Three Phase System   Unit-3 BEE   YEESHU

→ Poly means many (more than one) and phase means winding or circuit each of them having a single alternating voltage of the same magnitude and frequency.

→ Polyphase is a combination of two or more than two voltages having same magnitude and frequency but displaced from one another by equal electrical angle

→ $$\text{Phase angle} = \frac{360^\circ}{\text{No. of Phases}}$$

→ For generation, transmission and distribution of electric power 3-Phase system has been universally adopted

Advantages of 3-Phase over 1-Phase system :-

1) Constant Power : In single phase circuit, power delivered is pulsating. whereas in polyphase system power delivered is almost constant when the loads are balanced

2) High Rating : The rating of a 3-phase machine is nearly 1.5 times the rating of a single phase machine of same size [Output of 3-Phase motor is 1.5 times the output of single phase motor of same size]

3) Power Transmission Economics : To transmit same amount of power over a fixed distance at a given voltage, 3-Phase system requires only 75% of the weight of conducting material of that required by single Phase

4) Three Phase induction motors are self starting, whereas 1-Phase induction motors have no starting torque.

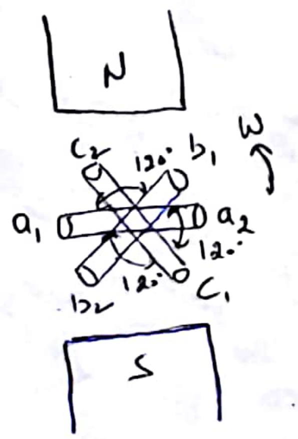
5) Three Phase induction motors have high power factor and efficiency than that of single phase.

Generation of 3-Phase Emf:

→ In 3-Phase System, there are three equal voltages of the same frequency having a phase difference of 120°. These voltages can be produced by three phase, ac generator having three identical winding displaced 120° electrical apart.

→ When these windings are rotated in stationary magnetic field or when these windings are kept stationary and magnetic field is rotated, an emf is induced in each winding or phase.

→ Consider three identical coils a_1a_2, b_1b_2 and c_1c_2 mounted on the same axis but displaced from each other by 120° rotating in anti-clockwise direction at ω radian/sec



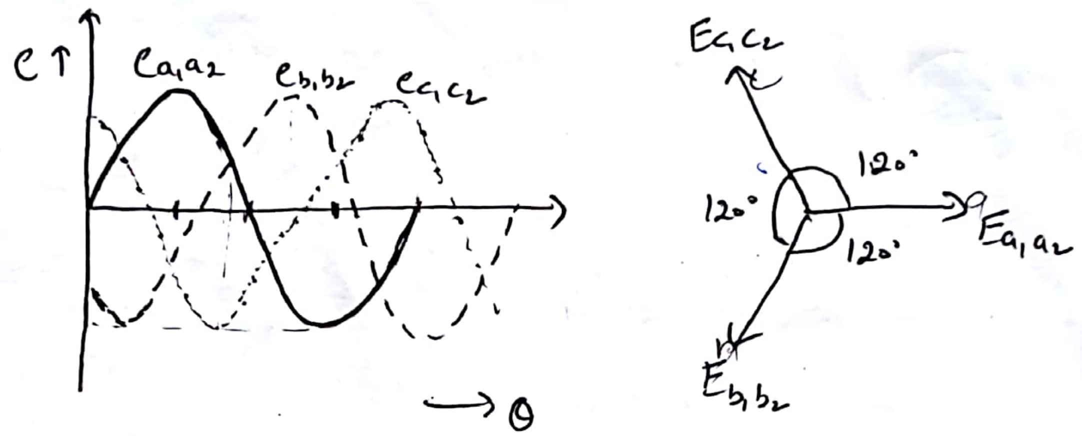
→ Three emf's are induced in three coils

$$e_{a_1a_2} = E_m \sin \omega t$$

$$e_{b_1b_2} = E_m \sin(\omega t - 120°)$$

$$e_{c_1c_2} = E_m \sin(\omega t - 240°)$$

→ The emf's induced in three coils are of same magnitude (3.2) and frequency but are displaced by 120° from each other.



→ The three phases may be named out by numbers (1,2,3) or by colours (Red, Yellow, Blue R,Y,B).

→ Resultant Instantaneous Emf $= e_a + e_b + e_c$

$$= E_m \sin \omega t + E_m \sin(\omega t - 120°) + E_m \sin(\omega t - 240°)$$

$$= E_m \left[\sin \omega t + \sin \omega t \cos 120° - \cos \omega t \sin 120° + \sin \omega t \cos 240° - \cos \omega t \sin 240° \right]$$

