IS5 in R: Confidence Intervals for Means (Chapter 14)

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July 10, 2018

Introduction and background

This document is intended to help describe how to undertake analyses introduced as examples in the Fifth Edition of *Intro Stats* (2018) by De Veaux, Velleman, and Bock. More information about the book can be found at http://wps.aw.com/aw_deveaux_stats_series. This file as well as the associated R Markdown reproducible analysis source file used to create it can be found at http://nhorton.people.amherst.edu/is5.

This work leverages initiatives undertaken by Project MOSAIC (http://www.mosaic-web.org), an NSF-funded effort to improve the teaching of statistics, calculus, science and computing in the undergraduate curriculum. In particular, we utilize the mosaic package, which was written to simplify the use of R for introductory statistics courses. A short summary of the R needed to teach introductory statistics can be found in the mosaic package vignettes (http://cran.r-project.org/web/packages/mosaic). A paper describing the mosaic approach was published in the R Journal: https://journal.r-project.org/archive/2017/RJ-2017-024.

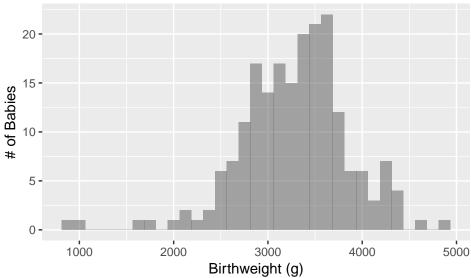
Chapter 14: Confidence Intervals for Means

```
library(mosaic)
library(readr)
library(janitor)
Babies <- read_csv("http://nhorton.people.amherst.edu/is5/data/Babysamp_98.csv") %>%
  clean_names()
## Parsed with column specification:
## cols(
##
     MomAge = col_integer(),
##
     DadAge = col_integer(),
##
     MomEduc = col_integer(),
##
     MomMarital = col_integer(),
     numlive = col_integer(),
##
##
     dobmm = col integer(),
##
     gestation = col_integer(),
##
     sex = col_character(),
     weight = col_integer(),
##
     prenatalstart = col integer(),
##
     orig.id = col integer(),
##
##
     preemie = col_logical()
## )
```

By default, read_csv() prints the variable names. These messages can be suppressed using the message=FALSE code chunk option to save space and improve readability.

Here we use the clean_names() function from the janitor package to sanitize the names of the columns (which would otherwise contain special characters or whitespace).

```
# Figure 14.1, page 441
gf_histogram(~ weight, data = Babies, binwidth = 125) %>%
gf_labs(x = "Birthweight (g)", y = "# of Babies")
```



```
# Figure 14.2
set.seed(12346) # For reproducibility
numsim <- 10000 # Number of simulations

# What does do() do?
mean(~ weight, data = sample(Babies, size = 100)) # Mean of a random sample of 100

## [1] 3214.53
mean(~ weight, data = sample(Babies, size = 100)) # Mean of another random sample

## [1] 3290.34
do(2) * mean(~ weight, data = sample(Babies, size = 100)) # Calculates the mean twice

## mean
## 1 3297.84
## 2 3286.70
# For the visualization, we need 10,000 means
WeightMeans <- do(numsim) * mean(~ weight, data = sample(Babies, size = 100))</pre>
```

Here, the do() function calculates, 10,000 times, the mean of a random sample of 100 baby weights.

```
gf_histogram(~ mean, data = WeightMeans, binwidth = 10)
```

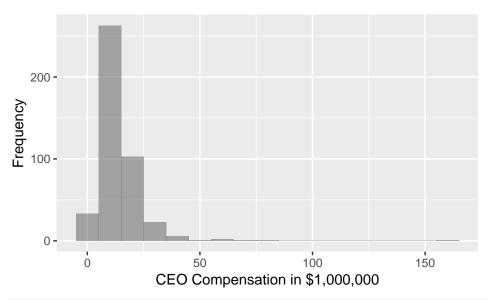
```
1000 - 750 - 250 - 250 - 3200 3300 3400 mean
```

```
favstats(~ mean, data = WeightMeans)

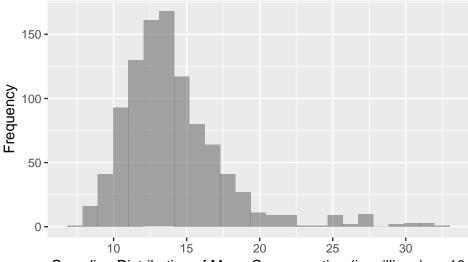
## min Q1 median Q3 max mean sd n missing
## 3132.17 3255.54 3282.14 3309.163 3422.09 3282.299 40.01813 10000 0
```

