IS5 in R: Multiple Regression (Chapter 9)

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June 27, 2018

Introduction and background

This document is intended to help describe how to undertake analyses introduced as examples in the Fifth Edition of *Intro Stats* (2018) by De Veaux, Velleman, and Bock. More information about the book can be found at http://wps.aw.com/aw_deveaux_stats_series. This file as well as the associated R Markdown reproducible analysis source file used to create it can be found at http://nhorton.people.amherst.edu/is5.

This work leverages initiatives undertaken by Project MOSAIC (http://www.mosaic-web.org), an NSF-funded effort to improve the teaching of statistics, calculus, science and computing in the undergraduate curriculum. In particular, we utilize the `mosaic` package, which was written to simplify the use of R for introductory statistics courses. A short summary of the R needed to teach introductory statistics can be found in the mosaic package vignettes (http://cran.r-project.org/web/packages/mosaic). A paper describing the mosaic approach was published in the *R Journal*: https://journal.r-project.org/archive/2017/RJ-2017-024.

Chapter 9: Multiple Regression

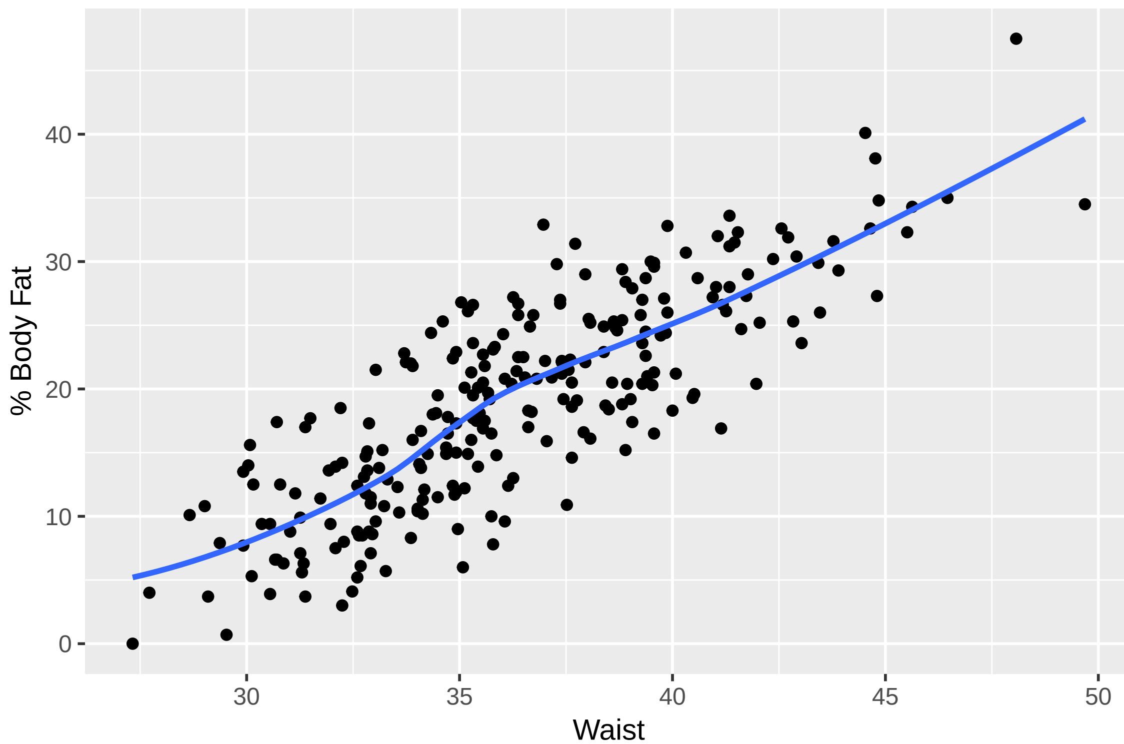
```
library(mosaic)
library(readr)
library(janitor)
BodyFat <- read_csv("http://nhorton.people.amherst.edu/is5/data/Bodyfat.csv") %>%
  clean_names()
```

```
## Parsed with column specification:
## cols(
##   Density = col_double(),
##   Pct.BF = col_double(),
##   Age = col_integer(),
##   Weight = col_double(),
##   Height = col_double(),
##   Neck = col_double(),
##   Chest = col_double(),
##   Abdomen = col_double(),
##   Waist = col_double(),
##   Hip = col_double(),
##   Thigh = col_double(),
##   Knee = col_double(),
##   Ankle = col_double(),
##   Bicep = col_double(),
##   Forearm = col_double(),
##   Wrist = col_double()
## )
```

By default, `read_csv()` prints the variable names. These messages can be suppressed using the `message=FALSE` code chunk option to save space and improve readability.
Here we use the `clean_names()` function from the `janitor` package to sanitize the names of the columns (which would otherwise contain special characters or whitespace).

```r
# Figure 9.1, page 276
gf_point(pct_bf ~ waist, data = BodyFat) %>%
  gf_labs(x = "Waist", y = "% Body Fat") %>%
  gf_smooth()
```

```
## `geom_smooth()` using method = 'loess' and formula 'y ~ x'
```



We've added `gf_smooth()` to demonstrate how to add a smoother.

Section 9.1: What is Multiple Regression?

Example 9.1: Modeling Home Prices

```r
RealEstate <- read_csv("http://nhorton.people.amherst.edu/is5/data/Real_Estate.csv") %>%
  clean_names()
```

```
## Parsed with column specification:
## cols(
##   Price = col_integer(),
##   `Living area` = col_integer(),
##   bedrooms = col_integer(),
##   bathrooms = col_double(),
##   year = col_integer(),
##   garage = col_integer(),
##   `date collected` = col_character(),
##   `location type` = col_character(),
##   Urban = col_integer(),
##   Suburb = col_integer(),
```

```
##   Rural = col_integer()
## )
```

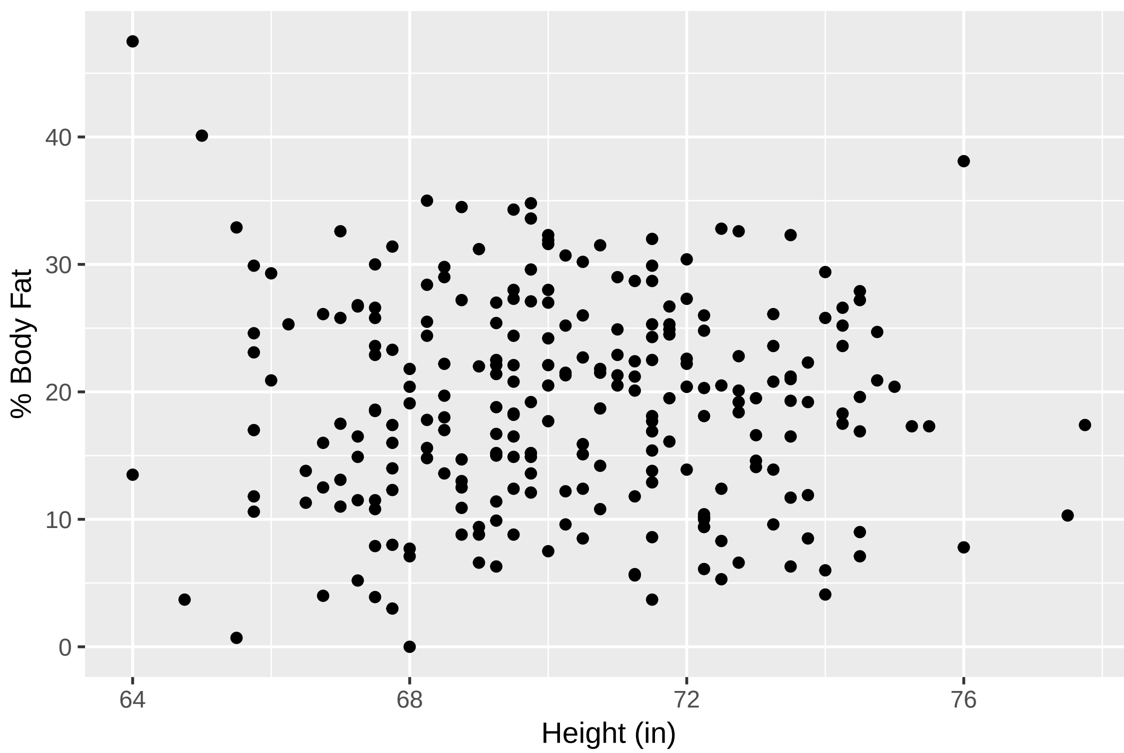
```
msummary(lm(price ~ living_area + bedrooms, data = RealEstate))
```

```
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 308100.44   41147.84   7.488 1.69e-13 ***
## living_area    135.09      11.48  11.771  < 2e-16 ***
## bedrooms    -43346.81   12844.14  -3.375 0.000771 ***
##
## Residual standard error: 266900 on 891 degrees of freedom
## Multiple R-squared:  0.1463, Adjusted R-squared:  0.1444
## F-statistic: 76.34 on 2 and 891 DF,  p-value: < 2.2e-16
```