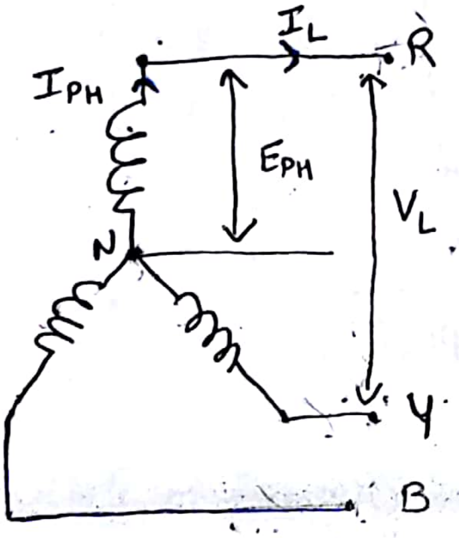
$$= E_m \left[\sin \omega t - 0.5 \sin \omega t - \cos \omega t \frac{\sqrt{3}}{2} + \sin \omega t (-0.5) - \cos \omega t \left(-\frac{\sqrt{3}}{2}\right) \right]$$

$$= 0$$

Star / Wye / Y connection

→ In star connection, the similar ends (either start or finish) of three windings are connected to a common point called star point or neutral point.

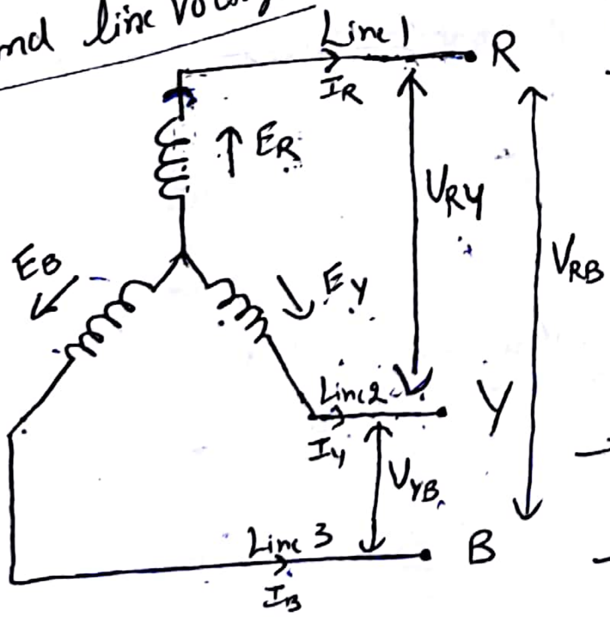
3-Phase 4-Wire Star Connected System



→ Voltage induced in each winding is known as Phase voltage (E_{PH}) and current in each winding is known as Phase current (I_{PH})

→ Voltage available b/w any pair of terminals is known as line voltage (V_L) and current flowing in each line is line current (I_L)

Relationship b/w Phase and line voltage



→ Since the system is balanced, the three voltages E_R, E_Y, E_B are equal in magnitude ~~by~~ but displaced from one another by 120°.

→ $E_R = E_Y = E_B = E_{PH}$ (in magnitude)

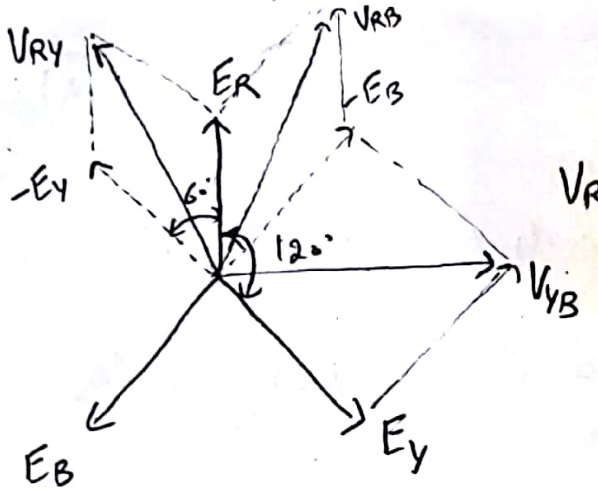
→ Line voltage, b/w line 1 and line 2 is the vector difference of E_R and E_Y.

$$\overline{V_{RY}} = \overline{E_R} - \overline{E_Y}$$

Similarly

$$\overline{V_{YB}} = \overline{E_Y} - \overline{E_B}$$

$$V_{RB} = \overline{E_R} - \overline{E_B}$$



$$V_{RY} = E_R - E_Y$$

$$V_{RY} = \sqrt{E_R^2 + E_Y^2 + 2E_R E_Y \cos 60^\circ}$$

$$= \sqrt{E_{PH}^2 + E_{PH}^2 + 2E_{PH}^2 \times \frac{1}{2}}$$

$$= \sqrt{3E_{PH}^2}$$

$$\boxed{V_{RY} = \sqrt{3}\, E_{PH}} \text{ or } \boxed{V_L = \sqrt{3}\, E_{PH}}$$

Similarly $V_{YB} = \sqrt{3}\, E_{PH}$

$V_{RB} = \sqrt{3}\, E_{PH}$

$$\boxed{\text{Line voltage} = \sqrt{3} \times \text{Phase voltage}}$$

Relationship b/w line current and Phase current

Same current flows through Phase winding as well as line

$$\boxed{\begin{matrix}\text{Line current} = \text{Phase Current} \\ I_L = I_{PH}\end{matrix}} \quad I_R = I_Y = I_B = I_L = I_{PH}$$

