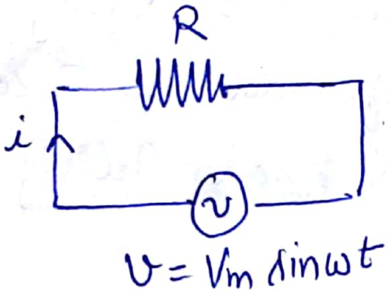
# Unit-2 (AC Circuits)

Ueshin

## AC through Pure resistance only




Let the applied voltage be $v = V_m \sin\omega t$

Let R = Resistance, $i$ = Instantaneous current

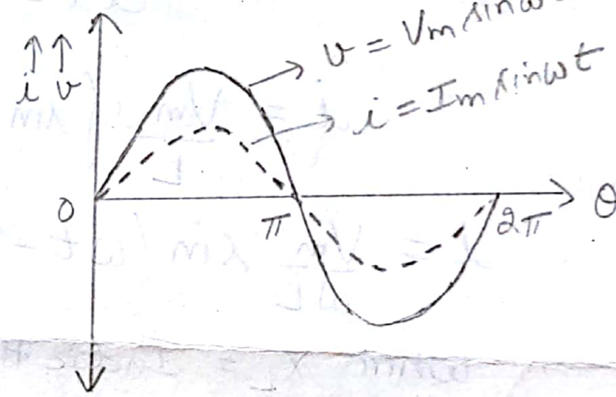
$$V = IR$$

$$V_m \sin\omega t = iR$$

$$i = \frac{V_m}{R} \sin\omega t$$

Current $i$ is max, when $\sin\omega t = 1$, $\therefore I_m = \frac{V_m}{R}$

$$i = I_m \sin\omega t$$




→ Voltage and current are in phase with each other

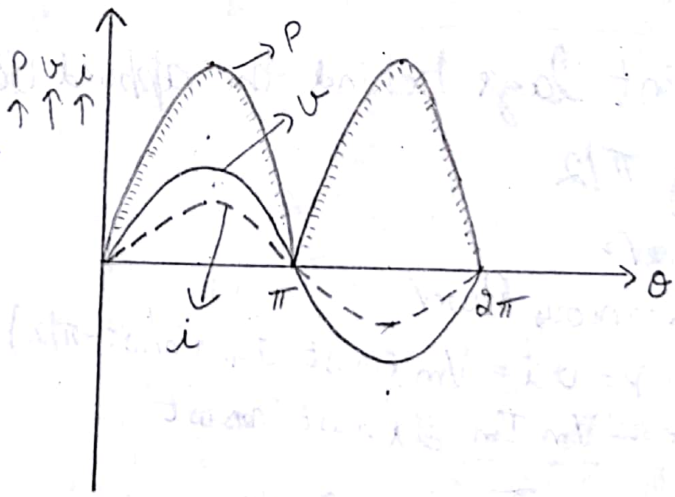
→ Power:

Instantaneous Power: $P = vi = V_m \sin\omega t \, I_m \sin\omega t$

$$P = V_m I_m \sin^2\omega t = \frac{V_m I_m}{2}(1 - \cos 2\omega t)$$

$$P = \underbrace{\frac{V_m I_m}{2}}_{\text{Constant part}} - \underbrace{\frac{V_m I_m}{2}\cos 2\omega t}_{\text{Pulsating Part}}$$

→ Average value of $\frac{V_m I_m}{2}\cos 2\omega t$ over complete cycle is zero



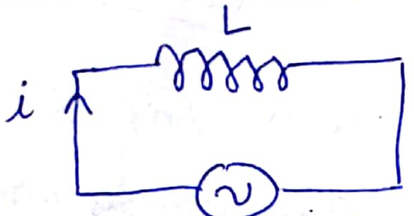

$\therefore$ Power for whole cycle

$$P = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}}$$

$$P = V_{rms} I_{rms}$$

In pure resistive ckt, power is never zero.

# AC through pure inductance only




→ When an alternating voltage is applied to the pure inductive coil, a back emf is produced due to self inductance of coil. The back emf opposes the rise or fall of current through coil

$$v = L\frac{di}{dt}$$

$$V_m \sin\omega t = L\frac{di}{dt}$$

$$di = \frac{V_m}{L}\sin\omega t\, dt$$

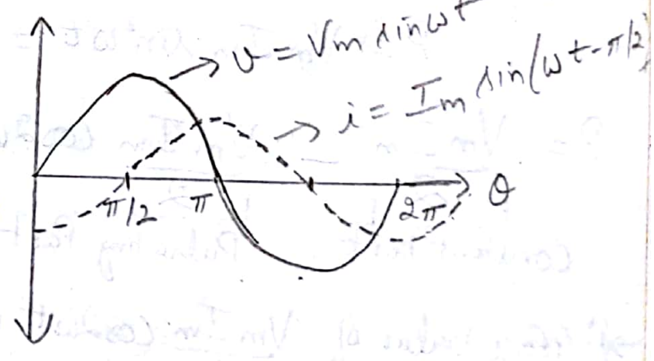
$$i = \frac{V_m}{L}\int \sin\omega t\, dt = \frac{V_m}{\omega L}(-\cos\omega t)$$

$$i = \frac{V_m}{\omega L}\sin(\omega t - \pi/2) = \frac{V_m}{X_L}\sin(\omega t - \pi/2)$$

Where $X_L$ = Inductive Reactance $= 2\pi f L$

Max. value of $i$ is $I_m = \frac{V_m}{\omega L}$ when $\sin(\omega t - \pi/2) = 1$

$$\boxed{i = I_m \sin(\omega t - \pi/2)}$$




→ Current lags behind the applied voltage by $\pi/2$

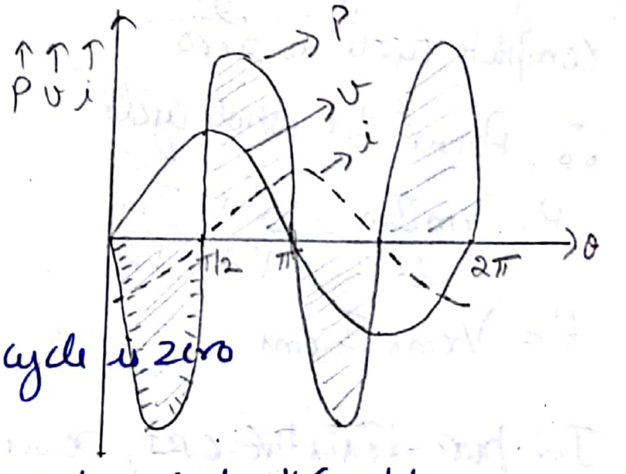
→ Power:

Instantaneous Power

$$P = vi = V_m \sin\omega t\; I_m \sin(\omega t - \pi/2)$$

$$= -\frac{V_m I_m}{2}\, 2\sin\omega t \cos\omega t$$

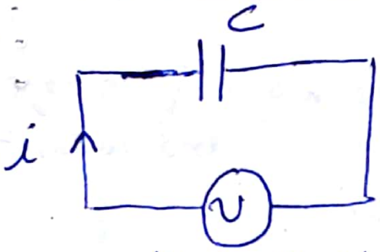
$$= -\frac{V_m I_m}{2}\sin 2\omega t$$




Average value of $\frac{V_m I_m}{2}\sin 2\omega t$ over complete cycle is zero

Average Power = 0 → No Power is consumed in pure inductive ckt

# AC through pure capacitance only




$v = V_m \sin\omega t$

When an alternating voltage is applied to the plates of capacitor, the capacitor is charged first in one direction and then in opposite direction.