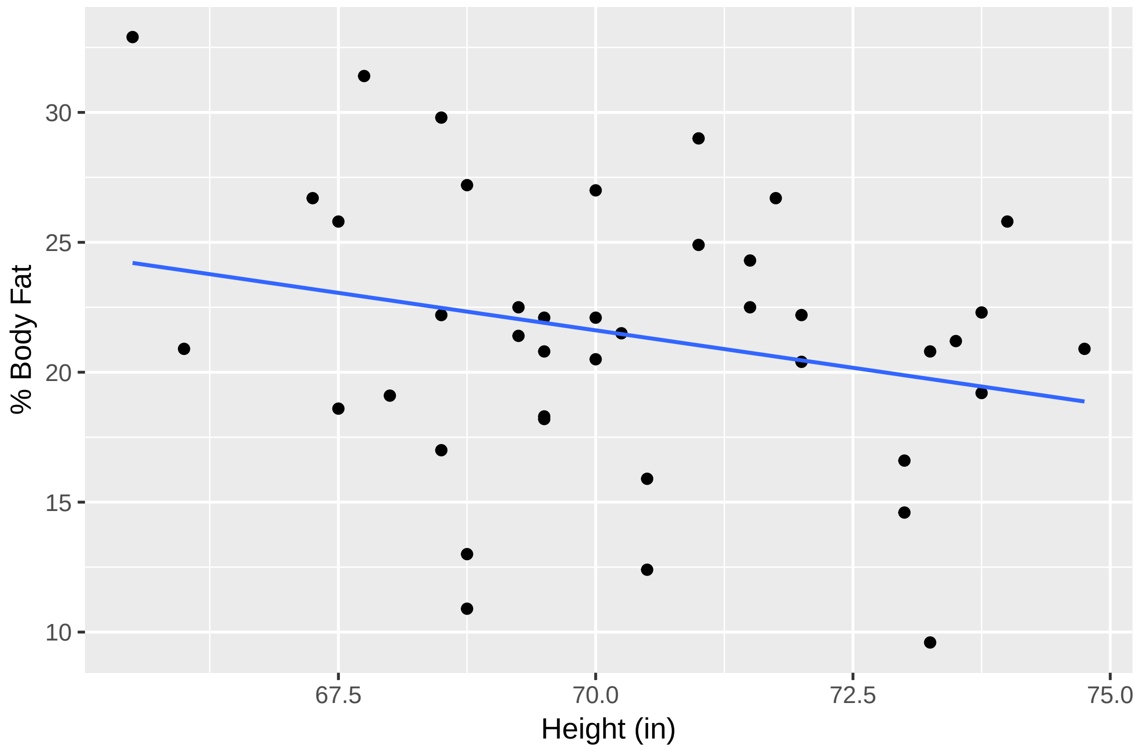
The coefficients for the model can be seen in the `Estimate` column.

Section 9.2: Interpreting Multiple Regression Coefficients

```
# Figure 9.2, page 279
gf_point(pct_bf ~ height, data = BodyFat) %>%
  gf_labs(x = "Height (in)", y = "% Body Fat")
```




```
# Figure 9.3
BodyFat %>%
  filter(waist >= 36 & waist <= 38) %>%
  gf_point(pct_bf ~ height) %>%
  gf_labs(x = "Height (in)", y = "% Body Fat") %>%
  gf_lm()
```

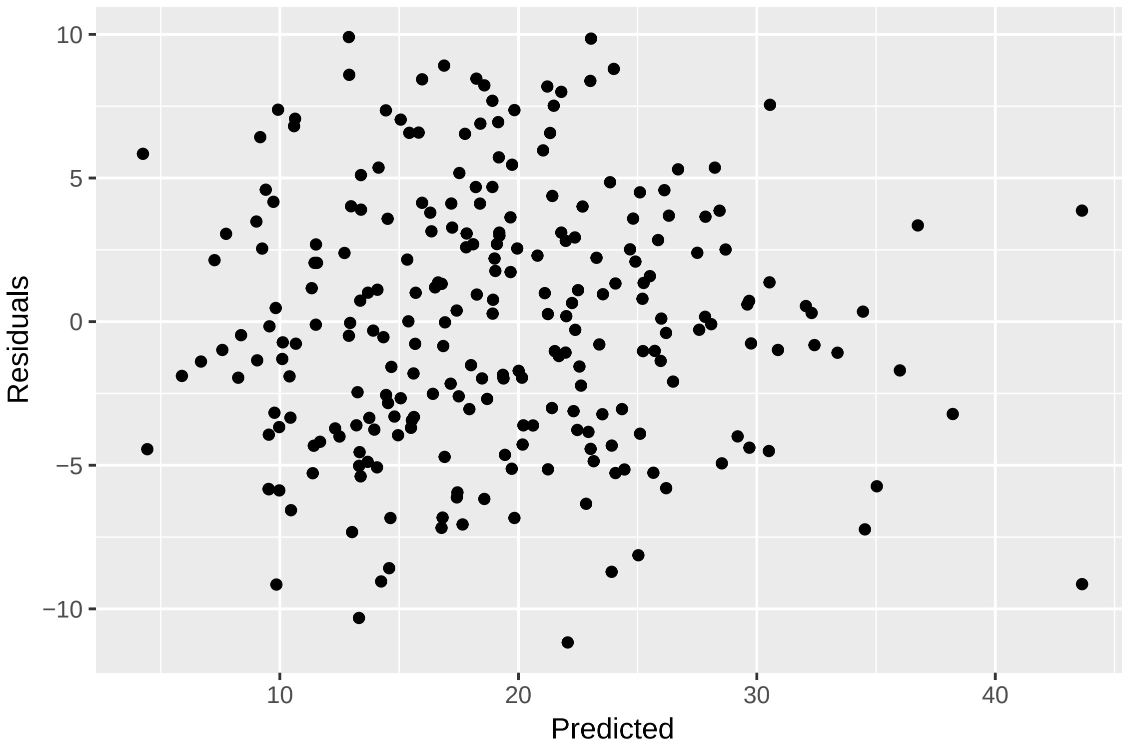


Section 9.3: The Multiple Regression Model–Assumptions and Conditions

Linearity Assumption

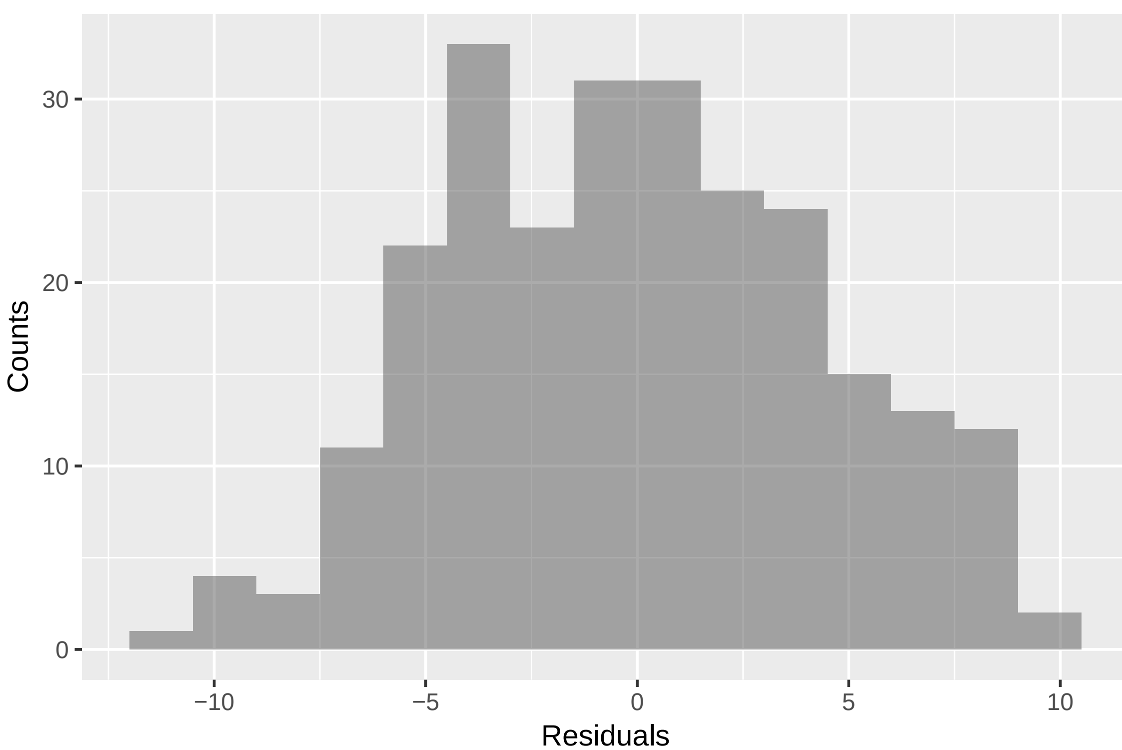
Equal Variance Assumption

```
bodyfatlm <- lm(pct_bf ~ waist + height, data = BodyFat)
# Figure 9.4, page 282
gf_point(resid(bodyfatlm) ~ fitted(bodyfatlm)) %>%
  gf_labs(x = "Predicted", y = "Residuals")
```

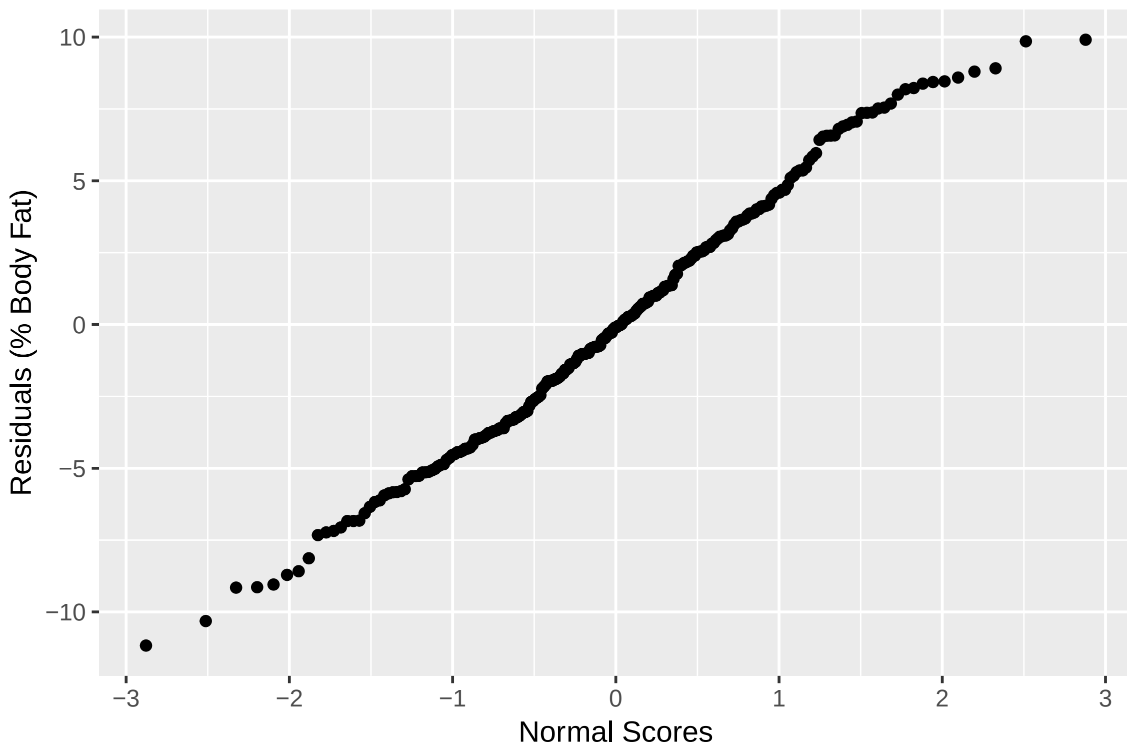


Check the Residuals

```
# Figure 9.5
gf_histogram(~ resid(bodyfatlm), binwidth = 1.5, center = 0.75) %>%
  gf_labs(x = "Residuals", y = "Counts")
```



```r
gf_qq(~ resid(bodyfatlm)) %>%
  gf_labs(x = "Normal Scores", y = "Residuals (% Body Fat)")
```

