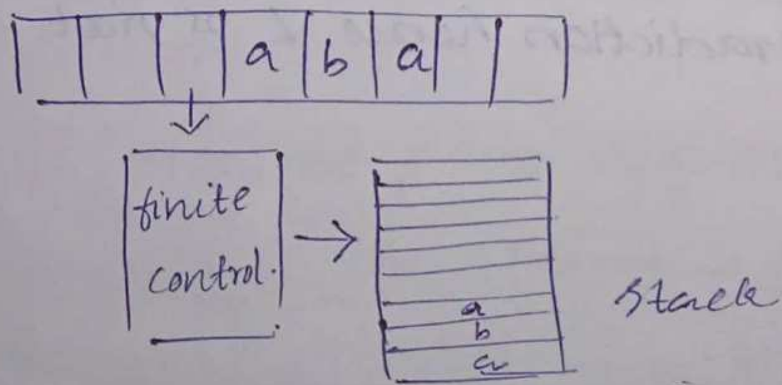
Push down Automata

A push down automata is a fina automata with input file that consists of input symbol and stack that stores the symbol that read from the input tape. The tape. The finate state control reads the input symbol and places the present symb on the top of the stack. The PDA observe the symbol it makes a transition to a new state and add the symbol to the top of the stack



Defenition

A PDA is defined by 7 types

$m = (Q, \varepsilon, \Gamma, \delta, q_0, z, F)$

$Q \rightarrow$ finite state
$\Sigma \rightarrow$ input alphabets
$\Gamma \rightarrow$ stack alphabets / stack input symbol
$\delta \rightarrow$ Transition function: $Q \times (\Sigma \cup \epsilon) \times \Gamma \rightarrow Q \times \Gamma^*$
$q_0 \rightarrow$ initial state.
$Z \rightarrow$ Represent initial stack symbol.
$F \rightarrow$ final state.

Moves of pushdown automata (PDA)

The value of δ takes an argument

$$\delta(q, a, x) = (P, Y)$$

where, $q \rightarrow$ current state
$a \rightarrow$ current input symbols
$x \rightarrow$ current symbol on the top of the stack

This mean in state q on reading an input symbol a with x on the top of the stack it reaches a set if $\delta(q, a, x) = (p, \epsilon)$ in this the state q reading an ~~root~~ input symbol a with x on the top of the stack it reaches a state p & pop x

? if a PDA $M = (\{q_0, q_1, q_2\}, \{a, b, c\}, \{a, b, z\}, \delta, q_0, z, \{q_3\})$

where δ is defined as

$$\delta(q_0, a, z) = (q_0, az)$$
$$\delta(q_0, a, a) = (q_0, aa)$$
$$\delta(q_0, a, b) = (q_0, ab)$$
$$\delta(q_0, c, a) = (q_1, a)$$
$$\cancel{\delta(q_0, z, a) = (q_1, z)}$$
$$\delta(q_0, c, z) = (q_1, z)$$
$$\delta(q_1, a, a) = \delta(q_1, b, b) = (q_1, \varepsilon)$$
$$\delta(q_1, \varepsilon, z) = (q_2, z)$$
$$\delta(q_0, b, z) = (q_0, ba)$$
$$\delta(q_0, b, b) = (q_0, bb)$$
$$\delta(q_0, c, b) = (q_1, b)$$

consider the input string bacab check the acceptability?

ans) $\delta(q_0, bacab, z) = (q_0, acab, bz)$

$= (q_0, cab, abz)$

$= (q_1, ab, abz)$

$= (q_1, b, bz)$

$= (q_1, \varepsilon, z)$

$= (q_2, z)$

it has reached the final state q_2
hence the given string is accepted by the PDA

? if the PDA $P = (\{q_1, q_2\}\{a, b\}, \{z, a\}, \delta, q, z, \{q_2\}$

$\delta(q_1, 0, z) = (q_1, az)$

$\delta(q_1, 0, a) = (q_1, aa)$

$\delta(q_1, 1, a) = (q_1, a)$

$\delta(q_1, \varepsilon, a) = (q_2, \varepsilon)$

$\delta(q_2, 1, a) = (q_2, aa)$

$\delta(q_2, \varepsilon, z) = (q_2, \varepsilon)$

Consider the [illegible] string 0011

ans): $\delta(q_0, 0011, z) = (q_1, 011, az)$
$= (q_1, 11, az)$
$= (q_1, 1, az)$
$= (q_1, az)$

Languages accepted by a PDA

There are 2 ways of acceptance of languages By PDA. The language is accepted if.

