PMPC Tutorial Sheet 5

1. Plot the beta distribution.[1] Vary the parameters, α and β, systematically to get a feeling for how they change the distribution. What do you observe? What will happen if you use the beta distribution to learn about the probability q that a thumbtack will land on its head and you get more and more data?

2. There are two candidates for an election. In a poll of 1000 people 533 were in favor of candidate A and the rest preferred candidate B. Do you think candidate A is going to win? How sure are you? What is your prior distribution for the proportion of people voting for A? Can you express your prior belief as a beta distribution? Make a plot of your posterior probability density function and cumulative density function.

3. In 1994 CNN announced a poll of 500 people aged 18-34. An incredible 46% (230/500) say they believe in unidentified flying objects (UFOs). Assuming a beta distribution with $\alpha = 1$ and $\beta = 9$ prior density for the population proportion who believe in UFOs, find a 95% posterior probability interval for this proportion. Produce a plot with the prior and the posterior probability distribution. Mark the central 95% posterior probability interval.

4. Null hypothesis significance testing. Is it possible to distinguish Coca Cola from Pepsi? On each trial a subject is either given Coca Cola or Pepsi and has to decide which one it is. This is repeated for 25 trials and the subject has to give a response on each trial ("no idea" is not an option, only "Coca Cola" or "Pepsi" are possible responses). The experimenter decides to reject the null-hypothesis that the subject is merely guessing if she gets more then 20 correct responses. Imagine that the subject cannot distinguish the two drinks at all. Imagine also that the experiment was repeated a great many times (every time with 25 trials). How often do you expect it will happen that the subject got more than 20 correct responses despite a total inability to discriminate the drinks? Run a simulation to answer the question.

5. Usually adopted interpretations of $p < .01$ by 70 academic psychologists [3].

Statement	f	%
1. The null hypothesis is absolutely disproved.	1	1.4
2. The probability of the null hypothesis has been found.	32	45.7
3. The experimental hypothesis is absolutely proved.	2	2.9
4. The probability of the experimental hypothesis can be deduced.	30	42.9
5. The probability that the decision taken is wrong is known.	48	68.6
6. A replication has a .99 probability of being significant.	24	34.3
7. The probability of the data given the null hypothesis is known.	8	11.3

   Which of these statements are correct and which are wrong? Explain why!

6. Is most research wrong [2]? Psychologists almost universally adopt an α level of 5% to reject the null hypothesis of no effect. Let's assume the power $1 - \beta$ of most studies in psychology is 75%.

[1] The beta distribution and related functions are `betapdf, betacdf, betainv` in matlab and octave. The parameters that we called α and β in class are called `A` and B in the functions. See `help betapdf`.

We don't know what the proportion q of non-null effects is among all the effects that are tested in psychology. An effect was found significant by null hypothesis significance testing (NHST). For what proportion q is the probability that the effect is real less than $\frac{1}{2}$?

7. Some people think that null hypothesis significance testing (NHST) should be banned. If you want to read up on the controversy surrounding NHST I highly recommend the (very polemic) paper by Cohen [1] and the paper by Ioannides [2]. For a more balanced account you should read the paper by Krantz [4].

8. The expectation of a function $f(X)$ of a random variable X over a discrete sample space Ω is defined as
$$E(f(X)) = \sum_{x \epsilon \Omega} p(x) f(x).$$
You can think about this as a weighted average: Which values of $f(X)$ will you see on average. Show that
$$E(a \cdot f(X) + b) = a \cdot E(f(X)) + b.$$
For two random variables X and Y with sample spaces Ω_X and Ω_Y and a function $f(X, Y)$ of both variables, the joint expectation is defined as
$$E(f(X, Y)) = \sum_{x \epsilon \Omega_X} \sum_{y \epsilon \Omega_Y} p(x, y) f(x, y).$$
Show that if X and Y are independent
$$E(X + Y) = E(X) + E(Y).$$
The mean of a random variable is defined as $E(X)$. The variance of a random variable is defined as
$$\mathrm{var}(X) = E\left((X - E(X))^2\right),$$
the expectation of the squared distance from the mean. Show that
$$\mathrm{var}(X) = E\left(X^2\right) - E\left(X\right)^2.$$
Show that for two independent random variables X and Y
$$\mathrm{var}(X + Y) = \mathrm{var}(X) + \mathrm{var}(Y).$$
A thumbtack has probability p of landing on its head. You toss it n times and get the outcomes $X_1 ... X_n$. What is the mean and the variance of each X_i? What is the mean and the variance of $N = \sum_{i=1}^{n} X_i$?

References

[1] J. Cohen. The earth is round (p<.05). *American Psychologist*, 49(12):997–1003, 1994.

[2] J. P.A. Ioannides. Why most published research findings are false. *PLoS Medicine*, 2(8), 2005.

[3] R. B. Kline. *Beyond Significance Testing*. American Psychological Association, Washington, DC, 2004.

[4] D. H. Krantz. The null hypothesis testing controversy in psychology. *Journal of the American Statistical Association*, 94(448):1372–1381, 1999.