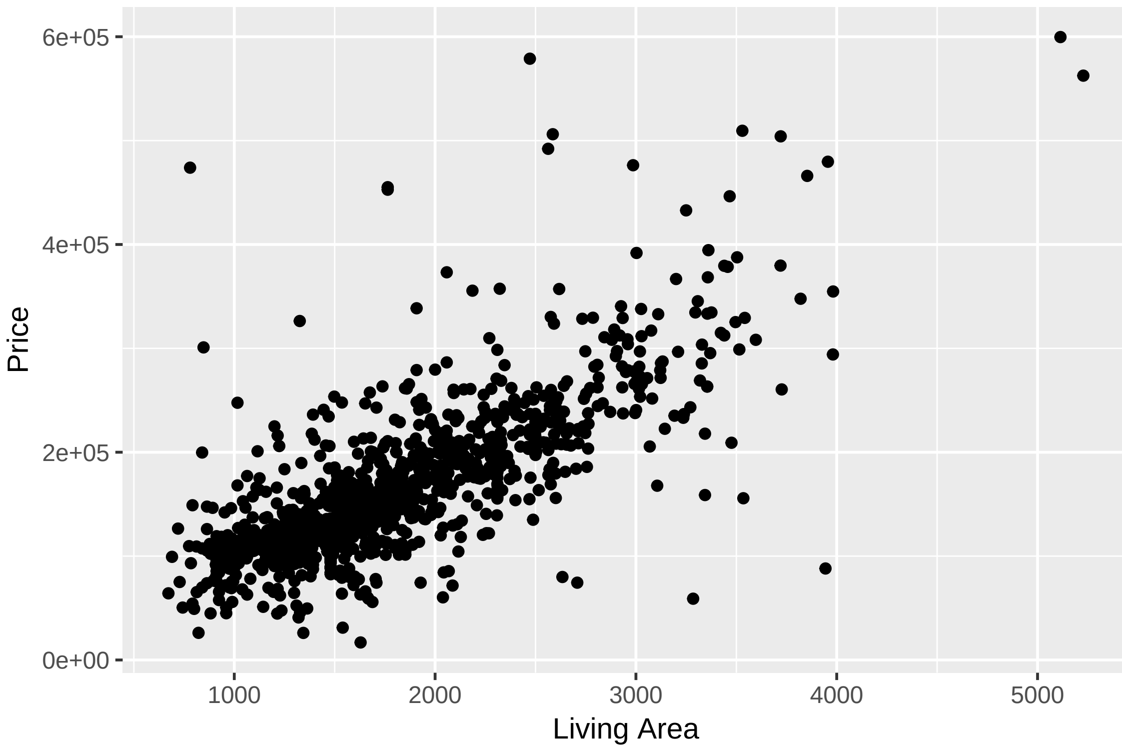


Step-By-Step Example: Multiple Regression

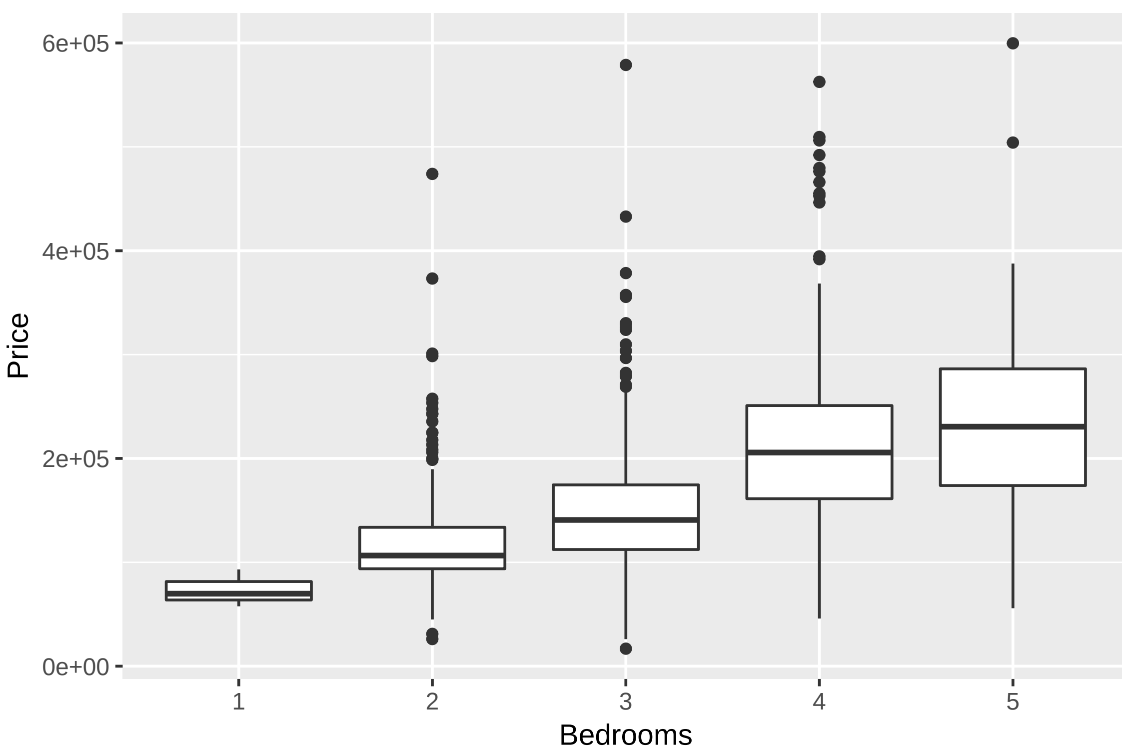
```r
HousingPrices <- read_csv("http://nhorton.people.amherst.edu/is5/data/Housing_prices.csv") %>%
  clean_names()
```

```
## Parsed with column specification:
## cols(
##   Price = col_integer(),
##   Living.Area = col_integer(),
##   Bedrooms = col_integer(),
##   Bathrooms = col_double(),
##   Fireplaces = col_integer(),
##   Age = col_integer()
## )
```

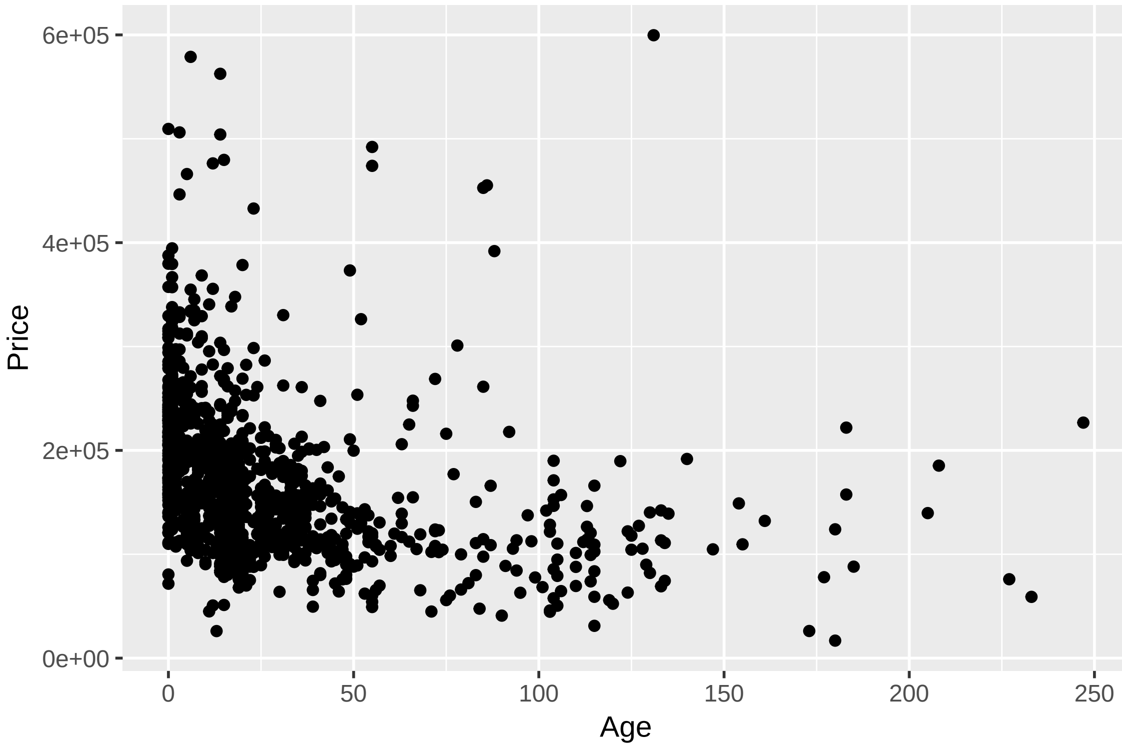
```r
gf_point(price ~ living_area, data = HousingPrices) %>%
  gf_labs(x = "Living Area", y = "Price")
```



