1 Signal Detection Theory III *2014-06-20*

1.1 From YN to 2AFC

In comparison to simple Yes-No-task there exists an alternative task design which is the 2-Alternative-Forced-Choice-task. In each trial the subject is presented with two intervals with a light stimulus in one of it, therefore there are two "stimulations" X_1,X_2. The probabilities for this experiment are given in the following.

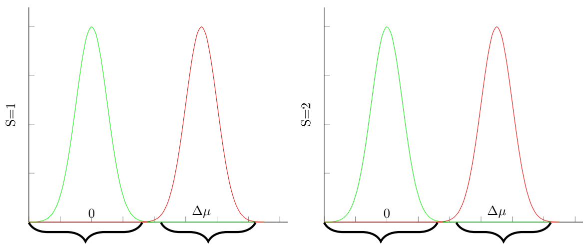
$(X_1|S=1) \sim N(\Delta\mu, \sigma^2)$ - The probability distribution for a stimulation in the first interval given that the signal was in the first interval

$(X_2|S=1) \sim N(0, \sigma^2)$

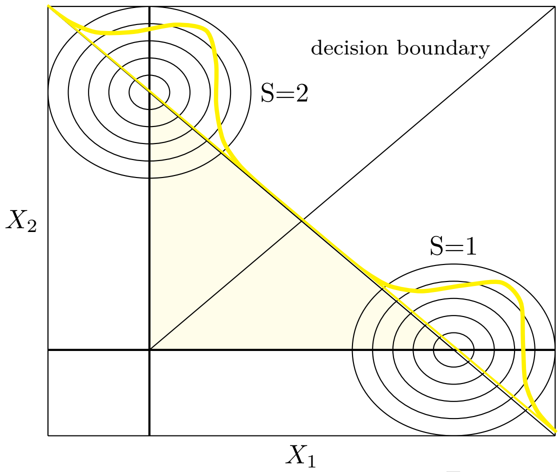
} equal variance signal detection model

$(X_1|S=2) \sim N(0, \sigma^2)$

$(X_2|S=2) \sim N(\Delta\mu, \sigma^2)$



$2^{nd}interval$ $1^{st}interval$ $1^{st}interval$ $2^{nd}interval$



Distance of the $\Delta\mu$: $\Delta\mu_{2AFC} = \sqrt{2}\Delta\mu$

Best strategy for the best performance in 2AFC:

- Say 1 if $X_1 > X_2$
- Say 2 if $X_2 \geq X_1$

$\Rightarrow \Delta X = X_2 - X_1 \overset{!}{>} 0$

d' = Detection Performance in YN-task
= signal-to-noise-ratio

What is the distribution of ΔX?

$(\Delta X|S=1) \sim N(-\Delta\mu, 2\sigma^2)$

$(\Delta X|S=2) \sim N(\Delta\mu, \sigma^2 + ((-1)\sigma)^2) = N(\Delta\mu, 2\sigma^2)$

Helpful rules:

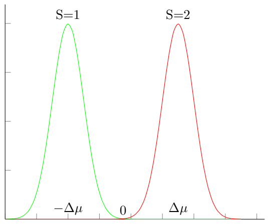
X_1, X_2 are normal (N)

$X_1 \sim N(\mu_1, \sigma_1^2)$

$X_2 \sim N(\mu_2, \sigma_2^2)$

$aX_1 \sim N(a\mu_1, (a\sigma_1)^2)$

$X_1 + X_2 \sim N(\mu_1 + \mu 2, \sigma_1^2 + \sigma_2^2)$



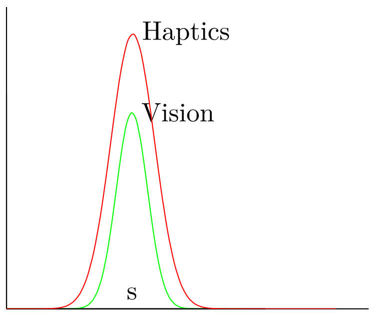
$$
\begin{aligned}
\Delta\mu_{2AFC} &= \sqrt{2}\Delta\mu| : \sigma \\
\Leftrightarrow d'_{2AFC} &= \sqrt{2}d'
\end{aligned}
$$

$$
\begin{aligned}
\frac{\Delta\mu_{2AFC}}{\sigma} &= \frac{2\Delta\mu}{\sqrt{2}\sigma} = \sqrt{2}d' \\
d'_{2AFC} &= \frac{2d'}{\sqrt{2}\sigma} = \sqrt{2}d' \text{(same result as in the geometric solution)}
\end{aligned}
$$

