Name/Surname: Version A

November ..., 2019

Student ID Number:

Graph Theory, Discrete Mathematics, and Optimization
Graph Theory and Discrete Mathematics module

1	2	3	4	5	6

Exam duration: one hour. The test is composed by 6 multiple-choice questions (one point each), and one open question (up to two points). For the multiple-choice questions please insert the correct answer into the above grid. No cheating or collaboration is allowed. Good luck!

1. (2 point) The following system

$$\begin{cases} 2x_1 & -2x_2 & & = 0 \\ x_1 & +2x_2 & +2x_3 & = 1 \\ \alpha x_1 & -\alpha x_2 & +x_3 & = 1 \end{cases}$$

is consistent for

a) $\alpha = 0$ c) $\alpha > 0$
b) $\alpha < 0$ d) for all real values α

2. (2 point) Let

$$A = \left\{ \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 0 \\ 2 \end{pmatrix} \right\};\ B = \left\{ \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right\};$$

a) A and B are linearly independent c) A is linearly independent and B is linearly dependent

b) A and B are linearly dependent d) B is linearly independent and A is linearly dependent

3. (2 point) Let

$$A = \begin{bmatrix} 1 & 2 & -1 & 1 \\ 2 & 4 & -3 & 0 \\ 1 & 2 & 1 & 5 \end{bmatrix}$$

and $N(A) = \{\mathbf{x} \ : \ A\mathbf{x} = \mathbf{0}\ \}$, $dim(N(A)) =$

a) 0 c) 1
b) 3 d) 2

4. (2 point) Define the linear transformation $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^2$ by

$$T(\mathbf{x}) = \begin{bmatrix} x_1 + x_2 - x_3 \\ x_1 + x_3 \end{bmatrix},$$

the corresponding matrix A is

$$\text{(a) } \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \text{ (b) } \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ -1 & 1 \end{bmatrix} \text{ (c) } \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \text{ (d) } \begin{bmatrix} 1 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix}$$

5. (2 point) Let $\mathbf{x} = [1\ 1]^T$, $\mathbf{y} = [-2\ 0]^T$, $\mathbf{v}$ the vector projection of $\mathbf{x}$ onto $\mathbf{y}$, and $\mathbf{w}$ the vector projection of $\mathbf{y}$ onto $\mathbf{x}$, then

a) $\mathbf{v} = [1\ 0]^T$, $\mathbf{w} = [-1\ -1]^T$ c) $\mathbf{v} = [0\ 1]^T$, $\mathbf{w} = [-1\ -1]^T$

b) $\mathbf{v} = [1\ 0]^T$, $\mathbf{w} = [1\ 1]^T$ d) $\mathbf{v} = [-1\ 0]^T$, $\mathbf{w} = [1\ 1]^T$

6. (2 point) The following vector is one eigenvector for the given matrix corresponding to the given eigenvalue,

$$\begin{bmatrix} 2 & 0 & 0 \\ 1 & 3 & 0 \\ 4 & 1 & 3 \end{bmatrix}, \ \lambda = 3,$$

a) $[0\ 0\ 0]^T$ c) $[1\ 1\ 1]^T$

b) $[1\ 1\ 0]^T$ d) $[0\ 0\ 2]^T$

Exercise.(4 points) Find the eigenvalues and eigenvectors of the following matrix A

$$A = \begin{bmatrix} 3 & -2 \\ -2 & 3 \end{bmatrix}.$$