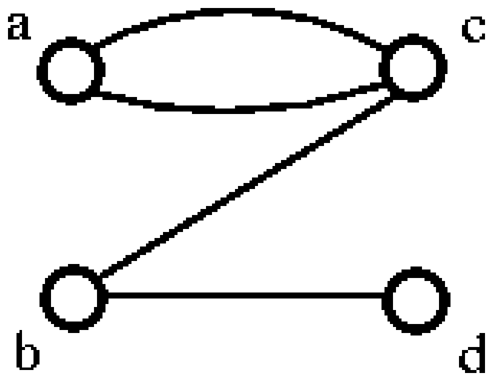
Graph Theory

Cut Edge

- **Cut Edge (Bridge)** A bridge is a single edge whose removal disconnects a graph.



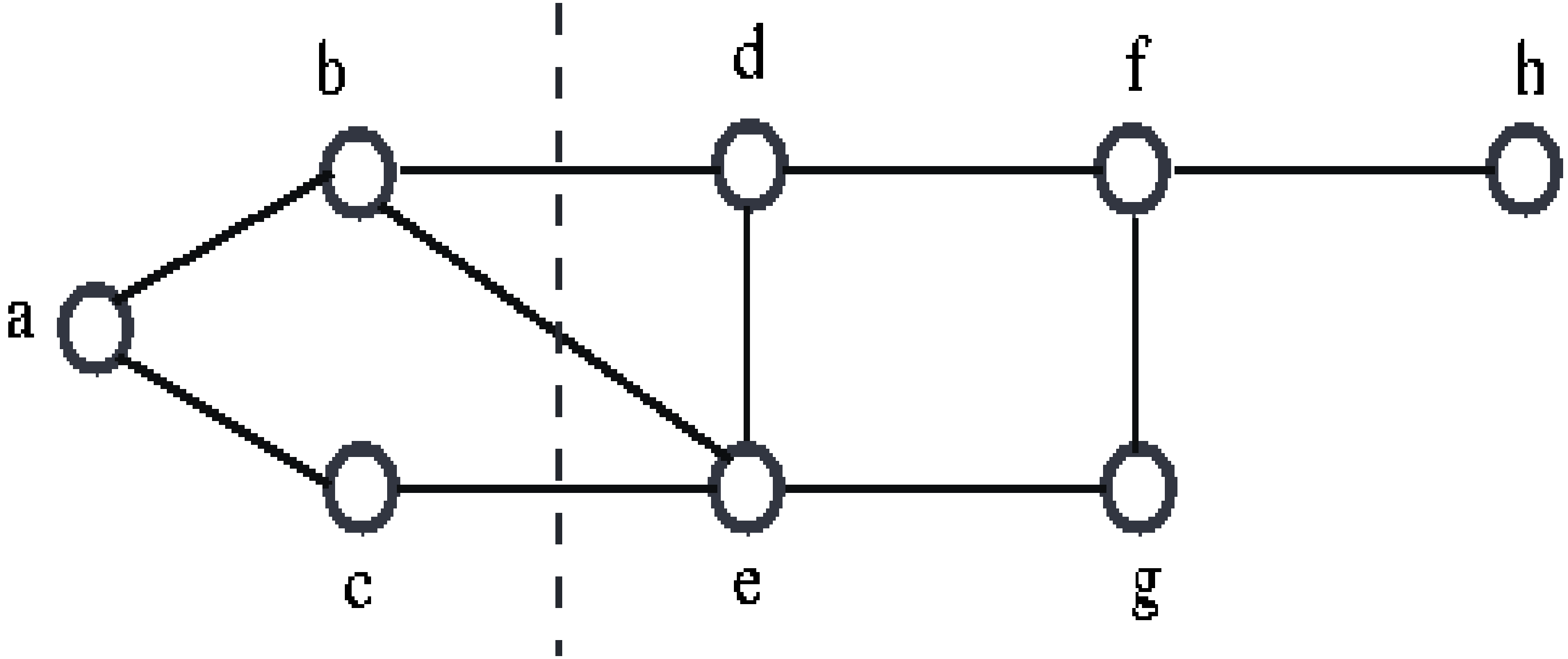
The above graph G1 can be split up into two components by removing one of the edges bc or bd. Therefore, edge bc or bd is a bridge.

