

1. Greek Letters Used

α	alpha	ϕ	phi	κ	kappa	τ	tau
β	beta	ψ	psi	μ	mu	χ	chi
γ	gamma	ξ	xi	ν	nu	ω	omega
δ	delta	η	eta	π	pi	Δ	cap. delta
ϵ	epsilon	ζ	zeta	ρ	rho		
i	iota	λ	lambda	σ	sigma		
θ	theta	Γ	cap. gamma	Σ	cap. sigma		

2. Some Notations

\in	belongs to	\Leftrightarrow	implies and implied by
\notin	does not belong to	iff	if and only if
\Rightarrow	implies	\cup	union
		\cap	intersection

3. Useful Data

$\sqrt{2} = 1.4142$	$\frac{1}{\pi} = 0.3183$	$1 \text{ rad.} = 57^\circ 17' 45''$	$\log_{10} e = 0.4343$
$\sqrt{3} = 1.732$	$e = 2.7183$	$1^\circ = 0.0174 \text{ rad.}$	$\log_e 2 = 0.6931$
$\pi = 3.1416$	$\frac{1}{e} = 0.3679$	$\log_e 10 = 2.3026$	$\log_e 3 = 1.0986$

4. Quadratic Equation

Roots of quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$

are $\frac{-b \pm \sqrt{D}}{2a}$, where $D = b^2 - 4ac$ is called discriminant

Sum of roots $= -\frac{b}{a}$, Product of roots $= \frac{c}{a}$

If $D > 0$, roots are real and distinct.

If $D = 0$, roots are equal.

If $D < 0$, roots are imaginary.

5. Progressions

(i) For the A.P. (Arithmetic Progression) $a, a + d, a + 2d, \dots$

$$T_n = a + (n-1)d, \quad S_n = \frac{n}{2} [2a + (n-1)d] = \frac{n}{2} (a + l).$$

(ii) For the G.P. (Geometric Progression) a, ar, ar^2, \dots

$$T_n = ar^{n-1}, \quad S_n = \begin{cases} \frac{a(1-r^n)}{1-r}, & \text{when } r \neq 1 \\ na, & \text{when } r = 1 \end{cases}$$

$$S_{\infty} = \frac{a}{1-r} \quad \text{provided } |r| < 1 \quad \text{i.e., } -1 < r < 1$$

(iii) A sequence is said to be in H.P. (Harmonic Progression) if the reciprocals of its terms are in A.P.

$$\text{For the H.P. } \frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, \dots, T_n = \frac{1}{a+(n-1)d}$$

(iv) For two numbers a and b ,

$$\text{A.M.} = \frac{a+b}{2}, \quad \text{G.M.} = \sqrt{ab}, \quad \text{H.M.} = \frac{2ab}{a+b}$$

(v) For natural numbers $1, 2, 3, \dots, n$

$$\Sigma n = \frac{n(n+1)}{2}, \quad \Sigma n^2 = \frac{n(n+1)(2n+1)}{6}, \quad \Sigma n^3 = \left[\frac{n(n+1)}{2} \right]^2$$

6. Permutations and Combinations

$${}^n P_r = \frac{n!}{(n-r)!}, \quad {}^n C_r = \frac{{}^n P_r}{r!} = \frac{n!}{r!(n-r)!}$$

$${}^n P_n = n!, \quad {}^n C_r = {}^n C_{n-r}, \quad {}^n C_n = {}^n C_0 = 1$$

7. Binomial Theorem

(i) When n is a positive integer

$$(a+b)^n = {}^n C_0 a^n + {}^n C_1 a^{n-1} b + {}^n C_2 a^{n-2} b^2 + \dots + {}^n C_r a^{n-r} b^r + \dots + {}^n C_n b^n$$

$$(1+x)^n = 1 + {}^n C_1 x + {}^n C_2 x^2 + \dots + {}^n C_n x^n$$

(ii) When n is a negative integer or a fraction

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!} x^2 + \frac{n(n-1)(n-2)}{3!} x^3 + \dots \infty$$

provided $|x| < 1$.

