Transformation Operators

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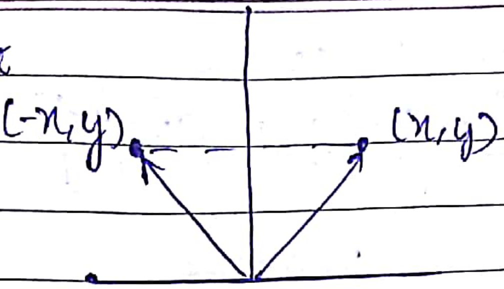
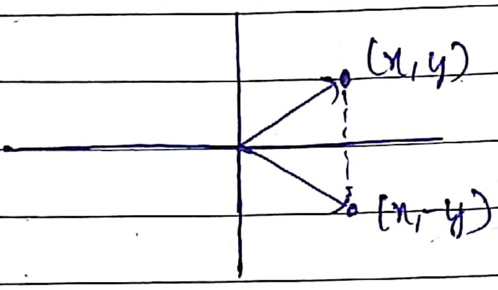
1. Reflection about y axis.



$x_1 = -x$
$x_2 = y$

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

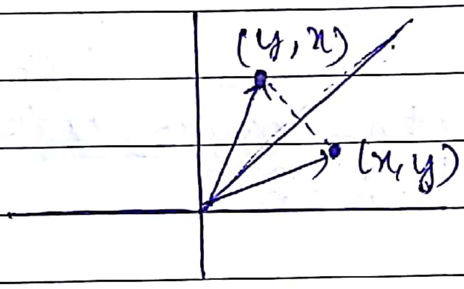
2. Reflection about x axis



$x_1 = x$
$x_2 = -y$

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

3. Reflection about line y=x



$x_1 = y$
$x_2 = x$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

4. Reflection about xy plane

$x_1 = x$
$x_2 = y$
$x_3 = -z$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

5. Reflection about yz plane.

$x_1 = -x$
$x_2 = y$
$x_3 = z$

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

6. Reflection about xz plane

$x_1 = x$
$x_2 = -y$
$x_3 = z$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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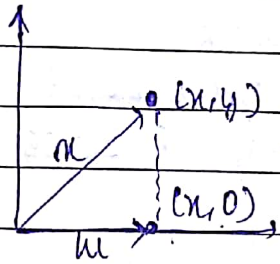
Projection Operators

1. Orthogonal projection on the x axis

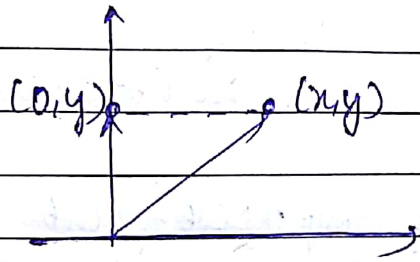


$w_1 = x$

$w_2 = 0$

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

2. Orthogonal proj^n on y axis



$w_1 = 0$

$w_2 = y$

$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

3. Orthogonal proj^n on xy plane.

$w_1 = x$

$w_2 = y$

$w_3 = 0$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

4. Orthogonal proj^n on yz plane

$w_1 = 0$

$w_2 = y$

$w_3 = z$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

5. Orthogonal proj^n on xz plane

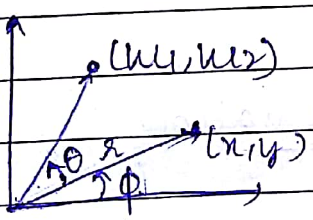
$w_1 = x$

$w_2 = 0$

$w_3 = z$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Rotation



$x = r\cos\phi \quad y = r\sin\phi$

$w_1 = r\cos(\phi+\theta). \quad w_2 = \cancel{r\cos} r\sin(\phi+\theta)$

$w_1 = r\cos\theta\cos\phi - r\sin\theta\sin\phi.$

$w_2 = r\cos\theta\sin\phi + r\cos\phi\sin\theta.$

$w_1 = x\cos\theta - \cancel{x} y\sin\theta$

$w_2 = x\sin\theta + y\cos\theta$

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}.$$

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1. Counterclockwise rotⁿ about x axis [θ].

$w_1 = x$
$w_2 = y\cos\theta - z\sin\theta$
$w_3 = y\sin\theta + z\cos\theta$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

2. Counterclockwise rotⁿ about y axis [θ].

$w_1 = x\cos\theta + z\sin\theta$
$w_2 = y$
$w_3 = -x\sin\theta + z\cos\theta$

$$\begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}.$$

3. Counterclockwise rotⁿ about z axis [θ].

$w_1 = x\cos\theta - y\sin\theta$
$w_2 = x\sin\theta + y\cos\theta$
$w_3 = z.$

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Contraction and Dilation

1. Contraction with factor k on R^2
2. Dilation with factor k on R^2

$\begin{cases} w_1 = kx \\ w_2 = ky \end{cases}$

$$\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}.$$

3. Contraction with factor k on R^3
4. Dilation with factor k on R^3

$w_1 = kx$
$w_2 = ky$
$w_3 = kz$

$$\begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix}.$$

Composition of linear transformation

$$(T_B \circ T_A)(x) = T_B(T_A(x))$$
$$\therefore T_B \circ T_A = T_{BA}$$

Composition of two rotⁿ $(T_2 \circ T_1)(x) = T_2(T_1(x))$

$T = T_3 \circ T_2 \circ T_1$.

$((M_1 M_2)(M_3 M_4))M_5$

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To show W is a subspace of V, it must be closed under addition and scalar multiplication

Case.

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