

## Unit-2

### Quantum Theory →

#### Advancements

↓  
Atomic and Subatomic particles

Quantum mechanics deals with microscopic particles i.e  
atomic and subatomic particles.

classical M. ✓

$$F = ma$$
$$S = ut + \frac{1}{2}at^2$$

Certain phenomena can not be explained with the help of  
classical theory i.e by taking the concept of Continuity  
of Energy and momentum.

So to explain those phenomena we consider that energy and momentum have discrete values. That new theory is called Quantum theory.

Quantum theory developed in two stages.

In first stage we have Max Planck's hypothesis.

According to which any material will absorb or emit energy in the form of packets called quanta.

In second stage Heisenberg and Schrodinger gave an idea that energy and momentum will have discontinuous values.

# Wave-Particle Duality

Radiation  $\rightleftharpoons$

- ① Wave Nature
- ② Particle Nature

Interference, Diffraction  
Polarisation etc

Interaction of Radiation with Radiation

→ Interaction of Radiation with matter  
eg → Photoelectric Effect

Matter  $\rightleftharpoons$

- ① Wave nature
- ② Particle nature

Every particle is associated  
with a wave of wavelength

$$\lambda = \frac{h}{mv}$$

Radiation possesses dual nature ie particle as well as wave.

Certain phenomena like Interference, diffraction, polarization can be explained by taking wave nature of radiation. Here we have interaction of radiation with radiation.

But some phenomena like photoelectric effect cannot be explained by taking wave nature. Here we have interaction of radiation with matter. Photoelectric effect is explained by taking particle nature of radiation ie radiation consists of stream of particles called quanta. Thus we conclude that radiation possesses dual nature.

Similarly material particle moving with velocity  $v$  is associated with a wave of wavelength  $\lambda$  given by

$$\lambda = \frac{h}{mv} \rightarrow \text{de-Broglie hypothesis.}$$

- Stream of electrons will also show diffraction effects.  
Thus matter have dual character.

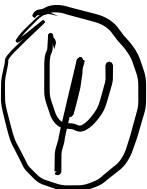
Need and Origin of Quantum Mechanics →

OR

Difficulties of Classical Mechanism:

- ① Stability of atom
- ② Photoelectric effect.
- ③ Line spectra of hydrogenation.

Classical Physics failed to explain certain phenomena, which were explained by new theory of discreteness called Quantum theory. Here are some difficulties of classical mechanics, which were explained by using Quantum Concepts.



① Classical Physics failed to explain the behaviour or motion of microscopic particles of atomic or subatomic dimension. Schrodinger derived his wave equations called Schrodinger wave equations to study the motion of microscopic particles. This wave equation is based upon Quantum theory.

② Atomic Stability → Classical Physics failed to explain atomic stability. According to classical Electromagnetic moving charged particle

will radiate energy. In atom moving electron will radiate energy and falls to the nucleus according to classical theory.

Thus atom is not stable as explained by classical concepts.

But we know that atom is stable. This stability is explained by Bohr, according to him electrons revolve around nucleus <sup>the nucleus in</sup> and only those orbits are possible in which angular momentum is integral multiple of  $\frac{h}{2\pi}$ .

$$\text{ie } mvr = n \frac{h}{2\pi} .$$

Thus atomic stability is explained by Quantum Theory.

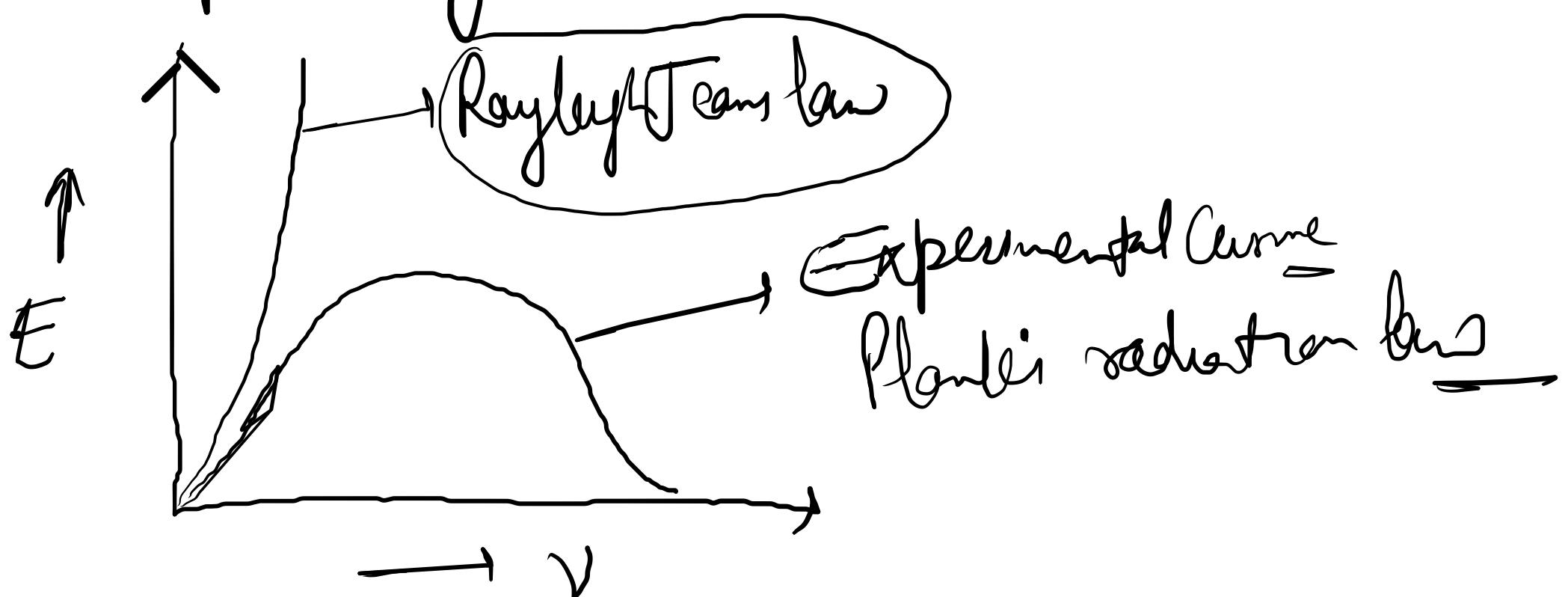
- ③ Classical theory could not explain line spectra i.e. emission of radiation of some particular wavelengths. not all.  
e.g. in case of line spectra of hydrogen atom we will get some particular wavelengths of Lyman, Balmer, Paschen, Brackett and Pfund series. This is explained by taking the quantum concept of discreteness.
- ④ Classical theory failed to explain Photoelectric effect. According to classical theory energy is proportional to Intensity. But experimentally we found that Energy is proportional to frequency of incident radiation. But according to Planck's hypothesis Energy is proportional to frequency. Also ejection of electrons is an instantaneous process.