Cut Set

A **cut set** of a connected graph *G* is a set S of edges with the following properties

- The removal of all edges in S disconnects *G*.
- The removal of some (but not all) of edges in S does not disconnects *G*.

As an example consider the following graph



We can disconnect *G* by removing the three edges bd, bc, and ce, but we cannot disconnect it by removing just two of these edges.

Edge Connectivity

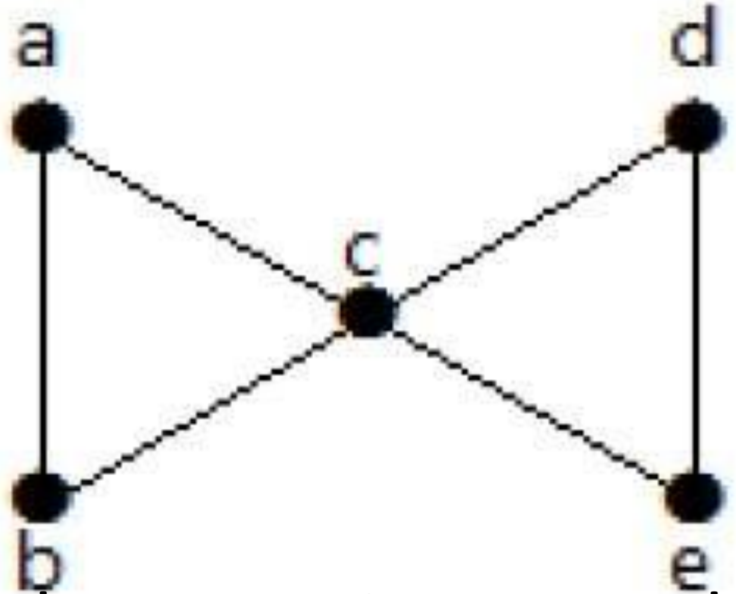
Let ‘G’ be a connected graph. The minimum number of edges whose removal makes ‘G’ disconnected is called edge connectivity of G.

Notation − λ(G)

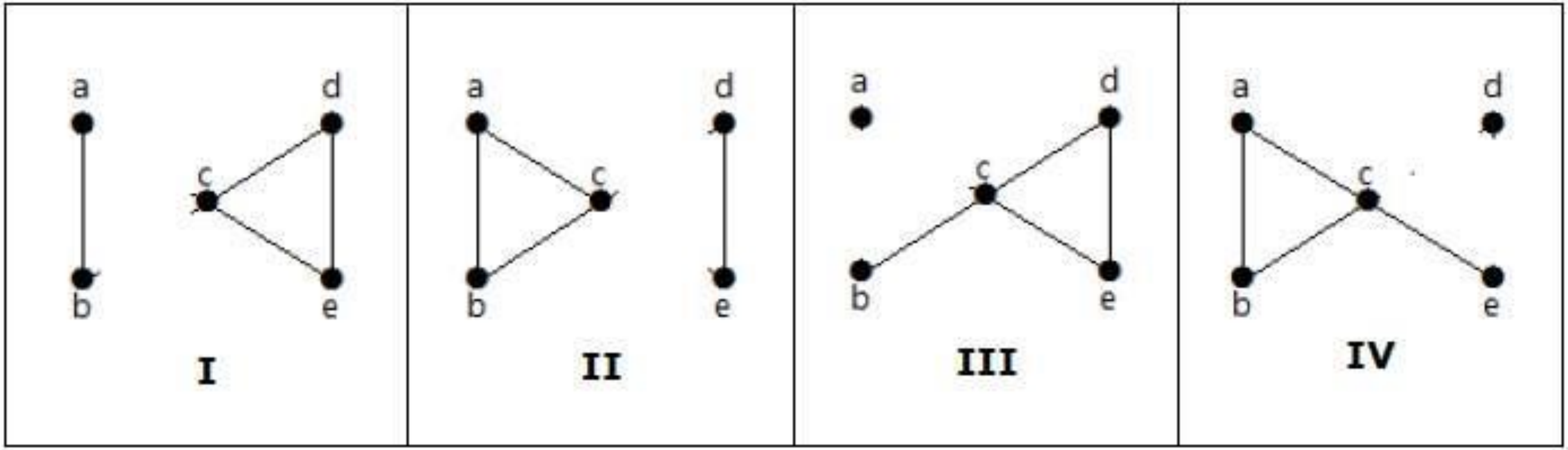
In other words, the **number of edges in a smallest cut set of G** is called the edge connectivity of G.

