

1. Greek Letters Used

| | | | | | | | |
|------------|---------|-----------|------------|----------|------------|----------|------------|
| α | alpha | ϕ | phi | κ | kappa | τ | tau |
| β | beta | ψ | psi | μ | mu | χ | chi |
| γ | gamma | ξ | xi | ν | nu | ω | omega |
| δ | delta | η | eta | π | pi | Δ | cap. delta |
| ϵ | epsilon | ζ | zeta | ρ | rho | | |
| i | iota | λ | lambda | σ | sigma | | |
| θ | theta | Γ | cap. gamma | Σ | cap. sigma | | |

2. Some Notations

| | | | |
|---------------|--------------------|-------------------|------------------------|
| \in | belongs to | \Leftrightarrow | implies and implied by |
| \notin | does not belong to | iff | if and only if |
| \Rightarrow | implies | \cup | union |
| | | \cap | intersection |

3. Useful Data

| | | | |
|---------------------|--------------------------|--------------------------------------|------------------------|
| $\sqrt{2} = 1.4142$ | $\frac{1}{\pi} = 0.3183$ | $1 \text{ rad.} = 57^\circ 17' 45''$ | $\log_{10} e = 0.4343$ |
| $\sqrt{3} = 1.732$ | $e = 2.7183$ | $1^\circ = 0.0174 \text{ rad.}$ | $\log_e 2 = 0.6931$ |
| $\pi = 3.1416$ | $\frac{1}{e} = 0.3679$ | $\log_e 10 = 2.3026$ | $\log_e 3 = 1.0986$ |

4. Quadratic Equation

Roots of quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$

are $\frac{-b \pm \sqrt{D}}{2a}$, where $D = b^2 - 4ac$ is called discriminant

Sum of roots $= -\frac{b}{a}$, Product of roots $= \frac{c}{a}$

If $D > 0$, roots are real and distinct.

If $D = 0$, roots are equal.

If $D < 0$, roots are imaginary.

5. Progressions

(i) For the A.P. (Arithmetic Progression) $a, a+d, a+2d, \dots$

$$T_n = a + (n-1)d, \quad S_n = \frac{n}{2} [2a + (n-1)d] = \frac{n}{2} (a + l).$$

(ii) For the G.P. (Geometric Progression) a, ar, ar^2, \dots

$$T_n = ar^{n-1}, \quad S_n = \begin{cases} \frac{a(1-r^n)}{1-r}, & \text{when } r \neq 1 \\ na, & \text{when } r = 1 \end{cases}$$

$$S_{\infty} = \frac{a}{1-r} \quad \text{provided } |r| < 1 \quad i.e., -1 < r < 1$$

(iii) A sequence is said to be in H.P. (Harmonic Progression) if the reciprocals of its terms are in A.P.

For the H.P. $\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, \dots, T_n = \frac{1}{a+(n-1)d}$

(iv) For two numbers a and b ,

$$\text{A.M.} = \frac{a+b}{2}, \quad \text{G.M.} = \sqrt{ab}, \quad \text{H.M.} = \frac{2ab}{a+b}$$

(v) For natural numbers $1, 2, 3, \dots, n$

$$\Sigma n = \frac{n(n+1)}{2}, \quad \Sigma n^2 = \frac{n(n+1)(2n+1)}{6}, \quad \Sigma n^3 = \left[\frac{n(n+1)}{2} \right]^2$$

6. Permutations and Combinations

$${ }^n P_r = \frac{n!}{(n-r)!}, \quad { }^n C_r = \frac{{ }^n P_r}{r!} = \frac{n!}{r!(n-r)!}$$

$${ }^n P_n = n!, \quad { }^n C_r = { }^n C_{n-r}, \quad { }^n C_n = { }^n C_0 = 1$$

7. Binomial Theorem

(i) When n is a positive integer

$$(a+b)^n = { }^n C_0 a^n + { }^n C_1 a^{n-1} b + { }^n C_2 a^{n-2} b^2 + \dots + { }^n C_r a^{n-r} b^r + \dots + { }^n C_n b^n$$

$$(1+x)^n = 1 + { }^n C_1 x + { }^n C_2 x^2 + \dots + { }^n C_n x^n$$

(ii) When n is a negative integer or a fraction

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!} x^2 + \frac{n(n-1)(n-2)}{3!} x^3 + \dots \infty$$

provided $|x| < 1$.

