# Multiple Linear Regression

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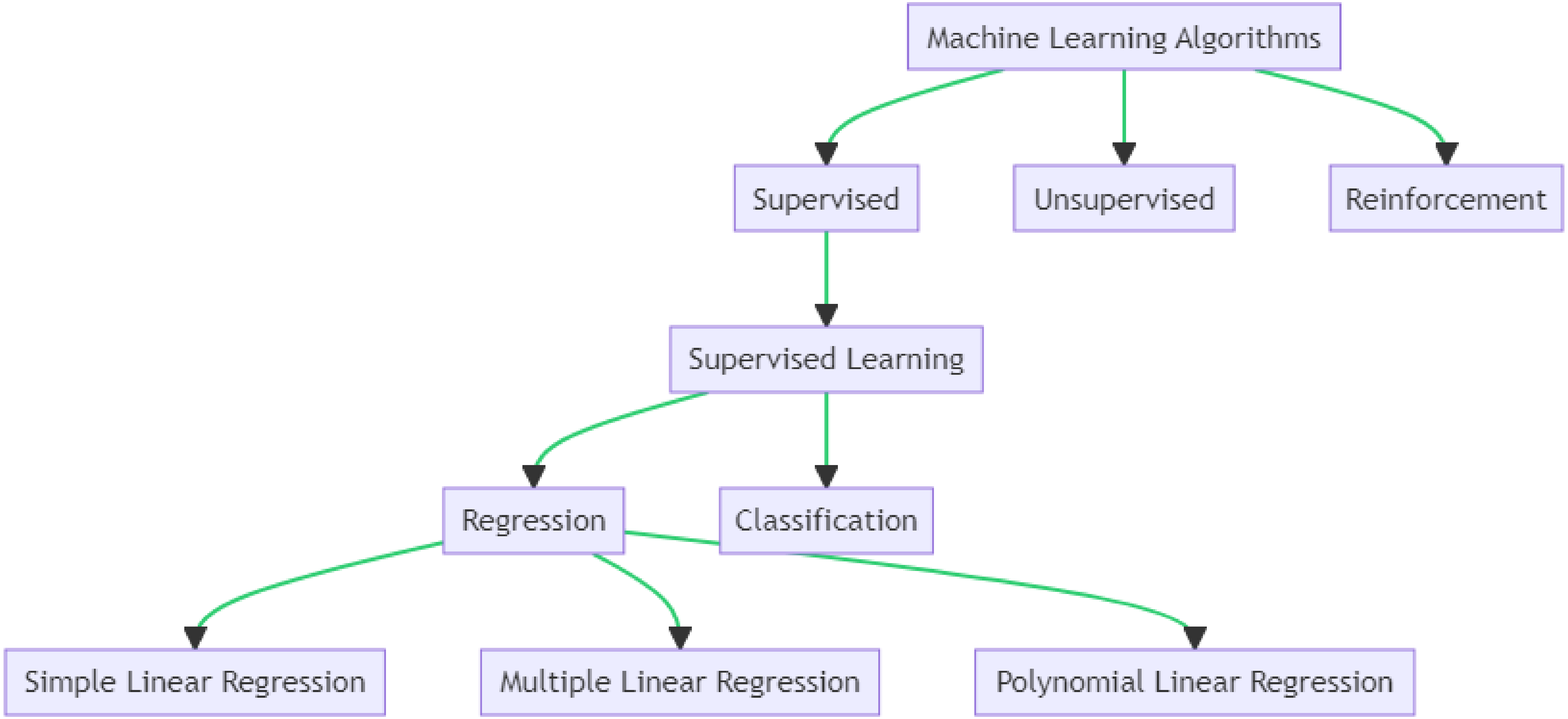
# AGENDA

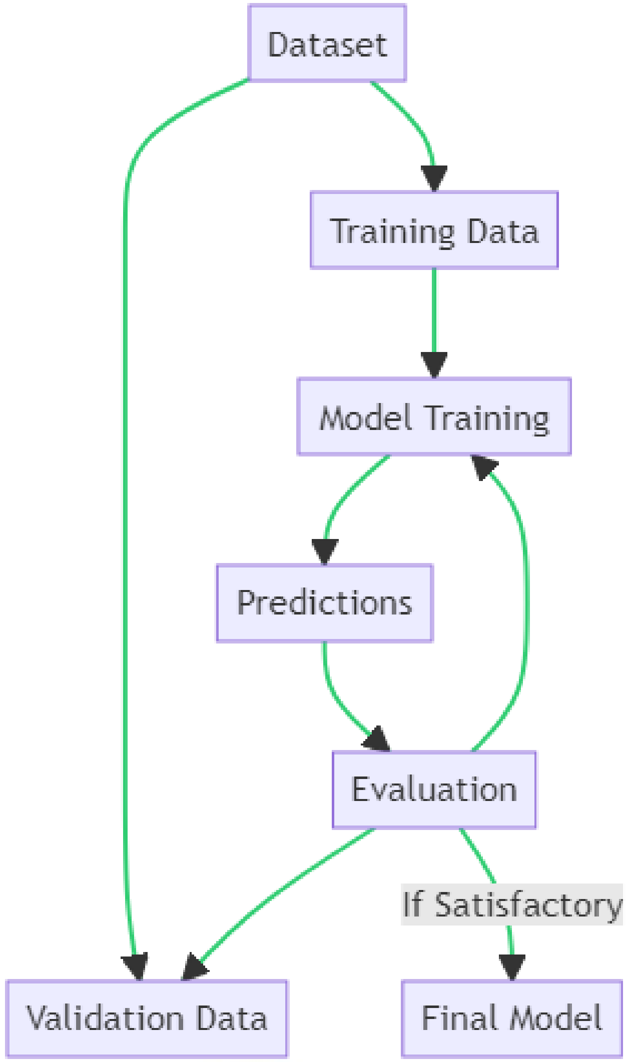
- Hierarchy of ML Algorithms
- Simple Linear Regresion
- Linear vs, Multiple Regression
- Multiple Linear Regression
- Mathematical Concept
- Benefits and Limitations
- Real-life Applications
- Summary

