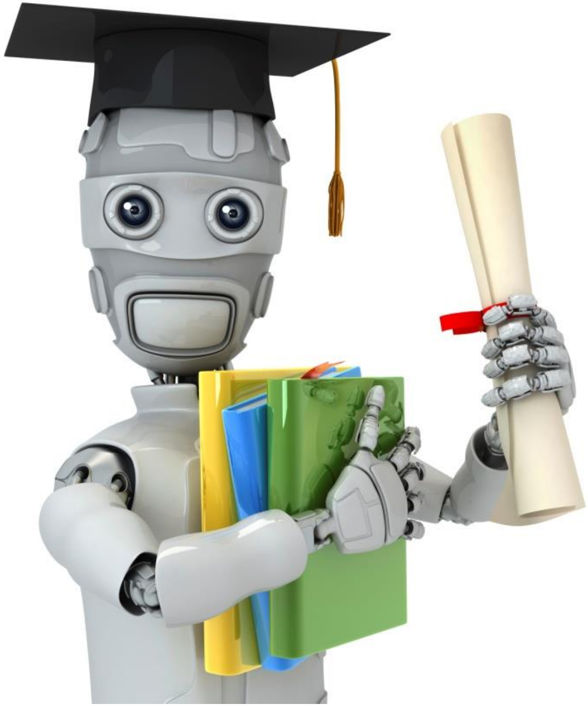
Logistic Regression

Classification



Machine Learning

Classification

Email: Spam / Not Spam?

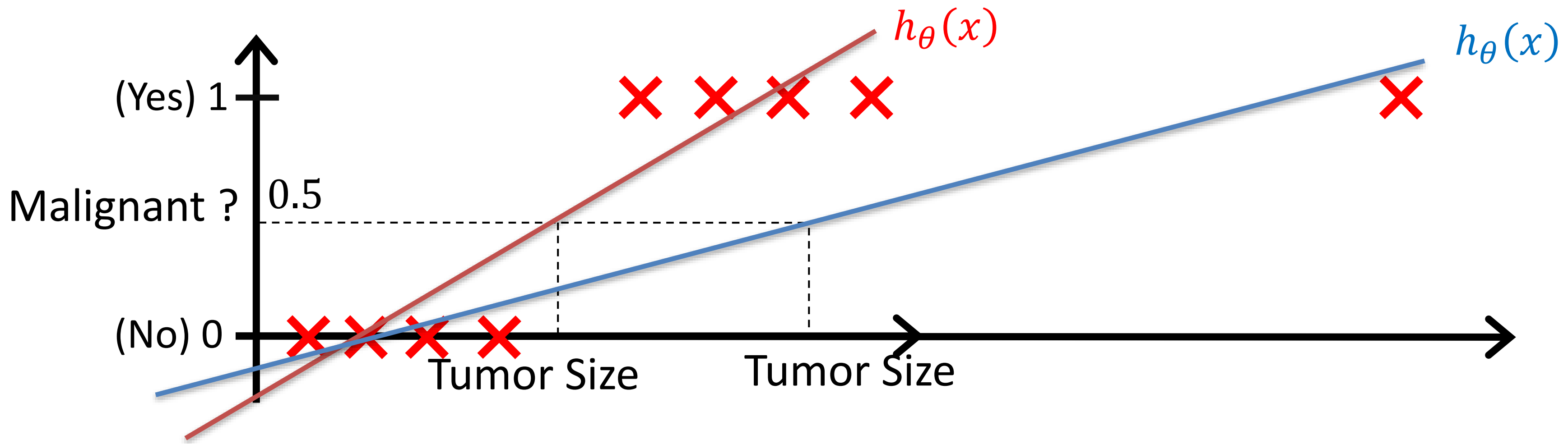
Online Transactions: Fraudulent (Yes / No)?

Tumor: Malignant / Benign ?

$$y \in \{0, 1\}$$

0: “Negative Class” (e.g., benign tumor)

1: “Positive Class” (e.g., malignant tumor)



$$h_\theta(x) = \theta^T x$$

Threshold classifier output $h_\theta(x)$ at 0.5:

If $h_\theta(x) \geq 0.5$, predict "y = 1"

If $h_\theta(x) < 0.5$, predict "y = 0"

