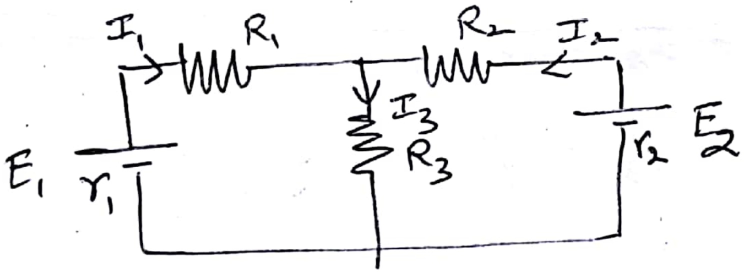
Superposition Theorem

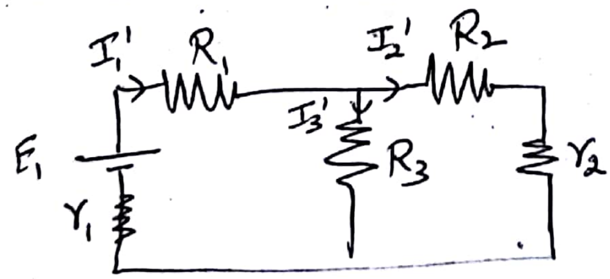
YEESHU Unit-1 Part-2 B (1)

→ Acc. to this theorem, if there are two or more than two sources of emf's acting simultaneously in a linear bilateral n/w, the current flowing through any section is the algebric sum of all the currents which should flow in that section if each source of emf were considered separately and all other sources are replaced by their internal resistance.

→ When both the sources are acting the currents flowing in R_1, R_2 and R_3 are I_1, I_2 and I_3 respectively



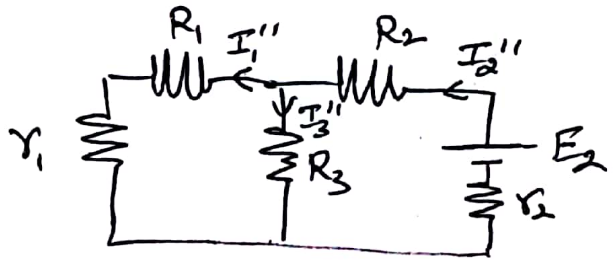
Step 1:- Short circuit voltage source E_2 and replace it by its internal resistance r_2



Now, Find out branch currents I_1', I_2' and I_3'

Step 2

Short circuit the voltage source E_1 and replace it by its internal resistance r_1



Now, find out branch currents I_1'', I_2'' and I_3''

Step 3

Add algebrically the above currents and we find actual currents

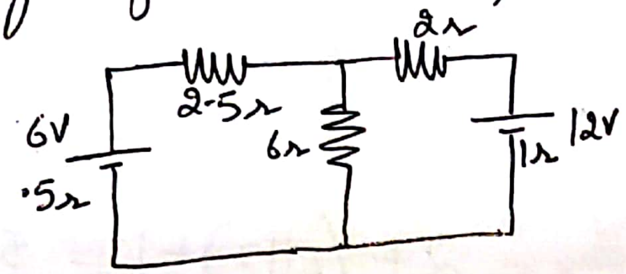
$$I_1 = I_1' - I_1''$$

$$I_2 = I_2' - I_2''$$

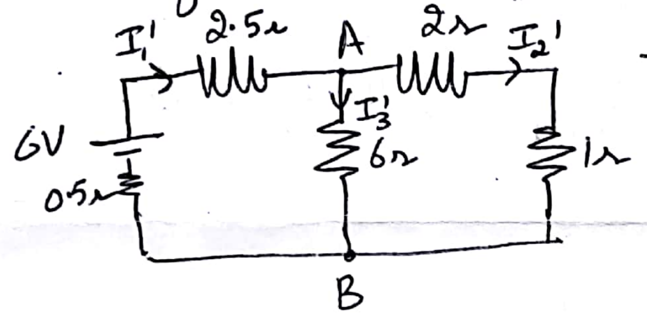
$$I_3 = I_3' + I_3''$$

Q1 - Find different currents flowing in the branches and voltage across 6 ohm resistor using superposition theorm in n/w shown

Let battery emf be 6 V and 12 V, their internal resistance $\cdot 5\Omega$ and 1Ω



Ans Step 1. 12 Volt battery has been removed through its internal resistance of 1Ω.



→ Two Parallel Paths between A and B, 6Ω and 3Ω

Equivalent Resistance = $3 \| 6 = 2\Omega$

Total Resistance $= 2\cdot5 + 2 + 0\cdot5 = 5\Omega$

$$\therefore I_1' = \frac{6V}{5\Omega} = 1\cdot2A$$

$$I_2' = \frac{6}{6+3} \times 1\cdot2 = 0\cdot8A$$

$$I_3' = I_1' - I_2' = 1\cdot2 - 0\cdot8 = 0\cdot4A$$

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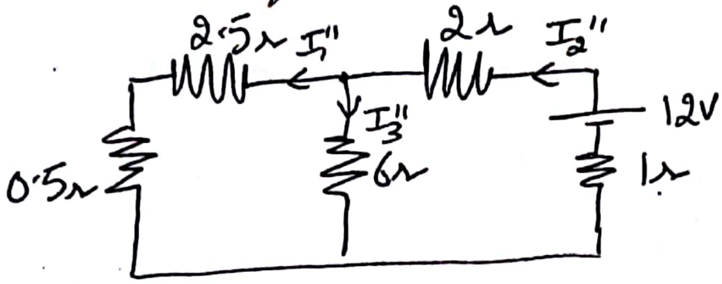
Step 2 6 volt battery has been ~~removed~~ replaced by its internal resistance



$$\text{Total Resistance} = 2 + (6 \| 3) + 1 = 5\Omega$$

$$I_2'' = \frac{12}{5} = 2.4\,A$$

$$I_1'' = \frac{6}{9} \times 2.4 = 1.6\,A$$

$$I_3'' = I_2'' - I_1'' = 2.4 - 1.6 = 0.8A$$

Step 3 Net current

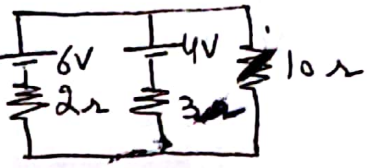
$$I_1 = I_1' - I_1'' = 1.2 - 1.6 = -0.4A$$

$$I_2 = I_2'' - I_2' = 1.6A$$

$$I_3 = I_3' + I_3'' = 1.2A$$

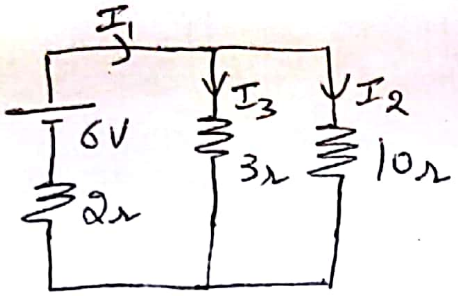
B (3)

(1) Apply the principle of Superposition to the network shown to find out current in the 10Ω resistance.



