Question Bank

Ans 1 $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$A = \{1, 3, 5, 7, 9\}$

$B = \{2, 3, 4, 5\}$

$C = \{5, 6, 7\}$

(i) $A \cap B = \{3, 5\}$

(ii) $A \cup B = \{1, 2, 3, 4, 5, 7\}$

(iii) complement of A $(U - A) = \{2, 4, 6, 8, \cancel{9}\}$

(iv) $A - C = \{1, 3, 9\}$

(v) $\{C - A\} = \{6\}$

(vi) $C \cap (A \cup B)$

$\{5, 6, 7\} \cap \{1, 2, 3, 4, 5, 7\}$

$= \{5, 7\}$

(vii) $(C \cap A) \cup (C \cap B)$

$\{5, 7\} \cup \{5\}$

$= \{5, 7\}$

(viii) $C \cup (A \cap B)$

$\{5, 6, 7\} \cup \{3, 5\} = \{3, 5, 6, 7\}$

(ix) $(C \cup A) \cap (C \cup B)$

$\{1, 3, 5, 6, 7, 9\} \cap \{2, 3, 4, 5, 6, 7\}$

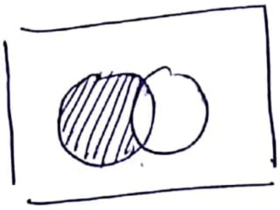
$= \{3, 5, 6, 7\}$

Ans 2

(i) $A - B = A \cap B'$

$\downarrow$

$A - B$ means the elements present in A only not B



$\therefore \cancel{A - (A \cap B)}$

$= A \cap B'$

$= A \cap (U - B)$

$= (A \cap U) - (A \cap B)$

$= A - (A \cap B)$

$= \cancel{(A - A) \cup (A - B)}$

$= \cancel{\phi \cup (A - B)}$

$= A - B$

Hence Proved

$A - B = A - (A \cap B)$

$= (A \cap U) - (A \cap B)$

$= A \cap (U - B)$ (By distributive law)

$= A \cap B'$

(ii) $(X \cup Y) - Z = (X - Z) \cup (Y - Z)$

In distributive law

$(A \cup B) \cup C = \cancel{(A \cup B)}\ A \cup (B \cup C)$

$(A \cap B) \cup C = (A \cup C) \cap (A \cup B)$

$\therefore$ By Appleying distributive

$\Rightarrow (X - Z) \cup (Y - Z)$

(Series)3

$(A \cup C) \cap (A \cup D) \cap (B \cup C) \cap (B \cup D)$

$[A \cup (C \cap D)] \cap [B \cup (C \cap D)]$ (by distributive law)

$(A \cap B) \cup (C \cap D)$ (By distributive law)

(4) $1 + 2 + \dots n = \frac{n(n+1)}{2}$.

for $n = 1$

$\therefore$ LHS = 1

RHS = $\frac{1(1+1)}{2} = 1$

LHS = RHS

$\therefore$ it is true for $n = 1$

let it is true for $n = k$

$1 + 2 + \dots k = \frac{k(k+1)}{2}$ —— (1)

To Prove for $k+1 = n$

To prove we have to prove - $1 + 2 + \dots, k+1 = \frac{(k+1)(k+2)}{2}$.

LHS $\underbrace{1 + 2 + \dots k}$, $+ k + 1$

$\frac{k(k+1)}{2} + k+1$ from (1)

$k(k+1) + 2k + 2 / 2$

$k^2 + k + 2k + 2 / 2$

$\frac{k(k+2) + (k+2)}{2}$

$= \frac{(k+1)(k+2)}{2}$

= RHS

Hence it is true for $k+1$

$\therefore$ it is true for every integer

(5) $1^3 + 2^3 + \dots n^3 = \left[\frac{n(n+1)}{2}\right]^2.$

For $n = 1$

$LHS = 1^3 = 1$

$RHS = \left[\frac{1(1+1)}{2}\right]^2 = 1$

$\therefore$ true for $n = 1$

[For $n = k$] assume it is true for $n = k$

$$1^3 + 2^3 + \dots + k^3 = \left[\frac{k(k+1)}{2}\right]^2. \quad \text{—} (1)$$

~~To~~ For $n = k+1$

To prove ~~this~~ for $n = k+1$ proove: $1^3 + 2^3 + \dots = \left[\frac{(k+1)(k+2)}{2}\right]^2$

$$LHS = \underbrace{1^3 + 2^3 + \dots k^3}+ (k+1)^3$$

$$\left[\frac{k(k+1)}{2}\right]^2 + (k+1)^3$$

$$(k+1)^2 \left[\frac{k^2}{4} + (k+1)\right]$$

$$\frac{(k+1)^2}{4}\left[k^2 + 4k + 4\right]$$

$$\frac{(k+1)^2(k+2)^2}{4}$$

$$= \left[\frac{(k+1)(k+2)}{2}\right]^2.$$

$= RHS$

$\therefore$ it is true for $n = k+1$

Hence it is true for every integer.

(5) $r^0, r^1, r^2, r^3, r^4 \dots = \left(\frac{r^{n+1}-1}{r-1}\right)$

~~let is~~ for $n=1$

$LHS = r^0 = 1$

$RHS = \left(\frac{r^1-1}{r-1}\right) = 1$

$LHS = RHS$

$\therefore$ it is true for $n=1$

Assume that it is true for $n=k$.

$r^0 + r^1 + r^2 \dots r^k = \left(\frac{r^{k+1}-1}{r-1}\right)$ —— (1)

Prove for $n=k+1$

(i) i.e. $r^0 + r^1 + r^2 + \dots r^k, r^{k+1} = \left(\frac{r^{k+2}-1}{r-1}\right)$

$LHS = \underbrace{r^0 + r^1 + r^2 + \dots r^k}_{} + r^{k+1}$

$\left(\frac{r^{k+1}-1}{r-1}\right) + r^{k+1}$

$\left(r^{k+1}-1\right) + r^{k+1}(r-1) \,/\, r-1$

$\frac{\left(r^{k+1}-1\right) + \left(r^{k+2} - r^{k+1}\right)}{r-1}$

$= \frac{\cancel{r^{k+1}} - 1 + r^{k+2} - \cancel{r^{k+1}}}{r-1}$

$= \left(\frac{r^{k+2}-1}{r-1}\right)$

$\left.\frac{}{}\right)\Big]^2$

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(7) For every $n \geq 5$ whe have $2^n > n^2$

~~let it is true~~ for $n = 5$

$2^5 > (5)^2$

$32 > 25$

$\therefore$ true for $n = 5$

let it is true for $n = k$

$\Rightarrow \cancel{2^n}$ $\boxed{2^k \geq k^2}$ — (1)