Classification: y = 0 or 1

$h_\theta(x)$ can be > 1 or < 0

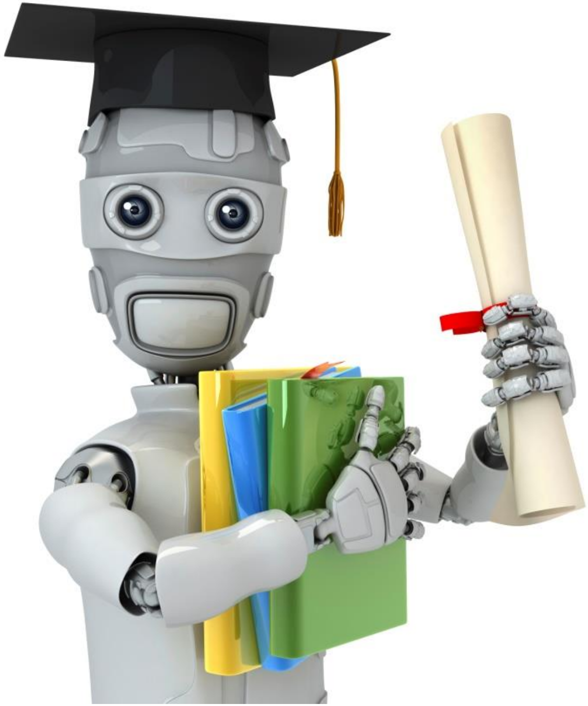
Logistic Regression: $0 \leq h_\theta(x) \leq 1$

1.逻辑回归是监督学习算法吗?

A. 是　　B.不是

2. 逻辑回归主要用来做回归吗?

A. 是　　B.不是



Machine Learning

Logistic Regression

Hypothesis Representation

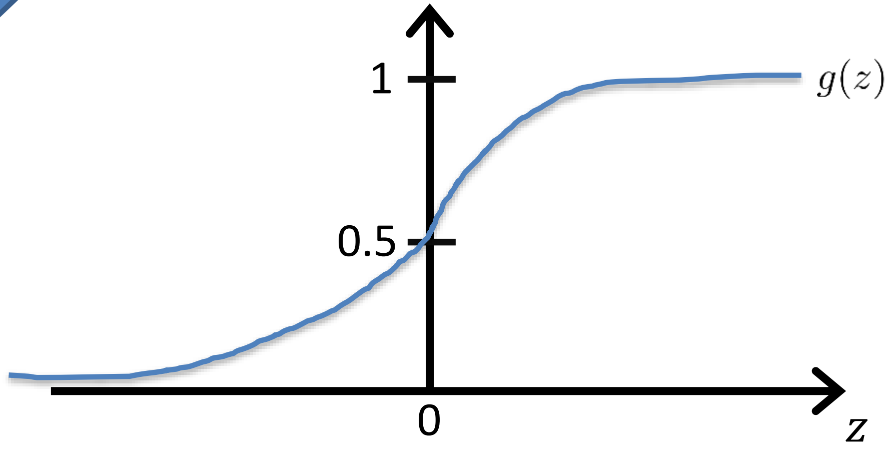
Logistic Regression Model

Want $0 \leq h_\theta(x) \leq 1$

$$h_\theta(x) = g(\ \theta^T x\ )$$

$$g(z) = \frac{1}{1+e^{-z}}$$

$$h_\theta(x) = \frac{1}{1+\mathrm{e}^{-\theta^{\mathrm{T}}x}}$$



Sigmoid function
Logistic function

Interpretation of Hypothesis Output

$h_\theta(x)$ = estimated probability that y = 1 on input x

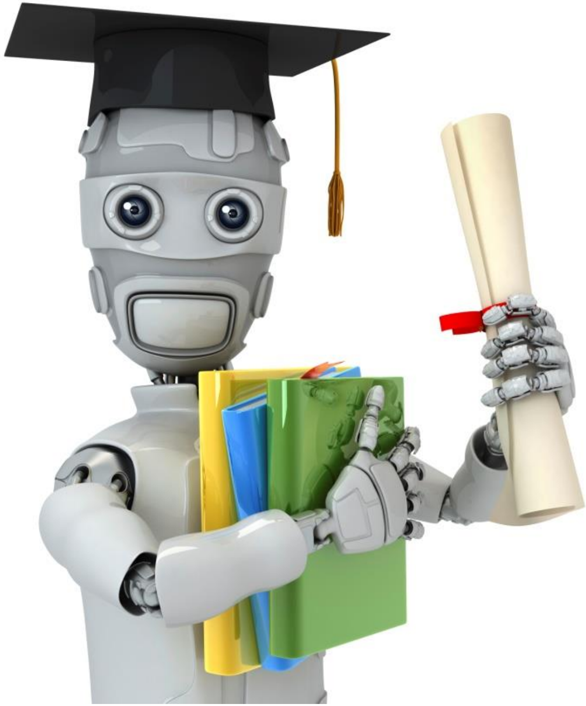
Example: If $x = \begin{bmatrix} x_0 \\ x_1 \end{bmatrix} = \begin{bmatrix} 1 \\ \text{tumorSize} \end{bmatrix}$

$$h_\theta(x) = 0.7$$

Tell patient that 70% chance of tumor being malignant

"probability that y = 1, given x, parameterized by θ"

$$P(y=0|x;\theta) + P(y=1|x;\theta) = 1$$
$$P(y=0|x;\theta) = 1 - P(y=1|x;\theta)$$