Charge on plates, $Q = CV$

$$Q = C V_m \sin\omega t$$

Current, $i = \frac{dQ}{dt} = \frac{d}{dt}(C V_m \sin\omega t) = C V_m \omega \cos\omega t$

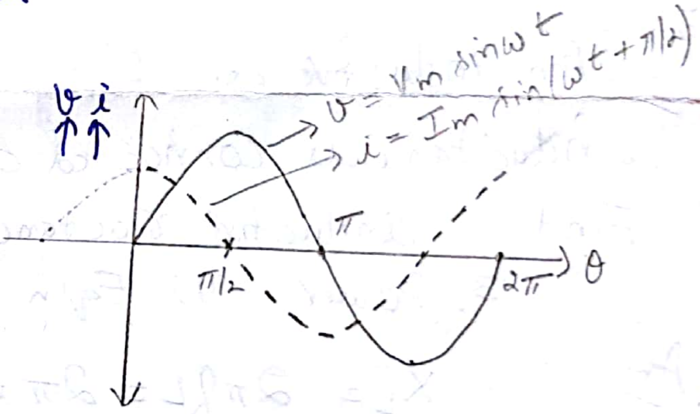
$$= \frac{V_m}{1/\omega C} \cos\omega t = \frac{V_m}{1/\omega C} \sin(\omega t + \pi/2) = \frac{V_m}{X_c} \sin(\omega t + \pi/2)$$

Max. value of current is $I_m = \frac{V_m}{1/\omega C}$ when $\sin(\omega t + \pi/2) = 1$

Where $X_c$ = Capacitive Reactance $= \frac{1}{2\pi f C}$

→ Current leads the voltage by $\pi/2$

→ Power




Instantaneous Power $= P = vi$

$$P = V_m \sin\omega t \, I_m \sin(\omega t + \pi/2)$$

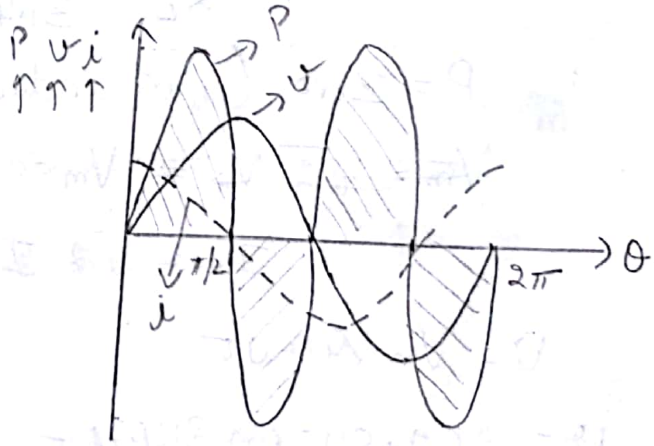
$$= V_m I_m \sin\omega t \cos\omega t$$

$$= \frac{V_m I_m}{2} 2 \sin\omega t \cos\omega t$$

$$= \frac{V_m I_m}{2} \sin 2\omega t$$

Average value of $\frac{V_m I_m}{2} \sin 2\omega t$ over complete cycle is zero

Average Power = 0




→ No Power is consumed in pure capacitive ckt.

Q1 An ac ckt consists of pure resistance of 10Ω and is connected across an ac supply of 230V, 50 Hz.
Calculate (1) Current (2) Power consumed (3) Write down eq'n of voltage and current.

Ans (1) Current in the ckt, $I = \frac{V}{R} = \frac{230}{10} = 23A$

(2) Power consumed, $P = V_{rms} I_{rms} = 230 \times 23 = 5290W$

Max. value of applied voltage, $\left[\frac{V_m}{\sqrt{2}} = V_{rms}\right]$, $V_m = \sqrt{2} \times 230$
$= 325.27V$

Max. value of current $\left[\frac{I_m}{\sqrt{2}} = I_{rms}\right]$ $I_m = \sqrt{2} \times 23 = 32.53A$

$\omega = 2\pi f = 2\pi \times 50 = 314.16$ rad/s

(3) $v = V_m \sin \omega t$
$v = 325.27 \sin 314.16t$

$i = I_m \sin \omega t$
$i = 32.53 \sin 314.16t$

Q2 An inductive coil having negligible resistance and 0.1 Henry inductance is connected across 200V, 50 Hz supply
Find (1) inductive reactance (2) rms value of current
(3) Power (4) Eq'n for voltage and current.

Ans $X_L = 2\pi f L = 2\pi \times 50 \times 0.1 = 31.416\Omega$

Current, $I = \frac{V}{X_L} = \frac{200}{31.416} = 6.366$ A

P = zero (Pure inductive ckt doesn't consume power)

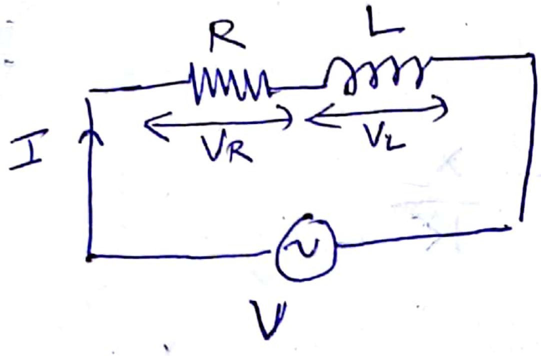
$\cancel{V_m = \sqrt{2} V_m}$ = $V_m = \sqrt{2} V_{rms} = \sqrt{2} \times 200 = 282.84V$

$I_m = \sqrt{2} I_{rms} = \sqrt{2} \times 6.366 = 9A$

$v = V_m \sin \omega t$
$v = 282.84 \sin 314.16t$

$i = I_m \sin(\omega t - \pi/2)$
$i = 9 \sin(314.16t - \pi/2)$

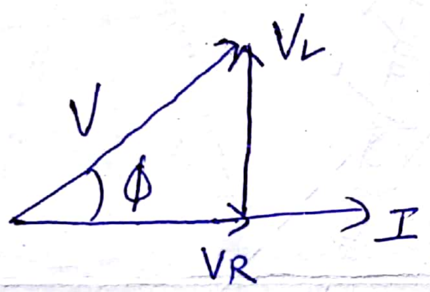
## R–L Series Circuit:




V = RMS value of voltage
I = RMS value of current

$V_R = IR$ (Voltage drop in resistance is in phase with current)

$V_L = I X_L$ (Voltage drop in inductive reactance) ~~lags~~ leads current by 90°




The vector sum of $V_R$ and $V_L$ is equal to applied voltage V.

$$V = \sqrt{(V_R)^2 + (V_L)^2} = \sqrt{(IR)^2 + (IX_L)^2}$$

$$V = I\sqrt{R^2 + X_L^2}$$

$$V = I Z$$

$$I = \frac{V}{Z}$$

where $Z = \sqrt{R^2 + X_L^2}$ is the total opposition offered to the flow of alternating current by an R-L series circuit and is known as Impedance.

