Closure Conversion for Dependent Type Theory, With Type-Passing Polymorphism*

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Closure conversion is an early translation step in the compilation of functional languages, which converts functions with potential free variable occurrences to pairs consisting of environments and closed functions. Environments carry values corresponding to free variables of functions. The closed functions abstract over the captured environments as well as the arguments of the original functions. This representation allows an efficient environment-based execution model where closed functions are compiled to shared immutable code segments.

Minamide et al. [1] described type-preserving closure conversion for a polymorphic language. They considered an *intensional* or *type-passing* implementation of polymorphism, which enables different memory layouts for differently typed runtime objects, and necessitates that runtime type representations are passed to polymorphic functions. In contrast, *type-erasing* polymorphism (as in [2]) removes types during compilation, mandating uniform runtime representations (although with potential layout-changing optimizations, such as unboxing).

Generalizing type-passing polymorphism to dependent type theories would allow precise specification of memory layout using dependent types. For example, Σ-types may represent two values next to each other in memory, where the size and layout of the second field depends on the value of the first field. Hence, runtime objects would be described by type-theoretic universes instead of simple statically known layout schemes. Also, a closure-converted type theory with precise control over memory layout could be useful as an intermediate language even if types are erased somewhere on the way to machine code.

The current work is a first step in this direction. I describe a dependent type theory with a predicative universe hierarchy, Σ-types, Π-types with *closed* inhabitants and a novel construction of closures. Consistency for this theory is proved with a standard type-theoretic model. Then, it is proved that the general function space with term formation in non-empty contexts is admissible in this theory. General functions are represented as closures and term formation corresponds to closure building. The expected β and η rules also hold for this function space. Then, a closure conversion translation into this theory is presented, from a source theory with predicative universes and dependent functions. Injectivity, preservation of typing and preservation of conversion are proven for the translation.

Closures and type codes in the target theory

The target theory has predicative universes U_i with decoding El, Σ-types, closed function types (denoted $(a : A) \to B$) and closure types $\mathsf{Cl}\,(a : A)\,B$. Closed functions differ from usual functions only in the term formation rule: λ-abstraction is only valid in the empty context (denoted $\cdot$). For closures, there are rules for type and term formation, elimination, and η and β conversion, presented in this order:

$$\frac{\Gamma \vdash A\,\mathsf{type}_i \quad \Gamma, a : A \vdash B\,\mathsf{type}_j}{\Gamma \vdash \mathsf{Cl}\,(a : A)\,B\;\mathsf{type}_{\mathrm{max}(i,j)}} \qquad \frac{\cdot \vdash E : \mathsf{U}_i \quad \Gamma \vdash \mathit{env} : \mathsf{El}\,E \quad \cdot \vdash t : (ea : \Sigma(e : \mathsf{El}\,E).A) \to B}{\Gamma \vdash \mathsf{pack}\,E\,\mathit{env}\;t : \mathsf{Cl}\,(a : A[e \mapsto \mathit{env}])\,(B[ea \mapsto (\mathit{env}, a)])}$$

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$$\frac{\Gamma \vdash t : \mathsf{Cl}\,(a : A)\,B \quad \Gamma \vdash u : A}{\Gamma \vdash t\,u : B[a \mapsto u]} \qquad \frac{\Gamma \vdash t : \mathsf{Cl}\,(a : A)\,B \quad \Gamma \vdash u : \mathsf{Cl}\,(a : A)\,B \quad \Gamma,\, a : A \vdash t\,a \equiv u\,a}{\Gamma \vdash t \equiv u}$$

$$(\mathsf{pack}\,E\,env\;t)\,u \equiv t\,(env,\,u)$$

We use a primitive construction, in contrast to [1] where closures are derived from existential types and translucent functions. The main reason is that universe levels of captured environments can be arbitrarily high and need to be hidden, which rules out Σ representations. We can prove consistency (and thus universe consistency) of Cl by a standard type-theoretic model which unpacks environments and functions from pack, and applies the latter to the former, and elsewhere acts as expected.

The type code inhabitants of U_i are themselves closure-converted: for instance, a code for a Cl type contains a code for the domain and a closure which computes the codomain type. This is because in a type-passing implementation, type dependencies need to be computed at runtime in a efficient manner. Decoding with El computes types from codes by applying closures as needed. Rules for Cl codes are listed below; cases for other types are analogous.

$$\frac{\Gamma \vdash A : \mathsf{U}_i \quad \Gamma \vdash B : \mathsf{Cl}\,(\mathsf{El}\,A)\,(\mathsf{U}_j)}{\Gamma \vdash \mathsf{Cl}'\,A\,B : \mathsf{U}_{\max(i,\,j)}} \qquad \mathsf{El}\,(\mathsf{Cl}'\,A\,B) \equiv \mathsf{Cl}\,(a : \mathsf{El}\,A)\,(\mathsf{El}\,(B\,a))$$

Admissibility of general function space

The main goal is to build a term of $\mathsf{Cl}\,(a : A)\,B$ from some $\Gamma, a : A \vdash t : B$, in a way such that β, η and substitution rules hold. Closure building is defined mutually with quoting operations on well-formed contexts and types:

- From each Γ, we construct a closed code $\mathsf{quote}\,\Gamma$ for the corresponding iterated Σ-type, along with an isomorphism between Γ and the singleton context containing $\mathsf{El}\,(\mathsf{quote}\,\Gamma)$, consisting of two back-and-forth substitutions.
- From each $\Gamma \vdash A\ \mathsf{type}_i$, we construct $\Gamma \vdash \mathsf{quote}\,A : \mathsf{U}_i$, such that El retracts quote, and quote is natural with respect to type substitution. Quoting to type codes here involves building closures which compute type dependencies, as we have seen for the Cl example.
- Closures are built by pack-ing together quote-ed environment types, environments (given from $\Gamma \to \mathsf{El}\,(\mathsf{quote}\,\Gamma)$ substitutions) and closed function bodies (given by closing the t input function bodies using the $\mathsf{El}\,(\mathsf{quote}\,\Gamma) \to \Gamma$ substitutions).

Closure conversion translation

With admissible general function space at hand, closure conversion is given by mutual induction on well-formed source syntax, converting source functions to closures.

References

[1] Yasuhiko Minamide, Greg Morrisett, and Robert Harper. Typed closure conversion. In *Proceedings of the 23rd ACM SIGPLAN-SIGACT symposium on Principles of programming languages*, pages 271–283. ACM, 1996.

[2] Greg Morrisett, David Walker, Karl Crary, and Neal Glew. From system f to typed assembly language. *ACM Transactions on Programming Languages and Systems (TOPLAS)*, 21(3):527–568, 1999.