To prove for $n = k+1$

to prove this $2^{k+1} \geq (k+1)^2$

~~LHS~~ $\cancel{2^k \cdot 2 - k^2 + 1 + 2k}$

$\cancel{2^k \cdot 2 - 2k >}$

$2^k \geq k^2$

$2 \cdot 2^k \geq 2k^2$

$2^{k+1} \geq 2k^2$

$2k^2 \geq k^2 + 1$

$(k+1)^2$

$k^2 + 1 + 2k$

$k(k+2) + 1$

$(k+1)(k+1)$

$2^k \geq k^2$

$2 \cdot 2^k \geq 2k^2$

$2^{k+1} \geq 2k^2$

$2k^2$ can be replaced by

$\boxed{2^{k+1} \geq (k+1)^2}$

As $2k^2 \geq (k+1)^2$

for $2(5)^2 \geq (5+1)^2$

$2 \times 25 \geq (6)^2$ $\therefore$

$50 \geq 36$

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(8) $X = \{1,2\}$ $Y = \{a,b,c\}$

(i) Power set of $X = \{ \cancel{(1,1)} \cancel{(2,2)}, \phi \cdot \{1\}, \{2\}, \phi$
$\{1,2\}$

(ii) $(X \times Y) = \{(1,a)(1,b)(1,c)(2,a)(2,b)(2,c)\}$.

(iii) $(Y \cap Z) = (b\ c)$, $X = \{1,2\}$
$X \times (Y \cap Z) = \{(1,b)(1,c)(2,b)(2,c)\}$

(iv) $(X \times Y) \cup (X \times Z)$
$\{(1,a),(1,b)(1,c)(2,a)(2,b)(2,c)\} \cup \{(1,b)(1,c)(1,d)(2,b)(2,c)(2,d)\}$
$= \{(1,a)(1,b)(1,c)(1,d)(2,a)(2,b)(2,c)(2,d)\}$.

(9) (i) $X \times (Y \cap Z) = (X \times Y) \cap (X \times Z)$

let $(x,y) \in X \times (Y \cap Z)$ $x \in X$
$y \in Y \cap Z$

$\Leftrightarrow x \in X$ and $y \in Y \cap Z$.
$\Leftrightarrow x \in X$ and $(y \in Y$ and $y \in Z.)$
$\Leftrightarrow (x \in X$ and $y \in Y)$ & $(x \in X$ and $y \in Z)$
$\Leftrightarrow (X \times Y) \cap (X \times Z)$

$\Rightarrow X \times (Y \cap Z) \subseteq (X \times Y) \cap (X \times Z)$ —— (1)

IIIly $(X \times Y) \cap (X \times Z) \subseteq X \times (Y \cap Z)$ —— (2)

from (1) and (2)

$$\boxed{X \times (Y \cap Z) = (X \times Y) \cap (X \times Z)}$$

(iii) $Power(X \cap Y) = Power(X) \cap Power(Y)$

let $X = \{1, 2, 3\}$.

$Y = \{3, 2, 5, 8\}$.

$X \cap Y = \{3, 2\}$

Power of $X \cap Y = \{\phi, \{3\}, \{2\}, \{3, 2\}\}$ —— (1)

Power of $X = \{\{3\}, \{2\}, \{1\}, \{1, 2\}, \{2, 3\}, \{3, 1\}, \phi, \{1, 2, 3\}\}$

Power of $Y = \{\{3\}, \{2\}, \{5\}, \{3, 2\}, \{2, 5\}, \{5, 3\}, \phi, \{3, 2, 5\}\}$

$Pow(X) \cap Pow(Y) = \{\{3\}, \{2\}, \{3, 2\}, \phi\}$ —— (2)

from (1) and (2)

Power of $(X \cap Y) =$ $Pow(X) \cap Pow(Y)$

OR.

let $x \in X$; ~~y~~ $\cancel{x \in X \cap Y}$ $x \in X \cap Y$

$\cancel{z \in P(X \cap Y)}$ $\{x\} \in P(X \cap Y)$

$z \in$ $\therefore x \in X \cap Y$

$x \in X$ and $x \in Y$

$x \in Y$ and $x \in Y$

$\Leftrightarrow \{x\} \in (X)$ and $\{x\} \in (Y)$

$\Leftrightarrow$

$\Leftrightarrow P(X) \cap P(Y)$

$\Leftrightarrow P(X \cap Y) \subseteq P(X) \cap P(Y)$

Illy $\boxed{P(X) \cap P(Y) = P(X \cap Y)}$

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Ques 10

$A = \{p, q, r\}$

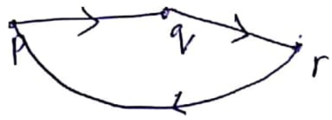
$R = \{(p,q)\ (q,r)\ (r,p)\}$

$S = \{(q,p)\ (r,p)\ (p,r)\}$ on A

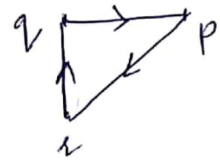
(i)

$$R = \begin{array}{c} \\ p \\ q \\ r \end{array} \begin{array}{c} \begin{array}{ccc} p & q & r \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \end{array} \qquad S = \begin{array}{c} \\ p \\ q \\ r \end{array} \begin{array}{c} \begin{array}{ccc} p & q & r \end{array} \\ \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \end{array}$$

(ii) (a) $R \Rightarrow$



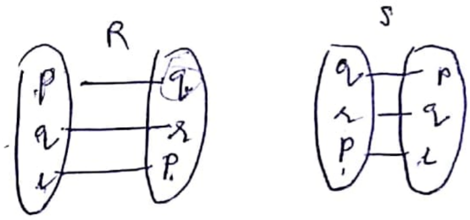
(b) $S \Rightarrow$



(iii) composition of $R \circ S$

$\Rightarrow R(S)$

$\{(p,p)\ (q,q)\ (r,r)\}$



(iv) $R^2 = R \circ R$

$p \to q$	$p - q$
$q \to r$	$q - r$
$r \to p$	$r - p$.

$R^2 \Rightarrow (p,r)\ (q,p)\ (r,q)$

$R^3 \Rightarrow R \circ (R^2)$

R	R2
$p \to q$	$p \to r$
$q - r$	$q \to p$
$r \to p$	$r \to q$

$R \circ R \circ R = R^3 \Rightarrow$

$(p,p)\ (r,r)\ (q,q)$

Ques11

(i) Reflexive
ROS, (R, S, R^3, ~~R~~)
~~R^2~~

(ii) symmetric.
~~R~~ = P, (R^3), ROS.