Section 14.1: The Central Limit Theorem

```
CEOComp <- read_csv("http://nhorton.people.amherst.edu/is5/data/CEO_Compensation_2014.csv") %>%
  clean_names()
## Parsed with column specification:
##
     Employer = col_character(),
##
     CEO = col_character(),
     CEO_Compensation = col_integer(),
##
##
    Median_Worker_Comp = col_integer(),
     Ratio = col_integer(),
##
##
     Company_Rating = col_double(),
##
     `CEO_Compensation_($M)` = col_double()
## )
# Figure 14.3
gf_histogram(~ ceo_compensation_m, data = CEOComp, binwidth = 10) %>%
 gf_labs(x = "CEO Compensation in $1,000,000", y = "Frequency")
```

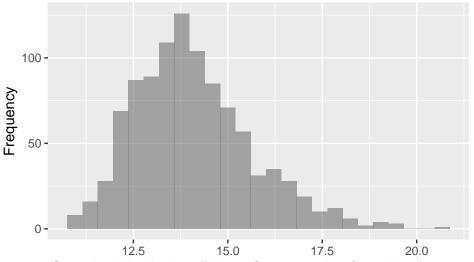


```
# Figure 14.4
set.seed(14354)
numsim <- 1000 # Here the number of simulations is 1,000
CEOMeans <- do(numsim) * mean(~ ceo_compensation_m, data = sample(CEOComp, size = 10))
gf_histogram(~ mean, data = CEOMeans) %>%
    gf_labs(x = "Sampling Distribution of Mean Compensation (in millions), n=10", y = "Frequency")
```



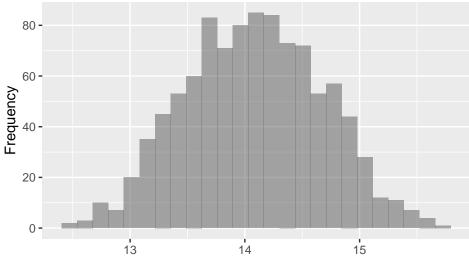
Sampling Distribution of Mean Compensation (in millions), n=10

```
# Figure 14.5
CEOMeans2 <- do(numsim) * mean(~ ceo_compensation_m, data = sample(CEOComp, size = 50))
gf_histogram(~ mean, data = CEOMeans2) %>%
gf_labs(x = "Sampling Distribution of Mean Compensation (in millions), n=50", y = "Frequency")
```



Sampling Distribution of Mean Compensation (in millions), n=50

```
# Figure 14.7 (skipped 14.6 because it's similar)
CEOMeans3 <- do(numsim) * mean(~ ceo_compensation_m, data = sample(CEOComp, size = 200))
gf_histogram(~ mean, data = CEOMeans3) %>%
gf_labs(x = "Sampling Distribution of Mean Compensation (in millions), n=200", y = "Frequency")
```

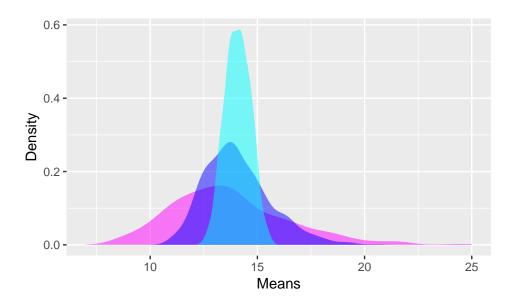


Sampling Distribution of Mean Compensation (in millions), n=200

Here, in each example, the do() function calculates the mean of random samples of 10, 50, and 200 CEO compensations.

```
# Samples as overlaid density plots
gf_density(~ mean, data = CEOMeans, fill = 6) %>%
gf_density(~ mean, data = CEOMeans2, fill = 4) %>%
gf_density(~ mean, data = CEOMeans3, fill = 5) %>%
gf_labs(y = "Density", x = "Means") +
xlim(7, 25)
```

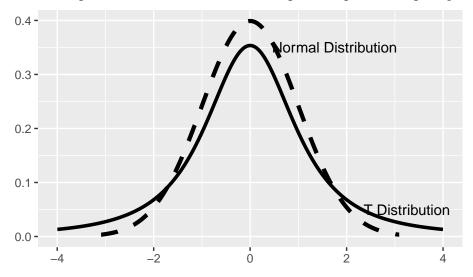
Warning: Removed 27 rows containing non-finite values (stat_density).