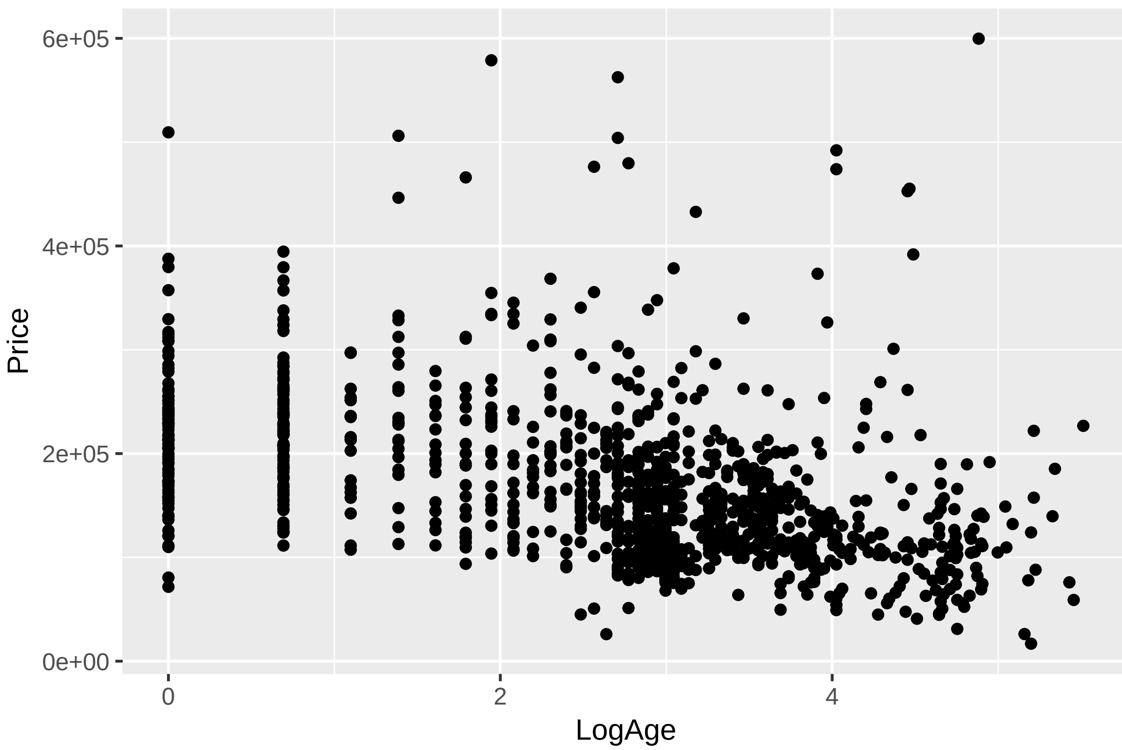

```
gf_boxplot(price ~ as.factor(bedrooms), data = HousingPrices) %>%
  gf_labs(x = "Bedrooms", y = "Price")
```



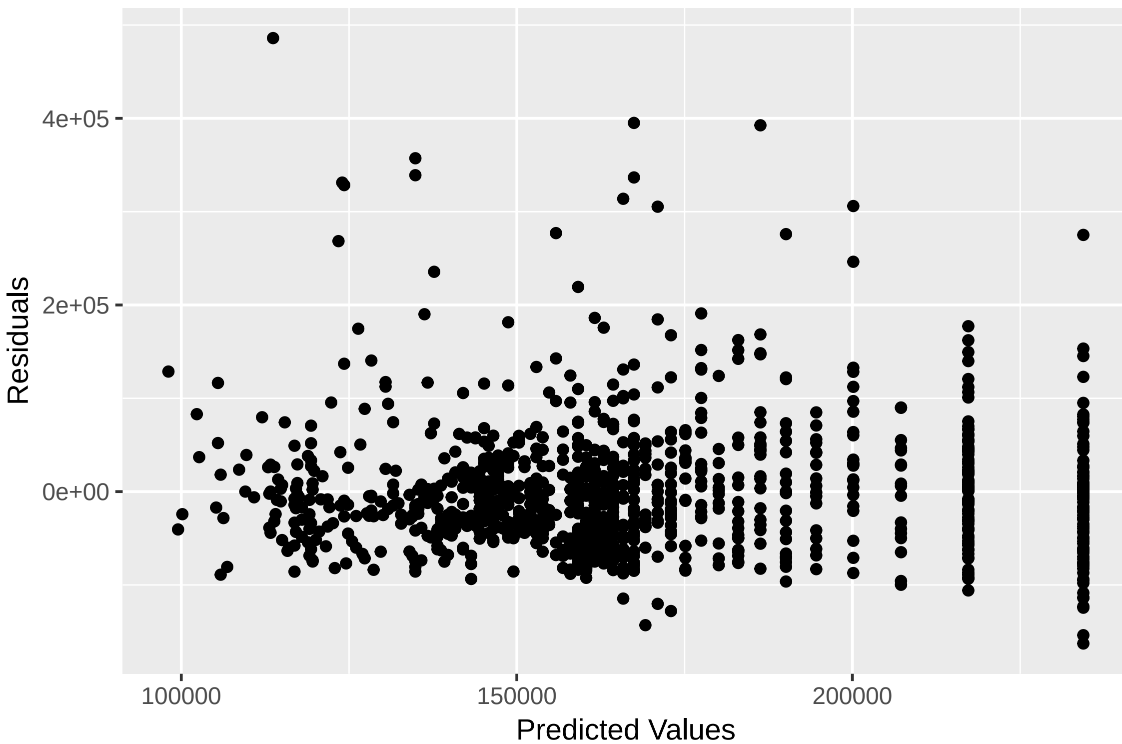
```
gf_point(price ~ age, data = HousingPrices) %>%
  gf_labs(x = "Age", y = "Price")
```




```
gf_point(price ~ log(age + 1), data = HousingPrices) %>%
  gf_labs(x = "LogAge", y = "Price")
```



```
housinglm <- lm(price ~ log(age + 1), data = HousingPrices)
gf_point(resid(housinglm) ~ fitted(housinglm)) %>%
  gf_labs(x = "Predicted Values", y = "Residuals")
```



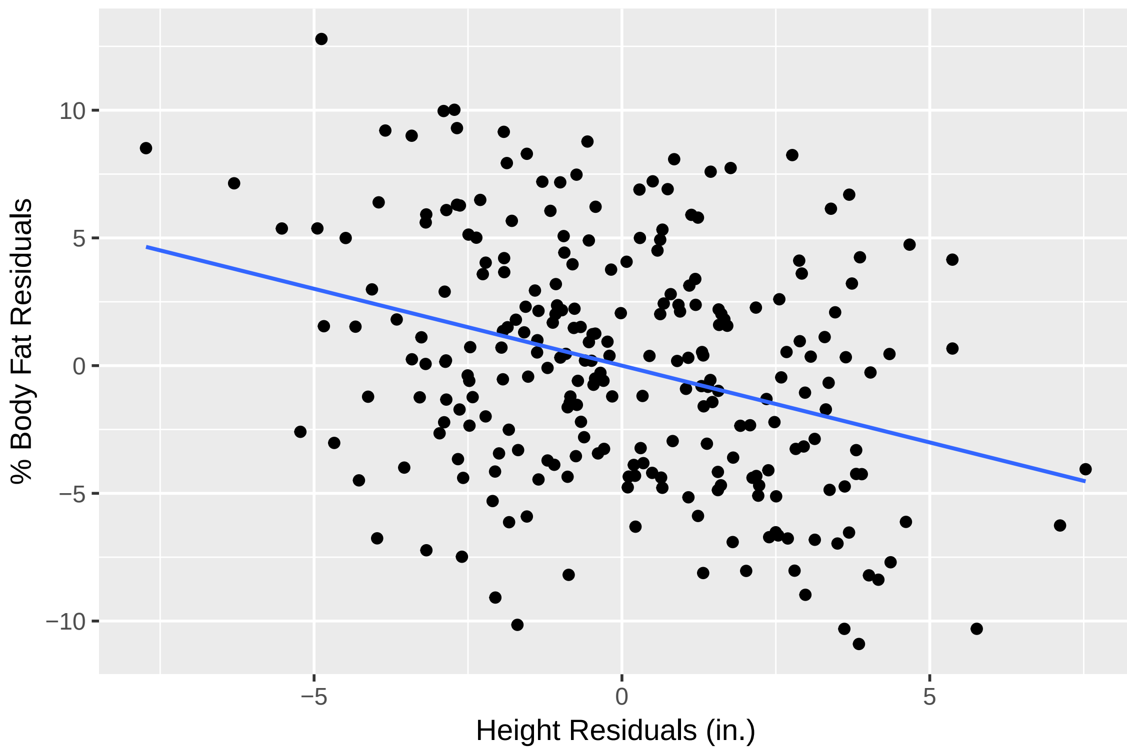

```
housinglm2 <- lm(price ~ living_area + log(age + 1) + bedrooms, data = HousingPrices)
msummary(housinglm2)
```

```
##                Estimate Std. Error t value Pr(>|t|)
## (Intercept)   44797.165   8356.609   5.361 1.02e-07 ***
## living_area      87.260      3.365  25.928  < 2e-16 ***
## log(age + 1) -6270.813   1299.133  -4.827 1.59e-06 ***
## bedrooms     -5902.756   2773.934  -2.128   0.0336 *
##
## Residual standard error: 49620 on 1053 degrees of freedom
## Multiple R-squared:  0.5876, Adjusted R-squared:  0.5864
## F-statistic: 500.1 on 3 and 1053 DF,  p-value: < 2.2e-16
```

Section 9.4: Partial Regression Plots

```
# Figure 9.6 (instructions on 287)
# Step 1
otherthanheightlm <- lm(pct_bf ~ waist, data = BodyFat)
# Step 2
residualsoflm <- resid(otherthanheightlm)
# Step 3
yheightlm <- lm(height ~ waist, data = BodyFat)
```

```r
# Step 4
residualsoflm2 <- resid(yheightlm)
# Step 5
gf_point(residualsoflm ~ residualsoflm2) %>%
  gf_lm() %>%
  gf_labs(x = "Height Residuals (in.)", y = "% Body Fat Residuals")
```