8. Logarithms

(i) Natural logarithm of a positive real number x is denoted by $\log_e x$ or simply $\log x$ or $\ln x$. It is the inverse of e^x .

Common logarithm of a positive real number x is denoted by $\log_{10} x$.

Relation: (i) $\log_{10} x = 0.4343 \log_e x$

$$(ii) \log_a 1 = 0, \quad \log_a a = 1, \quad \log_a 0 = -\infty \quad (a > 1)$$

$$(iii) \log(mn) = \log m + \log n, \quad \log\left(\frac{m}{n}\right) = \log m - \log n$$

$$\log(m^n) = n \log m, \quad \log_n m \times \log_m n = 1$$

9. Matrices and Determinants

(i) Two matrices $A = [a_{ij}]$ and $B = [b_{ij}]$ are equal if they have same order and corresponding elements are equal. i.e., $a_{ij} = b_{ij}$ for all i and j .

(ii) If $A = [a_{ij}]$ and $B = [b_{ij}]$ are two matrices of the **same order**, then $A + B$ is defined and $A + B = [a_{ij} + b_{ij}]$, i.e., add corresponding elements.

(iii) If $A = [a_{ij}]$ is a matrix and k is a scalar, then kA is another matrix obtained by multiplying each element of A by the scalar k . Thus, $kA = [ka_{ij}]$.

- (iv) The product AB of two matrices A and B is defined if the number of columns in A is equal to the number of rows in B . If A is an $m \times n$ matrix and B is an $n \times p$ matrix then AB is a matrix of order $m \times p$. The $(i, j)^{\text{th}}$ element of AB is obtained by multiplying the corresponding elements of i^{th} row of A and j^{th} column of B and adding all these products.

Matrix multiplication is not commutative, i.e., $AB \neq BA$ in general.

If the product of two matrices is a zero matrix, it is not necessary that one of the matrices is a zero matrix.

$$(AB)C = A(BC), \quad A(B + C) = AB + AC$$

whenever both sides of equality are defined.

- (v) For every square matrix A , there exists an identity matrix I of same order such that $AI = IA = A$.
- (vi) The matrix obtained by interchanging the rows and columns of a matrix A is called the transpose of A . Transpose of A is denoted by A' or A^T .

If $A = [a_{ij}]_{m \times n}$, then $A' = [a_{ji}]_{n \times m}$

$$(A')' = A, \quad (kA)' = kA', \quad (A + B)' = A' + B',$$

$$(AB)' = B'A'.$$

- (vii) A square matrix A is called symmetric if $A' = A$ and skew symmetric if $A' = -A$. All the diagonal elements of a skew symmetric matrix are zero.
- (viii) A square matrix A is said to be invertible if there exists a square matrix B of same order as A , such that $AB = BA = I$. The matrix B is called the inverse of A and it is denoted by A^{-1} .

Thus, $AA^{-1} = A^{-1}A = I$

Also, $(A^{-1})^{-1} = A, \quad (AB)^{-1} = B^{-1}A^{-1}.$

- (ix) A determinant is a function which associates each square matrix with a unique number (real or complex). The determinant of a square matrix A is denoted by $|A|$ or $\det. A$ or Δ .

$$(x) \quad \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb.$$

$$\begin{aligned} \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} &= a_1 \begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix} \\ &= a_1(b_2c_3 - b_3c_2) - a_2(b_1c_3 - b_3c_1) + a_3(b_1c_2 - b_2c_1) \\ \begin{vmatrix} a_1 & a_2 & a_3 \\ 0 & b_2 & b_3 \\ 0 & 0 & c_3 \end{vmatrix} &= \begin{vmatrix} a_1 & 0 & 0 \\ b_1 & b_2 & 0 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1 b_2 c_3 = \text{product of diagonal elements.} \end{aligned}$$

- (xi) If A is a square matrix of order n , then $|kA| = k^n |A|$.
- (xii) The value of a determinant remains unchanged if its rows and columns are interchanged, i.e., $|A'| = |A|$.
- (xiii) If any two-rows (or columns) of a determinant are interchanged, then sign of determinant changes.

