

IS5 in R: Multiple Regression (Chapter 9)

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Introduction and background

This document is intended to help describe how to undertake analyses introduced as examples in the Fifth Edition of *Intro Stats* (2018) by De Veaux, Velleman, and Bock. This file as well as the associated Quarto reproducible analysis source file used to create it can be found at <http://nhorton.people.amherst.edu/is5>.

This work leverages initiatives undertaken by Project MOSAIC (<http://www.mosaic-web.org>), an NSF-funded effort to improve the teaching of statistics, calculus, science and computing in the undergraduate curriculum. In particular, we utilize the `mosaic` package, which was written to simplify the use of R for introductory statistics courses. A short summary of the R needed to teach introductory statistics can be found in the `mosaic` package vignettes (<https://cran.r-project.org/web/packages/mosaic>). A paper describing the `mosaic` approach was published in the *R Journal*: <https://journal.r-project.org/archive/2017/RJ-2017-024>.

We begin by loading packages that will be required for our analyses.

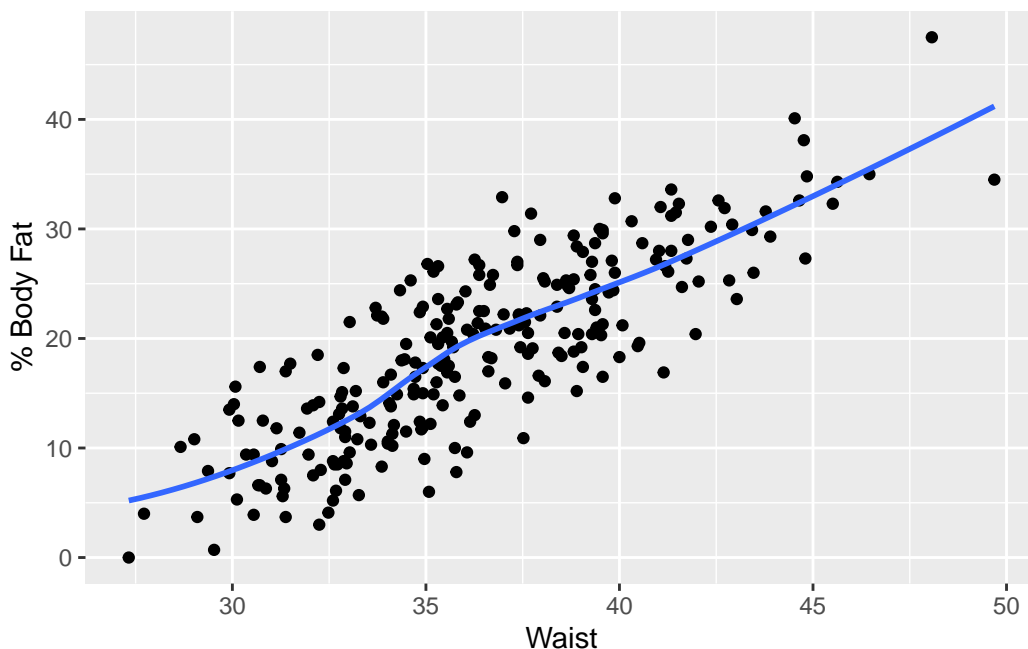
```
library(mosaic)
library(tidyverse)
library(broom) # We'll use this for augment() later
```

Chapter 9: Multiple Regression

```
BodyFat <- read_csv("http://nhorton.people.amherst.edu/is5/data/Bodyfat.csv") |>
  janitor::clean_names()
```

By default, `read_csv()` prints the variable names. These messages have been suppressed using the `message: false` code chunk option to save space and improve readability. Here we use the `clean_names()` function from the `janitor` package to sanitize the names of the columns (which would otherwise contain special characters or whitespace).

```
# Figure 9.1, page 276
gf_point(pct_bf ~ waist, data = BodyFat) |>
  gf_labs(x = "Waist", y = "% Body Fat") |>
  gf_smooth()
```



We've added `gf_smooth()` to demonstrate how to add a smoother.

Section 9.1: What is Multiple Regression?

```
# Table 9.1, page 277
multiplereg <- lm(pct_bf ~ waist + height, data = BodyFat)
summary(multiplereg)
```

```
Call:
lm(formula = pct_bf ~ waist + height, data = BodyFat)
```

```
Residuals:
```

	Min	1Q	Median	3Q	Max
	-11.1692	-3.4133	-0.0977	3.0995	9.9082

```
Coefficients:
```

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	-3.10088	7.68611	-0.403	0.687
waist	1.77309	0.07158	24.770	< 2e-16 ***
height	-0.60154	0.10994	-5.472	1.09e-07 ***

```
---
```

```
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
Residual standard error: 4.46 on 247 degrees of freedom
```

```
Multiple R-squared:  0.7132,    Adjusted R-squared:  0.7109
```

```
F-statistic: 307.1 on 2 and 247 DF,  p-value: < 2.2e-16
```

The `summary()` function provides the multiple R-squared along with the regression coefficients.

```
msummary(multiplereg)
```

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	-3.10088	7.68611	-0.403	0.687
waist	1.77309	0.07158	24.770	< 2e-16 ***
height	-0.60154	0.10994	-5.472	1.09e-07 ***

```
Residual standard error: 4.46 on 247 degrees of freedom
```

```
Multiple R-squared:  0.7132,    Adjusted R-squared:  0.7109
```

```
F-statistic: 307.1 on 2 and 247 DF,  p-value: < 2.2e-16
```

The `msummary()` function in the `mosaic` package provides a pruned version of the same output.

```
broom::tidy(multiplereg)
```

```
# A tibble: 3 x 5
```

term	estimate	std.error	statistic	p.value
------	----------	-----------	-----------	---------

	<chr>	<dbl>	<dbl>	<dbl>	<dbl>
1 (Intercept)		-3.10	7.69	-0.403	6.87e- 1
2 waist		1.77	0.0716	24.8	6.79e-69
3 height		-0.602	0.110	-5.47	1.09e- 7

The `tidy()` function in the `broom` package provides similar information as a `tibble/dataframe`.

Example 9.1: Modeling Home Prices

```
RealEstate <- read_csv("http://nhorton.people.amherst.edu/is5/data/Real_Estate.csv") |>
  janitor::clean_names()
realestatelm <- lm(price ~ living_area + bedrooms, data = RealEstate)
summary(realestatelm)
```

Call:

```
lm(formula = price ~ living_area + bedrooms, data = RealEstate)
```

Residuals:

Min	1Q	Median	3Q	Max
-433211	-198136	-63249	137183	1054177

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	308100.44	41147.84	7.488	1.69e-13 ***
living_area	135.09	11.48	11.771	< 2e-16 ***
bedrooms	-43346.81	12844.14	-3.375	0.000771 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 266900 on 891 degrees of freedom

Multiple R-squared: 0.1463, Adjusted R-squared: 0.1444

F-statistic: 76.34 on 2 and 891 DF, p-value: < 2.2e-16

Here we demonstrate how to create a function in R that can be used to calculate predicted values from a regression model.

```
# Predicted Values
realestatefn <- makeFun(realestatelm) # Making a function to find predicted values
# Predicted price for a home with 2800 sq ft living area and 5 bedrooms
realestatefn(living_area = 2800, bedrooms = 5)
```

```
1
469614.9
```

```
# Predicted price for a home with 2801 sq ft living area and 5 bedrooms
realestatefn(living_area = 2801, bedrooms = 5)
```

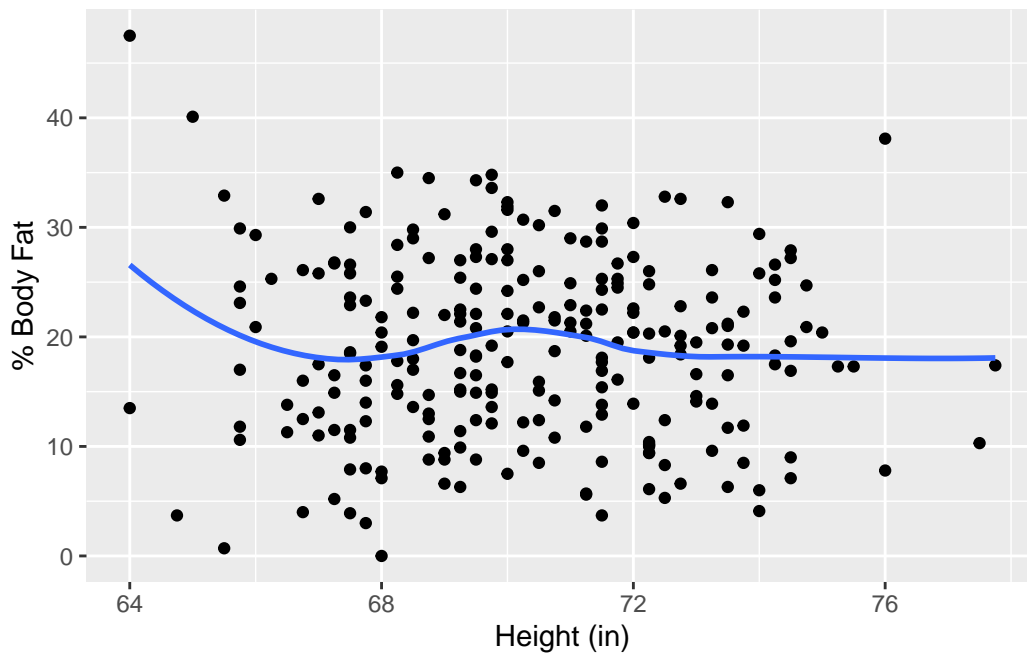
```
1
469750
```

```
# If we subtract predicted values one value apart, we get the slope
realestatefn(living_area = 2801, bedrooms = 5) -
realestatefn(living_area = 2800, bedrooms = 5)
```

```
1
135.0887
```