# Hierarchy of Machine Learning Algorithms



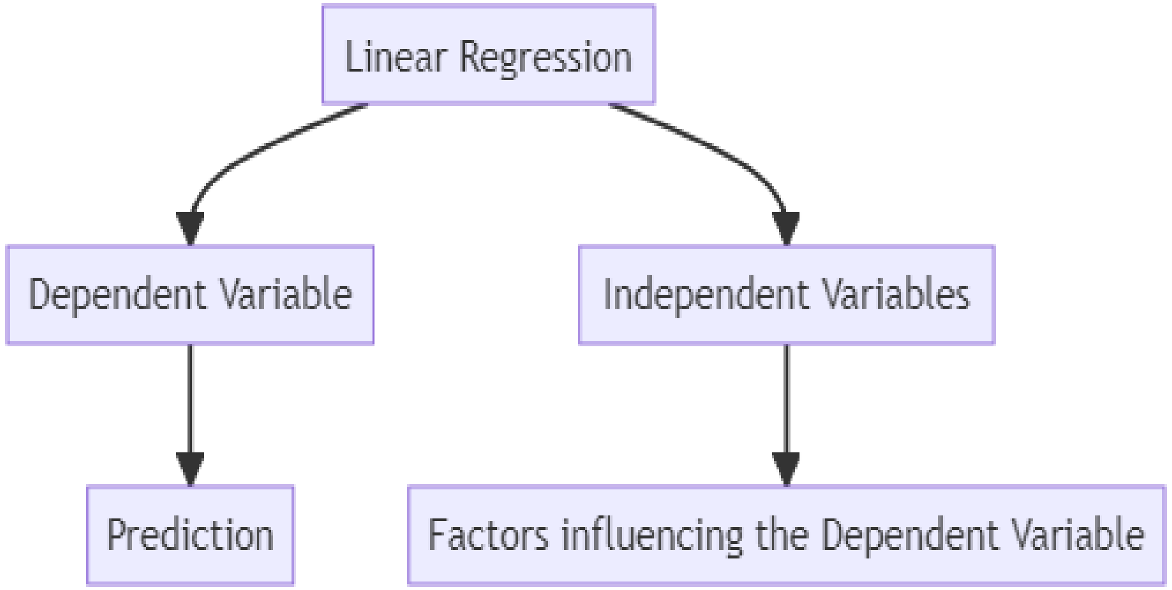
# Supervised Learning Workflow

- Dataset is split into Training Data and Validation Data.
- Training Data is used for Model Training.
- The trained model makes Predictions.
- Predictions are Evaluated using the Validation Data.
- If the evaluation is satisfactory, we get the Final Model, else the model is retrained.



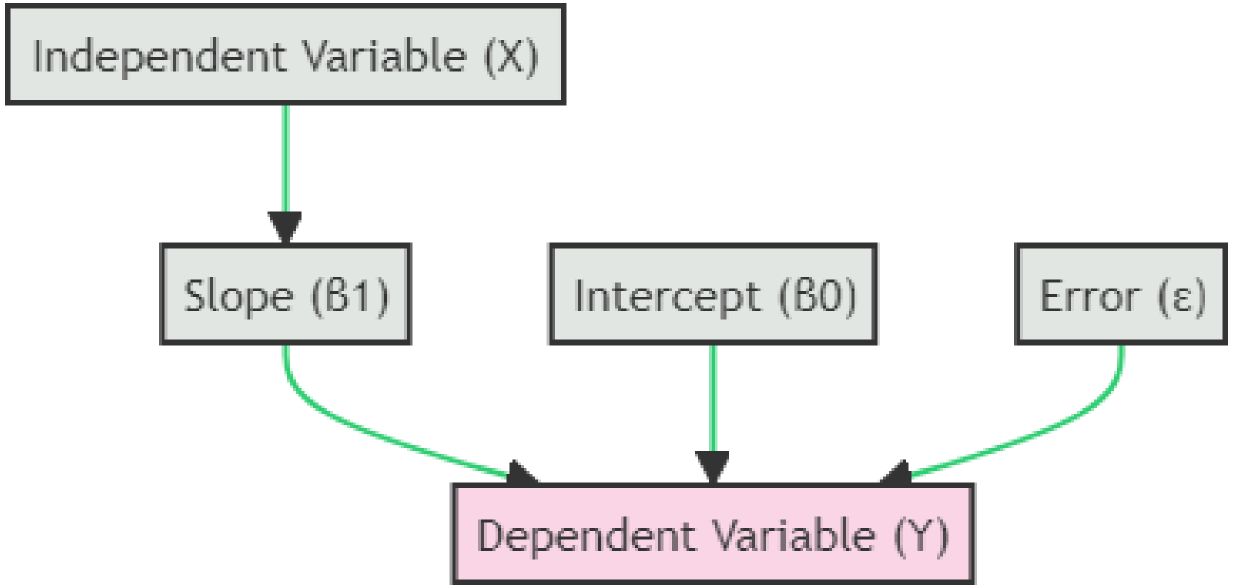
# Simple Linear Regression

Linear regression is a statistical technique that finds the best-fitting straight line to show how a dependent variable (the one we want to predict, often denoted as "Y") is related to one independent variables (the factor or input we use to make predictions, often denoted as "X").



# Linear Regression Equation

Linear regression is a statistical technique that finds the best-fitting straight line to show how a dependent variable (Y) is related to one independent variable (X). The equation is given by $Y=\beta 0+\beta 1X+\epsilon$, where $\beta 0$ is the y-intercept, $\beta 1$is the slope, and $\epsilon$ is the error term.



$$Y=\beta 0+\beta 1X+\epsilon$$

# Example - Correlation Between Study Hours and Grades

The dataset shows a potential positive correlation between study hours and grades. More study hours might lead to higher grades. This data is pivotal for further analyses like linear regression.

| Hours Studied | Grades Received |
|---|---|
| 1.0 | 35 |
| 3.0 | 55 |
| 2.3 | 42 |
| 6.0 | 94 |
| 1.5 | 36 |
| 7.0 | 96 |
| 5.0 | 90 |

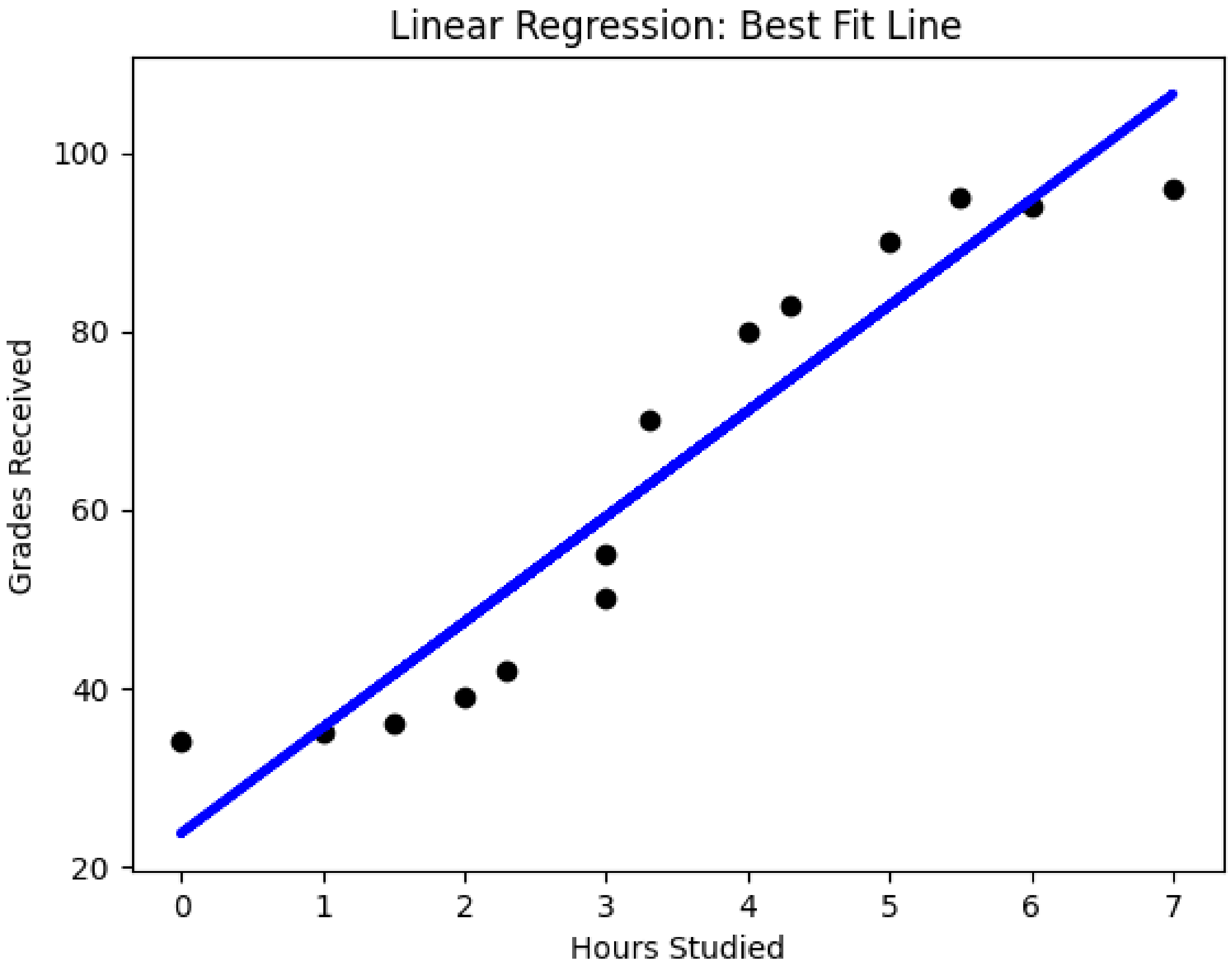
| Hours Studied | Grades Received |
|---|---|
| 3.3 | 70 |
| 4.0 | 80 |
| 2.0 | 39 |
| 3.0 | 50 |
| 0,0 | 34 |
| 5.5 | 95 |
| 4.3 | 83 |

