Program: **B.Tech. (CSE)**
Subject name: **Discrete Mathematical Structures**
Number of credits: **3**

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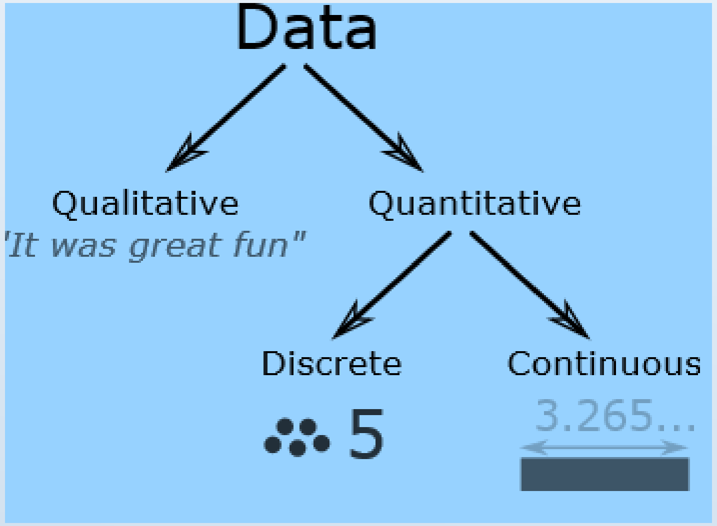
What is Data?

Data is a collection of facts, such as numbers, words, measurements, observations or even just descriptions of things.

Qualitative vs Quantitative

Data can be qualitative or quantitative.

- **Qualitative data** is descriptive information (it *describes* something)
- **Quantitative data**, is numerical information (numbers).



Discrete and Continuous Data

Data can be Descriptive (like "high" or "fast") or Numerical (numbers).
And Numerical Data can be **Discrete** or **Continuous**:

Discrete data is counted,
Continuous data is measured

Discrete Data

Discrete Data can only take certain values.
Example: the number of students in a class (you can't have half a student).

Continuous Data

Continuous Data can take any value (within a range)
Examples:
A person's height: could be any value (within the range of human heights), not just certain fixed heights,
Time in a race: you could even measure it to fractions of a second,
A dog's weight,
The length of a leaf etc.

What is Discrete Mathematics?

Discrete mathematics is the study of mathematical **structures** that are fundamentally **discrete** rather than continuous. ... **Discrete mathematics** therefore excludes topics in "continuous **mathematics**" such as calculus and analysis. **Discrete** objects can often be enumerated by integers.

Why do we study Discrete Mathematics?

The mathematics of modern computer science is built almost entirely on discrete math, in particular. This means that in order to learn the **fundamental** algorithms used by computer programmers, students will need a solid background in this subjects.

COURSE OUTLINE:

S. No.	Units	Contents
1.	Unit – I	Set Theory, Relation and Functions & theorem Proving Techniques
2	Unit – II	Vector algebra and Matrices Transformation
3	Unit – III	Trees and Basics of Graph Theory
4	Unit – IV	Graph Theory and its Applications

Text Book:

1. Discrete Mathematics and its applications by Kenneth Rosen, McGraw Hill Publications.
2. Discrete Mathematics, by R. K. Bisht & H.S.Dhami Oxford University Press.

Reference books:

1. Discrete Mathematics and its Applications, Trembley and Manohar TMH publications.
2. Elementary Linear Algebra, 9^{th} Edition by Howard Anton & Chris Rorres, published by wiley Publication.
3. Discrete Mathematics, Schaum's outline, by Seymour Lipschutz, McGraw Hill Publication.
4. Graph Theory with its Applications, Narsingh Deo, PHI

SET THEORY

Set Theory - Definitions and notation

- A ***set*** is an **unordered** collection of **distinct** objects referred to as elements.

Some Example of Sets

- A set of all positive integers
- A set of all the planets in the solar system
- A set of all the states in India
- A set of all the lowercase letters of the alphabet

Notation. Usually we denote sets with **upper-case letters**, elements with **lower-case letters**. The following notation is used to show set membership

- $x \in \mathrm{A}$ means that x is a member of the set A
- $x \notin \mathrm{A}$ means that x is not a member of the set A

Representation of a Set

1 – Explicitly: listing the elements of a set (Roster method or Tabular method)

{1, 2, 3} is the set containing "1" and "2" and "3." list the members between braces.

{1, 1, 2, 3, 3} = {1, 2, 3} since repetition is irrelevant.

