Normal forms (Discrete Maths)

Let $A(P_1, P_2 -- P_n)$ and $B(P_1, P_2 -- P_n)$ are two statement formulas or compound propositions. To determine whether a given stmt formulas are a Tautology or a contradiction and whether two formulas A and B are equivalent, we have to construct the truth tables and compare them.

Why we use Normal forms?

→ When the no. of primary propositions or variables $(P_1, P_2 -- P_n)$ increases, then the construction of the truth tables is very difficult.

→ A better method is to reduce the stmt formulas A & B to some standard forms called "Normal forms" or "Canonical forms."

Note:- for convinceance, we use the word "product" in place of conjunction and "sum" in place of Disjunction.

Elementary Product:- A product of the variables (propositions) and their negations is called an Elementary product.

Example:- $\sim P \wedge Q$, $P \wedge \sim Q$, $\sim P \wedge P \wedge Q$, $\sim P$, $\sim P \wedge \sim Q$ are some examples of elementary products

Elementary Sum:- A sum of the variables (propositions) and their negations is called an Elementary sum.

Example: $\sim P$, $\sim P \vee Q$, $P \vee \sim Q$, $\sim P \vee \sim Q$ are some examples of Elementary ~~product~~ sum.

$\wedge$ = product
$\vee$ = sum

Page. No. ________ Expt. No. ________ Date ________

Types of Normal forms :- There are 4 types of normal forms :-

(1.) Disjunctive Normal forms (DNF)
(2.) Conjunctive Normal forms (CNF)
(3.) ~~Principal Disjunctive Normal forms (PDNF)~~
(4.) ~~Principal Conjunctive Normal forms (PCNF)~~

(1) Disjunctive Normal forms :- A formula which is equivalent to a given formula and which consists of a sum of elementary products is called a "Disjunctive Normal form" of the given formula. DNF = Sum of Elementary products

Eg: Obtain the DNF of $p \wedge (p \to q)$

Soln
$p \wedge (p \to q)$ $\quad$ ($p \to q = \sim p \vee q$)

$\Leftrightarrow p \wedge (\sim p \vee q)$

$\Leftrightarrow (p \wedge \sim p) \vee (p \wedge q)$ $\longrightarrow$ Elementary products; $\longrightarrow$ sum of Elementary products

$\Leftrightarrow F \vee (p \wedge q)$

$\Leftrightarrow p \wedge q$

Procedure to obtain DNF :- (1.) If the given formula contains '$\to$' and "$\leftrightarrow$" connections, then replace the given formula with equivalent formulas by using $\wedge, \vee, \sim$ connectives.

(2.) If the '$\sim$' (Negation) is applied to the

SUNGRACE

Teacher's Signature : ________

formula or to a part of the formula, then replace them by using Demorgan's laws.

(3.) Now, apply distributive law until some of the elementary products are obtained.

Eg 2) Obtain DNF of $\sim(P \to (Q \wedge R))$

Soln $\sim(P \to (Q \wedge R))$ $\left(\because P \to Q \cong \sim P \vee Q\right)$

$\Leftrightarrow \sim(\sim P \vee (Q \wedge R))$

$\Leftrightarrow$ ~~$\sim(\sim P \vee Q$~~ $\sim(\sim P) \wedge \sim(Q \wedge R)$

$\Leftrightarrow P \wedge (\sim Q \vee \sim R)$

$\Leftrightarrow (P \wedge \sim Q) \vee (P \wedge \sim R)$ is Equivalent DNF.

(Elementary Product) sum of (Elementary Product)

Elementary products.

Eg 3.) Convert the following into DNF!

$P \to ((P \to Q) \wedge \sim(\sim Q \vee \sim P)) \to (1.)$

$\Leftrightarrow P \to ((\sim P \vee Q) \wedge \sim(\sim Q \vee \sim P))$ $\left[\because \sim(\sim P) = P, \ \sim(\sim Q) = Q\right]$

$\Leftrightarrow P \to ((\sim P \vee Q) \wedge (Q \wedge P))$

$\Leftrightarrow P \to ((\sim P \wedge Q \wedge P) \vee (Q \wedge Q \wedge P))$ $\left[\sim P \wedge P \cong F, \ \& \ Q \wedge Q \cong Q\right]$

$\Leftrightarrow P \to ((\underline{\sim P \wedge P} \wedge Q) \vee (Q \wedge Q \wedge P))$

