

Name/Surname:

Version B
November ..., 2019

Student ID Number:

Graph Theory, Discrete Mathematics, and Optimization
Graph Theory and Discrete Mathematics module

1	2	3	4	5	6

Exam duration: one hour. The test is composed by 6 multiple-choice questions (one point each), and one open question (up to two points). For the multiple-choice questions please insert the correct answer into the above grid. No cheating or collaboration is allowed. Good luck!

1. (2 point) The rank of the following matrix

$$\begin{bmatrix} 1 & -3 & 2 & -4 \\ -3 & -9 & -1 & 5 \\ 2 & -6 & 4 & -3 \\ -4 & 12 & 2 & 7 \end{bmatrix}$$

is

- a) 1 c) 3
b) 2 d) 4

2. (2 point) The following system (t is a real parameter)

$$\begin{cases} -tx_1 & +(t-1)x_2 & +x_3 & = 1 \\ & +(t-1)x_2 & +tx_3 & = 1 \\ 2x_1 & & +x_3 & = 5 \end{cases}$$

is consistent for

- a) $t < 0$ c) $t > 0$
b) $t \neq 1, 2$ d) for all real values t

3. (2 point) The dimension of the subspace S of \mathbb{R}^3 spanned by the vector

$$\mathbf{v}_1 = \begin{bmatrix} 2 \\ -8 \\ 6 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 3 \\ -7 \\ -1 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} -1 \\ 6 \\ -7 \end{bmatrix},$$

is

- a) = 1 c) = 4
b) = 2 d) = 3

4. (2 point) Let $\mathbf{x} = [1 \ 2 \ 3]^T$, and

$$\mathbf{y}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{y}_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \quad \mathbf{y}_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}.$$

We have $\mathbf{x} = c_1\mathbf{y}_1 + c_2\mathbf{y}_2 + c_3\mathbf{y}_3$,

- a) only for $c_1 = 1, c_2 = 2, c_3 = 3$ c) only for $c_1 = 2, c_2 = -1, c_3 = 2$
b) only for $c_1 = 3, c_2 = 2, c_3 = 1$ d) for no possible value of c_1, c_2 and c_3

5. (2 point) The following vector is one eigenvector for the given matrix corresponding to the given eigenvalue,

$$\begin{bmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{bmatrix}, \quad \lambda = 2,$$

- a) $[1 \ 1 \ 1]^T$ c) $[1 \ 2 \ 2]^T$
b) $[-5 \ 2 \ 2]^T$ d) $[0 \ 0 \ 0]^T$

6. (2 point) The following real matrix (α , and β are real parameters)

$$A = \begin{bmatrix} 0 & \beta & 0 \\ \alpha & -\alpha & 1 \\ \beta & 0 & 0 \end{bmatrix}$$

is orthogonal

- a) only for $\alpha = 0, \beta = 1$ c) for any value of β , and $\alpha = 0$
b) for any value of α , and $\beta = 0$ d) for no possible value of α and β

Exercise.(4 points) Find the eigenvalues and eigenvectors of the following matrix A

$$A = \begin{bmatrix} 7 & 2 \\ -4 & 1 \end{bmatrix}.$$