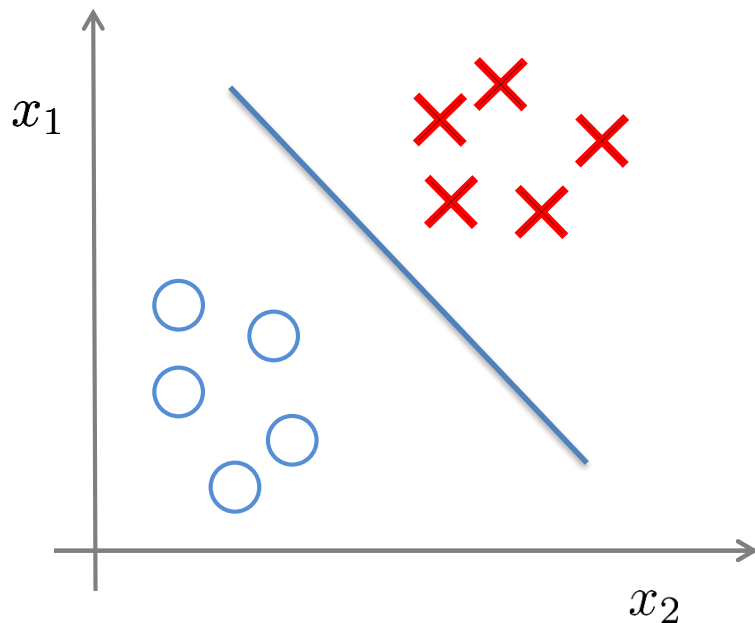


Machine Learning

Clustering

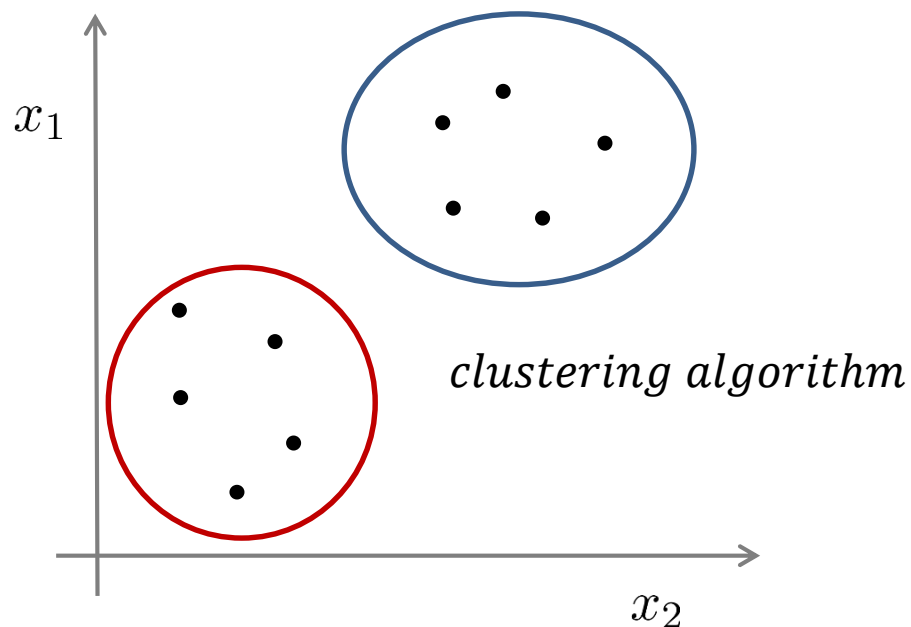
Unsupervised learning
introduction

Supervised learning



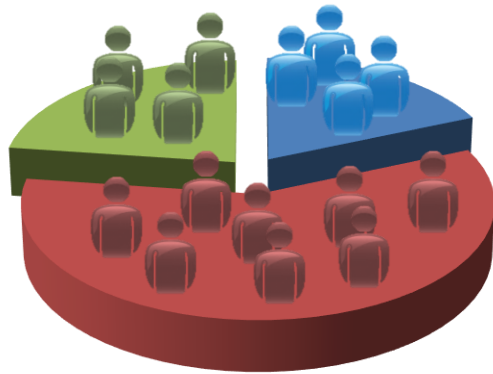
Training set: $\{(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), (x^{(3)}, y^{(3)}), \dots, (x^{(m)}, y^{(m)})\}$

Unsupervised learning

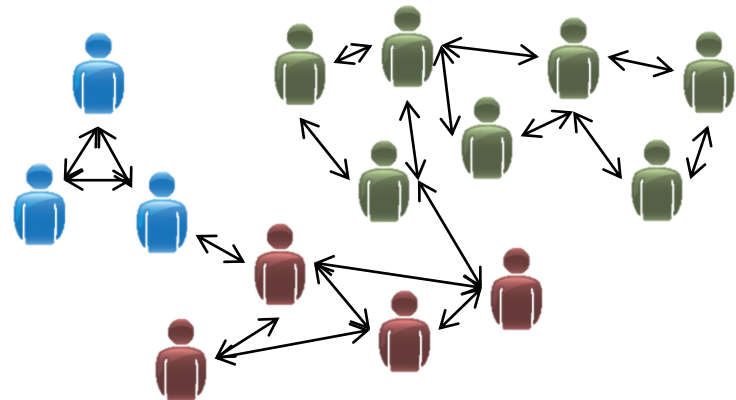


Training set: $\{x^{(1)}, x^{(2)}, x^{(3)}, \dots, x^{(m)}\}$

Applications of clustering



Market segmentation



Social network analysis



Organize computing clusters

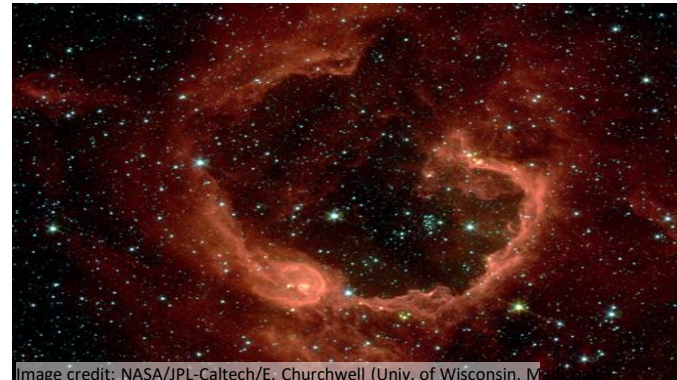
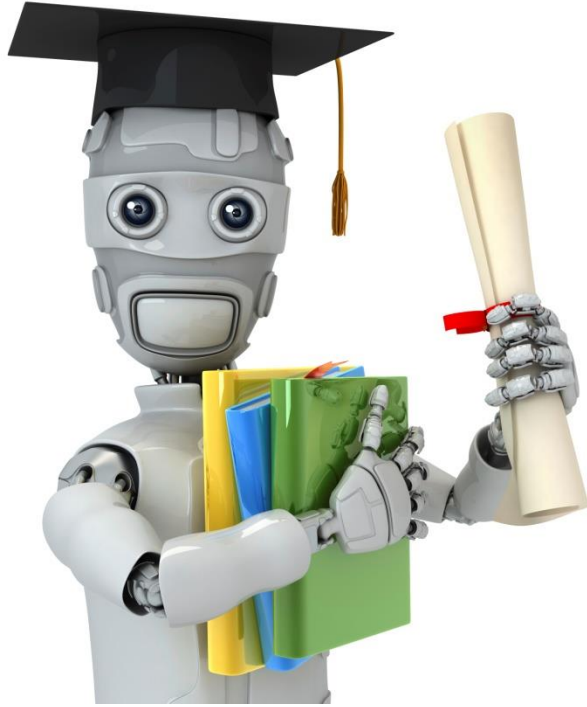


