PMPC Tutorial Sheet 3

1. Go back to last week's tutorial sheet. Why did you make the mistakes you did? What did you not understand before?

2. DNA evidence in court [1]. In a rape-murder case the police found traces of DNA on the victim that did not belong to the victim. The police had a couple of suspects but their DNA did not match the trace on the victim. Further examination revealed that the DNA did match with DNA found in an unsolved murder case that happened in the same region a year before. The police decides to ask all men in the region that are aged between 20 and 50 to participate in voluntary DNA testing. There are about 100,000 of them—but of course not all of them participate. If two DNA samples come from the same person a test will reveal this with practically perfect certainty. Sometimes a match will be found even if the samples do not come from the same person. With current DNA technology this happens in about 0.001 percent of the cases. A match is found in one of the men who were screened. What is the probability that the DNA on the victim is really from this man? If you were his lawyer how would you argue?

3. If you are philosophically inclined and want to know more about the subjective inter-preation of probabilities and how probabilities, betting, and logic relate to each other, read chapter 1 of [2]. I closely followed the presentation given there in class.

4. In 2004 the European soccer championships took place in Portugal. In the final Greece won over Portugal with a score of one to zero. Before Greece reached the final hardly anyone had thought it possible that Greece could become European champion. In fact the odds for this to happen were judged to be about 1:100 before the tournament started. Even though Greece had beaten Portugal in the opening game the bookmakers set the odds for the final as: 7:2 for Portugal to win and 2:5 for Greece to win. Two men (betters are mostly men) placed bets with the bookmaker. Adam bet 10 Euros on Portugal and Boris bet 10 Euros on Greece. Use the odds to calculate how much money Boris has won and how much money Adam would have won.

5. If the odds for Portugal are 7:2 then the odds for Greece should be 2:7. Why is this not so? Charly placed two bets: 4.90 Euros on Portugal and 1.80 Euros on Greece. How much did he win and how much would he have won in case Portugal had become champion?

6. Before the match your belief in Greece winning was p. For which values of p would you have been tempted to place a bet? Why do people bet at all?

7. The conjunction fallacy revisited. Someone believes that $P(A \cap B) > P(A)$. Accord-ingly, he is willing to buy or sell a ticket worth \$1 if $A \cap B$ and \$0 otherwise at a price of \$$P(A \cap B)$. And similarly for $P(A)$. How can you take advantage of this person?

8. Geometric distribution. Repeatedly toss a thumbtack until it lands on the pin. Let N be the random variable that gives the number of tosses needed until the thumbtack landed on the pin. What is the sample space for N? What is the probability distribution $p(N)$? Convince yourself that the distribution is normalized to 1. What is the probability for N being odd?

9. St. Petersburg paradox. Someone offers you the following gamble: A fair coin is tossed until head comes up. Call the number of tosses needed N. You get 2^N Euros. But before you are allowed to play the game you have to pay 1000 Euros. For which N would you win money? What is the probability that you lose money? Would you play? What is your expected gain for this gamble? What does this tell you about using fair bets as a way to elicit beliefs?

10. Imagine playing the following game with a friend. She has two dice, a red one and a blue one. Behind your back she rolls the dice. After each roll she tells you a number between 1 and 6. The rules of the game are that your friend either always reports the number that the red die shows or she always reports the minimum of the two dice. Your task is to figure out what she is doing. The results for the first 10 rolls have been 4, 2, 5, 6, 6, 3, 2, 6, 6, 1. What are your beliefs given these data?

References

[1] G. Gigerenzer. *Reckoning with Risk*. Penguin Books Ltd, 2003.

[2] R. Jeffrey. *Subjective Probability. The Real Thing*. Cambridge University Press, 2004.