Power in 3-Phase circuits

Power in single phase $= V_{PH}\, I_{PH} \cos\phi$

Power in 3-Phase (Balanced Load) $= 3 \times V_{PH}\, I_{PH} \cos\phi$

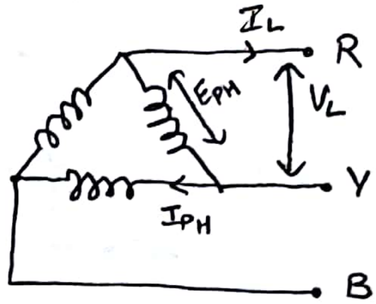
$$= 3 \times \frac{V_L}{\sqrt{3}} I_L \cos\phi$$

$$P = \underline{\sqrt{3}\, V_L I_L} \cos\phi$$

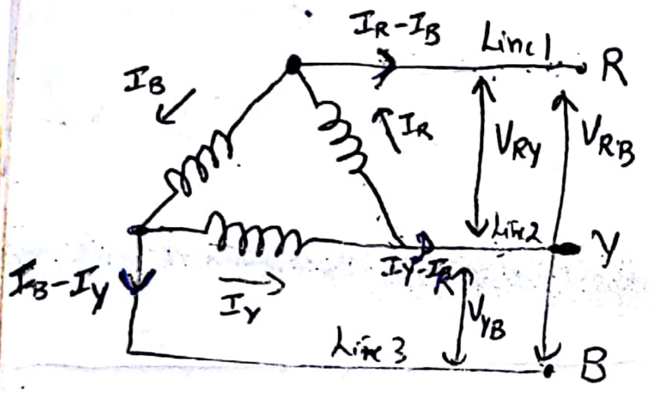
MESH / DELTA (Δ) Connection

3.4

→ In delta connection, finish terminal of one winding is connected to start terminal of the other Phase and so on which give a closed circuit.



Relationship b/w line and Phase voltage



Line voltage = Phase voltage

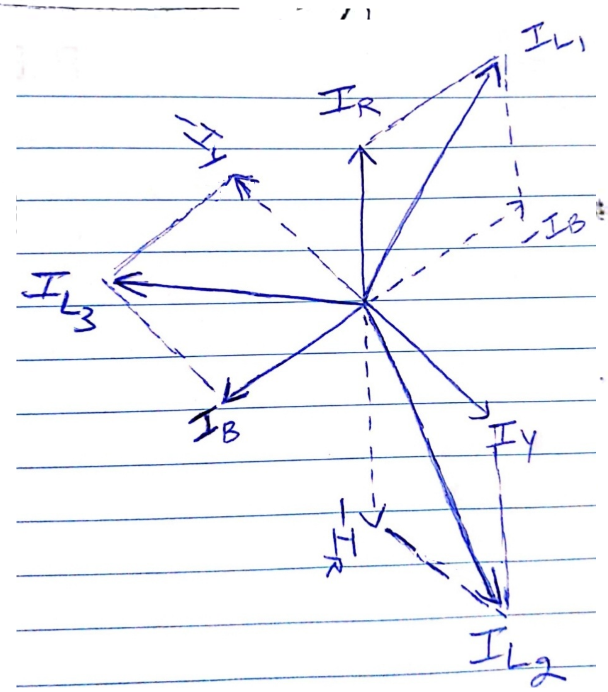
Relationship b/w line current and Phase current

$I_R = I_Y = I_B = I_{PH}$

I_{L_1}, Current in Line 1 $= I_R - I_B$

I_{L_2}, " Line 2 $= I_Y - I_R$

I_{L_3} " Line 3 $= I_B - I_Y$



$$I_{L_1} = \sqrt{I_R^2 + I_Y^2 + 2 I_R I_Y \cos 60}$$

$$= \sqrt{I_{PH}^2 + I_{PH}^2 + 2 I_{PH}^2 \times \frac{1}{2}}$$

$$\boxed{I_{L_1} = \sqrt{3}\, I_{PH}}$$

$$\boxed{\text{Line Current} = \sqrt{3} \times \text{Phase Current}}$$

Power in 3- Phase delta Connection

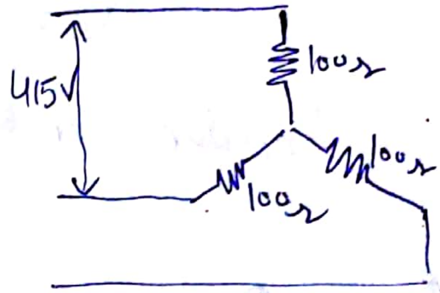
$$P = 3 \times V_{PH} I_{PH} \cos\phi$$

$$= 3 \times V_L \frac{I_L}{\sqrt{3}} \cos\phi$$

$$\underline{P = \sqrt{3}\, V_L I_L \cos\phi}$$

Q. Three 100Ω resistors are connected first in star and then in delta across 415V, 3-Phase supply. (3.5)
Calculate the line & Phase current in each and also the Power taken from the source.