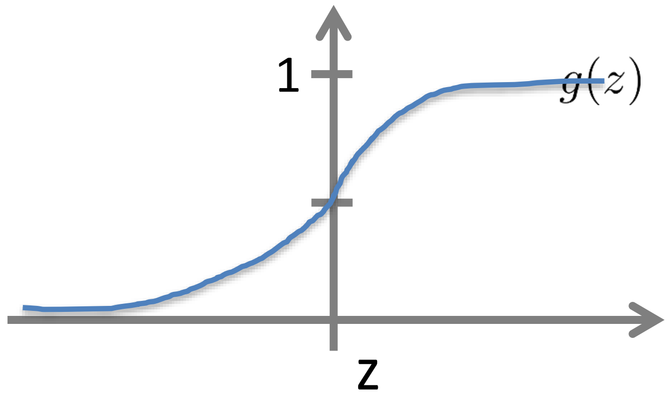


Machine Learning

Logistic Regression

Decision boundary

Logistic regression



$$h_\theta(x) = g(\theta^T x)$$

$$g(z) = \frac{1}{1+e^{-z}}$$

Suppose predict "$y = 1$" if $h_\theta(x) \geq 0.5$

$z \geq 0$， $g(z) \geq 0.5$

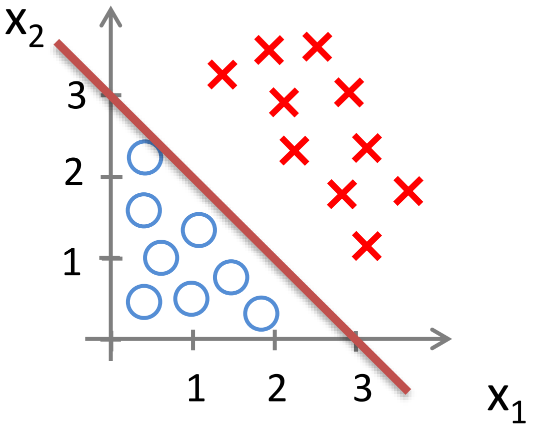
$h_\theta(x) = g(\theta^{\mathrm{T}} x) \geq 0.5$ $\quad\quad$ $\theta^{\mathrm{T}} x \geq 0$

predict "$y = 0$" if $h_\theta(x) < 0.5$

$z < 0$， $g(z) < 0.5$

$h_\theta(x) = g(\theta^{\mathrm{T}} x) < 0.5$ $\quad\quad$ $\theta^{\mathrm{T}} x < 0$

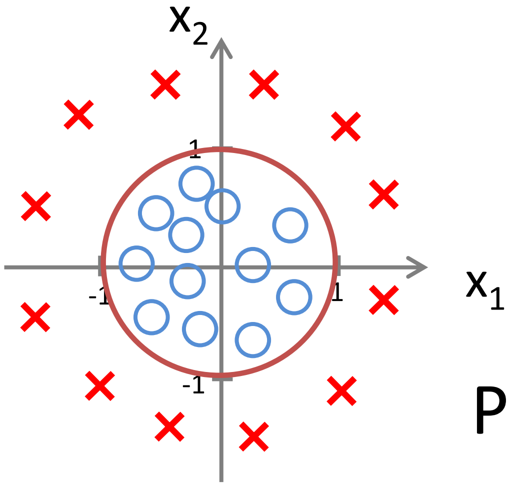
Decision Boundary



$$h_\theta(x) = g(\theta_0 + \theta_1 x_1 + \theta_2 x_2)$$

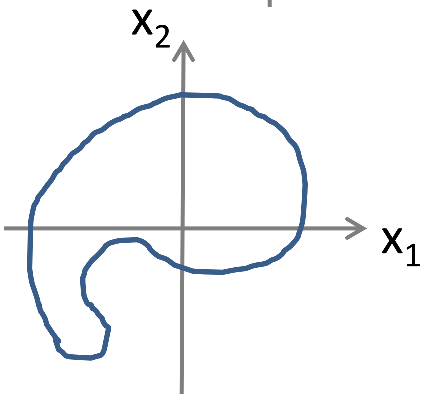
Predict "$y = 1$" if $-3 + x_1 + x_2 \geq 0$

Non-linear decision boundaries



$$h_\theta(x) = g(\theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_1^2 + \theta_4 x_2^2)$$

Predict "$y = 1$" if $-1 + x_1^2 + x_2^2 \geq 0$



$$h_\theta(x) = g(\theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_1^2 + \theta_4 x_1^2 x_2 + \theta_5 x_1^2 x_2^2 + \theta_6 x_1^3 x_2 + \dots)$$

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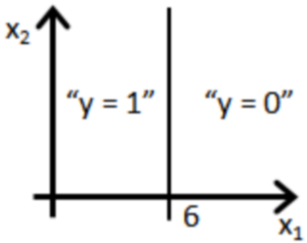
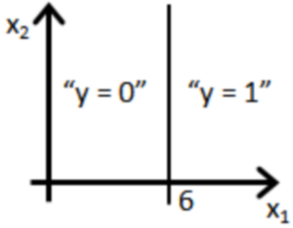
Suppose you train a logistic classifier $h_\theta(x) = g(\theta_0 + \theta_1 x_1 + \theta_2 x_2)$. Suppose $\theta_0 = -6, \theta_1 = 1, \theta_2 = 0$. Which of the following figures represents the decision boundary found by your classifier?

