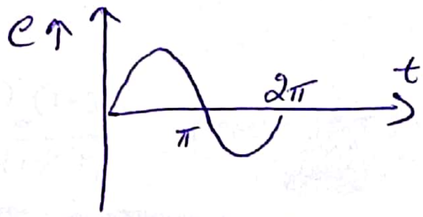
# Unit-2 AC Fundamentals

BEE Unit 2 A1

→ In direct current (dc) the flow of current is steady and in one direction only.

→ For large scale power generation, transmission and distribution, ac system is adopted.

→ A voltage that changes its polarity and magnitude at regular intervals of time is alternating voltage.




## Generation of Alternating voltage and current

An alternating voltage can be generated either

(1) By rotating a coil in uniform magnetic field at constant speed

(2) By rotating a uniform magnetic field within stationary coil

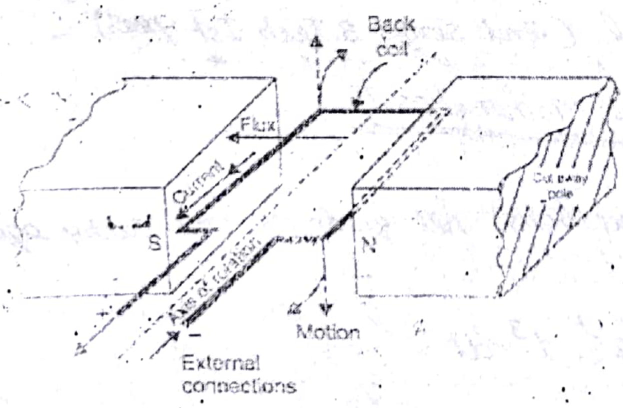



Fig. 1. Rotating a coil in a stationary magnetic field.

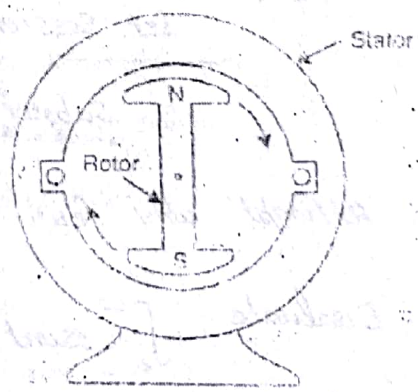
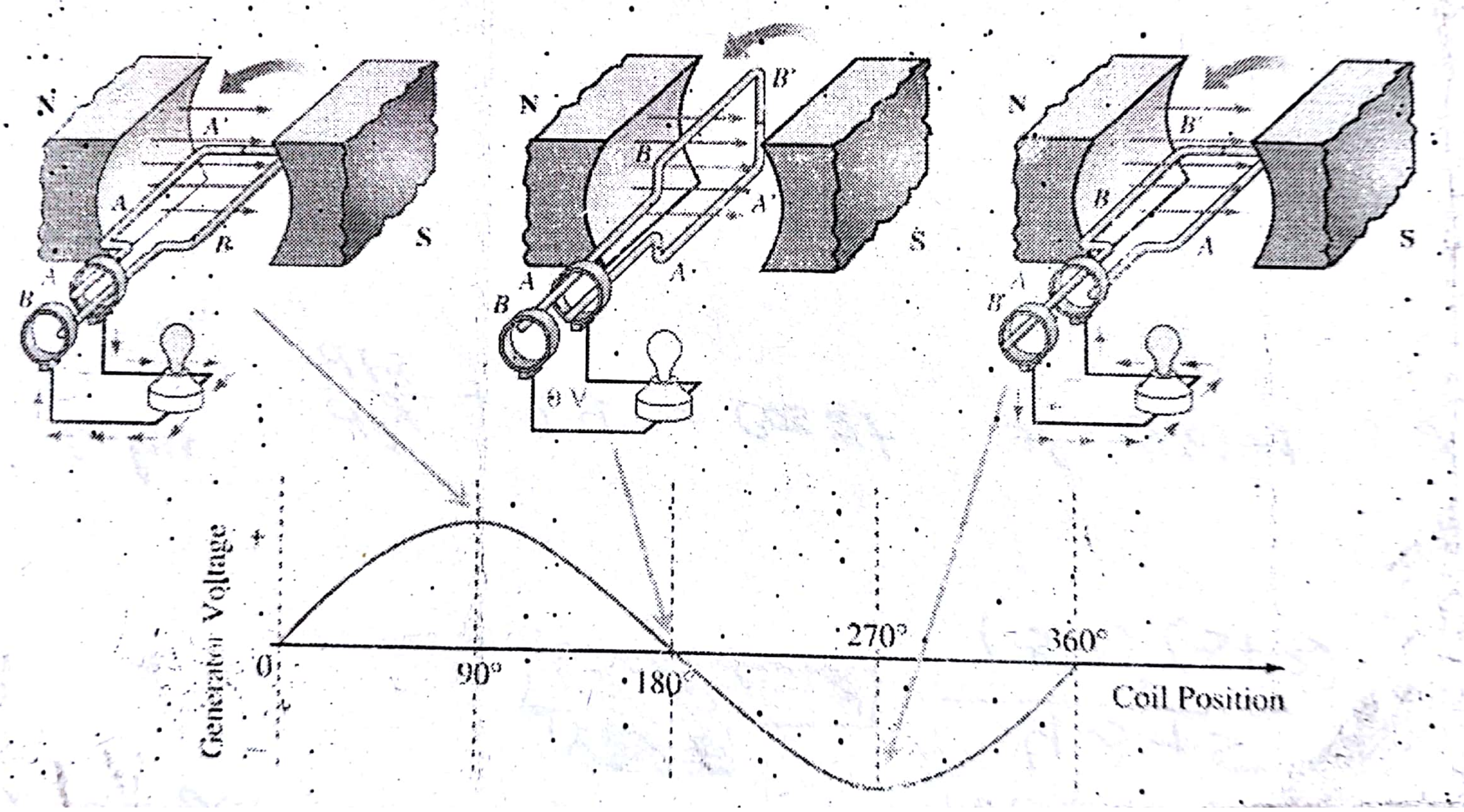