- If ‘G’ has a cut edge, then *λ(G)* is 1.

If we remove two minimum edges, the following connected graph becomes disconnected. Hence, its edge connectivity (λ(G)) is 2.



Here are the four ways to disconnect the graph by removing two edges –



Cut-Vertex

A cut-vertex is a single vertex whose removal disconnects a graph.

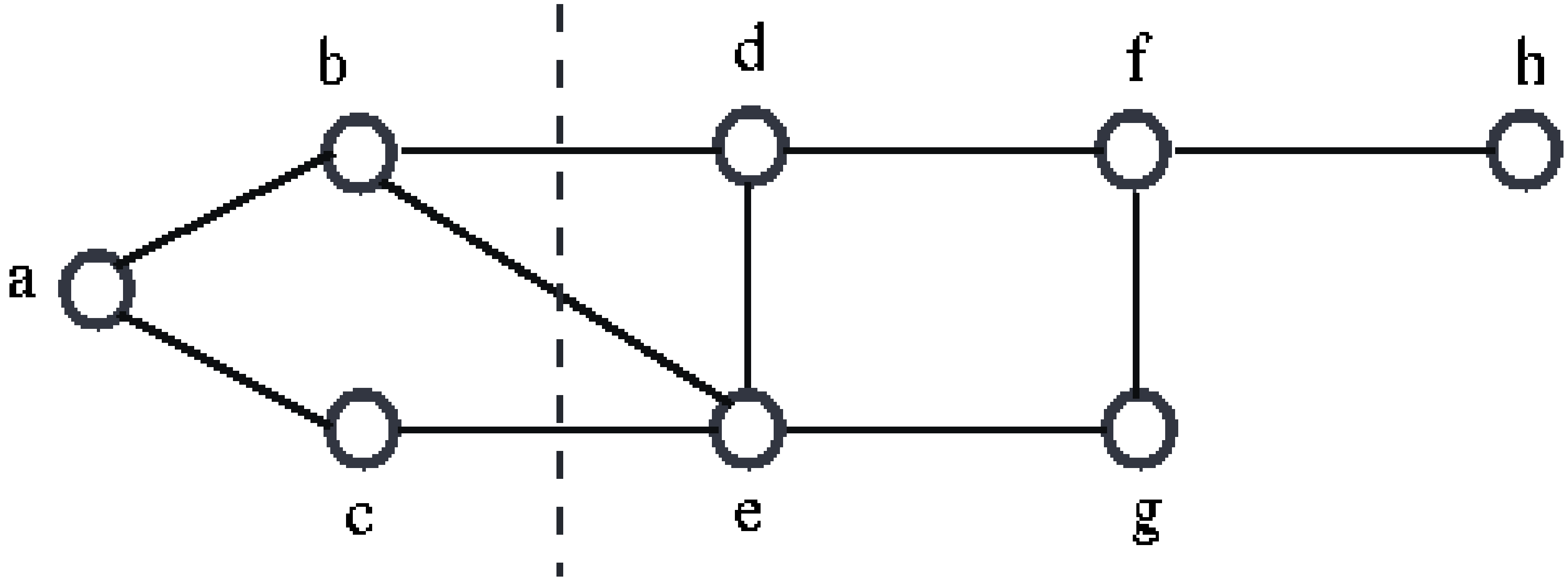
If G has a cut vertex, then $K(G) = 1$.

Vertex-Cut set

A vertex-cut set of a connected graph G is a set S of vertices with the following properties.

- the removal of all the vertices in S disconnects G.
- the removal of some (but not all) of vertices in S does not disconnects G.