Figure:



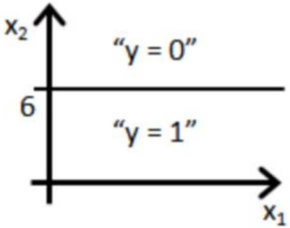
A.

Figure:



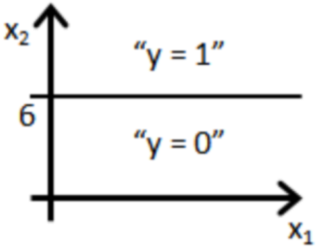
B.

Figure:



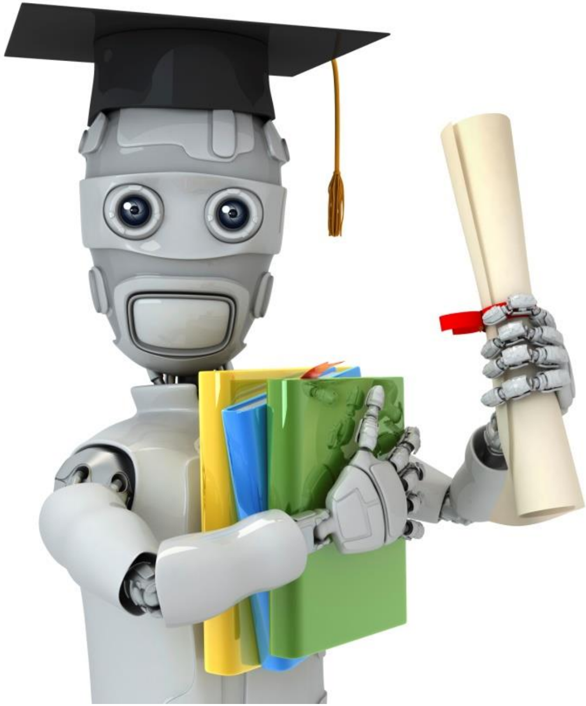
C.

Figure:



D.

B



Machine Learning

Logistic Regression

Cost function

Training set: $\{(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \cdots, (x^{(m)}, y^{(m)})\}$

m examples $\quad x \in \begin{bmatrix} x_0 \\ x_1 \\ \dots \\ x_n \end{bmatrix} \quad x_0 = 1, y \in \{0, 1\}$

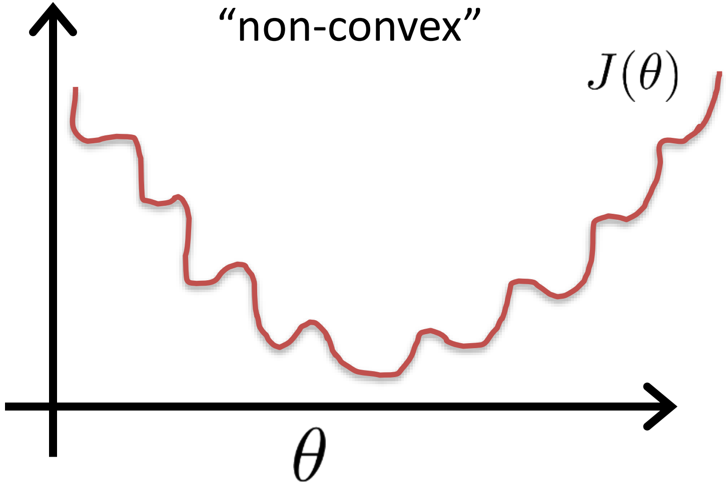
$$h_\theta(x) = \frac{1}{1 + e^{-\theta^T x}}$$

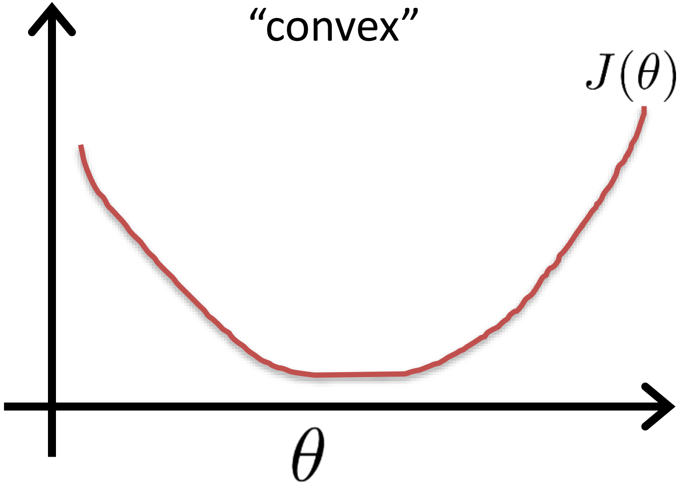
How to choose parameters θ ?