Fig. 2. Rotating a magnetic field within a stationary coil.

# Generating AC Voltage

- Rotating a coil in fixed magnetic field generates sinusoidal voltage.

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## Equation of Alternating Emf and Current

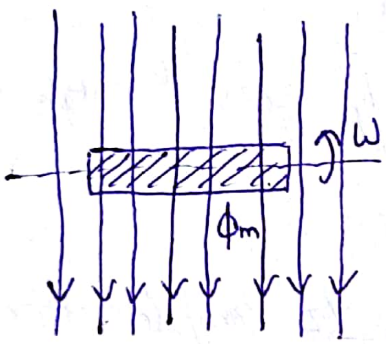


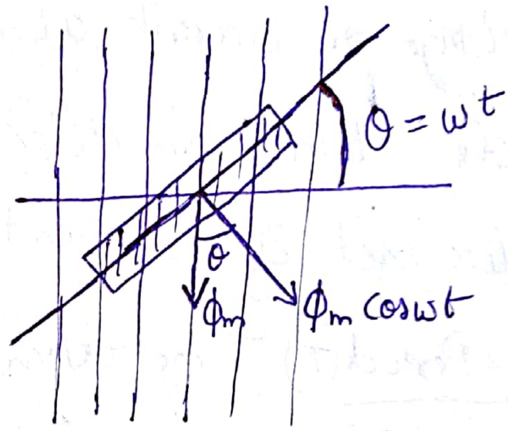
→ Consider a coil having N turns rotating in uniform magnetic field at angular velocity ω rad/sec

→ Maximum Flux $\phi_m$ is linking with the coil.

→ After t sec, coil is rotated through angle $\theta = \omega t$

⇒




Component of flux linking with the coil is $\phi_m \cos\omega t$

Instantaneous value of emf induced in coil, $e = -N \dfrac{d\phi}{dt}$

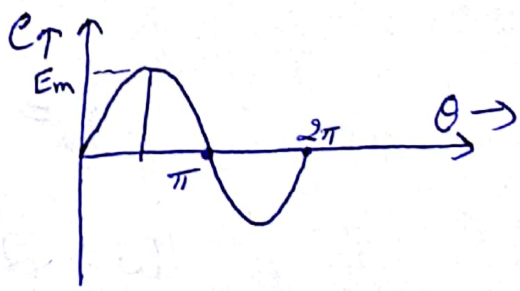
$$e = -N \frac{d(\phi_m \cos\omega t)}{dt}$$

$$= -N\phi_m(-\omega \sin\omega t)$$

$$e = \omega N \phi_m \sin\omega t$$

Value of e will be max. when $\omega t = 1$, $E_m = \omega N \phi_m$

$$\boxed{e = E_m \sin\omega t}$$




$$e = E_m \sin\theta$$

Eq'n of Alternating current

$$\boxed{i = I_m \sin\omega t}$$

## Important Terms:

1. **Waveform:** Shape of the curve obtained by plotting the instantaneous values of alternating quantity (voltage or current) along y-axis and time or angle along x-axis.

2. **Instantaneous value:** Value of alternating quantity i.e. voltage or current at any instant.

3. **Cycle:** When an alternating quantity goes through a complete set of +ve and -ve values.

4. **Time Period (T)** Time taken in seconds to complete one cycle by an alternating quantity

5. **Frequency (f)** No. of cycles made per sec. by alternating quantity

   Unit → Hertz (Hz)

6. **Amplitude:** Max. value (+ve or -ve) attained by an alternating quantity in one cycle.

   Denoted by $E_m / V_m / I_m$

$$\boxed{f = \frac{1}{T}}$$

$$\boxed{\omega = 2\pi f}$$

# Values of Alternating voltage and current

Three ways are adopted to express alternating quantities:
(1) Peak value (2) Average/Mean value (3) RMS/Effective value

## (1) Peak value

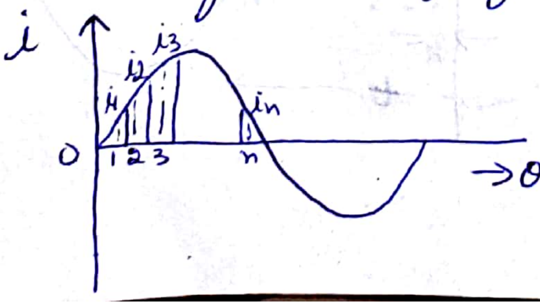
Max. value attained by an alternating quantity during one cycle is called its peak value.
This is also called maximum value / Crest / Amplitude.
Peak value of alternating voltage and current is represented by $E_m$ and $I_m$.