(iii) antisymmetric
ROS R, S, R^3, R^2

(iv) transitive: R^3, ROS.

R = (p, q) (q, r) (r, p)
S = (q, p) (r, p) (p, r)
ROS (p, p) (q, q) (r, r)
R^2 (p, r) (q, p) (r, p)
R^3 = (p, p) (q, q) (r, r)

ans12

(a) (a, b) ∈ R (a / b)

Reflexive.
~~R~~ a ∈ R
⇒ (a, a) ∈ R
(a divides a).

symmetric
~~a ∈ R~~, (a, b) ∈ R.
⇒ ~~a/b~~ b/a
⇏ a/b
⇏ (b, a) ∈ R
∴ not symmetric

Transitive
(a, b) ∈ R, (b, c) ∈ R
(b/a), (c/b)
⇒ (c/a)
⇒ transitive.

(b) (a, b) ∈ R a + b = even

~~let~~
Reflexive
let a ∈ R
a + a = 2a
= even
∴ Reflexive

symmetric
let ~~a ∈ R, b ∈ Z~~
⇒ if (a, b) ∈ Z
⇒ a + b = 2n
⇒ b + a = 2n
⇒ (b, a) ∈ Z
⇒ symmetric.

(c) Transitive

if $(a,b) \in Z$, $(b,c) \in Z$.

$\Rightarrow a+b = 2n$

$\Rightarrow b+c = 2m$

$$a+c+2b = 2n+2m$$

$$a+c = 2n+2m-2b$$

$$a+c = 2(n+m-b)$$

$$a+c = \underbrace{2k}_{\text{even}}$$

$\therefore$ transitive

$\therefore$ the Relation is equivalence

Q13 $\quad A = \{1, 2, 4, 5, 7, 11, 13\}$

$$x - y = 3(k)$$

$$R = \{ \underbrace{(4,1)(5,2)(7,4)(11,5)(13,7)}_{3} \underbrace{(7,1)\ (11-5)}_{6}^{(13-7)} \underbrace{(11-2)(13-}_{9}$$

$$(1,1)(2,2)(4,4)(5,5)(7,7)(11,11)(13,13)\}$$

Symmetric Reflexive

$a \cdot a \in R$

$\Rightarrow (a,a) \in R$

$a - a = 0 = 3(0)$

$\therefore$ Reflexive.

Symmetric

$a \in R$, $b \in R$

if $(a,b) \in R$

$\Rightarrow a-b = 3k$

$\Rightarrow -(b-a) = 3k$

$b - a = -3k$

$\Rightarrow (b,a) \in R$

$\therefore$ symmetric

transitive

$(a,b), (b,c) \in R$

$\Rightarrow a - b = 3k$

$\quad b - c = 3t$

$+$

$a - c = 3(k+t)$

$\therefore a - c = 3p$

$\Rightarrow (a,c) \in R.$

$\therefore$ transitive relation

$\therefore$ The relation is ~~equa~~ equivalence on R.

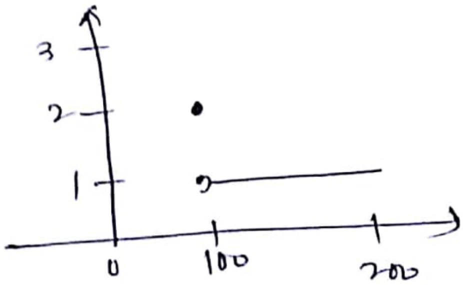
(14) f, g, and h $(N \rightarrow N)$.

$f(x) = \begin{cases} 1 & \text{if } x > 100 \\ 2 & \text{if } x = 100. \end{cases}$

$g(x) = x^2 + 1$

$h(x) = 2x + 1$

$f(x) =$



f(x) is neither one-one nor onto as 4 value of x is not having the distinct image in y and not every value in y have preimage in y.

$g(x) = x^2 + 1$

one-one.

~~x^2+1~~ let $x_1, x_2 \in N$ such that $f(x_1) = f(x_2)$

$x_1^2 + 1 = x_2^2 + 1$

$\Rightarrow x_1 = x_2$ as $x_1, x_2 \in N$

$\therefore$ One-one.

onto

let $f(x) = k$ and $k \in N$

$x^2+1 = k$

$x^2 = k-1$

$x = \sqrt{k-1}$

$f(x) = f(\sqrt{k-1}) = f(\sqrt{k-1})^2+1$

$= k-1+1$

$= k$

$\boxed{f(x) = k}$

$\Rightarrow$ The function is onto.

$h(x) = 2x+1$

one - one

let $x_1, x_2 \in N$

$f(x_1) = f(x_2)$

$2x_1+1 = 2x_2+1$

$\Rightarrow x_1 = x_2$

$\Rightarrow$ The fnx is one one

onto

let $f(x) = k$, $k \in N$

$\Rightarrow 2x+1 = k$

$x = \frac{k-1}{2}$

$f(x) = f\left(\frac{k-1}{2}\right) = \left(\frac{k-1}{2}\right)2+1$

$= k.$

$\therefore$ onto.

(b) $h \circ (f \circ g)$.

$= h(f(g(x)))$

$= h(f(x^2+1))$

$f(x^2+1) = \begin{cases} \text{if } x^2+1 > 100 \text{ the } f(x^2+1) = 1 \\ \text{if } x^2+1 = 100 \text{ then } f(x^2+1) = 2. \end{cases}$

$\Rightarrow h(3)$ if $x^2+1 > 100$ $= 3 \} - 10$ —— (1)

$h(3)$ if $x^2+1 = 100$ $= 8 \}. - 26$

$(h \circ g) \circ f$

$= h(g(x)) \circ f$

$= h(x^2+1) \circ f$

$[2(x^2+1)+1] \circ f$

$h \circ f$

$h(f(x))$

$= \begin{cases} 3 & \text{if } x > 100 \quad (\text{As } x=1) \\ 5 & \text{if } x = 100 \quad (\text{As } x=2) \end{cases}$

$2[(1)^2+1]+1 = 10$

$2((2)^2+1)+1 = 26$

$f(1) \quad x > 100$

$f(2) \quad x = 100$

— ②

from ① and ② Ass. law verified.

⑮ $f: N \to N \quad f(x) = x+1 \ \forall x \in N$

(a) Domain : N

(b) Range : N

(c) one-one

let $x_1, x_2 \in N$ such that $f(x_1) = f(x_2)$

$x_1 + 1 = x_2 + 1$

$\Rightarrow x_1 = x_2$

$\therefore$ the fnx is one-one.