$\Leftrightarrow P \to ((F \wedge Q) \vee (Q \wedge P))$

$\Leftrightarrow P \to (F \vee (Q \wedge P))$ $\left[\because F \wedge Q \cong F\right]$

$\Leftrightarrow P \to (Q \wedge P)$ $\left[\because F \vee (Q \wedge P) = Q \wedge P\right]$

$\Leftrightarrow \sim P \vee (Q \wedge P) \to (2.)$ $\left[\because F \vee \text{Any stmt} = \text{stmt}\right]$

$\Leftrightarrow$ ~~$(\sim P \vee Q) \wedge (\sim P \wedge P)$~~

= ~~sum of elementary products~~

~~So, $(\sim P \vee Q)$~~

Page. No. .... Expt. No. .... Date ....

$\Leftrightarrow (\sim P \vee Q) \wedge P$

$\Leftrightarrow (\sim P \wedge P) \vee (Q \wedge P) \rightarrow (3)$

= sum of elementary products

So, (3) is required DNF of (1).

(Eg4) Obtain the DNF of $\sim(P \vee Q) \Leftrightarrow (P \wedge Q)$ using Truth Tables

Sol:

P	Q	$P \vee Q$	$\sim(P \vee Q)$	$P \wedge Q$	$\sim(P \vee Q) \Leftrightarrow (P \wedge Q)$
T	T	T	F	T	F
T	F	T	F	F	$\boxed{T}$
F	T	T	F	F	$\boxed{T}$
F	F	F	T	F	F

From the above table, we are considering only True (T) values from the final column. After that the corresponding Truth values of P & Q are considered.

$\therefore (P \wedge \sim Q) \vee (\sim P \wedge Q)$

This is the required DNF.

(Eg5) Obtain DNF of $(Q \vee (P \wedge R)) \wedge \sim((P \vee R) \wedge Q) \rightarrow (1)$

$\Leftrightarrow (Q \vee (P \wedge R)) \wedge (\sim(P \vee R) \vee \sim Q)$

$\Leftrightarrow (Q \vee (P \wedge R)) \wedge ((\sim P \wedge \sim R) \vee \sim Q)$

$\Leftrightarrow [(Q \wedge (\sim P \wedge \sim R)) \vee (Q \wedge \sim Q)] \vee [(P \wedge R) \wedge (\sim P \wedge \sim R) \vee (P \wedge R) \wedge \sim Q]$

$\Leftrightarrow (Q \wedge (\sim P \wedge \sim R) \vee F) \vee [(P \wedge R) \wedge (\sim P \wedge \sim R) \vee (P \wedge \sim Q \wedge R)]$

SUNGRACE

Teacher's Signature :

$$\Leftrightarrow (Q \land (\sim P \land \sim R)) \lor [((P \land R) \land (\sim P \land \sim R)) \lor (P \land \sim Q \land R)]$$

$$\Leftrightarrow (Q \land \sim P \land \sim R) \lor [((P \land \sim P) \land (R \land \sim R)) \lor (P \land \sim Q \land R)]$$

$$\Leftrightarrow (\sim P \land Q \land \sim R) \lor [(F \land F) \lor (P \land \sim Q \land R)]$$

$$\Leftrightarrow (\sim P \land Q \land \sim R) \lor [F \lor (P \land \sim Q \land R)]$$

$$\Leftrightarrow \underbrace{[(\sim P \land Q \land \sim R)]}_{\text{Elementary Product}} \underbrace{\lor}_{\text{sum}} \underbrace{[P \land \sim Q \land R]}_{\text{Elementary Product}} \rightarrow (2)$$

So, (2) is the required DNF of (1).

Conjunctive Normal form (CNF)

A formula which is equivalent to a given formula and which consists of "Product of Elementary sums" is called CNF of given formula.

CNF = product of Elementary sums

<u>Procedure to obtain CNF:-</u> Same as DNF.

<u>Procedure to obtain CNF by Truth Tables:-</u>

(1) Construct the Truth Table for given formula.

(2) Consider only false values from the final col. of truth table.

(3) Then the negation of the corresponding truth values can be considered.

(4) Finally, we get the product of Elementary sums of the given formula.