Ans

Step 1: 4V has been removed



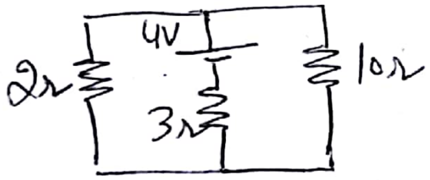
$$I_1 = \frac{6}{2 + 3\|10} = \frac{6}{\frac{56}{13}} = \frac{39}{28} A$$

$$I_2 = \frac{3}{3+10} \times I_1 = \frac{3}{13} \times \frac{39}{28}$$

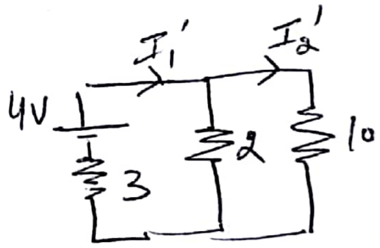
$$= \frac{9}{28} A$$

Step 2

6V has been removed



OR



$$I_1' = \frac{4}{(2\|10)+3} = \frac{6}{7}$$

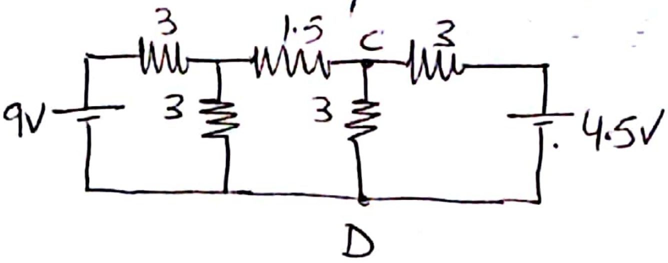
$$I_2' = \frac{2}{2+10} \times I_1' = \frac{2}{12} \times \frac{6}{7} = \frac{1}{7}$$

Step 3

Net current through 10Ω resistor

$$= I_2 + I_2' = \frac{9}{28} + \frac{1}{7} = 0.464 A$$

Q: Find current through 3Ω resistance connected b/w C and D.



Ans

Step 1. 4.5V battery has been removed

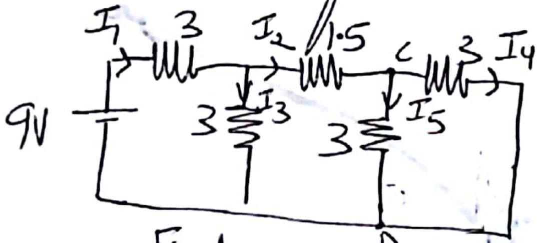


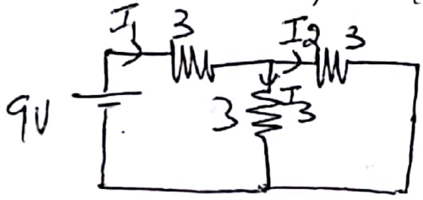
Fig 1

Equivalent Resistance =

$$[(3||3)+1.5]||3+3 = 4.5$$

$$I_1 = \frac{9}{4.5} = 2A$$

For current division, we should have two parallel branches

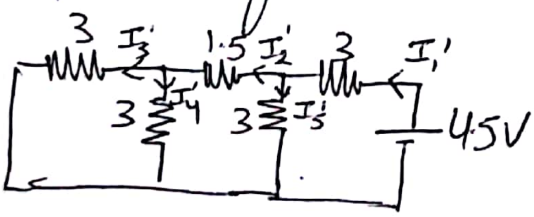


$$I_2 = \frac{3}{3+3} \times 2 = 1A$$

and Now from fig. 1 find out value of I_5 (By current Division)

$$I_5 = \frac{3}{3+3} \times 1 = 0.5A$$

Step 2 9V battery has been removed

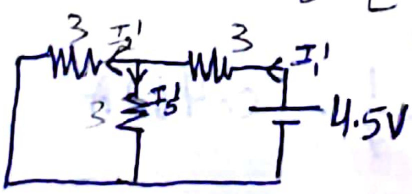


EQUIVALENT RESISTANCE $= [(3||3)+1.5]||3+3$

$= 4.5\Omega$

$$I_1' = \frac{4.5}{4.5} = 1A$$

Find current I_5' [By current Division]



$$I_5' = \frac{3}{3+3} \times I_1'$$

$= 0.5A$

Step 3

Net current flowing through 3Ω resistor connected b/w C and D

$= I_5 + I_5'$

$= 0.5 + 0.5$

$= 1A$

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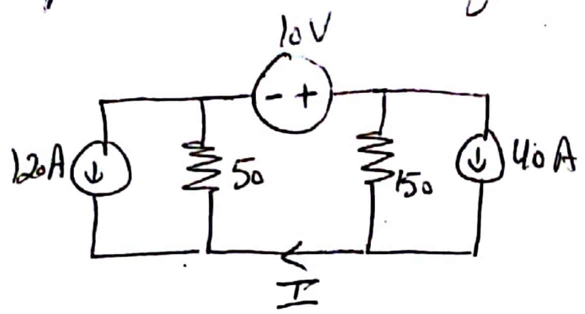
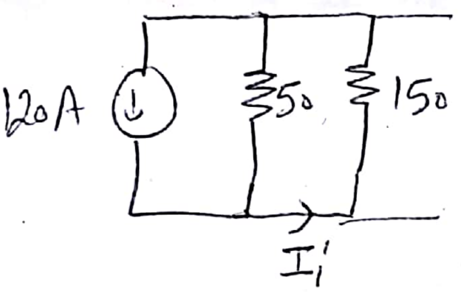
1: Use superposition theorem to find current I in the circuit shown

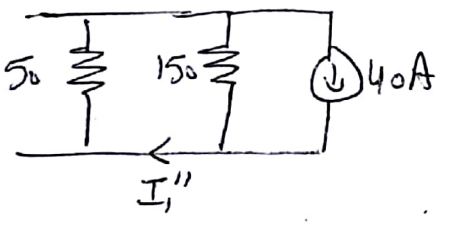


Ans Step 1: 10V voltage source has been replaced by short circuit and 40A current source by open circuit