Just Checking

```r
Hurricanes <- read_csv("http://nhorton.people.amherst.edu/is5/data/Hurricanes_2015.csv") %>%
  clean_names()
```

```
## Parsed with column specification:
## cols(
##   Name = col_character(),
##   Year = col_integer(),
##   `Max.Wind.Speed(kts)` = col_integer(),
##   `Central.Pressure(mb)` = col_integer(),
##   Category = col_integer()
## )
```

```r
hurricanelm <- lm(max_wind_speed_kts ~ year + central_pressure_mb, data = Hurricanes)
msummary(hurricanelm)
```

```
##                          Estimate Std. Error t value Pr(>|t|)
## (Intercept)             1.032e+03  3.852e+01  26.789   <2e-16 ***
## year                   -3.132e-04  9.075e-03  -0.035    0.973
```

```
## central_pressure_mb -9.750e-01  3.287e-02 -29.666   <2e-16 ***
##
## Residual standard error: 8.199 on 217 degrees of freedom
##   (7 observations deleted due to missingness)
## Multiple R-squared:  0.8056, Adjusted R-squared:  0.8038
## F-statistic: 449.6 on 2 and 217 DF,  p-value: < 2.2e-16
```

Section 9.5: Indicator Variables

```
library(broom) # We'll use this for augment() later
Coasters <- read_csv("http://nhorton.people.amherst.edu/is5/data/Coasters_2015.csv")
```

```
## Parsed with column specification:
## cols(
##   Name = col_character(),
##   Park = col_character(),
##   Track = col_character(),
##   Speed = col_double(),
##   Height = col_double(),
##   Drop = col_double(),
##   Length = col_double(),
##   Duration = col_integer(),
##   Inversions = col_integer()
## )
```

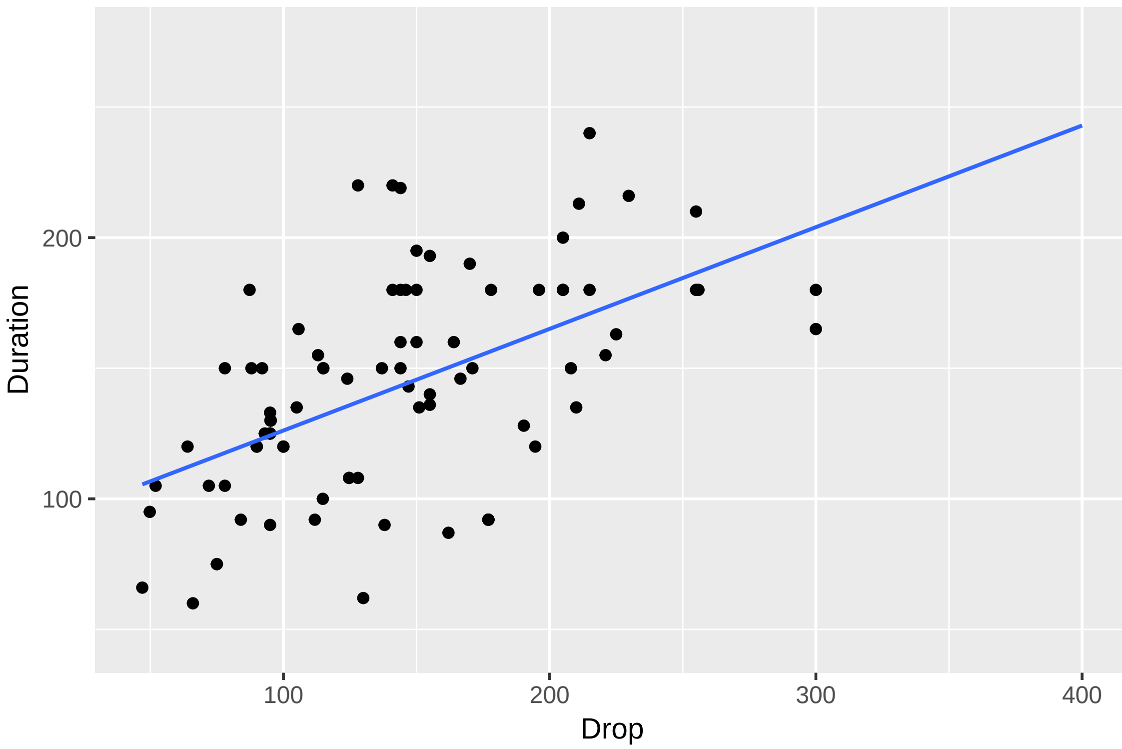
```
# Table 9.2, page 288
head(Coasters)
```

```
## # A tibble: 6 x 9
##   Name      Park       Track Speed Height  Drop Length Duration Inversions
##   <chr>     <chr>      <chr> <dbl>  <dbl> <dbl>  <dbl>    <int>      <int>
## 1 Top Thri~ Cedar Poi~ Steel   120    420  400    2800       NA          0
## 2 Superman~ Six Flags~ Steel   100    415  328.   1235       NA          0
## 3 Millenni~ Cedar Poi~ Steel    93    310  300    6595      165          0
## 4 Goliath   Six Flags~ Steel    85    235  255    4500      180          0
## 5 Titan     Six Flags~ Steel    85    245  255    5312      210          0
## 6 Phantom'~ Kennywood~ Steel    82    160  228    3200       NA          0
```

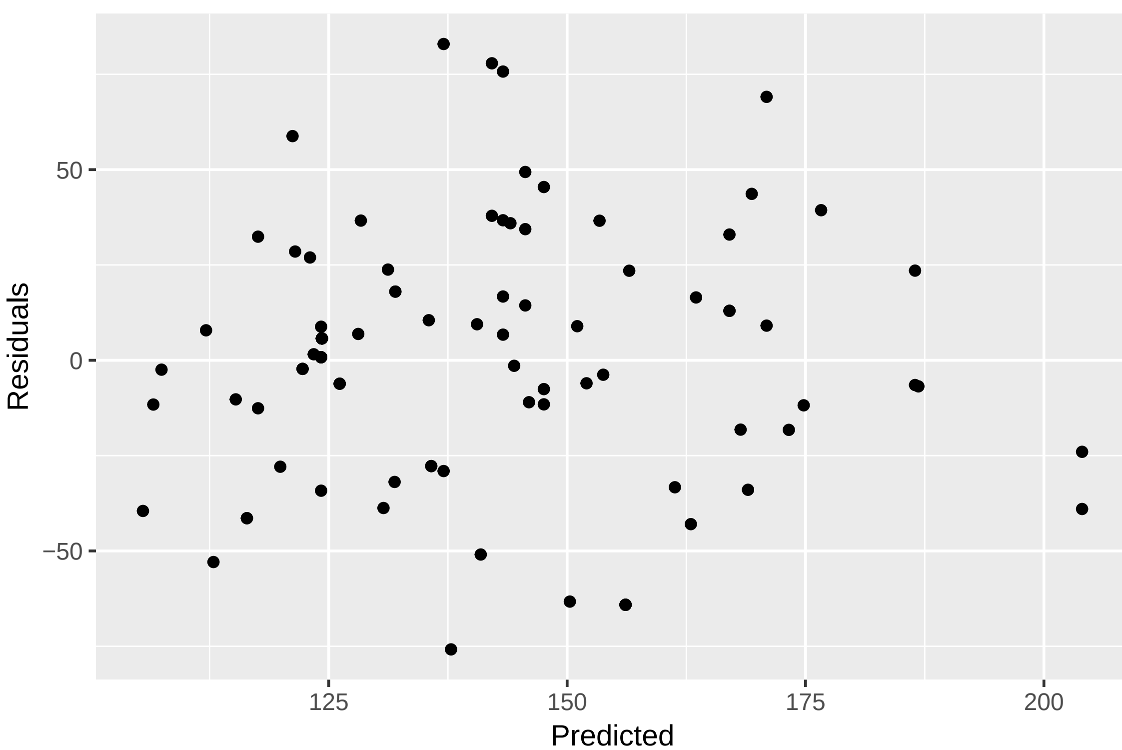
```
# Figure 9.7
# Coasters[72, ], Tower of Terror isn't included by the book
Coasters <- Coasters %>%
  filter(Name != "Tower of Terror") %>%
  mutate(Inversions = as.factor(Inversions))
gf_point(Duration ~ Drop, data = Coasters) %>%
  gf_lm()
```

```
## Warning: Removed 150 rows containing non-finite values (stat_lm).
```

```
## Warning: Removed 150 rows containing missing values (geom_point).
```




```r
coasterlm <- lm(Duration ~ Drop, data = Coasters)
gf_point(resid(coasterlm) ~ fitted(coasterlm)) %>%
  gf_labs(x = "Predicted", y = "Residuals")
```



