Lecture 11 - 20-04-2020

1.1 Analysis of K_{NN}

$$\mathbb{E}\ \ell_d(\hat{\ell}_s)\Big] \leq 2 \cdot \ell_D\left(f^*\right) + c \cdot \mathbb{E}\left[\ \|X = x_{\Pi(s,x)\|}\right]$$

At which rate this thing goes down? If number of dimension goes up then a lot of point are far away from X.
So this quantity must depend on the space in which X live.
Some dependence on number of depends and incresaing number of traning points close to X
This expecation is fucniton of random variable X and $X_{\pi(s,x)}$

We are going to use the assumption that:
$|X_t| \leq 1 \qquad \forall$ cordinates $i = 1, ..., d$
— DISEGNO —
Hyper box in bydimension. All point live in this box and we exploit that. Look at the little suare in which is divided and we assume that we are dividing the box in small boxes of size ε. Now the training points will be a strincle of point distributed in the big square.
Our training points are distributed in the box (this is our S).
Now we added a point x and given this two things can happned: falls in the square with training points or in a square without training points.
What is going to be the distance $X_{\pi(s,x)}$ in this two cases?
We have c_1 up to c_r How big is this when we have this two cases? (We lookjing at specific choices of x and s)

$$\|X - X_{s,x}\| leq \begin{cases} \varepsilon\sqrt{d} & c_i \cup S \neq 0 \\ \sqrt{d} & c_i \cup S = 0 \end{cases}$$

were $X \in C_i$
We have to multiply by the lenght of the cube. Will be $\varepsilon\sqrt{d}$
– DISEGNO —
If things go badly can be very far away like the length of the domain. Lenght is 2 and diagonal is$\sqrt{d}$
if close they are going to be $\varepsilon close$ or far as domain.
We can split that the expression inside the expectation according to the two cases.

$$\mathbb{E}\left[\|X - X_{\Pi(s,x)}\|\right] \leq \mathbb{E}\ \varepsilon\sqrt{d} \sum_{i=1}^{r} I\{X \in C_i\} \cdot I\{C_1 \cap S \neq 0\}\Bigg] + 2\sqrt{d} \sum_{i=1}^{r} I\{X \in C_i\} \cdot I\{C_i \cap S \neq 0\} =$$

$$= \varepsilon\sqrt{d}\,\mathbb{E}\left[\sum_{t=1}^{r} I\{X \in C_i\} I\{c_1 \cap S \neq 0\}\right] + 2\sqrt{d}\sum_{i=1}^{r}\mathbb{E}\left[I\{X \in C_1\}\cdot I\{C_1 \cap S \neq 0\}\right]$$

I don't care about this one $\sum_{t=1}^{r} I\{X \in C_i\} I\{c_1 \cap S \neq 0\}$
Can be either 0 or 1 (if for some i, X belong to some C_i
So at most 1 the expectation

$$\leq \ \varepsilon\sqrt{d} +$$

We can bound this square. Are the event I in the summation of the term after $+$. If they are indepednt the product will be the product of the two expectation. If I fix the cube. X and S are independent.
Now the two events are independent
$X \in C_1$ is inepdend of $C_i \cap S \neq$

$$\mathbb{E}\left\{X \in C_i\right\} \cdot I\{C_1 \cap S \neq 0\}] = \mathbb{E}\left[I\{X \in C_i\}\right] \cdot \mathbb{E}\left[I\{C_i\right.$$

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$$\mathbb{P}\left(C_i \cap S\right) = \left(1 - \mathbb{P}\left(X \in C_1\right)\right)^m \leq \exp(-m\mathbb{O}(x \in C_1))$$

The probability of the point fall there and will be the probability of falling in the cube.
Probability of Xs to fall in the cube with a m (samples?)
Now use inequality $(1-p)^m \in e^{-pm} \longrightarrow 1 + x \leq e^x$ – IMMAGINE –

$$\sum_{t=1}^{r}\mathbb{E}\left[\mathbb{P}(X \in C_1)\mathbb{P}(C_1 \cap S \neq)\right] \leq sum_{i=1}^{r} p_i\, e^{-m\,p_i} \leq$$

given that $p_i = \mathbb{P}(X \in C_i)$ I can upper bound this

$$\leq \sum_{t=1}^{r}\left(\max_{0 \leq p \leq 1} p\, e^{-m\,p}\right) \leq r \max_{0 \leq p \leq 1} \mathbf{p} \quad \mathbf{e}^{-m\,p}\mathbf{p} \quad \mathbf{e}^{-m\,p}\mathbf{p} \quad \mathbf{e}^{-m\,p}\mathbf{p} \quad \mathbf{e}^{-m\,p} = where\mathbf{p} \quad \mathbf{e}^{-m\,p} \text{ is } F(p) \text{ it is concave functi}$$