8. Logarithms

(i) Natural logarithm of a positive real number x is denoted by $\log_e x$ or simply $\log x$ or $\ln x$. It is the inverse of e^x .

Common logarithm of a positive real number x is denoted by $\log_{10} x$.

Relation: (i) $\log_{10} x = 0.4343 \log_e x$

$$(ii) \log_a 1 = 0, \quad \log_a a = 1, \quad \log_a 0 = -\infty \quad (a > 1)$$

$$(iii) \log(mn) = \log m + \log n, \quad \log\left(\frac{m}{n}\right) = \log m - \log n$$

$$\log(m^n) = n \log m, \quad \log_n m \times \log_m n = 1$$

9. Matrices and Determinants

(i) Two matrices $A = [a_{ij}]$ and $B = [b_{ij}]$ are equal if they have same order and corresponding elements are equal. i.e., $a_{ij} = b_{ij}$ for all i and j .

(ii) If $A = [a_{ij}]$ and $B = [b_{ij}]$ are two matrices of the **same order**, then $A + B$ is defined and $A + B = [a_{ij} + b_{ij}]$, i.e., add corresponding elements.

(iii) If $A = [a_{ij}]$ is a matrix and k is a scalar, then kA is another matrix obtained by multiplying each element of A by the scalar k . Thus, $kA = [ka_{ij}]$.

- (iv) The product AB of two matrices A and B is defined if the number of columns in A is equal to the number of rows in B. If A is an $m \times n$ matrix and B is an $n \times p$ matrix then AB is a matrix of order $m \times p$. The $(i, j)^{\text{th}}$ element of AB is obtained by multiplying the corresponding elements of i^{th} row of A and j^{th} column of B and adding all these products.

Matrix multiplication is not commutative, i.e., $AB \neq BA$ in general.

If the product of two matrices is a zero matrix, it is not necessary that one of the matrices is a zero matrix.

$$(AB)C = A(BC), \quad A(B + C) = AB + AC$$

whenever both sides of equality are defined.

- (v) For every square matrix A, there exists an identity matrix I of same order such that $AI = IA = A$.

- (vi) The matrix obtained by interchanging the rows and columns of a matrix A is called the transpose of A. Transpose of A is denoted by A' or A^T .

If $A = [a_{ij}]_{m \times n}$, then $A' = [a_{ji}]_{n \times m}$

$$(A')' = A, \quad (kA)' = kA', \quad (A + B)' = A' + B',$$

$$(AB)' = B'A'.$$

- (vii) A square matrix A is called symmetric if $A' = A$ and skew symmetric if $A' = -A$. All the diagonal elements of a skew symmetric matrix are zero.

- (viii) A square matrix A is said to be invertible if there exists a square matrix B of same order as A, such that $AB = BA = I$. The matrix B is called the inverse of A and it is denoted by A^{-1} .

$$\text{Thus, } AA^{-1} = A^{-1}A = I$$

$$\text{Also, } (A^{-1})^{-1} = A, \quad (AB)^{-1} = B^{-1}A^{-1}.$$

- (ix) A determinant is a function which associates each square matrix with a unique number (real or complex). The determinant of a square matrix A is denoted by $| A |$ or $\det. A$ or Δ .

$$(x) \quad \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb.$$

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}$$

$$= a_1(b_2c_3 - b_3c_2) - a_2(b_1c_3 - b_3c_1) + a_3(b_1c_2 - b_2c_1)$$

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ 0 & b_2 & b_3 \\ 0 & 0 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & 0 & 0 \\ b_1 & b_2 & 0 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1 b_2 c_3 = \text{product of diagonal elements.}$$

- (xi) If A is a square matrix of order n , then $| kA | = k^n | A |$.