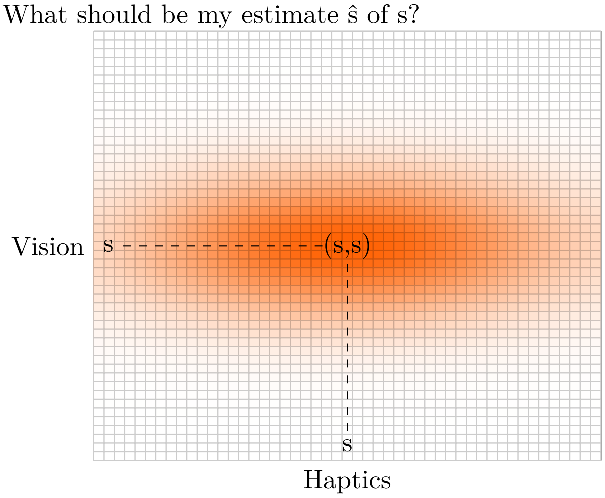
1.2 Cue Combination

Ernst & Banks (2002) - Visuo-haptic cie combination

- Judge the size of a bar when you can see and feel it
- You get two measurements of s
  - $V \sim N(s, \sigma_V^2)$
  - $H \sim N(s, \sigma_H^2)$



What should be my estimate ŝ of s?



$p(V = v; H = h|s) = \frac{1}{\sqrt{2\pi}\sigma_V} e^{-\frac{1}{2}\left(\frac{v-s}{\sigma_V}\right)^2} \frac{1}{\sqrt{2\pi}\sigma_H} e^{-\frac{1}{2}\left(\frac{h-s}{\sigma_H}\right)^2}$

ML-Estimate ŝ: Maximize log-likelihood of $p(V = v; H = h|s)$

$\Rightarrow -\frac{1}{2}\left(\left(\frac{v-\hat{s}}{\sigma_V}\right)^2 + \left(\frac{h-\hat{s}}{\sigma_H}\right)^2\right) = -\frac{1}{2}\left(\frac{v-\hat{s}}{\sigma_V}\right)^2 - \frac{1}{2}\left(\frac{h-\hat{s}}{\sigma_H}\right)^2$

First derivative:

$$
\begin{aligned}
& & \left(\frac{v-\hat{s}}{\sigma_V}\right)\frac{2}{2\sigma_V} + \left(\frac{h-\hat{s}}{\sigma_H}\right)\frac{2}{2\sigma_H} &\overset{!}{=} 0 \\
&\Leftrightarrow & \frac{v-\hat{s}}{\sigma_V^2} + \frac{h-\hat{s}}{\sigma_H^2} &= 0 \\
&\Leftrightarrow & \frac{v}{\sigma_V^2} + \frac{h}{\sigma_H^2} - \hat{s}\left(\frac{1}{\sigma_V^2} + \frac{1}{\sigma_H^2}\right) &= 0 \\
&\Leftrightarrow & \frac{v}{\sigma_V^2} + \frac{h}{\sigma_H^2} &= \hat{s}\left(\frac{1}{\sigma_V^2} + \frac{1}{\sigma_H^2}\right) \\
&\Leftrightarrow & \hat{s} &= \left(\frac{v}{\sigma_V^2} + \frac{h}{\sigma_H^2}\right)\frac{\sigma_V^2\sigma_H^2}{\sigma_V^2+\sigma_H^2} \\
&\Leftrightarrow & \hat{s} &= \frac{v\sigma_H^2}{\sigma_V^2+\sigma_H^2} + \frac{h\sigma_V^2}{\sigma_V^2+\sigma_H^2}
\end{aligned}
$$