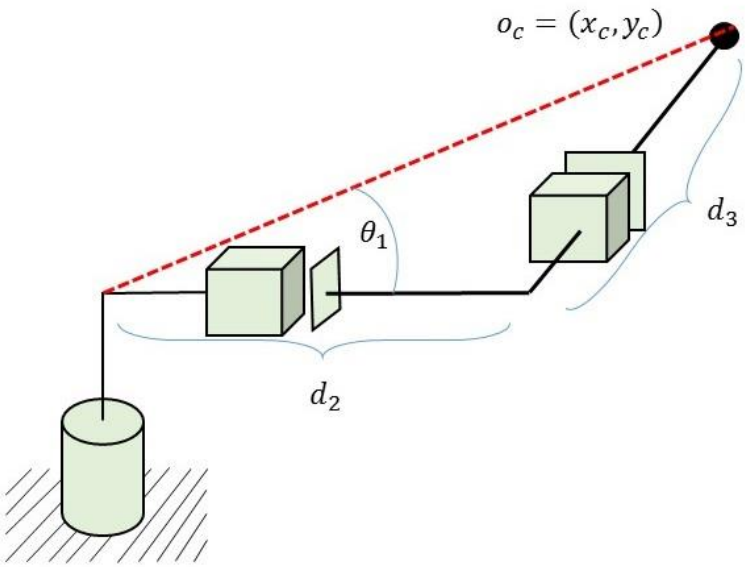
ECE 5397/6397: Introduction to Robotics, Spring 2016

HW #3 due Feb. 23

Discrete Grading Policy. 5 points for each: 2 points for trying, 3 points if partial answer, 5 point if correct.
Parameters and variables are generally defined by DH convention.

1. Given a wrist center o_c, find the inverse position kinematics ($d_2, d_3 \geq 0,\ 0 \leq \theta_1 \leq 90^\circ$).

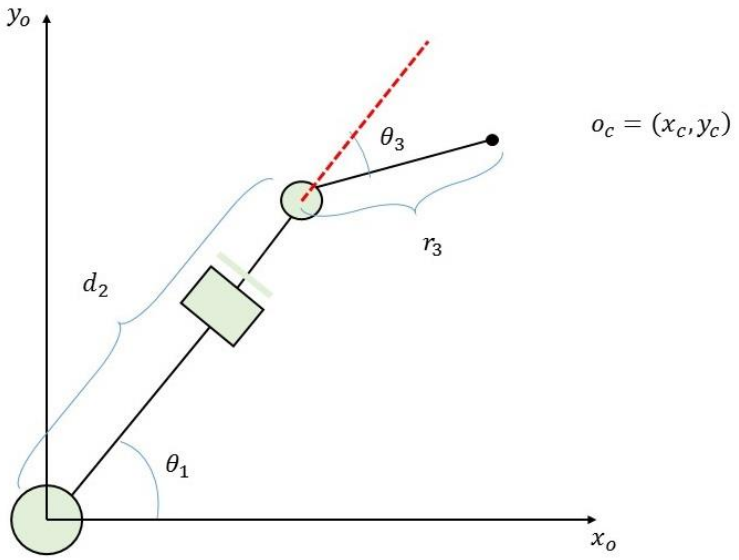


Solution:

$$\theta_1 = A\tan 2(x_{c,} y_c)$$

$$(d_2)^2 + (d_3)^2 = (x_c)^2 + (y_c)^2$$

2. Given a wrist center o_c, how many solutions are there to the inverse position kinematics? Where does the number of solutions change? ($0 \leq \theta_i \leq 360^\circ, d_2 > r_3$)



Solution:

On the upper/lower boundary of the system, there's only a unique solution, where

$$\begin{cases} \theta_1 = A\tan 2(x_{c,} y_c) \\ d_2 = \max(d_2) \\ \theta_3 = 0 \end{cases} \quad \text{or} \quad \begin{cases} \theta_1 = A\tan 2(x_{c,} y_c) \\ d_2 = \min(d_2) \\ \theta_3 = 180^\circ \end{cases}$$

Inside the area where $\min(d_2) - r_3 < \sqrt{(x_c)^2 + (y_c)^2} < \max(d_2) + r_3$, there are infinite number of solutions.

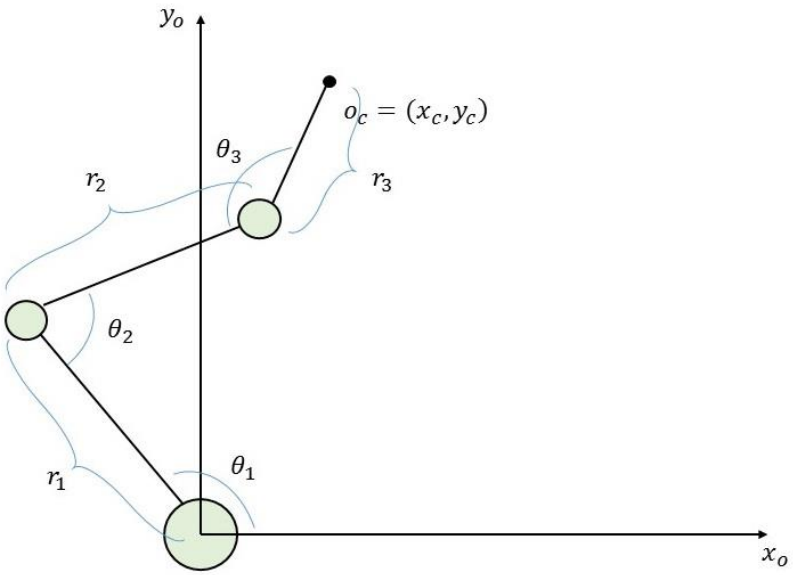
3. Given a wrist center o_c, with $r_1 = 5, r_2 = 3, r_3 = 1$, how many solutions are there to the inverse position kinematics? Where does the number of solutions change? ($0 \le \theta_i \le 360^\circ$, *i*=1, 2, 3)

Solution:

On the upper/lower boundary of the system, there's only a unique solution, where

$$\begin{cases} \theta_1 = A\tan 2(x_{c,} y_c) \\ \theta_2 = 180^\circ \\ \theta_3 = 180^\circ \end{cases} \quad \text{or} \quad \begin{cases} \theta_1 = A\tan 2(x_{c,} y_c) \\ \theta_2 = 0^\circ \\ \theta_3 = 180^\circ \end{cases}$$

Inside the area where $r_1 - r_2 - r_3 = 1 < \sqrt{(x_c)^2 + (y_c)^2} < 9 = r_1 + r_2 + r_3$, there are infinite number of solutions.


y_o
$o_c = (x_c, y_c)$
θ_3
r_2
r_3
θ_2
θ_1
r_1
x_o