Image credit: NASA/JPL-Caltech/E. Churchwell (Univ. of Wisconsin, M

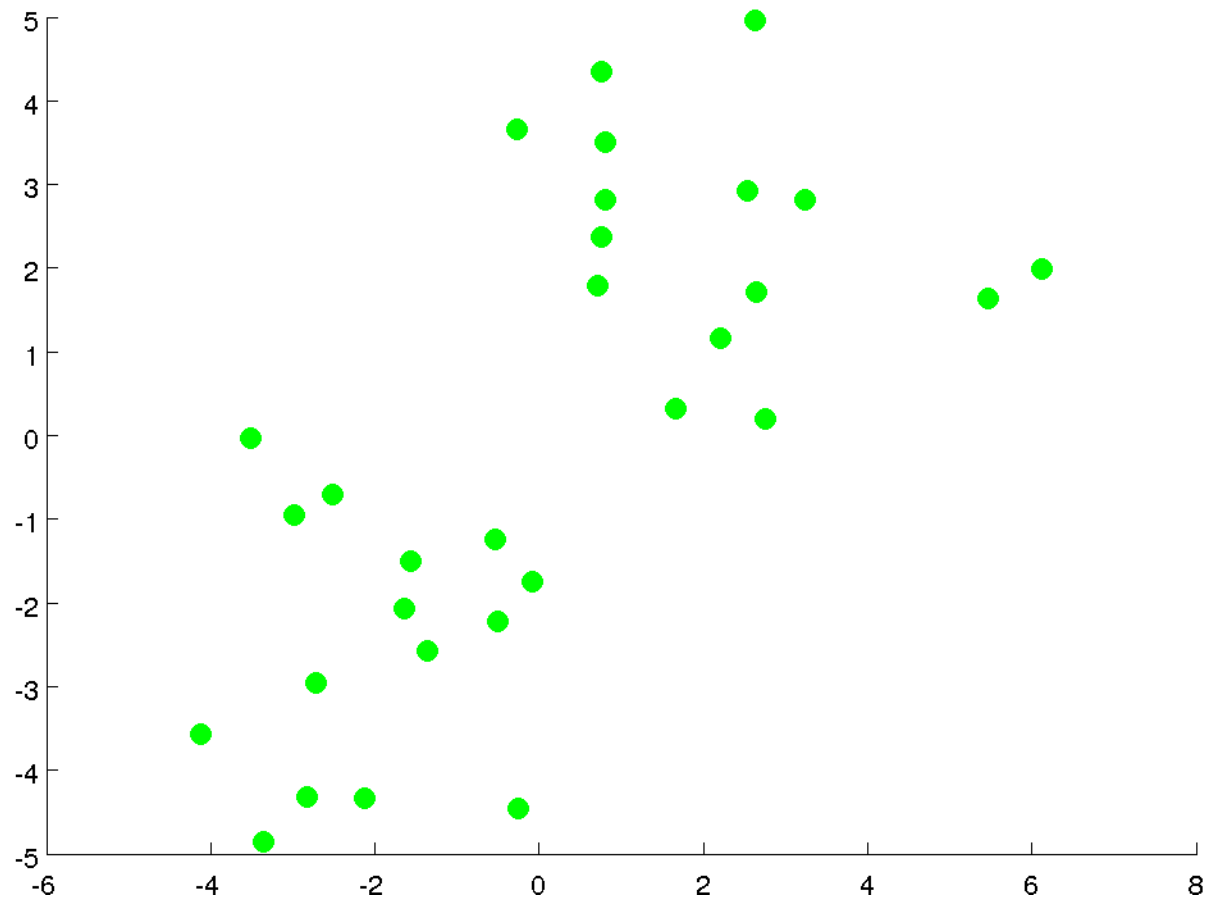
Astronomical data analysis

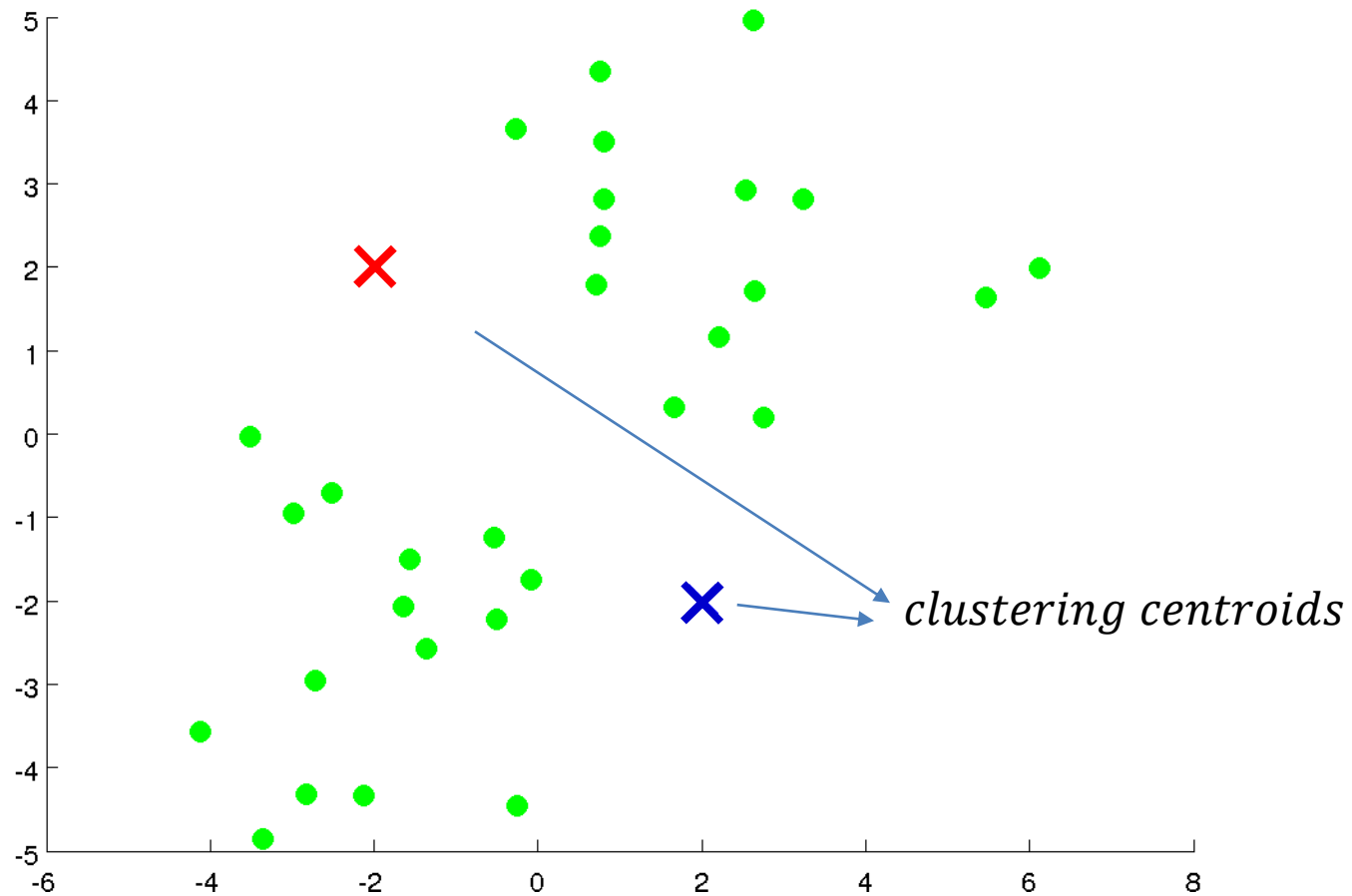


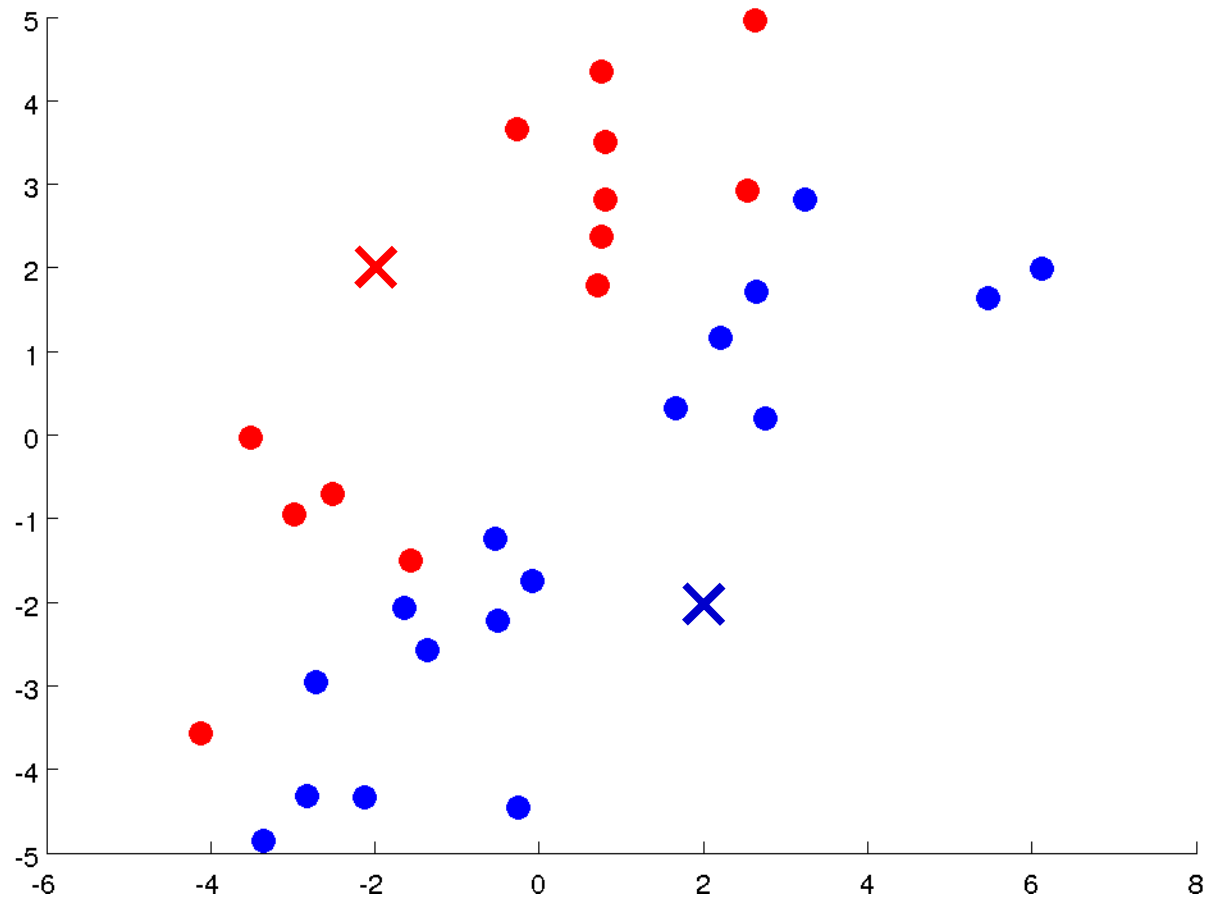
Machine Learning

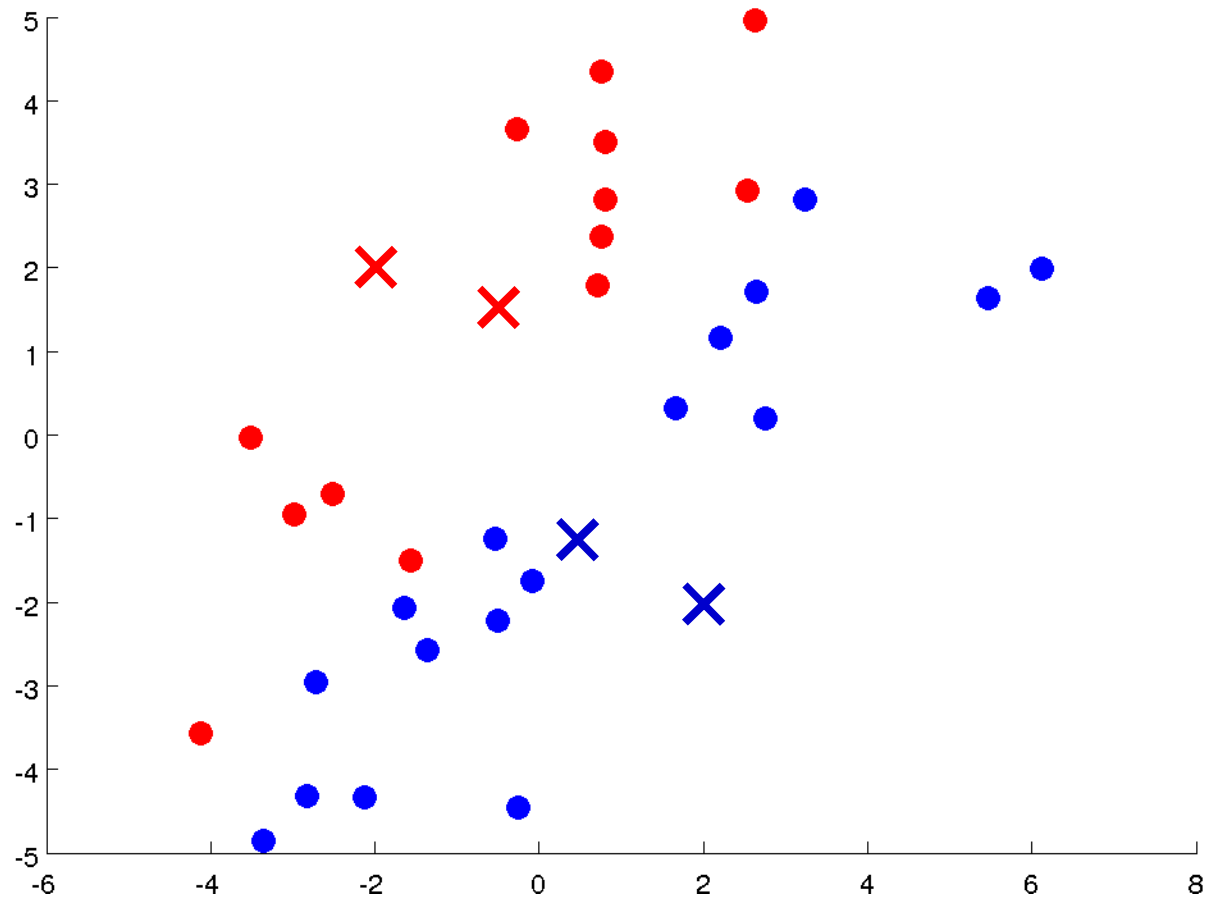
Clustering