(d) onto

let ~~x_1, x_2~~ $k \in N$, $f(x) = k$.

$x+1 = k$

$x = k-1$

$f(x) = f(k-1) = k-1+1$

$= k$

$\therefore$ onto.

(20)

$(A \cup B)' = A' \cap B'$

let $x \in (A \cup B)'$

$\Rightarrow x \notin (A \cup B)$

$\Rightarrow x \notin A$ and $x \notin B$

$\Rightarrow x \in A'$ and $x \in B'$

$\Rightarrow x \in A' \cap B'$

$\Rightarrow (A \cup B)' \subseteq A' \cap B'$

— (1)

let $A' \cap B'$

let $x \in A' \cap B'$

$x \notin$ $x \in A'$ and $x \in B'$

$\Rightarrow x \notin A$ or $x \notin B$

$\Rightarrow x \notin A$ or B

$\Rightarrow x \in (A \cup B)'$

$\Rightarrow A' \cap B' \subseteq (A \cup B)'$.

— (2)

from (1) and (2)

$(A \cup B)' = A' \cap B'$.

(21) $X = \{a, b, c\}$ $f : X \to X$

such that: $f : \{(a,b), (b,a), (c,c)\}$

(a) $f^{-1} = (b,a)(a,b)(c,c) = f$.

(b) $f^2 = \begin{array}{l} a \to b \\ b \to a \\ c \to c \end{array} \left| \begin{array}{l} a \to b \\ b \to (a) \\ c \to c \end{array} \right. = (a,a)(b,b)(c,c)$

(c) $= f^3 = \begin{array}{l} a \to b \\ b \to a \\ c \to c \end{array} \left| \begin{array}{l} a \to a \\ b \to b \\ c \to c \end{array} \right. = (a,b)(b,a)(c,c)$

(d) $f^4 = (a,a)(b,b)(c,c)$

$a_r = 2a_{r-1}$

$a_{r-1} = 2a_{r-2}$

$a_{r-2} = 2a_{r-3}$

$a_r = a_{r-3}$

$r^{\phi} = r + 4D$

$r^{\phi} = 4D$

① $a_n = a_{n-1} + 6a_{n-2}$; $n >= 2$

② $a_n = 7a_{n-1} - 10_{n-2}$; $n >= 2$.

③ $a_n = 5a_{n-1} - 6a_{n-2} + 1$

④ $a_n = 6a_{n-1} + 8a_{n-2} = 3$

⑤ $a_{n+2} - 2a_{n+1} + a_n = 3n+5$

α^2

$a_r = a_{r-1} + a_{r-1}$

① → $\alpha^2 = \alpha - 6$

$\alpha^2 - \alpha + 6 = 0$

$\alpha^2 - 3\alpha - 3\alpha + 6$

$\alpha(\alpha-3) - 3(\alpha-3)$

$(\alpha-3)(\alpha-3)$

② → $\alpha^2 = 7\alpha - 10$

$\alpha^2 - 7\alpha + 10 = 0$

$\alpha^2 - 5\alpha - 2\alpha + 10 = 0$

$\alpha(\alpha-5) - 2(\alpha-5) = 0$

$(\alpha-2)(\alpha-5) = 0$

③ $a_n = 5a_{n-1} - 6a_{n-2} + 1$

$f(r) = 1$

$f(r) = 1 \cdot 1^r$

$\beta = 1, B_0 = 1$

$a_r^P = 1^r(d)$

$\alpha^2 - 5\alpha + 6$

$\alpha^2 - 3\alpha + 2\alpha + 6$

$\alpha(\alpha-6) + 1(\alpha-6)$

$\cancel{(\alpha-6)}$ $\alpha = 2, 3$

$\alpha \neq \beta$

④. $\alpha^2 - 6\alpha + 8 - f(r) = 0$

$f(r) = 3$

$3 \cdot 1^r$

$a_r^P = 1^r(d)$

$\alpha^2 - 6\alpha + 8$

$\alpha^2 - 4\alpha - 2\alpha + 8$

$\alpha(\alpha-4) - 2(\alpha-4)$

$\alpha = 2, 4$

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(5) $\alpha_{n+2} - 2\alpha_{n+1} + \alpha_n = 3n+5$

C. Equation $\alpha^2 - 2\alpha + 1 = 0$

$\alpha^2 - \alpha - \alpha + 1 = 0$

$\alpha(\alpha-1) - (\alpha-1) = 0$

$(\alpha-1)(\alpha-1)$

$a_{n+2} - 2a_{n+1} + a_n = 3n+5$

$f(r) = (3n+5) \cdot 1^r$

$\beta = 1$ $\qquad (\alpha = \beta = 1)$

$\alpha_r^P = r^m (d_1 r + d_0) \beta^r$

$\alpha_r^P = r(3r+5) \; 1^r$

$\alpha_r^P = n(3n+5) 1^n.$

Q $(a_n + 4a_{n-1} + 4a_{n-2} = [n^2 + 3n + 5][1]^m.$ $\quad \beta = 1$

$\alpha^2 + 4\alpha + 4 = n^2 + 3n + 5$

C. Equation $\alpha^2 + 4\alpha + 4 = (\alpha+2)^2 = 0$

$(\alpha+2)(\alpha+2) = 0$

$\alpha = -2, -2. = (C_1 + C_2 n)(-2)^n$

$f(r) = n^2 + 3n + 5$

$f(r) = (n^2 + 3n + 5) 1^r.$

$\beta = 1,$ as $\alpha \neq \beta$

$\boxed{\alpha_r^P = (d_2 n^2 + d_1 3n + 5d_0) 1^r}$

ans 31

$a_n = 2^{n+1}$ is a sol of $a_n = 3a_{n-1} + 2^n$

Char. Equation = $\cancel{\alpha - 3\alpha = 0}$ $\alpha = 3$ $c_1 3^n$

$(\quad) \beta^n \alpha^t$

$f(n) = 2^n (1)$ $f(n) = (\quad 1 \quad) \beta^n$ $\beta = 2$

$= \beta = 2 (1)$ $f(n)$ —— (2^n)

as $\alpha = \beta$ $d_0 t$

$\therefore$ sol. $\rightarrow \alpha_r^p = (2^n)(d)$.

when $d = 2$:

$\alpha_r^p = (2^n)(2)$

sol : $\rightarrow$ $\boxed{\alpha_r^p = 2^{n+1}}$

ans 32 $a_n = c_1 + c_2 2^n - n$. sol of $a_n - 3a_{n-1} + 2a_{n-2} = 1$

Char eq: $\rightarrow$ $\alpha^2 - 3\alpha + 2 = 0$

$\alpha^2 - 2\alpha - \alpha + 2 = 0$

$\alpha(\alpha - 2) - 1(\alpha - 2) = 0$

$(\alpha - 1)(\alpha - 2) = 0$

$\alpha = 1, \alpha = 2$

Homogeneous sol. $c_1 1^n + c_2 2^n$

$= c_1 + c_2 2^n$

For particular sol: $f(n) = 1$

$1^n (1)$

$\beta = 1$

$(\alpha = \beta)$

$= n^{(1)} (d) (\beta 1^n)$

$-n$ (when $d = -1$)

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ans:- (i) (1, 1, 1, 1 . . . .)