Cost function

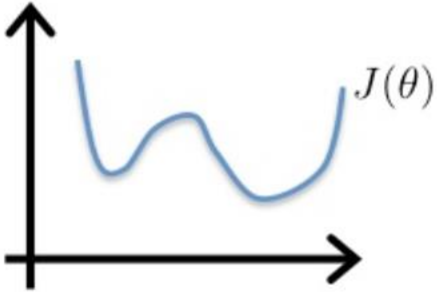
Linear regression: $J(\theta) = \frac{1}{m} \sum_{i=1}^{m} \frac{1}{2} \left(h_\theta(x^{(i)}) - y^{(i)} \right)^2$

$$\mathrm{Cost}(h_\theta(x^{(i)}), y^{(i)}) = \frac{1}{2} \left(h_\theta(x^{(i)}) - y^{(i)} \right)^2$$

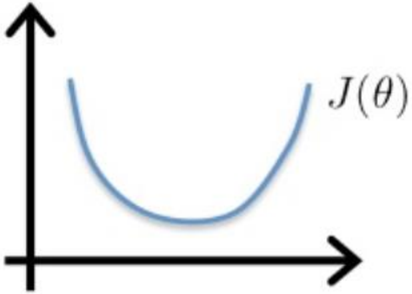




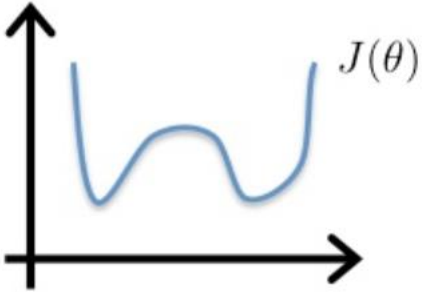
Consider minimizing a cost function $J(\theta)$， Which one of these functions is convex?



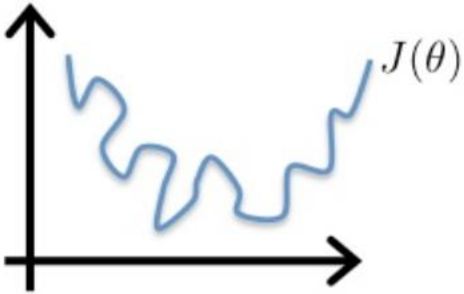
A.



B.



C.

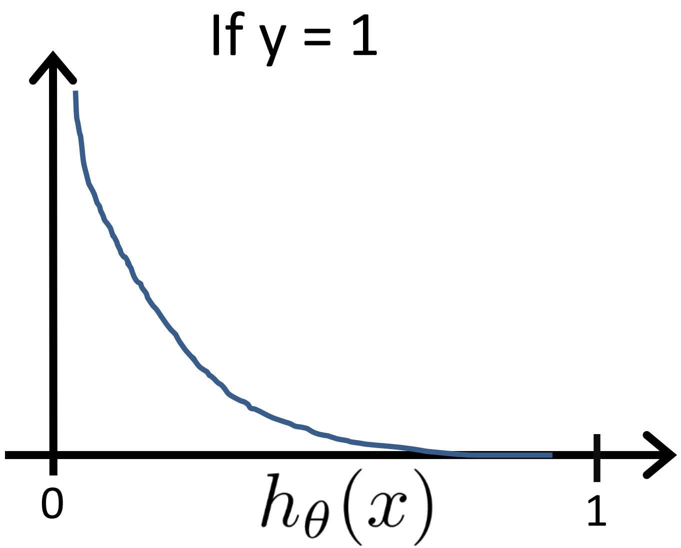


D.

B

Logistic regression cost function

$$\text{Cost}(h_\theta(x), y) = \begin{cases} -\log(h_\theta(x)) & \text{if } y = 1 \\ -\log(1 - h_\theta(x)) & \text{if } y = 0 \end{cases}$$