(5) Classical theory failed to explain dispersion.

(6) Classical ideas failed to explain the distribution of energy in black body.

Black body is one which absorbs all the wavelengths fall over it and when heated emits all the wavelengths.

The energy distribution curve according to classical theory given by Rayleigh and Jeans is not compatible with experimental curve explained by Planck's radiation law.



## Concept of Phase Velocity and Group Velocity

Phase Velocity OR Wave Velocity →

Wave is a form of disturbance which passes through a medium due to repeated periodic motion of particles about their mean position.

The individual oscillators which make up the medium only execute simple harmonic motion and do not themselves move . . . . .

So here a plane of constant phase moves in forward direction that we observe as a wave. The velocity with which this wave moves is called Phase Velocity or wave velocity given by  $v_p$ .

We know equation of plane progressive wave is given by

$$y = A \cos(\omega t - kx) \quad \text{---}$$

Where  $y \rightarrow$  displacement

$A \rightarrow$  amplitude

$\omega \rightarrow 2\pi\nu$  Angular velocity

$k \rightarrow \frac{2\pi}{\lambda}$

$\phi = (\omega t - kx) \rightarrow \underline{\text{Phase}}$

$$\phi = \omega t - kx = \omega \left( t - \frac{k}{\omega} x \right) \quad \text{---} \textcircled{1}$$

From eq \textcircled{1}  $\frac{k}{\omega} = \frac{2\pi/\lambda}{2\pi\nu} = \frac{1}{\lambda\nu} = \frac{1}{v_p} \Rightarrow v_p = \frac{\omega}{k}$

$\phi$  is constant ✓  
 $\Rightarrow \frac{d\phi}{dt} = 0$

$$\omega_p = \frac{\omega}{k}$$

$$\frac{d(\omega t - kx)}{dt} = 0$$
$$\Rightarrow \omega - k \frac{dx}{dt} = 0$$

$$\Rightarrow \frac{dx}{dt} = \frac{\omega}{k}$$

$$\Rightarrow \boxed{\omega_p = \frac{\omega}{k}}$$

prove that Phase velocity of de-Broglie wave exceeds velocity of light →

We know that  $v_p = \sqrt{\frac{E}{m}}$  — ①

According to de-Broglie hypothesis material particle moving with velocity  $v$  is associated with a wave of wavelength

$$\lambda = \frac{h}{mv} \quad \text{--- ②}$$

Also  $E = hv \Rightarrow v = \frac{E}{h} \quad \text{--- ③}$

Put ② and ③ in eq. ①

$$v_p = \frac{E}{h} \cdot \frac{h}{mv} = \frac{E}{mv} \quad \text{--- ④}$$

According to Einstein mass energy relation

$$E = mc^2 \quad \checkmark \quad \checkmark$$

Put in eq. ④

$$v_p = \frac{mc^2}{mv}$$

$$v_p = \frac{c^2}{\nu}$$

$$\Rightarrow v_p \gg c \rightarrow \text{velocity of light}$$

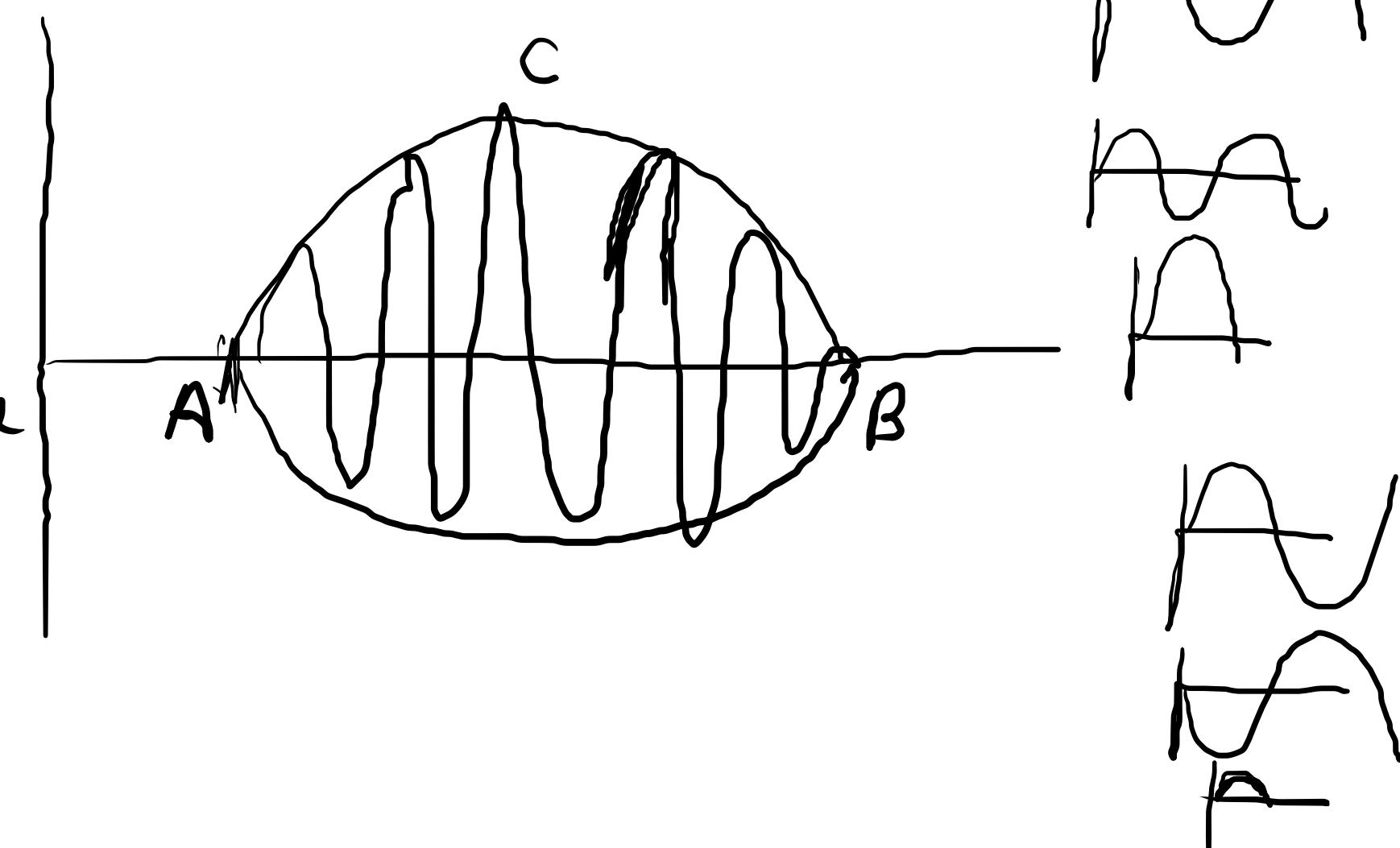
Thus phase velocity of de-Broglie wave exceeds ~~velocity of light.~~

