PMPC Tutorial Sheet 2

1. What is the sample space in roulette? What is the event space in roulette? How many possible events are there? What is the difference between betting $18 on the even numbers and betting $1 on each even number? How does the bank make money?

2. Prove the so-called union bound:

$$P\left(\bigcup_{i=1}^{n} E_i\right) \leq \sum_{i=1}^{n} P\left(E_i\right)$$

3. The following expression is a useful variant of Bayes' rule:

$$\frac{P(A \mid B)}{P(\text{not } A \mid B)} = \frac{P(B \mid A)}{P(B \mid \text{not } A)} \cdot \frac{P(A)}{P(\text{not } A)}.$$

   The left-hand side are the posterior odds that A is true. The first term on the right-hand side is called the likelihood ratio; the second term are the prior odds. Convince yourself that it is correct.

4. In a famous series of studies Daniel Kahneman and Amos Tversky examined judgements under uncertainty. In one of their experiments participants were given the following judgement task [4]:
   A cab was involved in a hit and run accident at night. Two cab companies, the Green and the Blue, operate in the city. You are given the following data:
   (a) 85% of the cabs in the city are Green and 15% are Blue
   (b) a witness identified the cab as Blue. The court tested the reliability of the witness under the same circumstances that existed on the night of the accident and concluded that the witness correctly identified each one of the two colors 80% of the time and failed 20% of the time. What is the probability that the cab involved in the accident was Blue rather than Green?
   A large number of subjects were tested and the mode and the median of the answers is 80%. What is the correct answer? What is going wrong?

5. Is medical screening sensible [1, 2]? To diagnose colorectal cancer the hemocult test is conducted to detect occult in the stool. For symptom-free people over 50 years old who participate in screening using the hemocult

test the following information is available:

Thirty out of 10,000 people have colorectal cancer. Of these 30 people with the cancer, 15 will have a positive test result (The hemocult test is not very sensitive, its hit-rate is only 50%). Of the remaining 9,970 people without the cancer, 300 will still have a positive hemocult test (the false-alarm-rate is about 3%). Imagine a sample of people (over 50, no symptoms) who have positive hemocult tests. How many of these people actually have colorectal cancer?

What are the advantages of screening, i.e. testing a large number of people who have no symptoms? What are the disadvantages?

6. For the last exercise write out the full joint probability table. Write out both conditional distributions. Calculate both marginal distributions. What would the joint distribution look like if the result of the hemocult test was independent of the patient having colorectal cancer but the marginal distributions were the same? What would the conditional distributions look like in this case?

7. The Monty Hall Problem (the presentation that Gerd Gigerenzer [1, p. 217] gives is worthwhile reading). For about three decades, Monty Hall hosted a popular American game show called *Let's Make a Deal*. The final contestant is given the choice of three doors. Behind one door is a car, behind the others goats. The contestant picks a door, say number 1, and the host, who knows what's behind the doors, opens another door, say number 3, which has a goat. He asks the contestant whether he or she wants to switch to door number 2. Should the contestant switch? Why?

8. Read the paper on learning Bayes' Rule by Sedlmeier and Gigerenzer [3].

References

[1] G. Gigerenzer. *Reckoning with Risk*. Penguin Books Ltd, 2003.

[2] G. Gigerenzer, W. Gaissmaier, E. Kurz-Milcke, L. M. Schwartz, and S. Woloshin. Helping doctors and patients make sense of health statistics. *Psychological Science in the Public Interest*, 8(2):53–96, 2007.

[3] P. Sedlmeier and G. Gigerenzer. Teaching bayesian reasoning in less than two hours. *Journal of Experimental Psychology: General*, 130(3):380–400, 2001.

[4] A. Tversky and D. Kahneman. Evidential impact of base rates. In D. Kahneman, P. Slovic, and A. Tversky, editors, *Judgment under Uncertainty: Heuristics and Biases*, pages 153–160. Cambridge University Press, 1982.