- (xiv) If any two rows (or columns) of a determinant are identical or proportional, then value of determinant is zero.
- (xv) If each element of a row (or column) of a determinant is multiplied by a constant k , then its value gets multiplied by k . By this property, we can take out any common factor from any one row (or column) of a given determinant.
- (xvi) If each element of a row (or column) of a determinant is the sum of m terms, then the determinant can be expressed as the sum of m determinants.
- (xvii) If, to each element of a row (or column) of a determinant, be added equi-multiples of the corresponding elements of some other row (or column), then value of determinant remains the same.

(xviii) If $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ then $\text{adj. } A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}'$, where A_{ij} is co-factor of a_{ij} .

(xix) For a square matrix of order n ,

$$A (\text{adj. } A) = (\text{adj. } A) A = |A| I$$

where I is the identity matrix of order n .

(xx) If A is a non-singular matrix of order n , then $|\text{adj. } A| = |A|^{n-1}$

(xxi) A square matrix A is invertible if and only if A is a non-singular matrix.

$$A^{-1} = \frac{1}{|A|} \text{adj. } A$$

10. Trigonometry

(i)

x°	0°	30°	45°	60°	90°
$\sin x$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan x$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

(ii) The figure shows a unit circle with centre at origin. If $\angle XOP = x$, then

$$P = (\cos x, \sin x).$$

$$\begin{aligned} \therefore \quad \cos 0 &= 1, & \sin 0 &= 0 \\ \cos \frac{\pi}{2} &= 0, & \sin \frac{\pi}{2} &= 1 \\ \cos \pi &= -1, & \sin \pi &= 0 \\ \cos \frac{3\pi}{2} &= 0, & \sin \frac{3\pi}{2} &= -1 \\ \cos 2\pi &= 1, & \sin 2\pi &= 0 \end{aligned}$$

(iii) Any t -ratio of $(n \cdot 90^\circ \pm x) = \pm$ same t -ratio of x , when n is even $= \pm$ co-ratio of x , when n is odd.

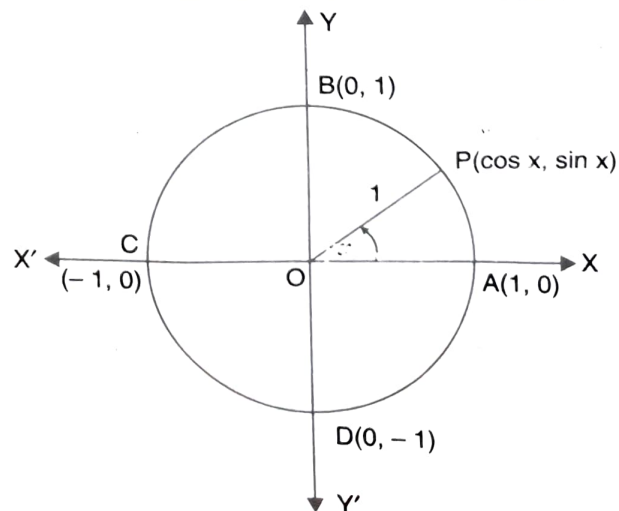


Fig. 1

where co-ratio of x is obtained by dropping co if present and adding co if absent. Thus, $\sin \rightleftharpoons \cos$, $\tan \rightleftharpoons \cot$, $\sec \rightleftharpoons \operatorname{cosec}$.

The sign \pm or $-$ is decided from the quadrant in which $n. 90^\circ \pm x$ lies.