Section 14.2: A Confidence Interval for the Mean

```
# Figure 14.9, page 446
gf_dist(dist = "norm", linetype = 2, lwd = 1.5) %>%
    gf_refine(annotate(geom = "text", x = 1.75, y = .35, label = "Normal Distribution")) %>%
    gf_labs(x = "", y = "") %>%
    gf_dist(dist = "t", df = 2, lwd = 1.25) %>%
    gf_refine(annotate(geom = "text", x = 3.25, y = .05, label = "T Distribution")) +
    xlim(-4, 4)
```

Warning: Removed 4104 rows containing missing values (geom_path).



Example 14.1: A One-Sample t-Interval for the Mean

```
Salmon <- read_csv("http://nhorton.people.amherst.edu/is5/data/Farmed_salmon.csv") %>%
    clean_names()
```

Parsed with column specification:

```
## cols(
##
     Kind = col_character(),
##
     Location = col_character(),
##
     Mirex = col_double(),
##
     Hexachlorobenzene = col_double(),
     HCH gamma = col double(),
##
     `Heptachlor Epoxide` = col double(),
##
     Dieldrin = col_double(),
##
##
     Endrin = col_double(),
##
     `Total Chlordane` = col_double(),
     `Total DDT` = col_double(),
     Dioxin = col_double(),
##
##
     `Total Pesticides` = col_double(),
     `Total PCBs` = col_integer()
##
## )
Salmon <- Salmon %>%
  filter(mirex != "NA")
favstats(~ mirex, data = Salmon)
## min
           Q1 median
                          Q3 max
                                      mean
      0 0.056 0.079 0.13475 0.194 0.09134 0.04952388 150
t.test(~ mirex, data = Salmon)
##
##
   One Sample t-test
##
## data: mirex
## t = 22.589, df = 149, p-value < 2.2e-16
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
## 0.08334978 0.09933022
## sample estimates:
## mean of x
     0.09134
salmonlm <- lm(mirex ~ 1, data = Salmon) # equiv to t.test(~ mirex)</pre>
# replication of interval from page 448
confint(salmonlm, data = Salmon)
                              97.5 %
##
                    2.5 %
## (Intercept) 0.08334978 0.09933022
tstats <- xqt(df = 149, p = c(.025, .975), plot = FALSE) # t statistics
sey <- sd(~ mirex, data = Salmon)/(150^(1/2)) # standard error</pre>
mean(~ mirex, data = Salmon) + sey * tstats # calculations match those from confint
## [1] 0.08334978 0.09933022
```

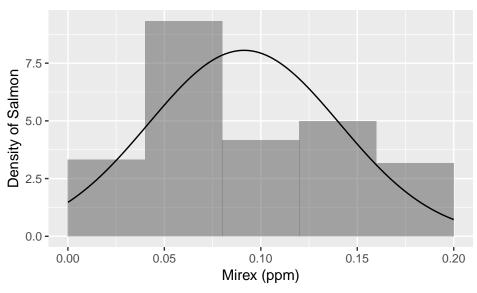
The confint() function takes an object, in this case a linear regression model, as an argument.

Example 14.2: Checking Assumptions and Conditions for Student's t

```
# replicate figure in Example 14.2 with a normal distribution (page 450)
gf_dhistogram(~ mirex, data = Salmon, binwidth = 0.04, center = 0.02) %>%
gf_dist(dist = "norm", mean = mean(~ mirex, data = Salmon), sd = sd(~ mirex, data = Salmon)) %>%
```

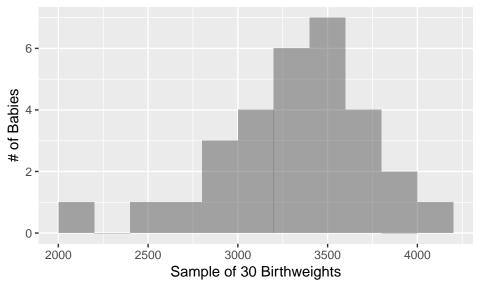
```
gf_labs(x = "Mirex (ppm)", y = "Density of Salmon") +
xlim(0, .2)
```

Warning: Removed 1733 rows containing missing values (geom_path).