# Relationship between Hours Studied and Grades Received

Grade=β0+β1×Study Time

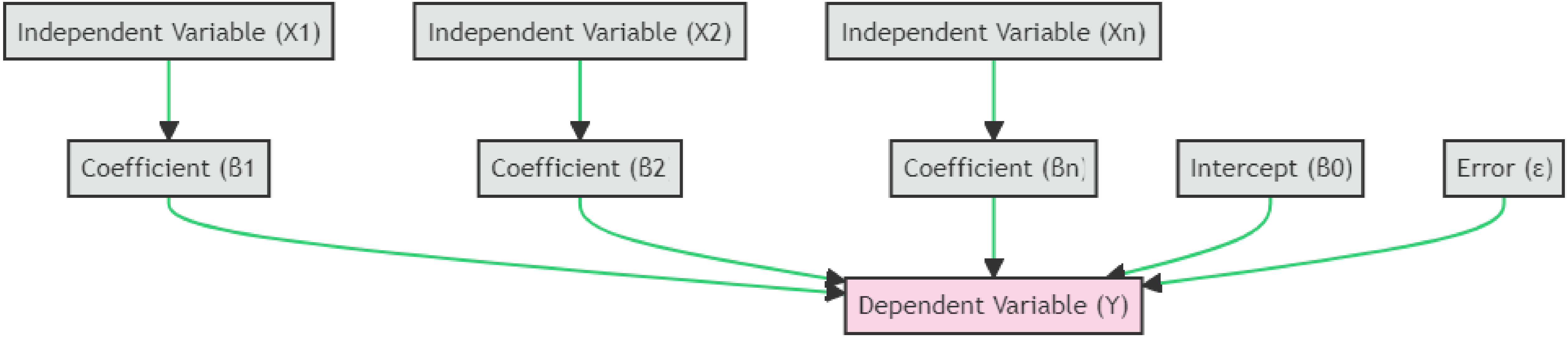
Where:

- β0 is the y-intercept (grade when study time is zero).
- β1 is the slope, indicating the change in grade for each additional hour of study.



# Multiple Linear Regression

Multiple Linear Regression is an extension of simple linear regression. It is used to predict the value of a dependent variable based on the values of multiple independent variables. The relationship is represented by the equation:



$$Y=\beta0+\beta1x1+\beta2x2+\ldots+\beta nxn+\epsilon$$

# Multiple Linear Regression

In multiple linear regression, we model the relationship between multiple independent variables (features) and a dependent variable.

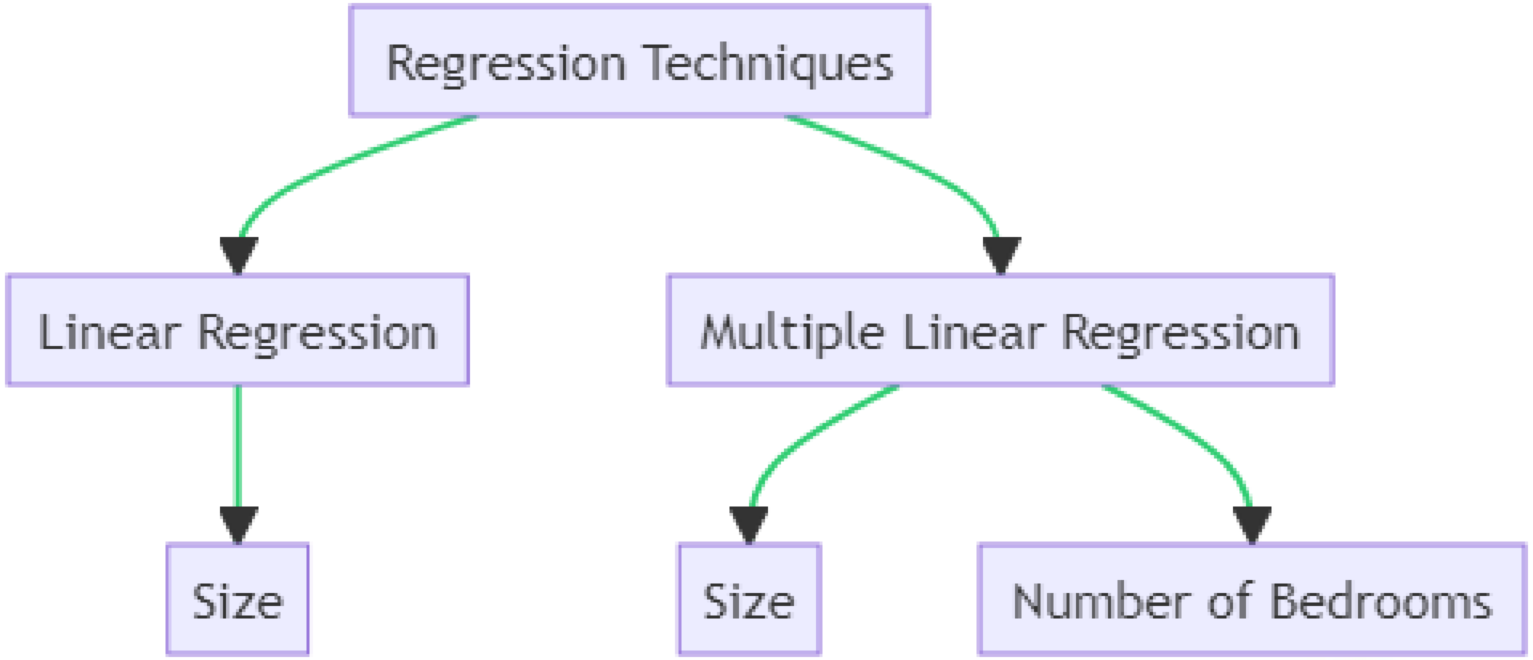
The equation for multiple linear regression is:

$$y=\beta 0+\beta 1x1+\beta 2x2+\ldots+\beta nxn+\epsilon$$

- y is the dependent variable (what we are trying to predict).
- β0  is the y-intercept.
- β1,β2,…,βn   are the coefficients of the independent variables.
- x1,x2,…,xn   are the independent variables (features).
- ϵ is the error term.

