IS5 in R: Comparing Counts (Chapter 19)

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Introduction and background

This document is intended to help describe how to undertake analyses introduced as examples in the Fifth Edition of *Intro Stats* (2018) by De Veaux, Velleman, and Bock. More information about the book can be found at http://wps.aw.com/aw_deveaux_stats_series. This file as well as the associated R Markdown reproducible analysis source file used to create it can be found at http://nhorton.people.amherst.edu/is5.

This work leverages initiatives undertaken by Project MOSAIC (http://www.mosaic-web.org), an NSF-funded effort to improve the teaching of statistics, calculus, science and computing in the undergraduate curriculum. In particular, we utilize the mosaic package, which was written to simplify the use of R for introductory statistics courses. A short summary of the R needed to teach introductory statistics can be found in the mosaic package vignettes (http://cran.r-project.org/web/packages/mosaic). A paper describing the mosaic approach was published in the R Journal: https://journal.r-project.org/archive/2017/RJ-2017-024.

Chapter 19: Comparing Counts

```
library(mosaic)
library(readr)
library(janitor)
Zodiac <- read_csv("http://nhorton.people.amherst.edu/is5/data/Zodiac.csv")

## Parsed with column specification:
## cols(
## Month = col_character(),
## Births = col_integer(),
## Expected = col_double(),
## Residual = col_double()</pre>
```

By default, read_csv() prints the variable names. These messages can be suppressed using the message=FALSE code chunk option to save space and improve readability.

```
Zodiac %>%
select(Month, Births)
```

```
## # A tibble: 12 x 2
##
      Month
                   Births
##
      <chr>
                    <int>
##
    1 Pisces
                       29
##
                       24
    2 Aquarius
##
    3 Aries
                       23
##
    4 Cancer
                       23
                       22
##
    5 Capricorn
##
   6 Scorpio
                       21
   7 Taurus
                       20
##
    8 Leo
                       20
   9 Saggitarius
                       19
```

```
## 10 Virgo 19
## 11 Libra 18
## 12 Gemini 18
```

Section 19.1: Goodness-of-Fit Tests

Example 19.1: Finding Expected Counts

```
# page 611
BaseballBirths <- read_csv("http://nhorton.people.amherst.edu/is5/data/Ballplayer_births.csv") %>%
  clean_names() # doesn't contain national birth %
## Parsed with column specification:
## cols(
##
    Month = col_integer(),
     `Ballplayer Count` = col_integer()
##
## )
Here we use the clean_names() function from the janitor package to sanitize the names of the columns
(which would otherwise contain special characters or whitespace).
natbirth <- c(.08, .07, .08, .08, .08, .08, .09, .09, .09, .09, .09, .09)
BaseballBirths <- cbind(BaseballBirths, natbirth) # adding a column for national birth %
totaln <- sum(~ ballplayer_count , data = BaseballBirths)</pre>
totaln
## [1] 1478
BaseballBirths <- BaseballBirths %>%
  mutate(expected = totaln * natbirth,
         observed = ballplayer_count,
         contrib = (observed - expected)^2/expected)
sum(~ contrib, data = BaseballBirths)
## [1] 26.48442
```

Assumptions and Conditions

Calculations

Chi-Square P-values

```
# Examples of chisq p-values
qchisq(df = 2, p = .1, lower.tail = FALSE)

## [1] 4.60517
qchisq(df = 10, p = .05, lower.tail = FALSE)

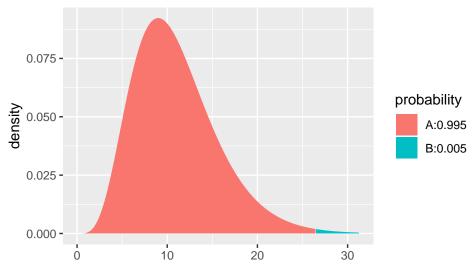
## [1] 18.30704
```

Example 19.3: Doing a Goodness-of-Fit Test

```
# page 614
df <- nrow(BaseballBirths) - 1
df</pre>
```

[1] 11

```
chisq <- sum(~ contrib, data = BaseballBirths)
xpchisq(q = chisq, df = df, lower.tail = FALSE)</pre>
```



