Network Flows

Max. Flow-Min. Cut Theorem

Minimum Cut Problem

Network: abstraction for material FLOWING through the edges.

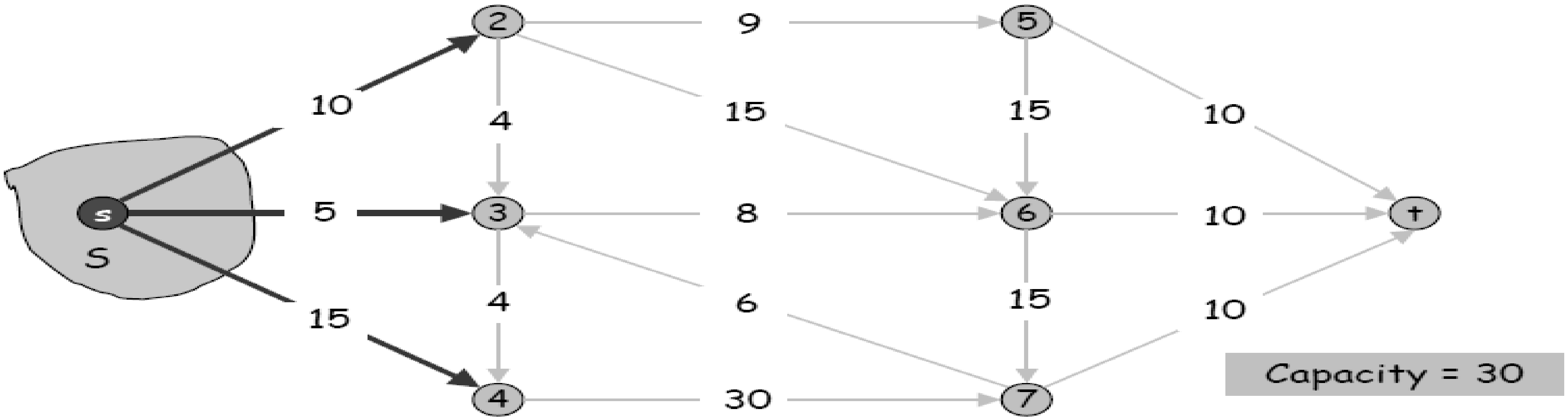
- Directed graph.
- Capacities on edges.
- Source node s, sink node t.