1. it searches a final state
2. if the stack is empty

Acceptance by final state

let PDA $m = (Q, \Sigma, \Gamma, \delta, q_0, F)$ language accepted by the PDA By final state is given as

$$\{ w \in \Sigma^* \mid (q_0, w, z) \vdash (q_f, \varepsilon, \alpha) \}$$

instantaneous description

for some $q_f \in F$, $\alpha \in \Gamma^*$

Acceptance by empty stack.

let the PDA be given as $M=(Q,\Sigma,\Gamma,\delta,q_0, ...)$ the language accepted by the PDA by empty stack is given as.

$$\{ w \in \Sigma^* \mid (q_0, w, Z) \vdash (q_f, \varepsilon, \varepsilon) \}$$

Q. Design a PDA that accepts equal no. of a's & equal no. of b's

ans): $\delta(q_0, abab, Z) = (q_0, bab, ax)$

$= (q_0, ab, Z)$

$= (q_0, b, ax)$

$= (q_0, \varepsilon, Z)$

PDA & CFL.

for every CFL there is a PDA that accepts it and the language accepted by the PDA is context free.

Conversion of grammar to PDA

if "L" is a CFL then we can construct a PDA & $L = L(M)$ $G = (V, T, P, S)$ is a CFG & we can construct a PDA proof

The transcleon fun. is defined by the following rules.

$$\delta(q_0, \varepsilon, A) = \{(q_0, \alpha)\}$$

$A \rightarrow \alpha$ is in P.

$$\delta(q_0, a, a) = \{(q_0, \varepsilon)\}$$

for every $a \in \varepsilon$

The symbols present in the stack are variables & terminals. if the PDA reads a variable A. on the top of the stack then it makes a null move (ε). by placing the terminals of ~~that~~ right side of A on the top of the stack by erasing the symbol which is present in the stack.

if the PDA reads the terminal symbol a from the top of the stack & if it matches the correct input symbol then the PDA erases a.

? Design a PDA for the given CFG.

$A \rightarrow a/b/c$

$B \rightarrow A/aB.$

ans): $M = (Q, \Sigma, \delta, \Gamma, q_0, Z_0, F)$

$= q_0, (a, b, c), \delta, \Gamma, q_0, Z_0, F)$

$\delta(q_0, \varepsilon, A) = \{(q_0, a), (q_0, b), (q_0, c)\}$

$\delta(q_0, \varepsilon, B) = \{(q_0, A), (q_0, aB).\}$

$\delta(q_0, a, a) = \delta(q_0, b, b) = \delta(q_0, c, c) = (q_0, \varepsilon)$

? Design a PDA for the grammar

$S \rightarrow aSa$

$S \rightarrow bSb$

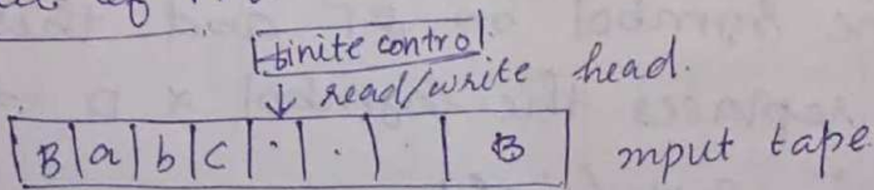
$S \rightarrow c$

$M = ($

Turing Machine

A·M Turing ~~machine~~ purpose the turing machine which is capable of performing any calculation that can be performed by any computing machine The PDA is more powerful than finite automata. The difference b/w finite automata & PDA is the temporary storage called stack which is used in PDA. A turing machine was inverted to carry out any possible computation. it chooses the correct location & decides wether accepted by the turing machine is called reccursively enumerable language

Model of TM.



Turing machine is a finite state automata that consists of finete state control which points to the state. The TM consists of a temp storage called input tape. which is divided into no. of cells. Each cell consist of one symbol the symbol present in the tale may be a blank symbol or input symbol it also consist of read write head.

it can travel either left/right and can read a single symbol on each move.

Defenition of a TM.

$M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$

$Q \rightarrow$ Represent finite set of states.
$\Sigma \rightarrow$ Represent finite set of input symbol.
$\Gamma \rightarrow$ Tape input symbol.
$\delta \rightarrow$ Transition function $Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}$

eg: $\delta(q, x) \rightarrow (P, Y, D)$

q is the current state x is the current input symbol. it reaches the next state P & y is the symbol in Γ and this symbol replaces the symbol x D. is the direction ie, left/right.

$q_0 \rightarrow$ initial state
$B \rightarrow$ Blank symbol
$F \rightarrow$ final state.

Representation of TM

TM are represented by 3 methods

1. instantaneous description (ID)
2. Transition table
3. Transition Diagram

Transition table

Present state	Type symbol b	0	1
q_1	$1 < q_2$	$0 R q_1$	
q_2	$b R q_3$	$0 < q_2$	$1 < q_2$
q_3		$b R q_4$	$b R q_5$
q_4	$0 R q_5$	$0 R q_4$	$1 R q_4$
q_5	$0 L q_2$		

the table consists of a column to represent the state of a TM & another column for each i/p symbol including blank symbol. if the transition given by $\delta(q,a) = (\alpha, \beta, \gamma)$. q represent the state ~~of~~ & a is the current state. α represent new state, β represent new symbol. γ is the direction.

