Vector Subspace

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Vector spaces may be formed from subsets of other vectors spaces. These are called subspaces.

A subspace of a vector space V is a subset H of V that has three properties:

a. The zero vector of V is in H.

b. For each u and v are in H, u + v is in H. (In this case we say H is closed under vector addition.)

c. For each u in H and each scalar c, cu is in H. (In this case we say H is closed under scalar multiplication.)

Problem 1

Let $H = \left\{ \begin{bmatrix} a \\ 0 \\ b \end{bmatrix} : \ a \text{ and } b \text{ are real} \right\}$. Show that H is a subspace of $\mathbf{R}^3$.

Problem 2

- Following sets are not a subspace of the specified vector space. Give a reason why it is not a subspace.

$$S_1 = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3 \;\middle|\; x_1 \geq 0 \right\}$$

in the vector space $\mathbb{R}^3$.

Problem 3

- Following sets are not a subspace of the specified vector space.,Give a reason why it is not a subspace.

$$S_2 = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3 \;\middle|\; x_1 - 4x_2 + 5x_3 = 2 \right\}$$

in the vector space $\mathbb{R}^3$.

Problem 4

Is $V = \{(a + 2b, 2a - 3b) : a \text{ and } b \text{ are real}\}$ a subspace of R^2? Why or why not?

Problem 5

Is $H = \left\{ \begin{bmatrix} a+2b \\ a+1 \\ a \end{bmatrix} : \text{ } a \text{ and } b \text{ are real} \right\}$ a subspace of $\mathbf{R}^3$?

Why or why not?

Problem 6

Is the set H of all matrices of the form $\begin{bmatrix} 2a & b \\ 3a+b & 3b \end{bmatrix}$ a subspace of $M_{2\times 2}$? Explain.

- Where a, b are real numbers.