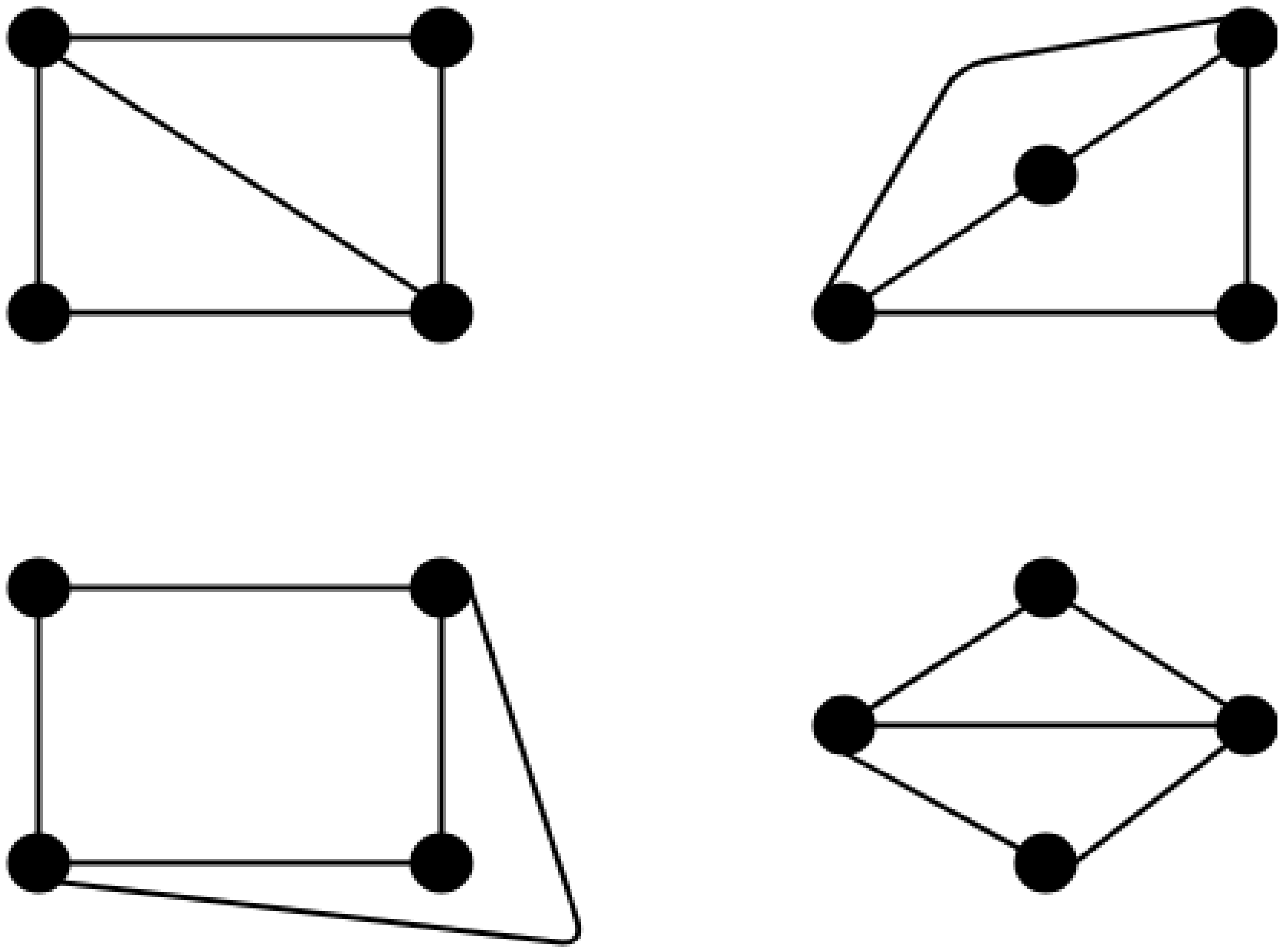
Consider the following graph



We can disconnects the graph by removing the two vertices b and e, but we cannot disconnect it by removing just one of these vertices. the vertex-cutset of G is {b, e}.

Graph Isomorphism

Graph Isomorphism is a phenomenon of existing the same graph in more than one forms. Such graphs are called as **isomorphic graphs**.