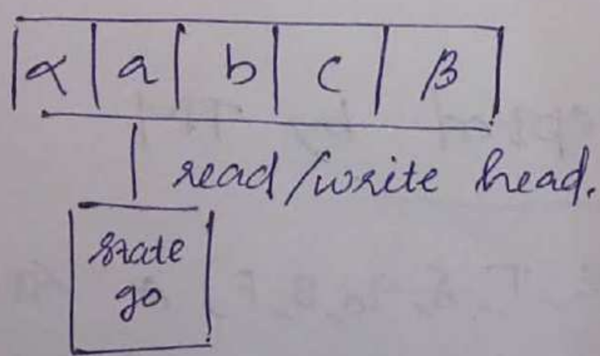
? consider the string oob. check wether it is accepted by the TM or not

ans: $q_1 00b \vdash 0q_1 0b \vdash 00q_1 b \vdash 0q_2 01 \vdash q_2 001$
$\vdash q_2 b001 \vdash bq_3 001 \vdash bbq_4 01 \vdash bb0q_4 1 \vdash$
$bb01q_4 b \vdash bb010q_5 b \vdash bb0q_2 100 \vdash bb0q_2 100$
$\vdash bbq_2 0100 \vdash bq_2 b0100 \vdash bbq_3 0100 \vdash$
$bbbq_4 100 \vdash bbb1q_4 00 \vdash bbb10q_4 0 \vdash$
$bbb100q_4 \vdash bbb1000q_5 b \vdash bb100q_2 00 \vdash$
$bbb10q_2 0001 \vdash bbb1q_2 000 \vdash bbbq_2 10000$
$\vdash bbq_2 b10000 \vdash bbq_3 100000 \vdash bbbbq_5 000$

instantaneous description of TM

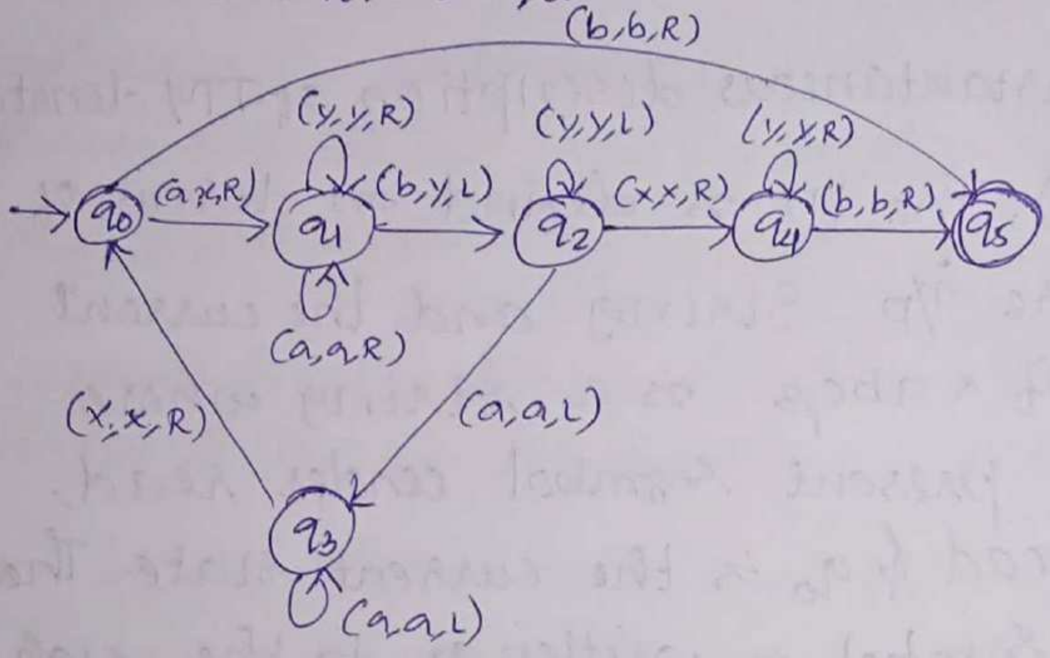
instantaneous description of TM denoted by ID($\vdash$). The ID is defined in terms of the entire i/p string and the current state. if α.abcβ is a string where a is the present symbol under read, write head & q_0 is the current state. The present symbol is written as to the right of state. q_0 the blank symbol is written as to the left of q_0

$\therefore$ The ID of the string is given by $\alpha q_0 abBc\beta$



Representatⁿ by Transition diagram

Transition diagram is a pictorial representation to denote the transition in a TM. it consists of states & the directed edges are used to represent the transitions b/w the states



$q_0 aabb \vdash xq_1abb \vdash xaq_1bb \vdash xq_2ayb \vdash$
$q_3xayb \vdash xq_0ayb \vdash xxq_1yb \vdash xxyq_1b \vdash$
$xxq_2yy \vdash xq_2xyy \vdash xxq_4yy \vdash xxyq_4y \vdash$
$xxyyq_4b \vdash xxyybq_5$.

Language accepted by TM

let $M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$ A string w in Σ is said to be accepted by a TM if $q_0w \vdash \alpha p \beta$ for some state $p \in F$ & tape symbols $\alpha, \beta \in \Gamma$.

in other words language accepted by a TM is the set of strings