## (2) Average value

The arithmatic average of all instantaneous value considered of an alternating quantity over one cycle is called average value

→ In case of symmetrical waves (sinusoidal), +ve half is equal to -ve half, therefore avg. value over complete cycle is zero.

→ There only +ve half cycle is considered

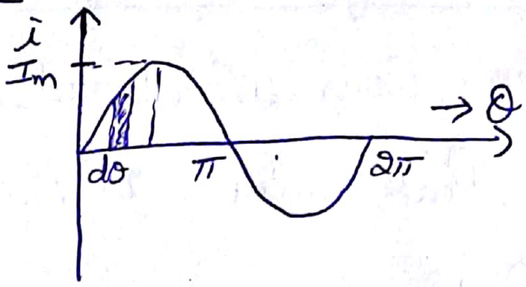



Let $i_1, i_2, i_3 \ldots i_n$ be mid ordinates

$$I_{av} = \frac{i_1 + i_2 + i_3 + \cdots i_n}{n}$$

## Average value of sinusoidal current

$$\boxed{I_{av} = \frac{1}{T}\int_0^T i \, d\theta}$$




Where $i = I_m \sin\theta$ [Eq'n of Alternating Current]

$$I_{av} = \frac{1}{\pi}\int_0^{\pi} I_m \sin\theta \, d\theta = \frac{I_m}{\pi}\int_0^{\pi} \sin\theta \, d\theta$$

$$= \frac{I_m}{\pi}\left[-\cos\theta\right]_0^{\pi} = -\frac{I_m}{\pi}\left[\cos\pi - \cos 0\right]$$

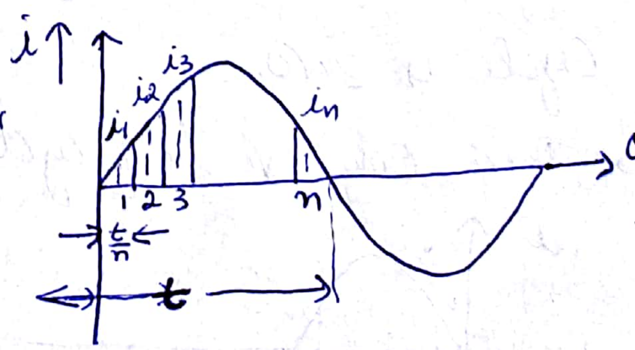
$$I_{av} = -\frac{I_m}{\pi}\left[-1-1\right] = \boxed{\frac{2I_m}{\pi}}$$

Avg value of current = $\cdot 637 \times$ Max. value of current

## (3) Effective or RMS value

That steady current which when flows through a resistor of known resistance for a given time produces the same amount of heat as produced by the alternating current when flows through the same resistor for the same time is known as effective or RMS value

Let $i$ be the alternating current flowing through a resistance $R$ for $t$ sec which produces the same amount of heat as produces by $I_{eff}$



Each Intervals is of $\frac{t}{n}$ seconds

Let $i_1, i_2 \cdots i_n$ be the mid ordinates

Heat produced in Ist Intervals $= i_1^2 R \frac{t}{n}$

" " " 2nd Interval $= i_2^2 R \frac{t}{n}$

" " " $n^{th}$ Interval $= i_n^2 R \frac{t}{n}$

Total Heat produced $= i_1^2 R \frac{t}{n} + i_2^2 R \frac{t}{n} + \cdots i_n^2 R \frac{t}{n}$

$$= Rt\left(\frac{i_1^2 + i_2^2 + \cdots i_n^2}{n}\right)$$

$I_{eff}$ is considered as effective value of this current

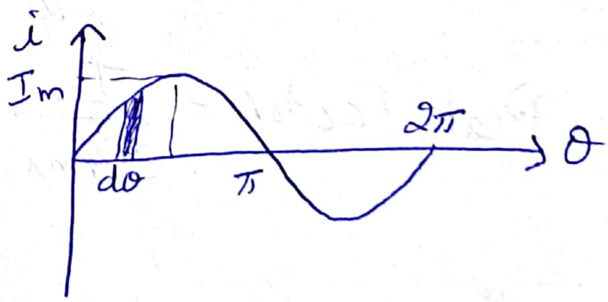
$$I_{eff}^2 Rt = Rt\left(\frac{i_1^2 + i_2^2 + \cdots i_n^2}{n}\right)$$

$$\boxed{I_{eff} = \sqrt{\frac{i_1^2 + i_2^2 + \cdots i_n^2}{n}}}$$

## RMS value of sinusoidal current

Eq'n of Alternating current

$$i = I_m \sin\theta$$




$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2 d\theta}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} (I_m \sin\theta)^2 d\theta} = \sqrt{\frac{I_m^2}{2\pi}\int_0^{2\pi} \sin^2\theta \, d\theta}$$

$$= \sqrt{\frac{I_m^2}{2\pi}\int_0^{2\pi} \left(\frac{1-\cos 2\theta}{2}\right) d\theta}$$

$$I_{rms} = \sqrt{\frac{I_m^2}{4\pi}\left([\theta]_0^{2\pi} - \left[\frac{\sin 2\theta}{2}\right]_0^{2\pi}\right)}$$

$$= \sqrt{\frac{I_m^2}{4\pi}\left[(2\pi - 0) - (0-0)\right]} = \sqrt{\frac{I_m^2}{4\pi} \times 2\pi}$$

$$= \sqrt{\frac{I_m^2}{2}}$$

$$\boxed{I_{rms} = \frac{I_m}{\sqrt{2}}}$$

RMS value of current = .707 × Max. value of current

**Form Factor** = Ratio of RMS value to average value of an alternating quantity

$$\text{Form Factor} = \frac{I_{rms}}{I_{av}} = \frac{I_m/\sqrt{2}}{2I_m/\pi} = 1.11$$