Ans **Star**



$$V_L = 415V$$

$$V_{PH} = \frac{415V}{\sqrt{3}} = 239.6V$$

$$I_{PH} = \frac{V_{PH}}{Z_{PH}} = \frac{239.6}{100} = 2.396A$$

$$I_L = I_{PH} = 2.396A$$

$$Power = \sqrt{3}V_L I_L \cos\phi = \sqrt{3}\times 415 \times 2.396 \times \frac{R}{Z} = \sqrt{3}\times 415 \times 2.396 \times 1$$

$$= 1722W$$

Delta

$$V_L = 415V$$

$$V_{PH} = V_L = 415V$$

$$I_{PH} = \frac{V_{PH}}{Z_{PH}} = \frac{415}{100} = 4.15A$$

$$I_L = \sqrt{3}\times I_{PH} = \sqrt{3}\times 4.15 = 7.188A$$

$$P = \sqrt{3}V_L I_L \cos\phi = \sqrt{3}\times 415 \times 7.188 \times 1$$

$$= 5166W$$

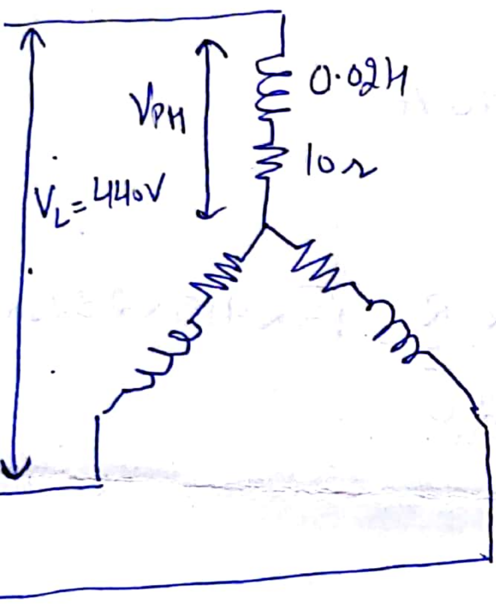
Q2 Three coils each having resistance of 10Ω and inductance 0.02H are connected in star across 440V, 50Hz, 3-Phase supply. Calculate the line current and total Power consumed.

Ans

$$X_L = 2\pi f L = 2\pi\, 50 \times 0.02 = 6.283\,\Omega$$

Impedance Per Phase, $Z_{PH} = \sqrt{R^2 + X_L^2}$

$$= \sqrt{10^2 + (6.283)^2} = 11.81\,\Omega$$



$$V_{PH} = \frac{V_L}{\sqrt{3}} = \frac{440}{\sqrt{3}} = 254.03V$$

$$I_{PH} = \frac{V_{PH}}{Z_{PH}} = \frac{254.03}{11.81} = \underline{\underline{21.51A}}$$

$$\cos\phi = \frac{R}{Z_{PH}} = \frac{10}{11.81} = 0.8467$$

$$P = \sqrt{3}\, V_L\, I_L \cos\phi$$

$$= \sqrt{3} \times 440 \times 21.51 \times 0.8467$$

$$= \underline{13880W}$$

Measurement of power in three phase circuits

Unit-3

36

→ In ac circuit, power is measured with the help of wattmeter.

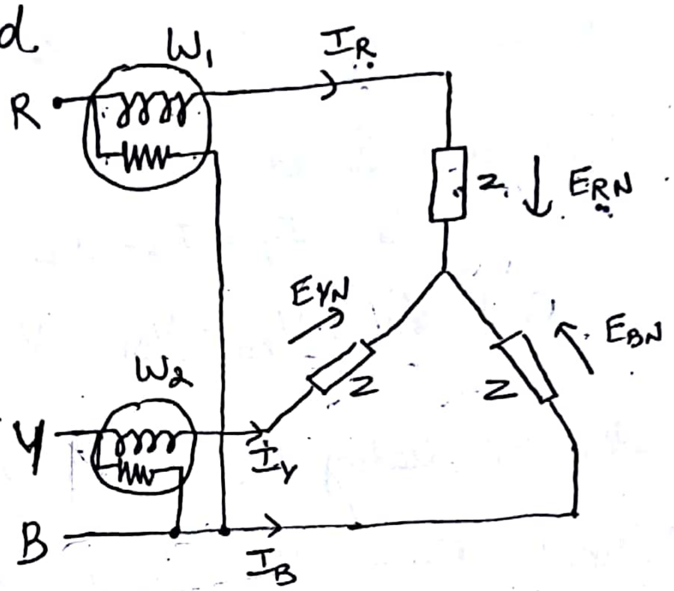
→ A wattmeter is an instruments which consists of two coils called current coil and potential coil.

→ The current coil having low resistance is connected in series with the load so that it carries the load current.

→ The potential coil having high resistance is connected across the load.

Two Wattmeter Method (Star Connected Balanced Load) (3.7)

→ . We shall prove that power measured by two wattmeters is equal to $\sqrt{3}\, V_L I_L \cos\phi$ which is actual power consumed in 3- phase balanced load.