## Group Velocity or Concept of Wave packet:- ✓

Schrodinger postulated that a moving material particle is associated with group of waves.

These group of waves will interfere constructively or destructively to give a resultant form in which there is variation in Amplitude. This resultant form called wave packet will move with velocity ( $v_g$ ) called group velocity.

Between any two consecutive minima ie A and B, there is a position of maximum Amplitude say C.



This wave packet moves in forward direction with velocity  $v_g$ , called group velocity.

Let a wave packet is formed by superposition of two waves of same amplitude  $A$  and slight different frequencies  $\omega$  and  $\omega + d\omega$  and propagation constant  $k$  and  $k + dk$ .

f.i.: The displacement of two waves can be written as

$$y_1 = A \cos(\omega t - kx) \quad \text{--- } ①$$

$$y_2 = A \cos[(\omega + d\omega)t - (k + dk)x] \quad \text{--- } ②$$

According to superposition principle - resultant displacement will be

$$y = y_1 + y_2$$

$$y = A \left[ \cos(wt - kx) + \cos\{(w+dw)t - (k+dk)x\} \right]$$

$\cos C + \cos D = 2 \cos \frac{C+D}{2} \cos \frac{C-D}{2}$

$$\Rightarrow y = 2A \left[ \cos \left\{ \frac{wt - kx + (w+dw)t - (k+dk)x}{2} \right\} \cos \left\{ \frac{wt - kx - (w+dw)t + (k+dk)x}{2} \right\} \right]$$

$$\Rightarrow y = 2A \left[ \cos \left\{ \frac{wt - kx + wt + dwt - kx - dkx}{2} \right\} \cos \left\{ \frac{wt - kx - wt - dwt + kx + dkx}{2} \right\} \right]$$

$$\Rightarrow y = 2A \left[ \cos \left\{ \frac{2wt + dwt - 2kx - dkx}{2} \right\} \cos \left\{ - \left\{ \frac{dw t - dk x}{2} \right\} \right\} \right]$$

$$\Rightarrow y = 2A \left[ \cos \left\{ \frac{(2w + dw)t - (2k + dk)x}{2} \right\} \cos \left\{ \frac{dw t - dk x}{2} \right\} \right]$$

$$\omega + d\omega \approx \omega, k + dk \approx k$$

$$y = A \cos(\omega t - kx) \cos\left[\frac{d\omega}{2}t - \frac{dk}{2}x\right]$$

This equation represents a wave of angular frequency  $\omega$  and wave no.  $k$

which has been superimposed upon by a modulated wave of angular frequency  $\frac{d\omega}{2}$  and wave no.  $\frac{dk}{2}$

This modulated wave moves with velocity  $\frac{d\omega/2}{dk/2} = \frac{d\omega}{dk}$  called

group velocity  $v_g$ .

$$v_g = \frac{d\omega}{dk}$$

## Relation between phase velocity ( $v_p$ ) and group velocity ( $v_g$ )

We know that phase velocity is

$$v_p = \frac{\omega}{k}$$

or we can write.

$$\omega = k v_p \quad \text{--- (1)}$$

Differentiate eq. (1) wrt.  $k$ .

$$\frac{d\omega}{dk} = v_p + k \frac{dv_p}{dk} \quad \text{--- (2)}$$

We know  $v_g = \frac{d\omega}{dk}$

$\therefore$  from eq. (2)  $v_g = v_p + k \frac{dv_p}{dk}$  --- (3)



$$v_g = v_p + k \frac{dv_p}{dr} \frac{dn}{dk} - \textcircled{4}$$

we know  $k = \frac{2\pi}{\lambda}$

$$2\pi n^{-1} = -2\pi \lambda = -\frac{2\pi}{\lambda}$$

$$\Rightarrow \frac{dk}{da} = -\frac{2\pi}{\lambda^2} \Rightarrow \frac{da}{dk} = -\frac{\lambda^2}{2\pi}$$

Put in Eq. no. ④

$$v_g = v_p + \frac{2\pi}{\lambda} \frac{dv_p}{da} \left( -\frac{\lambda^2}{2\pi} \right)$$

$$\Rightarrow v_g = v_p - n \frac{d v_p}{d n}$$

This is the relation between group velocity and phase velocity for dispersive medium.

For non-dispersive medium  $\frac{dV_p}{dp} = 0$

$$\Rightarrow \underline{\underline{V_g = V_p}}$$

To prove group velocity is particle velocity

$$V_g = V$$

Particle Velocity.

1) For non-relativistic particle  $\rightarrow$

Consider a non-relativistic particle of mass  $m$  moving with velocity  $V$ .

Its Energy and momentum are connected as

$$E = \frac{P^2}{2m}$$

Differentiate this equation wrt  $P$

$$\frac{dE}{dP} = \frac{2P}{2m} = \frac{P}{m} = \frac{mv}{m} = v \quad \checkmark \quad ①$$

According to Planck hypothesis

$$E = h\nu = \frac{h\nu \cdot 2\pi}{2\pi} = \frac{h\nu}{2\pi} = \hbar\nu \quad ②$$

$$\text{where } \hbar = \frac{h}{2\pi}$$

$$E = \frac{P^2}{2m}$$
$$E = \sqrt{P_C^2 + m_0^2 c^2}$$

Also Acc to de Broglie hypothesis

$$\checkmark \quad n = \frac{h}{P}$$

$$\Rightarrow P = \frac{h}{n} \frac{2\pi}{2\pi} = \boxed{\frac{h}{2\pi}} \frac{2\pi}{n} = \underline{\hbar k}$$

$$E = \hbar \omega$$

$$\frac{dE}{d\omega} = \hbar$$

$$\frac{dP}{dk} = \hbar$$

$$dE = \hbar d\omega$$

$$dP = \hbar dk$$

$$\Rightarrow \checkmark \frac{dE}{dP} = \frac{d\omega}{dk} = v_g : \textcircled{3}$$