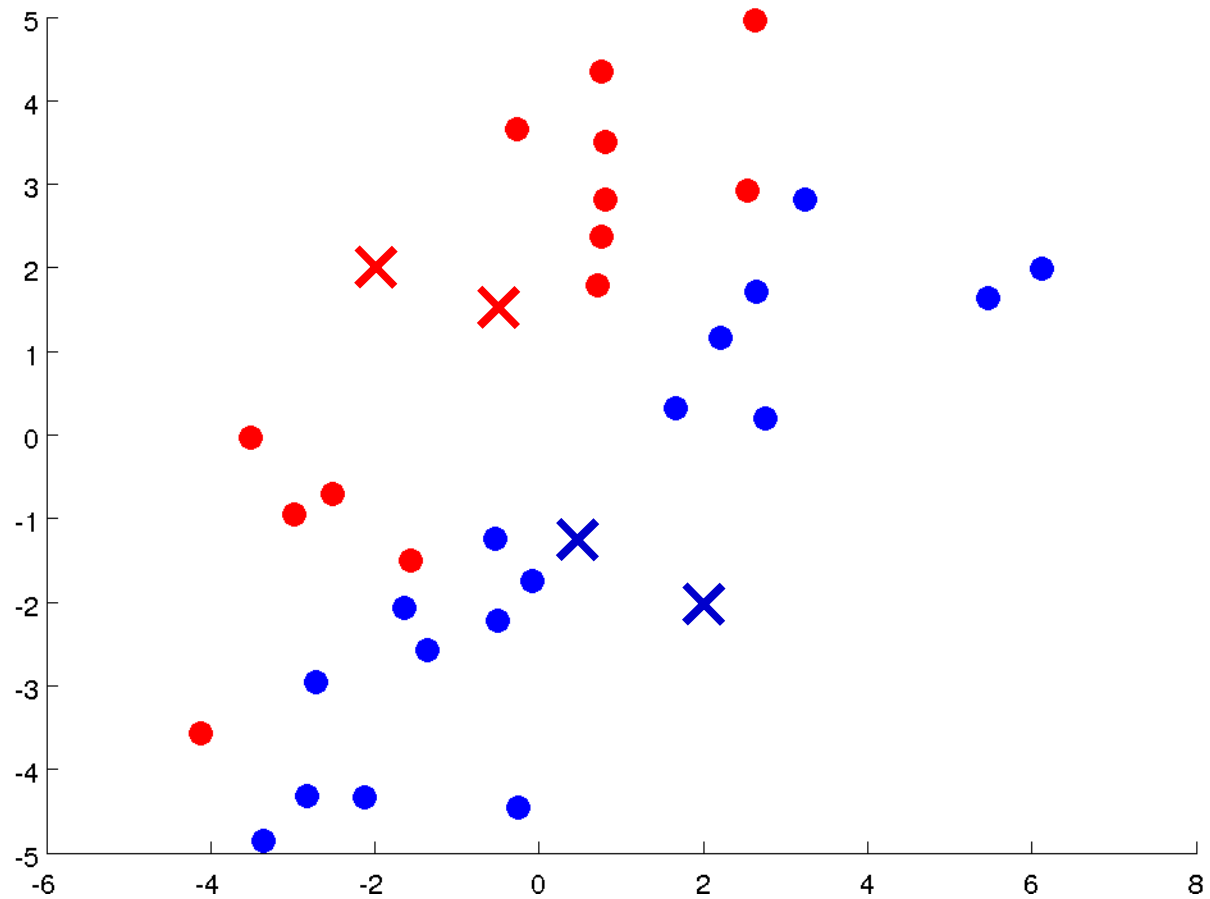
K-means algorithm

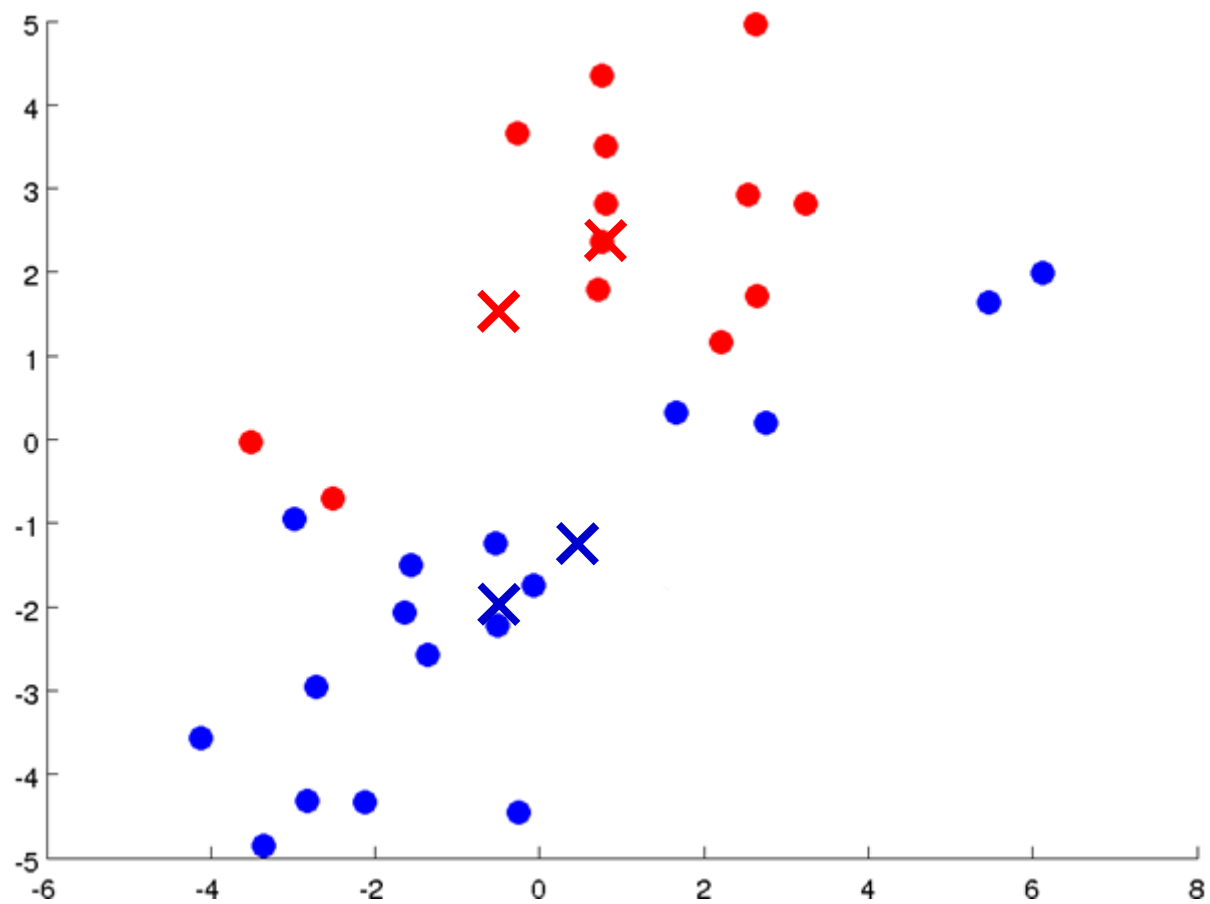


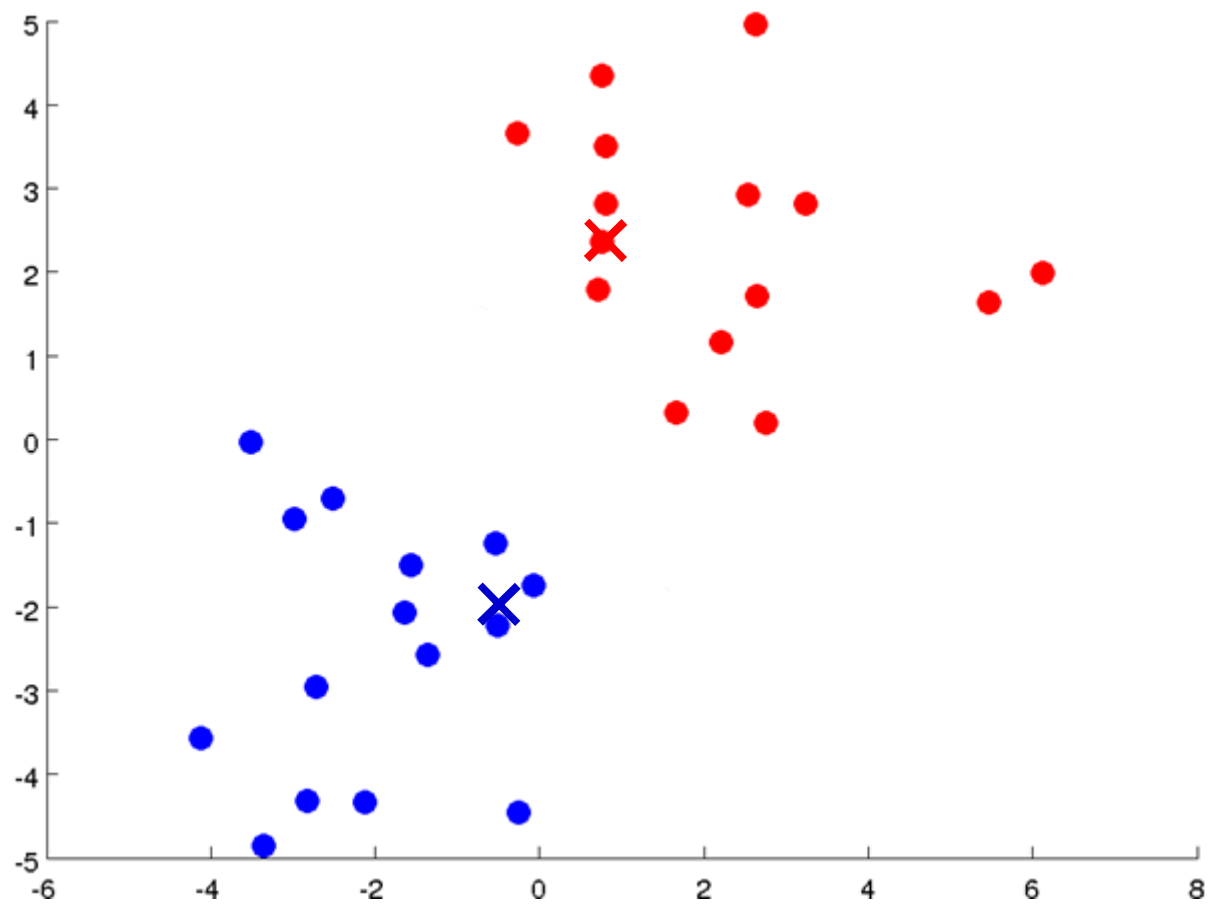


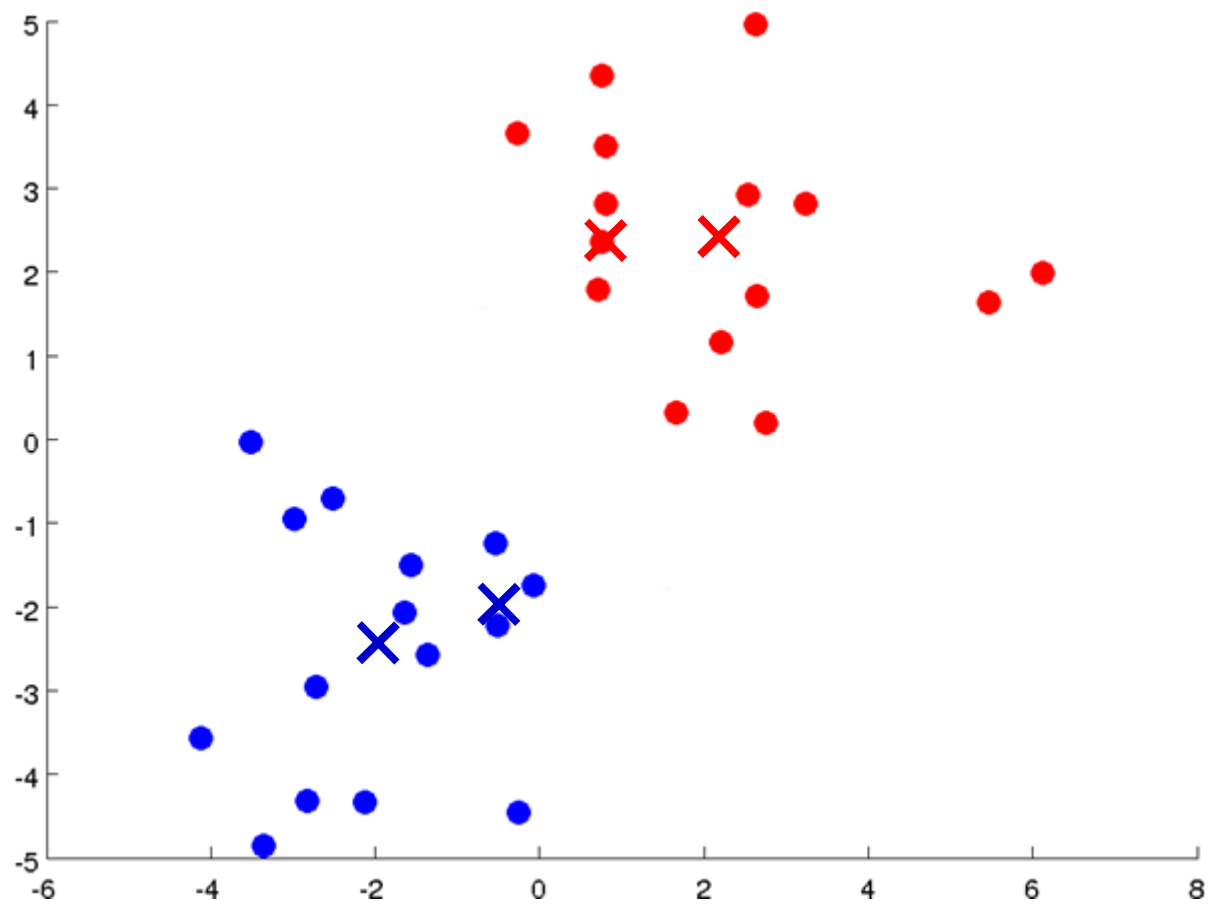


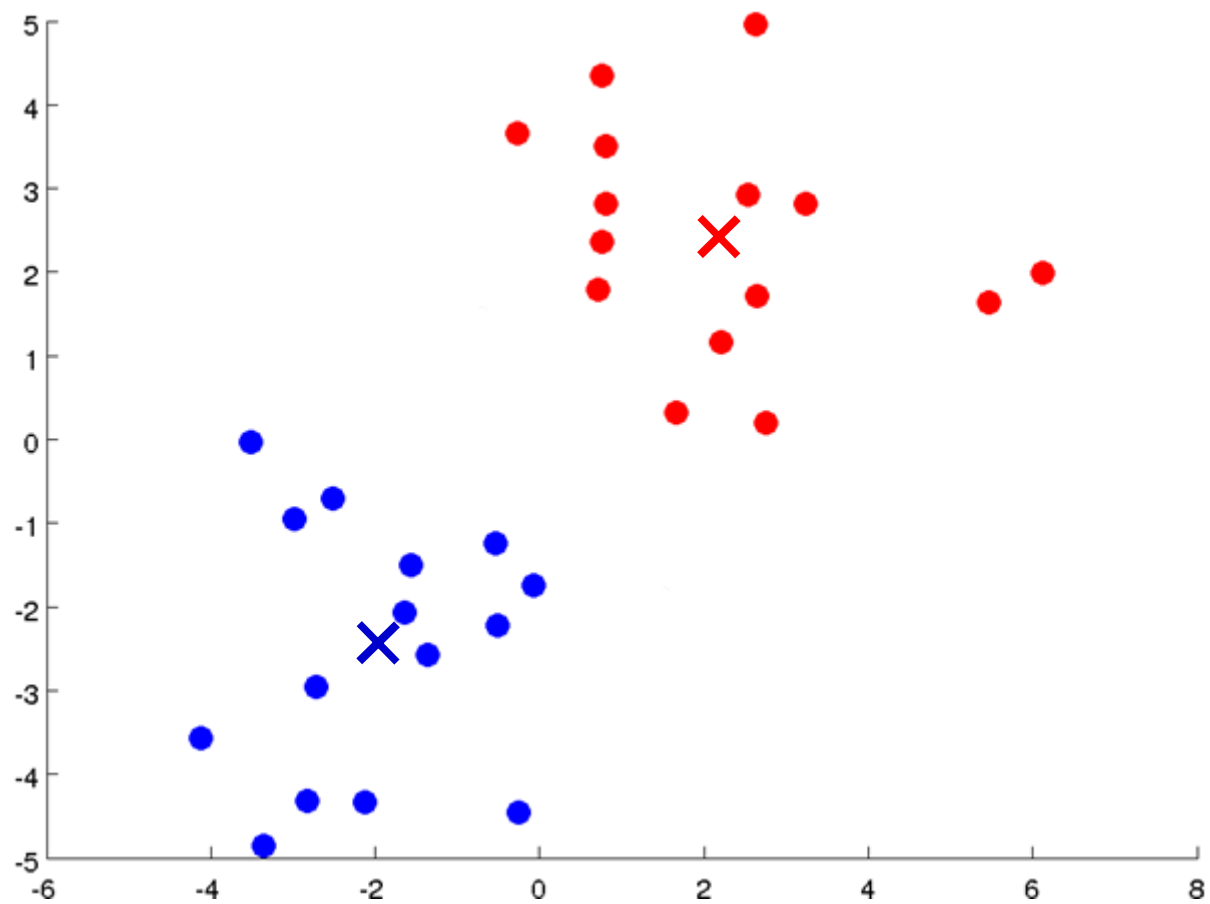












K-means algorithm

Input:

- K (number of clusters)