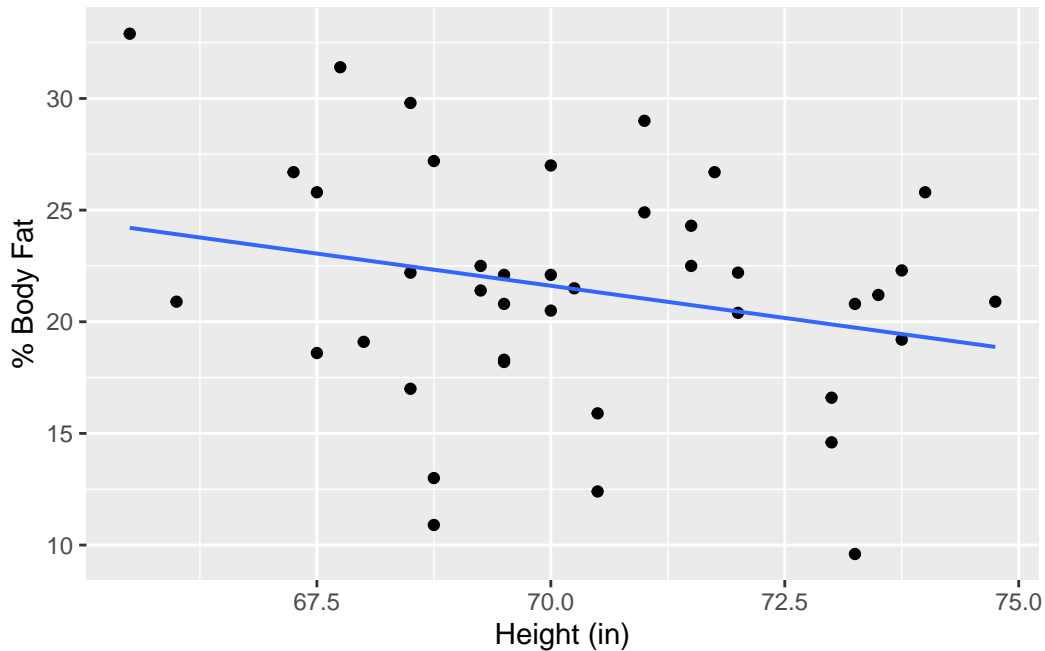
Section 9.2: Interpreting Multiple Regression Coefficients

```
# Figure 9.2, page 279
gf_point(pct_bf ~ height, data = BodyFat) |>
  gf_smooth() |> # Added a smoother to assess linearity
  gf_labs(x = "Height (in)", y = "% Body Fat")
```

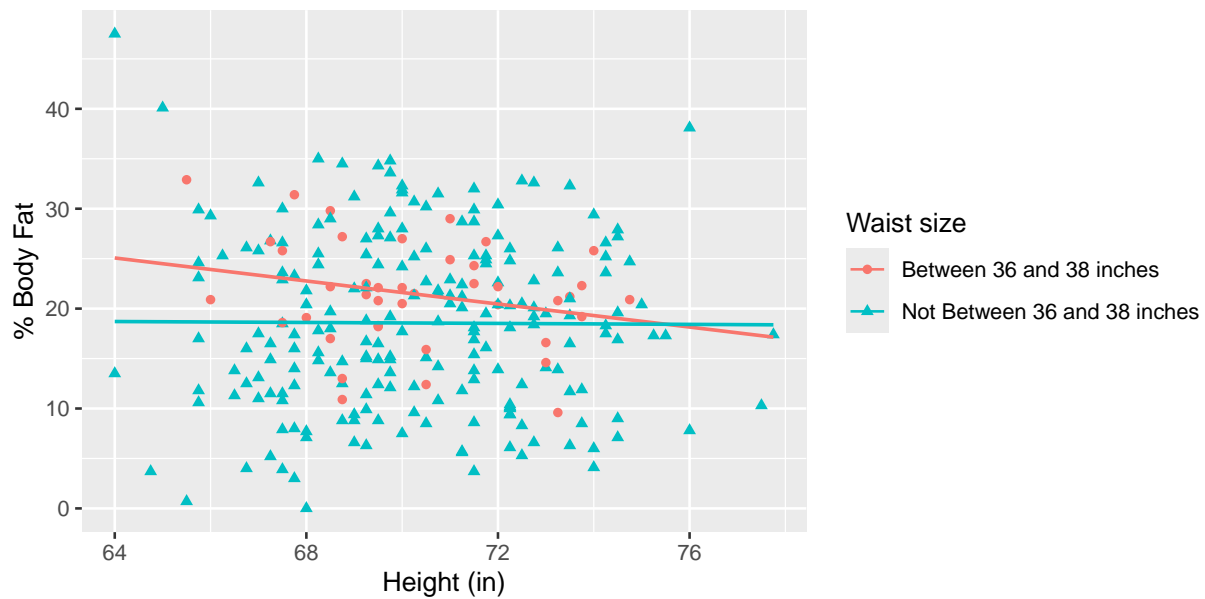


A message about the default smoother option was suppressed by adding `message: false` as a code chunk option.

```
# Figure 9.3
BodyFat |>
  filter(waist >= 36 & waist <= 38) |> # Just plotting waist sizes between 36 and 38 inches
  gf_point(pct_bf ~ height) |>
  gf_labs(x = "Height (in)", y = "% Body Fat") |>
  gf_lm()
```



```
# Plotting all points
BodyFat |>
  mutate(waistsize = ifelse(waist >= 36 & waist <= 38, "Between 36 and 38 inches",
    "Not Between 36 and 38 inches"
  )) |> # Subsetting
  gf_point(pct_bf ~ height, shape = ~ waistsize, color = ~ waistsize) |>
  gf_labs(
    x = "Height (in)",
    y = "% Body Fat",
    shape = "Waist size",
    color = "Waist size"
  ) |>
  gf_lm()
```



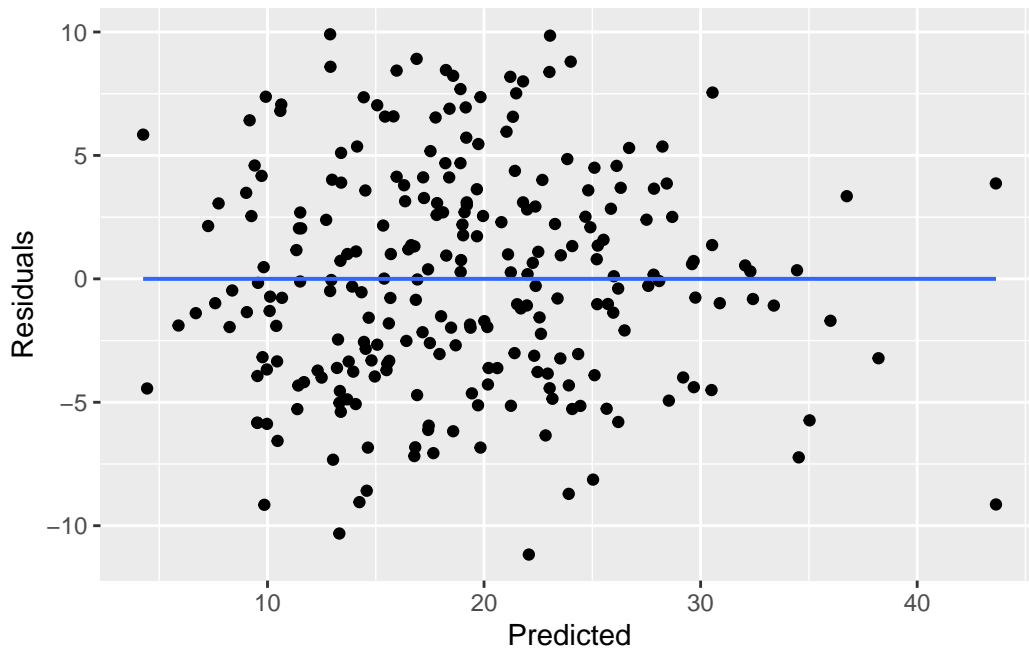
Section 9.3: The Multiple Regression Model—Assumptions and Conditions

Linearity Assumption

Equal Variance Assumption

We can assess the equal variance assumption in several ways. The simplest is through a scatterplot of residuals vs. fitted values.