- (xii) The value of a determinant remains unchanged if its rows and columns are interchanged, i.e., $| A' | = | A |$.

- (xiii) If any two-rows (or columns) of a determinant are interchanged, then sign of determinant changes.

- (xiv) If any two rows (or columns) of a determinant are identical or proportional, then value of determinant is zero.
- (xv) If each element of a row (or column) of a determinant is multiplied by a constant k , then its value gets multiplied by k . By this property, we can take out any common factor from any one row (or column) of a given determinant.
- (xvi) If each element of a row (or column) of a determinant is the sum of m terms, then the determinant can be expressed as the sum of m determinants.
- (xvii) If, to each element of a row (or column) of a determinant, be added equi-multiples of the corresponding elements of some other row (or column), then value of determinant remains the same.

(xviii) If $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ then $\text{adj. } A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$, where A_{ij} is co-factor of a_{ij} .

(xix) For a square matrix of order n ,

$$A(\text{adj. } A) = (\text{adj. } A)A = |A|I$$

where I is the identity matrix of order n .

(xx) If A is a non-singular matrix of order n , then $|\text{adj. } A| = |A|^{n-1}$

(xxi) A square matrix A is invertible if and only if A is a non-singular matrix.

$$A^{-1} = \frac{1}{|A|} \text{adj. } A$$

10. Trigonometry

(i)

| x° | 0° | 30° | 45° | 60° | 90° |
|-----------|-----------|----------------------|----------------------|----------------------|------------|
| $\sin x$ | 0 | $\frac{1}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{\sqrt{3}}{2}$ | 1 |
| $\cos x$ | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{2}$ | 0 |
| $\tan x$ | 0 | $\frac{1}{\sqrt{3}}$ | 1 | $\sqrt{3}$ | ∞ |

(ii) The figure shows a unit circle with centre at origin. If $\angle XOP = x$, then

$P = (\cos x, \sin x)$.

$$\therefore \cos 0 = 1,$$

$$\sin 0 = 0$$

$$\cos \frac{\pi}{2} = 0,$$

$$\sin \frac{\pi}{2} = 1$$

$$\cos \pi = -1,$$

$$\sin \pi = 0$$

$$\cos \frac{3\pi}{2} = 0,$$

$$\sin \frac{3\pi}{2} = -1$$

$$\cos 2\pi = 1,$$

$$\sin 2\pi = 0$$

(iii) Any t -ratio of $(n \cdot 90^\circ \pm x) = \pm$ same t -ratio of x , when n is even $= \pm$ co-ratio of x , when n is odd.

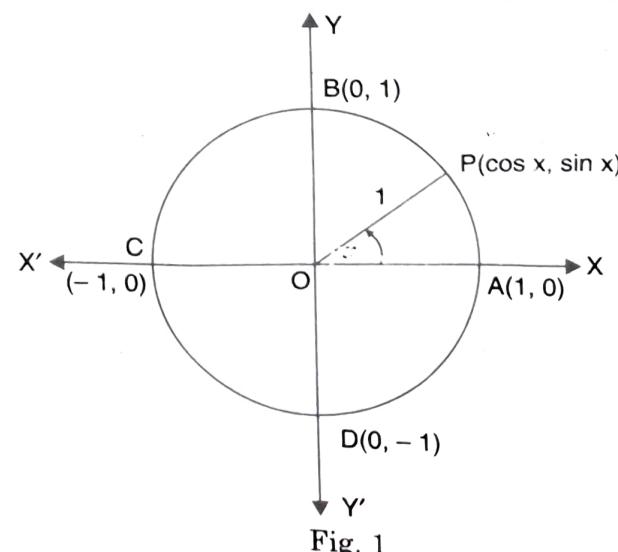


Fig. 1

where co-ratio of x is obtained by dropping co if present and adding co if absent. Thus, $\sin \rightleftharpoons \cos$, $\tan \rightleftharpoons \cot$, $\sec \rightleftharpoons \cosec$.