# Linear vs. Multiple Linear Regression

Linear Regression and Multiple Linear Regression are both statistical techniques used to predict the value of a dependent variable based on the values of independent variable(s), but they differ in the number of independent variables used.



Regression Techniques

Linear Regression

Multiple Linear Regression

Size

Size

Number of Bedrooms

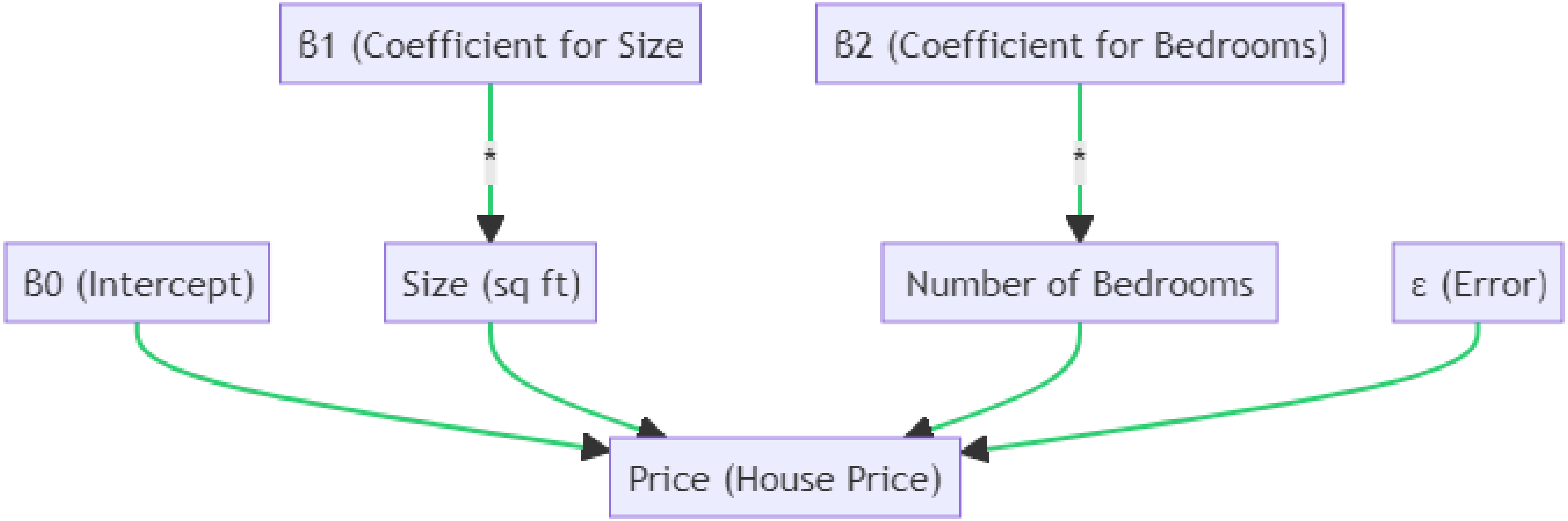
# Linear vs. Multiple Linear Regression

Regression techniques are statistical methods used to predict the value of a dependent variable based on one or more independent variables. In the context of house price prediction:

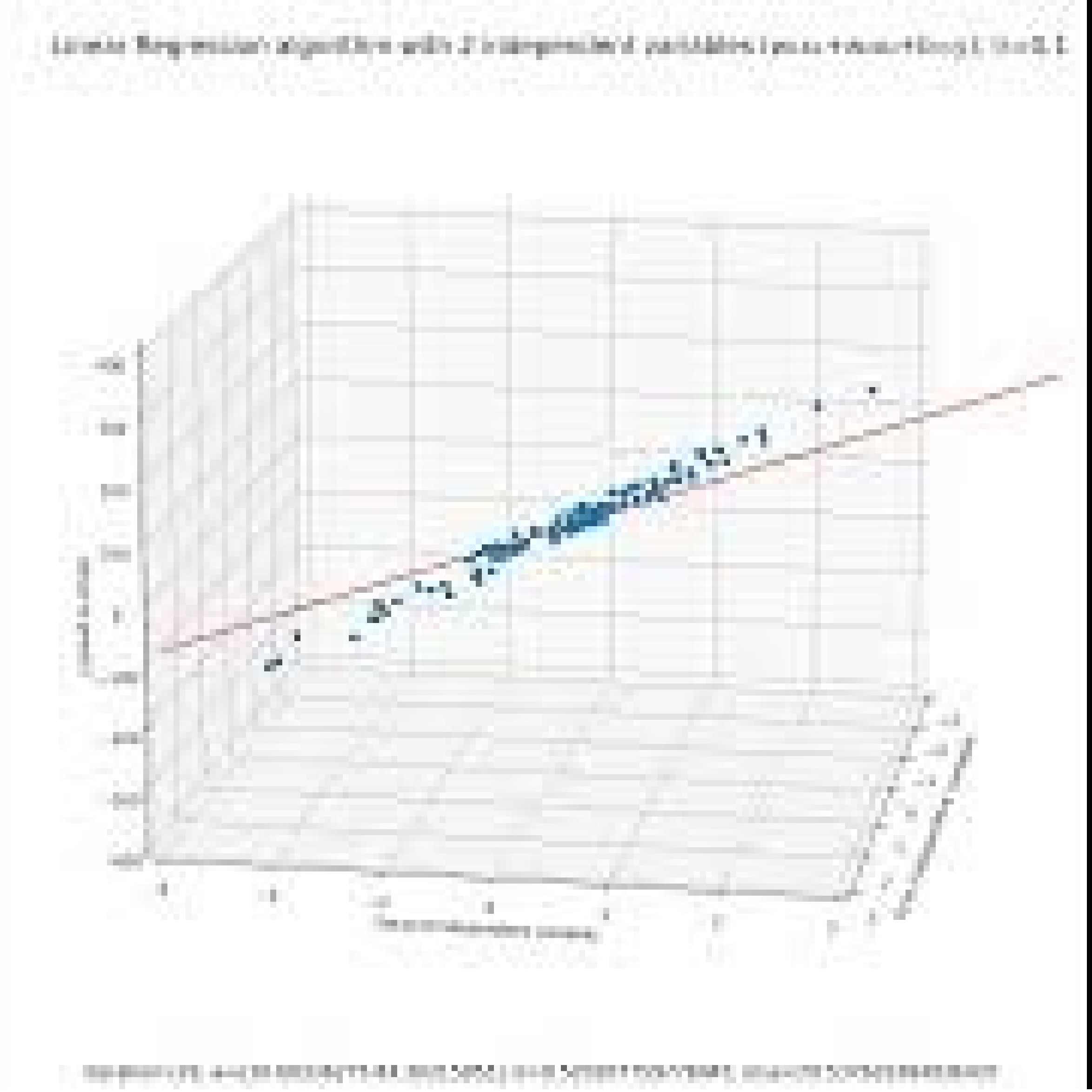
- Linear Regression: A method that predicts house prices based solely on the "Size" of the house. It establishes a relationship between the house price and its size.
- Multiple Linear Regression: An advanced technique that considers multiple factors, such as the "Size" and the "Number of Bedrooms", to predict house prices. It provides a more comprehensive model by incorporating multiple features of the house.