$a_0 = 1,\ a_1 = 1,\ a_3 = 1$

$\therefore G(x) = 1 + x + x^2 + x^3 + \ldots\ldots$

So This form a G.P

$\therefore$ sum of G.P $= \dfrac{a}{1-r}$ $\quad a = 1;\ r = x$

$= \left(\dfrac{1}{1-x}\right)$

(ii) 1, 2, 3 . . . . . .

$a_0 = 1,\ a_1 = 2,\ a_3 = 3$

$\therefore G(x) = 1 + 2x + 3x^2 + \ldots\ldots$

$-\ x\,G(x) = x + 2x^2 + 4x^2 + \ldots$

$G(x)(1-x) = 1 + x + x^2 + x^3 + \ldots\ldots$

$(1-x)\,G(x) = \left(\dfrac{1}{1-x}\right)$

$G(x) = \dfrac{1}{(1-x)^2}.$

(iii) 0, 0, 1, 1, 1 . . . .

$a_0 = 0,\ a_1 = 0\quad a_2 = 1,\ a_3 = 1\ \ldots$

$\therefore G(x) = 0 + 0 + x^2 + x^3 + \ldots\ldots$

$G(x) = \left(\dfrac{x^2}{1-x}\right)$

(iv) 1, 3, 3^2 . . . .

$\therefore G(x) = \left(\dfrac{1}{1-3x}\right)$

Ans 39

(i) $a_n = 3a_{n-1}$ $n >= 1$ $a_0 = 1$

$a_1 = 3a_0 = 3(1) = 3$

$a_2 = 3(a_1) = 3(3) = 3^2$

$a_3 = 3(3^2) = 3^3$

$\therefore G(x) = 1 + 3x + 3^2 x^2 + 3^3 x^3 + \dots$

$G(x) = \left(\frac{1}{1-3x}\right)$

(ii)

$a_n = 3a_{n-1} + 2.$ $a_0 = 1$

$a_1 = 3a_0 + 2. = 3(1) + 2 = 5$

$a_2 = 3a_1 + 2 = 3(5) + 2 = 17$

$a_3 = 3(17) + 2 = 3(17) + 2 = 53$

$= 3(53) + 2$

$G(x) = 1 + 5x + 17x^2 + 53x^3 + \dots$

(iii) $a_n - 9a_{n-1} + 20a_{n-2} = 0$

$a_n = 9a_{n-1} - 20a_{n-2}$ $a_0 = 3$

$a_1 = -10$

$a_2 = 9a_1 - 20a_0$

$a_2 = 9(-10) - 20(3)$

$a_2 = -90 - 60$

$a_2 = -150$

$a_3 = 9(a_2) - 20a_1$

$a_3 = 9(-150) - 20(-10)$

$a_3 = -1350 + 200$

$a_3 = -1150$

$\underline{G(x) = 3 - 10x - 150x^2 - 1150x^3}$

$\begin{array}{r}17 \\ \times 3 \\ \hline 51 \\ 2\end{array}$

$(5 \times 3 + 2$

$(17 -$

$\begin{array}{r}53 \\ -17 \\ \hline 36\end{array}$

$\begin{array}{r}53 \\ \times 3 \\ \hline 159 \\ 53 \\ \hline 126\end{array}$

$\begin{array}{r}13 \\ 2 \\ \hline 11\end{array}$

(iv) $a_{n+2} - 2a_{n+1} + a_n = 2^n$

$a_{n+2} = 2^n + 2a_{n+1} - a_n.$

$a_2 = 2^0 + 2(a_1) - a_0$

$a_2 = 1 + 2(1) - 2 = 0$

$a_3 = 2^1 + 2(a_2) - a_1$

$a_3 = 2 + 2(0) - 1 = (1)$

$a_4 = 2^2 + 2a_3 - a_2.$

$a_4 = 4 + 2(1) - (0)$

$a_4 = 4 + 2 = 6$

$a_5 = 2^3 + 2a_4 - a_3$

$a_5 = 8 + 2(6) - 1$

$a_5 = 8 + 12 - 1 = 19$

$G(x) = \overset{a_0}{2} + \overset{a_1}{1}x + \overset{a_2}{0}x^2 + \overset{a_3}{1}x^3 + 6x^4 + 19x^5 + \ldots$

Ques 35 $a_n = 3a_{n-1}, n>=1$

By iterative and recursive. , $a_0 = 1$

↓

$a_1 = 3a_0 = 3(1) = 3$

$a_2 = 3a_1 = 3(3) = 3^2$

$a_3 = 3a_2 = 3(3^2) = 3^3$

$\vdots$

$a_r = 3^r.$

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Recursive

$a_r = 3a_{r-1}$

& $a_{r-1} = 3a_{r-2}$; $a_{r-1} = 3(3a_{r-3})$

$a_{r-2} = 3a_{r-3}$; $a_r = 3(3(3(a_{r-3})))$

$a_r = 3^3 a_{r-3}$

$a_k = 3^k(a_{r-k})$

when

$k = r$

$a_r = 3^r a_0$

$\boxed{a_r = 3^r(1)}$

Ques 36 $a_n = a_{n-1} + 3$ $n = 1$, $a_0 = 1$

By iterative

$a_1 = a_0 + 3$

$1 + 3 = 4$ $= 1 + 3(1)$

$a_2 = a_1 + 3 = 4 + 3 = 7$ $1 + 3(2)$

$a_3 = a_2 + 3 = 7 + 3 = 10$ $1 + 3(3)$

$a_4 = a_3 + 3 = 10 + 3 = 13$ $(1 + 3(4)$

$\Rightarrow$

$\boxed{a_r = 1 + 3(r)}$

By Recursive —

$a_n = a_{n-1} + 3$

$a_{n-1} = a_{n-2} + 3$

$a_{n-2} = a_{n-3} + 3$

$a_{n-2} =$

$a_{n-1} = (a_{n-3} + 3) + 3$

$a_n = (a_{n-1}) + 3$

$a_n = [(a_{n-3} + 3) + 3] + 3$

$a_n = \ldots + 3(\ldots)$

$$a_n = a_{n-k} + 3(k)$$

when ~~let~~ $k = n$

$$a_n = a_0 + 3(n)$$

$$\boxed{a_n = 1 + 3n}$$

ans 6

$f(x) = 2x+1 : (R \to R)$

(i)