**Phase Angle:** Current in the circuit lags behind the applied voltage by an angle $\phi$.

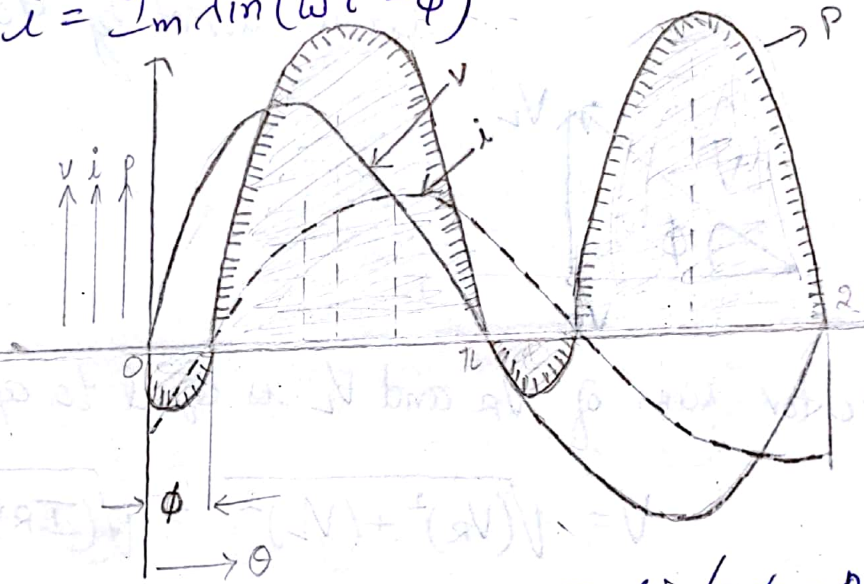
$$\tan\phi = \frac{V_L}{V_R} = \frac{I X_L}{I R} = \frac{X_L}{R}$$

$$\phi = \tan^{-1}\frac{X_L}{R}$$

**Power**

If $v = V_m \sin\omega t$

then $i = I_m \sin(\omega t - \phi)$




Instantaneous Power, $P = v i = V_m \sin\omega t \; I_m \sin(\omega t - \phi)$

$$= \frac{V_m I_m}{2} \, 2\sin\omega t \sin(\omega t - \phi)$$

$$\boxed{\begin{aligned} &2\sin A \sin B \\ &= \cos(A-B) - \cos(A+B) \end{aligned}}$$

$$= \frac{V_m I_m}{2}\left[\cos\phi - \cos(2\omega t - \phi)\right]$$

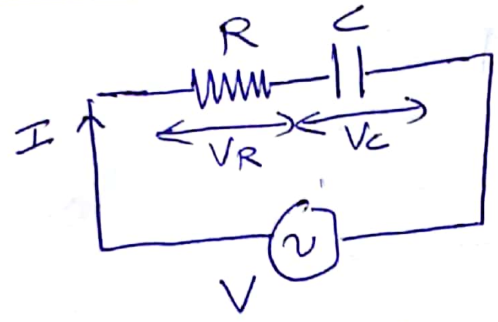
$$= \underbrace{\frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\phi}_{\text{constant part}} - \underbrace{\frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos(2\omega t - \phi)}_{\text{pulsating component}}$$

Average Power, $P_{av} = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\phi - 0$

$$P = VI\cos\phi \quad [\cos\phi \text{ is Power Factor}]$$

Scanned by CamScanner

## RC Series Circuit:

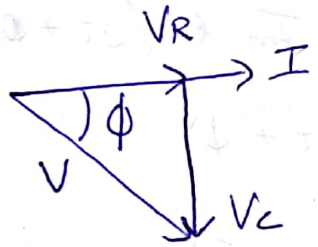



V = RMS value of applied voltage

I = RMS value of current

$V_R = IR$ (Voltage drop in resistance is in phase with current)

$V_C = IX_C$ (Voltage drop in capacitive reactance lags current by 90°)




$$V = \sqrt{V_R^2 + V_C^2}$$

$$= \sqrt{(IR)^2 + (IX_C)^2}$$

$$= I\sqrt{R^2 + X_C^2}$$

$$V = IZ$$

$$I = \frac{V}{Z}$$

Where $Z = \sqrt{R^2 + X_C^2}$ is total opposition offered to the flow of alternating current by RC series circuit.

### Phase Angle:

Current in the circuit leads the applied voltage by angle $\phi$.

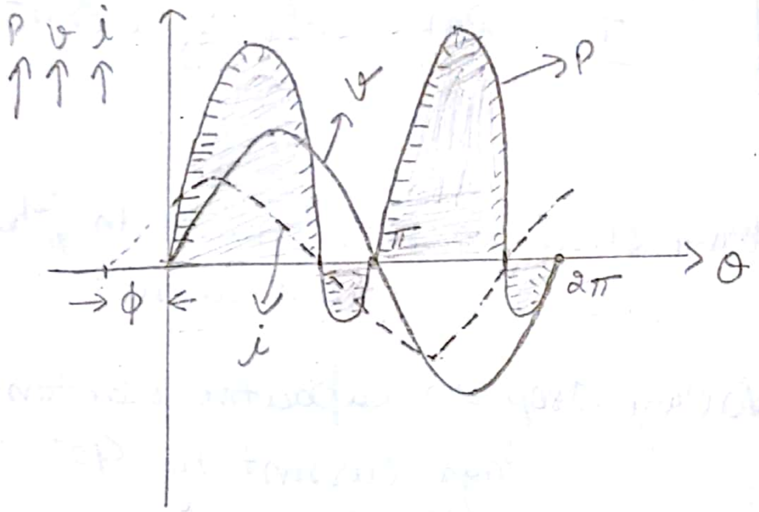
$$\tan\phi = \frac{V_C}{V_R} = \frac{IX_C}{IR}$$

$$\phi = \tan^{-1}\left(\frac{X_C}{R}\right)$$

# Power

If $v = V_m \sin \omega t$

$i = I_m \sin(\omega t + \phi)$




Instantaneous Power, $P = vi = V_m \sin\omega t \; I_m \sin(\omega t + \phi)$

$$= \frac{V_m I_m}{2} 2 \sin\omega t \sin(\omega t + \phi)$$

$$= \frac{V_m I_m}{2}\left[\cos\phi - \cos(2\omega t + \phi)\right]$$

$$= \underbrace{\frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\phi}_{\text{Constant part}} - \underbrace{\frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos(2\omega t + \phi)}_{\text{Pulsating component}}$$

Average Power $P_{av} = \text{avg of } \frac{V_m I_m}{\sqrt{2}\sqrt{2}} \cos\phi - \text{Avg. of } \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos(2\omega t + \phi)$

$$= \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\phi - \text{zero}$$

$$\boxed{P = VI\cos\phi}$$

$$P = (IZ) I \frac{V_R}{V} = (IZ) I \frac{IR}{IZ}$$

$$\boxed{P = I^2 R}$$

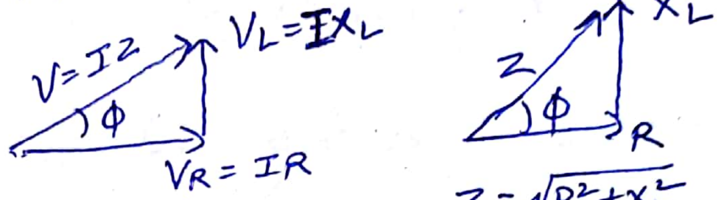
This shows Power is consumed in resistance only, Capacitor doesn't consume any power.