The sign \pm or $-$ is decided from the quadrant in which $n, 90^\circ \pm x$ lies.

(iv) Signs of t -ratios in different quadrants (Fig. 2)

$$(v) \cos^2 x + \sin^2 x = 1, \sec^2 x - \tan^2 x = 1, \\ \cosec^2 x - \cot^2 x = 1$$

$$(vi) \sin(x \pm y) = \sin x \cos y \pm \cos x \sin y \\ \cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

$$(vii) \sin 2x = 2 \sin x \cos x = \frac{2 \tan x}{1 + \tan^2 x}$$

$$(viii) \cos 2x = \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x = \frac{1 - \tan^2 x}{1 + \tan^2 x}$$

$$(ix) \sin 3x = 3 \sin x - 4 \sin^3 x, \cos 3x = 4 \cos^3 x - 3 \cos x$$

$$\tan 3x = \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x}$$

$$(x) \sin x \cos y = \frac{1}{2} [\sin(x+y) + \sin(x-y)]$$

$$\cos x \sin y = \frac{1}{2} [\sin(x+y) - \sin(x-y)]$$

$$\cos x \cos y = \frac{1}{2} [\cos(x+y) + \cos(x-y)]$$

$$\sin x \sin y = \frac{1}{2} [\cos(x-y) - \cos(x+y)]$$

$$(xi) \sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$$

$$\sin x - \sin y = 2 \cos \frac{x+y}{2} \sin \frac{x-y}{2}$$

$$\cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}$$

$$\cos x - \cos y = -2 \sin \frac{x+y}{2} \sin \frac{x-y}{2} = 2 \sin \frac{x+y}{2} \sin \frac{y-x}{2}$$

(xii) $a \sin x + b \cos x = r \sin(x+\theta), a \cos x + b \sin x = r \cos(x-\theta)$, where $a = r \cos \theta, b = r \sin \theta$

$$\text{so that } r = \sqrt{a^2 + b^2}, \theta = \tan^{-1} \left(\frac{b}{a} \right).$$

(xiii) In any ΔABC :

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

(sine formula)

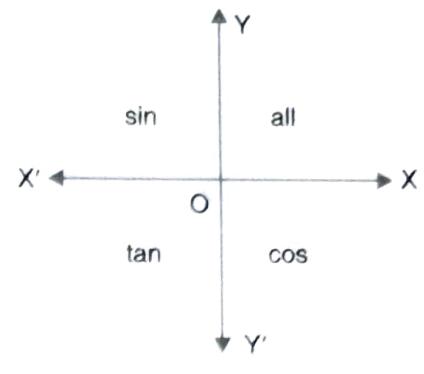


Fig. 2

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} \quad (\text{cosine formula})$$

$$a = b \cos C + c \cos B \quad (\text{projection formula})$$

$$\text{Area of } \Delta ABC = \frac{1}{2} bc \sin A = \sqrt{s(s-a)(s-b)(s-c)}, \text{ where } s = \frac{1}{2}(a+b+c)$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R, \text{ where } R \text{ is the circum-radius of } \Delta ABC.$$

$$R = \frac{abc}{4\Delta}$$

$$r = \frac{\Delta}{s}, \text{ where } r \text{ is the radius of inscribed circle of } \Delta ABC.$$

11. De Moivre's Theorem: $(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta$

Euler's Theorem: $\cos \theta + i \sin \theta = e^{i\theta}$.

12. Hyperbolic Functions

$$\sinh x = \frac{e^x - e^{-x}}{2}, \cosh x = \frac{e^x + e^{-x}}{2}, \tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\operatorname{sech} x = \frac{1}{\sinh x}, \operatorname{sech} x = \frac{1}{\cosh x}, \coth x = \frac{1}{\tanh x}$$

$$\tanh x = \frac{\sinh x}{\cosh x}, \coth x = \frac{\cosh x}{\sinh x}, \cosh^2 x - \sinh^2 x = 1$$

$$\sin ix = i \sinh x, \cos ix = \cosh x, \tan ix = i \tanh x$$

$$\sinh^{-1} x = \log \left(x + \sqrt{x^2 + 1} \right), \cosh^{-1} x = \log \left(x + \sqrt{x^2 - 1} \right),$$

$$\tanh^{-1} x = \frac{1}{2} \log \frac{1+x}{1-x}.$$

13. Series

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \infty$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots \infty, \quad \sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots \infty$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots \infty, \quad \cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots \infty$$