**Peak Factor** = Ratio of Maximum value to RMS value of an alternating quantity

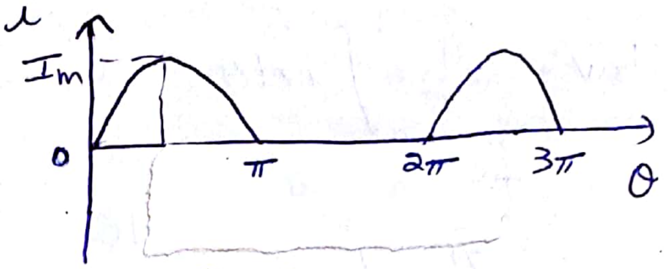
$$\text{Peak Factor} = \frac{I_m}{I_{rms}} = \frac{I_m}{I_m/\sqrt{2}} = \sqrt{2} = 1.4142$$

# Avg and RMS value of Half Wave Rectified output

$$I_{av} = \frac{1}{T}\int_0^T i\,d\theta$$

$i = I_m \sin\theta \qquad 0 < \theta < \pi$

$i = 0 \qquad \pi < \theta < 2\pi$




$T = 2\pi$

$$I_{av} = \frac{1}{2\pi}\int_0^{2\pi} i\,d\theta = \frac{1}{2\pi}\left[\int_0^{\pi} I_m \sin\theta\,d\theta + \int_{\pi}^{2\pi} 0\,d\theta\right]$$

$$= \frac{1}{2\pi}\int_0^{\pi} I_m \sin\theta\,d\theta = \frac{I_m}{2\pi}\left[-\cos\theta\right]_0^{\pi} = \frac{-I_m}{2\pi}\left[\cos\pi - \cos 0\right]$$

$$I_{av} = \frac{-I_m}{2\pi}\left[-1-1\right]$$

$$\boxed{I_{av} = \frac{I_m}{\pi}}$$

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2\,d\theta} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} I_m^2 \sin^2\theta\,d\theta}$$

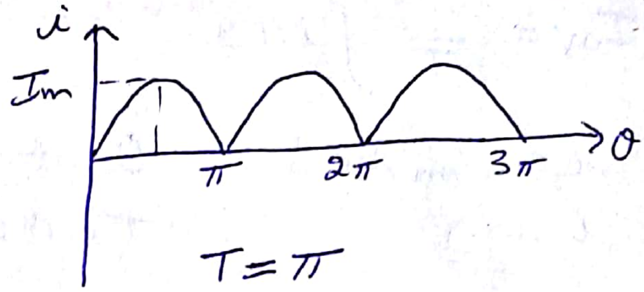
$$= \sqrt{\frac{I_m^2}{2\pi}\int_0^{\pi}\left(\frac{1-\cos 2\theta}{2}\right)d\theta} = \sqrt{\frac{I_m^2}{4\pi}\left(\left[\theta\right]_0^{\pi} - \left[\frac{\sin 2\theta}{2}\right]_0^{\pi}\right)}$$

$$= \sqrt{\frac{I_m^2}{4\pi}\left[(\pi-0)-(0-0)\right.} = \sqrt{\frac{I_m^2}{4}}$$

$$\boxed{I_{rms} = \frac{I_m}{2}}$$

Form Factor $= \dfrac{\frac{I_m}{2}}{\frac{I_m}{\pi}} = \dfrac{\pi}{2} = 1.5714$, Peak Factor $= \dfrac{I_m}{\frac{I_m}{2}} = 2$

## Average and RMS Value of Full Wave Rectified Output :




$$I_{av} = \frac{1}{T}\int_0^T i\,d\theta$$

$$= \frac{1}{\pi}\int_0^{\pi} I_m \sin\theta\, d\theta$$

$$= \frac{I_m}{\pi}\left[-\cos\theta\right]_0^{\pi}$$

$$= -\frac{I_m}{\pi}\left[\cos\pi - \cos 0^\circ\right] = \frac{2I_m}{\pi}$$

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2\, d\theta}$$

$$= \sqrt{\frac{1}{\pi}\int_0^{\pi} I_m^2 \sin^2\theta\, d\theta}$$

$$= \sqrt{\frac{I_m^2}{\pi}\int_0^{\pi}\left(\frac{1-\cos 2\theta}{2}\right)d\theta} = \sqrt{\frac{I_m^2}{2\pi}\left(\left[\theta\right]_0^{\pi} - \left[\frac{\sin 2\theta}{2}\right]_0^{\pi}\right)}$$

$$= \sqrt{\frac{I_m^2}{2\pi}\times\pi}$$

$$I_{rms} = \frac{I_m}{\sqrt{2}}$$

$$\text{Form Factor} = \frac{I_m/\sqrt{2}}{2I_m/\pi} = 1.11$$

$$\text{Peak Factor} = \frac{I_m}{I_m/\sqrt{2}} = \sqrt{2} = 1.4142$$

Q1 An alternating current is represented by $i = 70.7 \sin 520t$. Determine (1) frequency (2) current at time 0.0015 sec. (3) RMS and avg. value.