From  $\textcircled{1}$  and  $\textcircled{3}$

$$\boxed{v_g = v}$$

Amp. Velocity = Particle Velocity

Q2) For Relativistic particle

We know

$$E^2 = P^2 c^2 + m_b^2 c^4$$

Dif. wrt P

$$\cancel{\delta E} \frac{d\bar{E}}{dP} = \cancel{\delta P} c^2$$

$$\Rightarrow \frac{d\bar{E}}{dP} = \frac{Pc^2}{E} = \frac{P\cancel{c^2}}{m\cancel{c^2}} = \frac{m\omega}{m\omega} = \cancel{\omega} = \vartheta \quad \text{--- } \textcircled{1}$$

$$E = \hbar\omega, \quad P = \hbar k \quad \Rightarrow \quad d\bar{E} = \hbar d\omega, \quad dP = \hbar dk$$

$$\Rightarrow \frac{d\bar{E}}{dP} = \frac{d\omega}{dk} = \vartheta$$

From  $\textcircled{1}$  and  $\textcircled{2}$

$\vartheta = v_-$

Relation between Phase Velocity and Group velocity for  
Non-Relativistic free particle → ✓

Phase velocity of wave associated with non-relativistic free particle  
is  $v_p = \sqrt{n}$  —①

According to de-Broglie hypothesis  $\lambda = \frac{h}{m\omega} = \frac{h}{mv_g}$  —②

Also  $E = h\nu$  ✓

$$\nu = \frac{E}{h} = \frac{\frac{1}{2}mv^2}{h} = \frac{\frac{1}{2}mv_g^2}{h} —③$$

Put eq. ② and ③ in eq. ①

$$v_p = \frac{v_g}{2}$$

## Schrodinger Wave Equations → ✓

It was given by Schrodinger.

[These are the equation of motion for studying the motion of microscopic particles of atomic dimensions.]

Let us consider a System of waves associated with a microscopic particle moving with velocity v. Let  $\Psi$  be the displacement of these waves. This  $\Psi$  is called wave function which is a complex variable depending upon space and time.

The general differential equation of these waves is given by

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 \Psi}{\partial t^2} \quad \text{--- } ①$$

$$\frac{d^2 y}{dx^2} = \frac{1}{v^2} \frac{d^2 y}{dt^2}$$

OR

$$\nabla^2 \psi = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2} \quad \text{--- (2)}$$

where  $\nabla^2$  is called Laplacian operator

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

The solution for differential eq. (2) is given by

$$\psi = \psi_0 e^{-i\omega t} \quad \text{--- (3)}$$

so double differentiating eq (3) wrt. t.

$$\frac{\partial \psi}{\partial t} = \psi_0 e^{-i\omega t} (-i\omega)$$

$$\frac{\partial^2 \psi}{\partial t^2} = \psi_0 e^{-i\omega t} (-i\omega)^2$$

(1)

$$\nabla = i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} + k \frac{\partial}{\partial z}$$

$\nabla^2 \rightarrow$  Laplacian Operator

$$\begin{aligned} \nabla \cdot \nabla &= \nabla^2 \\ &= \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \end{aligned}$$

$$e^{ax} = e^{ax} \cdot a$$

From eq. 3  $\frac{\partial^2 \psi}{\partial t^2} = (-i\omega)^2 \psi = \psi_0 e^{-i\omega t}$   $i^2 = -1$

$$\frac{\partial^2 \psi}{\partial t^2} = -\psi \omega^2$$

Put this value in Eq. 2

$$\nabla^2 \psi = -\frac{\omega^2}{\mu} \psi$$

$$\omega = 2\pi\nu$$

$$\Rightarrow \nabla^2 \psi = -\frac{4\pi^2 \nu^2}{\mu} \psi$$

$$\nu = \underline{\mu}$$

$$\Rightarrow \nabla^2 \psi = -\frac{4\pi^2 \cancel{\mu}}{\cancel{\mu} n^2} \psi = -\frac{4\pi^2}{n^2} \psi \quad \textcircled{4}$$

According to de-Broglie hypothesis

$$\lambda = \frac{h}{mv} \quad \checkmark$$

Put in eq. ④

$$\nabla^2 \psi =$$

$$-\frac{4\pi^2 m v^2}{h^2} \psi$$

$$\begin{aligned} n &= \frac{h}{\lambda} \\ n^2 &= \frac{h^2}{m v^2} \\ \frac{1}{n^2} &= \frac{m^2 v^2}{h^2} \end{aligned}$$

⑥

If  $E$  be the total energy and  $V$  be the potential energy.

$$E = \frac{1}{2}mv^2 + V$$

$$\Rightarrow \frac{1}{2}mv^2 = E - V$$

$$\Rightarrow mv^2 = 2(E - V)$$

$$mv^2 = 2m(E - V) \quad \checkmark$$

Put this value in eq. ③

$$\nabla^2 \psi = -\frac{4\pi^2}{\hbar^2} 2m(E-V) \psi$$

$$\frac{\hbar}{4\pi} = \hbar$$

$$\Rightarrow \nabla^2 \psi = -\frac{2m}{\hbar^2}(E-V) \psi$$

$$\frac{\hbar^2}{4\pi^2} = \hbar^2$$

or

$$\nabla^2 \psi + \frac{2m}{\hbar^2}(E-V)\psi = 0$$

A  
CRM

This is Schrodinger Time-Independent equation

From eq. ③  $\psi = \psi_0 e^{-i\omega t}$  ✓

Differentiate this equation wrt. t

$$\frac{d\psi}{dt} = \psi_0 e^{-i\omega t} (-i\omega)$$

$$\frac{\partial \Psi}{\partial t} = -i\omega \underbrace{\Psi_0 e^{-i\omega t}}$$

$$\begin{aligned}\frac{\partial \Psi}{\partial t} &= -i\omega \Psi \\ &= -i(2\pi\nu) \Psi \quad [\text{where } \omega = 2\pi\nu]\end{aligned}$$

$$= -i \frac{(2\pi E)}{\hbar} \nu \Psi \quad \begin{aligned}E &= \hbar\nu \\ \Rightarrow \nu &= \frac{E}{\hbar}\end{aligned}$$

$$\frac{\partial \Psi}{\partial t} = -i \frac{E}{\hbar} \Psi$$

$$\Rightarrow E\Psi = i \frac{\hbar}{i(E-i)} \frac{\partial \Psi}{\partial t} = \frac{i\hbar}{-i^2} \frac{\partial \Psi}{\partial t} = i\hbar \frac{\partial \Psi}{\partial t} \quad \textcircled{7}$$