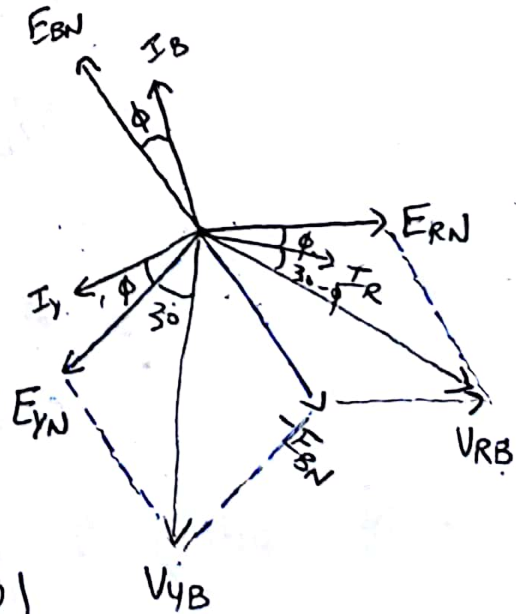


(i) Inductive load

→ Considering load to be an inductive one (R-L Load)

The three phase voltages E_{RN}, E_{YN} and E_{BN} displaced by angle 120°

→ The phase current lag behind their respective phase voltages by an angle ϕ



→ Current through current coil of $W_1 = I_R$

P.d. across potential coil of $W_1 = E_{RN} - E_{BN}$

$= V_{RB}$

Phase difference b/w V_{RB} and $I_R = 30 - \phi$

Power Measured by Wattmeter $W_1 = V_{RB} I_R \cos(30-\phi)$

→ Current through current coil of $W_2 = I_Y$

P.d. across potential coil of $W_2 = E_{YN} - E_{BN} = V_{YB}$ 4

Phase difference b/w V_{YB} and $I_Y = 30 + \phi$

Power measured by wattmeter $W_2 = V_{YB} I_Y \cos(30+\phi)$

→ Since load is balanced

$$I_R = I_Y = I_B = I_L$$

and $V_{RY} = V_{YB} = V_{BR} = V_L$

→ Wattmeter Reading $W_1 = V_L I_L \cos(30-\phi)$

Wattmeter Reading $W_2 = V_L I_L \cos(30+\phi)$

→ Sum of two wattmeter readings $= W_1 + W_2$

$$= V_L I_L \cos(30-\phi) + V_L I_L \cos(30+\phi)$$

$$= V_L I_L \left[\cos(30-\phi) + \cos(30+\phi)\right]$$

$$= V_L I_L \left[\cos 30 \cos\phi + \sin 30 \sin\phi + \cos 30 \cos\phi - \sin 30 \sin\phi\right]$$

$$= V_L I_L \left[2 \cos 30 \cos\phi\right]$$

$$= \sqrt{3}\, V_L I_L \cos\phi$$

= Total Power absorbed in 3-Phase balanced load

Sum of readings of two wattmeter is equal to power absorbed in 3-Phase balanced load.

(2) Capacitive load (R-C load)

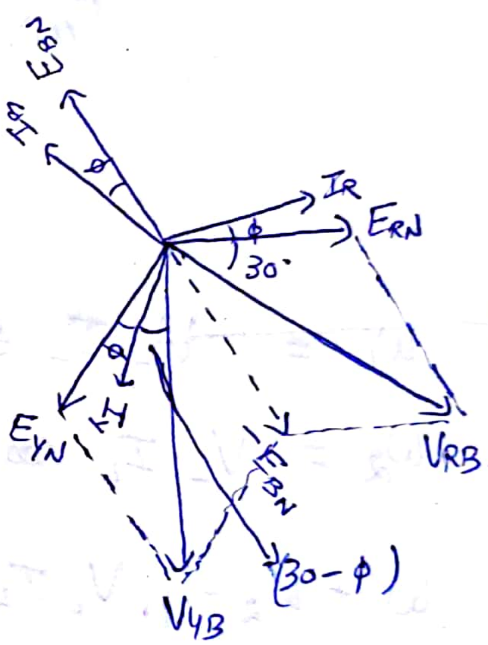
→ For capacitive load, the phase current lead their respective phase voltage by an angle ϕ.