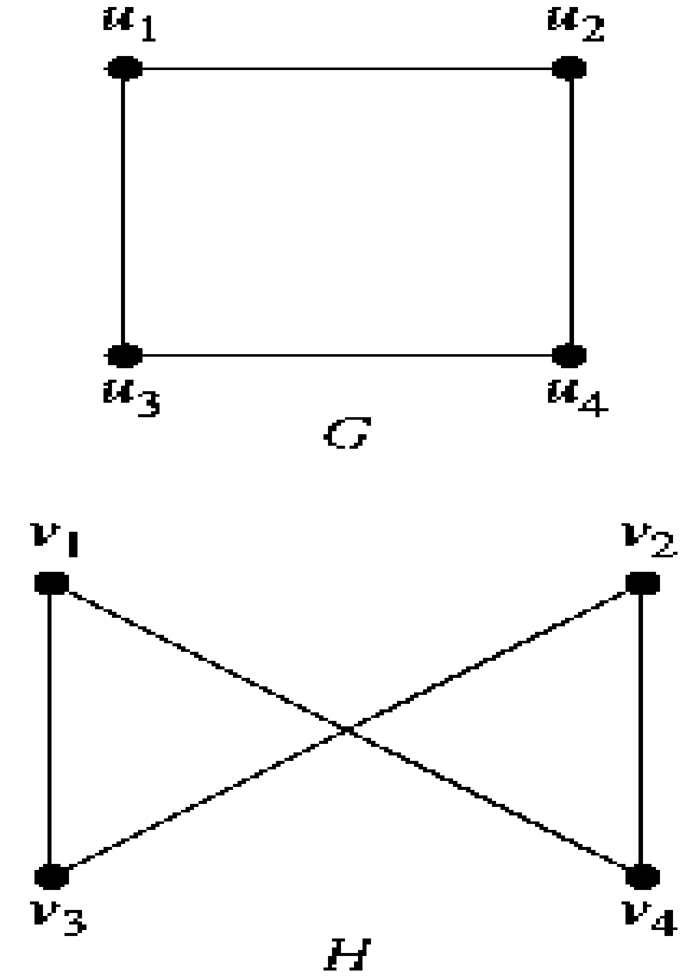
All these 4 graphs are Isomorphic Graphs

Isomorphic Graphs

The simple graphs $G1 = (V1,E1)$ and $G2 = (V2,E2)$ are *isomorphic* if there exists a one to-one and onto function f from $V1$ to $V2$ with the property that a and b are adjacent in $G1$ if and only if $f(a)$ and $f(b)$ are adjacent in $G2$, for all a and b in $V1$. Such a function f is called an *isomorphism*.

$*$ Two simple graphs that are not isomorphic are called *non-isomorphic*.

Example: isomorphic graphs



Necessary Conditions

If two graphs are isomorphic, they must have:

-the same number of vertices

-the same number of edges

-the same degrees for corresponding vertices

-the same number of connected components

-the same number of loops

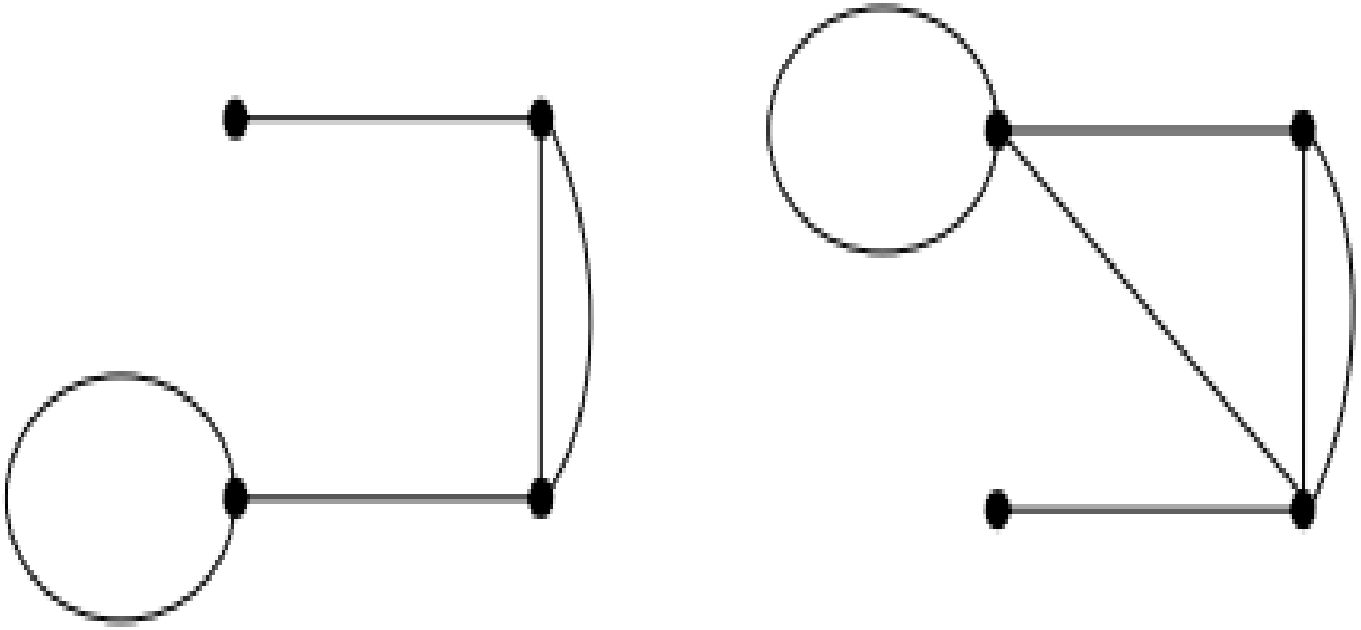
-the same number of multi-edges.