Put in eq. A

$$\nabla^2 \psi + \frac{2m}{\hbar^2} (\epsilon \psi - V \psi) = 0$$

$$\Rightarrow \nabla^2 \psi + \frac{2m}{\hbar^2} \left[ i\hbar \frac{\partial \psi}{\partial t} - V \psi \right] = 0 \quad \checkmark$$

This is Schrodinger's time dependent wave equation -

$$\nabla^2 \psi = - \frac{2m}{\hbar^2} \left[ i\hbar \frac{\partial \psi}{\partial t} - V \psi \right]$$

$$-\frac{\hbar^2}{2m} \nabla^2 \psi = i\hbar \frac{\partial \psi}{\partial t} - V \psi$$

$$-\frac{\hbar^2}{2m} \nabla^2 \psi + V \psi = i\hbar \frac{\partial \psi}{\partial t}$$

or

or

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\Psi = i\hbar \frac{\partial\Psi}{\partial t}$$

$$\hat{H}\Psi = E\Psi$$

$\hat{H} = -\frac{\hbar^2}{2m}\nabla^2 + V$  is called Hamiltonian Operator

$E = i\hbar \frac{\partial}{\partial t}$  is called Energy operator

20, 15

Q-1 Write a note on  
Translation vector

Lattice, Basis, crystal structure And

Q-2 Calculate Packing fraction for HCP. → 8 ✓

Q-3 What are Miller Indices. Write down the steps to  
find Miller Indices. Draw ..... these planes.

→ Semiconductors  
Physics

→ Applied Physics II  
Physics - II

May-June

90%

Sessional → 20

10  
15

Final

15

8  
7

6

Q-4 Derive an expression for Interplaner spacing.  
↓  
Numericals

$d_{hkl}$

Q-5 What are Point defects. Derive an expression for  
Concentration of Schottky/Frenkel defect.

or Show that Frenkel defects depend upon temperature.

Q-6

Discuss difficulties of classical mechanics.

Need and origin of  
Quantum mechanics

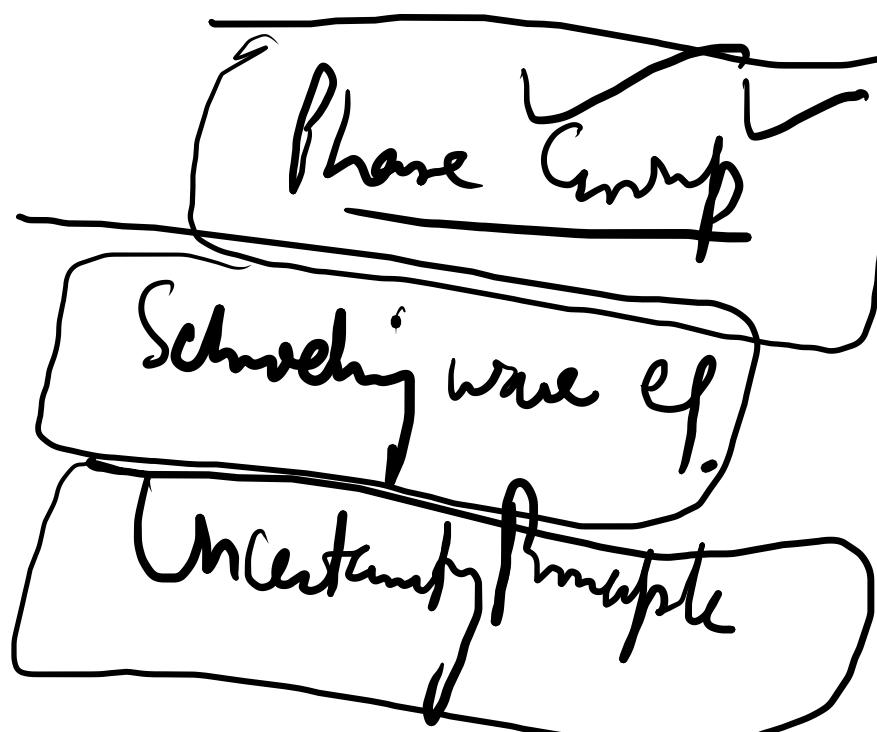
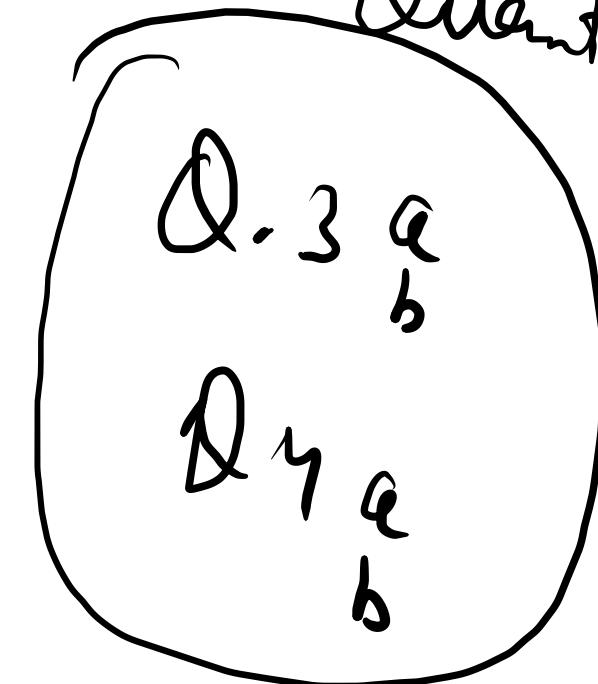
Q-7 Define wave Packet. Show that the velocity with which this wave packet moves is equal to particle velocity.

Q-8 Derive relation between Group velocity and Phase velocity.