→ Current through current coil of $W_1 = I_R$

P.D. across potential coil of $W_1 = E_{RN} - E_{BN}$
$= V_{RB}$

Phase difference b/w V_{RB} & $I_R = 30 + \phi$

Power Measured by Wattmeter $W_1 = V_{RB} I_R \cos(30+\phi)$



→ Current through current coil of $W_2 = I_Y$

P.d. across potential coil of $W_2 = E_{YN} - E_{BN} = V_{YB}$

Phase difference b/w V_{YB} & $I_B = 30 - \phi$

Power Measured by Wattmeter $W_2 = V_{YB} I_B \cos(30-\phi)$

→ Since load is balanced

$$I_R = I_Y = I_B = I_L$$

and $V_{RY} = V_{YB} = V_{BR} = V_L$

→ $W_1 + W_2 = V_L I_L \cos(30+\phi) + V_L I_L \cos(30-\phi)$

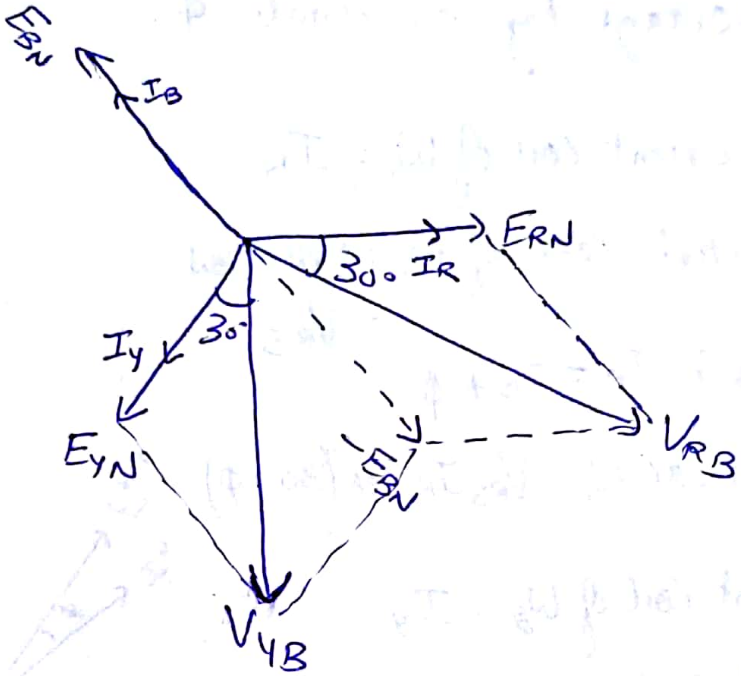
$$= V_L I_L \{\cos 30 \cos\phi - \sin 30 \sin\phi + \cos 30 \cos\phi + \sin 30 \sin\phi\}$$

$$\boxed{W_1 + W_2 = \sqrt{3}\, V_L I_L \cos\phi}$$

Scanned by CamScanner

(3) Pure Resistive load

For Pure Resistive load, current and voltage are in same phase.



$$W_1 = V_L I_L \cos 30°$$

$$W_2 = V_L I_L \cos 30°$$

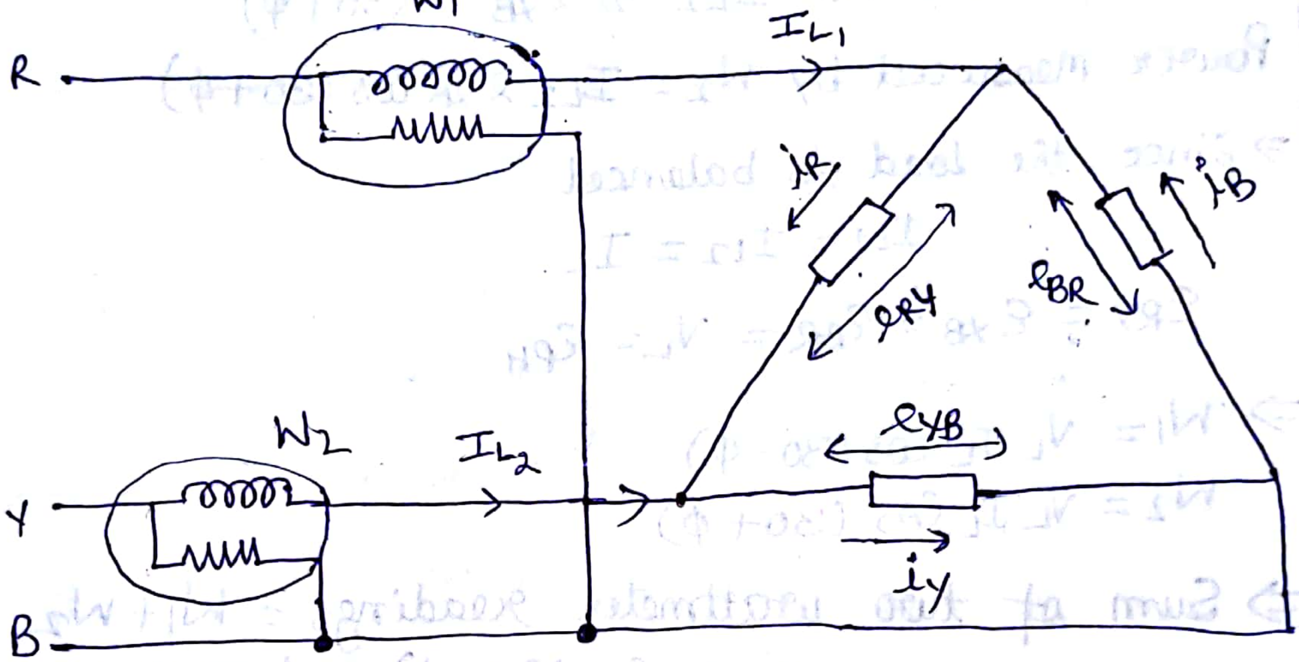
$$W_1 + W_2 = V_L I_L \cos 30 + V_L I_L \cos 30°$$

$$= \frac{\sqrt{3}}{2} V_L I_L + \frac{\sqrt{3}}{2} V_L I_L$$

$$\boxed{W_1 + W_2 = \sqrt{3}\, V_L I_L}$$

Two wattmeter method (Delta Connected Balanced ~~state~~ Load) (3.9B)

We have to prove that power measured by two wattmeters is equal to $\sqrt{3}\, V_L I_L \cos\phi$.



