

Assignement 2

Discrete Mathematical Structures

Q1. When doing a composite transformation, does the order in which you perform the transformations matter? Show with with suitable example.

Q2. What is vector space? Discuss the properties of vector space. Let V be the set of vectors $[2x - 3y, x + 2y, -y, 4x]$ with $x, y \in \mathbb{R}^2$. Addition and scalar multiplication are defined in the same way as on vectors. Prove that V is a vector space.

Q3. Determine whether the function $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is linear transformation. If yes, prove it; if not, provide a counterexample to one of the properties:

$$T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x + y \\ x - y \end{bmatrix}$$

Q4. Sketch the image of the square with vertices $(0, 0)$, $(3, 0)$, $(3, 3)$, and $(0, 3)$ under

- (a) A reflection about the x-axis
- (b) A reflection about the y-axis
- (c) A compression of factor $k=1/4$ in the y-direction
- (d) An expansion of factor $k=2$ in the x-direction
- (e) A shear of factor $k=3$ in the x-direction
- (f) A shear of factor $k=2$ in the y-direction

Q5. Use the decomposition theorem to find the chromatic polynomial of a complete graph with four nodes.

Q6. Explain Dijkstra's shortest path algorithm with suitable example.

Q7. Explain Ford-Fulkerson algorithm to find the maximum flow with example.

Q8. Write short note on:

- a. Euler's formula for planar graph
- b. Kuratowski graphs
- c. Chromatic numbers
- d. Linear Transformation