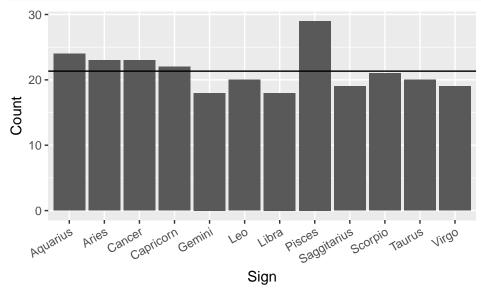
[1] 0.005494028

Step-By-Step Example: A Chi-Square Test for Goodness-of-Fit

```
expected <- mean(~ Births, data = Zodiac)
expected</pre>
```

[1] 21.33333

```
gf_col(Births ~ Month, data = Zodiac) %>%
gf_hline(yintercept = expected) %>%
gf_labs(x = "Sign", y = "Count") +
theme(axis.text.x = element_text(angle = 30, hjust = 1)) # to adjust the angle of the x axis labels
```



```
# Mechanics
df <- nrow(Zodiac) - 1</pre>
```

```
## [1] 11
Zodiac <- Zodiac %>%
    mutate(chisq = ((Births - Expected)^2)/Expected)
chisq <- sum(~ chisq, data = Zodiac)
chisq

## [1] 5.09383
xpchisq(q = chisq, df = df, lower.tail = FALSE)</pre>
probability
A:0.073
```

B:0.927

[1] 0.9265374

0

0.025 -

0.000

The Chi-Square Calculation

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```
Zodiac %>%
  mutate(residsq = Residual^2) %>%
  mutate(component = residsq/Expected)
```

30

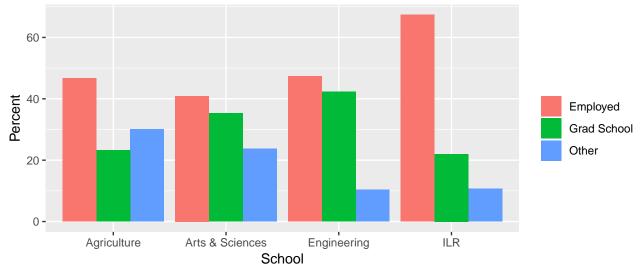
```
## # A tibble: 12 x 7
##
      Month
                  Births Expected Residual
                                              chisq residsq component
##
      <chr>
                   <int>
                             <dbl>
                                      <dbl>
                                              <dbl>
                                                       <dbl>
                                                                 <dbl>
##
   1 Pisces
                      29
                              21.3
                                      7.67 2.76
                                                      58.8
                                                               2.76
                      24
                              21.3
                                      2.67 0.333
##
  2 Aquarius
                                                       7.11
                                                               0.333
                      23
                              21.3
                                      1.67 0.130
                                                       2.78
                                                               0.130
## 3 Aries
##
  4 Cancer
                      23
                              21.3
                                      1.67 0.130
                                                       2.78
                                                               0.130
##
   5 Capricorn
                      22
                              21.3
                                      0.667 0.0209
                                                       0.445
                                                               0.0209
                             21.3
                                     -0.333 0.00520
                                                       0.111
##
  6 Scorpio
                      21
                                                               0.00520
## 7 Taurus
                      20
                              21.3
                                     -1.33 0.0833
                                                       1.78
                                                               0.0833
                      20
                             21.3
                                     -1.33 0.0833
                                                       1.78
## 8 Leo
                                                               0.0833
## 9 Saggitarius
                      19
                             21.3
                                     -2.33 0.255
                                                       5.44
                                                               0.255
## 10 Virgo
                      19
                              21.3
                                     -2.33 0.255
                                                       5.44
                                                               0.255
## 11 Libra
                              21.3
                                     -3.33 0.521
                                                               0.521
                      18
                                                      11.1
## 12 Gemini
                      18
                              21.3
                                     -3.33 0.521
                                                      11.1
                                                               0.521
```

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The Trouble with Goodness-of-Fit Tests: What's the Alternative?

Section 19.2: Chi-Square Test of Homogeneity