- Training set $\{x^{(1)}, x^{(2)}, \dots, x^{(m)}\}$

$x^{(i)} \in \mathbb{R}^n$ (drop $x_0 = 1$ convention)

K-means algorithm

Randomly initialize K cluster centroids $\mu_1, \mu_2, \dots, \mu_K \in \mathbb{R}^n$

Repeat {

 for $i = 1$ to m

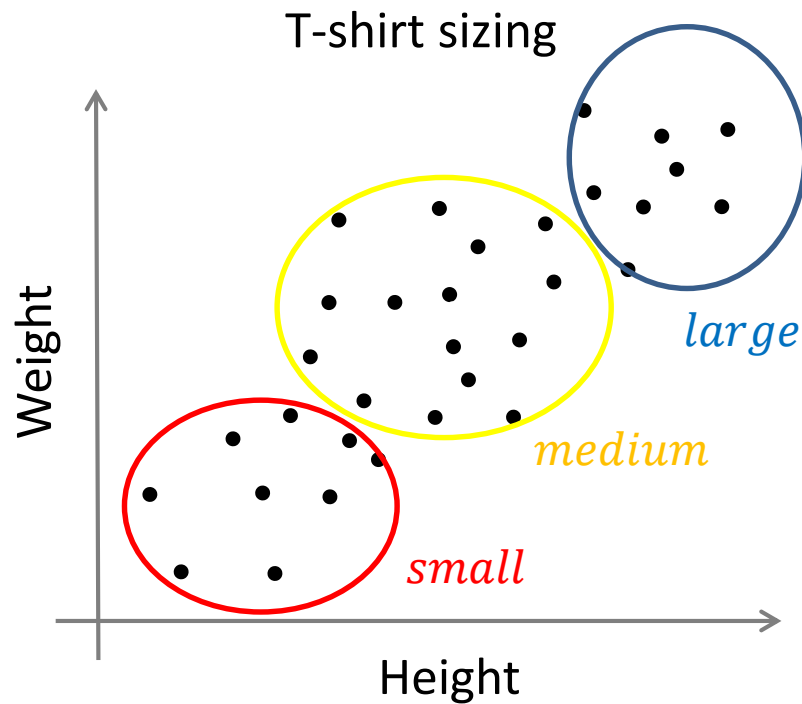