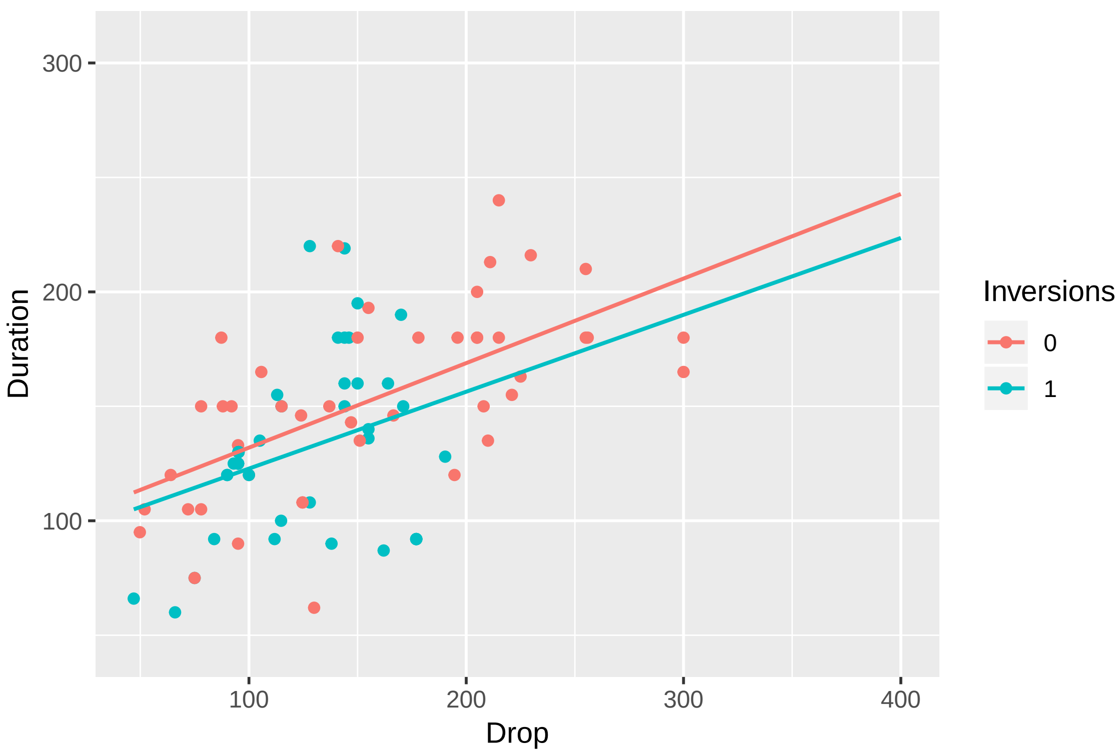
```r
msummary(coasterlm)
```

```
##             Estimate Std. Error t value Pr(>|t|)    
## (Intercept) 87.22005    9.73524   8.959 4.98e-14 ***
## Drop         0.38928    0.06428   6.056 3.36e-08 ***
## 
## Residual standard error: 34.06 on 88 degrees of freedom
##   (150 observations deleted due to missingness)
## Multiple R-squared:  0.2942, Adjusted R-squared:  0.2862 
## F-statistic: 36.68 on 1 and 88 DF,  p-value: 3.356e-08
```

```r
# Figure 9.8
gf_point(Duration ~ Drop, color = ~ Inversions, data = Coasters) %>%
  gf_lm() %>%
  gf_labs(color = "Inversions")
```

```
## Warning: Removed 150 rows containing non-finite values (stat_lm).

## Warning: Removed 150 rows containing missing values (geom_point).
```




```r
coasterlm2 <- lm(Duration ~ Drop + Inversions, data = Coasters)
msummary(coasterlm2)
```

```
##              Estimate Std. Error t value Pr(>|t|)    
## (Intercept)  96.14026   11.69140   8.223 1.74e-12 ***
## Drop          0.36215    0.06699   5.406 5.58e-07 ***
## Inversions1 -10.20093    7.48401  -1.363    0.176    
```

```
##
## Residual standard error: 33.9 on 87 degrees of freedom
##   (150 observations deleted due to missingness)
## Multiple R-squared:  0.3089, Adjusted R-squared:  0.293
## F-statistic: 19.45 on 2 and 87 DF,  p-value: 1.045e-07
```

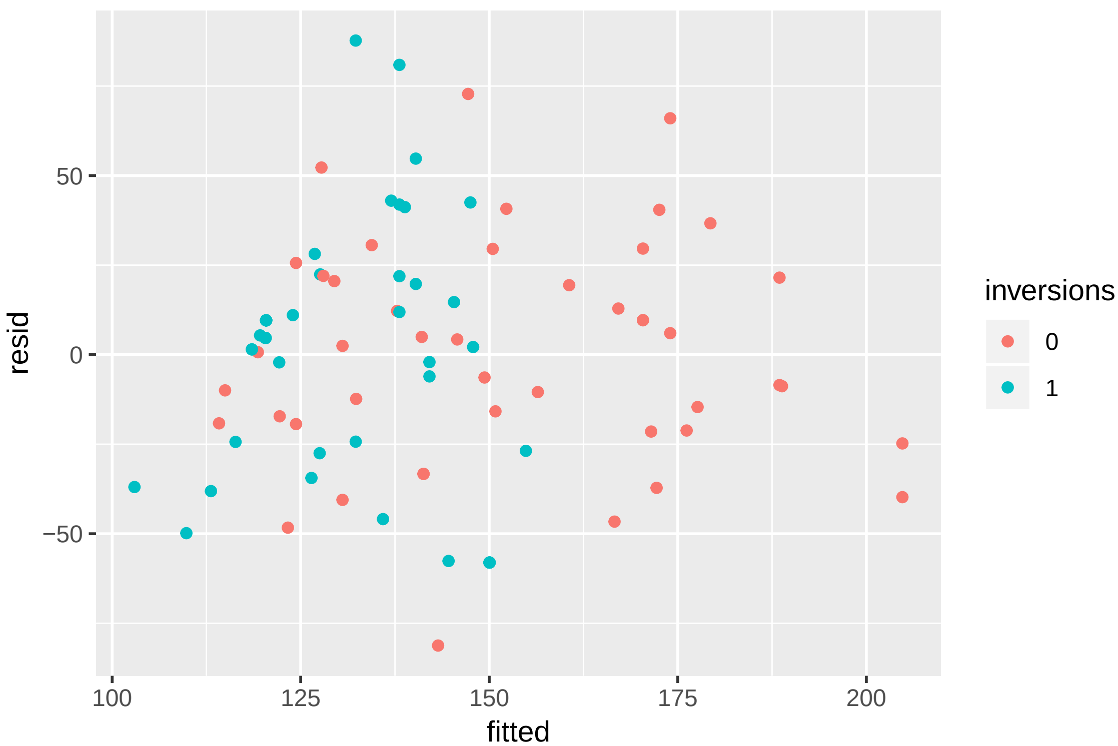
```
coasterlm2asdata <- augment(coasterlm2) %>%
  clean_names()
names(coasterlm2asdata)
```

```
##  [1] "rownames"    "duration"    "drop"        "inversions" "fitted"
##  [6] "se_fit"      "resid"       "hat"         "sigma"      "cooksd"
## [11] "std_resid"
```

```
glance(coasterlm2asdata)
```

```
##   nrow ncol complete.obs na.fraction
## 1   90   11           90           0
```

```
gf_point(resid ~ fitted, color = ~ inversions, data = coasterlm2asdata)
```



The augment() function creates a data frame from a linear model that includes a column for residuals, fitted values, etc. Here we use names() to check out the column names and glance() to view the structure of the data set.

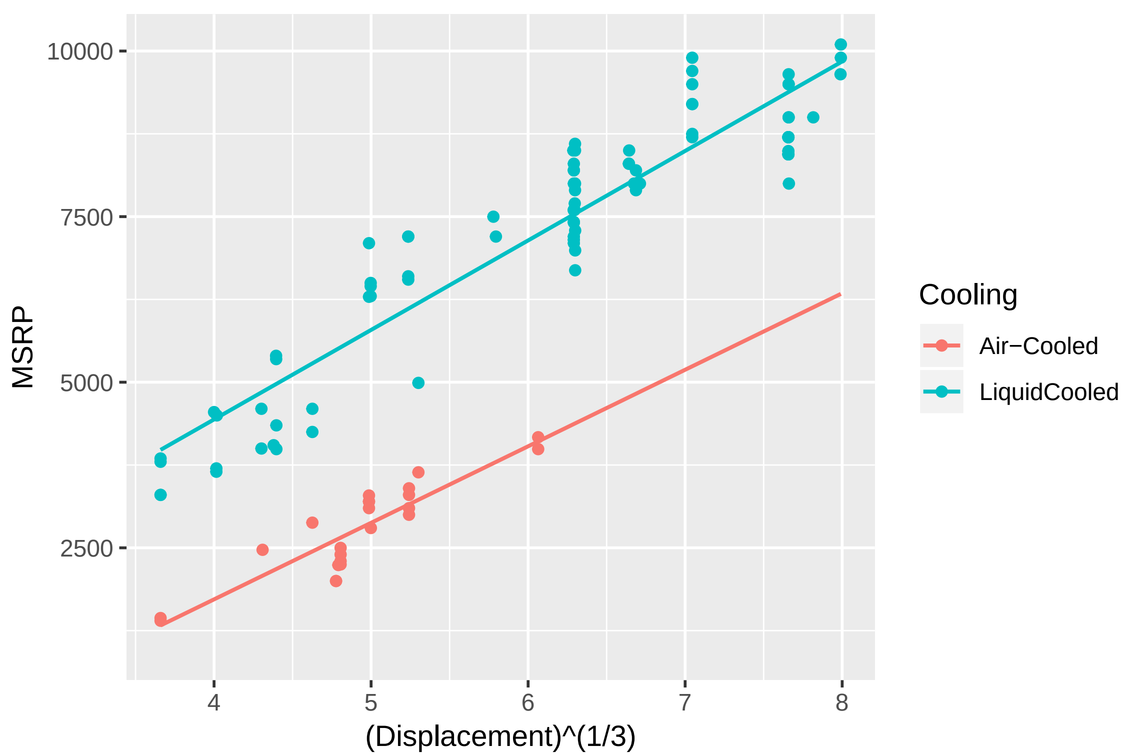
Example 9.3: Using Indicator Variables

```
DirtBikes <- read_csv("http://nhorton.people.amherst.edu/is5/data/Dirt_bikes_2014.csv")
```

```
## Parsed with column specification:
## cols(
##   .default = col_character(),
##   Year = col_integer(),
##   MSRP = col_integer(),
##   Displacement = col_double(),
##   `Wheel Base` = col_double(),
##   Bore = col_double(),
##   Stroke = col_double(),
##   Ratio = col_double(),
##   Weight = col_double(),
##   Rake = col_double(),
##   Trail = col_double(),
##   Tank = col_double(),
##   `Engine cooling` = col_integer()
## )

## See spec(...) for full column specifications.
```

```
DirtBikes <- DirtBikes %>%
  filter(Cooling != "NA") %>%
  mutate(Cooling = ifelse(Cooling == "Air-Cooled", "Air-Cooled", "LiquidCooled"))
gf_point(MSRP ~ (Displacement)^(1/3), color = ~ Cooling, data = DirtBikes) %>%
  gf_lm()
```



```r
bikeslm <- lm(MSRP ~ I(Displacement^(1/3)) + Cooling, data = DirtBikes)
msummary(bikeslm)
```

```
##                         Estimate Std. Error t value Pr(>|t|)
## (Intercept)              -3814.9      278.0  -13.72   <2e-16 ***
## I(Displacement^(1/3))    1341.4       50.4   26.61   <2e-16 ***
## CoolingLiquidCooled      2908.1      154.0   18.88   <2e-16 ***
##
## Residual standard error: 602.7 on 106 degrees of freedom
## Multiple R-squared:  0.9423, Adjusted R-squared:  0.9413
## F-statistic: 866.3 on 2 and 106 DF,  p-value: < 2.2e-16
```