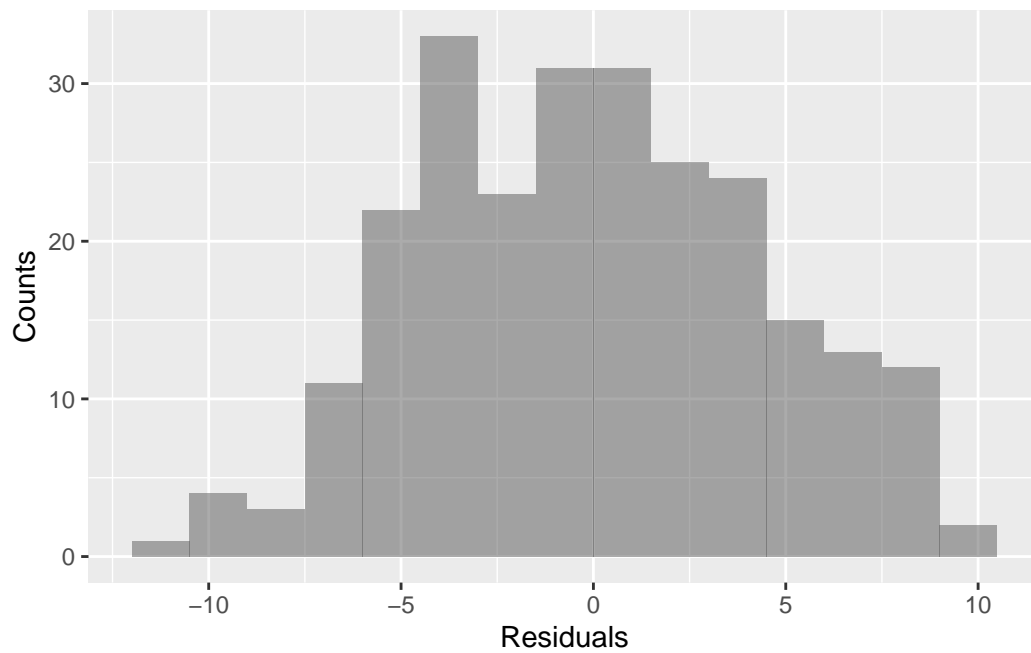
```
bodyfatlm <- lm(pct_bf ~ waist + height, data = BodyFat)
# Figure 9.4, page 282
gf_point(resid(bodyfatlm) ~ fitted(bodyfatlm)) |>
  gf_lm() |>
  gf_labs(x = "Predicted", y = "Residuals")
```



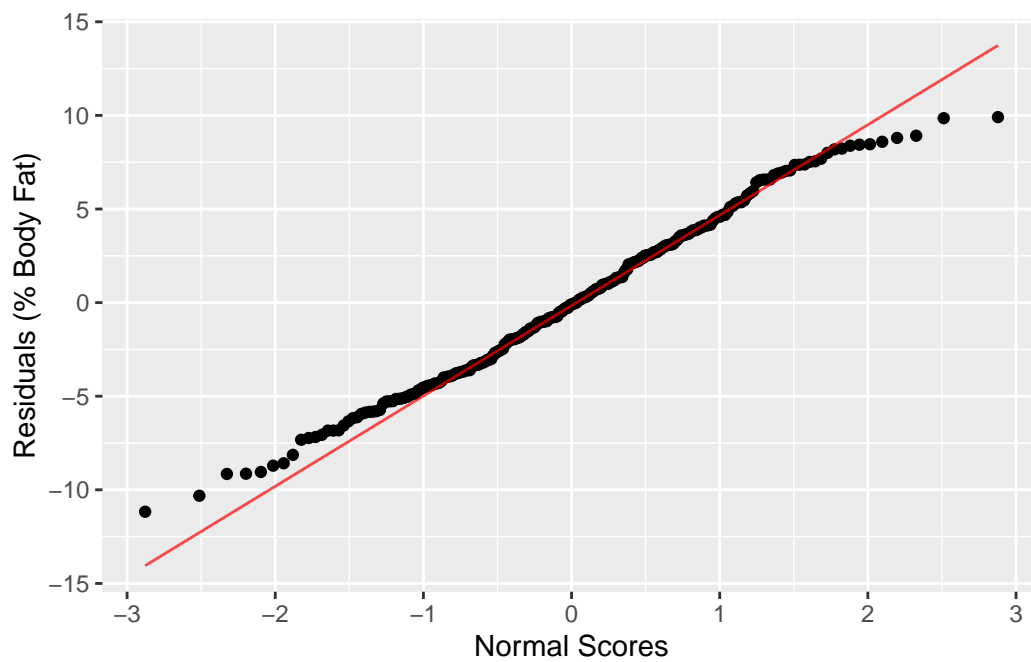
Check the Residuals

It's important to look at the residuals to see if the “Nearly Normal” condition is reasonable.

```
# Figure 9.5
gf_histogram(~ resid(bodyfatlm), binwidth = 1.5, center = 0.75) |>
  gf_labs(x = "Residuals", y = "Counts")
```

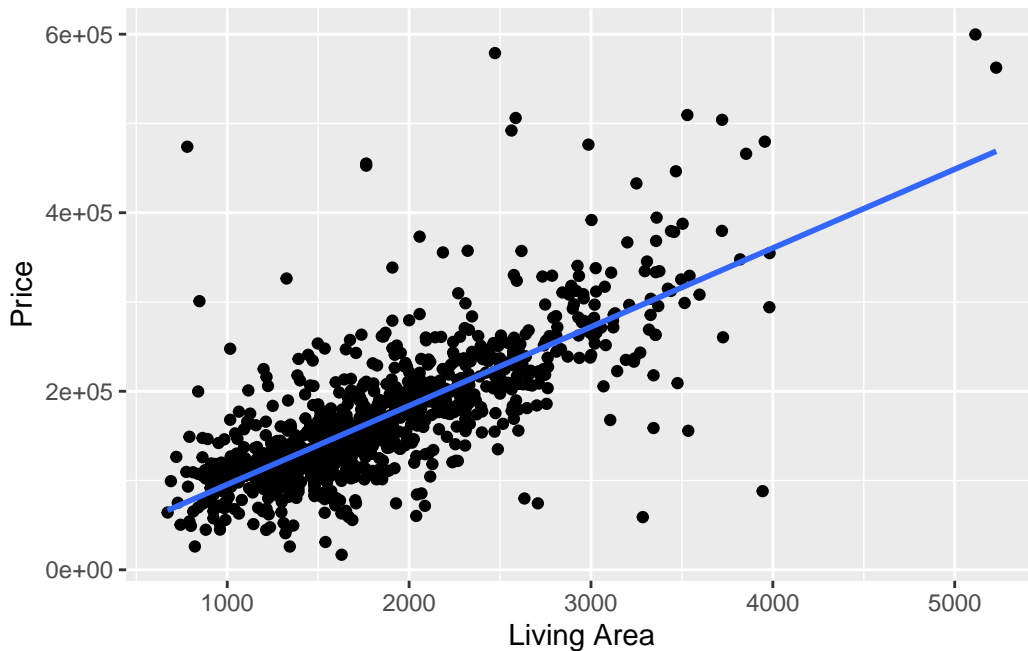
```
gf_qq(~ resid(bodyfatlm)) |>  
  gf_qqline(linetype = "solid", color = "red") |>  
  gf_labs(x = "Normal Scores", y = "Residuals (% Body Fat)")
```



Step-By-Step Example: Multiple Regression

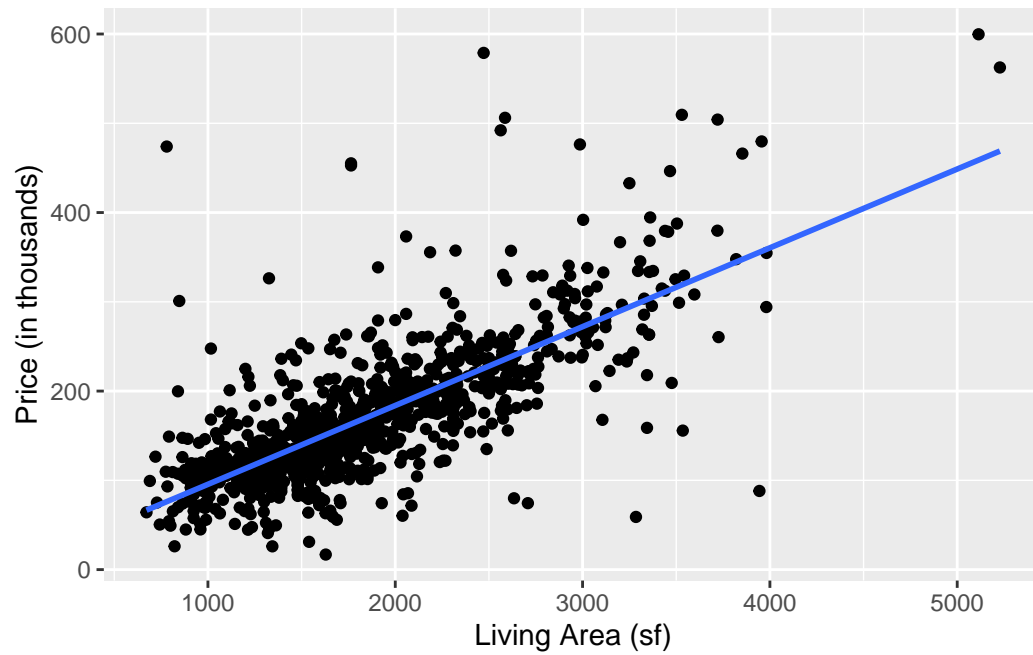
We begin by reading in the data for the step-by-step example.

```
HousingPrices <-  
  read_csv("http://nhorton.people.amherst.edu/is5/data/Housing_prices.csv") |>  
  janitor::clean_names()  
gf_point(price ~ living_area, data = HousingPrices) |>  
  gf_smooth() |>  
  gf_labs(x = "Living Area", y = "Price")
```