-the same circuits of given length

In general, it is easier to prove two graphs are not isomorphic by proving that one of the above properties fails.

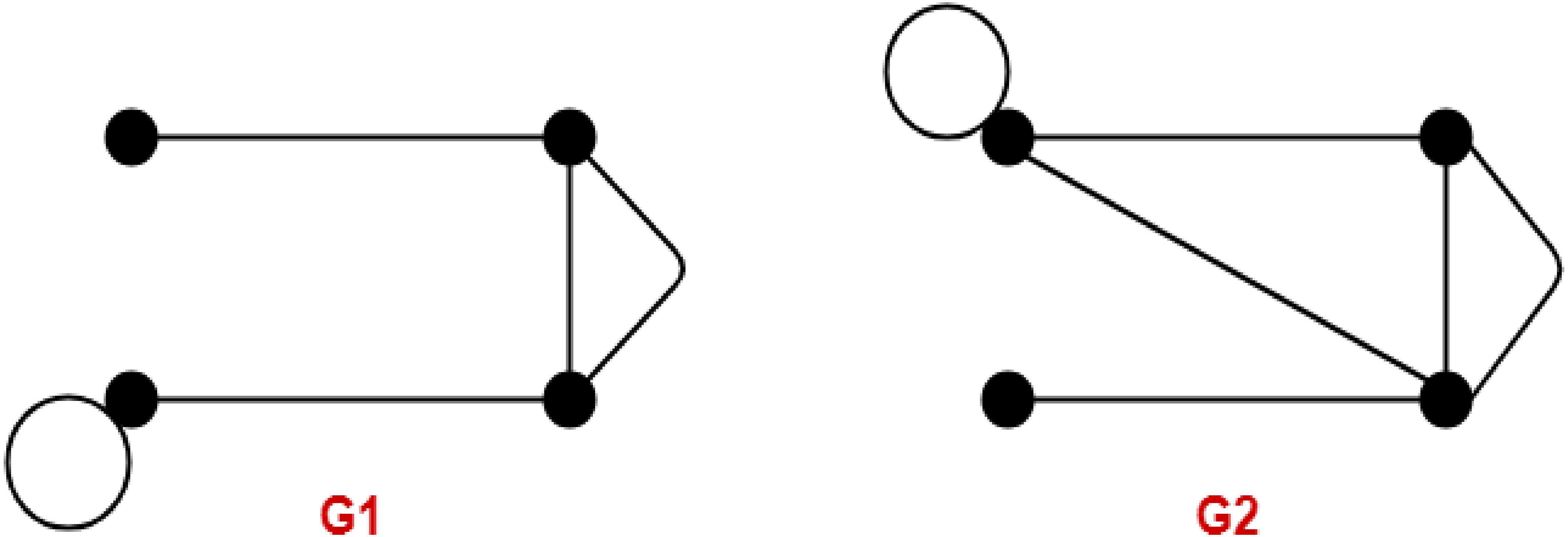
Example:

Consider the following graphs, are they the isomorphic, i.e. the “same”?