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots \infty, \quad \log(1-x) = - \left(x + \frac{x^2}{2} + \frac{x^3}{3} + \dots \infty \right)$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \dots \infty$$

$$\tanh^{-1} x = x + \frac{x^3}{3} + \frac{x^5}{5} + \dots \infty$$

(Gregory series)

14. Calculus

(a) Standard Limits

(i) $\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1}, n \in \mathbb{Q}$

(ii) $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

(iii) $\lim_{x \rightarrow 0} \frac{\sin^{-1} x}{x} = 1$

(iv) $\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$

(v) $\lim_{x \rightarrow 0} \frac{\tan^{-1} x}{x} = 1$

(vi) $\lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}} = e$

(vii) $\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \log_e a$

(viii) $\lim_{x \rightarrow \infty} x^{1/x} = 1$

(ix) If $f(a) = g(a) = 0$, then

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

(L' Hospital's Rule)

(Differentiate the numerator and denominator separately)

(b) Differentiation

(i) $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$ (Product Rule)

(ii) $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$ (Quotient Rule)

(iii) $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ (Chain Rule)

(iv) If $x = f(t), y = g(t)$, then $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$

(v) $\frac{d}{dx}(c) = 0$

(vi) $\frac{d}{dx}(x^n) = nx^{n-1}$

(vii) $\frac{d}{dx}(e^x) = e^x$

(viii) $\frac{d}{dx}(a^x) = a^x \log_e a$

(ix) $\frac{d}{dx}(\log_e x) = \frac{1}{x}$

(x) $\frac{d}{dx}(\log_a x) = \frac{1}{x \log a}$

(xi) $\frac{d}{dx}(\sin x) = \cos x$

(xii) $\frac{d}{dx}(\cos x) = -\sin x$

(xiii) $\frac{d}{dx}(\tan x) = \sec^2 x$

(xiv) $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

(xv) $\frac{d}{dx}(\sec x) = \sec x \tan x$

(xvi) $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$

(xvii) $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

(xviii) $\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$

$$(xix) \frac{d}{dx} (\tan^{-1} x) = \frac{1}{1+x^2}$$

$$(xxi) \frac{d}{dx} (\sec^{-1} x) = \frac{1}{x\sqrt{x^2 - 1}}$$

$$(xxii) \frac{d}{dx} (\sinh x) = \cosh x$$

(c) **Integration**

$$(i) \int x^n dx = \frac{x^{n+1}}{n+1} \quad (n \neq -1)$$

$$(iii) \int e^x dx = e^x$$

$$(v) \int \sin x dx = -\cos x$$

$$(vii) \int \tan x dx = -\log \cos x$$

$$(ix) \int \sec x \tan x dx = \sec x$$

$$(xi) \int \sec^2 x dx = \tan x$$

$$(xiii) \int \sec x dx = \log(\sec x + \tan x)$$

$$(xiv) \int \operatorname{cosec} x dx = \log(\operatorname{cosec} x - \cot x) = \log \tan \frac{x}{2}$$

$$(xv) \int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a}$$

$$(xvii) \int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log \frac{a+x}{a-x}$$

$$(xix) \int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log \frac{x-a}{x+a}$$

$$(xxi) \int \sqrt{a^2 - x^2} dx = \frac{x\sqrt{a^2 - x^2}}{2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a}$$

$$(xxii) \int \sqrt{a^2 + x^2} dx = \frac{x\sqrt{a^2 + x^2}}{2} + \frac{a^2}{2} \sinh^{-1} \frac{x}{a}$$

$$(xxiii) \int \sqrt{x^2 - a^2} dx = \frac{x\sqrt{x^2 - a^2}}{2} - \frac{a^2}{2} \cosh^{-1} \frac{x}{a}$$