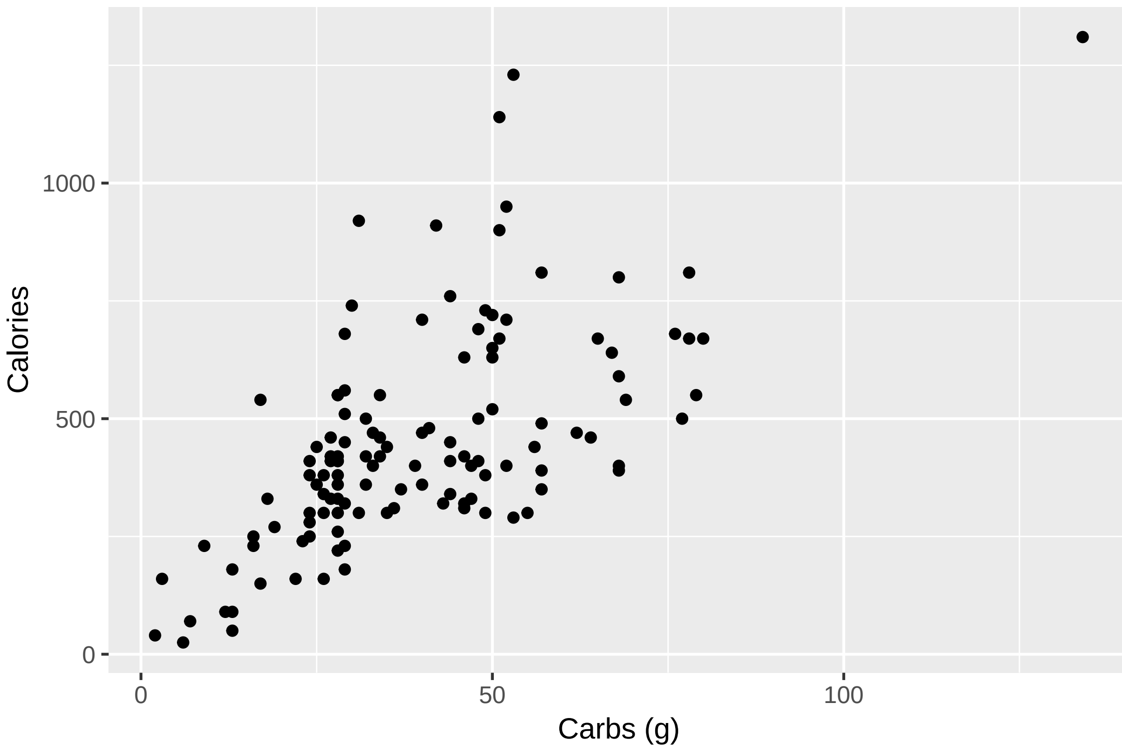
The `I()` function is used to keep the class of an object the same. Here we use it to keep the variable `Displacement` "as is" to prevent an error.

Adjusting for Different Slopes

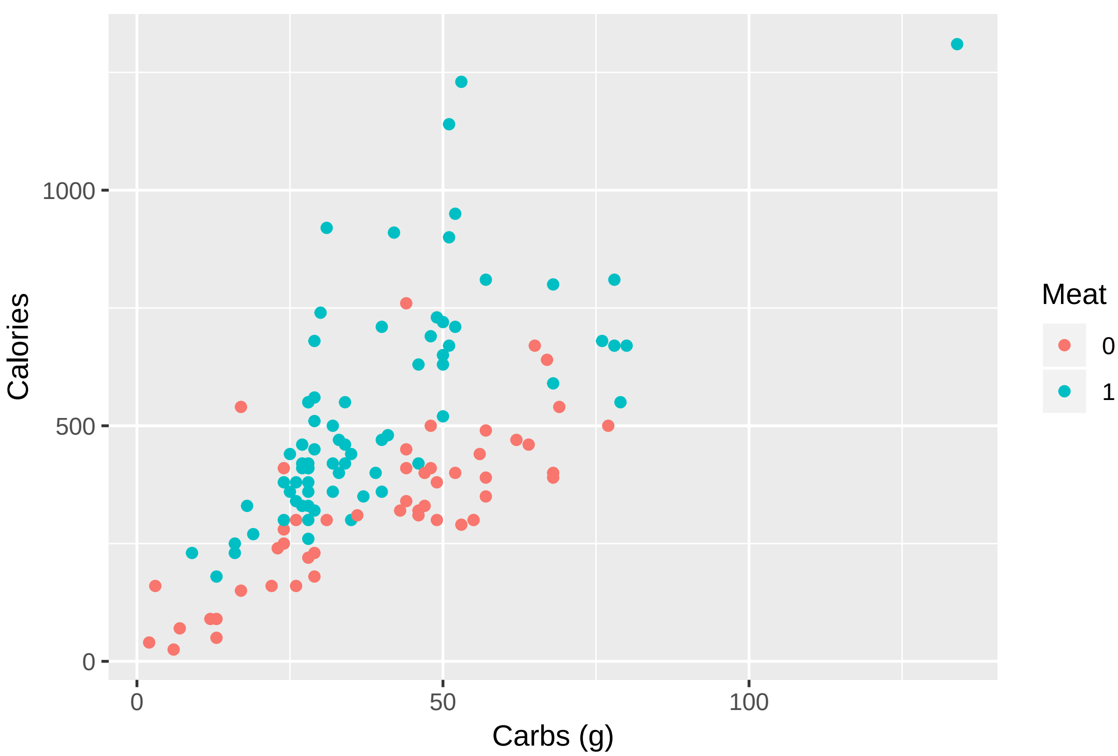
```r
BurgerKing <- read_csv("http://nhorton.people.amherst.edu/is5/data/Burger_King_items.csv") %>%
  clean_names()
```

```
## Parsed with column specification:
## cols(
##   Item = col_character(),
##   Serving.size = col_integer(),
##   Calories = col_integer(),
##   Fat.Cal = col_integer(),
##   `Protein(g)` = col_integer(),
##   `Fat(g)` = col_double(),
##   `Sat.Fat(g)` = col_double(),
##   `Trans.fat(g)` = col_double(),
##   `Chol(mg)` = col_integer(),
##   `Sodium(mg)` = col_integer(),
##   `Carbs(g)` = col_integer(),
##   `Fiber(g)` = col_integer(),
##   `Sugar(g)` = col_integer(),
##   Meat = col_integer(),
##   Breakfast = col_integer(),
##   `Not Breakfast` = col_integer(),
##   CarbsxMeat = col_integer()
## )
```

```r
# Figure 9.9, page 292
gf_point(calories ~ carbs_g, data = BurgerKing) %>%
  gf_labs(x = "Carbs (g)", y = "Calories")
```




```r
# Figure 9.10
gf_point(calories ~ carbs_g, color = ~ as.factor(meat), data = BurgerKing) %>%
  gf_labs(x = "Carbs (g)", y = "Calories", color = "Meat")
```



```r
msummary(lm(calories ~ carbs_g * as.factor(meat), data = BurgerKing))
```

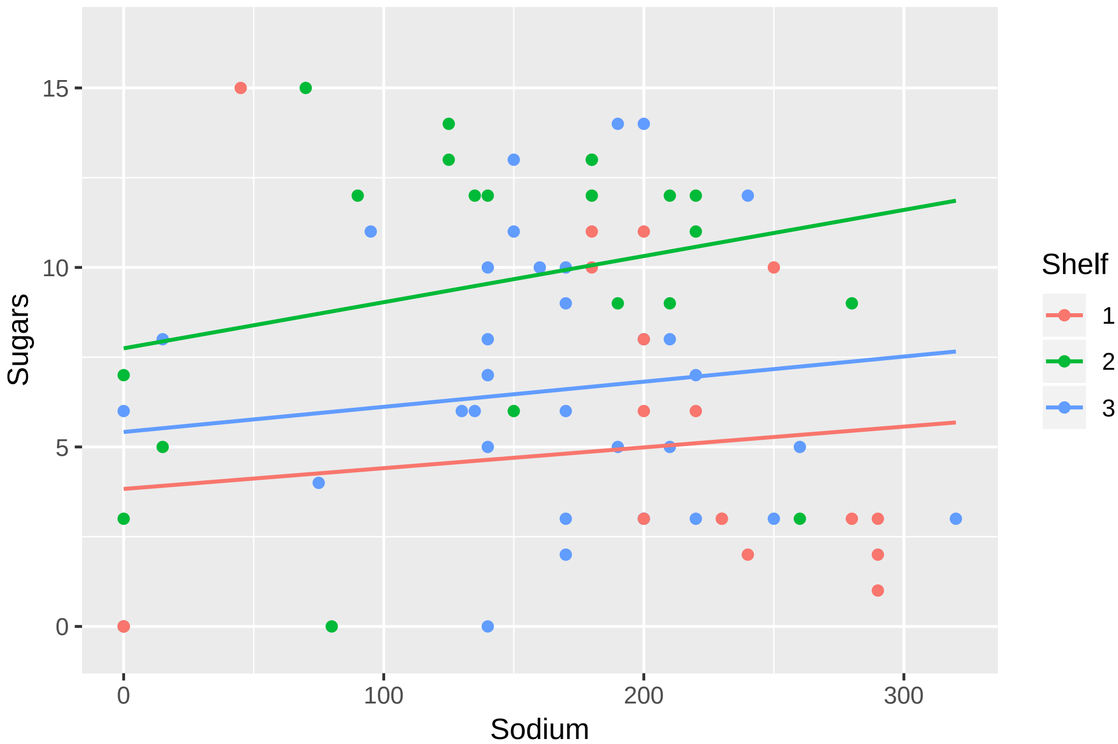
```
##                          Estimate Std. Error t value Pr(>|t|)
## (Intercept)                83.533     46.955   1.779   0.0778 .
## carbs_g                     6.255      1.063   5.885 3.81e-08 ***
## as.factor(meat)1          120.220     60.694   1.981   0.0499 *
## carbs_g:as.factor(meat)1    2.145      1.378   1.557   0.1222
##
## Residual standard error: 146.5 on 118 degrees of freedom
## Multiple R-squared:  0.6072, Adjusted R-squared:  0.5972
## F-statistic:  60.8 on 3 and 118 DF,  p-value: < 2.2e-16
```

