UNIVERSITY of HOUSTON | ECE

ECE 5397/6397: Introduction to Robotics, Spring 2016

HW #1 Due Feb. 2

Discrete Grading Policy. 5 points for each: 2 points for trying, 3 points if partial answer, 5 point if correct.

1. Consider the following sequence of rotations:
   1) Rotate by α_1 about the current x-axis.
   2) Rotate by α_2 about the world y-axis.
   3) Rotate by α_3 about the current z-axis.
   4) Rotate by α_4 about the world x-axis.

   Write the matrix product that will give the resulting rotation matrix (do not perform the matrix multiplication). Use the composition rule to derive each step.

2. Verify Equation (2-11) for similarity transformations, namely
$$B = \left(R_1^0\right)^{-1} A R_1^0 .$$
   (If A is the matrix representation of a given linear transformation in $o_0x_0y_0z_0$ and B is the representation of the same linear transformation in $o_1x_1y_1z_1$ then A and B are related as (2-11))

3. Suppose that three coordinate frames $o_1x_1y_1z_1$, $o_2x_2y_2z_2$, and $o_3x_3y_3z_3$ are given, and suppose
$$R_3^2 = \begin{bmatrix} c_\theta & 0 & s_\theta \\ 0 & 1 & 0 \\ -s_\theta & 0 & c_\theta \end{bmatrix}, \quad R_3^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_\phi & -s_\phi \\ 0 & s_\phi & c_\phi \end{bmatrix}$$
   Find the matrix R_2^1.

4. Given a rotation matrix $R \in SO(3)$
$$R = \begin{bmatrix} 0.0802 & -0.3801 & 0.2644 \\ 2.1764 & 1.0114 & -0.5641 \\ 0.2724 & 0.3050 & 0.9084 \end{bmatrix}$$
   Let V be a set of vectors, and $V \subset \mathbb{R}$, $V = \left\{ v \in V \mid \dfrac{Rv \cdot v}{\|v\|} = \dfrac{1}{2} \right\}$. You may use MATLAB, etc.
   1) Use two-argument arctangent function to specify the Euler angles (ϕ, θ, φ) of the rotation transformation R
   2) Give a physical interpretation of the set V.

5. Let $k = [0.5378,\ 0.2534,\ 0.8041]^T$, $\theta = 30^\circ$. Find $R_{k,\theta}$.
6. Consider the diagram of a rotary inverted pendulum in Figure 1. The center of a rotary wheel is set up at the origin of a fixed frame $oxyz$. A frame $o_1x_1y_1z_1$ is fixed to the edge of the first arm, and another frame $o_2x_2y_2z_2$ is fixed to the edge of the second arm. Lengths and angles are specified in Fig.1. Find the homogeneous transformations relating each of these frames the world frame $oxyz$. Find the homogeneous transformations relating $o_1x_1y_1z_1$ and $o_2x_2y_2z_2$.

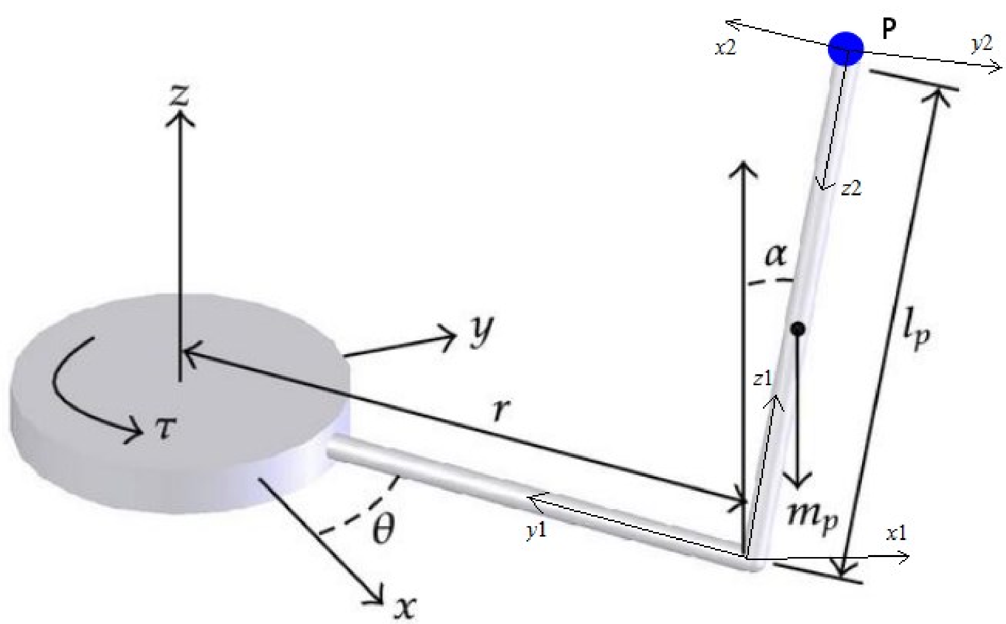

Figure 1. *A rotary inverted pendulum*