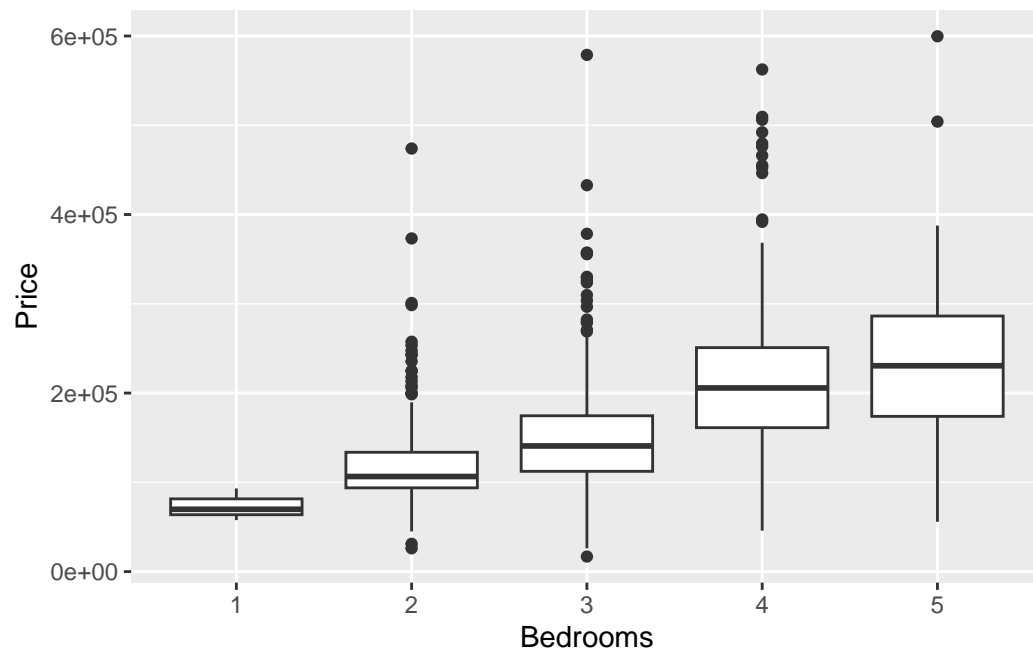


For this and other plots the y axis labels would be far easier to read if the values were rescaled. Here we demonstrate this but continue to mirror the book output for the other displays.

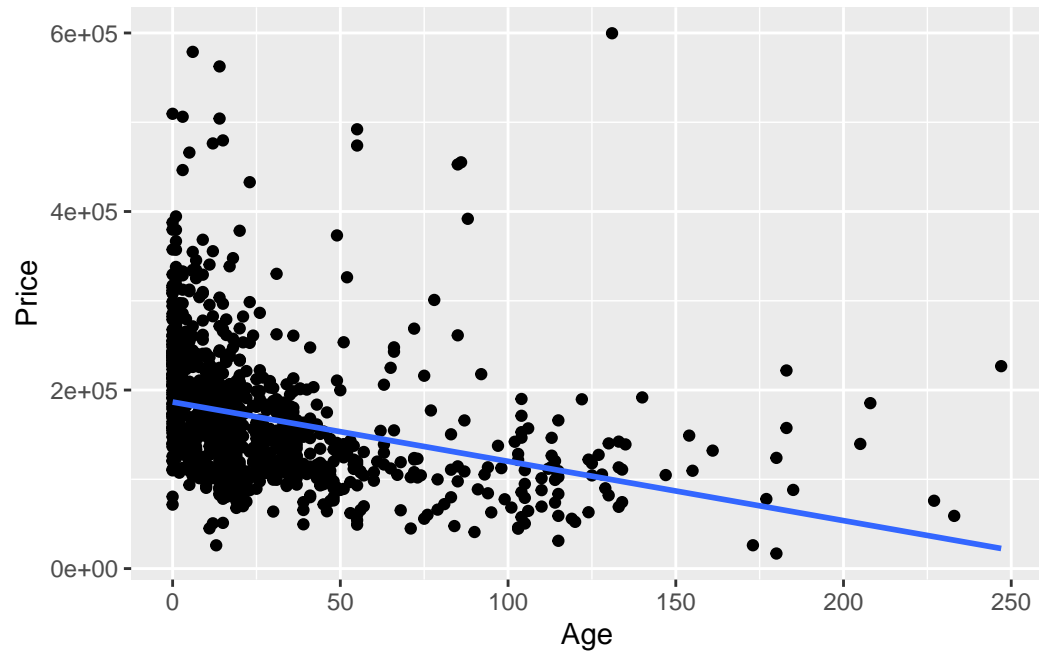
```
HousingRescaled <- HousingPrices |>  
  mutate(price1000 = price / 1000)  
gf_point(price1000 ~ living_area, data = HousingRescaled) |>  
  gf_smooth() |>  
  gf_labs(x = "Living Area (sf)", y = "Price (in thousands)")
```



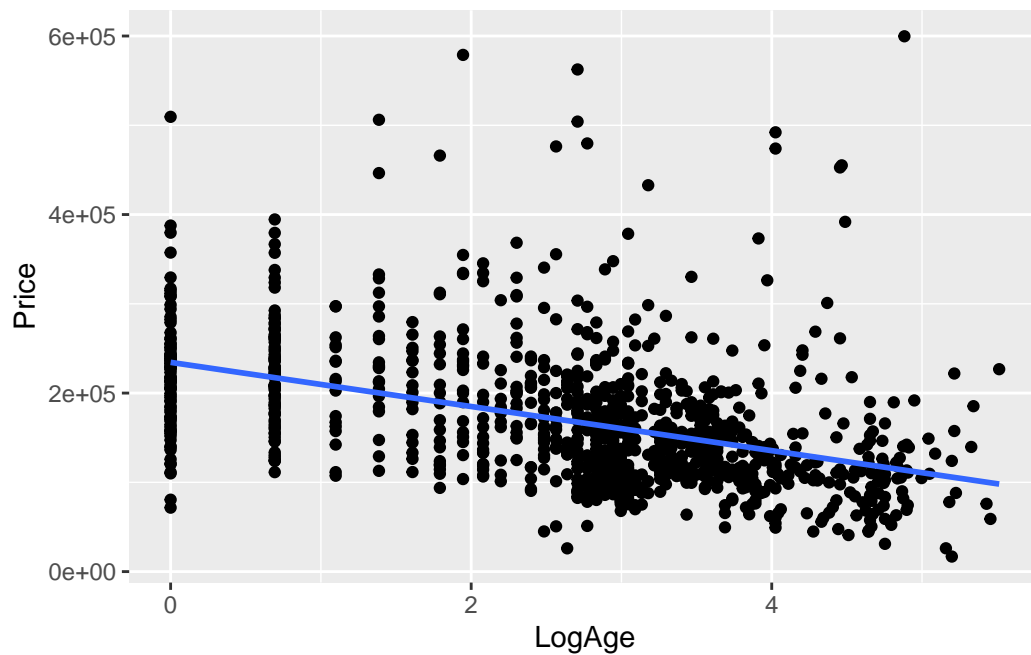
```
gf_boxplot(price ~ as.factor.bedrooms), data = HousingPrices) |>
  gf_labs(x = "Bedrooms", y = "Price")
```



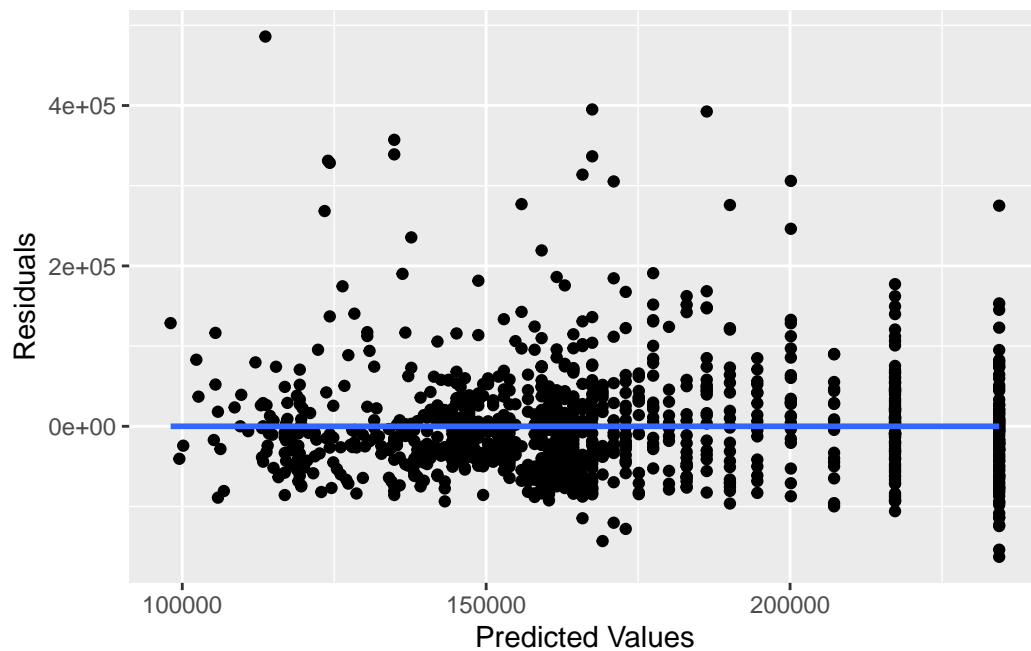
```
gf_point(price ~ age, data = HousingPrices) |>  
  gf_smooth() |>  
  gf_labs(x = "Age", y = "Price")
```



```
gf_point(price ~ log(age + 1), data = HousingPrices) |>  
  gf_smooth() |>  
  gf_labs(x = "LogAge", y = "Price")
```



```
housinglm <- lm(price ~ log(age + 1), data = HousingPrices)
gf_point(resid(housinglm) ~ fitted(housinglm)) |>
  gf_smooth() |>
  gf_labs(x = "Predicted Values", y = "Residuals")
```