One-one

let $x_1, x_2 \in R$

such that

$f(x_1) = f(x_2)$

$2x_1 + \cancel{1} = 2x_2 + \cancel{1}$

$\Rightarrow x_1 = x_2$

$\therefore$ fnx is one.

onto

let $(2x+1) = k$

$x = \left(\frac{k-1}{2}\right)$

$f\left(\frac{k-1}{2}\right) = f(x) = \left(\frac{k-1}{\cancel{2}}\right)\cancel{2} + 1$

$= k$

$\therefore$ fnx is onto

(ii) $f(x) = x^2+1$

one-one

let $x_1, x_2 \in R$

such that

$f(x_1) = f(x_2)$

$x_1^2+1 = x_2^2+1$

$\Rightarrow x_1^2 = x_2^2$

$\not\Rightarrow x_1 = x_2$

$\therefore$ fn is not one-one

onto

let $f(x) = k$.

$x^2+1 = k$

$x = \sqrt{k-1}$

$f(\sqrt{k-1}) = f(x) = (\sqrt{k-1})^2 + 1$

$k - \cancel{1} + \cancel{1}$

$= k$

fnx is onto

(iii) $f(x) = x^3$

let $x_1, x_2 \in R$

such that

$f(x_1) = f(x_2)$

$x_1^3 = x_2^3$

$\Rightarrow x_1 = x_2$

$\Rightarrow$ fn is one-one

onto

let $f(x) = k$

$x^3 = k$

$x = (k)^{1/3}$

$f(x) = f(k^{1/3}) = (k^{1/3})^3$

$= k$

$\therefore$ fn is onto

42 $[(a,b),(c,d)] \in R$

Reflexive

let $(a,a) \in R$.

$a+a = a+a$

$\Rightarrow [(a,a),(a,a)] \in R$

$\therefore$ Reflexive

if $[(a,b)(c,d)] \in R$

$\Rightarrow a+b = c+d$.

$\Rightarrow c+d = a+b$

$\Rightarrow [(c,d),(a,b)] \in R$

$\therefore$ symmetric

transitive

~~let~~ if $[(a,b)(c,d)] \in R$,

& $[(c,d)(e,f)] \in R$

then $[(a,b),(e,f)]$

$\Rightarrow$ a

$\Rightarrow a+b = c+d$.

$\Rightarrow c+d = e+f$

$\Rightarrow a+b = e+f$.

$\Rightarrow [(a,b),(e,f)] \in R$

Ques 43 $[(a,b),(c,d)]$ ~~$\in R$~~ if ~~ab~~ $- ad = bc$.

$\Rightarrow$ if $(a,a) \in R$ then $[(a,a),(a,a)] \in R$

~~$\Rightarrow$~~ $a \cdot a = a \cdot a$.

$\Rightarrow [(a,a),(a,a)] \in R$

$\therefore$ ~~trans~~ Reflexive

if $[(a,b),(c,d)] \in R$ then $[(c,d)(a,b)] \in R$

$(a \cdot d) = b \cdot c$

$\Rightarrow c \cdot b = d \cdot a$

$\Rightarrow [(c,d),(a,b)] \in R$

$\therefore$ Symmetric

if $[(a,b)(c,d)] \in R$. & $[(c,d),(e,f)] \in R$ then $[(a,b),(e,f)]$

if $af = be$

$\Rightarrow ad = bc$

$\frac{a}{b} = \frac{c}{d}$ — ①

$\Rightarrow cf = de$

$\frac{c}{d} = \frac{e}{f}$ — ①

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from ① and ②

$$\frac{a}{b} = \frac{e}{f}$$

$$af = be$$

$\Rightarrow$ $[(a,b),(e,f)] \in R.$

transitive. ✓

ans 4D $\{1,2,3,4\}$ (1,2) (1,4) (3,3) (4,1)

(i)

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 0 & 1 \\ 2 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 1 & 0 \\ 4 & 1 & 0 & 0 & 0 \end{array}$$

Reflexive closure: (1,1) (2,2) (3,3) (4,4)

(i) R = {(1,2) (1,4) (3,3) (4,1) (1,1) (2,2) (4,4)}.

(ii) symmetric closure.
= {(1,2) (1,4) (3,3) (4,1)}
= {(2,1) (4,1) (3,3) (1,4)}

transitive closure

(iii) R = (1,2) (1,4) (3,3) (4,1)

R^2 = (1,4) (3,3) (4,2) (4,4)

R^3 = (1,2) (1,4) (3,3) (4,4)

~~$R \cup R^2 \cup R^3$~~

R^4 = (1,4) (3,3) (4,2) (4,4)

$R^\infty = R \cup R^2 \cup R^3 \cup R^4$

R^∞ = (1,2) (1,4) (3,3) (4,1) (4,2) (4,4)

$1 \to 2$	$1 \to 2$
$1 \to 4$	$1 \to 4$
$3 \to 3$	$3 \to 3$
$4 \to 1$	$4 \to 1$

$1 \to 2$	$1 \to 4$
$1 \to 4$	$3 \to 3$
$3 \to 3$	$4 \to 2$
$4 \to 1$	$4 \to 4$

$1 \to 2$	$1 \to 2$
$1 \to 4$	$1 \to 4$
$3 \to 3$	$3 \to 3$

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X)

{ (1,1)(2,2)(3,3)(4,4) (1,2)(1,4)(3,3)

(a) Reflexive and transitive

{ (1,2)(1,4)(3,3)(4,1)(1,1)(2,2)(4,4) }

(b) symmetric and transitive

{ (1,2)(1,4)(3,3)(4,1) (2,1)(1,1) }

(c) Reflexive + transitive + symmetric.