Both techniques aim to offer accurate predictions, but the choice between them depends on the available data and the complexity of the relationship being studied.

# Example - House Price Prediction with Multiple Regression



**Price=β0+β1(Size)+β2(Bedrooms)+ϵ**


Linear Regression algorithm with 2 independent variables

# How well Regression Model Fits Data?

**R-squared** measures overall model goodness of fit.

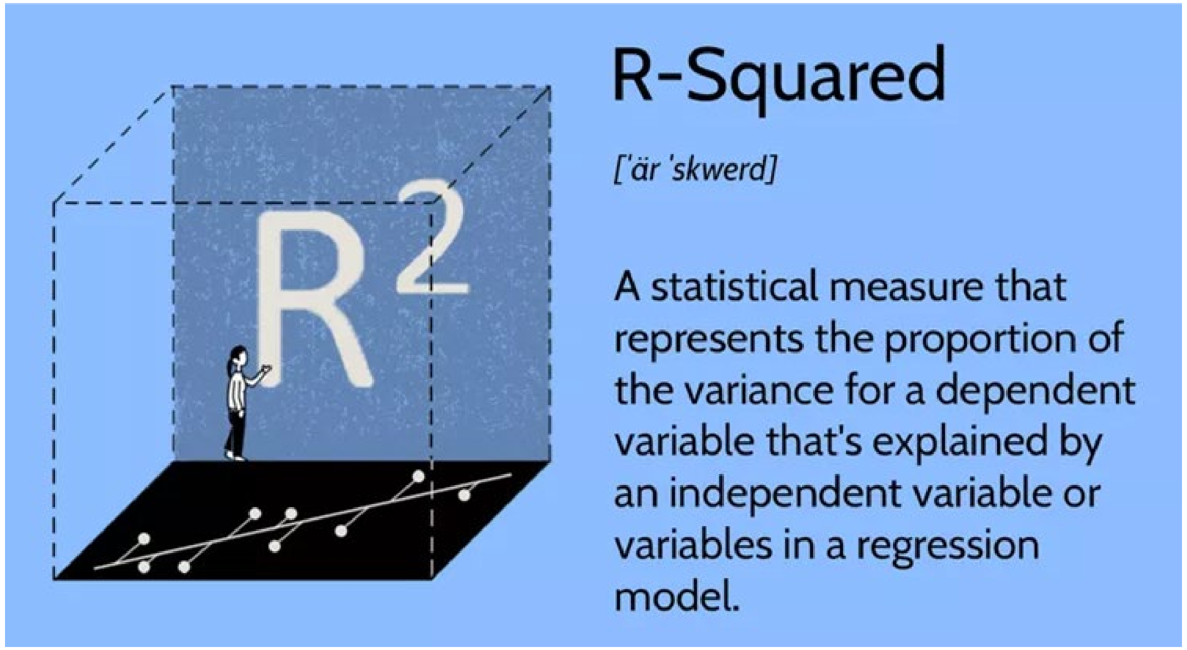
- Evaluates how well the model explains dependent variable variation.
- Higher $R^2$ indicates better overall fit (0 to 1).

**p-values** assess individual variable significance.

- Determines if each variable contributes significantly to the model.
- Low p-values ($<0.05$) imply significance.

# R-squared ($R^2$)

- R-squared ($R^2$) measures how well a regression model fits data.
- It ranges from 0 to 1, with 1 meaning a perfect fit.
- Higher $R^2$ values indicate better model fit.
- $R^2$ doesn't explain variable significance.



R-Squared

['är 'skwerd]

A statistical measure that represents the proportion of the variance for a dependent variable that's explained by an independent variable or variables in a regression model.

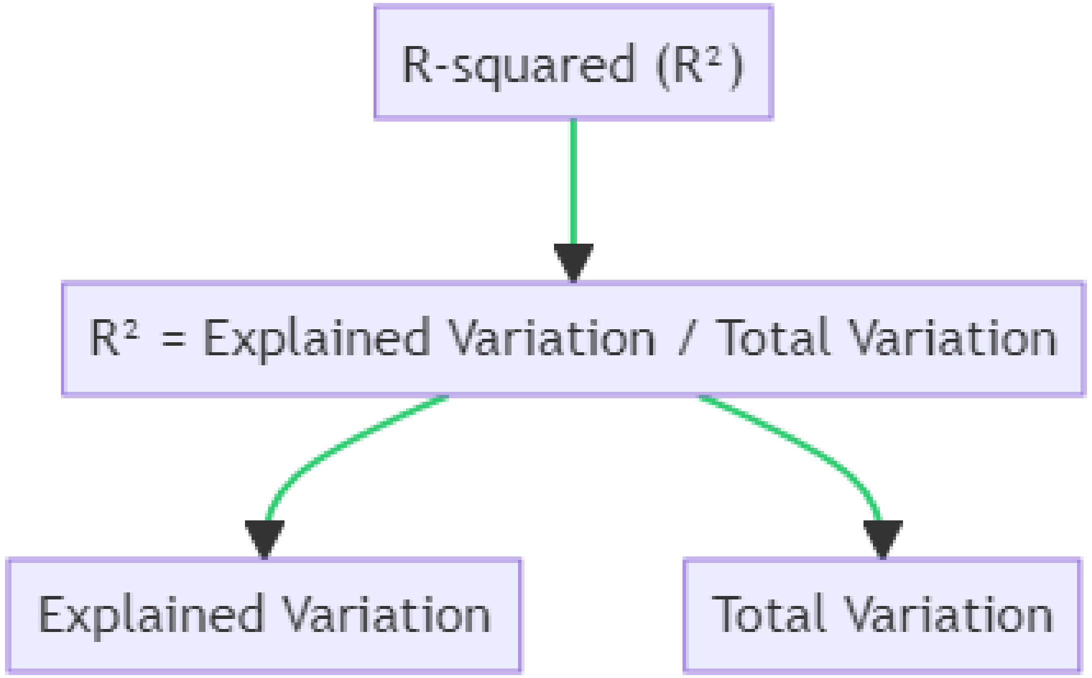
**Source:** J. Fernando, “R-Squared Definition,” Investopedia, Apr. 08, 2023. https://www.investopedia.com/terms/r/r-squared.asp

- **Note:**

  A high $R^2$ does not necessarily mean a good model; it could be overfitting. Always consider other model evaluation metrics.