Consider load to be an inductive (RL) one. The Three phase voltage E_{RY}, E_{YB}, E_{BR} are displaced by an angle $120°$. The Phase current lags behind their respective phase Voltages by an angle ϕ.

$\Rightarrow$ Current through current coil of wattmeter $W_1 = I_{L1}$

$\Rightarrow$ Potential difference across Potential coil of $W_1 = e_{RB}$

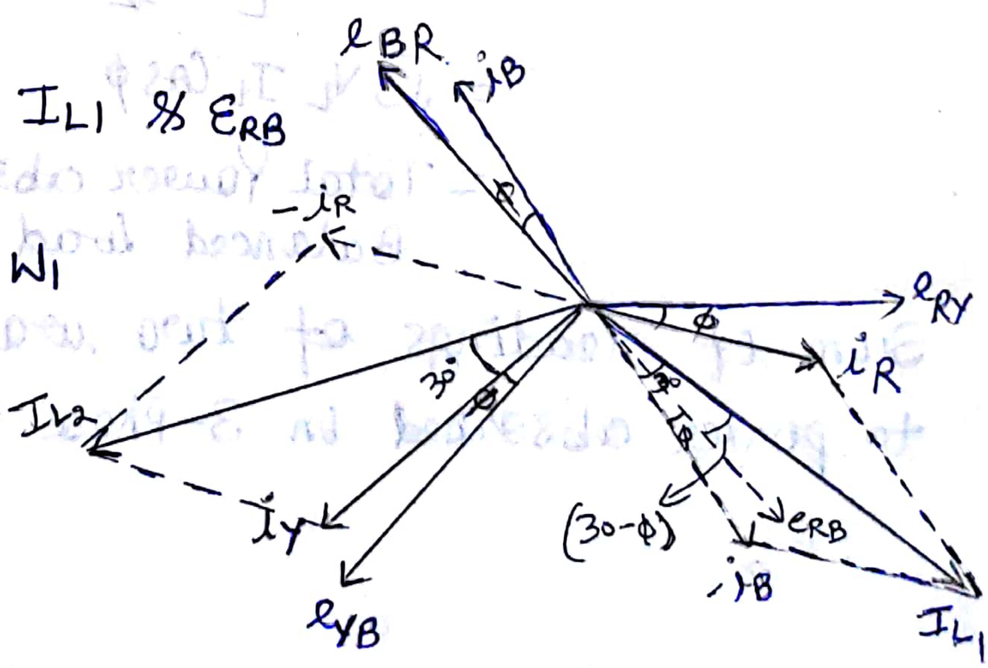
$I_{L1} = i_R - i_B$

Phase difference b/w I_{L1} & E_{RB}

$= (30 - \phi)$

Power measured by W_1

$= I_{L1} E_{RB} \cos(30 - \phi)$



⇒ Current through current coil of wattmeter $W_2 = I_{L2} = i_Y - i_R$

Potential difference across potential coil of $W_2 = e_{YB}$

Phase difference b/w I_{L2} & $e_{YB} = (30+\phi)$

Power measured by $W_2 = I_{L2}\, e_{YB} \cos(30+\phi)$

⇒ Since the load is balanced

$$I_{L1} = I_{L2} = I_L$$

$$\varepsilon_{RY} = \varepsilon_{YB} = \varepsilon_{BR} = V_L = \varepsilon_{PH}$$

⇒ $W_1 = V_L I_L \cos(30-\phi)$

$W_2 = V_L I_L \cos(30+\phi)$

⇒ Sum of two wattmeter readings $= W_1 + W_2$

$$= V_L I_L \cos(30-\phi) + V_L I_L \cos(30+\phi)$$

$$= V_L I_L [\cos(30-\phi) + \cos(30+\phi)]$$

$$= V_L I_L [\cos 30 \cos\phi + \cancel{\sin 30 \sin\phi} + \cos 30 \cos\phi - \cancel{\sin 30 \sin\phi}]$$

$$= V_L I_L [2 \cos 30 \cos\phi]$$

$$= V_L I_L \left[\cancel{2} \times \frac{\sqrt{3}}{\cancel{2}} \cos\phi\right]$$

$$= \sqrt{3} V_L I_L \cos\phi$$

= Total Power absorbed in 3-Phase Balanced load

Sum of readings of two wattmeters is equal to power absorbed in 3-Phase balanced load