Step-By-Step Example: A One-Sample t-Interval for the Mean

```
set.seed(34)
BabiesSample <- sample(Babies, size = 30)
gf_histogram(~ weight, data = BabiesSample, binwidth = 200, center = 100) %>%
gf_labs(x = "Sample of 30 Birthweights", y = "# of Babies")
```



```
favstats(~ weight, data = BabiesSample)
```

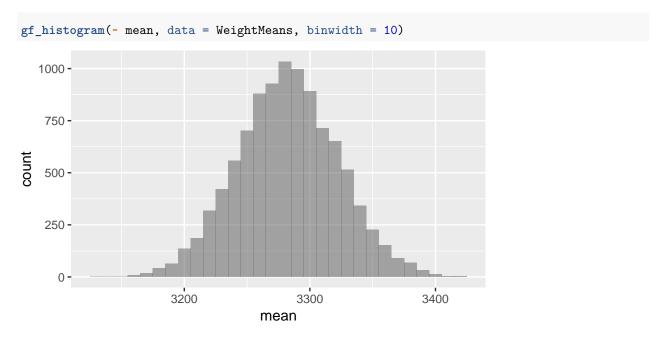
```
## min Q1 median Q3 max mean sd n missing ## 2182 3056.25 3382.5 3572 4054 3316.6 423.2446 30 0
```

```
babieslm <- lm(weight ~ 1, data = BabiesSample)
confint(babieslm, level = 0.90)

## 5 % 95 %
## (Intercept) 3185.302 3447.898</pre>
```

Section 14.3: Interpreting Confidence Intervals

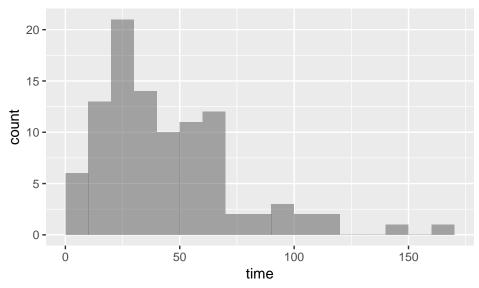
Section 14.4: Picking Our Interval up by Our Bootstraps



Step-By-Step Example: A Bootstrap Confidence Interval for the Mean

```
CommuteSample <- read_csv("http://nhorton.people.amherst.edu/is5/data/Commuter_sample.csv")
## Parsed with column specification:
## cols(
## time = col_integer()
## )

gf_histogram(~ time, data = CommuteSample, binwidth = 10, center = 5)</pre>
```



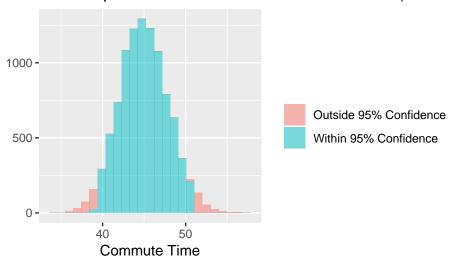
##

```
# Bootstrap
set.seed(134) # For reproducibility
numsim <- 10000
commutebootstrap <- do(numsim) * mean(~ time, data = resample(CommuteSample))</pre>
The resample() function samples of that data set size with replacement.
qdata(~ mean, p = c(.025, .975), data = commutebootstrap) # grab the percentiles with qdata
##
         quantile
## 2.5%
            39.28 0.025
## 97.5%
            50.98 0.975
confint(commutebootstrap, method = "quantile") # an equivalent quantile approach
```

```
name lower upper level
## 1 mean 39.28 50.98 0.95 percentile
                                          44.98
commutebootstrap <- commutebootstrap %>%
 mutate(interval = ifelse(mean > 39.28 & mean < 50.98, "Within 95% Confidence",</pre>
                           "Outside 95% Confidence")) # for fill
gf_histogram(~ mean, fill = ~ interval, data = commutebootstrap,
             title = "Bootstrap Estimates of Mean Commute Times (minutes)") %>%
  gf_labs(x = "Commute Time", y = "", fill = "")
```

method estimate

Bootstrap Estimates of Mean Commute Times (minutes



Section 14.5: Thoughts About Confidence Intervals