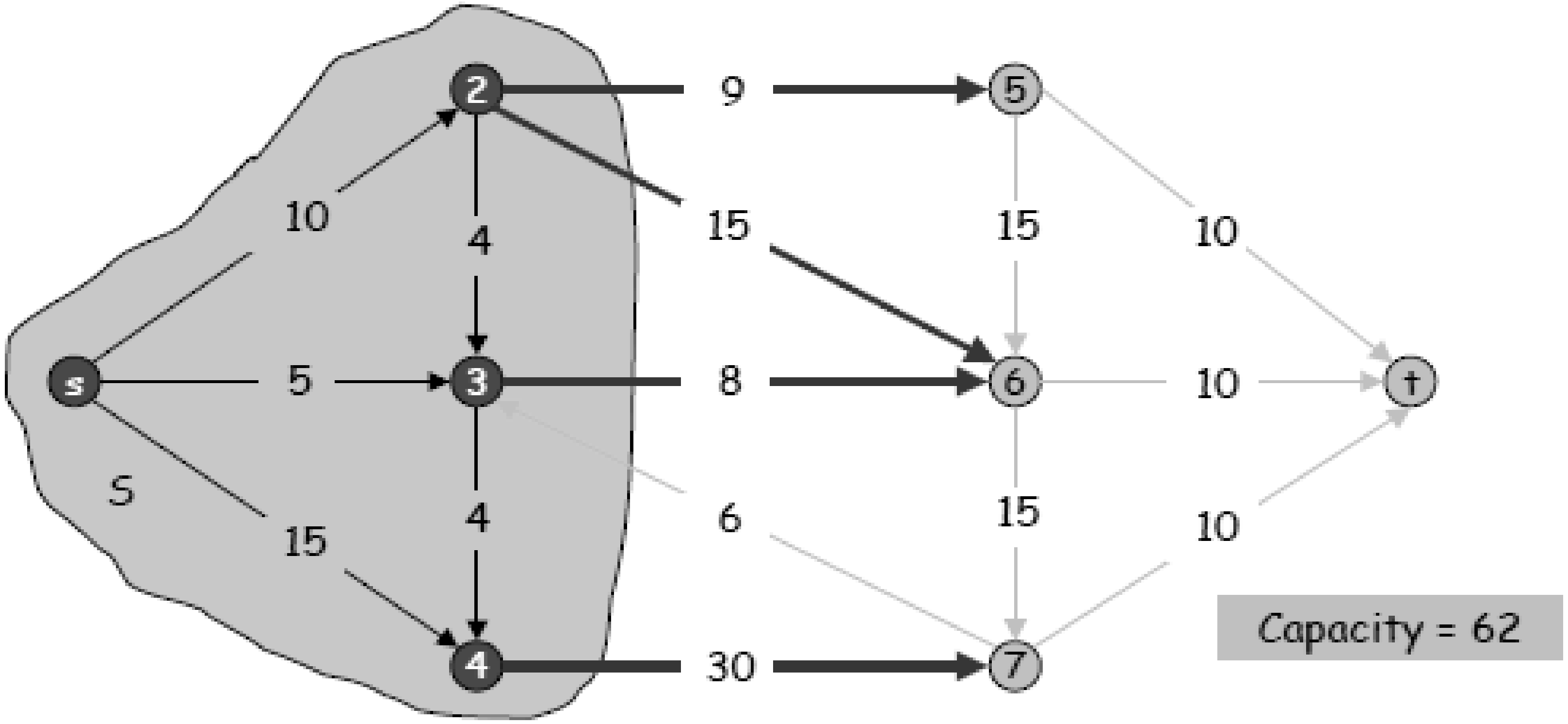
Min cut problem: Delete "best" set of edges to disconnect t from s (or identify a cut with minimum capacity).