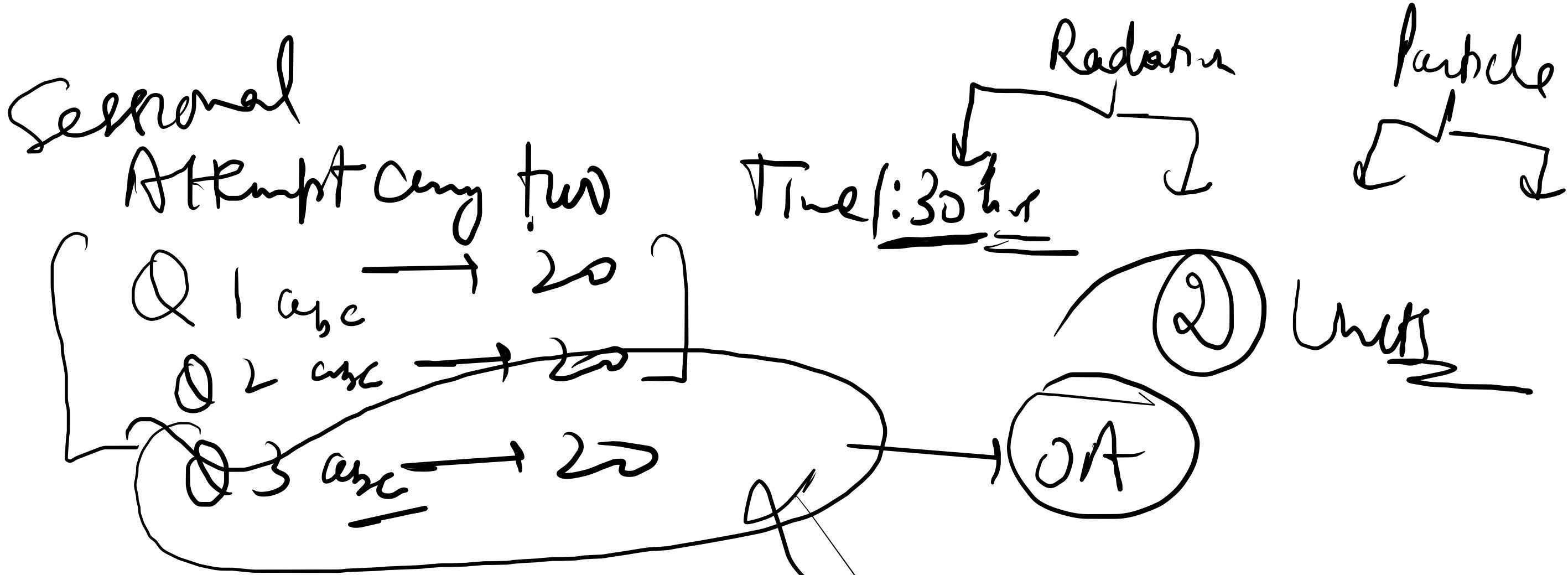
Q-9 Derive Schrödinger's time independent and time dependent wave equations.

Q-10 Bravais lattices in two and three dimensions.

Q-11 Discuss various types of bonding in solids.



Q-12 Write a note on Wave-Particle duality. 2018 → 8



Final

~~Q1~~

Unit I

Unit II

✓

Unit III

✓

Unit IV

✓

Wave function is a complex quantity of the form

$$\Psi = a + ib$$

Complex conjugate  $\Psi^* = a - ib$

$$\begin{aligned}\Psi^* \Psi &= |\Psi|^2 = (a+ib)(a-ib) \\ &= a^2 + \cancel{iab} - i^2 b^2 - \cancel{iab} \\ &= a^2 + b^2\end{aligned}$$

$$i^2 = -1$$

$|\Psi|^2$  will give probability of finding a microscopic particle  
within the wave packet.

~~Plank's Constant~~  $\underline{\underline{h}} \rightarrow$  Dimensional formulae

Elementary Quantum or Quantity of Action

$$\underline{\underline{ML^2T^{-1}}}$$

Means When action involved in a physical event of the order of Planck's Constant then we must use quantum physics to describe the event

Accurately.

$$P = m v \rightarrow LT^{-1}$$

$$MLT^{-1}$$

$$I = \vec{v} \times \vec{p}$$

$$\boxed{ML^2T^{-1}}$$

①  $E = \underline{\underline{hv}}$

②  $mvr = \frac{n \underline{\underline{h}}}{2\pi}$

③  $n = \frac{\underline{\underline{h}}}{P}$

④  $\underline{\underline{h}} = 6.626 \times 10^{-34} JS$

Q.1 Find de-Broglie wavelength of  
electron with K.E 15 eV  $[3.12 \text{ \AA}]$

Q.2 Find phase velocity and group velocity of an electron whose  
de-Broglie wavelength is  $1.2 \text{ \AA}$  (Neglect Relativistic effect)

$$\left\{ \begin{array}{l} v_g = 6.06 \times 10^6 \text{ m/s} \\ v_p = 3.03 \times 10^6 \text{ m/s} \end{array} \right.$$

---

## Nature of Photons

$3 \times 10^8 \text{ m/s}$

① Being quanta of light photon must travel with velocity of light.

② Energy of photon is linearly dependent on the frequency  $\nu$ .

$$E = h\nu$$

$$E = \frac{h(2\pi\nu)}{2\pi} \quad \frac{h}{2\pi}\omega = \hbar\omega$$

③ Linear momentum associated with photon

$$p = \hbar k$$

④ Rest mass of photon is zero but a photon can never be at rest.