$\text{Cost} = 0 \text{ if } y = 1, h_\theta(x) = 1$

But as $h_\theta(x) \to 0$

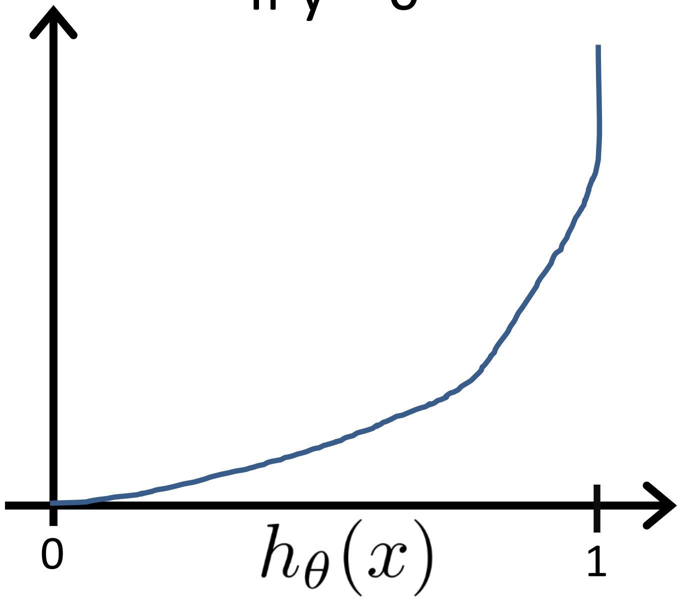
$Cost \to \infty$

Captures intuition that if $h_\theta(x) = 0$, (predict $P(y = 1|x; \theta) = 0$), but $y = 1$, we'll penalize learning algorithm by a very large cost.

Logistic regression cost function

$$\mathrm{Cost}(h_\theta(x), y) = \begin{cases} -\log(h_\theta(x)) & \text{if } y = 1 \\ -\log(1 - h_\theta(x)) & \text{if } y = 0 \end{cases}$$

If y = 0



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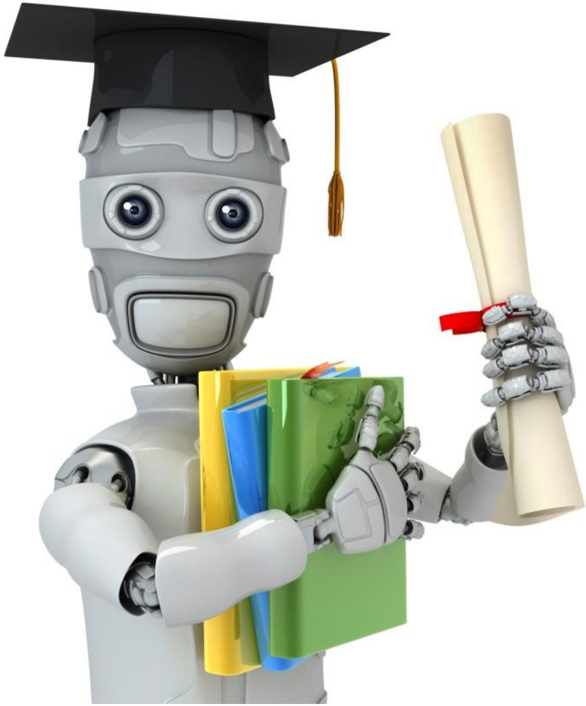


Logistic Regression

Simplified cost function and gradient descent

Machine Learning

Logistic regression cost function

$$J(\theta) = \frac{1}{m} \sum_{i=1}^{m} \mathrm{Cost}(h_\theta(x^{(i)}), y^{(i)})$$

$$\mathrm{Cost}(h_\theta(x), y) = \begin{cases} -\log(h_\theta(x)) & \text{if } y = 1 \\ -\log(1 - h_\theta(x)) & \text{if } y = 0 \end{cases}$$

Note: $y = 0$ or 1 always

$$Cost(h_\theta(x), y) = -y \times log\big(h_\theta(x)\big) - (1 - y) \times log\big(1 - h_\theta(x)\big)$$

$$If\ y = 1\text{:}\ Cost(h_\theta(x), y) = -log\big(h_\theta(x)\big)$$

$$If\ y = 0\text{:}\ Cost(h_\theta(x), y) = -log\big(1 - h_\theta(x)\big)$$

Logistic regression cost function

$$J(\theta) = \frac{1}{m}\sum_{i=1}^{m}\mathrm{Cost}(h_\theta(x^{(i)}), y^{(i)})$$

$$= -\frac{1}{m}\left[\sum_{i=1}^{m} y^{(i)}\log h_\theta(x^{(i)}) + (1-y^{(i)})\log\left(1-h_\theta(x^{(i)})\right)\right]$$

To fit parameters θ:

$$\min_\theta J(\theta)$$

To make a prediction given new x:

Output $h_\theta(x) = \frac{1}{1+e^{-\theta^T x}}$ $\qquad P(y=1|x;\theta)$

Gradient Descent

$$J(\theta) = -\frac{1}{m}[\sum_{i=1}^{m} y^{(i)} \log h_\theta(x^{(i)}) + (1 - y^{(i)}) \log (1 - h_\theta(x^{(i)}))]$$

Want $\min_\theta J(\theta)$:

Repeat $\{$

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta)$$

(simultaneously update all θ_j)

$\}$

Gradient Descent

$$J(\theta) = -\frac{1}{m}[\sum_{i=1}^{m} y^{(i)} \log h_\theta(x^{(i)}) + (1 - y^{(i)}) \log (1 - h_\theta(x^{(i)}))]$$

Want $\min_\theta J(\theta)$:

Repeat $\{$

$$\theta_j := \theta_j - \alpha \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_j^{(i)}$$

(simultaneously update all θ_j)

$\}$

Algorithm looks identical to linear regression!