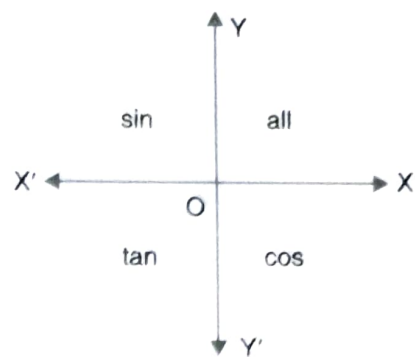


Fig. 2

(iv) Signs of t -ratios in different quadrants (Fig. 2)

$$(v) \quad \cos^2 x + \sin^2 x = 1, \sec^2 x - \tan^2 x = 1, \\ \operatorname{cosec}^2 x - \cot^2 x = 1$$

$$(vi) \quad \sin(x \pm y) = \sin x \cos y \pm \cos x \sin y \\ \cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

$$(vii) \quad \sin 2x = 2 \sin x \cos x = \frac{2 \tan x}{1 + \tan^2 x}$$

$$(viii) \quad \cos 2x = \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x = \frac{1 - \tan^2 x}{1 + \tan^2 x}$$

$$(ix) \quad \sin 3x = 3 \sin x - 4 \sin^3 x, \cos 3x = 4 \cos^3 x - 3 \cos x$$

$$\tan 3x = \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x}$$

$$(x) \quad \sin x \cos y = \frac{1}{2} [\sin(x+y) + \sin(x-y)]$$

$$\cos x \sin y = \frac{1}{2} [\sin(x+y) - \sin(x-y)]$$

$$\cos x \cos y = \frac{1}{2} [\cos(x+y) + \cos(x-y)]$$

$$\sin x \sin y = \frac{1}{2} [\cos(x-y) - \cos(x+y)]$$

$$(xi) \quad \sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$$

$$\sin x - \sin y = 2 \cos \frac{x+y}{2} \sin \frac{x-y}{2}$$

$$\cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}$$

$$\cos x - \cos y = -2 \sin \frac{x+y}{2} \sin \frac{x-y}{2} = 2 \sin \frac{x+y}{2} \sin \frac{y-x}{2}$$

$$(xii) \quad a \sin x + b \cos x = r \sin(x+\theta), a \cos x + b \sin x = r \cos(x-\theta), \text{ where } a = r \cos \theta, b = r \sin \theta$$

$$\text{so that } r = \sqrt{a^2 + b^2}, \theta = \tan^{-1} \left(\frac{b}{a} \right).$$

(xiii) In any $\triangle ABC$:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

(sine formula)

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} \quad (\text{cosine formula})$$

$$a = b \cos C + c \cos B \quad (\text{projection formula})$$

Area of $\triangle ABC = \frac{1}{2} bc \sin A = \frac{1}{2} \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{1}{2}(a+b+c)$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R, \text{ where } R \text{ is the circum-radius of } \triangle ABC.$$

$$R = \frac{abc}{4\Delta}$$

$r = \frac{\Delta}{s}$, where r is the radius of inscribed circle of $\triangle ABC$.

11. **De Moivre's Theorem:** $(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta$

Euler's Theorem: $\cos \theta + i \sin \theta = e^{i\theta}$.

12. Hyperbolic Functions

$$\sinh x = \frac{e^x - e^{-x}}{2}, \cosh x = \frac{e^x + e^{-x}}{2}, \tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\operatorname{cosech} x = \frac{1}{\sinh x}, \operatorname{sech} x = \frac{1}{\cosh x}, \coth x = \frac{1}{\tanh x}$$

$$\tanh x = \frac{\sinh x}{\cosh x}, \coth x = \frac{\cosh x}{\sinh x}, \cosh^2 x - \sinh^2 x = 1$$

$$\sin ix = i \sinh x, \cos ix = \cosh x, \tan ix = i \tanh x$$

$$\sinh^{-1} x = \log \left(x + \sqrt{x^2 + 1} \right), \cosh^{-1} x = \log \left(x + \sqrt{x^2 - 1} \right),$$

$$\tanh^{-1} x = \frac{1}{2} \log \frac{1+x}{1-x}.$$

13. Series

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \infty$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots \infty, \quad \sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots \infty$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots \infty, \quad \cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots \infty$$