Scanned by CamScanner

## Impedance Triangle

A right angled triangle whose base represents ckt resistance, perpendicular represents ckt reactance and hypotaneous represents ckt impedance

For RL ckt




$Z = \sqrt{R^2 + X_L^2}$

$\cos\phi = \frac{R}{Z}$

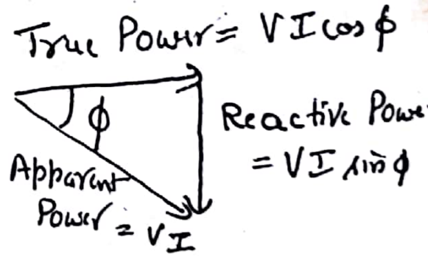
## Power Factor

→ Power factor is defined as cosine of angle b/w voltage and current in an ac circuit.

(1) $\cos\phi$

(2) $\frac{R}{Z} = \frac{\text{Resistance}}{\text{Impedance}}$

(3) $\frac{\text{True Power}}{\text{Apparent Power}} = \frac{W}{\text{Volt-amperes}} = \frac{W}{VA}$




→ In case of pure resistive circuit, current is in phase with circuit voltage i.e. $\phi = 0$. ∴ power factor of circuit, $\cos\phi = 1$.

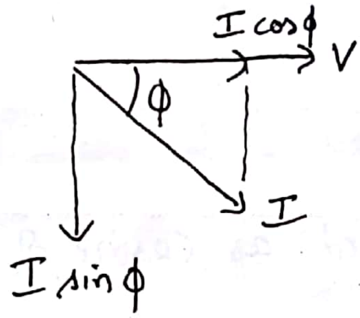
→ In case of pure inductive or capacitive circuit current is 90° out of phase with circuit voltage i.e. $\phi = 90$; ∴ $\cos\phi = 0$.

→ Value of p.f. can never be more than one.

## Active and Reactive components of Circuit Current

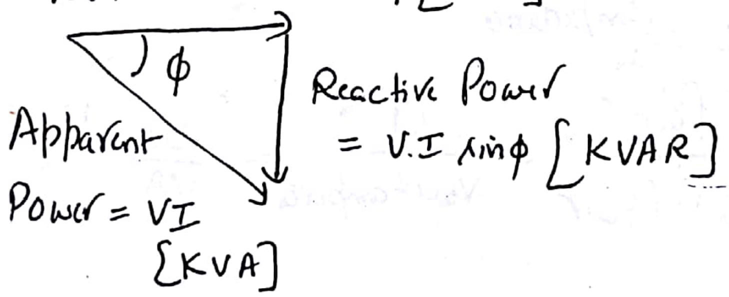
→ Active component is that which is in phase with the applied Voltage V i.e. $I\cos\phi$. It is also known as 'Wattful' component

→ Reactive component is that which is in quadrature with V i.e. $I\sin\phi$. It is also known as 'wattless' component.

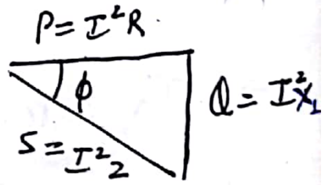



→ **True Power** :- Power which is actually consumed in an ac circuit. Power is consumed only in Resistance.

True Power = $VI\cos\phi$ [KW]




True Power = $VI\cos\phi = I^2R$




→ **Reactive Power**

Power developed in the inductive reactance of circuit

$$Q = VI\sin\phi \ [VAR] = I^2X_L \qquad \left[\because \sin\phi = \frac{X_L}{Z}\right]$$

Volt amperes reactive

→ **Apparent Power**

given by product of rms values of current and voltage

$$S = VI = I^2Z$$

Q1. A coil of 12 Ω resistance and 0.2H inductance is connected across a 220V, 50Hz supply. Find (a) the current (b) Phase angle (c) Power factor

Ans

$R = 12\,\Omega$, $L = 0.2H$

$X_L = 2\pi f L = 2 \times 3.14 \times 50 \times 0.2 = 62.8\,\Omega$

$Z = \sqrt{R^2 + X_L^2} = \sqrt{12^2 + (62.8)^2} = 63.936\,\Omega$

(a) Current, $I = \frac{V}{Z} = \frac{220}{63.96} = 3.44A$

(b) Phase Angle, $\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{62.8}{12}\right) = 79.18$

(c) $\cos\phi = \frac{R}{Z} = 0.188$

Q2 A voltage $v(t) = 150 \sin(2\pi f)t$, 50 Hz is applied to a series circuit consisting of 10 Ω resistance, 0.0318 H inductance. Determine (a) Expression for current i(t) (b) Phase angle b/w. voltage and current (c) Power Factor (d) Active Power Consumed $\cancel{(e)}\phi$

Ans

$v(t) = 150 \sin(2\pi f)t$

$f = 50Hz$, $R = 10\,\Omega$, $L = 0.0318 H$

$X_L = 2\pi f L = 2\pi\, 50 \times 0.0318 = 134.8\,\Omega$

$Z = \sqrt{R^2 + X_L^2} = \sqrt{10^2 + (134.8)^2} = 135.17\,\Omega$

(a) Current is given by

$i(t) = I_m \sin(\omega t - \phi)$ for R-L ckt.

$I_m = \frac{V_m}{Z} = \frac{150}{135.17} = 1.1097 A$

$i(t) = 1.1097 \sin(2\pi f t - \phi)$

(b) Phase angle, $\phi = \tan^{-1}\frac{X_L}{R} = \tan^{-1}\left(\frac{134.8}{10}\right) = 85.75°$

(c) Power Factor, $\cos\phi = \cos(85.75°) = 0.0741$

(d) Power Consumed $= VI\cos\phi = \frac{150}{\sqrt{2}} \times \frac{1.1097}{\sqrt{2}} \times 0.0741 = 6.169W$

Q3 An ac ckt consisting of resistance of 14 Ω in series with a Capacitor of 250 μF is connected across 220V, 50 Hz supply. Find (a) current flowing (b) power factor (c) voltage drop across each element (d) Power absorbed

Ans $V = 220V,\ f = 50Hz,\ R = 14\Omega,\ C = 250\mu F = 250\times10^{-6} F$

$X_C = \frac{1}{2\pi f C} = \frac{1}{2\times3.14\times50\times250\times10^{-6}} = 12.739\Omega$

$Z = \sqrt{R^2 + X_C^2} = \sqrt{14^2 + (12.739)^2} = 18.93\Omega$

(a) Current, $I = \frac{V}{Z} = \frac{220}{18.93} = 11.623 A$

(b) $\phi = \tan^{-1}\frac{X_C}{R} = 42.30°$

$P.F = \cos\phi = \cos(42.30°) = 0.739$ $\left[\text{or } \cos\phi = \frac{R}{Z}\right]$

(c) $V_R = IR = 11.623 \times 14 = 162.72V$

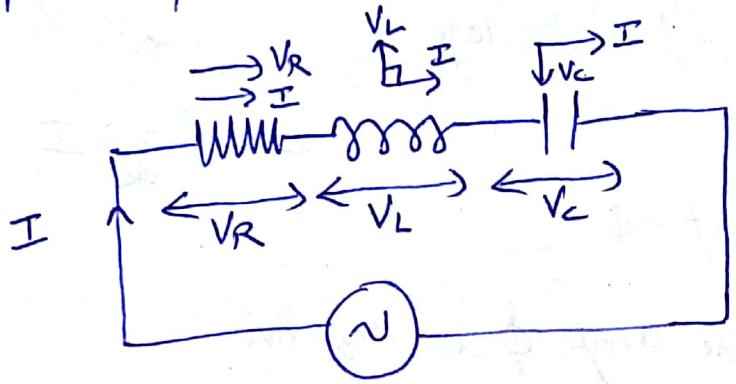
$V_C = IX_C = 11.623 \times 12.739 = 148.06V$

(d) Power absorbed $= VI\cos\phi = 220 \times 11.623 \times .739$

$= 1889.67W$

## RLC Series Ckt

A circuit that contains pure resistance, pure inductance and pure capacitance, all connected in series.