Suppose that you have trained a logistic regression classifier, and it outputs on a new example x a prediction $h_\theta(x)$ = 0.2. This means (check all that apply):

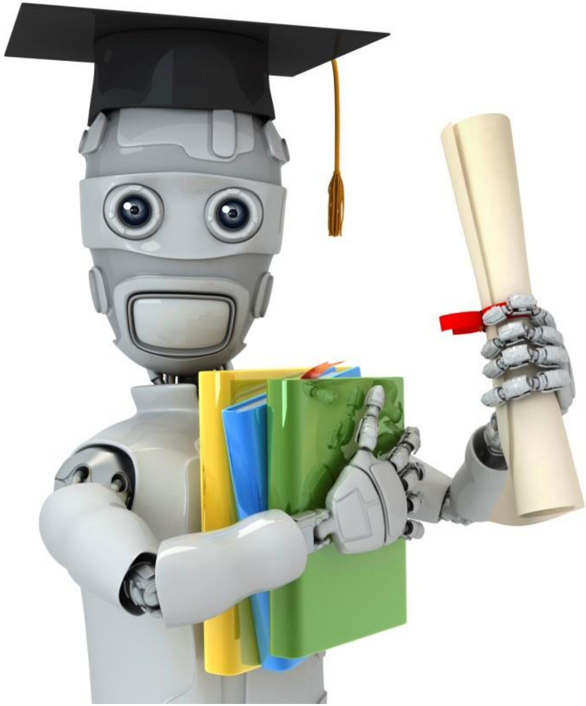
A. Our estimate for $P(y = 1|x;\theta)$ is 0.8.