Ans Standard Eq'n $i = I_m \sin \omega t$

$$i = 70.7 \sin 520t$$

$$I_m = 70.7 A$$

$$\omega t = 520t$$

$$2\pi f = 520$$

(1) $\boxed{f = 83 Hz}$

(2) $i = 70.7 \sin (520 \times 0.0015)$

$$i = 70.7 \sin (\underbrace{0.78}_{\text{radians}})$$

$\pi$ radian $= 180°$

1 radian $= \frac{180°}{\pi}$

$i = 70.7 \sin (44.71°)$

0.78 radian $= \frac{180}{\pi} \times 0.78$

$= 44.71°$

$i = 70.7 \cancel{\sin} \; .7033$

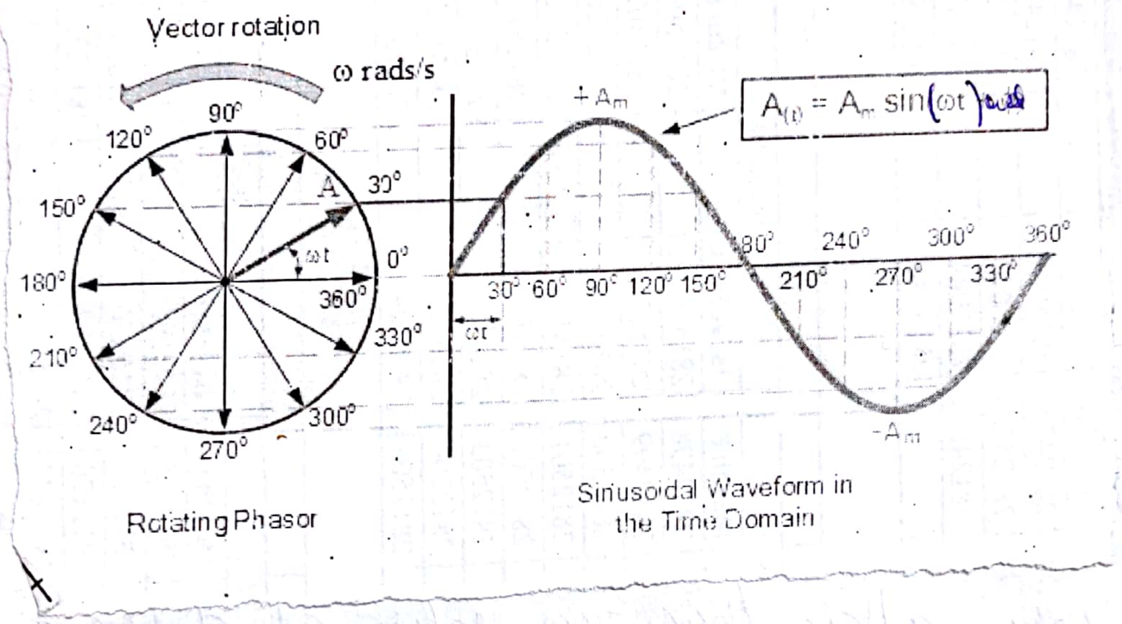
$\boxed{i = 49.72 A}$

(3) $I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{70.7}{\sqrt{2}} = 50 A$

$I_{av} = \frac{2 I_m}{\pi} = \frac{2 \times 70.7}{\pi} = 45 A$

# Phasor Representation of Alternating quantities

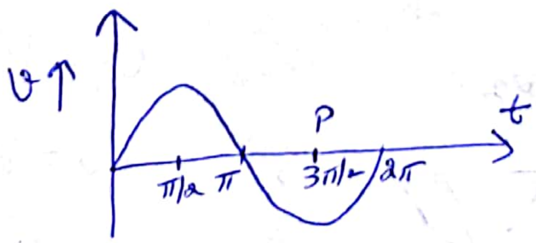
→ An AC signal can be represented by portion of rotating vector which shows its magnitude and direction.




→ Its projection on y-axis represents its Instantaneous value

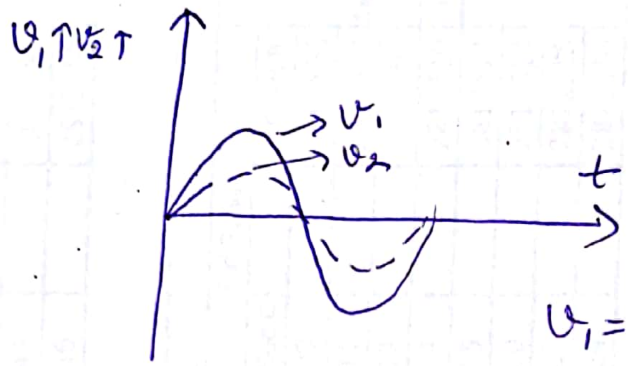
## Phase and Phase Difference:

The phase of an alternating quantity at an instant is defined as fractional part of cycle through which the quantity has advanced from selected origin.




