IS5 in R: Comparing Counts (Chapter 19)

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July 23, 2018

Introduction and background

This document is intended to help describe how to undertake analyses introduced as examples in the Fifth Edition of *Intro Stats* (2018) by De Veaux, Velleman, and Bock. More information about the book can be found at http://wps.aw.com/aw_deveaux_stats_series. This file as well as the associated R Markdown reproducible analysis source file used to create it can be found at http://nhorton.people.amherst.edu/is5.

This work leverages initiatives undertaken by Project MOSAIC (http://www.mosaic-web.org), an NSF-funded effort to improve the teaching of statistics, calculus, science and computing in the undergraduate curriculum. In particular, we utilize the mosaic package, which was written to simplify the use of R for introductory statistics courses. A short summary of the R needed to teach introductory statistics can be found in the mosaic package vignettes (http://cran.r-project.org/web/packages/mosaic). A paper describing the mosaic approach was published in the R Journal: https://journal.r-project.org/archive/2017/RJ-2017-024.

Chapter 19: Comparing Counts

```
library(mosaic)
library(readr)
library(janitor)
Zodiac <- read_csv("http://nhorton.people.amherst.edu/is5/data/Zodiac.csv")

## Parsed with column specification:
## cols(
## Month = col_character(),
## Births = col_integer(),
## Expected = col_double(),
## Residual = col_double()</pre>
```

By default, read_csv() prints the variable names. These messages can be suppressed using the message=FALSE code chunk option to save space and improve readability.

```
Zodiac %>%
select(Month, Births)
```

```
## # A tibble: 12 x 2
##
      Month
                   Births
##
      <chr>
                    <int>
##
    1 Pisces
                       29
##
                       24
    2 Aquarius
##
    3 Aries
                       23
##
    4 Cancer
                       23
                       22
##
    5 Capricorn
##
   6 Scorpio
                       21
   7 Taurus
                       20
##
    8 Leo
                       20
   9 Saggitarius
                       19
```

```
## 10 Virgo 19
## 11 Libra 18
## 12 Gemini 18
```

Section 19.1: Goodness-of-Fit Tests

Example 19.1: Finding Expected Counts

```
# page 611
BaseballBirths <- read_csv("http://nhorton.people.amherst.edu/is5/data/Ballplayer_births.csv") %>%
  clean_names() # doesn't contain national birth %
## Parsed with column specification:
## cols(
##
    Month = col_integer(),
     `Ballplayer Count` = col_integer()
##
## )
Here we use the clean_names() function from the janitor package to sanitize the names of the columns
(which would otherwise contain special characters or whitespace).
natbirth <- c(.08, .07, .08, .08, .08, .08, .09, .09, .09, .09, .09, .09)
BaseballBirths <- cbind(BaseballBirths, natbirth) # adding a column for national birth %
totaln <- sum(~ ballplayer_count , data = BaseballBirths)</pre>
totaln
## [1] 1478
BaseballBirths <- BaseballBirths %>%
  mutate(expected = totaln * natbirth,
         observed = ballplayer_count,
         contrib = (observed - expected)^2/expected)
sum(~ contrib, data = BaseballBirths)
## [1] 26.48442
```

Assumptions and Conditions

Calculations

Chi-Square P-values

```
# Examples of chisq p-values
qchisq(df = 2, p = .1, lower.tail = FALSE)

## [1] 4.60517
qchisq(df = 10, p = .05, lower.tail = FALSE)

## [1] 18.30704
```

Example 19.3: Doing a Goodness-of-Fit Test

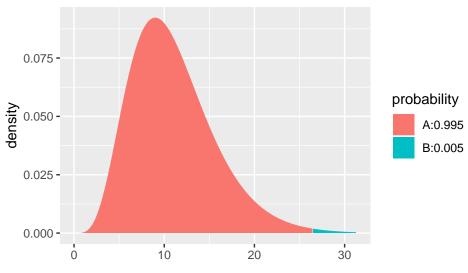
```
# page 614
df <- nrow(BaseballBirths) - 1
df</pre>
```

```
## [1] 11
```

```
# XX Can't seem to get the right value
# Getting the chisq statistic
# with(chisq.test(tally(~ ballplayer_count | expected, data = BaseballBirths)), statistic)
# with(chisq.test(tally(natbirth ~ ballplayer_count, data = BaseballBirths)), statistic)
# Alternate method
BaseballBirths %>%
    mutate(chisq = ((ballplayer_count - expected)^2)/expected) %>%
    summarise(sum(chisq))
```

sum(chisq) ## 1 26.48442

xpchisq(q = 26.5, df = df, lower.tail = FALSE)