Cut

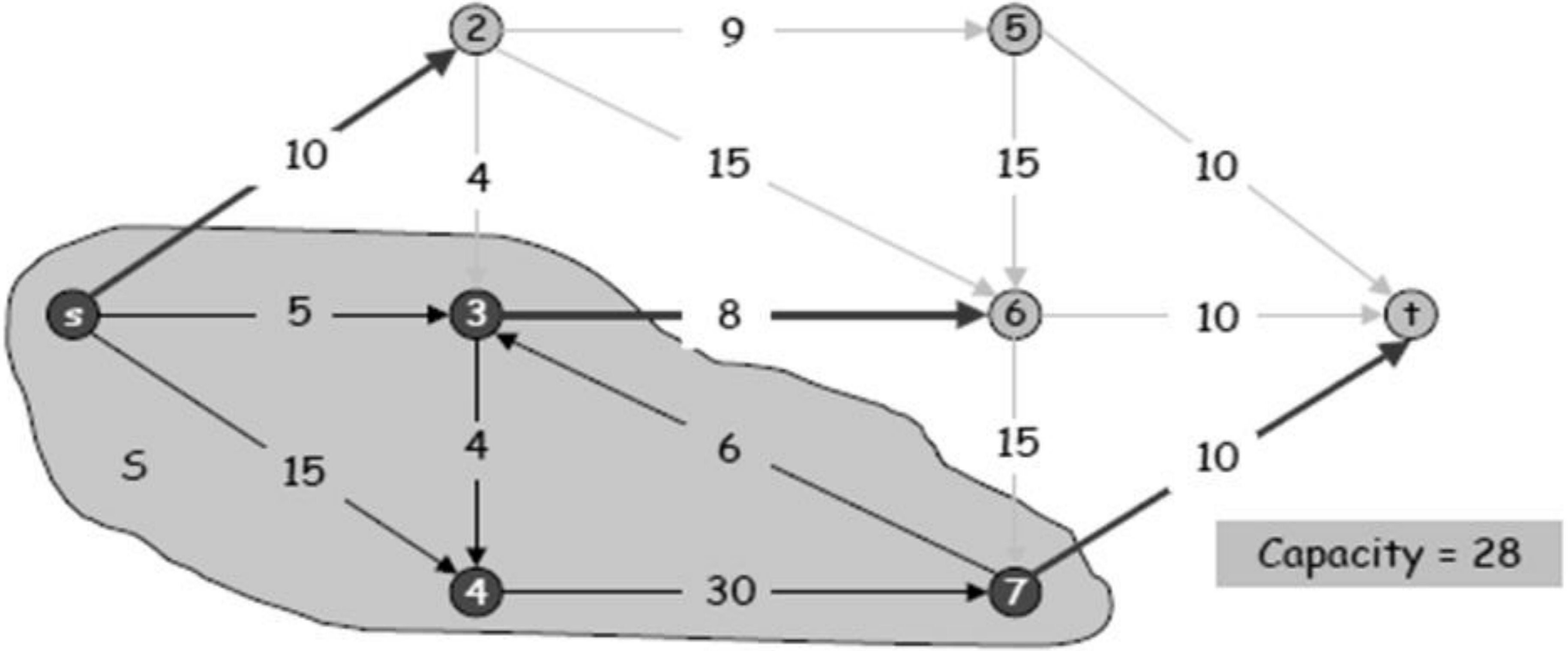
A cut is a node partition (S, T) such that s is in S and t is in T.

- capacity(S, T) = sum of weights of edges leaving S.





A cut with partitions {s,2,3,4} and {5, 6, 7, t}



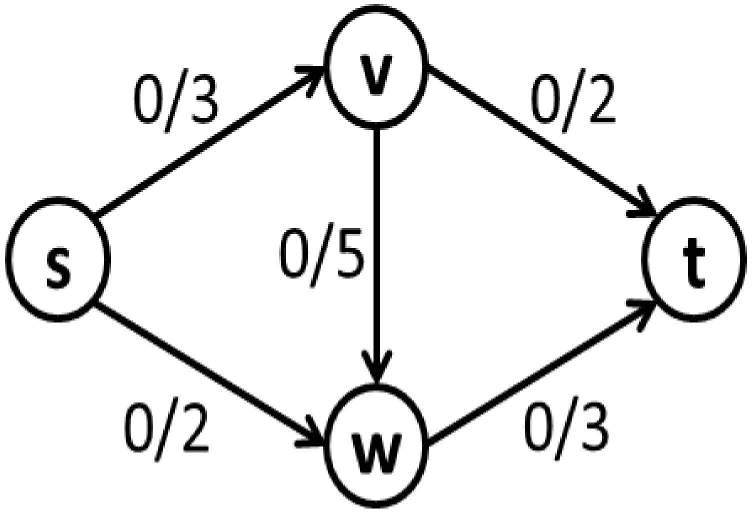
A cut with partitions {s,3,4,7} and {2, 5, 6, t}

Max-Flow Min-Cut Theorem

The value of the max flow is equal to the capacity of the min cut.

Example

Consider the following graph



Possible cuts are

S	T	Capacity
{s}	{v, w, t}	5
{s, v}	{w, t}	9
{s, w}	{v, t}	6
{s, v, w}	{t}	5

Out of four cuts the min cut has the capacity 5. So, according to Max flow- Min Cut Theorem the maximum flow which can be achieved is 5.

Exercise

Find the maximum flow in the following flow network graph using Ford-Fulkerson method. Also verify the max. flow-min. cut theorem.