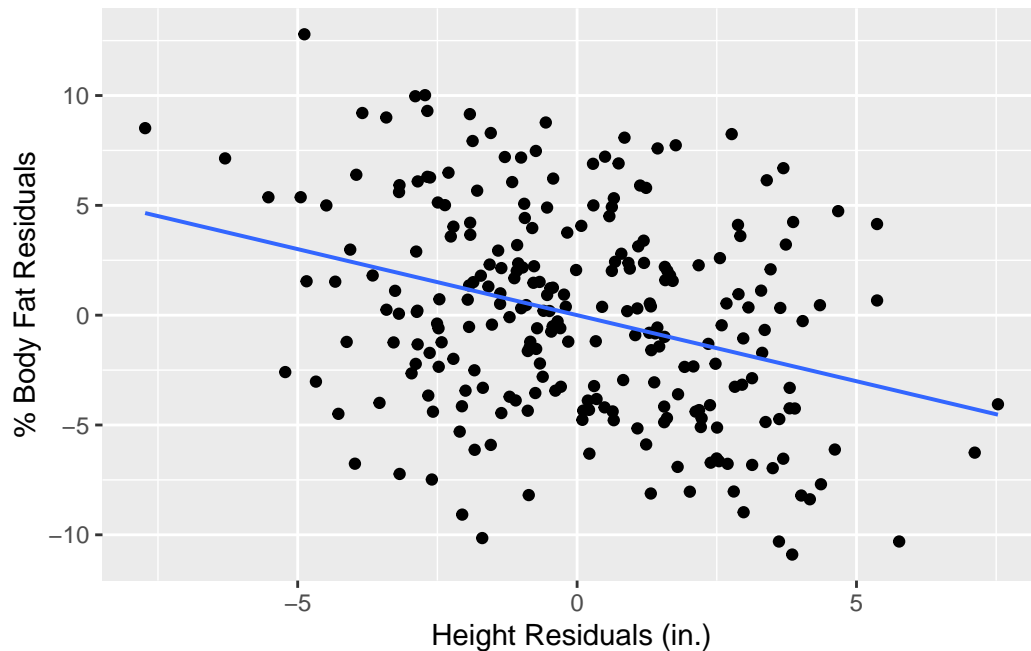
```
housinglm2 <- lm(price ~ living_area + log(age + 1) + bedrooms, data = HousingPrices)
msummary(housinglm2)
```

	Estimate	Std. Error	t value	Pr(> t)	
(Intercept)	44797.165	8356.609	5.361	1.02e-07	***
living_area	87.260	3.365	25.928	< 2e-16	***
log(age + 1)	-6270.813	1299.133	-4.827	1.59e-06	***
bedrooms	-5902.756	2773.934	-2.128	0.0336	*

Residual standard error: 49620 on 1053 degrees of freedom
Multiple R-squared: 0.5876, Adjusted R-squared: 0.5864
F-statistic: 500.1 on 3 and 1053 DF, p-value: < 2.2e-16

Section 9.4: Partial Regression Plots

```
# Figure 9.6 (instructions on 287)
# Step 1
otherthanheightlm <- lm(pct_bf ~ waist, data = BodyFat)
# Step 2
residualsoflm <- resid(otherthanheightlm)
# Step 3
yheightlm <- lm(height ~ waist, data = BodyFat)
# Step 4
residualsoflm2 <- resid(yheightlm)
# Step 5
gf_point(residualsoflm ~ residualsoflm2) |>
  gf_lm() |>
  gf_labs(x = "Height Residuals (in.)", y = "% Body Fat Residuals")
```



Just Checking

```
Hurricanes <- read_csv("http://nhorton.people.amherst.edu/is5/data/Hurricanes_2015.csv") |>
  janitor::clean_names()
hurricanelm <- lm(max_wind_speed_kts ~ year + central_pressure_mb, data = Hurricanes)
msummary(hurricanelm)
```

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	1.032e+03	3.852e+01	26.789	<2e-16 ***
year	-3.132e-04	9.075e-03	-0.035	0.973
central_pressure_mb	-9.750e-01	3.287e-02	-29.666	<2e-16 ***

Residual standard error: 8.199 on 217 degrees of freedom

(7 observations deleted due to missingness)

Multiple R-squared: 0.8056, Adjusted R-squared: 0.8038

F-statistic: 449.6 on 2 and 217 DF, p-value: < 2.2e-16

Section 9.5: Indicator Variables

```
Coasters <- read_csv("http://nhorton.people.amherst.edu/is5/data/Coasters_2015.csv")
# Table 9.2, page 288
head(Coasters)
```

```
# A tibble: 6 x 9
```

	Name	Park	Track	Speed	Height	Drop	Length	Duration	Inversions
	<chr>	<chr>	<chr>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>
1	Top Thrill Dragster	Cedar~	Steel	120	420	400	2800	NA	0
2	Superman The Escap	Six F~	Steel	100	415	328.	1235	NA	0
3	Millennium Force	Cedar~	Steel	93	310	300	6595	165	0
4	Goliath	Six F~	Steel	85	235	255	4500	180	0
5	Titan	Six F~	Steel	85	245	255	5312	210	0
6	Phantom's Revenge	Kenny~	Steel	82	160	228	3200	NA	0

```
# Figure 9.7
```

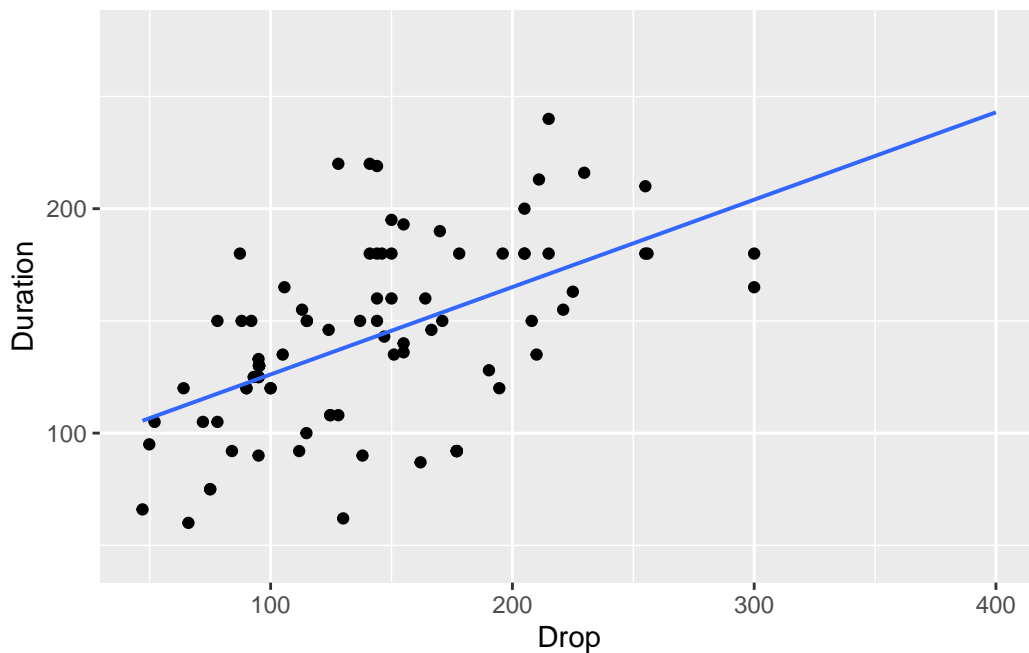
```
# Tower of Terror isn't included by the book, so we need to drop it
```

```
Coasters <- Coasters |>
```