$$(xx) \frac{d}{dx} (\cot^{-1} x) = \frac{-1}{1+x^2}$$

$$(xxii) \frac{d}{dx} (\operatorname{cosec}^{-1} x) = \frac{-1}{x\sqrt{x^2 - 1}}$$

$$(xxiv) \frac{d}{dx} (\cosh x) = \sinh x.$$

$$(ii) \int \frac{1}{x} dx = \log_e x$$

$$(iv) \int a^x dx = \frac{a^x}{\log_e a}$$

$$(vi) \int \cos x dx = \sin x$$

$$(viii) \int \cot x dx = \log \sin x$$

$$(x) \int \operatorname{cosec} x \cot x dx = -\operatorname{cosec} x$$

$$(xi) \int \operatorname{cosec}^2 x dx = -\cot x$$

$$(xvi) \int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a}$$

$$(xviii) \int \frac{dx}{\sqrt{a^2 + x^2}} = \sinh^{-1} \frac{x}{a}$$

$$(xx) \int \frac{dx}{\sqrt{x^2 - a^2}} = \cosh^{-1} \frac{x}{a}$$

$$(xxiv) \int e^{ax} \sin bx \, dx = \frac{e^{ax}}{a^2 + b^2} (a \sin bx - b \cos bx) = \frac{e^{ax}}{\sqrt{a^2 + b^2}} \sin \left(bx - \tan^{-1} \frac{b}{a} \right)$$

$$(xxv) \int e^{ax} \cos bx \, dx = \frac{e^{ax}}{a^2 + b^2} (a \cos bx + b \sin bx) = \frac{e^{ax}}{\sqrt{a^2 + b^2}} \cos \left(bx - \tan^{-1} \frac{b}{a} \right)$$

$$(xxvi) \int \sinh x \, dx = \cosh x$$

$$(xxvii) \int \cosh x \, dx = \sinh x$$

$$(xxviii) \int \tanh x \, dx = \log \cosh x$$

$$(xxix) \int \coth x \, dx = \log \sinh x$$

$$(xxx) \int \operatorname{sech}^2 x \, dx = \tanh x$$

$$(xxxi) \int \operatorname{cosech}^2 x \, dx = -\coth x$$

$$(xxxii) \int_a^b f(x) \, dx = - \int_b^a f(x) \, dx$$

$$(xxxiii) \int_a^b f(x) \, dx = \int_a^c f(x) \, dx + \int_c^b f(x) \, dx, \quad a < c < b$$

$$(xxxiv) \int_0^a f(x) \, dx = \int_0^a f(a-x) \, dx$$

$$(xxxv) \int_a^b f(x) \, dx = \int_a^b f(a+b-x) \, dx$$

$$(xxxvi) \int_{-a}^a f(x) \, dx = \begin{cases} 0, & \text{if } f \text{ is an odd function} \\ 2 \int_0^a f(x) \, dx, & \text{if } f \text{ is an even function} \end{cases}$$

$$(xxxvii) \int_0^{2a} f(x) \, dx = \begin{cases} 0, & \text{if } f(2a-x) = -f(x) \\ 2 \int_0^a f(x) \, dx, & \text{if } f(2a-x) = f(x) \end{cases}$$

$$(xxxviii) \int_0^{\pi/2} \sin^n x \, dx = \int_0^{\pi/2} \cos^n x \, dx \quad (n > 1)$$

$$= \frac{(n-1)(n-3)(n-5)\dots}{n(n-2)(n-4)\dots} \times \left(\frac{\pi}{2}, \text{only if } n \text{ is even} \right)$$

$$(xxxix) \int_0^{\pi/2} \sin^m x \cos^n x \, dx$$

$$= \frac{(m-1)(m-3)\dots \times (n-1)(n-3)\dots}{(m+n)(m+n-2)(m+n-4)\dots} \times \left(\frac{\pi}{2}, \text{only if both } m \text{ and } n \text{ are even} \right).$$