

Fundamental Haskell notes

Encyclopedcal handbook for learning and undersatanding fundamentals

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Contents

| | | |
|-----------|-----------------------------------|-----------|
| I | Introduction | 22 |
| II | Definitions | 25 |
| 1 | Algebra | 26 |
| 1.1 | * | 26 |
| 1.2 | Algebraic | 26 |
| 1.3 | Algebraic structure | 26 |
| 1.3.1 | * | 26 |
| 1.3.2 | Fundamental theorem of algebra | 27 |
| 1.4 | Modular arithmetic | 27 |
| 1.4.1 | * | 27 |
| 1.4.2 | Modulus | 27 |
| 1.4.2.1 | * | 27 |
| 2 | Category theory | 28 |
| 2.1 | * | 28 |
| 2.2 | Abelian category | 28 |
| 2.2.1 | * | 29 |
| 2.3 | Composition | 29 |
| 2.3.1 | * | 29 |
| 2.4 | Endofunctor category | 29 |
| 2.5 | Functor | 29 |
| 2.5.1 | * | 30 |
| 2.5.2 | Power set functor | 30 |
| 2.5.2.1 | * | 30 |
| 2.5.2.2 | Power set functor laws | 30 |
| 2.5.2.2.1 | * | 30 |
| 2.5.2.2.2 | Power set functor identity law | 31 |
| 2.5.2.2.3 | Power set functor composition law | 31 |
| 2.5.2.3 | Lift | 31 |
| 2.5.2.3.1 | * | 31 |
| 2.5.2.4 | Power set functor is a free monad | 31 |
| 2.5.3 | Functorial | 31 |
| 2.5.4 | Forgetful functor | 31 |
| 2.5.4.1 | * | 31 |
| 2.5.5 | Identity functor | 31 |
| 2.5.6 | Endofunctor | 31 |
| 2.5.6.1 | * | 31 |
| 2.5.7 | Applicative functor | 32 |
| 2.5.7.1 | * | 32 |
| 2.5.7.2 | Applicative law | 32 |
| 2.5.7.3 | * | 32 |

| | | |
|--------------|---|----|
| 2.5.7.3.1 | Applicative identity law | 32 |
| 2.5.7.3.2 | Applicative composition law | 32 |
| 2.5.7.3.3 | Applicative homomorphism law | 32 |
| 2.5.7.3.4 | Applicative interchange law | 32 |
| 2.5.7.4 | Applicative function | 33 |
| 2.5.7.4.1 | liftA* | 33 |
| 2.5.7.4.1.1 | liftA | 33 |
| 2.5.7.4.1.2 | liftA2 | 33 |
| 2.5.7.4.1.3 | «<liftA2 (<*>)»> | 33 |
| 2.5.7.4.1.4 | liftA2 (liftA2 (<*>)) | 33 |
| 2.5.7.4.1.5 | liftA3 | 33 |
| 2.5.7.4.2 | Conditional applicative computations | 33 |
| 2.5.7.5 | Special applicatives | 33 |
| 2.5.7.5.1 | Identity applicative | 33 |
| 2.5.7.5.2 | Constant applicative | 34 |
| 2.5.7.5.3 | Maybe applicative | 34 |
| 2.5.7.5.4 | Either applicative | 34 |
| 2.5.7.5.5 | Validation applicative | 34 |
| 2.5.7.6 | Monad | 34 |
| 2.5.7.6.1 | * | 35 |
| 2.5.7.6.2 | Monad law | 36 |
| 2.5.7.6.2.1 | * | 36 |
| 2.5.7.6.2.2 | Monad left identity law | 36 |
| 2.5.7.6.2.3 | Monad right identity law | 36 |
| 2.5.7.6.2.4 | Monad associativity law | 37 |
| 2.5.7.6.3 | Monad type class | 38 |
| 2.5.7.6.3.1 | MonadPlus type class | 38 |
| 2.5.7.6.4 | Functor -> Applicative -> Monad progression | 38 |
| 2.5.7.6.5 | Monad function | 39 |
| 2.5.7.6.5.1 | Return function | 39 |
| 2.5.7.6.5.2 | Join function | 39 |
| 2.5.7.6.5.3 | Bind function | 39 |
| 2.5.7.6.5.4 | Sequencing operator (>>) \equiv (>>): | 40 |
| 2.5.7.6.5.5 | Monadic versions of list functions | 40 |
| 2.5.7.6.5.6 | liftM* | 40 |
| 2.5.7.6.6 | Comonad | 40 |
| 2.5.7.6.7 | Kleisli arrow | 41 |
| 2.5.7.6.7.1 | * | 41 |
| 2.5.7.6.8 | Kleisli composition | 41 |
| 2.5.7.6.9 | Kleisli category | 41 |
| 2.5.7.6.10 | Special monad | 41 |
| 2.5.7.6.10.1 | Identity monad | 41 |
| 2.5.7.6.10.2 | Maybe monad | 42 |
| 2.5.7.6.10.3 | Either monad | 42 |
| 2.5.7.6.10.4 | Error monad | 42 |
| 2.5.7.6.10.5 | List monad | 43 |
| 2.5.7.6.10.6 | Reader monad | 43 |
| 2.5.7.6.10.7 | Writer monad | 44 |
| 2.5.7.6.10.8 | State monad | 45 |
| 2.5.7.6.11 | Monad transformer | 46 |
| 2.5.7.6.11.1 | MaybeT | 47 |
| 2.5.7.6.11.2 | EitherT | 47 |
| 2.5.7.6.11.3 | ReaderT | 47 |
| 2.5.7.6.11.4 | MonadTrans type class | 47 |
| 2.5.7.7 | Alternative type class | 49 |

