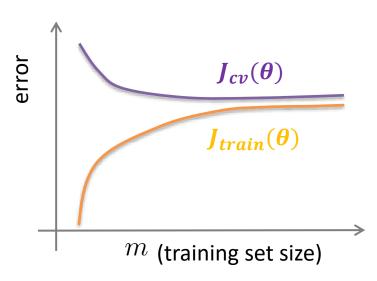
Learning curves

$$J_{train}(\theta) = \frac{1}{2m} \sum_{\substack{i=1 \ m_{cv}}}^{m} (h_{\theta}(x^{(i)}) - y^{(i)})^{2}$$
$$J_{cv}(\theta) = \frac{1}{2m_{cv}} \sum_{\substack{i=1 \ m_{cv}}}^{m} (h_{\theta}(x^{(i)}) - y^{(i)})^{2}$$

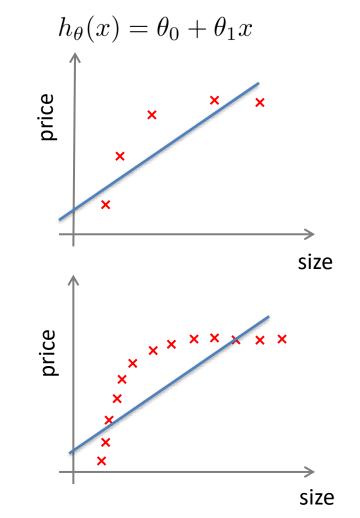




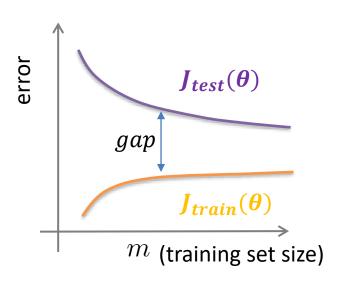
High bias



If a learning algorithm is suffering from high bias, getting more training data will not (by itself) help much.

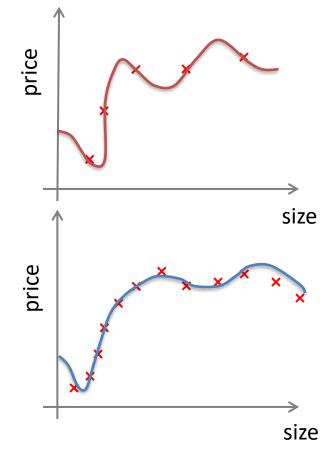


High variance

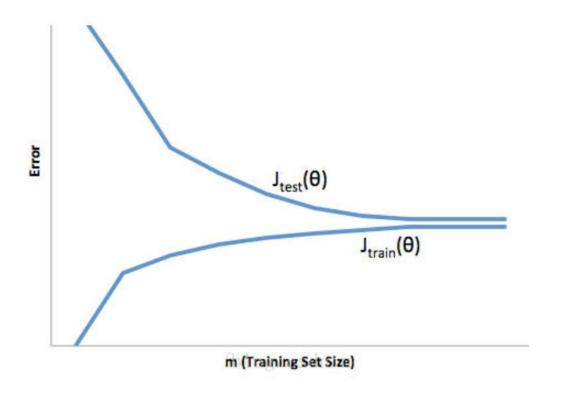


If a learning algorithm is suffering from high variance, getting more training data is likely to help.

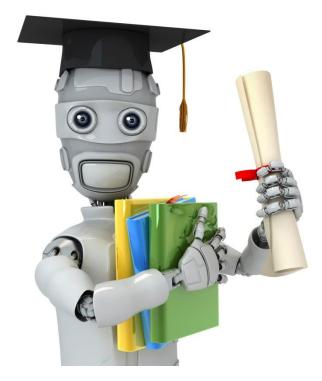
$$h_{\theta}(x) = \theta_0 + \theta_1 x + \dots + \theta_{100} x^{100}$$
 (and small λ)



你训练一个学习算法,发现它在测试集上的误差很高。绘制学习曲线,并获得下图。算法是否存在高偏差、高方差或两者都不存在?



A. 高偏差 B. 高方差 C. 两者都不



Machine Learning

Deciding what to try next (revisited)

Debugging a learning algorithm:

Suppose you have implemented regularized linear regression to predict housing prices. However, when you test your hypothesis in a new set of houses, you find that it makes unacceptably large errors in its prediction. What should you try next?

- Get more training examples fixes high variance
- Try smaller sets of features fixes high variance
- Try getting additional features $\longrightarrow fixes \ high \ bias$
- Try adding polynomial features $(x_1^2, x_2^2, x_1x_2, \text{etc}) \longrightarrow fixes \ high \ bias$
- Try decreasing $\lambda \longrightarrow fixes \ high \ bias$
- Try increasing λ \longrightarrow fixes high variance

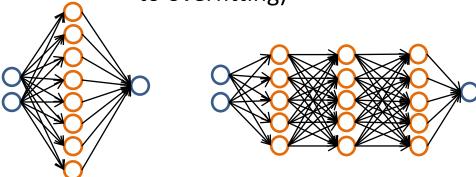
Neural networks and overfitting

"Small" neural network (fewer parameters; more prone to underfitting)



Computationally cheaper

"Large" neural network (more parameters; more prone to overfitting)



Computationally more expensive.

Use regularization (λ) to address overfitting.