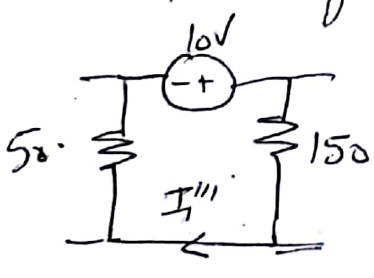
$$I_1' = \frac{50}{50+150} \times 120 = 30A$$

Step 2: 120A current source has been replaced by open circuit and 10V voltage source by short circuit



$$I_1'' = \frac{150}{50+150} \times 40 = 30A$$

Step 3: 120A current source and 40A current source have been replaced by open circuit



$$I_1''' = \frac{10}{50+150} = .05A$$

Step 4: Net current $I = -I_1' + I_1'' + I_1'''$

$$= -30 + 30 + 0.05$$

$$= 0.05A$$

Thevenin Theorem

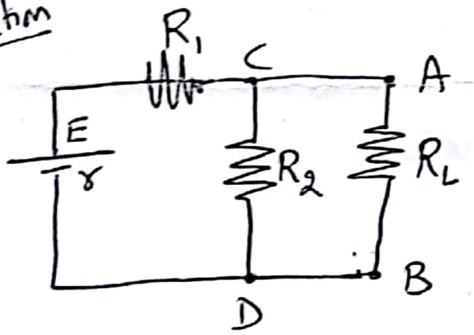
Statement → The current flowing through a load resistance R_L connected across any two terminals A and B of a linear, active bilateral network is given by $V_{oc} || (R_{th} + R_L)$

Where, V_{oc} is the open circuit voltage

i.e. voltage across the two terminals when R_L is removed

R_{th} is the internal resistance of network as viewed back into the open circuited n/w from terminals A and B with all voltage sources replaced by their internal resistance and current source by open circuit.

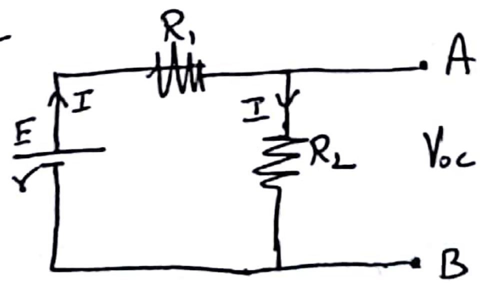
Explanation →



Suppose it is required to find current flowing through R_L

Step 1 →

Remove R_L from the circuit terminals A and B and redraw the circuit



Now, Calculate open circuit voltage V_{oc} which appears across terminals A and B when they are open

V_{oc} = Drop across R_2 $= IR_2$

$$I = \frac{E}{R_1+R_2+r}$$

$$V_{oc} = IR_2 = \frac{E R_2}{R_1+R_2+r}$$

It is also called Thevenin Voltage V_{Th}

Step 2 Remove the voltage source and replace it by internal resistance

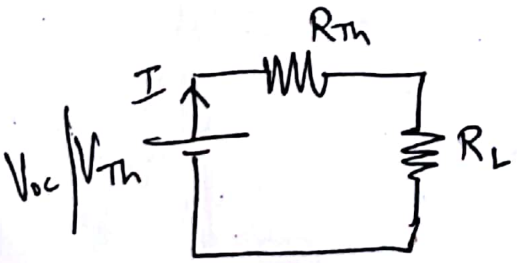
$$R_{Th} = (R_1+r) \| R_2$$

This is equivalent resistance of n/w when viewed from terminals A and B

This resistance is also called Thevenin Resistance

Step 3 Thevenin Equivalent Circuit

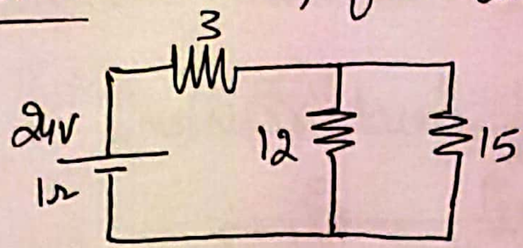
R_L is connected back across terminals A and B



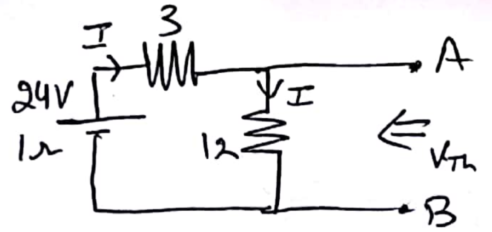
Current flowing through R_L is given by

$$\boxed{I = \frac{V_{Th}}{R_{Th}+R_L}}$$

Q: Using Thevenin Theorem, find current in 15Ω resistor B⑥.



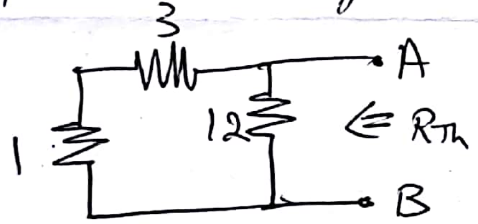
Ans. Step 1 15Ω has been removed and replaced by open circuit.



$$I = \frac{24}{12+3+1} = 1.5A$$

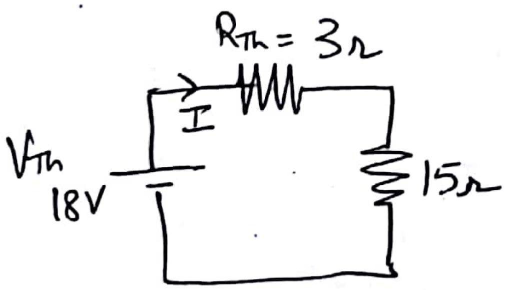
$$V_{Th} = 12 \times 1.5 = 18V$$

Step 2 Replace 24V voltage source by its internal resistance



$$R_{Th} = (1+3) \| 12 = 4 \| 12 = \frac{4 \times 12}{4+12} = 3\Omega$$

Step 3 Thevenin Equivalent Circuit



$$I = \frac{18}{15+3} = \underline{\underline{1A}}$$

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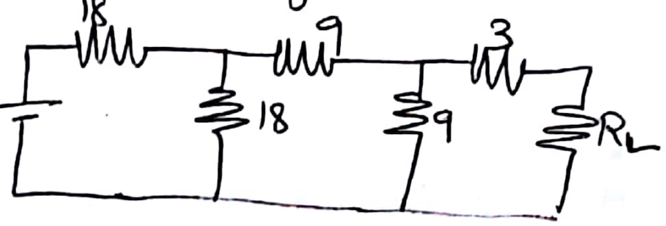
9)

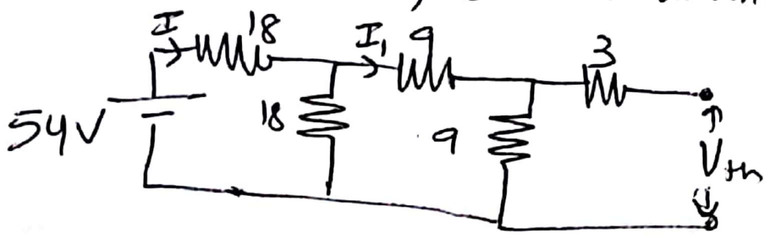
1) For a network shown determine the current flowing through R_L when value of load resistance is 3 ohm.