# Understanding R-squared (R²)

- **Equation:**



R-squared (R²)

R² = Explained Variation / Total Variation

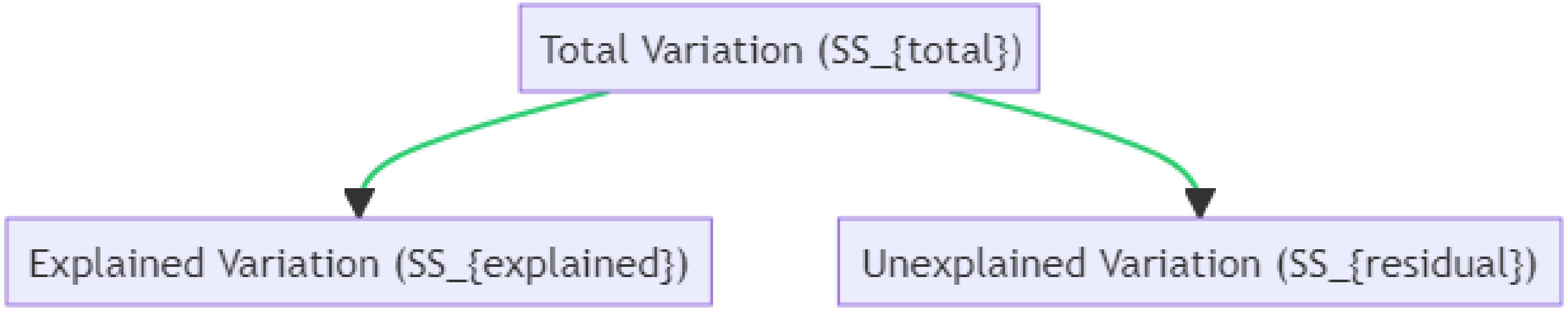
Explained Variation

Total Variation

Where,

- **Explained Variation:** The variation in the dependent variable explained by the regression model.
- **Total Variation:** The overall variability present in the the dependent variable.

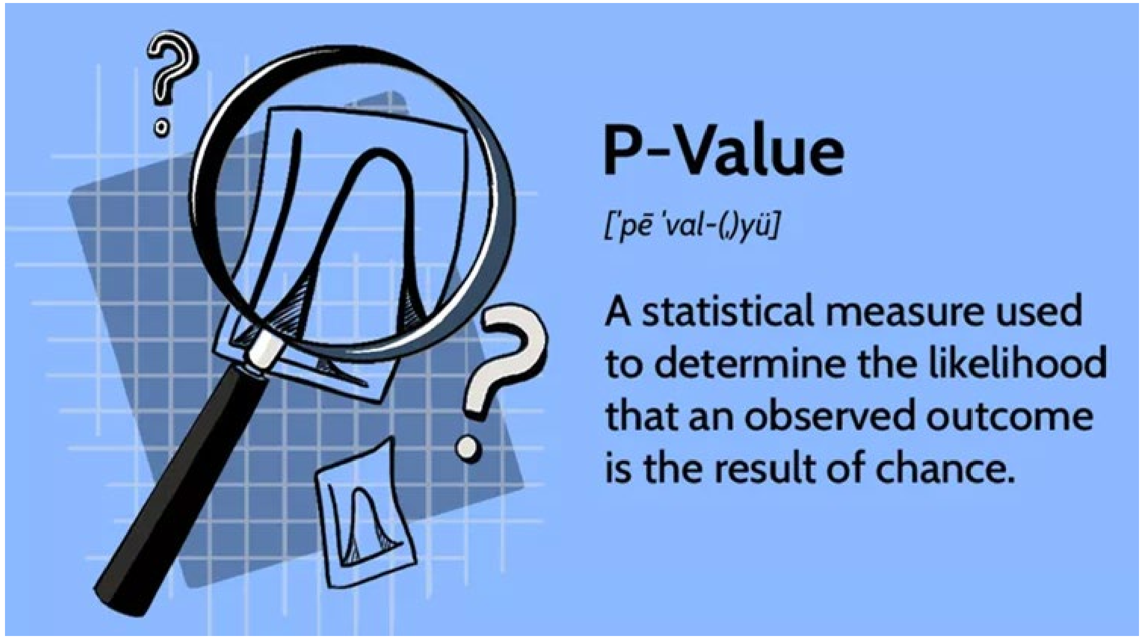
# Understanding R-squared ($R^2$)


Total Variation (SS_{total})
Explained Variation (SS_{explained})
Unexplained Variation (SS_{residual})

- **Explained Variation: (SSexplained):** Represents the portion of the total variation in the dependent variable that is captured by the regression model. It quantifies the variability attributed to the relationship with the independent variable(s).
- **Unexplained Variation (SSresidual):** The portion of the total variation that the model fails to explain. It's the difference between the observed values and the predicted values from the model.
- **Total Variation (SStotal):** The overall variability in the dependent variable without considering the effect of the independent variable(s). It's the sum of the Explained and Unexplained Variations.

# Understanding p-values

- P-values test variable significance in a regression model.
- Low p-values (<0.05) mean significance.
- High p-values suggest insignificance.
- Use p-values to decide variable inclusion.



**P-Value**

[ˈpē ˈval-(ˌ)yü]

A statistical measure used to determine the likelihood that an observed outcome is the result of chance.

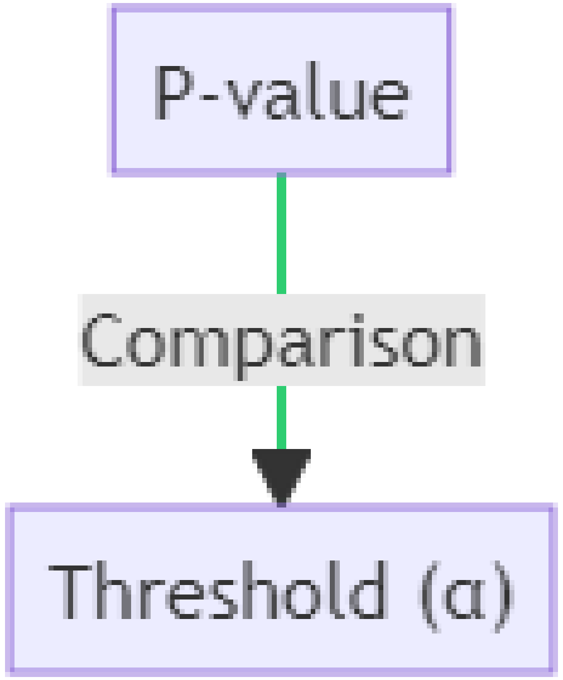
**Source:** B. Beers, "What P-Value Tells Us," Investopedia, May 18, 2022. https://www.investopedia.com/terms/p/p-value.asp

# Understanding p-values

The P-value is a measure used in hypothesis testing to determine the significance of results. It's compared against a predetermined threshold, often denoted as α (commonly set at 0.05).

- If the P-value is less than α, the result is considered statistically significant, and the null hypothesis is rejected.
- If the P-value is greater than or equal to α, there's insufficient evidence to reject the null hypothesis.

The P-value tells us if the observed data is consistent with the null hypothesis or if it's rare under the assumption that the null hypothesis is true.



P-value

Comparison

Threshold (α)

# Assumptions of Multiple Linear Regression

- A linear relationship between the dependent and independent variables.
- The independent variables are not highly correlated with each other.
- The variance of the residuals is constant (i.e., differences between predicted and actual values).
- Each data point shouldn't depend on others; they should be separate.
- All variables should be normally distributed.