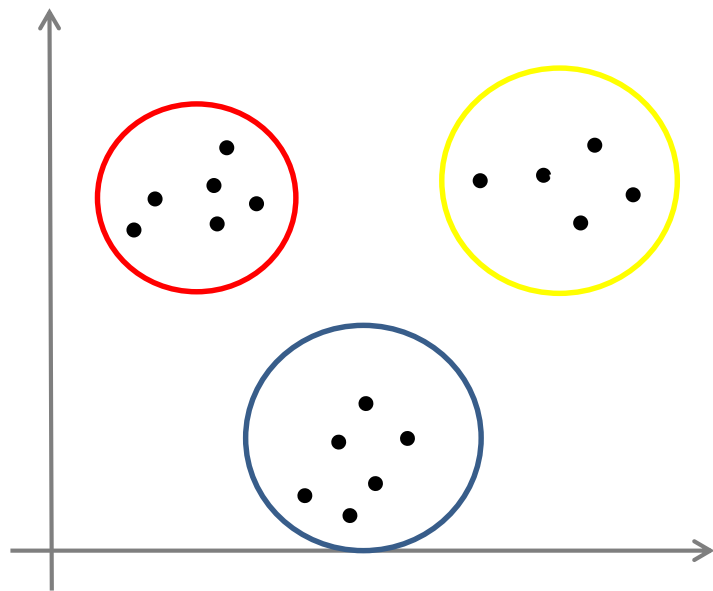
$c^{(i)} :=$ index (from 1 to K) of cluster centroid
 closest to $x^{(i)}$

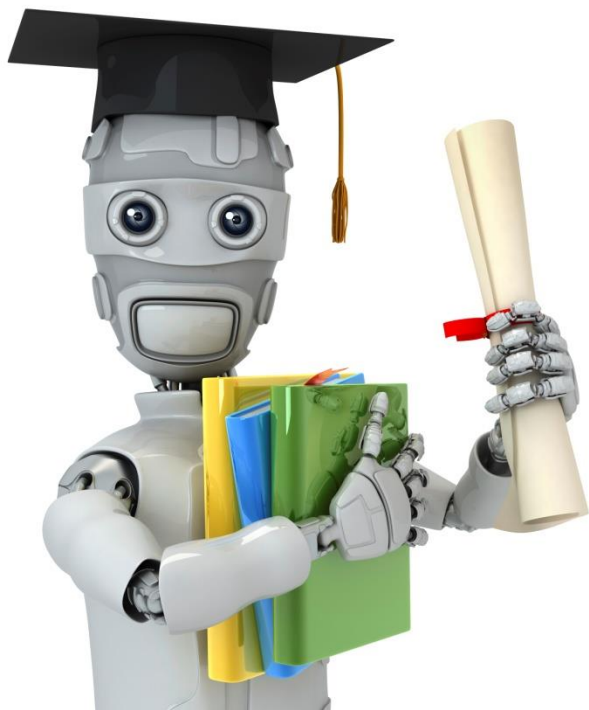
 for $k = 1$ to K

$\mu_k :=$ average (mean) of points assigned to cluster k

}

K-means for non-separated clusters





Machine Learning

Clustering

Optimization objective

K-means optimization objective

$c^{(i)}$ = index of cluster $(1, 2, \dots, K)$ to which example $x^{(i)}$ is currently assigned

μ_k = cluster centroid k ($\mu_k \in \mathbb{R}^n$)

$\mu_{c^{(i)}}$ = cluster centroid of cluster to which example $x^{(i)}$ has been assigned