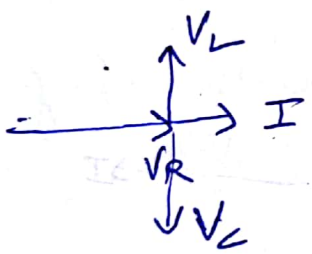



$V_R = IR$ i.e. voltage across R is in phase with I

$V_L = I X_L$ (voltage across L leads I by $\pi/2$)

$V_C = I X_C$ (voltage across C lags I by $\pi/2$)

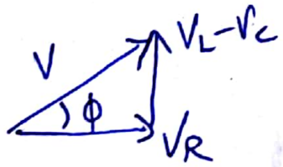
→ $V_L$ and $V_C$ are 180° out of phase with each other




$$V = \sqrt{V_R^2 + (V_L - V_C)^2} = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

$$V = I\sqrt{R^2 + (X_L - X_C)^2}$$

$$I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{Z}$$




$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{IX_L - IX_C}{IR}$$

$$\tan\phi = \frac{X_L - X_C}{R}$$

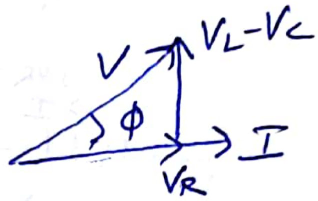
$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

Three Cases of RLC series ckt:

(i) When $X_L > X_C$, phase angle, $\phi$ is positive

Circuit behaves as an R-L series ckt.

Current lags behind the applied voltage.



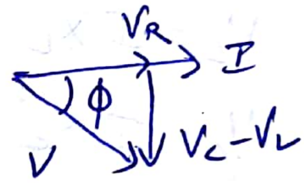

If $v = V_m \sin\omega t$

then $i = I_m \sin(\omega t - \phi)$

(2) When $X_L < X_C$, Phase angle $\phi$ is negative

Circuit behaves as an R-C series ckt

Current leads the applied voltage



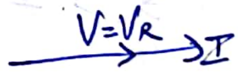

If $v = V_m \sin\omega t$

then $i = I_m \sin(\omega t + \phi)$

(3) When $X_L = X_C$, Phase angle is zero

~~Cur~~ Ckt behaves like pure resistive ckt

Current is in phase with applied voltage




If $v = V_m \sin\omega t$

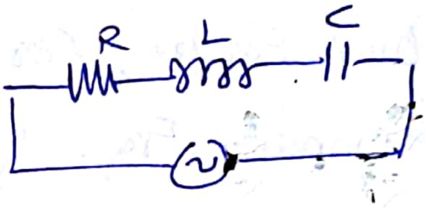
then $i = I_m \sin\omega t$

Power consumed by series RLC ckt, $P = VI\cos\phi$

# Series Resonance in RLC ckt (Voltage Resonance)

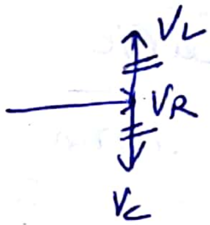
→ In RLC series ckt, when ckt current is in phase with the applied voltage, the circuit is said to be in series resonance.

→ This condition is obtained in an R-L-C ckt when $X_L = X_C$




Impedance at Resonance $Z_r = \sqrt{R^2 + (X_L - X_C)^2}$
$= R$

Current, $I_r = \dfrac{V}{R}$




→ When a series RLC ckt is in resonance, it possess minimum impedance ($Z = R$), Hence ckt current is maximum.

→ <u>Resonant Frequency</u>: The frequency at which net reactance of the series ckt is zero

$$X_L - X_C = 0 \Rightarrow X_L = X_C$$

$$2\pi f_r L = \frac{1}{2\pi f_r C}$$

$$f_r^2 = \frac{1}{4\pi^2 LC}$$

$$\boxed{f_r = \frac{1}{2\pi\sqrt{LC}}}$$

When an R-L-C ckt. is in resonance:

(1) Net Reactance of the ckt is zero i.e. $X_L - X_C = 0$

(2) Circuit Impedance is minimum i.e. $Z_r = R$

(3) Circuit Current is maximum i.e. $I_r = \frac{V}{R}$

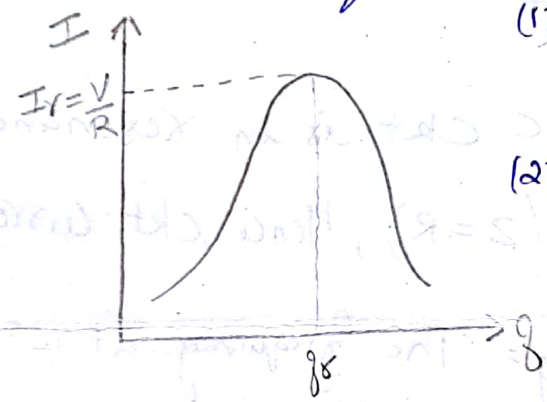
(4) Power Dissipated is maximum i.e. $P_r = I_r^2 R$

(5) Power Factor, $\cos\phi = 1$ [$\because \phi = 0$]

(6) Resonant Freq. $f_r = \frac{1}{2\pi\sqrt{LC}}$

## Resonance Curve

Graph b/w Current and frequency is known as Resonance Curve

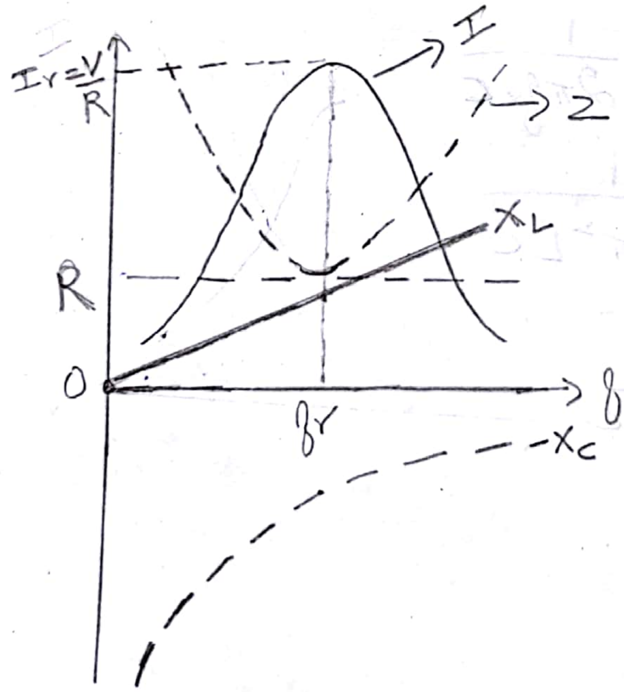



(1) For any freq. less than $f_r$, $X_L < X_C$, ckt acts as capacitive ckt

(2) For any freq. higher than $f_r$, $X_L > X_C$, ckt acts as inductive ckt

(3) For $f = f_r$, $X_L = X_C$, ckt behaves as resistive ckt.