At Point P, Phase is $\frac{3\pi}{2}$

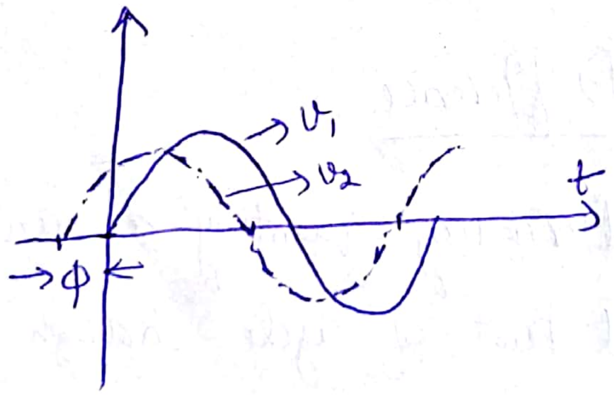
→ Two alternating quantities are said to be in phase, if they accomplish their zero value and max. value at the same time




$v_1 = V_{m_1} \sin\theta$

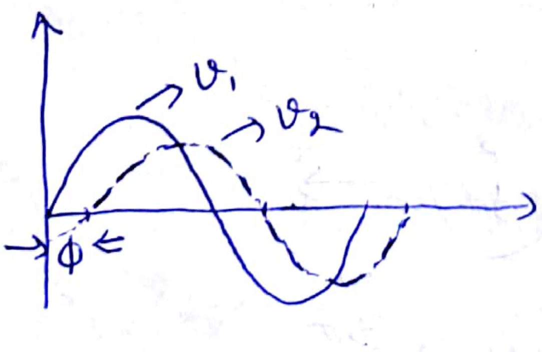
$v_2 = V_{m_2} \sin\theta$

→ Phase Difference: Two alternating quantities having same frequency when attain their zero value at different instants the quantities are said to have phase difference.




$v_1 = V_{m_1} \sin\omega t$

$v_2 = V_{m_2} \sin(\omega t + \phi)$




$v_1 = V_{m_1} \sin\omega t$

$v_2 = V_{m_2} \sin(\omega t - \phi)$.

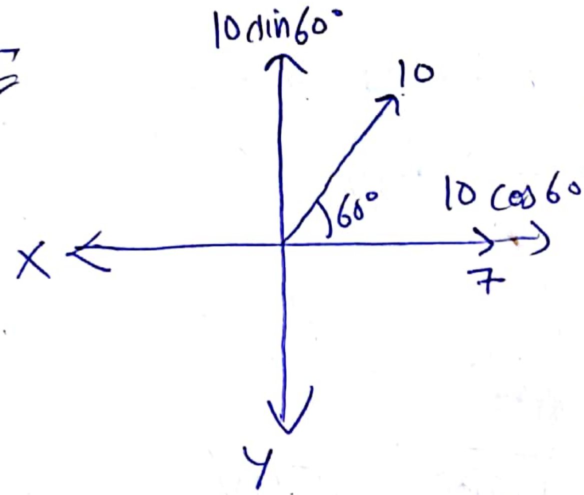
# Addition and Subtraction of Alternating Quantities



**Q1** Add the following currents

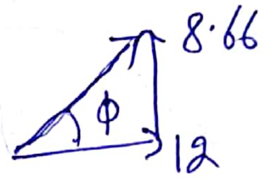
$$i_1 = 7 \sin \omega t, \quad i_2 = 10 \sin(\omega t + \pi/3)$$

**A**




X – Component $= 7 + 10\cos 60° = 12$

Y – Component $= 10 \sin 60° = 8.66$




$$I_m = \sqrt{12^2 + 8.66^2} = 14.8$$

$$\phi = \tan^{-1}\left(\frac{8.66}{12}\right) = 35.8°$$

$$\boxed{i_r = 14.8 \sin(\omega t + 35.8°)}$$

**Q2** Three voltages represented by $e_1 = 20 \sin \omega t$, $e_2 = 30 \sin(\omega t - \pi/4)$, $e_3 = 40 \cos(\omega t + \pi/6)$ act together in a circuit. Find an expression of resultant voltage

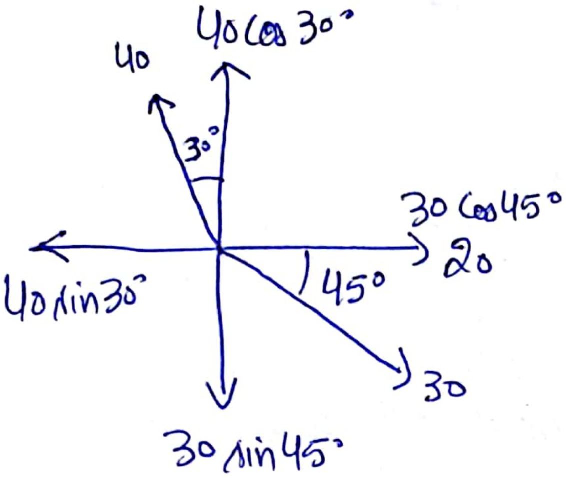
**Ans**

$$e_1 = 20 \sin \omega t$$

$$e_2 = 30 \sin(\omega t - \pi/4)$$

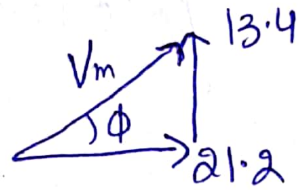
$$e_3 = 40 \cos(\omega t + \pi/6) = 40 \sin\left(\omega t + \frac{\pi}{6} + 90°\right)$$