Thevenin



Ans

Step 1 - To determine V_{th}, load resistance R_L is removed



$$R = (9+9) \| 18 + 18 = 27\,\Omega$$

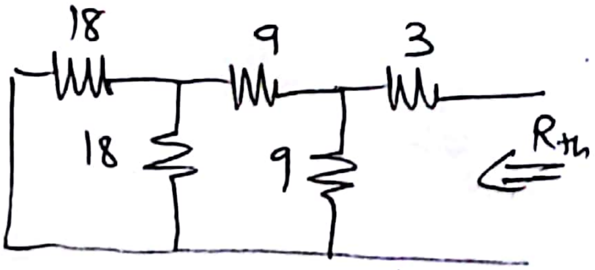
$$I = \frac{54}{27} = 2\,A$$

$$I_1 = \frac{18}{18+18} \times I = \frac{18}{36} \times 2 = 1\,A$$

$$V_{th} = 9 \times I_1 = 9\,V$$

Step 2

Battery is replaced by short circuit

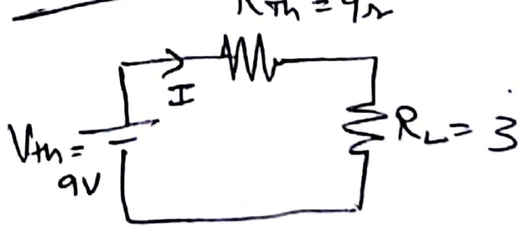


$$R_{th} = \left[(18 \| 18) + 9 \right] \| 9 + 3$$

$$= 9\,\Omega$$

Step 3

$R_{Th} = 9\,\Omega$

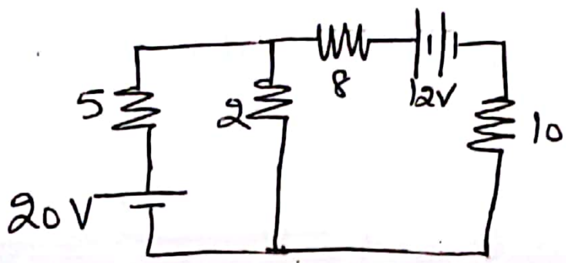


$$I = \frac{V_{Th}}{R_{Th} + R_L} = \frac{9}{9+3}$$

$$= 0.75\,A$$

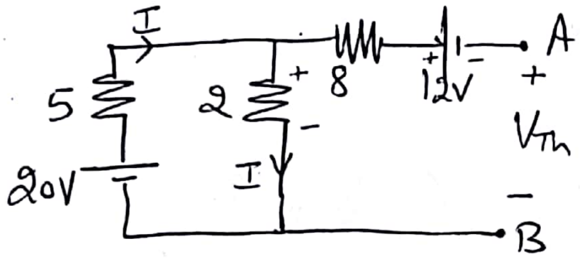
B 7

Q. For the circuit calculate the current in 10 Ω resistance using Thevenin Theorem.



Ans: Step 1

Remove 10 Ω resistance



$$I = \frac{20}{5+2} = \frac{20}{7}\ A$$

Applying KVL, $V_{Th} + 12 - 2I = 0$

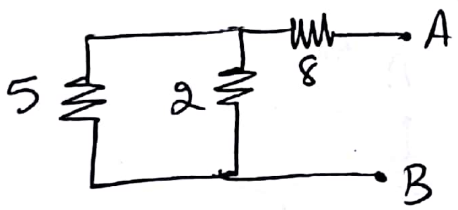
$V_{Th} = -12 + 2I$

$V_{Th} = -12 + 2 \times \frac{20}{7}$

$= -6.29V$

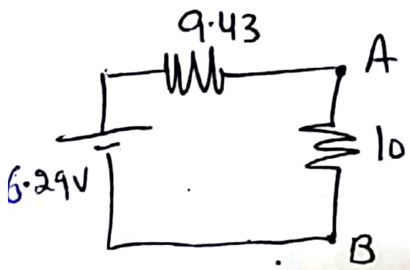
[No current will flow in 8 Ω resistance, so voltage drop across it is zero]

Step 2 Replace the ~~batteries~~ batteries by short circuit

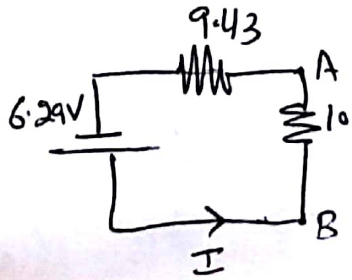


$R_{Th} = 5 \| 2 + 8 = 9.43\ \Omega$

Step 3 Thevenin Equivalent circuit



or



$$I = \frac{6.29}{9.43+10} = \underline{\underline{0.32A}}$$

Current flows from B to A.

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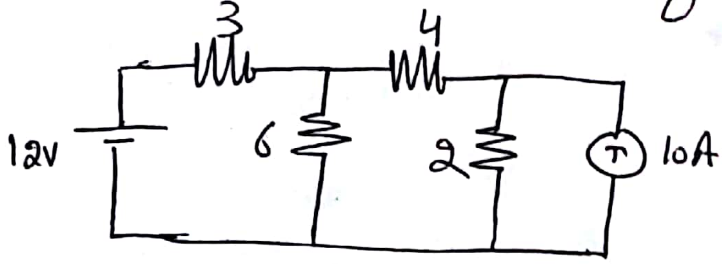
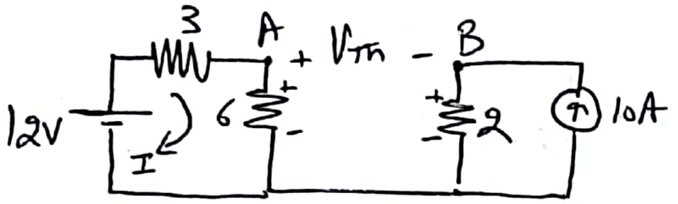
Q. Using Thevenin Theorem, calculate the current flowing through 4Ω resistor



