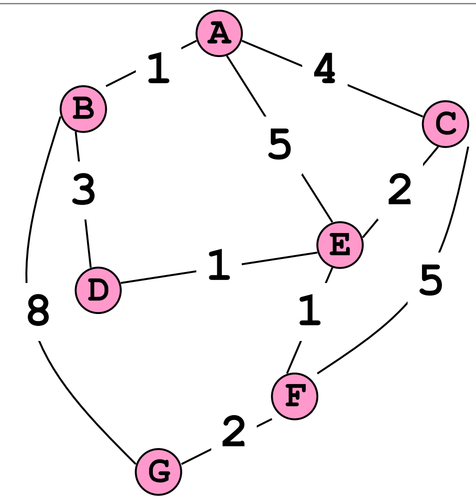
Shortest Path Algorithm

Shortest Path Problem

For weighted graphs it is often useful to find the shortest path between two vertices

- Here, the “shortest path” is the path that has the smallest sum of its edge weights



The shortest path between B and G is:

B–D–E–F–G and not

B–G (or B–A–E–F–G)

MST Vs Shortest Path

Consider the triangle graph with unit weights - it has three vertices x,y,z, and all three edges {x,y},{x,z},{y,z} have weight 1.
The shortest path between any two vertices is the direct path, but if you put all of them together you get a triangle rather than a tree. E
very collection of two edges forms a minimum spanning tree in this graph, yet if (for example) you choose {x,y},{y,z}, then you miss the shortest path {x,z}
.
In conclusion, if you put all shortest paths together, you don't necessarily get a tree.

Dijkstra’s Algorithm

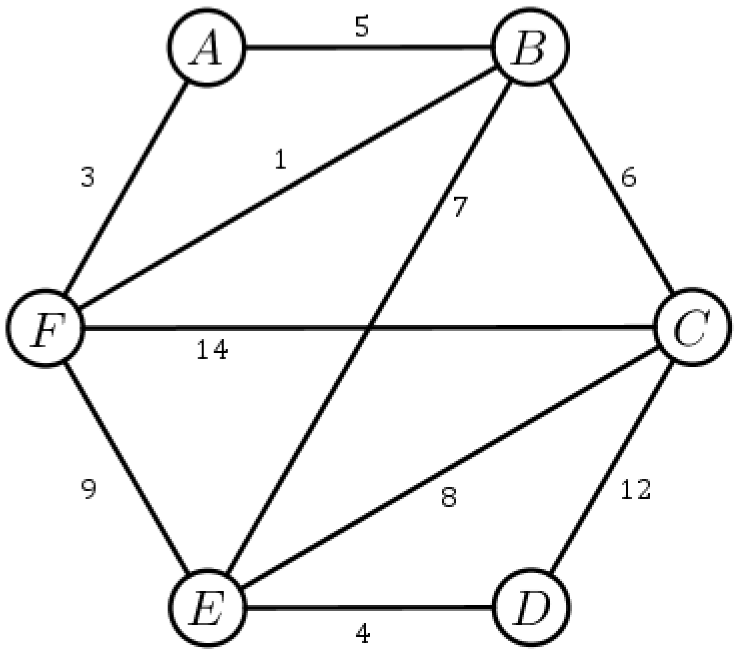
- Solution to the single-source shortest path problem in graph theory
- Both directed and undirected graphs
- All edges must have nonnegative weights
- Graph must be connected

Dijkstra's Algorithm

dist[s] $\leftarrow$0	(distance to source vertex is zero)
for all $v \in V-\{s\}$	
do dist[v] $\leftarrow\infty$	(set all other distances to infinity)
$S\leftarrow\emptyset$	(S, the set of visited vertices is initially empty)
$Q\leftarrow V$	(Q, the queue initially contains all vertices)
while $Q \neq\emptyset$	(while the queue is not empty)
do u $\leftarrow$ mindistance(Q,dist)	(select the element of Q with the min. distance)
$S\leftarrow S \cup \{u\}$, $Q= Q-\{u\}$	(add u to list of visited vertices)
for all $v \in$ neighbors[u]	
do if dist[v] > dist[u] + w(u, v)	(if new shortest path found)
then d[v] $\leftarrow$d[u] + w(u, v)	(set new value of shortest path)
	(if desired, add traceback code)
return dist	