# Benefits of Multiple Linear Regression

- **Predictive Power**
  - Predict dependent variable based on multiple independent variables.
- **Quantifying Relationships**
  - Measures strength and direction of relationships.
- **Control for Confounding Factors**
  - Accounts for other influencing variables.
- **Model Interpretability**
  - Coefficients offer insights into variable impact.
- **Assumption Testing**
  - Provides diagnostic tools for model quality.

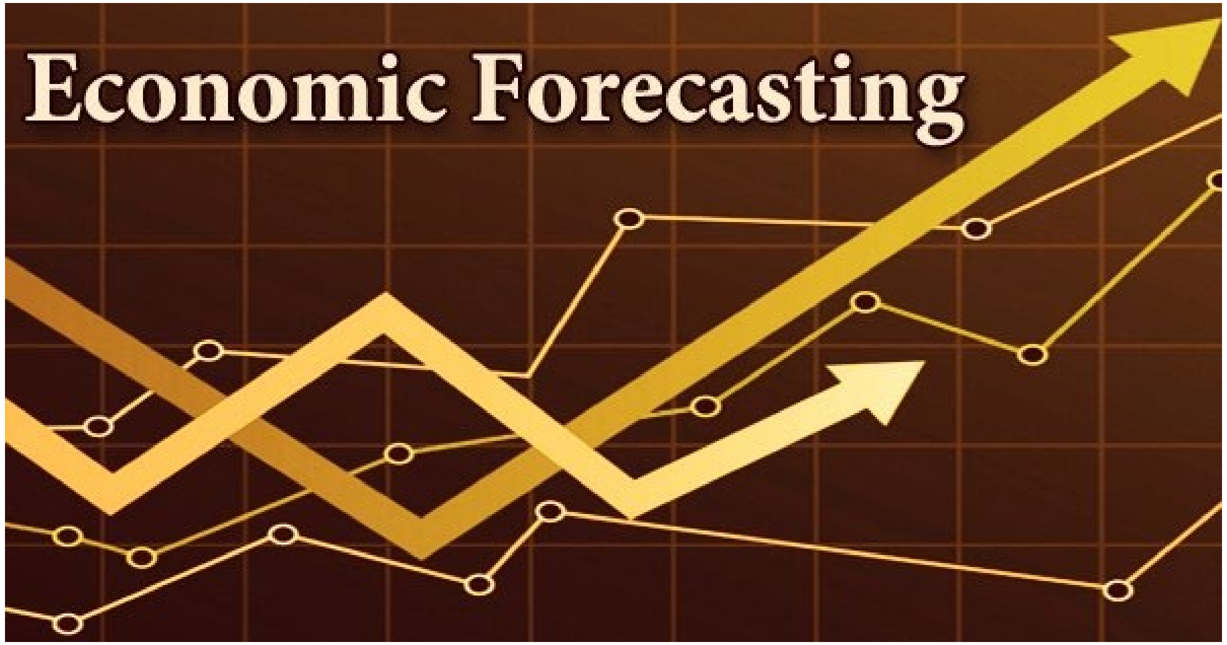
# Limitations of Multiple Linear Regression

- **Linearity Assumption**
  - Assumes linear relationships.
- **Multicollinearity**
  - High correlations among variables can lead to instability.
- **Overfitting**
  - Too many variables can cause poor generalization.
- **Assumption Violations**
  - Relies on normality, homoscedasticity, and independence.
- **Handling Categorical Variables**
  - Requires encoding, adding complexity.
- **Data Requirements**
  - Needs a large dataset for reliable results.

# Real-Life Applications of Multiple Regression

- **Economics and Finance:**

Stock Price Prediction



Economic Forecasting

# Real-Life Applications of Multiple Regression

- **Healthcare:**

Medical Research

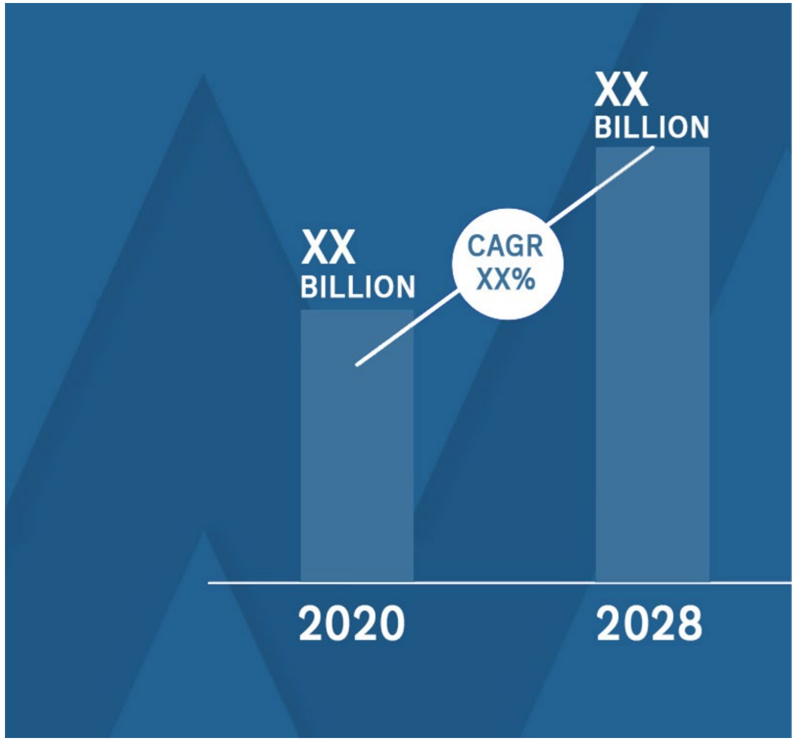
HOSPITAL
EXIT
ENTER
caregiver/ self-management
caregiver/ self-management
Non-adherence, comorbidities and more