```
# Create the data set
Postgrad <- rbind(</pre>
  do(209) * data.frame(activity = "Employed", school = "Agriculture"),
  do(198) * data.frame(activity = "Employed", school = "Arts & Sciences"),
  do(177) * data.frame(activity = "Employed", school = "Engineering"),
  do(101) * data.frame(activity = "Employed", school = "ILR"),
  do(104) * data.frame(activity = "Grad School", school = "Agriculture"),
  do(171) * data.frame(activity = "Grad School", school = "Arts & Sciences"),
  do(158) * data.frame(activity = "Grad School", school = "Engineering"),
  do(33) * data.frame(activity = "Grad School", school = "ILR"),
  do(135) * data.frame(activity = "Other", school = "Agriculture"),
  do(115) * data.frame(activity = "Other", school = "Arts & Sciences"),
  do(39) * data.frame(activity = "Other", school = "Engineering"),
  do(16) * data.frame(activity = "Other", school = "ILR")
)
# Table 19.1, page 618
tally(activity ~ school, data = Postgrad, margins = TRUE)
##
                school
## activity
                 Agriculture Arts & Sciences Engineering ILR
##
     Employed
                         209
                                         198
                                                     177 101
##
     Grad School
                         104
                                         171
                                                     158 33
##
     Other
                         135
                                         115
                                                      39 16
     Total
                         448
                                         484
##
                                                     374 150
# Table 19.2
tally(activity ~ school, format = "percent", data = Postgrad, margins = TRUE)
##
                school
## activity
                 Agriculture Arts & Sciences Engineering
                                                                ILR
##
                    46.65179
                                    40.90909
                                                47.32620 67.33333
     Employed
##
     Grad School
                    23.21429
                                    35.33058
                                                42.24599 22.00000
                    30.13393
##
     Other
                                    23.76033
                                                10.42781 10.66667
##
    Total
                   100.00000
                                   100.00000
                                               100.00000 100.00000
# Table 19.3
with(chisq.test(tally(activity ~ school, data = Postgrad, margins = TRUE)), expected)
##
                school
## activity
                 Agriculture Arts & Sciences Engineering
                                                                ILR
##
                   210.76923
                                    227.7060
                                               175.95467
                                                          70.57005
     Employed
##
     Grad School
                   143.38462
                                    154.9066
                                               119.70055
                                                          48.00824
##
     Other
                                    101.3874
                                                78.34478 31.42170
                   93.84615
##
     Total
                   448.00000
                                    484.0000
                                               374.00000 150.00000
Step-By-Step Example: A Chi-Square Test for Homogeneity
tally(activity ~ school, format = "percent", data = Postgrad) %>%
  data.frame() %>%
  gf col(Freq ~ school, fill = ~ activity, position = "dodge") %>%
  gf_labs(x = "School", y = "Percent", fill = "")
```



```
# Mechanics
tally(activity ~ school, data = Postgrad, margins = TRUE)
```

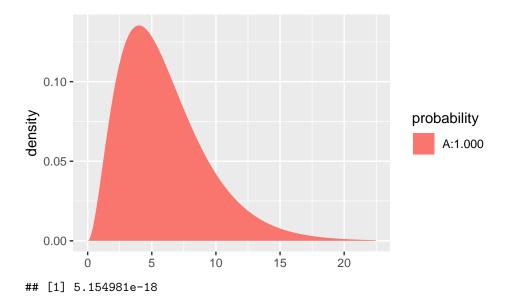
```
##
                 school
                 Agriculture Arts & Sciences Engineering ILR
## activity
##
                                           198
                                                       177 101
     Employed
                          209
     Grad School
                                                       158 33
##
                          104
                                           171
##
     Other
                          135
                                           115
                                                        39 16
                                                       374 150
##
     Total
                          448
                                           484
```

with(chisq.test(tally(activity ~ school, data = Postgrad, margins = TRUE)), expected)

```
##
                school
## activity
                 Agriculture Arts & Sciences Engineering
                                                               ILR
                   210.76923
##
     Employed
                                    227.7060
                                              175.95467
                                                          70.57005
##
     Grad School
                   143.38462
                                    154.9066
                                               119.70055 48.00824
##
     Other
                   93.84615
                                    101.3874
                                               78.34478 31.42170
     Total
                   448.00000
                                    484.0000
                                               374.00000 150.00000
##
with(chisq.test(tally(activity ~ school, data = Postgrad)), statistic)
```

```
## X-squared
## 93.65667
```

xpchisq(q = 93.7, df = 6, lower.tail = FALSE)



Section 19.3: Examining the Residuals

```
# Table 19.4, page 622
with(chisq.test(tally(activity ~ school, data = Postgrad, margins = TRUE)), residuals)
##
               school
## activity
                Agriculture Arts & Sciences Engineering
##
     Employed
                -0.12186553
                                -1.96860027 0.07880484 3.62235442
##
    Grad School -3.28908677
                                 1.29304319 3.50061599 -2.16606715
##
     Other
                 4.24817296
                                 1.35191804 -4.44510568 -2.75117035
##
     Total
                 0.00000000
                                 0.0000000 0.0000000 0.00000000
```

Example 19.4: Looking at χ^2 , Residuals

```
BaseballBirths %>%
mutate(residuals = (ballplayer_count - expected)/(expected^.5)) %>%
select(month, residuals)
```