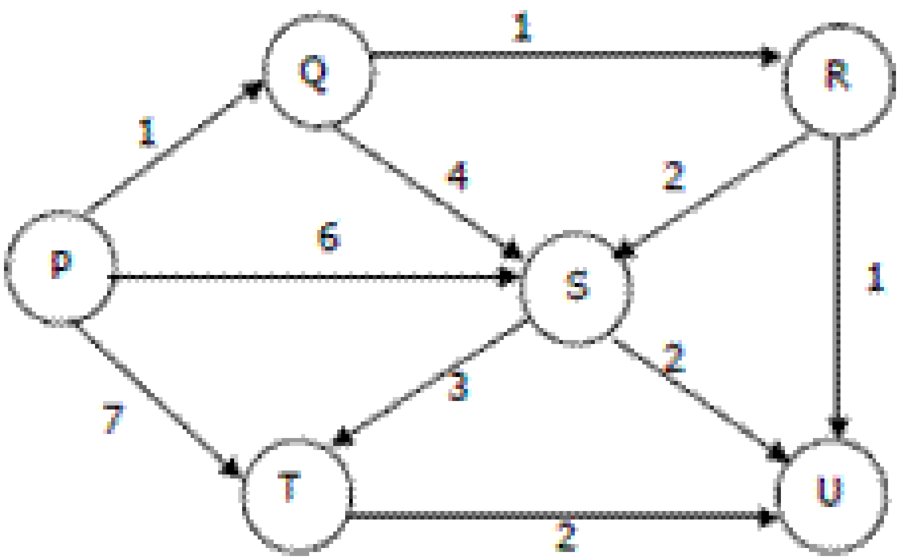
Class Exercise 1

Apply the Dijkstra's algorithm to find the shortest path between the vertex A and each of the other vertices.



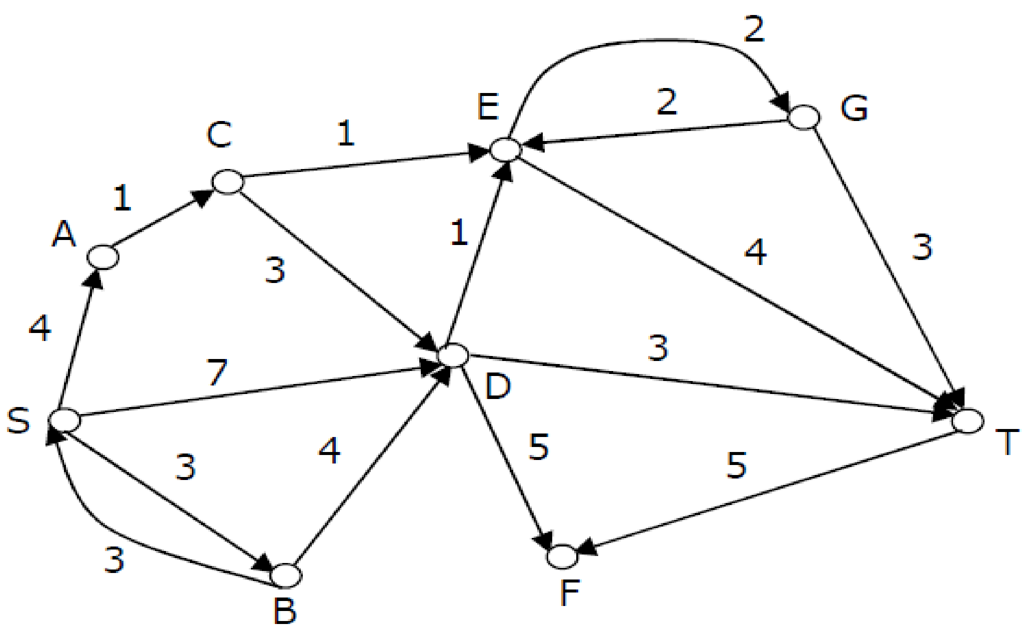
Class Exercise 2

Suppose we run Dijkstra’s single source shortest-path algorithm on the following edge weighted directed graph with vertex P as the source. In what order do the nodes get included into the set of vertices for which the shortest path distances are finalized?



Class Exercise 3

Consider the directed graph shown in the figure below. There are multiple shortest paths between vertices S and T. Which one will be reported by Dijkstra's shortest path algorithm? Assume that, in any iteration, the shortest path to a vertex v is updated only when a strictly shorter path to v is discovered.



i. SACDT
ii. SBDT
iii. SDT
iv. SACET