## Variation of Circuit Parameters with frequency:




(1) Resistance

Resistance is constant (fixed Value)

Represented by straight line.

(2) $X_L$ $\quad X_L = 2\pi f L$

$X_L \propto f$

$X_L$ is directly proportional to $f$

(3) $X_C$ $\quad X_C = \frac{1}{2\pi f C}$, $X_C \propto \frac{1}{f}$

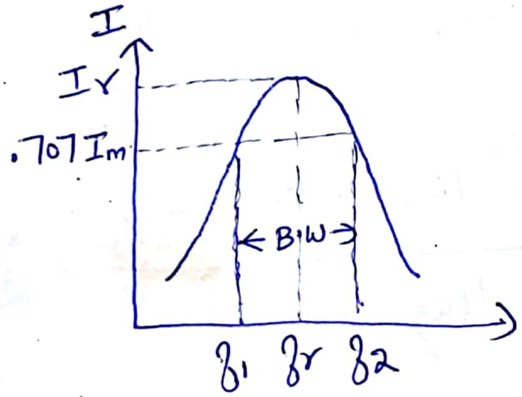
$X_C$ is inversely proportional to $f$

(4) Z Circuit Impedance

Z has min. value (R) at $f_r$

## Band width

Range of frequencies over which circuit current is equal to or more than 70.7% of max. current.




$$B.W = f_2 - f_1$$

$f_1$ = lower cut off freq.

$f_2$ = upper cut off freq

$$f_1 = f_r - \frac{B.W.}{2}$$

$$f_2 = f_r + \frac{B.W.}{2}$$

## Q- Factor of Series Resonant circuit (Quality Factor)

(i) It is given by voltage magnification produced in the circuit at resonance

$$Q\text{- Factor} = \frac{\text{Voltage across } L \text{ or } C}{\text{Applied voltage}} = \frac{I_o X_L}{I_o R} = \frac{X_L}{R}$$

$$= \frac{\omega_o L}{R} = \tan\phi$$

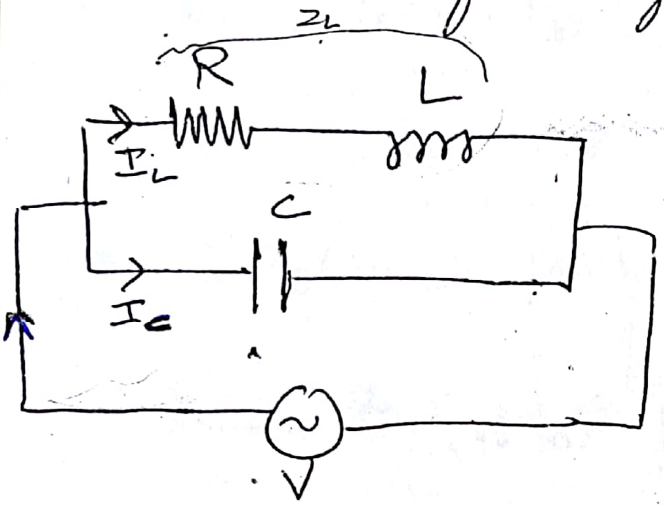
Where $\omega_o = 2\pi f_o = 2\pi \frac{1}{2\pi\sqrt{LC}} = \frac{1}{\sqrt{LC}}$

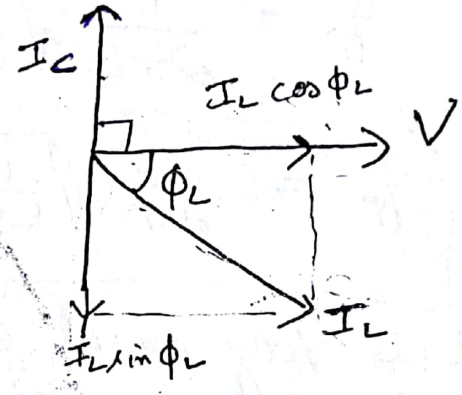
$$Q\text{- Factor} = \frac{L}{R} \times \frac{1}{\sqrt{LC}} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Scanned by CamScanner

# Parallel Resonance (Current Resonance)

→ Consider ~~and~~ an inductor of L Henry having some resistance R connected in parallel with a capacitor of capacitance C across a supply voltage of V volts.



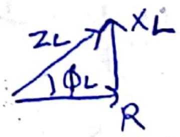

→ $I_L$ will lag the voltage by an angle $\phi$, where $\phi = \tan^{-1}\frac{X_L}{R}$

→ $I_C$ will lead the voltage by 90°.

→ For Resonance, the reactive component of circuit current must be zero

$$I_C - I_L \sin\phi_L = 0 \Rightarrow I_C = I_L \sin\phi_L$$

Where $I_L = \frac{V}{Z_L}$, $\sin\phi_L = \frac{X_L}{Z_L}$, $I_C = \frac{V}{X_C}$




$\sin\phi_L = \frac{X_L}{Z_L}$

$\cos\phi_L = \frac{R}{Z_L}$

$$\frac{V}{X_C} - \frac{V}{Z_L} \times \frac{X_L}{Z_L} = 0$$

$$\frac{V}{X_C} = \frac{V}{Z_L} \times \frac{X_L}{Z_L}$$

$$Z_L^2 = X_L X_C$$

$$Z_L^2 = X_L X_C \Rightarrow Z_L^2 = \frac{\omega L}{\omega C} = \frac{L}{C}$$

$$R^2 + X_L^2 = \frac{\cancel{\omega} L}{\cancel{\omega} C}$$

$$R^2 + (2\pi f_0 L)^2 = \frac{L}{C}$$

$$2\pi f_0 L = \sqrt{\frac{L}{C} - R^2}$$

$$f_0 = \frac{1}{2\pi L}\sqrt{\frac{L}{C} - R^2}$$

Resonant freq)
$$\boxed{f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}}$$

If R is very small compared to L,

then resonant frequency $f_0 = \frac{1}{2\pi\sqrt{LC}}$

## Current at Resonance

The circuit current is $I = I_L \cos\phi_L = \frac{V}{Z_L}\frac{R}{Z_L} = \frac{VR}{Z_L^2}$

Putting the value of $Z_L^2 = \frac{L}{C}$

$$\boxed{I = \frac{VR}{L/C} = \frac{V}{L/CR}}$$

$\frac{L}{CR}$ is known as equivalent/dynamic impedance of the parallel circuit at resonance

## Parallel Resonance (Important Points)

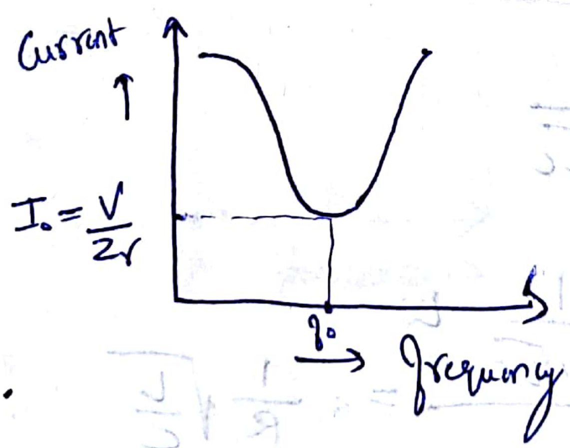
→ Net susceptance is zero i.e. $\frac{1}{X_C} = \frac{X_L}{Z^2}$ or $Z = \sqrt{\frac{L}{C}}$

→ Admittance equals conductance

→ Reactive (or wattless component) of line current is zero, hence circuit power factor is unity.