{ (1,1)(2,2)(3,3)(4,4) (1,2) (1,4) (4,1) (2,1) }

ans 41

$= \{ (1,2)\ (2,1)\ (2,3)\ (3,4)\ (4,1) \}$

(a)

(i)

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 \\ 3 & 0 & 1 & 0 & 1 \\ 4 & 1 & 0 & 0 & 0 \end{array}$$

Reflexive

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 \\ 3 & 0 & 1 & 1 & 1 \\ 4 & 1 & 0 & 0 & 1 \end{array}$$

→ Reflexive closure

$R = \{ (1,2)(2,1)(2,3)(3,4)(4,1)$
$(1,1)(2,2)(3,3)(4,4) \}$

Symmetric closure

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 0 & 1 \\ 2 & 1 & 0 & 1 & 0 \\ 3 & 0 & 1 & 0 & 1 \\ 4 & 1 & 0 & 1 & 0 \end{array}$$

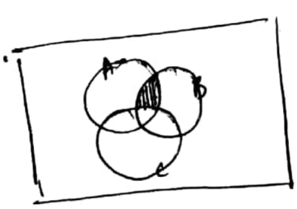
$R = \{ (1,2)\ (2,1)\ (2,3)(3,2)\ (3,4)(4,3)$
$(4,1)(1,4) \}$

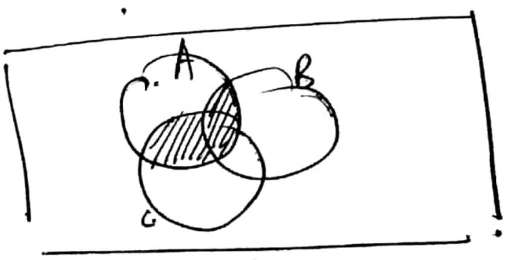
transitive Closure

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 1 & 1 & 1 \\ 2 & 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 0 & 1 \\ 4 & 1 & 0 & 0 & 0 \end{array}$$

$\{ (1,2)\ (2,1)\ (1,1)\ (2,3)\ (3,4)$
$(1,4)(2,4)(1,3)\ (4,1)\ (3,1) \}$

ans 50 (i) $A \cap (B - C)$ venn diagram (ii) $(A \cap B) \cup (A \cap C)$





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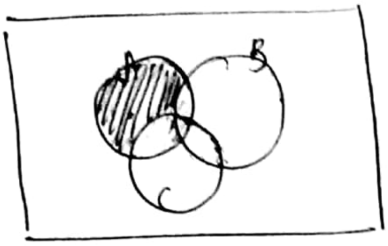
(iii) $(A \cap B') \cup (A \cap C') = \cancel{A \cap (B' \cup C')}$

$= \cancel{A \cap (B \cap C)'}$



ans 51

(i) $A - B = A \cap B'$

$A - B = A - (A \cap B)$

$(A \cap U) - (A \cap B)$

$A \cap (U - B)$

$= A \cap (B)'$

(ii) $(A \cap B) \cup (A \cap B')$

$A \cup (B \cap B')$

$A \cup (\phi)$

$A \cup \phi = A.$

$A \cup \phi = A$

$A \cap \phi =$

ans 52

$(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$

let $\cancel{\{x\}} \in (A \cap B) \times (C \cap D)$

$(x, y) \in (A \cap B) \times (C \cap D)$

$\Rightarrow x \in (A \cap B)$ and $y \in (C \cap D)$

$\Rightarrow (x \in A$ and $x \in B)$ and $(y \in C$ and $y \in D)$

$\Rightarrow (x \in A$ and $y \in C)$ and $(x \in B$ and $y \in D)$

$\Rightarrow x \cancel{\in A} (A \times C)$ and $y \in \cancel{y} (B \times D)$

$\Rightarrow (x, y) \in (A \times C) \cap (B \times D)$

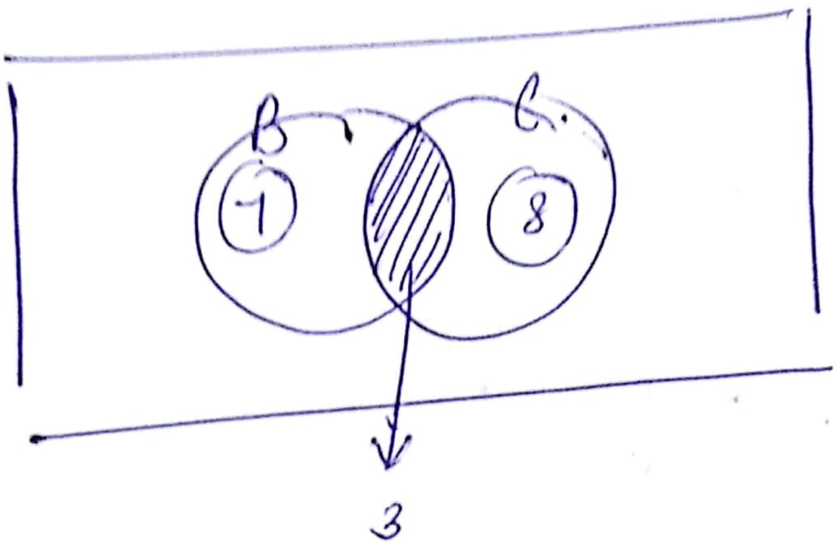
$\Rightarrow (A \cap B) \times (C \cap D \subseteq (A \times C) \cap (B \times D).$

IIIly $(A \times C) \cap (B \times D) \subseteq (A \times B) \cap (C \times D)$

$\therefore$ equal

(57)

7 = BasketBall
8 = Cricket
3 = B and C.



$(A \cup B) = (7+8) - 3$

$(A \cup B) = 12$

(55) 63 = Total Student.

33 - Ps   10 = P+C

25 - Chemy   9 = B+C

26 = Botany   8 = P+B

$(A \cup B \cup C) = A + B + C - (A \cap B) - (B \cap C) - (C \cap A) + A \cap B \cap C$

$63 = 33 + 25 + 26 + 10 + -(10 + 9 + 8) + (x)$

$\boxed{6 = x}$

ans 56

19 Grade   10 learners

G = 41

H = 36

M = 30

M and H = 16

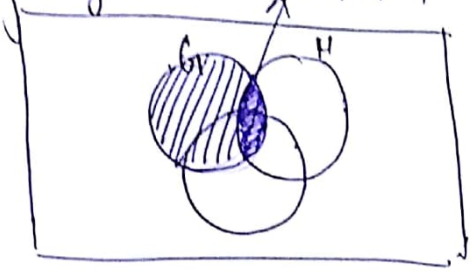
G and H = 6

Math only = 8

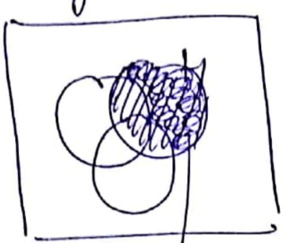
H only = 16

Geography

G and H



History



H only = 16

Maths

M∩H = 16

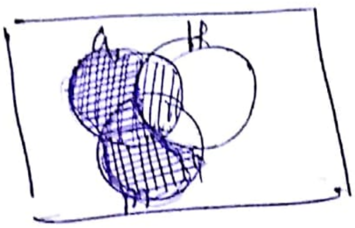
30

Maths only = 8

(ii)

only G ⇒ [G - ~~G∩H~~ - ~~(M∩G) + G∩H∩M∩H~~]

(i)



(only M) + [G - (G ∩ H)]

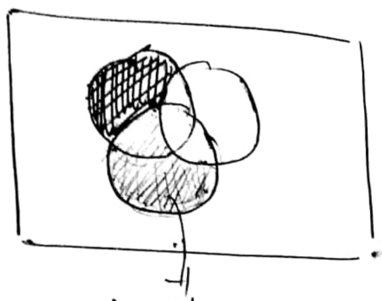
= 8 + [41 - 6]

(35) + 8

Math and geog only not History = 43.