B. Our estimate for $P(y = 0|x;\theta)$ is 0.2.

C. Our estimate for $P(y = 0|x;\theta)$ is 0.8.

D. Our estimate for $P(y = 1|x;\theta)$ is 0.2.

CD



Machine Learning

Logistic Regression

Multi-class classification: One-vs-all

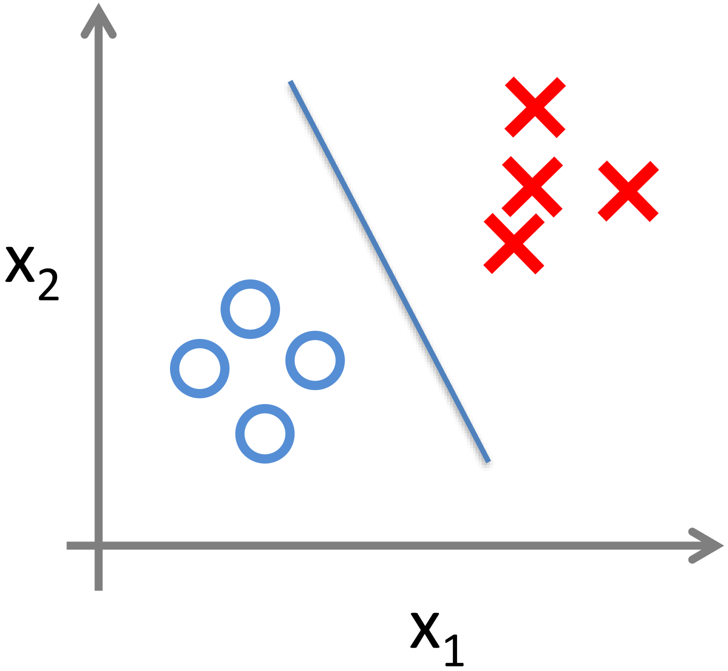
Multiclass classification

Email foldering/tagging: Work, Friends, Family, Hobby

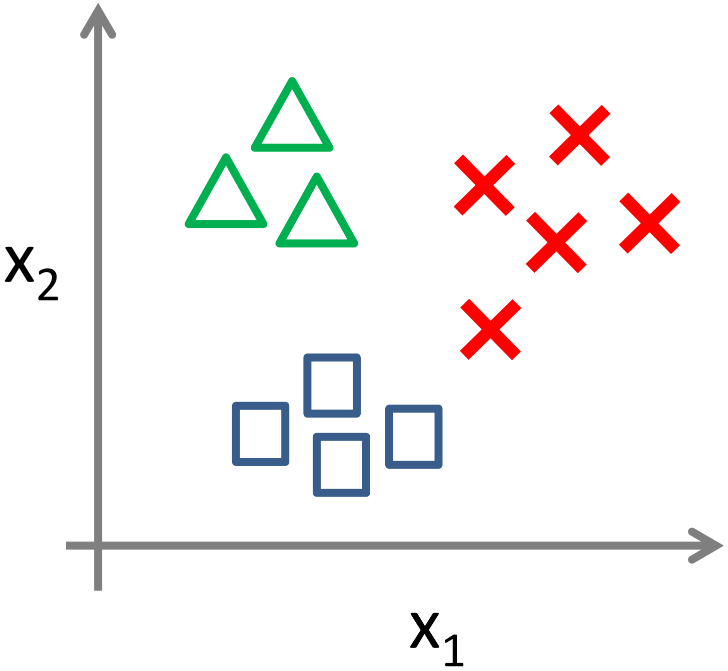
Medical diagrams: Not ill, Cold, Flu

Weather: Sunny, Cloudy, Rain, Snow

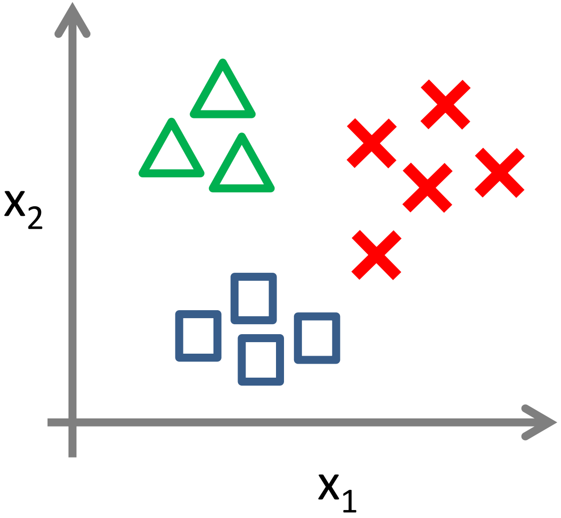
Binary classification:

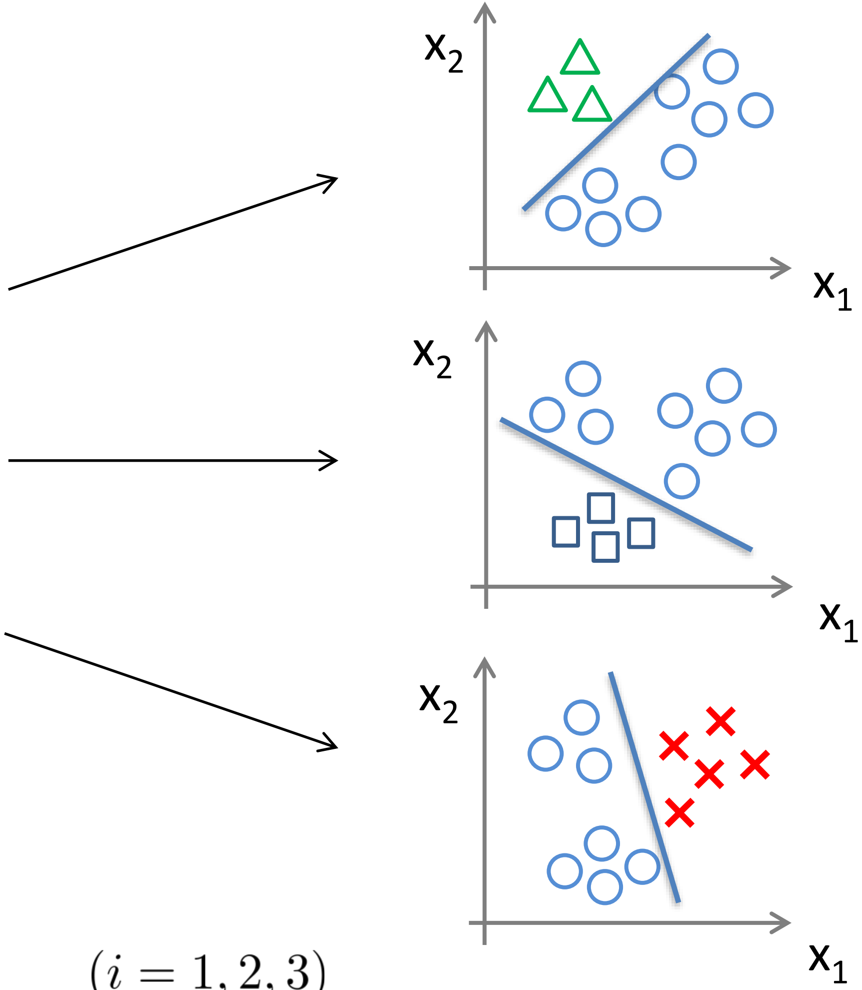


Multi-class classification:



One-vs-all (one-vs-rest):





Class 1:

Class 2:

Class 3:

$$h_\theta^{(i)}(x) = P(y = i|x;\theta) \qquad (i = 1, 2, 3)$$

One-vs-all

Train a logistic regression classifier $h_\theta^{(i)}(x)$ for each class i to predict the probability that $y = i$.

On a new input x, to make a prediction, pick the class i that maximizes

$$\max_i h_\theta^{(i)}(x)$$