→ Impedance is purely resistive i.e. $Z_r = \frac{L}{CR}$ because there is no frequency term.

→ Line current is minimum and is equal to $\frac{V}{L/CR}$ and is in phase with applied voltage



## Q Factor of Parallel resonant circuit :→

The ratio of current circulating b/w its two branches to the line current drawn from the supply.

$$Q\text{-factor} = \frac{\text{Current circulating b/w } L \,\&\, C}{\text{Line current}} = \frac{I_C}{I}$$

$$I_C = \frac{V}{X_C} = \frac{V}{1/\omega C} = \omega C V.$$

$$\& \; I = \frac{V}{L/CR}$$

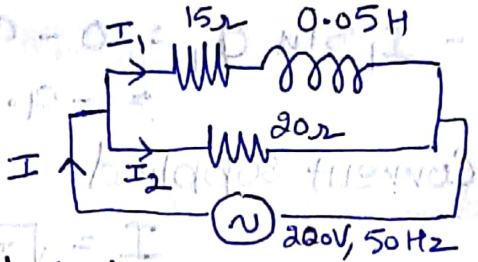
$$Q\text{-factor} = \frac{\omega C V}{\frac{V}{L/CR}} = \frac{\omega C V L}{V C R} = \frac{\omega L}{R} = \frac{2\pi f_0 L}{R}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$Q\text{-factor} = \frac{2\pi \frac{1}{2\pi\sqrt{LC}} \cdot L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

# AC Parallel circuits






## Phasor Method

Q) A coil resistance 15 ohm and inductance 0.05 H is connected in parallel with a non inductive resistance of 20 ohm. Find (i) Current in each branch (ii) Total Current Supplied (iii) phase angle and p.f of Combination when a voltage of 200 volt at 50 Hz is applied iv) Power Consumed in the Circuit

**Soltn** Applied Voltage $V = 200V$ Supplied frequency $f = 50Hz$

**Branch I**

$R_1 = 15\Omega$

$X_{L_1} = 2\pi f L_1 = 2\pi \times 50 \times 0.05 = 15.7\Omega$

$Z_1 = \sqrt{R_1^2 + X_L^2}$

$= \sqrt{(15)^2 + (15.7)^2} = 21.72$ ohm.

Current in the Coil

$I_1 = \frac{V}{Z_1} = \frac{200}{21.72} = 9.2A$ (Ans)

Phase angle

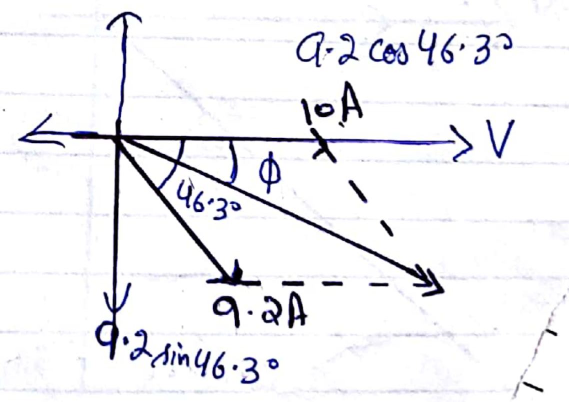
$\phi_1 = \tan^{-1}\left(\frac{X_{L_1}}{R}\right) = \tan^{-1}\frac{15.7}{15} = 46.3°$ lagging

**Branch II**

$R_2 = 20$ ohm.

$I_2 = \frac{V}{R_2} = \frac{200}{20} = 10A$ (Ans)

Phase angle $\phi_2 = 0$ ($I_2$ is in same phase vs




$I_{xx} = I_2 + I_1 \cos\phi_1$.

$= 10 + 9.2\cos 46.3°$

$= 10 + 9.2 \times 0.6909$

$= 16.356A$.

$$I_{YY} = 0 - I_1 \sin\phi_1 = 0 - 9.2 \sin 46.3$$
$$= -9.2 \times 0.723 = -6.65\ A$$

Total Current Supplied

$$I = \sqrt{I_{XX}^2 + I_{YY}^2}$$
$$= \sqrt{(16.356)^2 - (6.65)^2}$$
$$= 17.6566\ A \text{ (Ans)}$$

Phase angle $\phi = \tan^{-1} \cancel{\dfrac{I_{XX}}{I_{YY}}}\ \dfrac{I_{YY}}{I_{XX}} = \tan^{-1}\left(\dfrac{-6.65}{16.356}\right)$

$$= -22.126^\circ \text{ (Ans)}$$

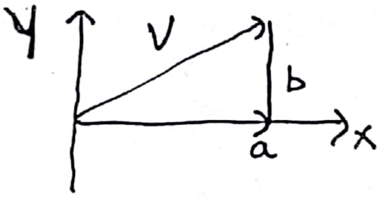
Power factor of the Circuit

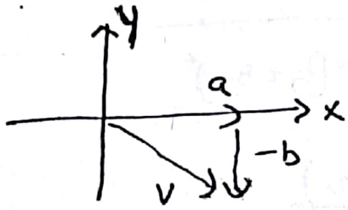
$$\cos\phi = \cos(-22.126^\circ)$$
$$= 0.9264 \text{ (lagging) Ans.}$$

Power $P = VI\cos\phi$ $= 200 \times 17.656 \times 0.9264$
$$= 3271.3\ W \text{ (Ans)}$$

# Phasor Algebra / Symbolic Method / j- Method

⇒ The symbol $j(\sqrt{-1})$ is used to represent the virtical components of phasor quantities.




Phasor V is represented by

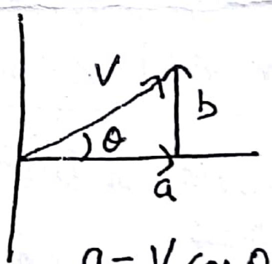
$V = a + jb$ ↓ $V = a - jb$

## Mathematical representation of Phasors

### a) Rectangular Form

$$\boxed{\bar{V} = a + jb}$$

### b) Trignometrical Form




$a = V\cos\theta$

$b = V\sin\theta$

$$\bar{V} = a + jb = V\cos\theta + j\cancel{b}\sin\theta$$

$$\boxed{\bar{V} = V(\cos\theta + j\sin\theta)}$$

where $V = \sqrt{a^2 + b^2}$

$\theta = \tan^{-1}\left(\frac{b}{a}\right)$

### c) Polar Form

$$\boxed{\bar{V} = V\angle\theta}$$

### d) Exponential Form

$$\boxed{\bar{V} = Ve^{j\theta}}$$

$e^{j\theta} = \cos\theta + j\sin\theta$

# Addition and Subtraction of Vector Quantities

[* Rectangular Form is best suited for addition and subtraction]