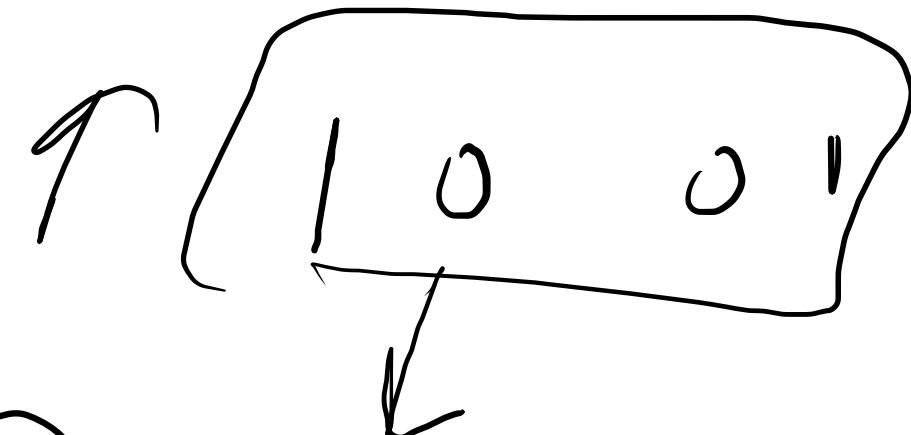
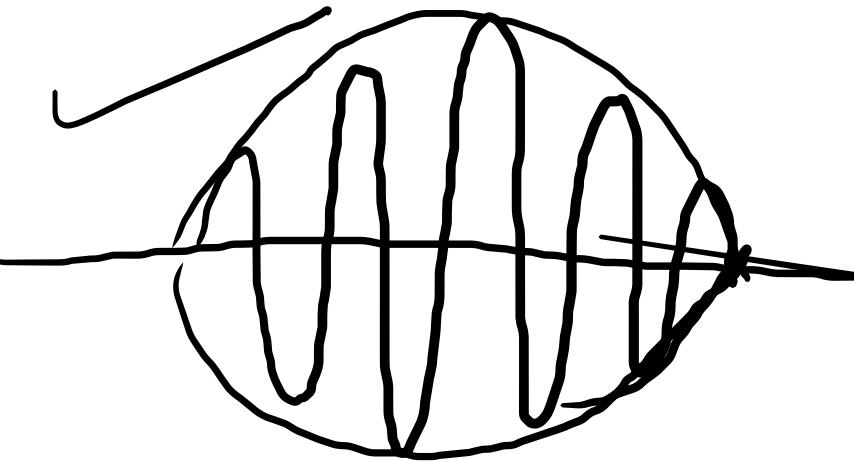
Its relativistic mass is given by

$$m = \frac{E}{c^2} = \frac{h\nu}{c^2} = \frac{h2\pi\nu}{2\pi c^2} = \frac{\hbar\omega}{c^2}$$

2012

# Particle Control in Quantum Mechanics

By  
Serge Haroche and David J. Wineland



11  
10  
01

## Course outcomes

### Semiconductor Physics →

C.O<sub>1</sub> → Recognise the fundamentals of Crystal Structure Unit I

C.O<sub>2</sub> → Understand the importance of Quantum Physics.

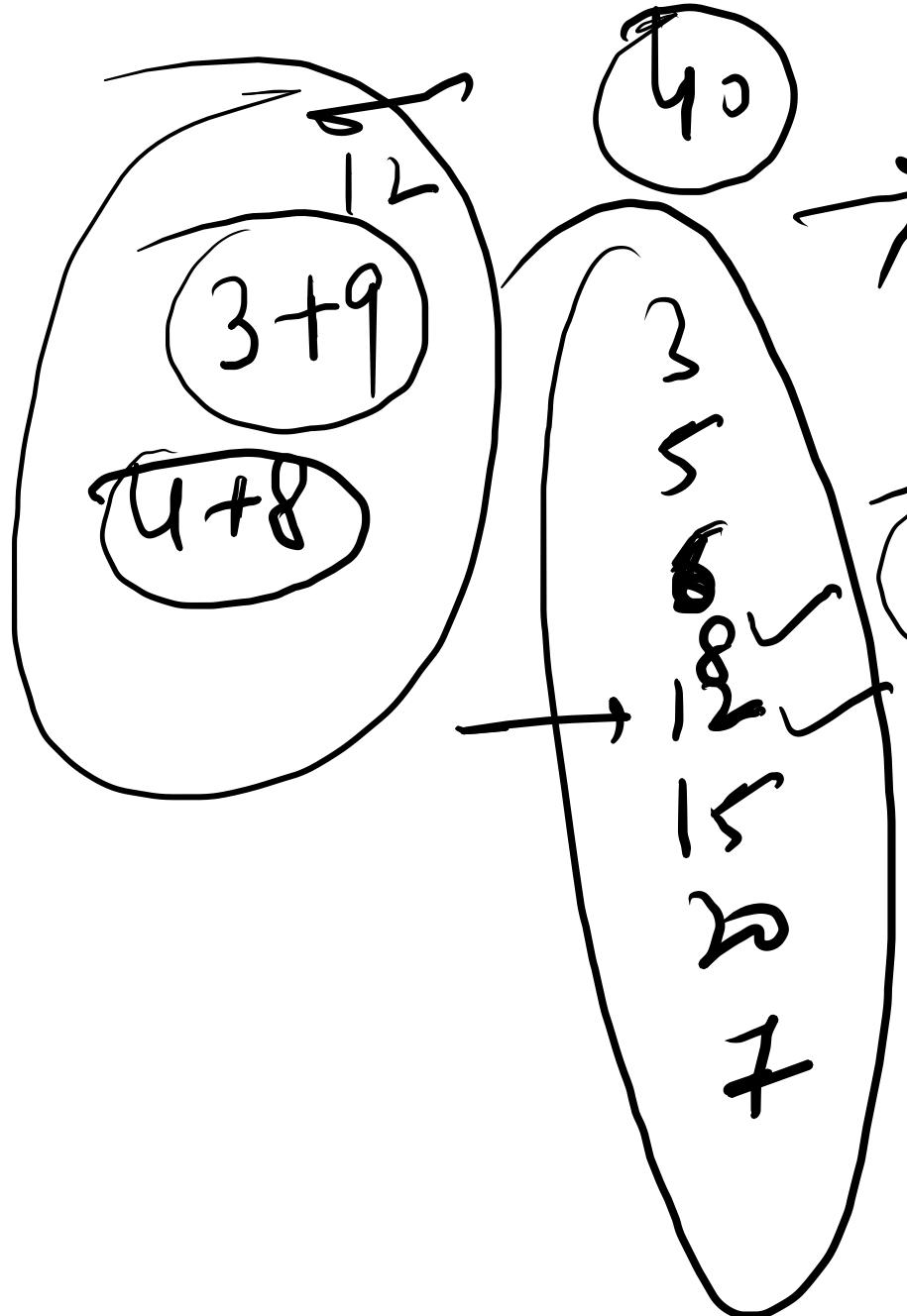
C.O<sub>3</sub> → Apply the concepts of Free Electron Theory and Band Theory of Solids in day to day life

C.O<sub>4</sub> → Implement the concepts of Semiconductor Physics to make Semiconductor Devices.

C.O<sub>5</sub> → Justify the basic theoretical concepts with Experimental results

C.O<sub>6</sub> → Develop the basic knowledge to create the models and projects..

