

$$= \frac{2}{\pi\sqrt{2}} \left(\frac{2}{3} \sin 2x + \frac{2}{5 \cdot 7} \sin 6x + \frac{2}{9 \cdot 11} \sin 10x \right. \quad \text{--- } \infty$$

$$= \frac{4}{\pi\sqrt{2}} \left(\frac{\sin 2x}{1 \cdot 3} - \frac{\sin 6x}{5 \cdot 7} + \frac{\sin 10x}{9 \cdot 11} \right. \quad \text{--- } \infty$$

Practice Questions

- ① Find the Fourier series to represent the function

$$f(x) = |\sin x|, \quad -\pi < x < \pi$$

- ② Find Fourier series of $f(x)$ in interval $(-\pi, \pi)$

$$f(x) = \begin{cases} \pi + x, & -\pi < x < 0 \\ \pi - x, & 0 < x < \pi \end{cases}$$

- ③ Obtain Fourier series of $f(x) = \sqrt{1 - \cos x}$ in $(0, 2\pi)$.

hence evaluate $\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \dots \infty$

- ④ Express $f(x) = |x|$ as Fourier series, $-\pi < x < \pi$

- ⑤ $f(x) = x \cos x$ in $(-\pi, \pi)$

$$(6) \quad f(x) = \begin{cases} -x+1, & -\pi \leq x \leq 0 \\ x+1, & 0 \leq x \leq \pi \end{cases}$$

Is the function even or odd? Find the Fourier series for $f(x)$. Hence evaluate

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \infty$$

(7) Obtain the Fourier series for function

$$f(x) = \begin{cases} \pi x, & 0 \leq x \leq 1 \\ \pi(2-x), & 1 \leq x \leq 2 \end{cases}$$

Hence show that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \infty = \frac{\pi^2}{8}$$

(8) Fourier series of $f(x) = x^2$ in $(-l, l)$

(9) $f(x) = e^x$ as Fourier series in $(-l, l)$

(10) $f(x) = x^2$ in $0 < x < 2l$. Hence show that

$$(i) \quad \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{6}$$

$$(ii) \quad \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots = \frac{\pi^2}{12}$$

(11) Find Half range Sine series of $\pi x - x^2$ in $(0, \pi)$.

(12)

$$f(x) = \begin{cases} x, & 0 < x < \pi/2 \\ \pi - x, & \pi/2 < x < \pi \end{cases}$$

Find Half range Sine series as well as half range cosine series.

(13)

Obtain Half range cosine series for

$$f(x) = \begin{cases} kx, & 0 \leq x \leq l/2 \\ k(l-x), & l/2 \leq x \leq l \end{cases}$$

hence evaluate $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots \infty$

(14)

Half range cosine series of $\sin x$ in $(0, \pi)$

(15)

Half range sine series for $f(x)$

$$f(x) = \begin{cases} \frac{1}{4} - x, & 0 < x < 1/2 \\ x - \frac{3}{4}, & 1/2 < x < 1 \end{cases}$$

Answers

$$(1) |f(x)| = \frac{2}{\pi} - \frac{4}{\pi} \left(\frac{\cos 2x}{3} + \frac{\cos 4x}{15} + \frac{\cos 6x}{35} \dots \right)$$

$$(2) f(x) = \frac{\pi}{2} + \frac{4}{\pi} \left(\frac{\cos x}{1^2} + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} \dots \infty \right)$$

$$(3) |x| = \frac{\pi}{2} - \frac{4}{\pi} \left(\cos x + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} \dots \infty \right)$$

$$(6) f(x) = \frac{\pi}{2} + 1 - \frac{4}{\pi} \left(\cos x + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} \dots \infty \right)$$

$\pi/8$

$$(7) f(x) = \frac{\pi}{2} - \frac{4}{\pi} \left(\frac{\cos \pi x}{1^2} + \frac{\cos 3\pi x}{3^2} + \frac{\cos 5\pi x}{5^2} \dots \infty \right)$$

$$(8) x^2 = \frac{2^2}{3} - \frac{4^2}{\pi^2} \left(\frac{\cos \frac{\pi x}{2}}{1^2} - \frac{\cos 2\pi x}{2^2} + \frac{\cos 3\pi x}{3^2} \dots \infty \right)$$

$$(9) e^{-x} = \sinh x \left[\frac{1}{e} - 2e \left(\frac{\cos \frac{\pi x}{2}}{e^2 + \pi^2} + \frac{\cos 2\pi x}{e^2 + 2^2\pi^2} + \frac{\cos 3\pi x}{e^2 + 3^2\pi^2} \dots \right) \right]$$

$$- 2\pi \left(\frac{\sin \frac{\pi x}{2}}{e^2 + \pi^2} - \frac{2}{e^2 + 2^2\pi^2} \sin 2\pi x + \frac{3}{e^2 + 3^2\pi^2} \sin 3\pi x \dots \right)$$

$$(10) f(x) = \frac{4l^2}{3} + \frac{4l^2}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \cos \frac{n\pi x}{l} - \frac{4l^2}{\pi^2} \sum_{n=1}^{\infty} \frac{\sin n\pi x}{l}$$

$$(11) \pi x - x^2 = \frac{8}{\pi} \left[\sin x + \frac{\sin 3x}{3^3} + \frac{\sin 5x}{5^3} - \dots \right]$$

(12) Half range sine Series

$$f(x) = \frac{4}{\pi} \left[\frac{\sin x}{1^2} - \frac{\sin 3x}{3^2} + \frac{\sin 5x}{5^2} - \dots \right]$$

Half range cosine Series

$$f(x) = \frac{\pi}{4} - \frac{2}{\pi} \left[\frac{\cos 2x}{1^2} + \frac{\cos 6x}{3^2} + \frac{\cos 10x}{5^2} - \dots \right]$$

$$(13) f(x) = \frac{kl}{4} - \frac{8kl}{\pi^2} \left[\frac{\cos \frac{2\pi x}{l}}{2^2} + \frac{\cos \frac{6\pi x}{l}}{6^2} + \frac{\cos \frac{10\pi x}{l}}{10^2} - \dots \right]$$

Cosine Series.

$$(14) \sin x = \frac{2}{\pi} - \frac{4}{\pi} \left[\frac{\cos 2x}{1 \cdot 3} + \frac{\cos 4x}{3 \cdot 5} + \frac{\cos 6x}{5 \cdot 7} - \dots \right]$$

$$(15) f(x) = \left(\frac{1}{\pi} - \frac{4}{\pi^2} \right) \sin \pi x + \left(\frac{1}{3\pi} + \frac{4}{3^2 \pi^2} \right) \sin 3\pi x \\ + \left(\frac{1}{5\pi} - \frac{4}{5^2 \pi^2} \right) \sin 5\pi x - \dots - \infty$$