| | | | |
|--------|-------------------|---------------------|-----------------------------------|
| | 2.5.7.7.1 | * | 49 |
| 2.5.8 | Monoidal functor | | 49 |
| 2.5.9 | Fusion | | 49 |
| 2.5.10 | \$> | | 49 |
| | 2.5.10.1 | * | 49 |
| 2.5.11 | Multifunctor | | 49 |
| | 2.5.11.1 | * | 50 |
| 2.6 | Hask category | | 50 |
| 2.6.1 | * | | 50 |
| 2.7 | Magma | | 50 |
| 2.7.1 | Mag category | | 50 |
| | 2.7.1.1 | * | 50 |
| 2.7.2 | Semigroup | | 50 |
| | 2.7.2.1 | * | 51 |
| | 2.7.2.2 | Monoid | 51 |
| | | 2.7.2.2.1 | * |
| | | 2.7.2.2.2 | Monoid laws |
| | | | 2.7.2.2.2.1 |
| | | | 2.7.2.2.2.2 |
| | | | 2.7.2.2.2.3 |
| | | | 2.7.2.2.3 |
| | | | 2.7.2.2.3.1 |
| | | | 2.7.2.2.4 |
| | | | 2.7.2.2.4.1 |
| | | | 2.7.2.2.4.2 |
| 2.8 | Morphism | | 52 |
| 2.8.1 | * | | 52 |
| 2.8.2 | Homomorphism | | 53 |
| | 2.8.2.1 | * | 53 |
| 2.8.3 | Identity morphism | | 53 |
| | 2.8.3.1 | Identity | 53 |
| | | 2.8.3.1.1 | Two-sided identity of a predicate |
| | | 2.8.3.1.2 | Left identity of a predicate |
| | | 2.8.3.1.3 | Right identity of a predicate |
| | 2.8.3.2 | Identity function | 53 |
| 2.8.4 | Monomorphism | | 54 |
| | 2.8.4.1 | * | 54 |
| 2.8.5 | Epimorphism | | 54 |
| | 2.8.5.1 | * | 54 |
| 2.8.6 | Isomorphism | | 54 |
| | 2.8.6.1 | * | 54 |
| | 2.8.6.2 | Lax | 55 |
| 2.8.7 | Endomorphism | | 55 |
| | 2.8.7.1 | Automorphism | 55 |
| | | 2.8.7.1.1 | * |
| | 2.8.7.2 | * | 55 |
| 2.8.8 | Catamorphism | | 55 |
| | 2.8.8.1 | * | 55 |
| | 2.8.8.2 | Catamorphism law | 55 |
| | | 2.8.8.2.1 | Hylomorphism |
| | | | 2.8.8.2.1.1 |
| | 2.8.8.3 | Anamorphism | 56 |
| | | 2.8.8.3.1 | * |
| 2.8.9 | Kernel | | 56 |
| | 2.8.9.1 | Kernel homomorphism | 56 |