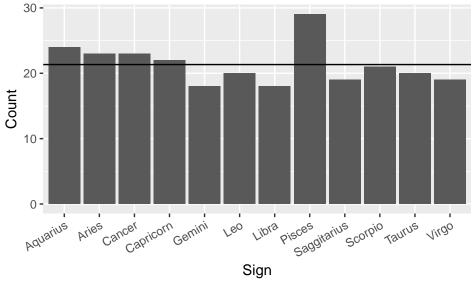
[1] 0.005464563

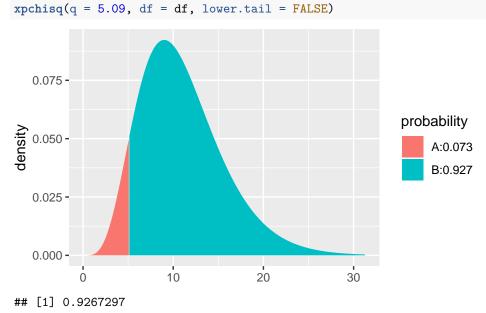
Step-By-Step Example: A Chi-Square Test for Goodness-of-Fit

```
expected <- mean(~ Births, data = Zodiac)
expected</pre>
```

```
## [1] 21.33333
```

```
gf_col(Births ~ Month, data = Zodiac) %>%
gf_hline(yintercept = expected) %>%
gf_labs(x = "Sign", y = "Count") +
theme(axis.text.x = element_text(angle = 30, hjust = 1)) # to adjust the angle of the x axis labels
```





The Chi-Square Calculation

```
Zodiac %>%
 mutate(residsq = Residual^2) %>%
 mutate(component = residsq/Expected)
## # A tibble: 12 x 6
##
     Month
                 Births Expected Residual residsq component
##
      <chr>
                 <int>
                           <dbl>
                                    <dbl>
                                            <dbl>
                                                     <dbl>
## 1 Pisces
                            21.3
                                    7.67
                                           58.8
                                                    2.76
                     29
## 2 Aquarius
                     24
                            21.3
                                   2.67
                                           7.11
                                                   0.333
## 3 Aries
                     23
                            21.3
                                   1.67
                                            2.78
                                                   0.130
                                                   0.130
## 4 Cancer
                     23
                            21.3
                                   1.67
                                           2.78
## 5 Capricorn
                     22
                            21.3
                                  0.667
                                           0.445
                                                  0.0209
## 6 Scorpio
                     21
                            21.3
                                   -0.333
                                           0.111
                                                   0.00520
## 7 Taurus
                     20
                            21.3
                                   -1.33
                                            1.78
                                                   0.0833
                            21.3
## 8 Leo
                     20
                                   -1.33
                                           1.78
                                                   0.0833
## 9 Saggitarius
                    19
                            21.3
                                   -2.33
                                           5.44
                                                   0.255
## 10 Virgo
                     19
                            21.3
                                   -2.33
                                           5.44
                                                   0.255
## 11 Libra
                     18
                                                   0.521
                            21.3
                                   -3.33
                                          11.1
## 12 Gemini
                     18
                            21.3
                                   -3.33
                                          11.1
                                                   0.521
```

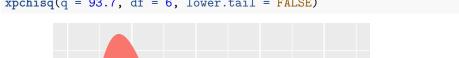
The Trouble with Goodness-of-Fit Tests: What's the Alternative?