Optimization objective:

$$J(c^{(1)}, \dots, c^{(m)}, \mu_1, \dots, \mu_K) = \frac{1}{m} \sum_{i=1}^m \|x^{(i)} - \mu_{c^{(i)}}\|^2$$

$$\min_{\substack{c^{(1)}, \dots, c^{(m)}, \\ \mu_1, \dots, \mu_K}} J(c^{(1)}, \dots, c^{(m)}, \mu_1, \dots, \mu_K)$$

K-means algorithm

Randomly initialize K cluster centroids $\mu_1, \mu_2, \dots, \mu_K \in \mathbb{R}^n$

Repeat {

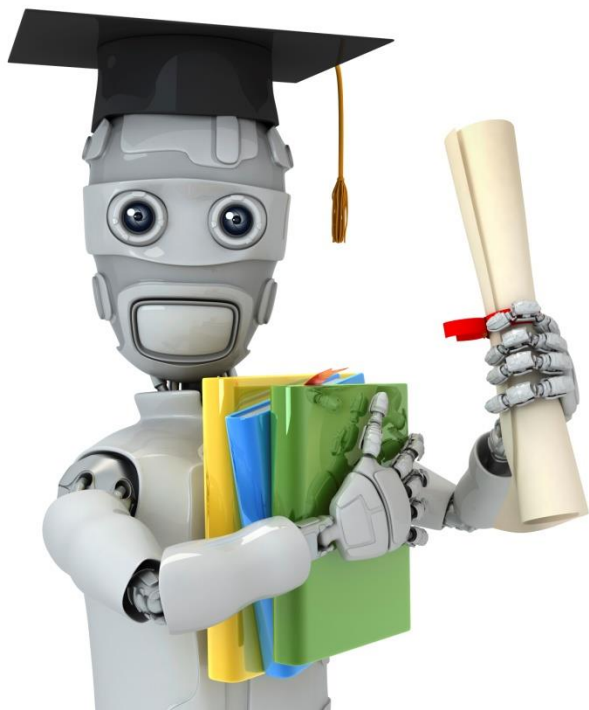
 for $i = 1$ to m

$c^{(i)} :=$ index (from 1 to K) of cluster centroid
 closest to $x^{(i)}$

 for $k = 1$ to K

$\mu_k :=$ average (mean) of points assigned to cluster k

}



Machine Learning

Clustering

Random initialization

K-means algorithm

Randomly initialize K cluster centroids $\mu_1, \mu_2, \dots, \mu_K \in \mathbb{R}^n$

Repeat {

 for $i = 1$ to m

$c^{(i)} :=$ index (from 1 to K) of cluster centroid
 closest to $x^{(i)}$

 for $k = 1$ to K

$\mu_k :=$ average (mean) of points assigned to cluster k

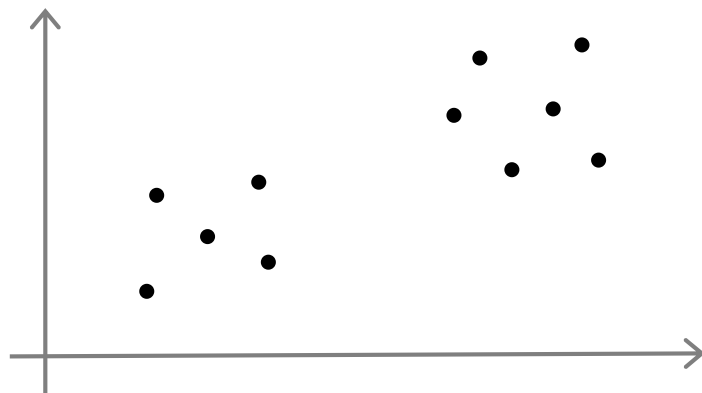
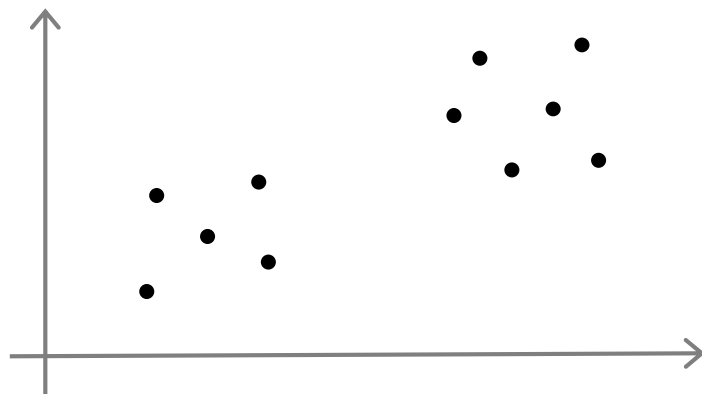
}

Random initialization

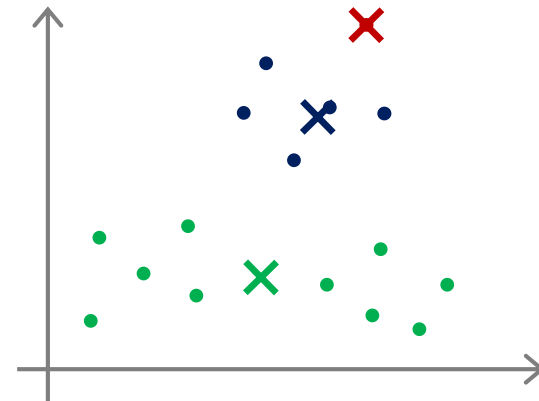
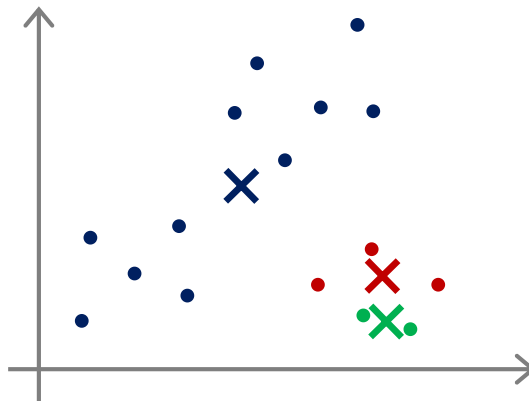
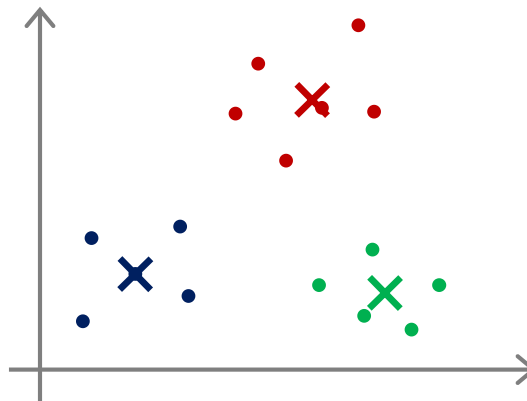
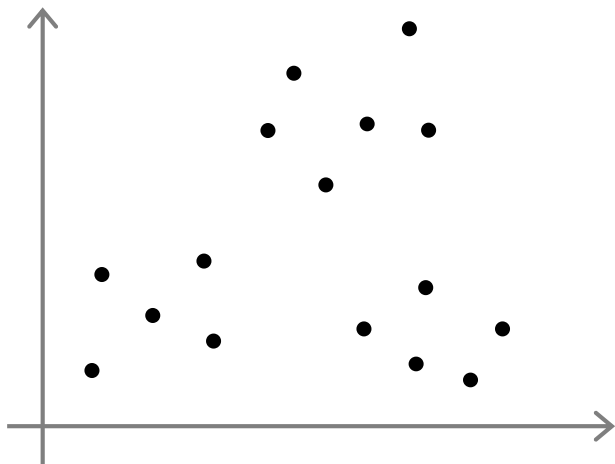
Should have $K < m$

Randomly pick K training examples.