No. The left-hand graph has 5 edges; the right hand graph has 6 edges.

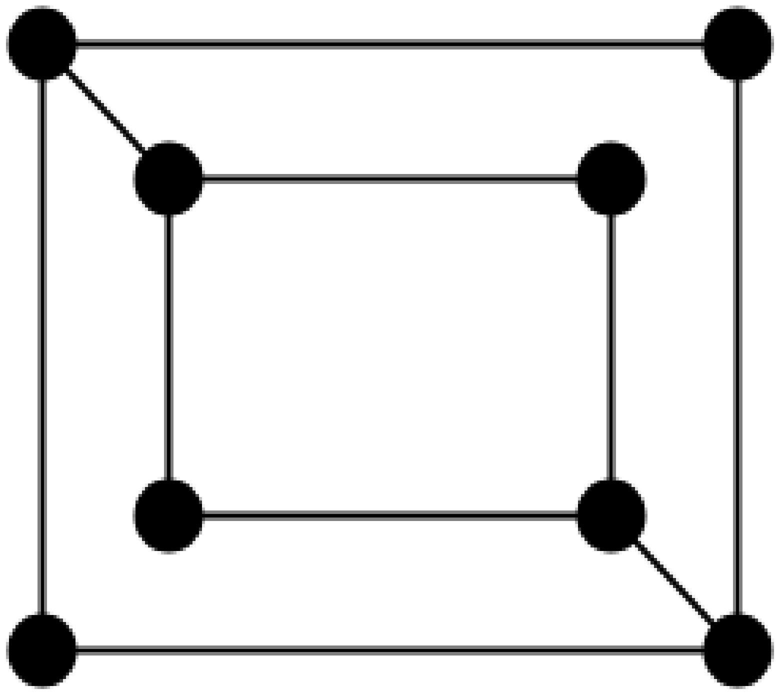
Example: Are the following two graphs isomorphic?



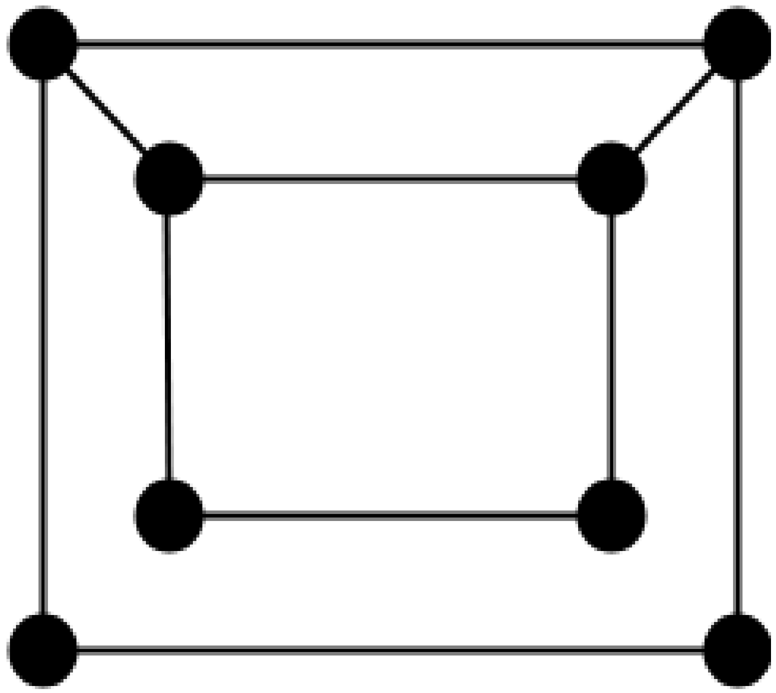
G1 G2

both the graphs G1 and G2 have different number of edges. So, they can not be isomorphic.

Example. Show that the following two graphs are not isomorphic.



G1



G2

- both the graphs G1 and G2 have same number of vertices. So, they may be isomorphic.
- both the graphs G1 and G2 have same number of edges. So, they may be isomorphic.
- both the graphs G1 and G2 have same degree sequence. So, they may be isomorphic.
- In graph G2, the degree-3 vertices form a cycle of length 4 but in graph G2, the degree-3 vertices do not form a 4-cycle as the vertices are not adjacent. both the graphs G1 and G2 do not contain same cycles in them. So, they can not be isomorphic.