Ans **Step 1**

Remove 4Ω resistor



$$I = \frac{12}{6+3} = \frac{4}{3}\,A$$

Drop across 6Ω $= 6 \times \frac{4}{3} = 8\,V$

Drop across 2Ω $= 10 \times 2 = 20\,V$

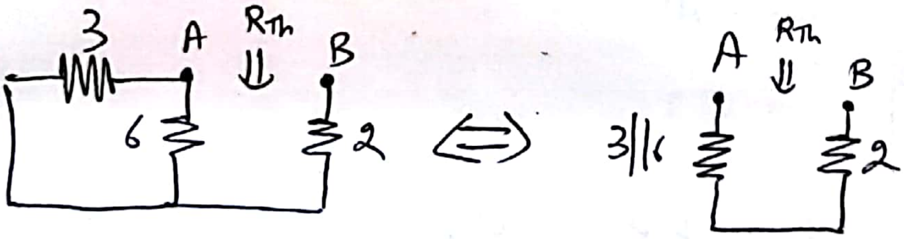
Applying KVL,

$$V_{Th} - 8 + 20 = 0$$

$$V_{Th} = -12\,V$$

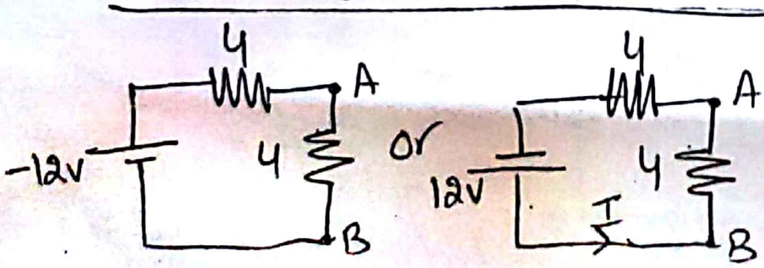
Step 2

12V battery has been short circuited
10 A Current source has been open circuited



$$R_{Th} = 3||6 + 2 = 4\Omega$$

Step 3 Thevenin Equivalent ~~Resistance~~ Circuit



$$I = \frac{12}{4+4} = \underline{\underline{1.5A}}$$

Current flows from B to A.

NORTON'S THEOREM

Statement: The current flowing through a resistance connected across any two terminals of a network can be determined by replacing the whole network by an equivalent circuit of a current source having a current output of I_N in parallel with resistance R_N.

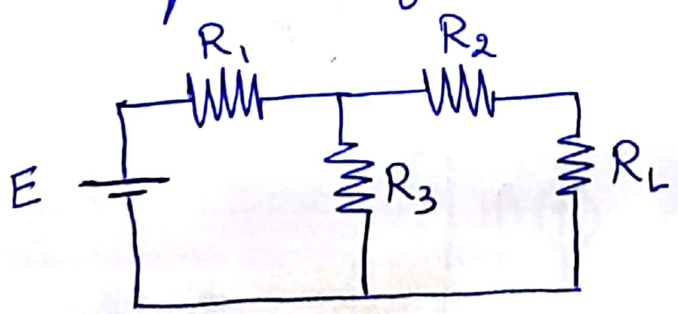
Where I_N = Short circuit current supplied by the source that would flow between the two selected terminals when they are short circuited

(Norton current)

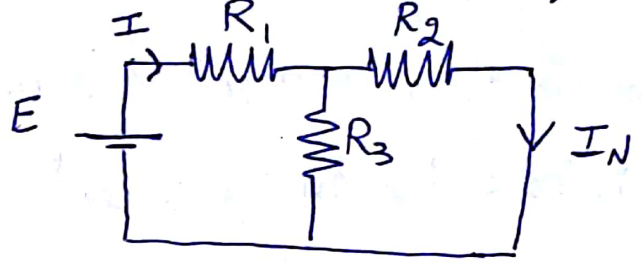
R_N = Equivalent Resistance of the network as seen from the two terminals with all other emf sources replaced by their internal resistance and current source replaced by open circuit.

Explanation:

Suppose it is required to find current in load resistance R_L



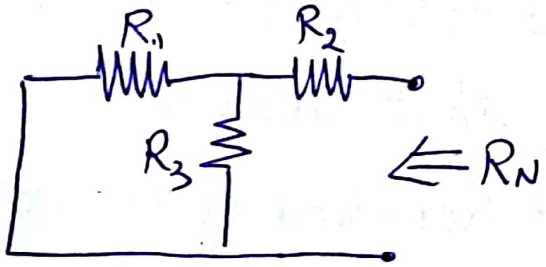
(1) Short circuit the terminals across which load resistor is connected and calculate current (I_N) which would flow between them.



$$I = \frac{E}{R_1 + (R_2 \| R_3)}$$

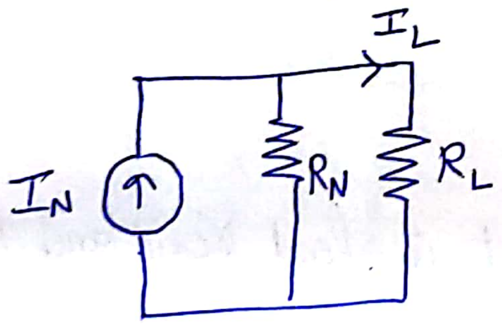
$$I_N = \frac{R_3}{R_2 + R_3} \times I$$

(2) Replace voltage source by short circuit and calculate R_N b/w terminals across which load is connected.



$$R_N = (R_1 \| R_3) + R_2$$

(3) Norton's Equivalent circuit



$$I_L = \frac{R_N}{R_N + R_L} \times I_N$$

(Applying current division formula)

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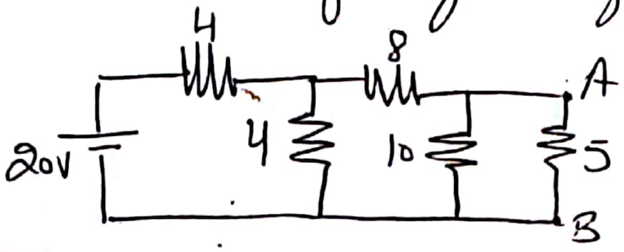
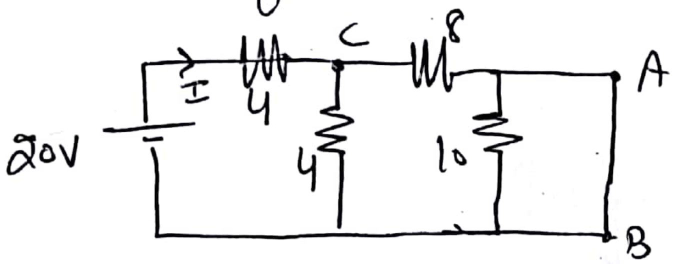
B (9)

1. Apply Norton's Theorem to calculate current flowing through 5 Ω resistor of n/w shown



Ans.