~~11~~

(ii)



~~(43)~~ 14

~~maths and~~ -
geography

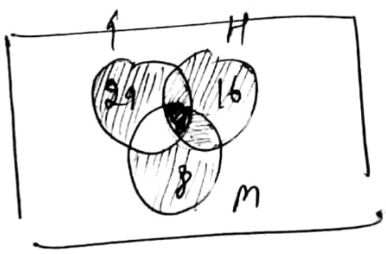
$(M) - (M \cap H) = 30 - 16$
$= 14$

maths and geog not history - $[M - M \cap H]$

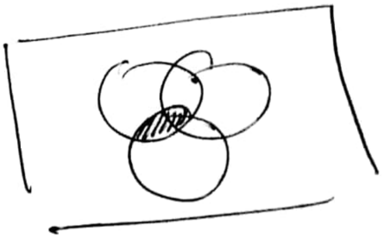
(14)

only Geo = 43 -
= 29

(iii) ~~A~~ $G \cap H \cap M$



$(M \cap G)$



$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| -$

Ques 57 $\sqrt{2}$ is irrational

let $\sqrt{2}$ = rational no

$\sqrt{2} = \frac{p}{q}$

$\sqrt{2}q = p$

squaring both side

$2q^2 = p^2$

$p^2 = 2(q^2)$

p^2 = even

$\therefore p^2 = 2k.$

squaring both side.

$\cancel{\frac{p^2}{q^2}} = \quad p^2 = 4k^2.$

$2q^2 = 4k^2.$

$q^2 = 2k^2$

$\Rightarrow q^2$ = even

$\Rightarrow q$ = even

As p and q both are even $\therefore$ they have 2 as common factor. $\therefore$ ~~th~~ $\therefore$ they cannot be in p/q

$\Rightarrow \sqrt{2}$ is irrational no.

(58) n = odd then n^2 = odd.

Let ~~n~~. p: n is an odd integer.

q: n^2 is odd integer.

Let n be even integer

$\therefore n = 2k.$

squaring both side

$n^2 = (2k)^2$

$n^2 = 4k^2$

$n^2 = 2(2k^2)$

b

As n^2 is also even

$\Rightarrow \sim p \rightarrow \sim q$

$\Rightarrow p \rightarrow q$ (by contrapositive)

3. n = integer.
$3n+2$ = even then is even

let p = n is integer and $3n+2$ = even.
q = n is even

(a) Contraposition
↳ It state that if $\sim p \rightarrow \sim q$ then $p \rightarrow q$.

$\therefore$ let $3n+2$ = odd

$3n+2 = 2k+1$

$3n+\cancel{1} = 2k$

$3n = 2k-1$

$n = \frac{\cancel{3}2k-1}{3}$

$\Rightarrow$ n is odd.

$\Rightarrow$ if $3n+2$ = odd then n = odd

$\Rightarrow$ if $3n+2$ = even then n = even

(b) Contradiction -

let ~~3n+2~~ let n is odd

$p \rightarrow \sim q$

let $3n+2$ = even

$3n+2 = 2k$

$n = \frac{2(k-1)}{3}$

$n = 2(m)$

$\boxed{n = 2(m)}$

$\Rightarrow$ n is even

$\therefore$ Our assumption is wrong a

$\Rightarrow$ $3n+2$ = even the n = even.

(62) n = integer if n^3+5 = odd then n is even.

(a) Contraposition

if $\sim p \to \sim q \Rightarrow p \to q$.

let n^3+5 = ~~od~~ even.

then n is odd

$\Rightarrow n^3+5 = 2k$

$\Rightarrow n^3 = 2k-5$

~~$\Rightarrow n$ = odd i~~

$\Rightarrow n^3$ = odd.

$\Rightarrow n$ = odd

$\Rightarrow \sim p \to \sim q$

$\Rightarrow p \to q$

$\therefore n^3+5$ = odd, n = even.

(b) Contradiction

(if $p \to \sim q$)

let n^3+5 = odd and n is ~~even~~ odd.

$\Rightarrow n^3+5 = 2k+1$

$\Rightarrow n^3+5 = 2k+1-5$

$\Rightarrow n^3 = 2k-4$

$\Rightarrow n^3 = 2(k-2)$

$\Rightarrow n^3$ = even

$\Rightarrow n$ = even

$\Rightarrow \therefore$ ~~our assumption was~~ w

$\Rightarrow n^3+5$ = odd then n = even.

(61)

$n =$ even , $n^2 =$ even

let $n =$ even

$\Rightarrow$ ~~squaring both side~~

$n = 2k$

squaring both side

$n^2 = 4k^2$

$n^2 = 2(2k^2)$

$\Rightarrow n^2 =$ even

Hence Proved.

(59)

$n, m =$ odd $n \cdot m =$ odd

let $n = 2k+1$ $m = (2l+1)$

$n \cdot m = (2k+1)(2l+1)$

$n \cdot m = 2 \cdot 2kl + 2k + 2l + 1$

$n \cdot m = 2(2kl + k + l) + 1$

$n \cdot m = 2c + 1$

$\Rightarrow n \cdot m =$ odd

Hence Proved.

(48)

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

let $x \in A \cap (B \cup C)$

$\Rightarrow x \in A$ and $x \in (B \cup C)$

$\Rightarrow x \in A$ and $(x \in B$ or $x \in C)$

$\Rightarrow (x \in A$ and $x \in B)$ or $(x \in A$ or $x \in C)$

$\Rightarrow x \in (A \cap B)$ or $x \in (A \cap C)$

$\Rightarrow x \in (A \cap B) \cup (A \cap C)$

$\Rightarrow A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$ — (1)

IIIly $(A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$ — (2)

from (1) and (2)

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$