

$$= \frac{2}{\pi \sqrt{2}} \left( \frac{2}{3} \sin 2x + \frac{2}{5 \cdot 7} \sin 6x + \frac{2}{9 \cdot 11} \sin 10x \right) \dots \infty$$

$$= \frac{4}{\pi \sqrt{2}} \left( \frac{\sin 2x}{1 \cdot 3} - \frac{\sin 6x}{5 \cdot 7} + \frac{\sin 10x}{9 \cdot 11} \right) \dots \infty$$

### Practice Questions

① Find the Fourier series to represent the function

$$f(x) = |\sin x|, -\pi < x < \pi$$

② Find Fourier series of  $f(x)$  in interval  $(-\pi, \pi)$

$$f(x) = \begin{cases} \pi + x, & -\pi < x < 0 \\ \pi - x, & 0 < x < \pi \end{cases}$$

③ Obtain Fourier series of  $f(x) = \sqrt{1 - \cos x}$  in  $(0, 2\pi)$

hence evaluate  $\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \dots \infty$

④ Express  $f(x) = |x|$  as Fourier series,  $-\pi < x < \pi$

⑤  $f(x) = x \cos x$  in  $(-\pi, \pi)$

⑥  $f(x) = \begin{cases} -x+1, & -\pi \leq x \leq 0 \\ x+1, & 0 \leq x \leq \pi \end{cases}$

Is the function even or odd? Find the Fourier series for  $f(x)$ . Hence evaluate

\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} - \dots = \infty

⑦ Obtain the Fourier series for function

$$f(x) = \begin{cases} \pi x, & 0 \leq x \leq 1 \\ \pi(2-x), & 1 \leq x \leq 2 \end{cases}$$

Hence show that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} - \dots = \frac{\pi^2}{8}$$

⑧ Fourier series of  $f(x) = x^2 \text{ in } (-l, l)$

⑨  $f(x) = e^x$  as Fourier series in  $(-l, l)$

⑩  $f(x) = x^2 \text{ in } 0 \leq x < 2l$ . Hence  
show that

$$(i) \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} - \dots = \frac{\pi^2}{6}$$

$$(ii) \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} - \dots = \frac{\pi^2}{12}$$

(11) Find Half range Sine series of  $\pi x - x^2$   
 in  $(0, \pi)$ .

(12)  $f(x) = \begin{cases} x, & 0 < x < \pi/2 \\ \pi - x, & \pi/2 < x < \pi \end{cases}$

Find Half range Sine series as well as  
 half range Cosine series.

(13) Obtain Half range Cosine series for

$$f(x) = \begin{cases} bx, & 0 \leq x \leq l/2 \\ b(l-x), & l/2 \leq x \leq l \end{cases}$$

Hence evaluate  $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} = \dots = \infty$

(14) Half range Cosine series of  $\sin x$  in  $(0, \pi)$ .

(15) Half range Sine series for  $f(x)$

$$f(x) = \begin{cases} \frac{1}{4} - x, & 0 < x < 1/2 \\ x - \frac{3}{4}, & 1/2 < x < 1 \end{cases}$$

## Answers

(1)  $|f(n)| = \frac{2}{\pi} - \frac{4}{\pi} \left( \frac{\cos 2n}{3} + \frac{\cos 4n}{15} + \frac{\cos 6n}{35} - \dots \right)$

(2)  $f(n) = \frac{\pi}{2} + \frac{4}{\pi} \left( \frac{\cos n}{1^2} + \frac{\cos 3n}{3^2} + \frac{\cos 5n}{5^2} - \dots \right)$

(3)  $|x| = \frac{\pi}{2} - \frac{4}{\pi} \left( \cos n + \frac{\cos 3n}{3^2} + \frac{\cos 5n}{5^2} - \dots \right)$

(4)  $f(n) = \frac{\pi}{2} + 1 - \frac{4}{\pi} \left( \cos n + \frac{\cos 3n}{3^2} + \frac{\cos 5n}{5^2} - \dots \right)$

(5)  $f(n) = \frac{\pi}{2} - \frac{4}{\pi} \left( \frac{\cos nx}{1^2} + \frac{\cos 3nx}{3^2} + \frac{\cos 5nx}{5^2} - \dots \right)$

(6)  $x^2 = \frac{\ell^2}{3} - \frac{4\ell^2}{\pi^2} \left( \frac{\cos \frac{\pi x}{\ell}}{1^2} - \frac{\cos 2\frac{\pi x}{\ell}}{2^2} + \frac{\cos 3\frac{\pi x}{\ell}}{3^2} - \dots \right)$

(7)  $e^{-x} = \sinh x \left\{ \frac{1}{e} - 2e \left( \frac{\cos \frac{\pi x}{e}}{e^2 + \pi^2} + \frac{\cos 2\frac{\pi x}{e}}{e^2 + 2^2 \pi^2} + \frac{\cos 3\frac{\pi x}{e}}{e^2 + 3^2 \pi^2} - \dots \right) \right.$   
 $\left. - 2\pi \left( \frac{\sin \frac{\pi x}{e}}{e^2 + \pi^2} - \frac{2}{e^2 + 2^2 \pi^2} \sin 2\frac{\pi x}{e} + \frac{3}{e^2 + 3^2 \pi^2} \sin 3\frac{\pi x}{e} - \dots \right) \right\}$

(10)  $f(n) = \frac{4l^2}{3} + \frac{4l^2}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \cos \frac{n\pi n}{l} - \frac{4l^2}{\pi} \sum_{n=1}^{\infty} \frac{\sin n\pi n}{l}$

(11)  $\pi n - n^2 = \frac{8}{\pi} \left( \ln n + \frac{\ln 3n}{3^2} + \frac{\ln 5n}{5^2} \right) \rightarrow \infty$

(12) Half range sine Series

$$f(n) = \frac{4}{\pi} \left[ \frac{\ln n}{1^2} - \frac{\ln 3n}{3^2} + \frac{\ln 5n}{5^2} \rightarrow \infty \right]$$

Half range cosine Series

$$f(n) = \frac{\pi}{4} - \frac{2}{\pi} \left[ \frac{\cos 2n}{1^2} + \frac{\cos 6n}{3^2} + \frac{\cos 10n}{5^2} \rightarrow \infty \right]$$

(3)  $f(n) = \frac{bl}{4} - \frac{8bl}{\pi^2} \left[ \frac{\cos 2\pi n}{2^2} + \frac{\cos 6\pi n}{6^2} + \frac{\cos 10\pi n}{10^2} \rightarrow \infty \right]$

Cosine Series :

(4)  $\sin n = \frac{2}{\pi} - \frac{4}{\pi} \left[ \frac{\cos 2n}{1 \cdot 3} + \frac{\cos 4n}{3 \cdot 5} + \frac{\cos 6n}{5 \cdot 7} \rightarrow \infty \right]$

(5)  $f(n) = \left( \frac{1}{\pi} - \frac{4}{\pi^2} \right) \ln \pi n + \left( \frac{1}{3\pi} + \frac{4}{3^2\pi^2} \right) \ln 3\pi n$

$$+ \left( \frac{1}{5\pi} - \frac{4}{5^2\pi} \right) \ln 5\pi n \rightarrow \infty$$