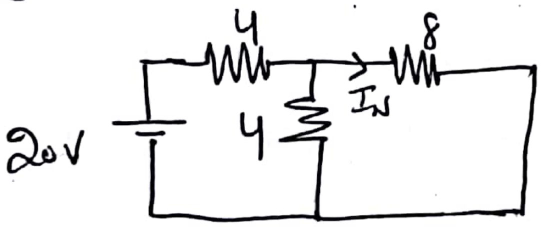
Step 1 Remove 5 Ω resistor and put short across terminals A and B and find I_N



Total Resistance seen by the battery $= 4 + (4 \| 8) = \frac{20}{3}\,\Omega$

Current $I = \dfrac{20}{\frac{20}{3}} = 3A$

(3 A) Current divides at point C

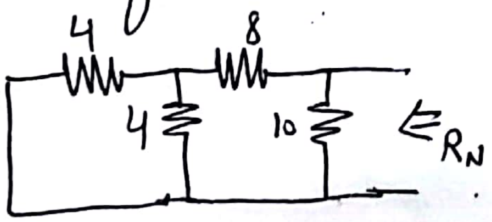


Current flowing through 10 Ω resistance is 0 A.

$I_N = \dfrac{4}{4+8} \times 3 = 1A$

Step 2

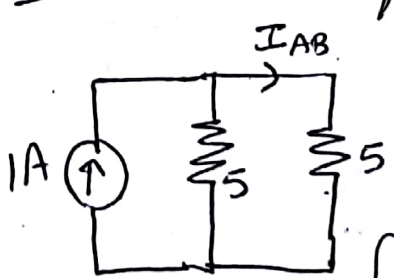
Battery has been removed and replaced by short circuit



$R_N = (4\|4 + 8)\|10$

$= (2+8)\|10 = 10\|10 = 5\,\Omega$

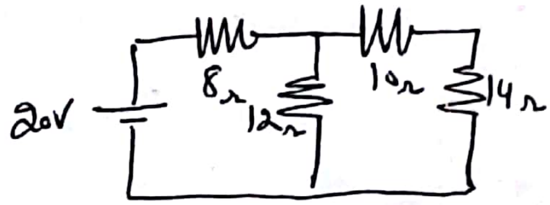
Step 3 Norton's Equivalent Circuit



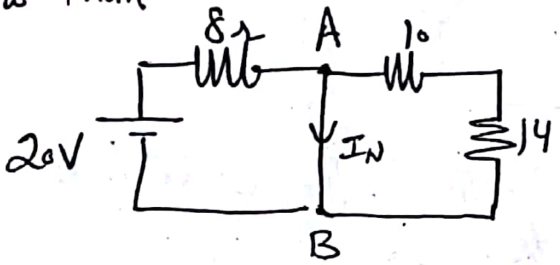
$I_{AB} = \dfrac{5}{5+5} \times 1$

$\boxed{I_{AB} = 0.5A}$

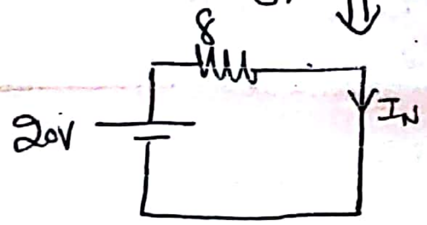
Q Using Norton's Theorem determine the current in $12\,\Omega$ in the n/w shown



Ans Step 1: $12\,\Omega$ resistor is removed and replaced by short circuit and calculate current I_N which would flow b/w them

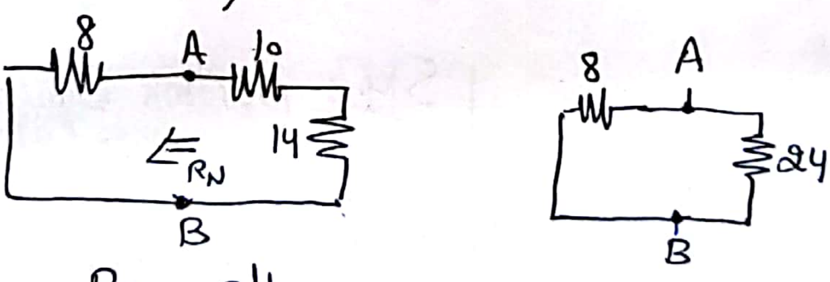


or ⇕



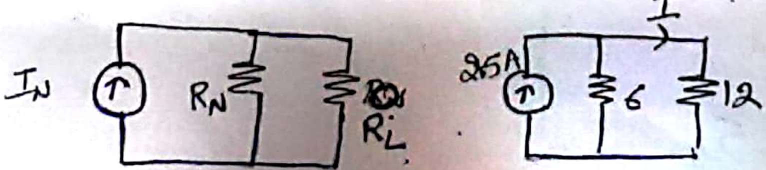
$$I_N = \frac{20}{8} = 2.5A$$

Step 2 Voltage Source is replaced by its internal resistance (zero in this case) and Calculate R_N



$$R_N = 8 \| 24 = 6\,\Omega$$

Step 3 Norton Equivalent Circuit



$$I = \frac{6}{6+12} \times 2.5$$
$$= 0.833A$$

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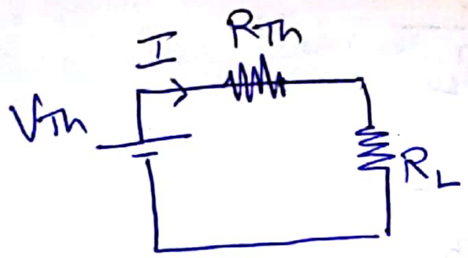
Maximum Power Transfer Theorem

→ Statement: The output obtained from a network is maximum when the load resistance R_L is equal to the internal resistance of the network as seen from the terminals of load resistance.