{1, 2, 3} = {3, 2, 1} since sets are unordered.

∅ = {} is the empty set, or the set containing no elements.

2 – Implicitly: by using a set builder notations, stating the property or properties of the elements of the set.

$S = \{m \mid 2 \le m \le 100, m \text{ is integer}\}$

S is
the set of
all m
such that
m is between 2 and 100
and
m is integer.

: and | are read "such that" or "where"

- **Important Sets**
- **N** = {1,2,3,…}, the set of **natural numbers**, non negative integers.
- **Z** = { …, -2, -1, 0, 1, 2,3, …), the set of **integers**
- $\mathbf{Z}^{+}$ = {1,2,3,…} set of **positive integers**
- **R**, the set of real numbers

Cardinality of a Set

Cardinality of a set S, denoted by |S|
It is the number of elements of the set. The number is also referred as the cardinal number. If a set has an infinite number of elements, its cardinality is ∞.

Example − $|\{1,4,3,5\}|=4$, $|\{1,2,3,4,5,\ldots\}|=\infty$

Finite Set
A set which contains a definite number of elements is called a finite set.
Example − $S=\{x \mid x \in N \text{ and } 70>x>50\}$

Infinite Set
A set which contains infinite number of elements is called an infinite set.
Example − $S=\{x \mid x \in N \text{ and } x>10\}$

Subset

A set X is a subset of set Y (Written as $X \subseteq Y$) if every element of X is an element of set Y.

Symbolically,

$$A \subseteq B \Leftrightarrow \forall x, \text{ if } x \in A \text{ then } x \in B.$$

$$A \not\subseteq B \Leftrightarrow \exists x \text{ such that } x \in A \text{ and } x \notin B.$$

Example 1 − Let, X={1,2,3,4,5,6}

and Y={1,2}. Here set Y is a subset of set X as all the elements of set Y is in set X. Hence, we can write $Y \subseteq X$.

Proper Subset

The term "proper subset" can be defined as "subset of but not equal to". A Set X is a proper subset of set Y (Written as $X \subset Y$) if every element of X is an element of set Y and $|X| < |Y|$.

Example − Let, X={1,2,3,4,5,6} and Y={1,2}. Here set $Y \subset X$ since all elements in Y are contained in X too and X has at least one element is more than set Y.

Universal Set

It is a collection of all elements in a particular context or application. All the sets in that context or application are essentially subsets of this universal set. Universal sets are represented as **U**.

Example − We may define U as the set of all animals on earth. In this case, set of all mammals is a subset of U, set of all fishes is a subset of U, set of all insects is a subset of U, and so on.

Empty Set or Null Set

An empty set contains no elements. It is denoted by $\emptyset$ or **{ }**

As the number of elements in an empty set is finite, empty set is a finite set. The cardinality of empt

set or null set is zero.

Example − $S=\{x \mid x \in N \text{ and } 7<x<8\}=\emptyset$

For all sets A,

1. $\emptyset \subseteq A$
2. $A \cup \emptyset = A$
3. $A \cap \emptyset = \emptyset$
4. $A \cap A^c = \emptyset$

Singleton Set or Unit Set

Singleton set or unit set contains only one element. A singleton set is denoted by {s}.
Example − $S=\{x \mid x \in N, 7<x<9\} = \{8\}$

Equal Set

If two sets contain the same elements they are said to be equal.

Symbolically,

$A=B \Leftrightarrow A \subseteq B$ and $B \subseteq A$

Example − If $A=\{1,2,6\}$

and $B=\{6,1,2\}$, they are equal as every element of set A is an element of set B and every element of set B is an element of set A.

Combining Sets – Set Union

- $$A \cup B$$

- "*A* union *B*" is the set of all elements that are in *A*, or *B*, or both.

- This is similar to the logical "or" operator.

Combining Sets – Set Intersection

- $$A \cap B$$

- "*A* intersect *B*" is the set of all elements that are in *both* *A* and *B*.

- This is similar to the logical "and"

Set Complement

$$\overline{A}$$

- "*A* complement," or "not *A*" is the set of all elements not in *A*.
- The complement operator is similar to the logical not.

Set Difference

- $$A - B$$
- The set difference "A minus B" is the set of elements that are in A, with those that are in B subtracted out. Another way of putting it is, it is the set of elements that are in A, *and* not in B.