Uncertainty Principle →



Test I Dated → 15/01/2021

Q-1 What are Miller Indices.  
Steps to find Miller Indices.  
Draw Planes. Take

Q. 2 → Derive an expression for  
Concentration of Schottky And  
Frenkel defects.

Q. 3 → Discuss various types of  
bonding in solids

Q. 4 Derive Schrodinger wave  
equations

Q. 5 Relation b/w group velocity  
and phase velocity.

## Uncertainty Principle →

$$\frac{Vg \approx h}{}$$

In classical Physics, one can find position and momentum of a moving body easily if initial values are known!

In Quantum Physics the particle is described in terms of a wavepacket. The particle is likely to found anywhere within the wavepacket moving with group velocity.

Hence it is impossible to know where exactly the particle is and what exactly is its momentum at any instant.

Now according to de-Broglie hypothesis  $\lambda = \frac{h}{P} = \frac{h}{mv}$  — ①

If  $P$  is large means there is uncertainty in the measurement of momentum. But according to eq. ①  $\lambda$  will be small means size of

wavepacket is small and hence we can easily locate the particle

If  $P$  is small, then we can measure momentum easily but from eq. ①  $\Delta x$  will become large ie size of wavepacket will be large. So we have uncertainty in the measurement of position.

Def: It is impossible to specify precisely and simultaneously the values of momentum and the position of the particle."

$$\therefore \Delta x \Delta p \geq \frac{\hbar}{2}$$

$$1E \Delta t \geq \frac{\hbar}{2}$$

$$E = \frac{1}{2} m v^2$$

$$\Delta E = \cancel{\frac{1}{2}} m v \Delta v$$

$$\Delta E = m v \Delta v$$

$$\Delta E = v \boxed{m \Delta v} \rightarrow \Delta p$$

$$\Delta E = v \Delta p$$

$$\Delta E = \frac{\Delta p}{\Delta t} \Delta p$$

$$\Delta E \Delta t = \Delta p \Delta p$$

$$\text{so } \Delta E \Delta t \geq \hbar$$

## Applications of Uncertainty Principle →

- ① The radius of Bohr first orbit. ✓
  - ② Binding energy of electron in an atom.
  - ③ Non-existence of electrons in the nucleus ✓
  - ④ Finite width of the spectral lines.
- ① The radius of Bohr first orbit →

Let  $\Delta x$  and  $\Delta p$  are the Uncertainties in the position and momentum of electron in first orbit then according to Uncertainty Principle.

$$\Delta x \Delta p \approx \hbar$$

$$\text{or } \Delta p = \frac{\hbar}{\Delta x} \quad \text{--- } ①$$

The Uncertainty in the Kinetic energy will be.

$$\Delta K = \frac{(\Delta p)^2}{2m} = \frac{\hbar^2}{2m(\Delta x)^2} - \textcircled{2}$$

Now The Uncertainty in the Potential energy of the same electron will be

$$\Delta V = \frac{1}{4\pi\epsilon_0} \frac{(-e)(ze)}{\Delta x} = -\frac{1}{4\pi\epsilon_0} \frac{ze^2}{\Delta x} - \textcircled{3}$$

Now Uncertainty in the total energy will be

$$\Delta E = \Delta K + \Delta V = \frac{\hbar^2}{2m(\Delta x)^2} - \frac{1}{4\pi\epsilon_0} \frac{ze^2}{\Delta x} - \textcircled{4}$$

The Uncertainty in the total energy would be minimum if

$$\frac{d(\Delta E)}{d(\Delta x)} = 0$$

$$\frac{d}{d(\Delta x)} \left[ \frac{\hbar^2}{2m(\Delta x)^2} - \frac{1}{4\pi\epsilon_0} \frac{Ze^2}{\Delta x} \right] = 0 \quad (\Delta x)^{-2} \quad \Delta x^{-1}$$

$$\Rightarrow \frac{\hbar^2}{2m} (-\Delta x^{-3}) - \frac{Ze^2}{4\pi\epsilon_0} (-1) \Delta x^{-2} = 0$$

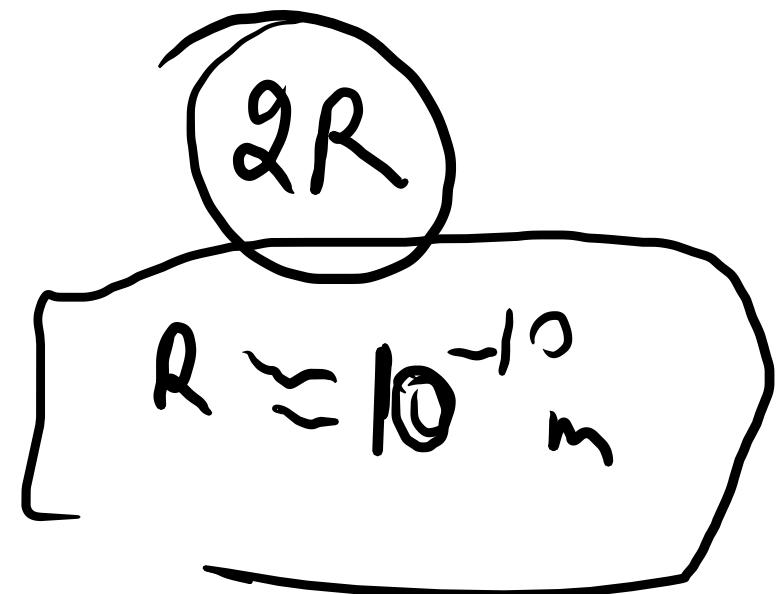
$$\Rightarrow -\frac{\hbar^2}{m \Delta x^3} + \frac{Ze^2}{4\pi\epsilon_0 \Delta x^2} = 0$$

$$\Rightarrow \frac{\hbar^2}{m \Delta x^2} = \frac{Ze^2}{4\pi\epsilon_0 \Delta x^2}$$

$$\Rightarrow \boxed{\frac{\hbar^2}{m Ze^2} \cdot 4\pi\epsilon_0 = \Delta x = R} \quad \text{Radius of Bohr first orbit} \approx$$

②

## Bonding energy of an electron in atom



$$k = 0.96 \text{ eV}$$

$$V = -14.4z \text{ eV}$$

$$V = -14.4 \text{ eV} \quad z=1$$

$$E = -14.4 + 0.96 = -13.44 \text{ eV}$$

$$\Delta x = 2R$$

$$\Delta P = ? \checkmark$$

$$\Delta K = \checkmark$$

$$\Delta V = \checkmark -ze^l$$

$$\rightarrow z=1 \text{ for hydrogen} \quad \cancel{4\pi\epsilon_0 R}$$

$$\Delta E = -13.4 \text{ eV}$$