→



$$\text{Current, } I = \frac{V_{Th}}{R_{Th}+R_L}$$

$$\text{Power Delivered to the load} = I^2 R_L$$

$$P = \left(\frac{V_{Th}}{R_{Th}+R_L}\right)^2 R_L$$

→ Power will be maximum if $\frac{dP}{dR_L} = 0$

$$\frac{d}{dR_L}\left(\frac{V_{Th}}{R_{Th}+R_L}\right)^2 R_L = 0$$

$$V_{Th}^2\left[\frac{(R_{Th}+R_L)^2 - 2R_L(R_{Th}+R_L)}{(R_{Th}+R_L)^4}\right] = 0$$

$$(R_{Th}+R_L)^2 - 2R_L(R_{Th}+R_L) = 0$$

$$(R_{Th}+R_L)^2 = 2R_L(R_{Th}+R_L)$$

$$R_{Th}+R_L = 2R_L$$

$$\boxed{R_{Th} = R_L}$$

Internal Resistance = Load Resistance

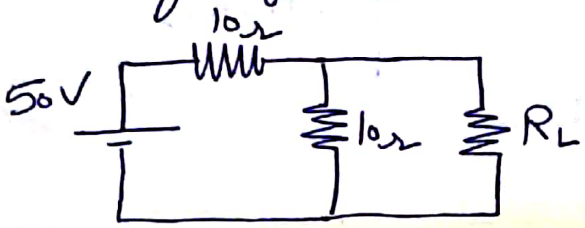
→ Hence, it is proved that max. power is transferred to the load resistance R_L, when R_L is equal to R_{Th}.

→ Max. Power Delivered to load $= \left(\frac{V_{Th}}{R_{Th}+R_L}\right)^2 R_L = \left(\frac{V_{Th}}{R_L+R_L}\right)^2 R_L$

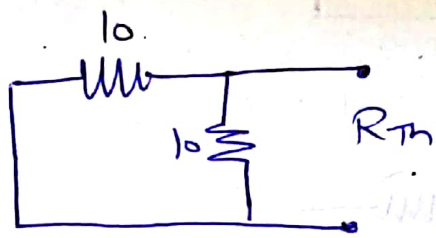
$$= \frac{V_{Th}^2}{4R_L}$$

$$= \frac{V_{Th}^2}{4R_{Th}}$$

Q1 Calculate the value of R_L so that max. Power is transferred from the battery for the circuit shown.



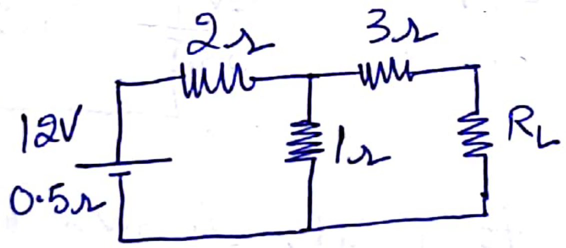
Ans Max. Power is delivered when $R_L = R_{Th}$



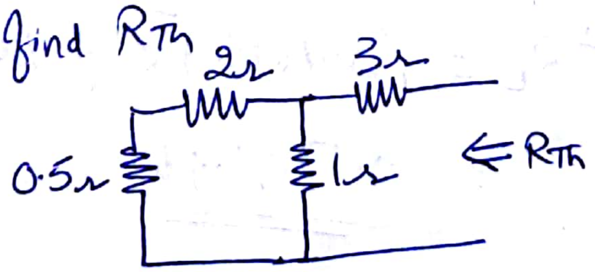
$$R_{Th} = \frac{10 \times 10}{10+10} = 5\Omega$$

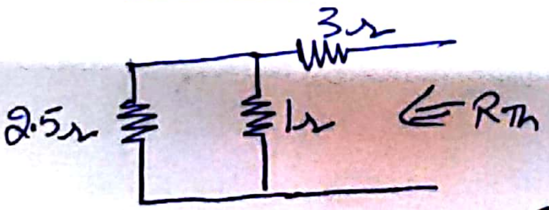
$$R_L = R_{Th} = 5\Omega$$

Q2 Find the value of R_L at which max. Power is transferred to R_L and hence max. Power transferred to R_L in the circuit :



Ans To find R_{Th}



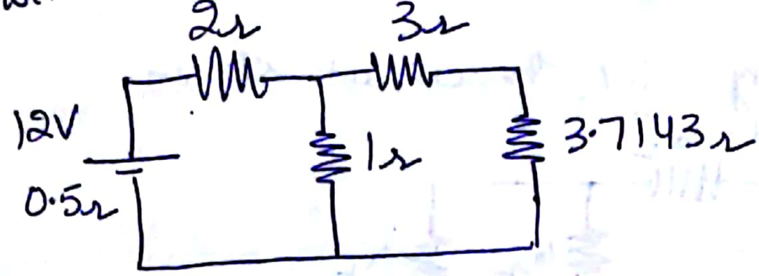


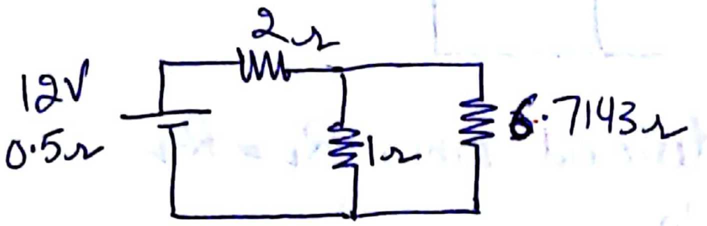
Parallel combination of 2.5 and 1 = 0.7143Ω