EXAMPLES

$$\Omega = \{1, 2, 3, 4, 5, 6\}$$

$$A = \{1, 2, 3\} \qquad B = \{3, 4, 5, 6\}$$

$$A \cap B = \{3\} \qquad A \cup B = \{1, 2, 3, 4, 5, 6\}$$

$$B - A = \{4, 5, 6\} \qquad \bar{B} = \{1, 2\}$$

SYMMETRIC DIFFERENCE

The symmetric difference of two sets S and T is the set of objects that are in one and only one of the sets. The symmetric difference is written S∆T .

$$S \Delta T = \{(S - T) \cup (T - S)\}$$

MUTUALLY EXCLUSIVE AND EXHAUSTIVE SETS

- *Definition.* We say that a group of sets is ***mutually exhaustive*** of another set if their union is equal to that set. For example, if $A \cup B = C$ we say that A and B are exhaustive with respect to C.

- *Definition.* We say that two sets A and B are ***mutually exclusive*** **(Disjoint Sets)** if $A \cap B = \varnothing$, that is, the sets have no elements in common.

➢ Sets $A_1, A_2, \ldots, A_n$ are called **mutually disjoint** *iff* for all i,j = 1,2,…, n
$A_i \cap A_j = \varnothing$ whenever $i \neq j$.

➢ *Examples:*

1) A={1,2} and B={3,4} are disjoint.
2) The sets of even and odd integers are disjoint.
3) A={1,4}, B={2,5}, C={3} are mutually disjoint.
4) A–B, B–A and A∩B are mutually disjoint.

Set Partitions

Definition: A collection of nonempty sets $\{A_1, A_2, \ldots, A_n\}$ is a **partition** of a set A *iff*

1. $A = A_1 \cup A_2 \cup \ldots \cup A_n$
2. $A_1, A_2, \ldots, A_n$ are mutually disjoint.

Example: $\{\mathbf{Z}^+, \mathbf{Z}^-, \{0\}\}$ is a partition of $\mathbf{Z}$.

POWER SETS

- **Definition:** Given a set A,

  the **power set** of A, denoted $\mathcal{P}$**(A)** , is the set of all subsets of A.
- *Example:* $\mathcal{P}(\{a,b\}) = \{\varnothing, \{a\}, \{b\}, \{a,b\}\}$.

- **Properties:**

  1) *If* $A \subseteq B$ *then* $\mathcal{P}(A) \subseteq \mathcal{P}(B)$.

  2) *If* a set A has n elements

     then $\mathcal{P}(A)$ has **2^n elements**.

CARTESIAN PRODUCTS

- Given two sets, A and B, we define the ***Cartesian Product***, $A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$.
- The element (a, b) is called **an *ordered pair***, since (a, b) and (b, a) are distinct if $a \neq b$.
- If $A = \{1, 2, 3\}$ and $B = \{8, 9\}$, then:

  $A \times B = \{(1, 8), (1, 9), (2, 8), (2, 9), (3, 8), (3, 9)\}$

  $B \times A = \{(8, 1), (8, 2), (8, 3), (9, 1), (9, 2), (9, 3)\}$

SOME TEST QUESTIONS

$$\mathbf{A} \cup \varnothing = ?$$

SOME TEST QUESTIONS

$$\mathbf{A} \cup \bar{\mathbf{A}} = ?$$

SOME TEST QUESTIONS

$$\mathbf{A} \cap \varnothing = ?$$

SOME TEST QUESTIONS

$$A - \bar{A} = ?$$

SOME TEST QUESTIONS

$$\mathbf{A} \cap \bar{\mathbf{A}} = ?$$

SOME TEST QUESTIONS

$$\mathrm{A} \cup \Omega = ?$$

SOME TEST QUESTIONS

$$\mathrm{A} \cap \Omega = ?$$

SOME TEST QUESTIONS

$$\text{If } A \subset B \text{ then}$$

$$A \cap B = ?$$

SOME TEST QUESTIONS

$$\text{If } A \subset B \text{ then}$$

$$A \cup B = ?$$

SOME TEST QUESTIONS

Let A = {x : x is a natural number and a factor of 18} and B = {x : x is a natural number and less than 6}.
Find $A \cup B$ and $A \cap B$.

SOME TEST QUESTIONS

Given three sets P, Q and R such that:
P = {x : x is a natural number between 10 and 16},
Q = {y : y is a even number between 8 and 20} and
R = {7, 9, 11, 14, 18, 20}
(i) Find the difference of two sets P and Q
(ii) Find Q - R
(iii) Find R - P
(iv) Find Q – P