① $f(x) = \underbrace{|\cos x|}$ in $\underbrace{(-\pi, \pi)}$

$|\cos x|$ is an even function

$$|\cos x| = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

where $a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} |\cos x|\, dx$

$$= \frac{2}{\pi}\int_{0}^{\pi} |\cos x|\, dx$$

$$= \frac{2}{\pi}\left[\int_0^{\pi/2} \cos x\, dx + \int_{\pi/2}^{\pi} -\cos x\, dx\right]$$

$$\int_{-a}^{a} f(x)\, dx = 2\int_0^a f(x)\, dx$$

if $f(x)$ is even.

$$a_0 = \frac{4}{\pi}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} \underbrace{\underbrace{|\cos x|}_{\text{Even } f^n} \underbrace{\cos nx}_{\text{Even } f^n}}_{\text{Even } f^n} dx$$

$$= \frac{2}{\pi}\int_0^{\pi} |\cos x| \cos nx\, dx$$

$$= \frac{2}{\pi}\left[\int_0^{\pi/2} \cos x \cos nx\, dx + \int_{\pi/2}^{\pi} -\cos x \cos nx\, dx\right]$$

$$2\cos A\cos B = \cos(A+B)+\cos(A-B)$$

$$a_n = \frac{1}{\pi}\left[\int_0^{\pi/2} 2\cos nx\cos x\,dx - \int_{\pi/2}^{\pi} 2\cos nx\cos x\,dx\right]$$

$$= \frac{1}{\pi}\left[\int_0^{\pi/2}\left(\cos(n+1)x+\cos(n-1)x\right)dx - \int_{\pi/2}^{\pi}\left(\cos(n+1)x+\cos(n-1)x\right)dx\right]$$

$$= \frac{1}{\pi}\left[\left(\frac{\sin(n+1)x}{n+1}+\frac{\sin(n-1)x}{n-1}\right)_0^{\pi/2} - \left(\frac{\sin(n+1)x}{n+1}+\frac{\sin(n-1)x}{n-1}\right)_{\pi/2}^{\pi}\right]$$

$(n \neq 1)$

$$= \frac{1}{\pi}\left[\left(\frac{\sin(n+1)\frac{\pi}{2}}{n+1}+\frac{\sin(n-1)\frac{\pi}{2}}{n-1}\right)+\left(\frac{\sin(n+1)\frac{\pi}{2}}{n+1}+\frac{\sin(n-1)\frac{\pi}{2}}{n-1}\right)\right] \quad (n \neq 1)$$

$$a_n = \frac{2}{\pi}\left[\frac{\sin(n+1)\frac{\pi}{2}}{n+1}+\frac{\sin(n-1)\frac{\pi}{2}}{n-1}\right]$$

$(n \neq 1)$

$$a_1 = \frac{2}{\pi}\int_0^{\pi}|\cos x|\cos x\,dx$$

$$= \frac{2}{\pi}\left[\int_0^{\pi/2}\cos x.\cos x\,dx + \int_{\pi/2}^{\pi}-\cos x.\cos x\,dx\right]$$

$$a_1 = \frac{2}{\pi}\left(\int_0^{\pi/2}\cos^2 x\,dx - \int_{\pi/2}^{\pi}\cos^2 x\,dx\right)$$

$a_1 = 0$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} \underbrace{|\cos x|}_{\text{Even } f^n} \underbrace{\sin nx}_{\text{odd } f^n}\, dx.$$

$\underbrace{\qquad\qquad}_{\text{odd } f^n}$

$\boxed{b_n = 0}$

$a_0 = \frac{4}{\pi}$, $a_n = \frac{2}{\pi}\left[\frac{\sin(n+1)\frac{\pi}{2}}{n+1} + \frac{\sin(n-1)\frac{\pi}{2}}{n-1}\right]$

$a_1 = 0$, $b_n = 0$

$\sin\frac{\pi}{2} = 1$ | $\sin\frac{3\pi}{2} = -1$ $\quad \sin\left(\pi + \frac{\pi}{2}\right)$

$= -\sin\frac{\pi}{2} = -1$

$\sin 5\frac{\pi}{2} = 1$, $\sin 7\frac{\pi}{2} = -1$ ......

$$\int_{-a}^{a} f(x)\,dx = 0 \text{ if } f(x) \text{ is odd } f^n.$$

Fourier series expansion :-

$$|\cos x| = \frac{2}{\pi} + a_1 \cos x + \sum_{n=2}^{\infty} a_n \cos nx$$

$$|\cos x| = \frac{2}{\pi} + \frac{2}{\pi}\sum_{n=2}^{\infty}\left[\frac{\sin(n+1)\frac{\pi}{2}}{n+1} + \frac{\sin(n-1)\frac{\pi}{2}}{n-1}\right] \cdot \cos nx$$

$$= \frac{2}{\pi} + \frac{2}{\pi}\left[\left(\frac{\sin 3\frac{\pi}{2}}{3} + \frac{\sin\frac{\pi}{2}}{1}\right)\cos 2x + \left(\frac{\sin 5\frac{\pi}{2}}{5} + \frac{\sin 3\frac{\pi}{2}}{3}\right)\cos 4x\right.$$

$$\left. + \left(\frac{\sin 7\frac{\pi}{2}}{7} + \frac{\sin 5\frac{\pi}{2}}{5}\right)\cos 6x \;\ldots\ldots\right]$$

Change of interval :-

$(-2, 2)$

$(0, 2)$

$[0, 3)$

$(-1, 1)$

$f(x)$, in $(\alpha, \alpha + 2l)$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{l} + \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l}$$

where $a_0 = \frac{1}{l} \int_{\alpha}^{\alpha+2l} f(x)\, dx$

$$a_n = \frac{1}{l} \int_{\alpha}^{\alpha+2l} f(x) \cos \frac{n\pi x}{l}\, dx$$

$$b_n = \frac{1}{l} \int_{\alpha}^{\alpha+2l} f(x) \sin \frac{n\pi x}{l}\, dx$$

$f(x) \to (\alpha, \alpha + 2\pi)$

$(-\pi, \pi)$ or $(0, 2\pi)$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

$$a_0 = \frac{1}{\pi} \int_{\alpha}^{\alpha+2\pi} f(x)\, dx$$

$$a_n = \frac{1}{\pi} \int_{\alpha}^{\alpha+2\pi} f(x) \cos nx\, dx$$

$$b_n = \frac{1}{\pi} \int_{\alpha}^{\alpha+2\pi} f(x) \sin nx\, dx$$

$$f(x) = x - x^2 \,, \quad -1 \le x \le 1$$

$$\alpha \le x \le \alpha + 2l$$

$$(0,2)$$

$$\alpha = -1$$

$$\alpha + 2l = 1$$

then $l = 1$

$$\alpha = 0$$

$$\alpha + 2l = 2$$

$$2l = 2$$

$$l = 1$$

$$(0,3)$$

then $l = 3/2$

$$x - x^2 = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}$$

here $l = 1$

$$x - x^2 = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\pi x + \sum_{n=1}^{\infty} b_n \sin n\pi x$$

$$b_n = \int_{-1}^{1} (x - x^2) \sin n\pi x \, dx$$

$$a_0 = \frac{1}{1}\int_{-1}^{1} (x - x^2)\, dx = \int_{-1}^{1} (x - x^2)\, dx =$$

$$a_n = \frac{1}{1}\int_{-1}^{1} (x - x^2) \cos n\pi x \, dx = \int_{-1}^{1} (x - x^2) \cos n\pi x$$