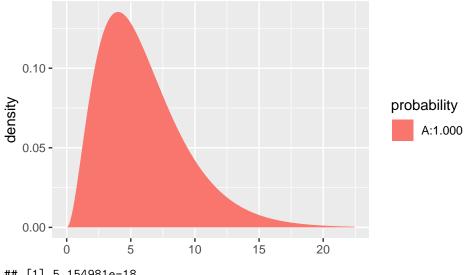
Section 19.2: Chi-Square Test of Homogeneity

```
# Create the data set
Postgrad <- rbind(</pre>
  do(209) * data.frame(activity = "Employed", school = "Agriculture"),
  do(198) * data.frame(activity = "Employed", school = "Arts & Sciences"),
  do(177) * data.frame(activity = "Employed", school = "Engineering"),
  do(101) * data.frame(activity = "Employed", school = "ILR"),
  do(104) * data.frame(activity = "Grad School", school = "Agriculture"),
  do(171) * data.frame(activity = "Grad School", school = "Arts & Sciences"),
  do(158) * data.frame(activity = "Grad School", school = "Engineering"),
  do(33) * data.frame(activity = "Grad School", school = "ILR"),
  do(135) * data.frame(activity = "Other", school = "Agriculture"),
  do(115) * data.frame(activity = "Other", school = "Arts & Sciences"),
  do(39) * data.frame(activity = "Other", school = "Engineering"),
  do(16) * data.frame(activity = "Other", school = "ILR")
)
# Table 19.1, page 618
tally(activity ~ school, data = Postgrad, margins = TRUE)
##
## activity
                 Agriculture Arts & Sciences Engineering ILR
##
    Employed
                         209
                                         198
                                                      177 101
     Grad School
##
                         104
                                         171
                                                      158 33
##
     Other
                         135
                                         115
                                                      39 16
##
    Total
                         448
                                         484
                                                      374 150
# Table 19.2
tally(activity ~ school, format = "percent", data = Postgrad, margins = TRUE)
```

```
##
                 school
                  Agriculture Arts & Sciences Engineering
## activity
                                                                  TI.R.
                                     40.90909
##
     Employed
                     46.65179
                                                  47.32620
                                                            67.33333
     Grad School
                     23.21429
                                     35.33058
                                                  42.24599
##
                                                            22.00000
##
     Other
                     30.13393
                                      23.76033
                                                  10.42781
                                                            10.66667
##
     Total
                    100.00000
                                     100.00000
                                                 100.00000 100.00000
# Table 19.3
with(chisq.test(tally(activity ~ school, data = Postgrad, margins = TRUE)), expected)
##
                 school
## activity
                  Agriculture Arts & Sciences Engineering
                                                                  ILR
##
     Employed
                    210.76923
                                     227.7060
                                                 175.95467
                                                             70.57005
##
     Grad School
                    143.38462
                                      154.9066
                                                 119.70055
                                                             48.00824
##
     Other
                     93.84615
                                     101.3874
                                                  78.34478
                                                            31.42170
##
     Total
                    448.00000
                                      484.0000
                                                 374.00000 150.00000
Step-By-Step Example: A Chi-Square Test for Homogeneity
tally(activity ~ school, format = "percent", data = Postgrad) %>%
  data.frame() %>%
  gf_col(Freq ~ school, fill = ~ activity, position = "dodge") %>%
  gf labs(x = "School", y = "Percent", fill = "")
  60 -
Percent
  40
                                                                                    Employed
                                                                                    Grad School
                                                                                    Other
  20 -
   0 -
                                                                 ILR
           Agriculture
                          Arts & Sciences
                                             Engineering
                                      School
# Mechanics
tally(activity ~ school, data = Postgrad, margins = TRUE)
##
                 school
## activity
                  Agriculture Arts & Sciences Engineering ILR
##
     Employed
                          209
                                           198
                                                        177 101
##
     Grad School
                          104
                                           171
                                                        158 33
##
     Other
                          135
                                           115
                                                        39 16
     Total
                          448
                                           484
                                                        374 150
with(chisq.test(tally(activity ~ school, data = Postgrad, margins = TRUE)), expected)
##
## activity
                  Agriculture Arts & Sciences Engineering
                                                                  ILR
     Employed
                    210.76923
                                     227.7060
                                                 175.95467 70.57005
```

```
Grad School
##
                  143.38462
                                    154.9066
                                               119.70055 48.00824
##
     Other
                   93.84615
                                    101.3874
                                              78.34478 31.42170
                  448.00000
                                    484.0000
##
     Total
                                               374.00000 150.00000
with(chisq.test(tally(activity ~ school, data = Postgrad)), statistic)
## X-squared
## 93.65667
xpchisq(q = 93.7, df = 6, lower.tail = FALSE)
```





[1] 5.154981e-18

Section 19.3: Examining the Residuals

```
# Table 19.4, page 622
with(chisq.test(tally(activity ~ school, data = Postgrad, margins = TRUE)), residuals)
##
                school
## activity
                 Agriculture Arts & Sciences Engineering
                                                                 ILR
                                 -1.96860027 0.07880484 3.62235442
##
     Employed
                 -0.12186553
##
     Grad School -3.28908677
                                  1.29304319 3.50061599 -2.16606715
##
     Other
                  4.24817296
                                  1.35191804 -4.44510568 -2.75117035
##
     Total
                  0.00000000
                                  0.00000000 0.00000000 0.00000000
```

Example 19.4: Looking at χ^2 , Residuals

```
BaseballBirths %>%
  mutate(residuals = (ballplayer_count - expected)/(expected^.5)) %>%
 select(month, residuals)
```

```
##
     month
            residuals
## 1
         1 1.72524439
## 2
         2 1.72442119
## 3
         3 -0.20599933
## 4
         4 0.25382060
## 5
         5 0.71364054
## 6
         6 -0.38992730
```

```
## 7 7 -2.68957291

## 8 8 2.77280921

## 9 9 0.08497039

## 10 10 -1.56241469

## 11 11 -1.21760318

## 12 12 -0.95548335
```

