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Matrix Assignment

Q. Show that the eq^n's

$$3x+4y+5z=a$$
$$4x+5y+6z=b$$
$$5x+6y+7z=c$$

do not have sol^n unless ~~c = a + b~~ $a+c=2b$

$$[A:B] = \begin{bmatrix} 3 & 4 & 5 & : & a \\ 4 & 5 & 6 & : & b \\ 5 & 6 & 7 & : & c \end{bmatrix}$$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_2$

$$\begin{bmatrix} 3 & 4 & 5 & : & a \\ 1 & 1 & 1 & : & b-a \\ 1 & 1 & 1 & : & c-b \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$\begin{bmatrix} 3 & 4 & 5 & : & a \\ 1 & 1 & 1 & : & b-a \\ 0 & 0 & 0 & : & c+a-2b \end{bmatrix} \to 0$$

$\rho(A) = 2$ $\therefore$ $\rho(A:B)$ must be 2

$$c+a-2b=0$$

Q. For what value of λ, does system of eq^n's

$$-x+2y+z=1$$
$$3x-y+2z=1$$
$$y+\lambda z=1$$

1. no sol^n
2. unique sol^n

$$[A:B] = \begin{bmatrix} -1 & 2 & 1 & : & 1 \\ 3 & -1 & 2 & : & 1 \\ 0 & 1 & \lambda & : & 1 \end{bmatrix}$$

$R_2 \to 3R_1 + R_2 \Rightarrow \begin{bmatrix} -1 & 2 & 1 & : & 1 \\ 0 & 5 & 5 & : & 4 \\ 0 & 1 & \lambda & : & 1 \end{bmatrix}$

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$R_3 \to 5R_3 - R_2 \Rightarrow \begin{bmatrix} -1 & 2 & 1 & : & 1 \\ 0 & 5 & 5 & : & 4 \\ 0 & 0 & 5\lambda-5 & : & 1 \end{bmatrix}$

1. no sol^n, $\lambda = 0$
2. unique ~~many~~ sol^n, $\lambda \neq 1$

$z = \frac{1}{5(\lambda-1)}$, $y \Rightarrow \left(4 - \frac{5 \times 1}{5(\lambda-1)}\right) \times \frac{1}{5}$

$x \Rightarrow \frac{3\lambda-4}{5(\lambda-1)}$

Q. show that vectors $\underset{x_1}{(1,0,0)}$, $\underset{x_2}{(0,1,0)}$, $\underset{x_3}{(0,0,1)}$ are linearly dependent.

let $K_1x_1 + K_2x_2 + K_3x_3 = 0$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} K_1 \\ K_2 \\ K_3 \end{bmatrix} = 0$, $K_1 = 0, K_2 = 0, K_3 = 0$

$\therefore x_1, x_2, x_3$ are linearly independent

Q. Check $(1,2,3)$, $(4,-2,7)$.

$K_1(1,2,3) + K_2(4,-2,7) = 0$

$K_1 + 4K_2 = 0$

$2K_1 - 2K_2 = 0$

$3K_1 + 7K_2 = 0$

$K_1 = 0, K_2 = 0$

Q. Find rank of a matrix

$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 4 \\ 7 & 10 & 12 \end{bmatrix}$

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$R_2 \to R_2 - 3R_1$, $R_3 \to R_3 - 7R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & -5 \\ 0 & -4 & -9 \end{bmatrix}$$

$R_3 \to 2R_2 - R_3$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & -5 \\ 0 & 0 & -1 \end{bmatrix} \to \rho(A) = 3$$

Q. Find rank of

$$A \Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 6 & 10 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow R_3 \to R_3 - R_2$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = 2$

Q. prove that

$$\begin{vmatrix} x+\lambda & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda \end{vmatrix} = \lambda^2 (3x+\lambda)$$

$R_1 \to R_1 + R_2 + R_3$

$$(3x+\lambda) \begin{vmatrix} 1 & 1 & 1 \\ x & x+\lambda & x \\ x & x & x+\lambda \end{vmatrix}$$

$C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$

$$(3x+\lambda) \begin{vmatrix} 1 & 0 & 0 \\ x & \lambda & 0 \\ x & 0 & \lambda \end{vmatrix} \Rightarrow (3x+\lambda)\lambda^2$$

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Q. Show that $\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix} = 4a^2b^2c^2$

taking a, b, c common from $(R_1, C_1), (R_2, C_2), (R_3, C_3)$

$$a^2b^2c^2 \begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix} \rightarrow \begin{matrix} R_1 \rightarrow R_1 + R_2 \\ R_2 \rightarrow R_2 - R_3 \end{matrix}$$

$$\Rightarrow 4a^2b^2c^2$$

Q. solve linear eq^{n}s $2x + 3y = 5$, $3x - 2y = 1$ with help of determinants.

$2x + 3y = 5$

$3x - 2y = 1$

$$D = \begin{vmatrix} 2 & 3 \\ 3 & -2 \end{vmatrix} \Rightarrow -13 \neq 0$$

$$D_1 \Rightarrow \begin{vmatrix} 5 & 3 \\ 1 & -2 \end{vmatrix} \Rightarrow -13$$

$$D_2 \Rightarrow \begin{vmatrix} 2 & 5 \\ 3 & 1 \end{vmatrix} \Rightarrow -13$$

$x = 1$, $y = 1$

Q. Solve following system using Cramer's rule.

$x + 4y - 2z = 3$

$3x + y + 5z = 7$

$2x + 3y + z = 5$

$$D = \begin{vmatrix} 1 & 4 & -2 \\ 3 & 1 & 5 \\ 2 & 3 & 1 \end{vmatrix} \Rightarrow 0$$

$-14 + 4 \times 7 - 2 \times 7$

$$\Delta_1 = \begin{vmatrix} 3 & 4 & -2 \\ 7 & 1 & 5 \\ 5 & 3 & 1 \end{vmatrix}$$

$\Rightarrow 3 \times -14 + 4 \times 18 - 2 \times 16$

$\Rightarrow -42 + 72 - 32 \Rightarrow -2$

$\therefore$ no solⁿ

Q. Find inverse by Gauss Jordan Method.

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$$

$$[A:I] = \left[\begin{array}{ccc:ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 3 & 2 & 3 & 0 & 1 & 0 \\ 1 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$$

$R_2 \to R_2 - 3R_1$, $R_3 \to R_3 - R_1$

$$\left[\begin{array}{ccc:ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & -4 & 0 & -3 & 1 & 0 \\ 0 & -1 & 1 & -1 & 0 & 1 \end{array}\right]$$

$R_2 \to (-1/4)R_2$

$$\left[\begin{array}{ccc:ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 3/4 & -1/4 & 0 \\ 0 & -1 & 1 & -1 & 0 & 1 \end{array}\right]$$

$R_1 \to R_1 - 2R_2$, $R_3 \to R_3 + R_2$

$$\left[\begin{array}{ccc:ccc} 1 & 0 & 1 & -1/2 & 1/2 & 0 \\ 0 & 1 & 0 & 3/4 & -1/4 & 0 \\ 0 & 0 & 1 & -1/4 & -1/4 & 1 \end{array}\right]$$

$R_1 \to R_1 - R_3$

$$\Rightarrow \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & -1/4 & 3/4 & -1 \\ 0 & 1 & 0 & 3/4 & -1/4 & 0 \\ 0 & 0 & 1 & -1/4 & -1/4 & 1 \end{array}\right]$$

A^{-1}