Q1) Obtain the CNF of the formula $\sim(P \vee Q) \leftrightarrow (P \wedge Q)$

We know that for $P \Leftrightarrow Q \cong (P \to Q) \wedge (Q \to P)$

where $P = \sim(P \vee Q)$

$Q = P \wedge Q$

So, $\sim(P \vee Q) \Longleftrightarrow (P \wedge Q) \to (1)$

$\Leftrightarrow (\sim(P \vee Q) \to (P \wedge Q)) \wedge ((P \wedge Q) \to \sim(P \vee Q))$

$\Leftrightarrow [(\sim P \wedge \sim Q) \to (P \wedge Q)] \wedge [(P \wedge Q) \to (\sim P \wedge \sim Q)]$

$\Leftrightarrow [\sim(\sim P \wedge \sim Q) \vee (P \wedge Q)] \wedge [\sim(P \wedge Q) \vee (\sim P \wedge \sim Q)]$

$[\because P \to Q \cong \sim P \vee Q]$

$\Leftrightarrow [(P \vee Q) \vee (P \wedge Q)] \wedge [(\sim P \vee \sim Q) \vee (\sim P \wedge \sim Q)]$

$\Leftrightarrow [P \vee (P \vee Q) \wedge Q \vee (P \vee Q)] \wedge [(\sim P \vee (\sim P \vee \sim Q)) \wedge (\sim Q \vee \sim P \vee \sim Q)]$

$\Leftrightarrow [(P \vee P \vee Q) \wedge (Q \vee P \vee Q)] \wedge [(\sim P \vee \sim P \vee \sim Q) \wedge (\sim P \vee \sim Q \vee \sim Q)]$

$\Leftrightarrow [(P \vee Q) \wedge (P \vee Q)] \wedge [(\sim P \vee \sim Q) \wedge (\sim P \vee \sim Q)]$

$\Rightarrow [(P \vee Q)] \wedge [\sim P \vee \sim Q] \to (2)$ $[\because P \wedge P = P$

Elementary sum

product of Elementary sums

Elementary sums

So, (2) is required CNF of (1).

Eg2. Using a CNF method, show that $Q \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$ is a Tautology.

Soln: Given formula: $Q \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$

$\Leftrightarrow Q \vee [(P \vee \sim P) \wedge \sim Q]$ (By distributive law)

$\Leftrightarrow Q \vee (P \vee \sim P) \wedge (Q \vee \sim Q)$ $\left[\because P \vee \sim P \cong T,\ Q \vee \sim Q \cong T\right]$

$\Leftrightarrow Q \vee (T \wedge T)$

$\Leftrightarrow Q \vee T$

$\Leftrightarrow T$

$\therefore Q \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$ is a Tautology.

Eg3. Obtain the CNF of $A \rightarrow (B \wedge C) \rightarrow (1)$

Soln:- Given formula is $A \rightarrow (B \wedge C)$

$\Leftrightarrow \sim A \vee (B \wedge C)$

$\Leftrightarrow \underbrace{(\sim A \vee B)}_{\text{Elementary sum}} \underbrace{\wedge}_{\text{Product}} \underbrace{(\sim A \wedge C)}_{\text{Elementary sum}} \rightarrow (2)$ = Product of Elem

So, (2) is required CNF of (1).

Q4. Obtain the CNF of $\sim(P \rightarrow Q) \vee (R \rightarrow P) \rightarrow (1)$

Soln $\sim(P \rightarrow Q) \vee (R \rightarrow P)$

$\Leftrightarrow \sim(\sim P \vee Q) \vee (\sim R \vee P)$

$\Leftrightarrow (P \wedge \sim Q) \vee (\sim R \vee P) \rightarrow (2)$

~~= Product of elementary sums~~

~~So, (2) is required CNF of (1)~~

Page. No. ________ Expt. No. ________ Date ________

$\Leftrightarrow [P \vee (\sim R \vee P)] \wedge (\sim Q \vee (\sim R \vee P))$

$\Leftrightarrow [P \vee P \vee \sim R] \wedge [P \vee \sim Q \vee \sim R]$

$\Leftrightarrow \underbrace{[P \vee \sim R]}_{\text{Elementary sum}} \underbrace{\wedge}_{\text{Product}} \underbrace{[P \vee \sim Q \vee \sim R]}_{\text{Elementary sum}} \rightarrow (2)$

which is product of Elementary sums.

So, (2) is the required CNF of (1).

Q.5) Obtain the CNF of $P \wedge (P \rightarrow Q)$

Sol) $P \wedge (P \rightarrow Q)$

$\Leftrightarrow P \wedge (\sim P \vee Q)$

$\Leftrightarrow (P \vee P) \wedge (\sim P \vee Q) \rightarrow (2)$ $[\because P = P \vee P]$

– product of elementary sums

So, (2) is the required CNF of (1).

SUNGRACE Teacher's Signature : ________