Sufficient conditions for two graphs to be Isomorphic

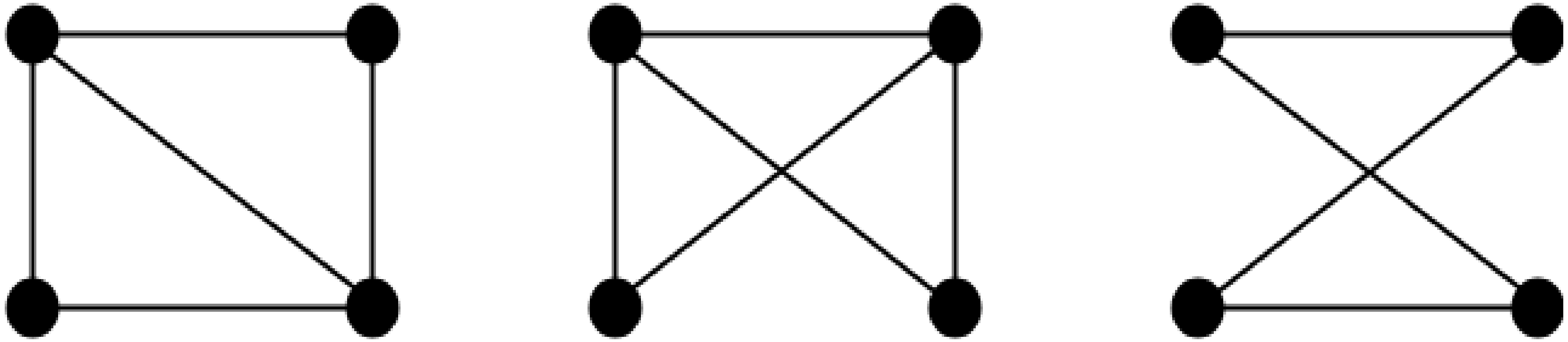
Condition-01:

Two graphs are isomorphic if and only if their complement graphs are isomorphic.

Condition-02:

Two graphs are isomorphic if their adjacency matrices are same.

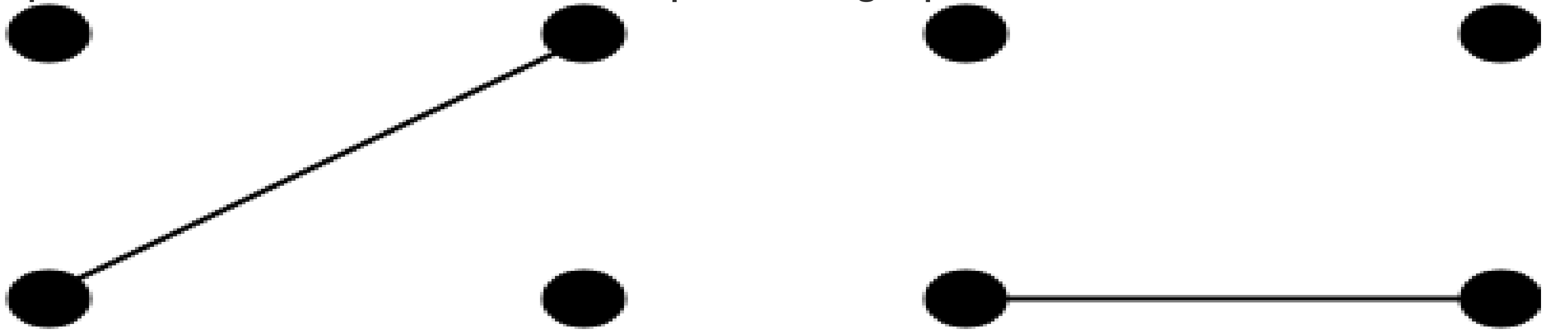
Example:



G1 G2 G3

G3 is not isomorphic to any graph because they have different number of edges.

•By sufficient conditions, we know that the two graphs are isomorphic if and only if their complement graphs are isomorphic. So, let us draw their complement graphs-



$\overline{G1}$ $\overline{G2}$

Complement Graphs of G1 and G2