Section 19.4: Chi-Square Test of Independence

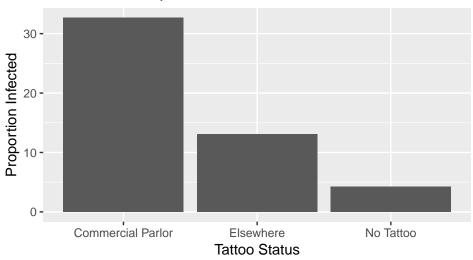
```
Tattoos <- read_csv("http://nhorton.people.amherst.edu/is5/data/Tattoos.csv", skip = 1) %>%
  clean names() # skip = 1 because first row is "Col1", "Col2"
## Parsed with column specification:
## cols(
     Location = col_character(),
##
     `Has hepatitis C` = col_character()
## )
# Table 19.5, page 623
tally(location ~ has_hepatitis_c, data = Tattoos, margins = TRUE)
##
                      has_hepatitis_c
## location
                        No Yes
##
     Commercial Parlor 35 17
##
     Elsewhere
                        53
                       491 22
##
    No Tattoo
                       579 47
##
    Total
```

Assumptions and Conditions

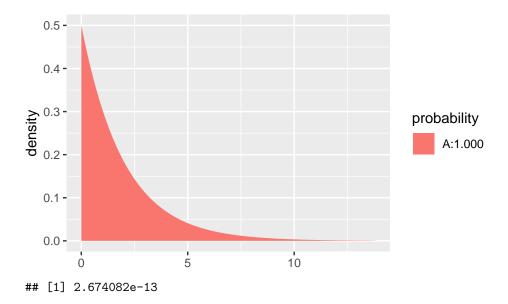
Step-By-Step Example: A Chi-Square Test for Independence

```
tally(has_hepatitis_c ~ location, format = "percent", data = Tattoos) %>%
  data.frame() %>%
  filter(has_hepatitis_c == "Yes") %>%
  gf_col(Freq ~ location) %>%
  gf_labs(x = "Tattoo Status", y = "Proportion Infected", title = "Tattoos and Hepatitis C")
```

Tattoos and Hepatitis C



```
# Observed
tally(location ~ has_hepatitis_c, data = Tattoos, margins = TRUE)
##
                      has_hepatitis_c
## location
                        No Yes
     Commercial Parlor
                        35
##
                           17
##
     Elsewhere
                        53
##
     No Tattoo
                       491 22
     Total
                       579
                           47
# Expected
with(chisq.test(tally(location ~ has_hepatitis_c, data = Tattoos, margins = TRUE)), expected)
## Warning in chisq.test(tally(location ~ has_hepatitis_c, data = Tattoos, :
## Chi-squared approximation may be incorrect
##
                      has_hepatitis_c
## location
     Commercial Parlor 48.09585 3.904153
##
##
     Elsewhere
                        56.42013 4.579872
##
     No Tattoo
                       474.48403 38.515974
                       579.00000 47.000000
##
     Total
# Mechanics
with(chisq.test(tally(location ~ has_hepatitis_c, data = Tattoos)), statistic)
## Warning in chisq.test(tally(location ~ has_hepatitis_c, data = Tattoos)):
## Chi-squared approximation may be incorrect
## X-squared
## 57.91217
xpchisq(q = 57.9, df = 2, lower.tail = FALSE)
```



Examine the Residuals

```
# Table 19.6, page 627
with(chisq.test(tally(location ~ has_hepatitis_c, data = Tattoos)), residuals)
## Warning in chisq.test(tally(location ~ has_hepatitis_c, data = Tattoos)):
## Chi-squared approximation may be incorrect
##
                     has_hepatitis_c
## location
                              No
                                        Yes
##
    Commercial Parlor -1.8883383 6.6278115
    Elsewhere -0.4553290 1.5981431
##
    No Tattoo
                       0.7582168 -2.6612383
##
# Table 19.7, page 628
Tattoos <- Tattoos %>%
 mutate(tattoo = ifelse(location == "No Tattoo", "None", "Tattoo"))
tally(tattoo ~ has_hepatitis_c, margins = TRUE, data = Tattoos)
##
          has_hepatitis_c
            No Yes
## tattoo
##
    None
            491 22
     Tattoo 88 25
##
##
     Total 579 47
```

Chi-Square and Causation