$$= 40 \sin(\omega t + 120°)$$




$X$ - component $= 20 + 30\cos 45° - 40\sin 30°$
$= 21.2V$

$Y$ - component $= 40\cos 30° - 30\sin 45°$
$= 13.4V$




$$V_m = \sqrt{21.2^2 + 13.4^2} = 25.1V$$

$$\phi = \tan^{-1}\left(\frac{13.4}{21.2}\right) = 32.3°$$

$$\boxed{e_r = 25.1 \sin(\omega t + 32.3°)}$$

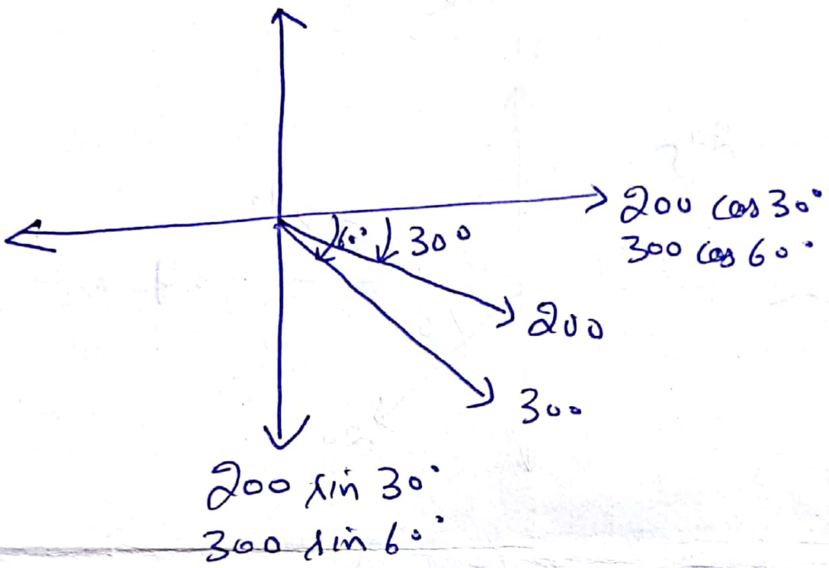
A9

Q2 A current $i_1 = 300\sin(\omega t - \pi/3)$ and $i_2 = 200\sin(\omega t - \pi/6)$ are flowing in a circuit. Determine resultant current.

Ans

$$i_1 = 300\sin(\omega t - 60^\circ)$$

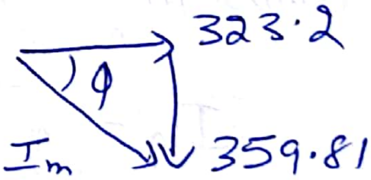
$$i_2 = 200\sin(\omega t - 30^\circ)$$




X - Component = $200\cos 30 + 300\cos 60^\circ = 323.2$

Y - Component = $200\sin 30^\circ + 300\sin 60^\circ = 359.81$

$$I_m = \sqrt{x^2+y^2}$$




$$= \sqrt{323.2^2 + 359.81^2}$$

$$= 483.654 \text{ A}$$

$$\phi = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{359.81}{323.2}\right) = 48.07^\circ$$

$$\boxed{i_r = 483.654\sin(\omega t - 48.07^\circ)}$$

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# Addition and Subtraction of Alternating Quantities

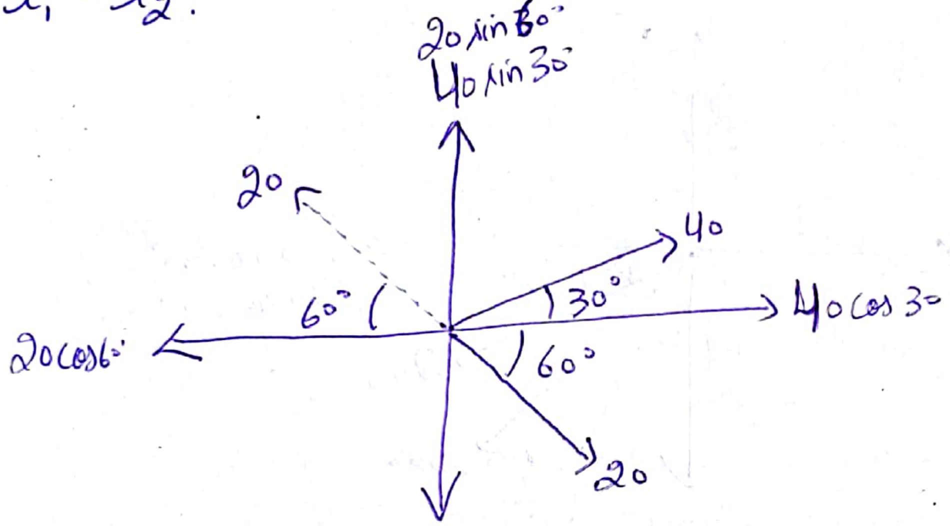
Q Two currents $i_1$ and $i_2$ are given by the expression

$$i_1 = 40 \sin(314t + \pi/6)$$
$$i_2 = 20 \sin(314t - \pi/3)$$

Subtraction

Find $i_1 - i_2$.