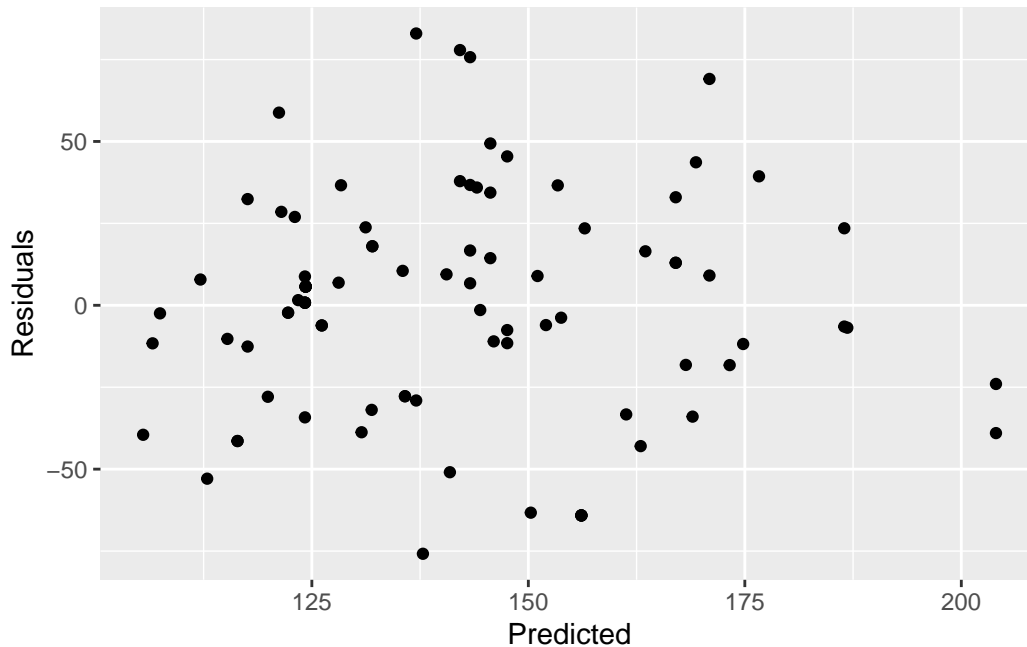
```
  filter(Name != "Tower of Terror") |>
```

```
  mutate(Inversions = as.factor(Inversions)) # turn the variable into a factor
```

```
gf_point(Duration ~ Drop, data = Coasters) |>
  gf_lm()
```




```
coasterlm <- lm(Duration ~ Drop, data = Coasters)
gf_point(resid(coasterlm) ~ fitted(coasterlm)) |>
  gf_labs(x = "Predicted", y = "Residuals")
```



```
msummary(coasterlm)
```

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	87.22005	9.73524	8.959	4.98e-14 ***
Drop	0.38928	0.06428	6.056	3.36e-08 ***

Residual standard error: 34.06 on 88 degrees of freedom

(150 observations deleted due to missingness)

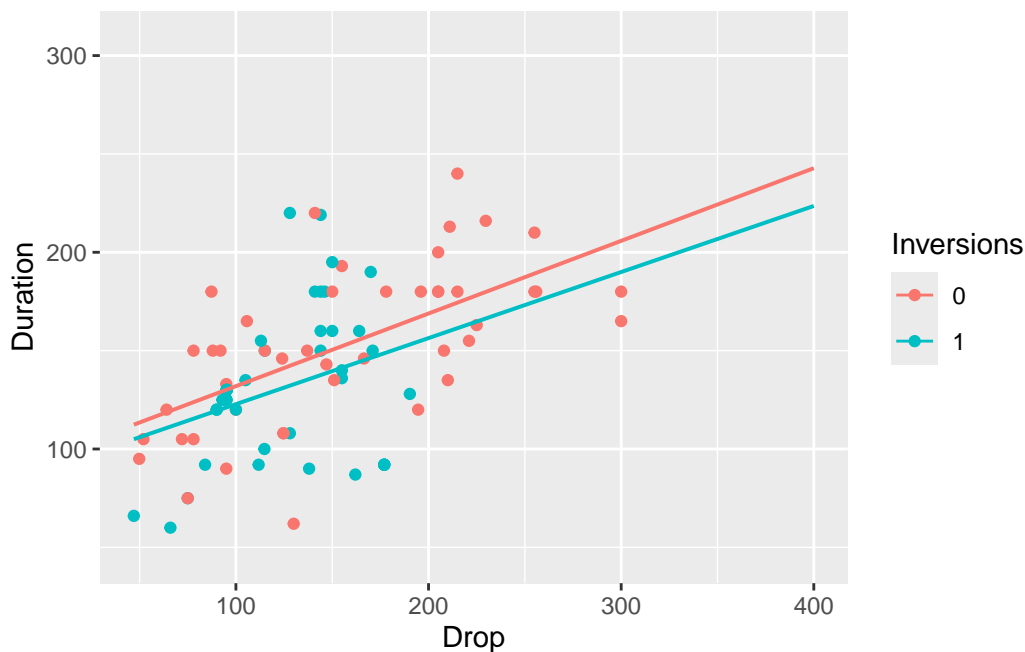
Multiple R-squared: 0.2942, Adjusted R-squared: 0.2862

F-statistic: 36.68 on 1 and 88 DF, p-value: 3.356e-08

```
# Figure 9.8
gf_point(Duration ~ Drop, color = ~Inversions, data = Coasters) |>
  gf_lm() |>
  gf_labs(color = "Inversions")
```

Warning: Removed 150 rows containing non-finite outside the scale range (`stat_lm()`).

Warning: Removed 150 rows containing missing values or values outside the scale range (``geom_point()``).



Here it would be appropriate to add `warning: false` as a code chunk option once we've verified that there are indeed 150 observations missing

```
coasterlm2 <- lm(Duration ~ Drop + Inversions, data = Coasters)
msummary(coasterlm2)
```

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	96.14026	11.69140	8.223	1.74e-12 ***
Drop	0.36215	0.06699	5.406	5.58e-07 ***
Inversions1	-10.20093	7.48401	-1.363	0.176

Residual standard error: 33.9 on 87 degrees of freedom

(150 observations deleted due to missingness)

Multiple R-squared: 0.3089, Adjusted R-squared: 0.293

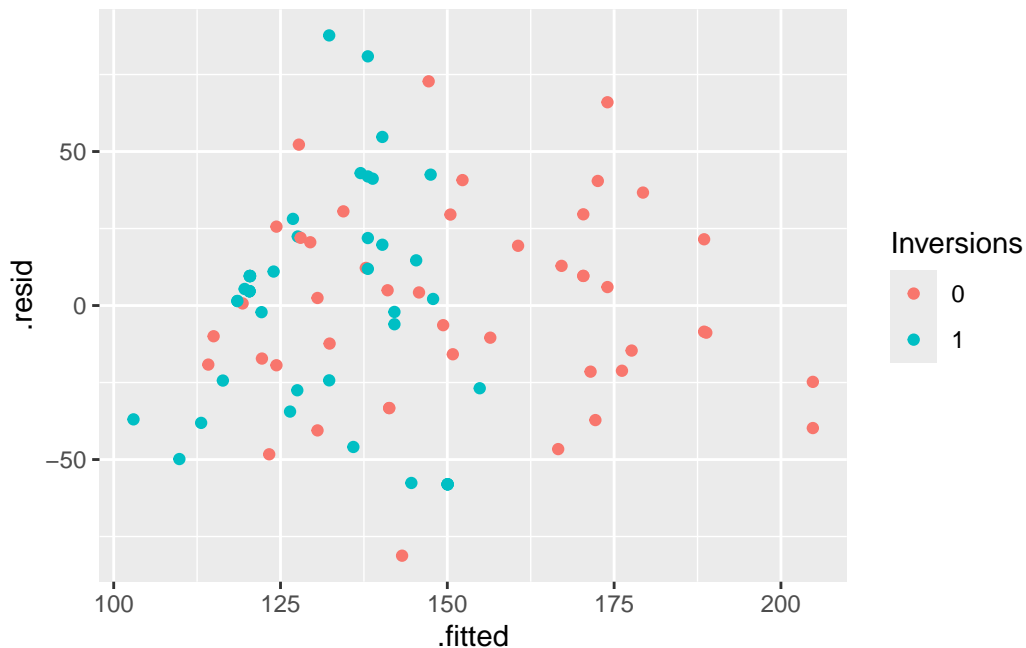
F-statistic: 19.45 on 2 and 87 DF, p-value: 1.045e-07

```
coasterlm2asdata <- broom::augment(coasterlm2) # another helpful function
broom::glance(coasterlm2) |> data.frame()
```

	r.squared	adj.r.squared	sigma	statistic	p.value	df	logLik	AIC
1	0.3089346	0.293048	33.89636	19.44628	1.04492e-07	2	-443.2766	894.5532

	BIC	deviance	df.residual	nobs
1	904.5524	99959.82	87	90

```
gf_point(.resid ~ .fitted, color = ~ Inversions, data = coasterlm2asdata)
```

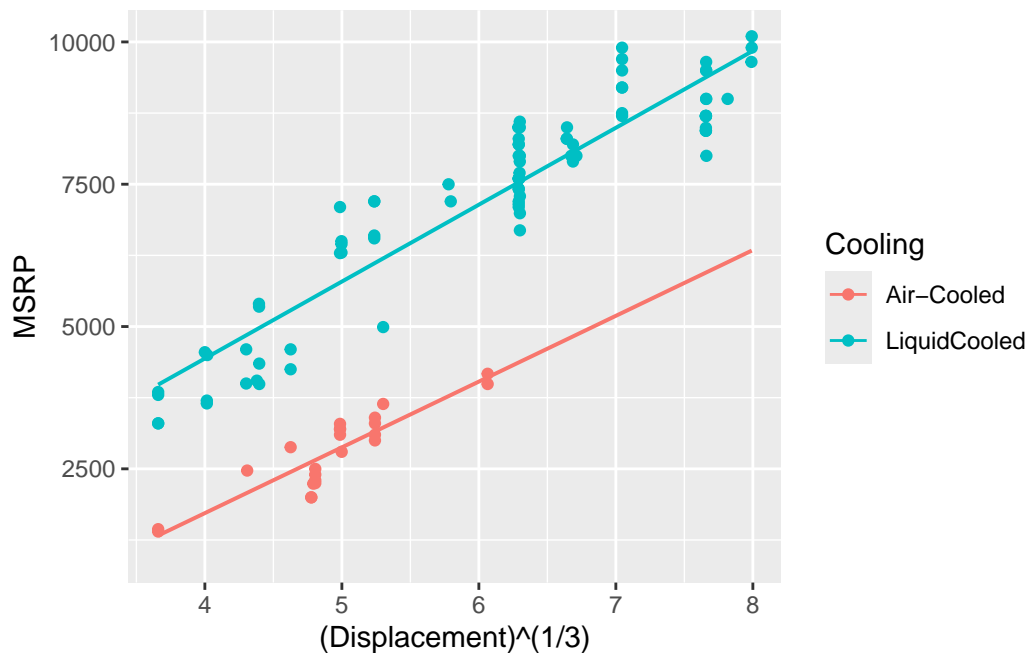


The `augment()` function from the `broom` package creates a data frame from a linear model that includes a column for residuals, fitted values, etc. Here we use `names()` to check out the column names and `glance()` to view the structure of the data set.