Series combination of 0.7143 and 3

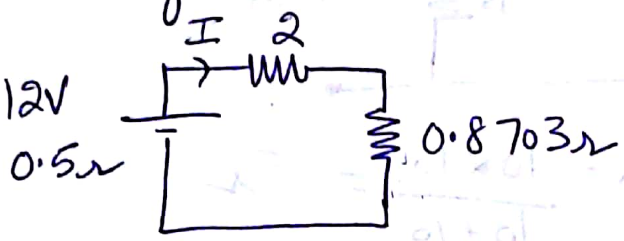
$R_{Th} = 3.7143\Omega$

Max Power



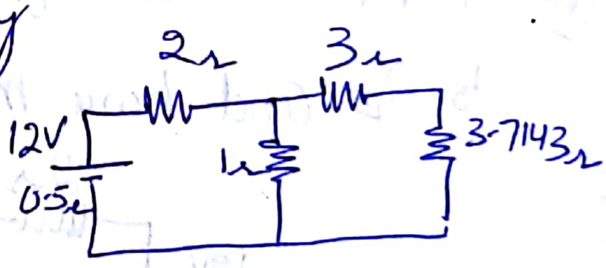


Parallel Combination of 1 and 6.7143Ω = 0.8703Ω



Main Current, $I = \dfrac{12}{0.5+2+0.8703} = 3.5604\ A$

→ Current through Resistance R_L is given by



$\dfrac{1}{1+6.7143} \times 3.5604$

$= 0.4615\ A$

→ Max. Power Transferred to resistor is given by

$= I_1^2 R_L = (0.4615)^2 \times 3.7143$

$= \underline{0.79119\ W}$

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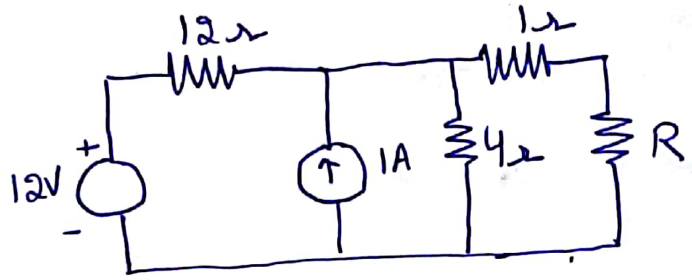
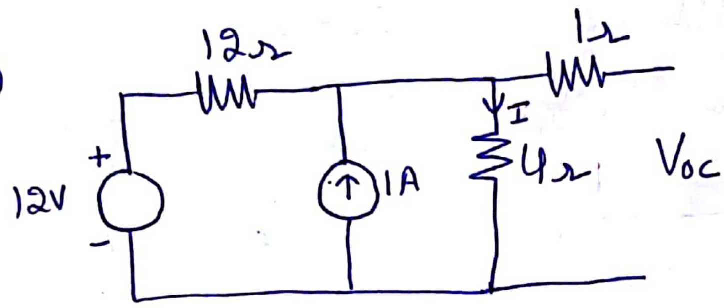
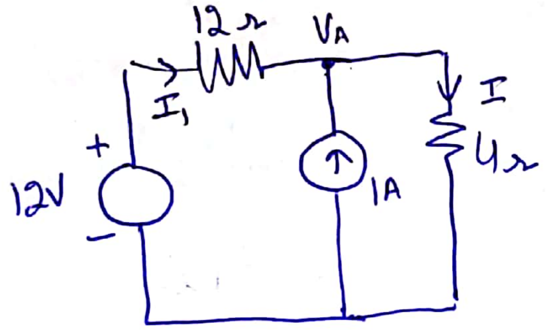
Q For the circuit shown, derive the Thevenin's and Norton's equivalent wrt load resistance R



Ans ①



$V_{oc} = 4I$



Solve for I using Nodal

$I_1 + 1 = I$

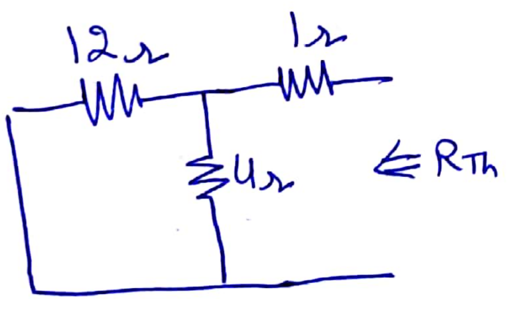
$\frac{12 - V_A}{12} + 1 = \frac{V_A}{4}$

$\boxed{V_A = 6V}$

$I = \frac{V_A}{4} = \frac{6}{4} A$

$\boxed{V_{oc} = \frac{6}{4} \times 4 = 6V}$

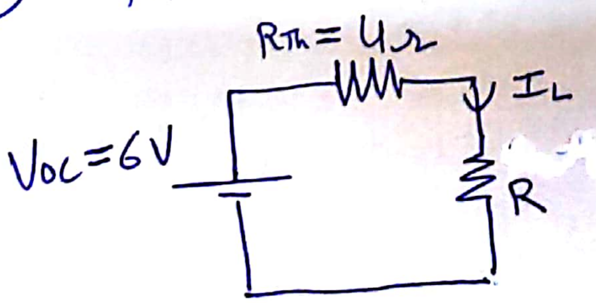
②



$R_{Th} = (12 \| 4) + 1$

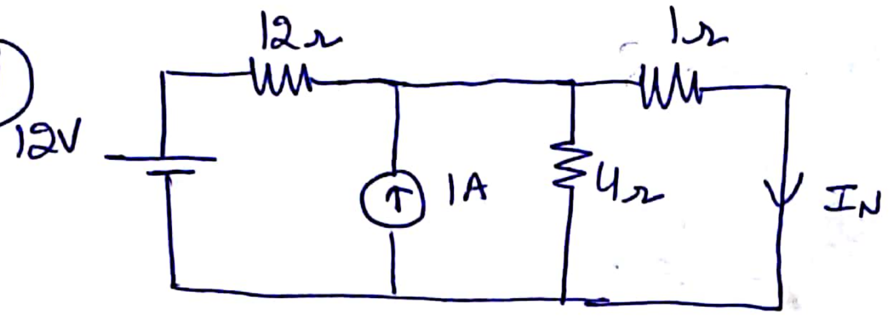
$= 3 + 1 = 4\Omega$

③ Thevenin's Equivalent circuit

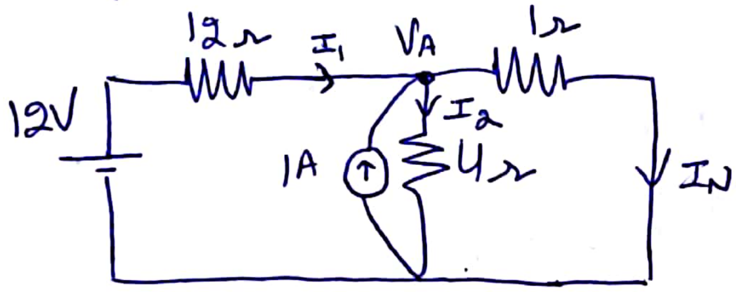


Norton's Theorem

1



Applying Nodal for I_N



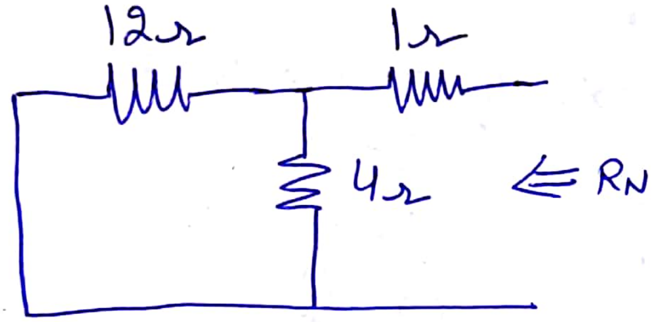
$$I_1 + 1 = I_2 + I_N$$

$$\frac{12 - V_A}{12} + 1 = \frac{V_A}{4} + \frac{V_A}{1}$$

$$V_A = \frac{3}{2} V$$

$$I_N = \frac{V_A}{1} = \frac{3}{2} A$$

2



$$R_N = (12 \| 4) + 1$$
$$= 4\Omega$$

3 Norton's Equivalent ckt.