One, Two, Many

```r
Cereal <- read_csv("http://nhorton.people.amherst.edu/is5/data/Cereals.csv")
```

```
## Parsed with column specification:
## cols(
##   name = col_character(),
##   mfr = col_character(),
##   calories = col_integer(),
##   sugars = col_integer(),
##   carbo = col_double(),
##   protein = col_integer(),
##   fat = col_integer(),
##   sodium = col_integer(),
##   fiber = col_double(),
##   potass = col_integer(),
##   shelf = col_integer(),
##   Middle = col_character(),
##   shelf_1 = col_integer(),
##   shelf_2 = col_integer(),
##   shelf_3 = col_integer()
## )
```

```r
ceреallm <- lm(sugars ~ sodium + as.factor(shelf), data = Cereal)
gf_point(sugars ~ sodium, color = ~ as.factor(shelf), data = Cereal) %>%
  gf_lm() %>%
  gf_labs(x = "Sodium", y = "Sugars", color = "Shelf")
```




```r
msummary(cereallm)
```

```
##                    Estimate Std. Error t value Pr(>|t|)
## (Intercept)        3.446740   1.345111   2.562 0.012457 *
## sodium             0.007962   0.005620   1.417 0.160818
## as.factor(shelf)2 5.012166   1.283154   3.906 0.000207 ***
## as.factor(shelf)3 1.818214   1.139384   1.596 0.114857
##
## Residual standard error: 4.07 on 73 degrees of freedom
## Multiple R-squared:  0.1866, Adjusted R-squared:  0.1532
## F-statistic: 5.583 on 3 and 73 DF,  p-value: 0.001669
```

Example 9.4: Indicators for Variables with Several Levels

```r
Diamonds <- read_csv("http://nhorton.people.amherst.edu/is5/data/Diamonds.csv") %>%
  clean_names()
```

```
## Parsed with column specification:
## cols(
##   Price = col_integer(),
##   `Carat Size` = col_double(),
##   Color = col_character(),
##   Clarity = col_character(),
##   Cut = col_character()
## )
```

```r
# Parallel Slopes
diamondlm <- lm(sqrt(price) ~ carat_size + color, data = Diamonds)
msummary(diamondlm)
```

```
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   13.1946     0.5488  24.043  < 2e-16 ***
## carat_size    61.2491     0.5032 121.722  < 2e-16 ***
## colorE        -2.1027     0.5399  -3.895 0.000101 ***
## colorF        -2.8640     0.5576  -5.136 3.00e-07 ***
## colorG        -3.6320     0.5769  -6.296 3.57e-10 ***
## colorH        -7.8948     0.5858 -13.477  < 2e-16 ***
## colorI       -11.8542     0.6261 -18.932  < 2e-16 ***
## colorJ       -16.6404     0.6637 -25.071  < 2e-16 ***
## colorK       -21.3577     0.8282 -25.787  < 2e-16 ***
##
## Residual standard error: 7.218 on 2681 degrees of freedom
## Multiple R-squared:  0.8583, Adjusted R-squared:  0.8579
## F-statistic:  2030 on 8 and 2681 DF,  p-value: < 2.2e-16
```

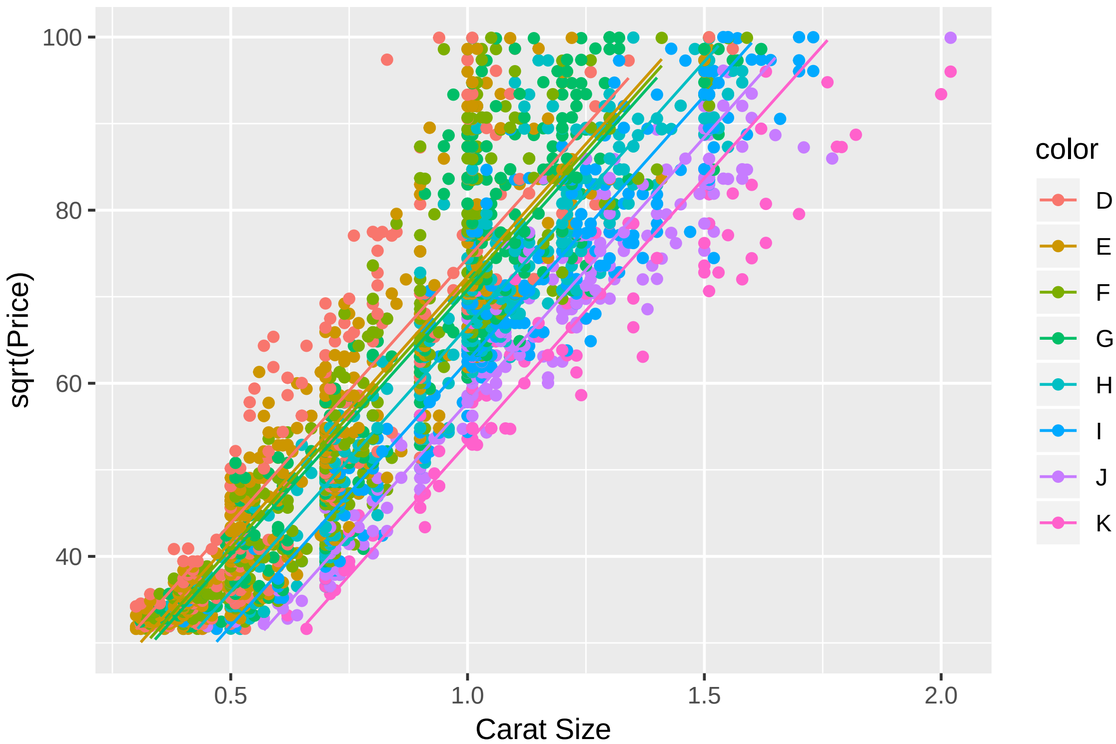
```r
diamondpredict <- makeFun(diamondlm)

diamonddata <- augment(diamondlm) %>% # To get fitted values
  clean_names()
str(diamonddata)
```

```
## 'data.frame':    2690 obs. of  10 variables:
##  $ sqrt_price: num  31.6 31.6 31.6 31.6 31.6 ...
##  $ carat_size: num  0.3 0.44 0.31 0.66 0.47 0.4 0.36 0.52 0.53 0.43 ...
##  $ color     : chr  "E" "E" "E" "K" ...
##  $ fitted    : num  29.5 38 30.1 32.3 34.1 ...
##  $ se_fit    : num  0.374 0.344 0.372 0.707 0.442 ...
##  $ resid     : num  2.156 -6.419 1.544 -0.639 -2.464 ...
##  $ hat       : num  0.00269 0.00226 0.00265 0.00959 0.00375 ...
##  $ sigma     : num  7.22 7.22 7.22 7.22 7.22 ...
##  $ cooksd    : num  2.68e-05 2.00e-04 1.35e-05 8.51e-06 4.89e-05 ...
##  $ std_resid : num  0.2991 -0.8902 0.2141 -0.0889 -0.342 ...
```

```r
gf_point(sqrt_price ~ carat_size, color = ~ color, data = diamonddata) %>%
  gf_line(fitted ~ carat_size) %>%
  gf_labs(x = "Carat Size", y = "sqrt(Price)") +
  ylim(30, 100)
```

```
## Warning: Removed 79 rows containing missing values (geom_path).
```




```r
# With interaction
diamondlm2 <- lm(sqrt(price) ~ carat_size * color, data = Diamonds)
msummary(diamondlm2)
```

```
##                    Estimate Std. Error t value Pr(>|t|)
## (Intercept)          9.3239     1.2142   7.679 2.23e-14 ***
## carat_size          67.0408     1.7025  39.379  < 2e-16 ***
## colorE              -0.5392     1.5075  -0.358  0.72063
## colorF              -2.3716     1.5627  -1.518  0.12922
## colorG              -2.6709     1.6643  -1.605  0.10867
## colorH              -3.9177     1.8248  -2.147  0.03189 *
## colorI              -2.5481     1.9301  -1.320  0.18689
## colorJ              -5.4176     2.0716  -2.615  0.00897 **
## colorK               0.5976     2.7815   0.215  0.82991
## carat_size:colorE   -2.4007     2.0999  -1.143  0.25305
## carat_size:colorF   -1.3211     2.0954  -0.630  0.52843
## carat_size:colorG   -2.5457     2.0868  -1.220  0.22260
## carat_size:colorH   -5.9017     2.1774  -2.710  0.00676 **
## carat_size:colorI  -10.9139     2.1812  -5.004 5.99e-07 ***
## carat_size:colorJ  -12.4948     2.2531  -5.546 3.22e-08 ***
## carat_size:colorK  -21.4477     2.6978  -7.950 2.72e-15 ***
##
## Residual standard error: 7.058 on 2674 degrees of freedom
## Multiple R-squared:  0.8649, Adjusted R-squared:  0.8641
## F-statistic:  1141 on 15 and 2674 DF,  p-value: < 2.2e-16
```

```r
gf_point(sqrt(price) ~ carat_size, color = ~ color, data = Diamonds) %>%
  gf_lm() %>%
```

```r
gf_labs(x = "Carat Size", y = "sqrt(Price)") +
ylim(30, 100)
```

```
## Warning: Removed 204 rows containing missing values (geom_lm).
```