Example 9.3: Using Indicator Variables

We can explore the use of indicator variables to model categorical variables.

```
DirtBikes <- read_csv("http://nhorton.people.amherst.edu/is5/data/Dirt_bikes_2014.csv")
DirtBikes <- DirtBikes |>
  filter(Cooling != "NA") |>
  mutate(Cooling = ifelse(Cooling == "Air-Cooled", "Air-Cooled", "LiquidCooled"))
gf_point(MSRP ~ (Displacement)^(1 / 3), color = ~ Cooling, data = DirtBikes) |>
  gf_lm()
```



```
bikeslm <- lm(MSRP ~ I(Displacement^(1 / 3)) + Cooling, data = DirtBikes)
msummary(bikeslm)
```

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	-3814.9	278.0	-13.72	<2e-16 ***
I(Displacement^(1/3))	1341.4	50.4	26.61	<2e-16 ***
CoolingLiquidCooled	2908.1	154.0	18.88	<2e-16 ***

Residual standard error: 602.7 on 106 degrees of freedom

Multiple R-squared: 0.9423, Adjusted R-squared: 0.9413

F-statistic: 866.3 on 2 and 106 DF, p-value: < 2.2e-16

The I() function is used to keep the class of an object the same. Here we use it to keep the variable Displacement “as is” to prevent an error.

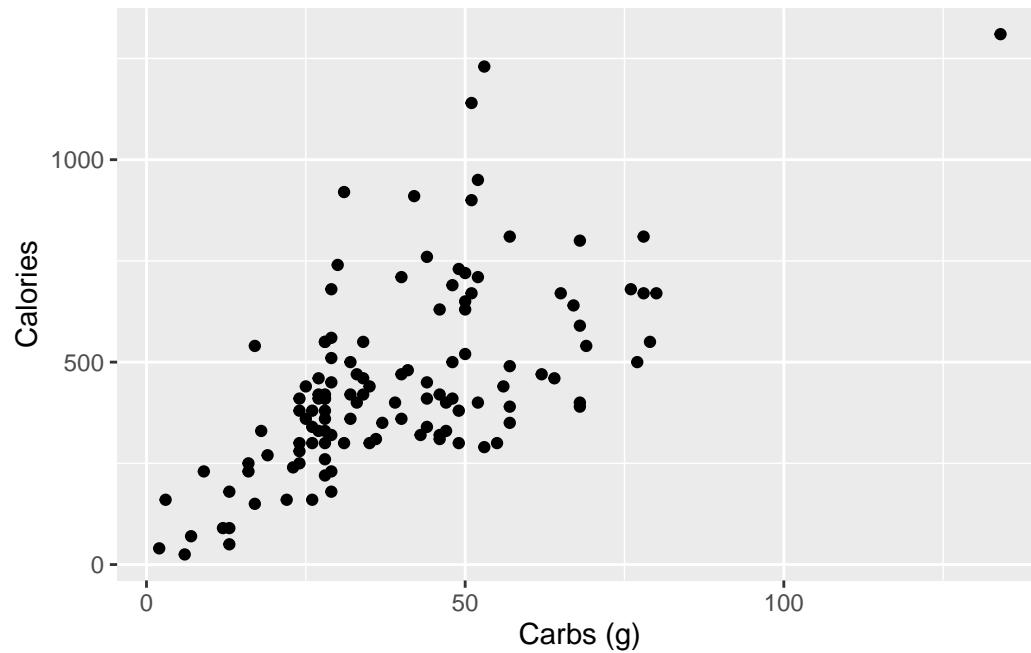
Adjusting for Different Slopes

We can fit a model with different slopes.

```
BurgerKing <-
  read_csv("http://nhorton.people.amherst.edu/is5/data/Burger_King_items.csv") |>
  janitor::clean_names()
```

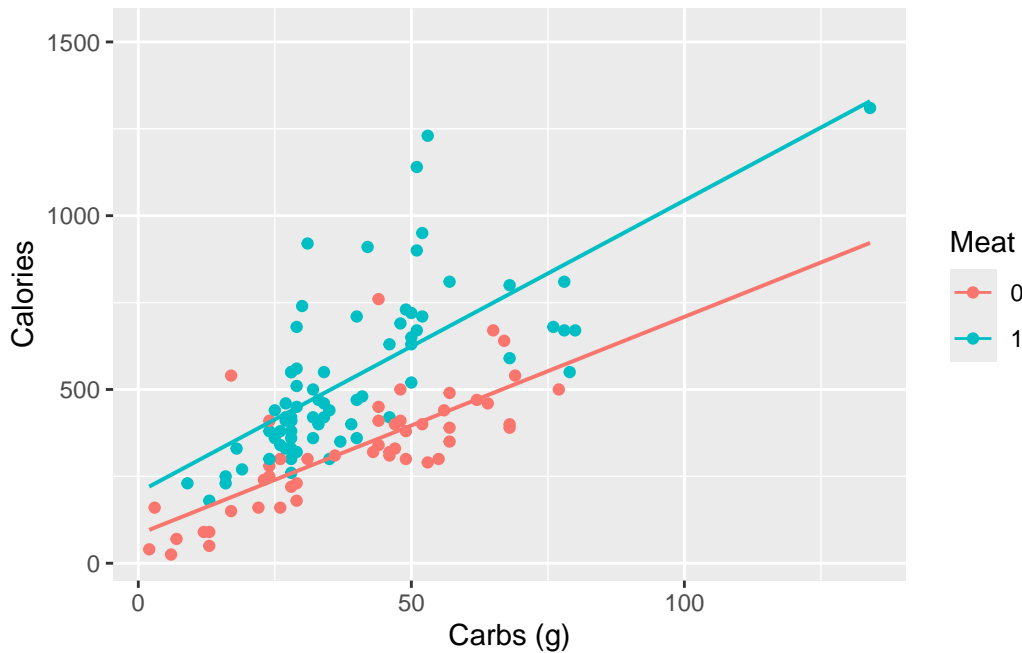
```
# Figure 9.9, page 292
```

```
gf_point(calories ~ carbs_g, data = BurgerKing) |>  
gf_labs(x = "Carbs (g)", y = "Calories")
```



```
# Figure 9.10
```

```
gf_point(calories ~ carbs_g, color = ~ as.factor(meat), data = BurgerKing) |>  
gf_labs(x = "Carbs (g)", y = "Calories", color = "Meat") |>  
gf_lm()
```



```
msummary(lm(calories ~ carbs_g * as.factor(meat), data = BurgerKing))
```

	Estimate	Std. Error	t value	Pr(> t)	
(Intercept)	83.533	46.955	1.779	0.0778	.
carbs_g	6.255	1.063	5.885	3.81e-08	***
as.factor(meat)1	120.220	60.694	1.981	0.0499	*
carbs_g:as.factor(meat)1	2.145	1.378	1.557	0.1222	

