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Sequnce and Series !

Sequence !-

→ list of things which are in order.

A sequence usualy have a rule, which is a way to find the value of each term-

e.g. $\{3, 5, 7, 9, ----\}$

$\{4, 3, 2, 1\}$

Sequence

- Infinite: $\{1, 2, 3, 4 --\}$
- finite: $\{a, b, c, d\}$
- Oscillating: $\{0, 1, 0, 1, 0, 1, --\}$

→ Sequence can also be defined as function whose domain is the set of all natural numbers and range may be any set S.

*Real sequence !- A real sequence is a function whose domain is the set of all natural numbers and range is the subset of set R of real numbers

$f: N \to R$

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* **Range of a sequence** :- The set of all distinct terms of a sequence is known as its range.

e.g. $\{-1, 1, -1, 1, -1, 1 - - - \infty\}$ (Infinite) is an oscillating sequence. Its range is $\{-1, 1\}$, which is a finite set

$\{x_n\}$ or $\{a_n\}$ or $\langle a_n \rangle$ Notation of a sequence

* **Constant sequence** :- A sequence $\{a_n\}$ defined by $a_n = c$, where c is a fixed real number, $\forall n \in N$ is called constant sequence.

$$\{a_n\} = \{c, c, c, c, - - - \infty\}$$

* **Bounded / Unbounded Sequence**

→ **Bounded above** :- A sequence $\{a_n\}$ is said to be bounded above if there exists a real number m such that

$$a_n \leq m \ \forall \ n \in N$$

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→ **Bounded below** :- A sequence $\{a_n\}$ is said to be bounded below if there exists a real number k such that

$$a_n \geq k \quad \forall n \in N$$

e.g. $\{a_n\} = \{\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \text{———} \infty\}$

Each term of above sequence lies b/w 0 and 1

i.e. $a_n \geq 0$ and $a_n \leq 1 \quad \forall n \in N$

↓ bounded below ↓ bounded above.

A sequence which is both bounded below and bounded above is know as **bounded sequence**.

→ **Unbounded sequence** :- The sequence which is neither bounded above nor bounded below is known as unbounded sequence.

e.g. Sequence $\{a_n\} = \{1, 2, 2^2, 2^3, \text{———} \infty\}$

$a_n = 2^{n-1}$

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here $a_n \geq 1 \quad \forall n \in N$

there exists no real number m such that
$a_n \leq m \quad \forall n \in N$

the sequence is unbounded above

Convergent Sequence :-

A sequence $\{a_n\}$ is said to be convergent if $\lim_{n \to \infty} a_n$ is finite
↓
n^{th} term of sequence

Divergent Sequence :- A sequence is said to be divergent if $\lim_{n \to \infty} a_n$ is not finite.

i.e. $\lim_{n \to \infty} a_n = +\infty$ or $-\infty$.

e.g

~~$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$~~

A sequence $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8} \dots\dots \infty$

or

$1, \frac{1}{2^1}, \frac{1}{2^2}, \frac{1}{2^3}, \dots \frac{1}{2^n}, \dots \infty$

here

$a_n = \frac{1}{2^n}$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{2^n} = \frac{1}{2^\infty} = \frac{1}{\infty} = 0 \text{ (finite)}$$

$\Rightarrow$ sequence $\{a_n\}$ i.e. $\{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, --\infty\}$ is Convergent

$\rightarrow$ Another sequence $\{a_n\} = \{1, 2, 3, 4, --n, --\infty\}$

here $a_n = n$

Now $\lim_{n\to\infty} a_n = \lim_{n\to\infty} n = \infty$ (not finite)

$\Rightarrow \{1, 2, 3, 4, ---\infty\}$ is divergent sequence.

Oscillatory Sequence :- A sequence which ~~is~~ neither Converges to a finite number nor diverges to $+\infty$ or $-\infty$ is known as oscillating sequences

Oscillate finitely   Oscillate infinitely

e.g. $\{a_n\} = \{-1, 1, -1, 1, -1, 1, --\infty\}$

$= \{(-1)^n\}$

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here $\lim_{n \to \infty} a_n = -1$ or 1

osillate finitely.

e.g.- Consider $\{a_n\} = \{(-1)^n . n\}$

$= \{-1, 2, -3, 4, -5, 6, - \text{———} \infty\}$

here $a_n = (-1)^n . n$

$\lim_{n \to \infty} a_n = +\infty$ or $-\infty$ (depends n is even or odd)

$\Rightarrow$ sequence oscillates infinitely.

Montonic Sequences

$\rightarrow$ A ~~monotonic~~ sequence $\{a_n\}$ is said to be monotonically increasing sequence if $a_{n+1} \geq a_n$ $\forall n \in N$

i.e. $a_1 \leq a_2 \leq a_3 - - \cdot \leq a_n \leq a_{n+1} \leq - - - -$

$\rightarrow$ A sequence $\{a_n\}$ is said to be monotonically decreasing sequence if $a_{n+1} \leq a_n$ $\forall n \in N$

i.e. $a_1 \geq a_2 \geq a_3 \geq - - a_n \geq a_{n+1} \geq$ ——

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