Set μ_1, \dots, μ_K equal to these K examples.



Local optima



Random initialization

For $i = 1$ to 100 {

Randomly initialize K-means.

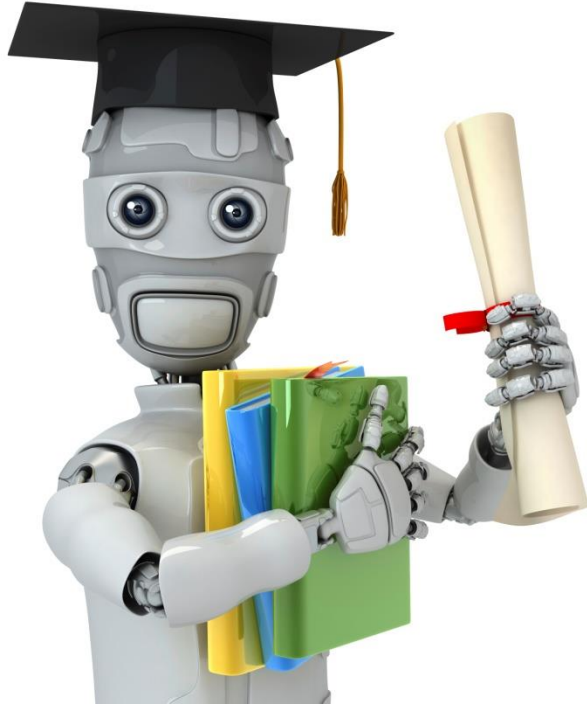
Run K-means. Get $c^{(1)}, \dots, c^{(m)}, \mu_1, \dots, \mu_K$.

Compute cost function (distortion)

$$J(c^{(1)}, \dots, c^{(m)}, \mu_1, \dots, \mu_K)$$

}

Pick clustering that gave lowest cost $J(c^{(1)}, \dots, c^{(m)}, \mu_1, \dots, \mu_K)$

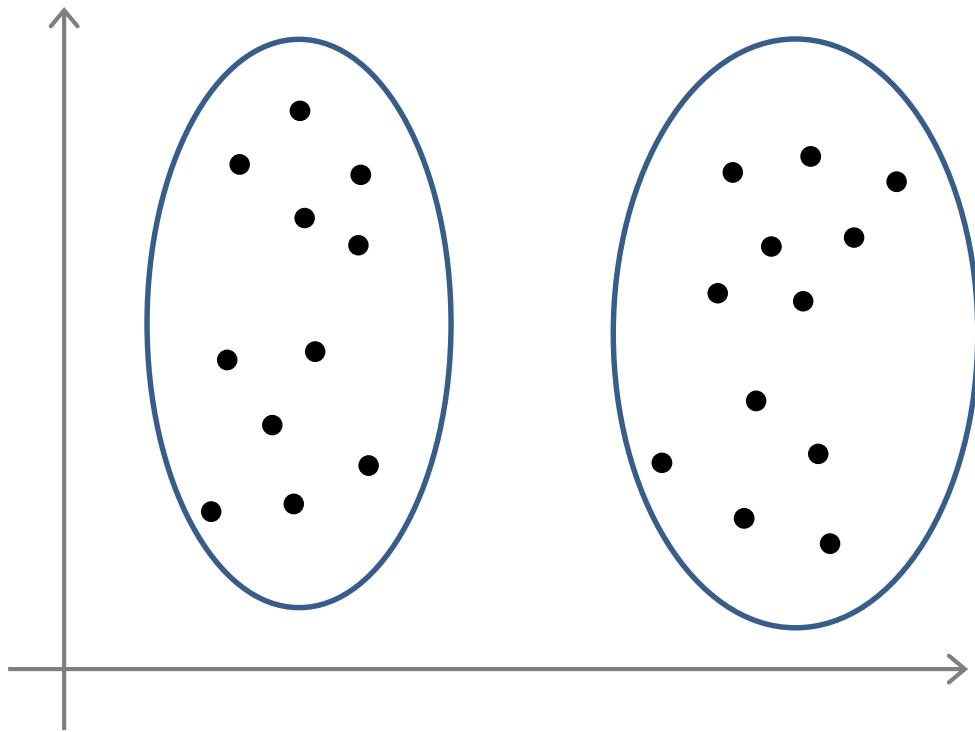


Machine Learning

Clustering

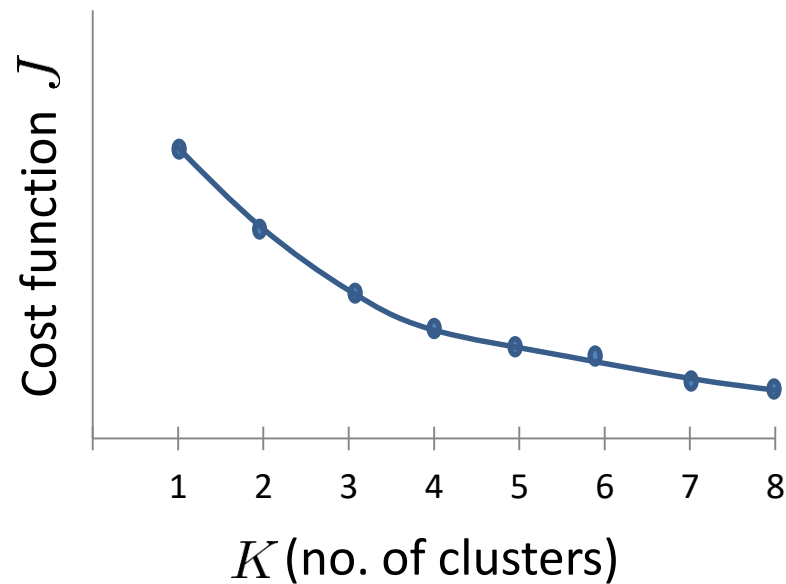
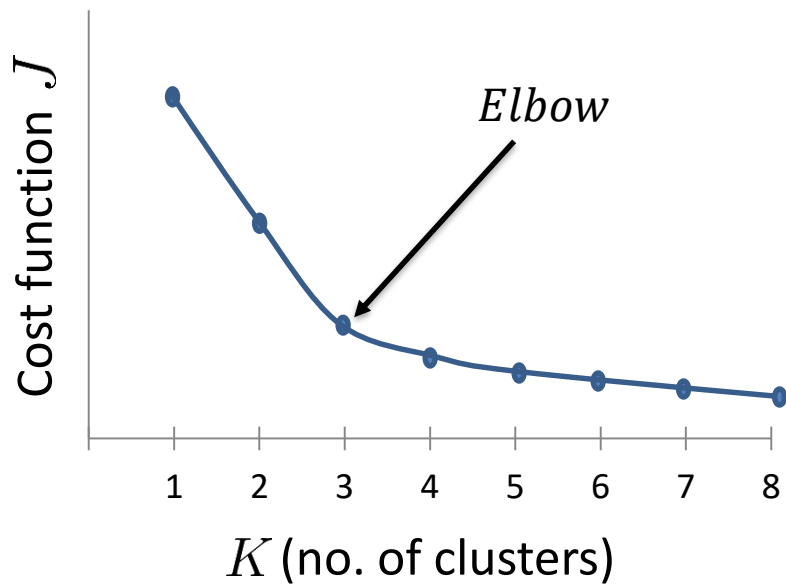
Choosing the
number of clusters

What is the right value of K?



Choosing the value of K

Elbow method:



Choosing the value of K

Sometimes, you're running K-means to get clusters to use for some later/downstream purpose. Evaluate K-means based on a metric for how well it performs for that later purpose.

E.g.