Ans

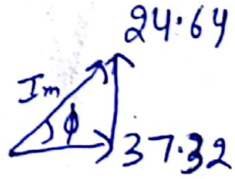



$$i_1 - i_2 = i_1 + (-i_2)$$

X - Component $= 40 \cos 30 - 20 \cos 60° = 24.64\,A$

Y - Component $= 40 \sin 30 + 20 \sin 60° = 37.32\,A$

$$I_m = \sqrt{x^2 + y^2} = \sqrt{24.64^2 + 37.32^2}$$

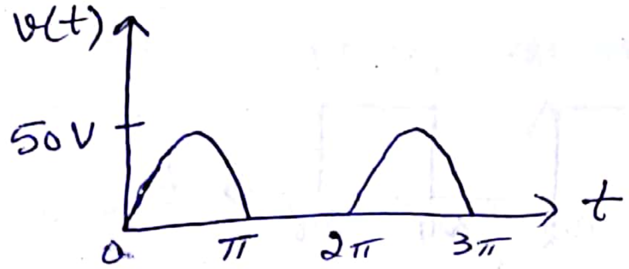



$$= 44.72\,A$$

$$\phi = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{37.32}{24.64}\right) = 56.56°$$

$$\boxed{I_r = 44.72 \sin(314t + 56.56°)}$$

Q1. Find the form factor and peak factor for the sine wave.




Ans

$$v(t) = \begin{cases} 50\sin\theta & 0<\theta<\pi \\ 0 & \pi<\theta<2\pi \end{cases}$$

Time Period, $T = 2\pi$

Average Value

$$V_{av} = \frac{1}{T}\int_0^T v(t)\,d\theta$$

$$= \frac{1}{2\pi}\int_0^{2\pi} v(t)\,d\theta$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} 50\sin\theta\,d\theta + \int_{\pi}^{2\pi} 0\,d\theta\right]$$

$$= \frac{1}{2\pi}\int_0^{\pi} 50\sin\theta\,d\theta = \frac{50}{2\pi}\left[-\cos\theta\right]_0^{\pi}$$

$$= \frac{50}{2\pi}\left[-(-1-1)\right] = \frac{50}{2\pi}\times 2 = \frac{50}{\pi} = 15.91V$$

RMS Value

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,d\theta}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{\pi}(50\sin\theta)^2 d\theta} = \sqrt{\frac{2500}{2\pi}\int_0^{\pi}\left(\frac{1-\cos 2\theta}{2}\right)d\theta}$$

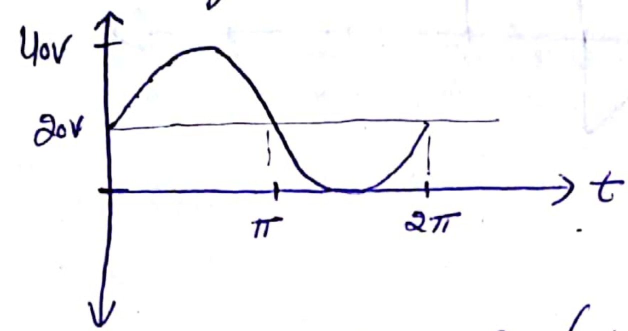
$$= \sqrt{\frac{2500}{4\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}} = \sqrt{\frac{2500}{4\pi}\left[\pi - 0 - (0-0)\right]}$$

$$= \sqrt{\frac{2500\times\pi}{4\pi}} = \sqrt{625} = 25V$$

Form Factor $= \frac{V_{rms}}{V_{av}} = 1.5713$, Peak Factor $= \frac{V_m}{V_{rms}} = 2$

All

Q6. A circuit carries a DC voltage of 20V and sinusoidal alternating voltage as shown. Find rms and avg value.




Ans

$$v = 20 + 20\sin\theta = 20(1+\sin\theta)$$

$$V_{av} = \frac{1}{T}\int_0^T v(\theta)\,d\theta = \frac{1}{2\pi}\int_0^{2\pi}\left[20(1+\sin\theta)\right]d\theta$$

$$= \frac{20}{2\pi}\left[\theta - \cos\theta\right]_0^{2\pi} = \frac{20}{2\pi}\left[(2\pi-0)-(\cos 2\pi - \cos 0)\right]$$

$$= \frac{20}{2\pi}\times 2\pi = 20\,V$$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(\theta)\,d\theta} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi}(20+20\sin\theta)^2\,d\theta}$$

$$= \sqrt{\frac{400}{2\pi}\int_0^{2\pi}(1+\sin\theta)^2\,d\theta} = \sqrt{\frac{400}{2\pi}\int_0^{2\pi}(1+\sin^2\theta+2\sin\theta)\,d\theta}$$

$$= \sqrt{\frac{400}{2\pi}\int_0^{2\pi}\left(1+\frac{1-\cos 2\theta}{2}+2\sin\theta\right)d\theta}$$

$$= \sqrt{\frac{400}{2\pi}\left[\theta+\frac{1}{2}\theta-\frac{\sin 2\theta}{4}-2\cos\theta\right]_0^{2\pi}}$$

$$= \sqrt{\frac{400}{2\pi}\left[(2\pi-0)+\frac{1}{2}(2\pi-0)-\left(\frac{\sin 4\pi-\sin 0}{4}\right)-2(\cos 2\pi-\cos 0)\right]}$$

$$= \sqrt{\frac{400}{2\pi}\times 3\pi} = \sqrt{600} = 24.5\,V$$

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