```
##
            residuals
     month
## 1
          1 1.72524439
## 2
          2 1.72442119
## 3
          3 -0.20599933
## 4
          4 0.25382060
         5 0.71364054
## 5
## 6
         6 -0.38992730
         7 -2.68957291
## 7
## 8
         8 2.77280921
## 9
         9 0.08497039
## 10
        10 -1.56241469
## 11
         11 -1.21760318
## 12
        12 -0.95548335
```

Section 19.4: Chi-Square Test of Independence

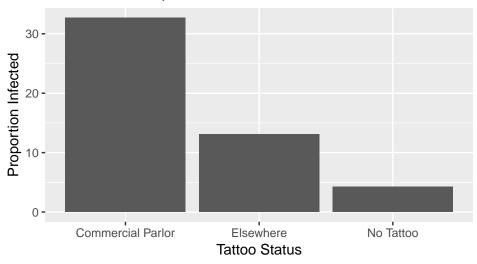
```
Tattoos <- read_csv("http://nhorton.people.amherst.edu/is5/data/Tattoos.csv", skip = 1) %>%
  clean_names() # skip = 1 because first row is "Col1", "Col2"
## Parsed with column specification:
## cols(
##
     Location = col_character(),
     `Has hepatitis C` = col_character()
## )
# Table 19.5, page 623
tally(location ~ has_hepatitis_c, data = Tattoos, margins = TRUE)
##
                      has_hepatitis_c
## location
                        No Yes
##
     Commercial Parlor
                       35
                           17
##
     Elsewhere
                        53
##
    No Tattoo
                       491 22
     Total
                       579 47
##
```

Assumptions and Conditions

Step-By-Step Example: A Chi-Square Test for Independence

```
tally(has_hepatitis_c ~ location, format = "percent", data = Tattoos) %>%
  data.frame() %>%
  filter(has_hepatitis_c == "Yes") %>%
  gf_col(Freq ~ location) %>%
  gf_labs(x = "Tattoo Status", y = "Proportion Infected", title = "Tattoos and Hepatitis C")
```

Tattoos and Hepatitis C



```
# Observed
tally(location ~ has_hepatitis_c, data = Tattoos, margins = TRUE)

## has_hepatitis_c
## location No Yes
```

```
Commercial Parlor 35 17
##
##
     Elsewhere
                        53
                             8
     No Tattoo
                       491 22
##
     Total
                       579 47
##
# Expected
with(chisq.test(tally(location ~ has_hepatitis_c, data = Tattoos, margins = TRUE)), expected)
## Warning in chisq.test(tally(location ~ has_hepatitis_c, data = Tattoos, :
## Chi-squared approximation may be incorrect
##
                      has_hepatitis_c
## location
                               No
                                        Yes
##
     Commercial Parlor 48.09585 3.904153
##
     Elsewhere
                        56.42013 4.579872
     No Tattoo
                       474.48403 38.515974
##
                       579.00000 47.000000
##
     Total
# Mechanics
with(chisq.test(tally(location ~ has_hepatitis_c, data = Tattoos)), statistic)
## Warning in chisq.test(tally(location ~ has_hepatitis_c, data = Tattoos)):
## Chi-squared approximation may be incorrect
## X-squared
## 57.91217
xpchisq(q = 57.9, df = 2, lower.tail = FALSE)
  0.5 -
  0.4 -
density
0.3
                                                         probability
                                                              A:1.000
  0.1 -
  0.0 -
                                       10
                       5
## [1] 2.674082e-13
Examine the Residuals
```

```
# Table 19.6, page 627
with(chisq.test(tally(location ~ has_hepatitis_c, data = Tattoos)), residuals)
## Warning in chisq.test(tally(location ~ has_hepatitis_c, data = Tattoos)):
## Chi-squared approximation may be incorrect
## has_hepatitis_c
## location No Yes
```

```
##
    Commercial Parlor -1.8883383 6.6278115
##
    Elsewhere -0.4553290 1.5981431
## No Tattoo
                     0.7582168 -2.6612383
# Table 19.7, page 628
Tattoos <- Tattoos %>%
 mutate(tattoo = ifelse(location == "No Tattoo", "None", "Tattoo"))
tally(tattoo ~ has_hepatitis_c, margins = TRUE, data = Tattoos)
##
          has_hepatitis_c
           No Yes
## tattoo
         491 22
##
    None
    Tattoo 88 25
##
    Total 579 47
```

Chi-Square and Causation