Let $\bar{V}_1 = a_1 + jb_1$ , $\bar{V}_2 = a_2 + jb_2$

Addition

$$\bar{V} = \bar{V}_1 + \bar{V}_2 = (a_1 + a_2) + j(b_1 + b_2)$$

$$\text{Magnitude} = \sqrt{(a_1+a_2)^2 + (b_1+b_2)^2}$$

$$\theta = \tan^{-1}\left(\frac{b_1+b_2}{a_1+a_2}\right)$$

Subtraction

$$\bar{V} = \bar{V}_1 - \bar{V}_2 = (a_1 - a_2) - j(b_1 - b_2)$$

$$\text{Magnitude} = \sqrt{(a_1-a_2)^2 + (b_1-b_2)^2}$$

$$\theta = \tan^{-1}\left(\frac{b_1-b_2}{a_1-a_2}\right)$$

# Multiplication and Division of Phasor Quantities

* Polar Form is best suited for multiplication and division

Let $\bar{V}_1 = V_1 \angle\theta_1$ , $\bar{V}_2 = V_2 \angle\theta_2$

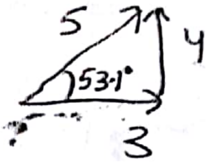
Multiplication $\quad \bar{V}_1 \times \bar{V}_2 = V_1 V_2 \angle(\theta_1 + \theta_2)$

Division $\quad \dfrac{\bar{V}_1}{\bar{V}_2} = \dfrac{V_1}{V_2} \angle(\theta_1 - \theta_2)$

Scanned by CamScanner

Q Write the equivalent exponential and polar form of vectors $3+j4$

$\therefore$ Magnitude $= \sqrt{3^2+4^2} = 5$




$\tan\theta = \frac{4}{3}$

$\theta = \tan^{-1}(4/3)$

$\theta = 53.1°$

Exponential Form $= 5e^{53.1°}$

Polar Form $= 5\angle 53.1°$

Q A vector is represented by $20e^{-j2\pi/3}$. Write the equivalent rectangular and polar form.

Rectangular form, $a = 20\cos(-120) = -10$

$b = 20\sin(-120) = -17.32$

$a+jb = -10-j17.32$

$\cos(-120) = -0.5$

$\sin(120) = 0.866$

Polar Form

$20\angle -120°$

Q Given the following two vectors

$A = 20\angle 60°$ and $B = 5\angle 30°$

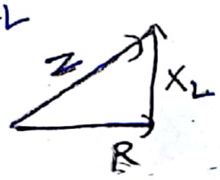
Find (1) $A \times B$ (2) $A/B$

$A \times B = 20\angle 60° \times 5\angle 30° = 100\angle 90°$

$\frac{A}{B} = \frac{20\angle 60°}{5\angle 30°} = 4\angle 30°$

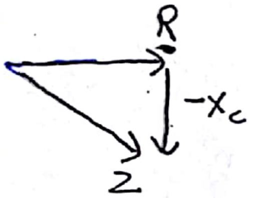
# RL and RC circuits in rectangular Form

R-L




$Z = R + jX_L$

R-C




$Z = R - jX_c$

An alternating voltage of $(160 + j120)$ is applied to a circuit and current in the circuit is given by $(6 + j8)$ A.

Find (i) Values of elements of the circuit

(2) Power factor of the circuit

(3) Power consumed

$\Rightarrow$ $V = 160 + j120$, $I = 6 + j8$

$V = 200\angle 36.87°$ $\left[\sqrt{160^2 + 120^2} = 200,\ \theta = \tan^{-1}\left(\frac{120}{160}\right) = 36.87°\right]$

$I = 10\angle 53.13°$ $\left[\sqrt{6^2 + 8^2} = 10,\ \theta = \tan^{-1}\left(\frac{8}{6}\right) = 53.13°\right]$

Impedance, $Z = \frac{V}{I} = \frac{200\angle 36.87°}{10\angle 53.13°} = 20\angle -16.26°$

$= 20\cos(-16.26) + j20\sin(-16.26)$

$= 19.2 - j5.6$

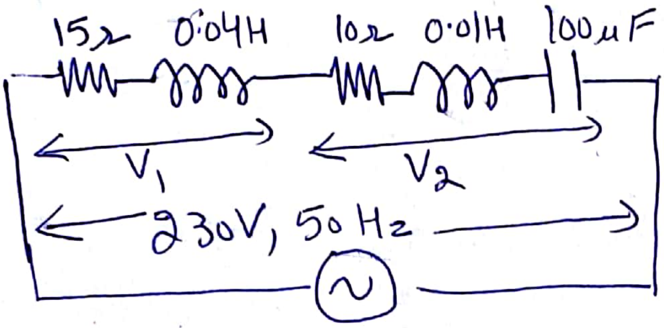
i) $R = 19.2$, $X_c = 5.6 \Rightarrow \frac{1}{2\pi f c} = 5.6 \Rightarrow \frac{1}{2\pi 50 C} = 5.6$

$\Rightarrow C = 568.4\,\mu F$

ii) $\cos\phi = \frac{19.2}{20}\left(\frac{R}{Z}\right) = 0.96$ (leading) (3) $P = VI\cos\phi = 200 \times 10 \times 0.96$

$= 1920\,W$

Q.




Find (i) Current drawn
(2) Voltages $V_1$ & $V_2$
(3) Power Factor

Draw also Phasor diagram indicating the voltages $V_1$, $V_2$ and supply voltage wrt current.

Ans

$$X_{L_1} = 2\pi f L_1 = 2\pi\, 50 \times 0.04 = 12.566\,\Omega$$

$$X_{L_2} = 2\pi f L_2 = 2\pi\, 50 \times 0.01 = 3.141\,\Omega$$

$$X_{C_2} = \frac{1}{2\pi f C} = \frac{1}{2\pi 50 \times 100 \times 10^{-6}} = 31.831\,\Omega$$

$$Z_1 = R_1 + jX_{L_1} = 15 + j12.566 = 19.568 \angle 39.95^\circ$$

$$Z_2 = R_2 + jX_{L_2} - jX_{C_2} = 10 + j3.1416 - j31.831$$

$$= 10 - j28.69 = 30.383 \angle -70.78^\circ$$

Total Impedance, $Z = Z_1 + Z_2 = 15 + j2.566 + 10 - j28.69$

$$= 25 - j16.124 = 29.748 \angle -32.82^\circ$$

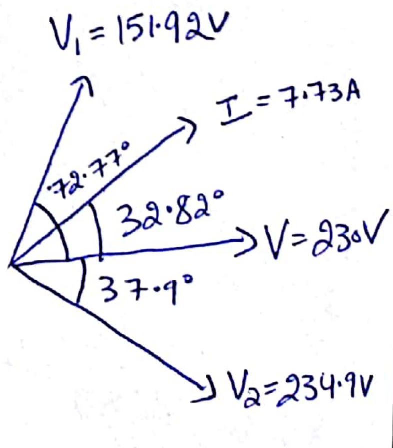
Current $I = \dfrac{V}{Z} = \dfrac{230\angle 0^\circ}{29.748\angle -32.82^\circ} = 7.73\angle 32.82^\circ$ A

$$V_1 = I Z_1 = 7.73 \times 19.568 \angle (32.82^\circ + 39.95^\circ)$$

$$= 151.29 \angle 72.77^\circ \text{ V}$$

$$V_2 = I Z_2 = 7.73 \times 30.383 \angle (32.82^\circ - 70.78^\circ)$$

$$= 234.9 \angle -37.96^\circ \text{ V}$$




Power Factor, $\cos\phi = \cos 32.82^\circ$

$$= 0.8404 \text{ leading}$$