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots \infty, \quad \log(1-x) = - \left(x + \frac{x^2}{2} + \frac{x^3}{3} + \dots \infty \right)$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \dots \infty$$

$$\tanh^{-1} x = x + \frac{x^3}{3} + \frac{x^5}{5} + \dots \infty$$

(Gregory series)

14. Calculus

(a) Standard Limits

$$(i) \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1}, n \in \mathbb{Q}$$

$$(ii) \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$(iii) \lim_{x \rightarrow 0} \frac{\sin^{-1} x}{x} = 1$$

$$(iv) \lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$$

$$(v) \lim_{x \rightarrow 0} \frac{\tan^{-1} x}{x} = 1$$

$$(vi) \lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}} = e$$

$$(vii) \lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \log_e a$$

$$(viii) \lim_{x \rightarrow \infty} x^{1/x} = 1$$

(ix) If $f(a) = g(a) = 0$, then

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

(L' Hospital's Rule)

(Differentiate the numerator and denominator separately)

(b) Differentiation

$$(i) \frac{d}{dx} (uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

(Product Rule)

$$(ii) \frac{d}{dx} \left(\frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

(Quotient Rule)

$$(iii) \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

(Chain Rule)

$$(iv) \text{ If } x = f(t), y = g(t), \text{ then } \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$(v) \frac{d}{dx} (c) = 0$$

$$(vi) \frac{d}{dx} (x^n) = nx^{n-1}$$

$$(vii) \frac{d}{dx} (e^x) = e^x$$

$$(viii) \frac{d}{dx} (a^x) = a^x \log_e a$$

$$(ix) \frac{d}{dx} (\log_e x) = \frac{1}{x}$$

$$(x) \frac{d}{dx} (\log_a x) = \frac{1}{x \log a}$$

$$(xi) \frac{d}{dx} (\sin x) = \cos x$$

$$(xii) \frac{d}{dx} (\cos x) = -\sin x$$

$$(xiii) \frac{d}{dx} (\tan x) = \sec^2 x$$

$$(xiv) \frac{d}{dx} (\cot x) = -\operatorname{cosec}^2 x$$

$$(xv) \frac{d}{dx} (\sec x) = \sec x \tan x$$

$$(xvi) \frac{d}{dx} (\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$$

$$(xvii) \frac{d}{dx} (\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

$$(xviii) \frac{d}{dx} (\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$$

$$(xix) \frac{d}{dx} (\tan^{-1} x) = \frac{1}{1+x^2}$$

$$(xxi) \frac{d}{dx} (\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$$

$$(xxiii) \frac{d}{dx} (\sinh x) = \cosh x$$

$$(xx) \frac{d}{dx} (\cot^{-1} x) = \frac{-1}{1+x^2}$$

$$(xxii) \frac{d}{dx} (\operatorname{cosec}^{-1} x) = \frac{-1}{x\sqrt{x^2-1}}$$

$$(xxiv) \frac{d}{dx} (\cosh x) = \sinh x.$$

(c) **Integration**

$$(i) \int x^n dx = \frac{x^{n+1}}{n+1} \quad (n \neq -1)$$

$$(iii) \int e^x dx = e^x$$

$$(v) \int \sin x dx = -\cos x$$

$$(vii) \int \tan x dx = -\log \cos x$$

$$(ix) \int \sec x \tan x dx = \sec x$$

$$(xi) \int \sec^2 x dx = \tan x$$

$$(xiii) \int \sec x dx = \log (\sec x + \tan x)$$

$$(xiv) \int \operatorname{cosec} x dx = \log (\operatorname{cosec} x - \cot x) = \log \tan \frac{x}{2}$$

$$(xv) \int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a}$$

$$(xvii) \int \frac{dx}{a^2-x^2} = \frac{1}{2a} \log \frac{a+x}{a-x}$$

$$(xix) \int \frac{dx}{x^2-a^2} = \frac{1}{2a} \log \frac{x-a}{x+a}$$

$$(xxi) \int \sqrt{a^2-x^2} dx = \frac{x\sqrt{a^2-x^2}}{2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a}$$