Residual standard error: 146.5 on 118 degrees of freedom

Multiple R-squared: 0.6072, Adjusted R-squared: 0.5972

F-statistic: 60.8 on 3 and 118 DF, p-value: < 2.2e-16

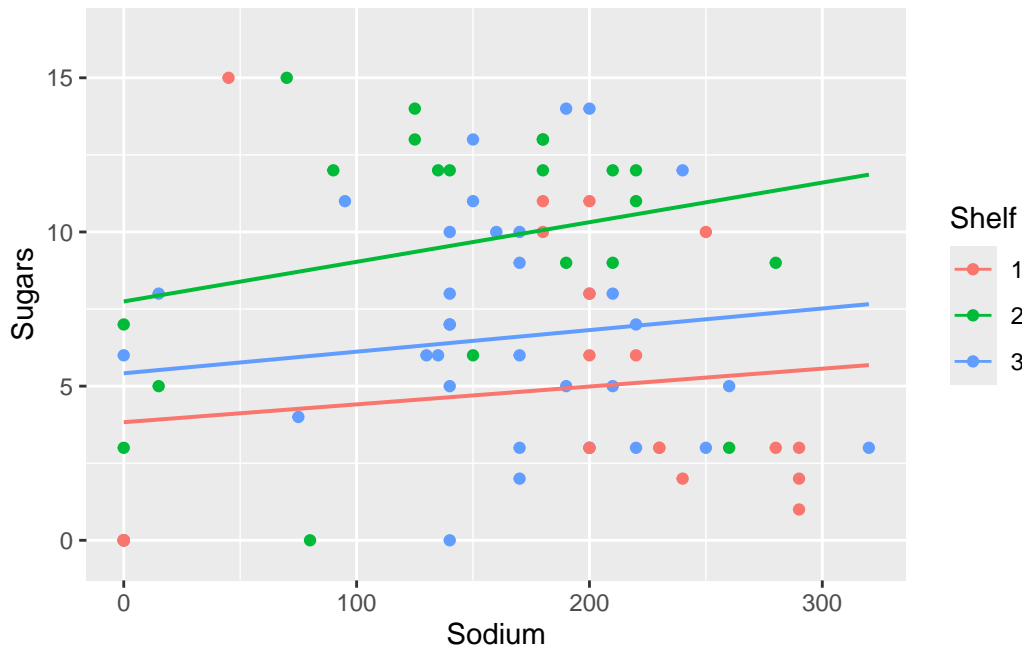
The output here is a bit ugly: it would be straightforward to create the new variables using `mutate()` to provide easier to read output.

One, Two, Many

We can also consider three level variables.

```
Cereal <- read_csv("http://nhorton.people.amherst.edu/is5/data/Cereals.csv")
cereal_lm <- lm(sugars ~ sodium + as.factor(shelf), data = Cereal)
gf_point(sugars ~ sodium, color = ~ as.factor(shelf), data = Cereal) |>
```

```
gf_lm() |>
gf_labs(x = "Sodium", y = "Sugars", color = "Shelf")
```



```
msummary(cereallm)
```

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	3.446740	1.345111	2.562	0.012457 *
sodium	0.007962	0.005620	1.417	0.160818
as.factor(shelf)2	5.012166	1.283154	3.906	0.000207 ***
as.factor(shelf)3	1.818214	1.139384	1.596	0.114857

Residual standard error: 4.07 on 73 degrees of freedom
Multiple R-squared: 0.1866, Adjusted R-squared: 0.1532
F-statistic: 5.583 on 3 and 73 DF, p-value: 0.001669

Example 9.4: Indicators for Variables with Several Levels

We will read in the diamonds data.

```
Diamonds <- read_csv("http://nhorton.people.amherst.edu/is5/data/Diamonds.csv") |>
janitor::clean_names()
```

```
# Parallel Slopes
diamondlm <- lm(sqrt(price) ~ carat_size + color, data = Diamonds)
msummary(diamondlm)
```

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	13.1946	0.5488	24.043	< 2e-16 ***
carat_size	61.2491	0.5032	121.722	< 2e-16 ***
colorE	-2.1027	0.5399	-3.895	0.000101 ***
colorF	-2.8640	0.5576	-5.136	3.00e-07 ***
colorG	-3.6320	0.5769	-6.296	3.57e-10 ***
colorH	-7.8948	0.5858	-13.477	< 2e-16 ***
colorI	-11.8542	0.6261	-18.932	< 2e-16 ***
colorJ	-16.6404	0.6637	-25.071	< 2e-16 ***
colorK	-21.3577	0.8282	-25.787	< 2e-16 ***

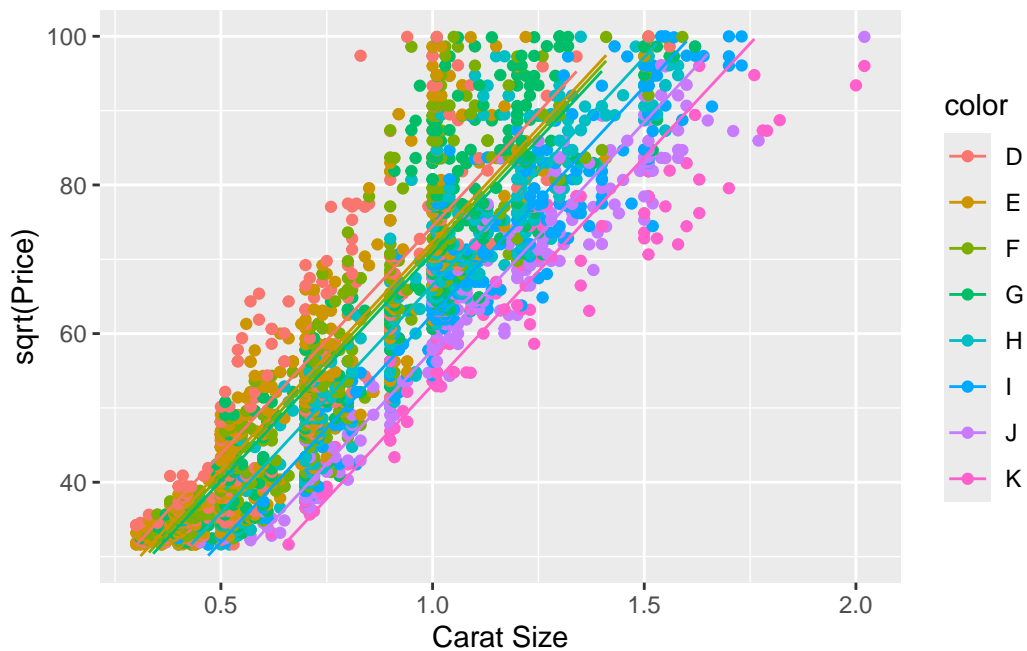
Residual standard error: 7.218 on 2681 degrees of freedom
Multiple R-squared: 0.8583, Adjusted R-squared: 0.8579
F-statistic: 2030 on 8 and 2681 DF, p-value: < 2.2e-16

```
diamondpredict <- makeFun(diamondlm)

diamonddata <- augment(diamondlm) |> # To get fitted values
  janitor::clean_names()
  glimpse(diamonddata)
```

```
Rows: 2,690
Columns: 9
$ sqrt_price <dbl> 31.62278, 31.62278, 31.62278, 31.62278, 31.62278, 31.62278,~
$ carat_size <dbl> 0.30, 0.44, 0.31, 0.66, 0.47, 0.40, 0.36, 0.52, 0.53, 0.43,~
$ color <chr> "E", "E", "E", "K", "H", "G", "D", "H", "D", "F", "F", "F",~
$ fitted <dbl> 29.46659, 38.04146, 30.07908, 32.26133, 34.08682, 34.06221,~
$ resid <dbl> 2.1561877, -6.4186795, 1.5436972, -0.6385503, -2.4640456, --
$ hat <dbl> 0.002687088, 0.002264823, 0.002650609, 0.009594672, 0.00374~
$ sigma <dbl> 7.219488, 7.218542, 7.219547, 7.219598, 7.219451, 7.219454,~
$ cooks_d <dbl> 2.678455e-05, 1.998882e-04, 1.354152e-05, 8.505241e-06, 4.8~
$ std_resid <dbl> 0.29911501, -0.89023651, 0.21414395, -0.08889063, -0.342003~
```

```
gf_point(sqrt_price ~ carat_size, color = ~ color, data = diamonddata) |>
  gf_line(fitted ~ carat_size) |>
  gf_labs(x = "Carat Size", y = "sqrt(Price)") +
  ylim(30, 100)
```

```
# With interaction
diamondlm2 <- lm(sqrt(price) ~ carat_size * color, data = Diamonds)
msummary(diamondlm2)
```

	Estimate	Std. Error	t value	Pr(> t)	
(Intercept)	9.3239	1.2142	7.679	2.23e-14	***
carat_size	67.0408	1.7025	39.379	< 2e-16	***
colorE	-0.5392	1.5075	-0.358	0.72063	
colorF	-2.3716	1.5627	-1.518	0.12922	
colorG	-2.6709	1.6643	-1.605	0.10867	
colorH	-3.9177	1.8248	-2.147	0.03189	*
colorI	-2.5481	1.9301	-1.320	0.18689	
colorJ	-5.4176	2.0716	-2.615	0.00897	**
colorK	0.5976	2.7815	0.215	0.82991	
carat_size:colorE	-2.4007	2.0999	-1.143	0.25305	
carat_size:colorF	-1.3211	2.0954	-0.630	0.52843	
carat_size:colorG	-2.5457	2.0868	-1.220	0.22260	
carat_size:colorH	-5.9017	2.1774	-2.710	0.00676	**
carat_size:colorI	-10.9139	2.1812	-5.004	5.99e-07	***
carat_size:colorJ	-12.4948	2.2531	-5.546	3.22e-08	***
carat_size:colorK	-21.4477	2.6978	-7.950	2.72e-15	***

Residual standard error: 7.058 on 2674 degrees of freedom

Multiple R-squared: 0.8649, Adjusted R-squared: 0.8641
F-statistic: 1141 on 15 and 2674 DF, p-value: < 2.2e-16

```
gf_point(sqrt(price) ~ carat_size, color = ~ color, data = Diamonds) |>  
  gf_lm() |>  
  gf_labs(x = "Carat Size", y = "sqrt(Price)") +  
  ylim(30, 100)
```