Determination of Power Factor from Wattmeter Readings

$$W_1 + W_2 = \sqrt{3}\, V_L I_L \cos\phi \qquad -①$$

$$W_1 - W_2 = V_L I_L \left[\cos(30-\phi) - \cos(30+\phi)\right]$$

$$= V_L I_L \left[\cos 30 \cos\phi + \sin 30 \sin\phi - \cos 30 \cos\phi + \sin 30 \sin\phi\right]$$

$$= 2 V_L I_L \sin 30 \sin\phi$$

$$W_1 - W_2 = V_L I_L \sin\phi \qquad -②$$

Dividing ② by ①

$$\frac{W_1 - W_2}{W_1 + W_2} = \frac{V_L I_L \sin\phi}{\sqrt{3}\, V_L I_L \cos\phi}$$

$$\tan\phi = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2} \Rightarrow \phi = \tan^{-1}\sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2}$$

Power factor, $\boxed{\cos\phi = \cos \tan^{-1}\sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2}}$

Effect of Power Factor on Wattmeter Readings

→ For Lagging Power Factor the Wattmeter Readings are :-

$$W_1 = V_L I_L \cos(30-\phi), \quad W_2 = V_L I_L \cos(30+\phi)$$

Case ① For pure resistive load, P.F. is unity, $\cos\phi = 1 \Rightarrow \phi = 0$

Two Wattmeter Readings

$$W_1 = V_L I_L \cos 30$$

$$W_2 = V_L I_L \cos 30 \qquad \text{i.e. } W_1 = W_2$$

Case ② When $\phi = 60 \Rightarrow$ P.F $= \cos 60 = 0.5$

$$W_1 = V_L I_L \cos(30-60) = V_L I_L \cos 30$$

$$W_2 = V_L I \cos(30+60) = 0$$

[Whole of Power is measured by Wattmeter W_1]

Case ③ When $\phi = 90$, P.F. $= \cos 90 = 0$

$$W_1 = V_L I_L \cos 60° \ (+ve) = \frac{V_L I_L}{2}$$

$$W_2 = V_L I_L \cos(30+90) = -V_L I_L \cos 60 \ (-ve)$$

$$= -\frac{V_L I_L}{2}$$

1) A - Balanced 3-Phase, star connected load draws power from 440V supply. The Two Wattmeters connected indicate $W_1 = 4.2$ KW and $W_2 = 0.8$ KW. (3.11)

Calculate Power, Power Factor and current in circuit.

Ans
$$W_1 = 4.2\,KW, \quad W_2 = 0.8\,KW$$

$$P = W_1 + W_2 = \underline{5\,KW}$$

$$\tan\phi = \frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2} = \frac{\sqrt{3}(4.2 - 0.8)}{4.2 + 0.8} = 1.1778$$

$$\cos\phi = \cos\tan^{-1}(1.1778) = \underline{0.6472}$$

$$P = \sqrt{3}\, V_L I_L \cos\phi$$

$$5000 = \sqrt{3} \times 440 \times I_L \times 0.6472$$

$$\boxed{I_L = 10.137A}$$

---

1) Two Wattmeters are used to measure the power in 3-Phase Balanced system. What is the power factor when

(1) Both the meters read equal

(2) Both the meters read equal but one is negative

(3) One reads twice the other.

Ans (1) Case I → $W_1 = W_2$

$$\tan\phi = \frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2} = 0 \Rightarrow \phi = 0^\circ$$

$$\boxed{\text{Power Factor} = \cos\phi = \underline{1}}$$

(2) Case II → $W_1 = -W_2$

$$\tan\phi = \frac{\sqrt{3}(-W_2 - W_2)}{-W_2 + W_2} = \infty \Rightarrow \phi = 90^\circ$$

$$\boxed{P.F = \cos 90 = 0}$$

(3) Case III → $W_1 = 2W_2$, $\tan\phi = \frac{\sqrt{3}(2W_2 - W_2)}{2W_2 + W_2} = \frac{1}{\sqrt{3}} \Rightarrow \phi = 30$

$$\boxed{P.F = \cos 30 = 0.866}$$

Q) A-3 Phase 500V motor has 0.4 power factor lagging. Two Wattmeters are connected to measure the input. They show the total input to be 30 KW. Find the reading of each Wattmeter

Ans

$$W_1 + W_2 = 30\ KW \quad -(1)$$

$$P.F = \cos\phi = 0.4$$

$$\phi = \cos^{-1}(0.4) = 66.42^\circ$$

$$\tan\phi = \tan 66.42^\circ = 2.29$$

$$2.29 = \frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2} = \frac{\sqrt{3}(W_1 - W_2)}{30}$$

$$W_1 - W_2 = 39.69\ KW \quad -(2)$$

Solving eq (1) & (2)

$$W_1 = 34.84\ KW$$

$$W_2 = -4.8\ KW$$

Q) Two Wattmeter connected to measure the input to a balanced 3-Phase circuit indicate 2000W & 500W. What will be the Power Factor when the later reading is obtained after reversing the connection to the current coil of ~~first instrument~~.

Ans

$$W_1 = 2000W$$

$$W_2 = -500W \text{ (After Reversing the Connection)}$$

$$\tan\phi = \frac{\sqrt{3}[2000 - (-500)]}{2000 + (-500)} = 2.886$$

$$\phi = \tan^{-1}(2.886) = 70.88^\circ$$

$$P.F = \cos\phi = \cos 70.88^\circ = 0.327$$