$$(xxii) \int \sqrt{a^2+x^2} dx = \frac{x\sqrt{a^2+x^2}}{2} + \frac{a^2}{2} \sinh^{-1} \frac{x}{a}$$

$$(xxiii) \int \sqrt{x^2-a^2} dx = \frac{x\sqrt{x^2-a^2}}{2} - \frac{a^2}{2} \cosh^{-1} \frac{x}{a}$$

$$(ii) \int \frac{1}{x} dx = \log_e x$$

$$(iv) \int a^x dx = \frac{a^x}{\log_e a}$$

$$(vi) \int \cos x dx = \sin x$$

$$(viii) \int \cot x dx = \log \sin x$$

$$(x) \int \operatorname{cosec} x \cot x dx = -\operatorname{cosec} x$$

$$(xii) \int \operatorname{cosec}^2 x dx = -\cot x$$

$$(xvi) \int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1} \frac{x}{a}$$

$$(xviii) \int \frac{dx}{\sqrt{a^2+x^2}} = \sinh^{-1} \frac{x}{a}$$

$$(xx) \int \frac{dx}{\sqrt{x^2-a^2}} = \cosh^{-1} \frac{x}{a}$$

$$(xxiv) \int e^{ax} \sin bx \, dx = \frac{e^{ax}}{a^2 + b^2} (a \sin bx - b \cos bx) = \frac{e^{ax}}{\sqrt{a^2 + b^2}} \sin \left(bx - \tan^{-1} \frac{b}{a} \right)$$

$$(xxv) \int e^{ax} \cos bx \, dx = \frac{e^{ax}}{a^2 + b^2} (a \cos bx + b \sin bx) = \frac{e^{ax}}{\sqrt{a^2 + b^2}} \cos \left(bx - \tan^{-1} \frac{b}{a} \right)$$

$$(xxvi) \int \sinh x \, dx = \cosh x$$

$$(xxvii) \int \cosh x \, dx = \sinh x$$

$$(xxviii) \int \tanh x \, dx = \log \cosh x$$

$$(xxix) \int \coth x \, dx = \log \sinh x$$

$$(xxx) \int \operatorname{sech}^2 x \, dx = \tanh x$$

$$(xxxi) \int \operatorname{cosech}^2 x \, dx = -\coth x$$

$$(xxxii) \int_a^b f(x) \, dx = -\int_b^a f(x) \, dx$$

$$(xxxiii) \int_a^b f(x) \, dx = \int_a^c f(x) \, dx + \int_c^b f(x) \, dx, \quad a < c < b$$

$$(xxxiv) \int_0^a f(x) \, dx = \int_0^a f(a-x) \, dx$$

$$(xxxv) \int_a^b f(x) \, dx = \int_a^b f(a+b-x) \, dx$$

$$(xxxvi) \int_{-a}^a f(x) \, dx = \begin{cases} 0, & \text{if } f \text{ is an odd function} \\ 2 \int_0^a f(x) \, dx, & \text{if } f \text{ is an even function} \end{cases}$$

$$(xxxvii) \int_0^{2a} f(x) \, dx = \begin{cases} 0, & \text{if } f(2a-x) = -f(x) \\ 2 \int_0^a f(x) \, dx, & \text{if } f(2a-x) = f(x) \end{cases}$$

$$(xxxviii) \int_0^{\pi/2} \sin^n x \, dx = \int_0^{\pi/2} \cos^n x \, dx \quad (n > 1)$$

$$= \frac{(n-1)(n-3)(n-5)\dots}{n(n-2)(n-4)\dots} \times \left(\frac{\pi}{2}, \text{ only if } n \text{ is even} \right)$$

$$(xxxix) \int_0^{\pi/2} \sin^m x \cos^n x \, dx$$

$$= \frac{(m-1)(m-3)\dots \times (n-1)(n-3)\dots}{(m+n)(m+n-2)(m+n-4)\dots} \times \left(\frac{\pi}{2}, \text{ only if both } m \text{ and } n \text{ are even} \right).$$