Hospital Readmission Prediction

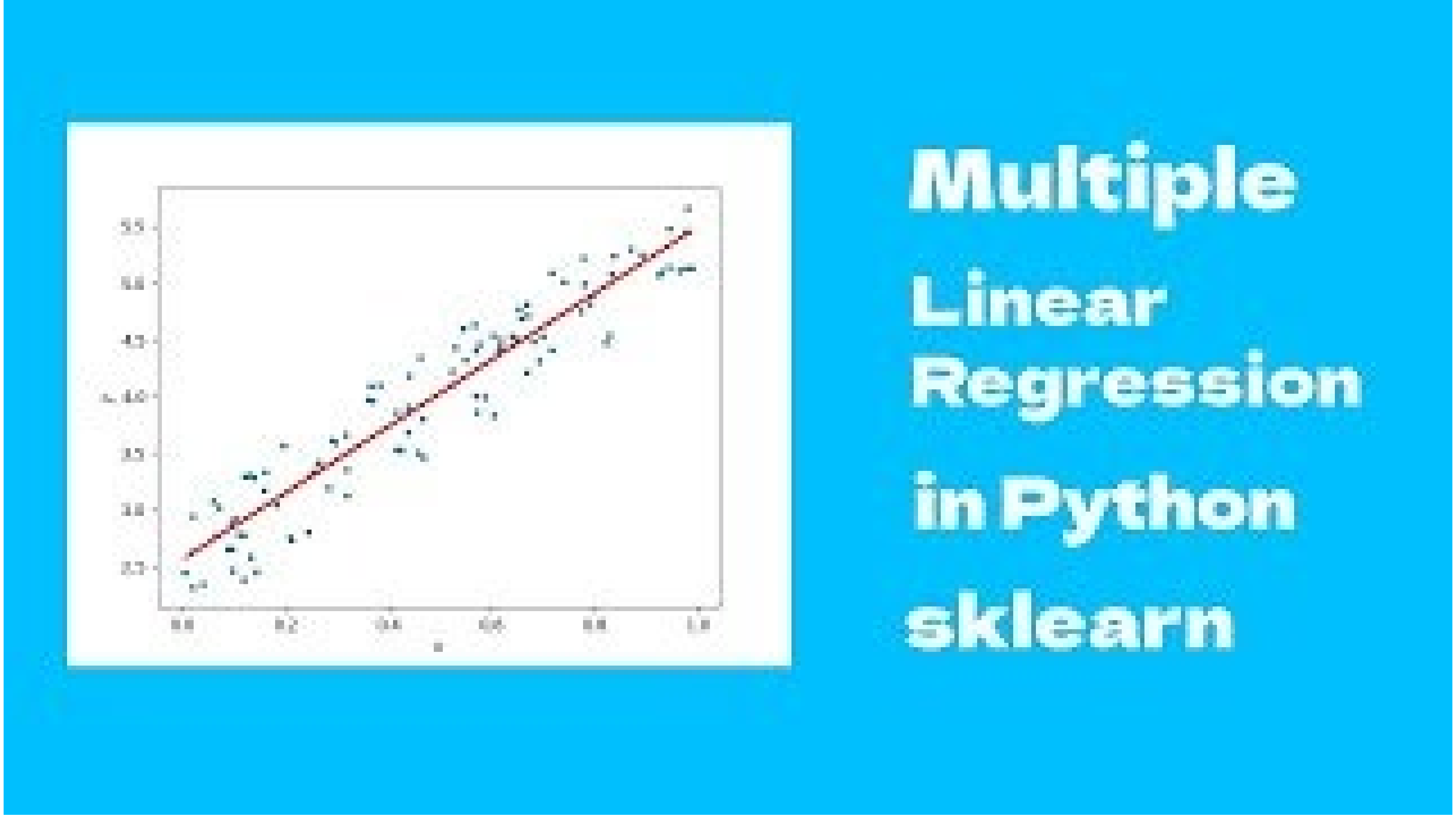
# Real-Life Applications of Multiple Regression

- **Marketing:**

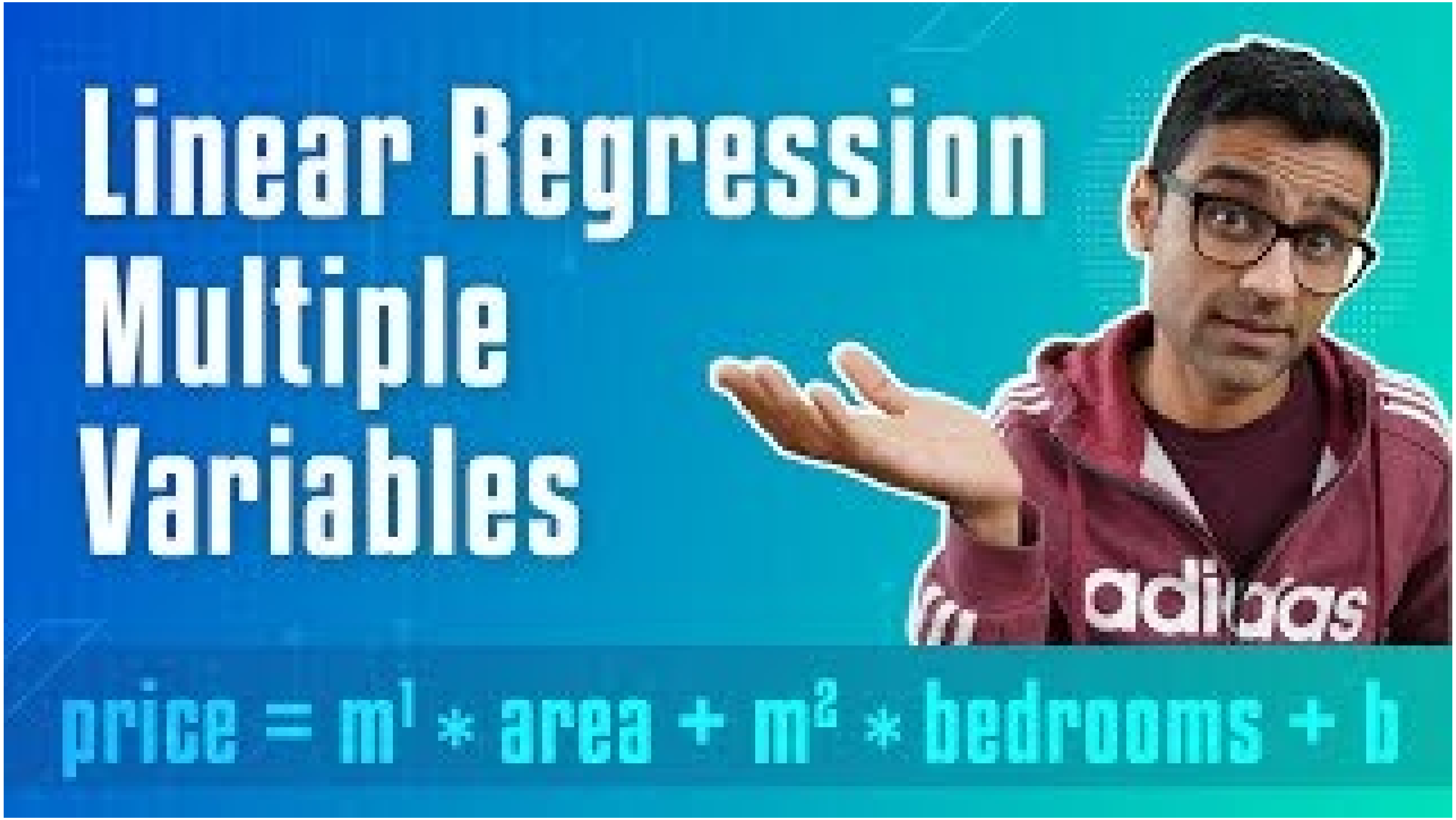
Sales Forecasting



Market Research



**Source:**“Multiple Linear Regression in Python - sklearn,” *www.youtube.com,* https://youtu.be/wH_ezgftiy0



**Source:** “Machine Learning Tutorial Python - 3: Linear Regression Multiple Variables,” *www.youtube.com*. https://youtu.be/J_LnPL3Qg70

# Summary

- Hierarchy: Explored ML algorithm hierarchy.
- Simple Linear Regression: Basics of modeling relationships.
- Linear vs. Multiple Regression: Single vs. multiple predictor variables.
- Multiple Linear Regression: Complex modeling with multiple predictors.
- Mathematical Concept: Key equations behind regression.
- Benefits and Limitations: Pros and cons of multiple regression.
- Real-life Applications: Practical use cases across domains.

# References

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# Thank You