| | | |
|------------|-----------------------------------|-----------|
| 2.9 | Set category | 56 |
| 2.10 | Natural transformation | 56 |
| 2.10.1 | * | 58 |
| 2.10.2 | Natural transformation component | 58 |
| 2.10.2.1 | * | 58 |
| 2.10.3 | Natural transformation in Haskell | 58 |
| 2.10.4 | Cat category | 58 |
| 2.10.4.1 | * | 58 |
| 2.10.4.2 | Bicategory | 58 |
| 2.11 | Category dual | 59 |
| 2.11.0.0.1 | * | 59 |
| 2.11.1 | Coalgebra | 59 |
| 2.12 | Thin category | 59 |
| 2.12.1 | * | 59 |
| 2.13 | Commuting diagram | 59 |
| 2.13.1 | * | 59 |
| 2.14 | Universal construction | 59 |
| 2.14.1 | * | 60 |
| 2.15 | Product | 60 |
| 2.15.1 | * | 60 |
| 2.16 | Coproduct | 60 |
| 2.16.1 | * | 60 |
| 2.17 | Free object | 61 |
| 2.18 | Internal category | 61 |
| 2.19 | Hom set | 61 |
| 2.19.1 | * | 61 |
| 2.19.2 | Hom-functor | 61 |
| 2.19.3 | Exponential object | 61 |
| 2.19.3.1 | * | 62 |
| 2.19.3.2 | Enriched category | 62 |
| 2.19.3.2.1 | * | 62 |
| 3 | Data type | 63 |
| 3.1 | * | 63 |
| 3.2 | Actual type | 63 |
| 3.3 | Algebraic data type | 63 |
| 3.3.1 | * | 63 |
| 3.4 | Cardinality | 63 |
| 3.4.1 | * | 63 |
| 3.5 | Data constant | 63 |
| 3.6 | Data constructor | 63 |
| 3.7 | data declaration | 64 |
| 3.8 | Dependent type | 64 |
| 3.8.1 | * | 64 |
| 3.9 | Gen type | 64 |
| 3.10 | Higher-kinded data type | 64 |
| 3.10.1 | * | 64 |
| 3.11 | newtype declaration | 64 |
| 3.12 | Principal type | 64 |
| 3.13 | Product data type | 65 |
| 3.13.1 | * | 65 |
| 3.13.2 | Sequence | 65 |
| 3.13.2.1 | * | 65 |
| 3.13.2.2 | List | 65 |
| 3.14 | Proxy type | 66 |

| | | |
|----------|-------------------------|----|
| 3.15 | Static typing | 66 |
| 3.16 | Structural type | 66 |
| 3.16.1 | * | 66 |
| 3.17 | Structural type system | 66 |
| 3.17.1 | * | 66 |
| 3.18 | Sum data type | 66 |
| 3.19 | Type alias | 66 |
| 3.20 | Type class | 67 |
| 3.20.1 | * | 67 |
| 3.20.2 | Arbitrary type class | 67 |
| 3.20.2.1 | Arbitrary function | 67 |
| 3.20.3 | CoArbitrary type class | 67 |
| 3.20.3.1 | * | 67 |
| 3.20.4 | Typeable type class | 67 |
| 3.20.4.1 | * | 67 |
| 3.20.5 | Type class inheritance | 67 |
| 3.20.6 | Derived instance | 67 |
| 3.20.6.1 | * | 68 |
| 3.21 | Type constant | 68 |
| 3.22 | Type constructor | 68 |
| 3.23 | type declaration | 68 |
| 3.24 | Typed hole | 68 |
| 3.24.1 | * | 68 |
| 3.25 | Type inference | 68 |
| 3.25.1 | * | 69 |
| 3.26 | Type class instance | 69 |
| 3.27 | Type rank | 69 |
| 3.27.1 | * | 69 |
| 3.28 | Type variable | 69 |
| 3.29 | Unlifted type | 69 |
| 3.29.1 | * | 69 |
| 3.30 | Linear type | 70 |
| 3.30.1 | * | 70 |
| 3.31 | NonEmpty list data type | 70 |
| 3.32 | Session type | 70 |
| 3.33 | Binary tree | 70 |
| 3.34 | Bottom value | 70 |
| 3.34.1 | * | 70 |
| 3.35 | Bound | 70 |
| 3.35.1 | * | 70 |
| 3.36 | Constructor | 71 |
| 3.36.1 | * | 71 |
| 3.37 | Context | 71 |
| 3.37.1 | * | 71 |
| 3.38 | Inhabit | 71 |
| 3.39 | Maybe | 71 |
| 3.39.0.1 | * | 71 |
| 3.40 | Expected type | 71 |
| 3.41 | ADT | 71 |
| 3.42 | Concrete type | 72 |
| 3.43 | Type punning | 72 |
| 3.44 | Kind | 72 |
| 3.44.1 | * | 72 |
| 3.45 | IO | 72 |

| | | |
|----------|------------------------|-----------|
| 4 | Expression | 73 |
| 4.1 | * | 73 |
| 4.2 | Closed-form expression | 73 |
| 4.3 | RHS | 73 |
| 4.4 | LHS | 73 |
| 4.5 | Redex | 73 |
| 4.6 | Concatenate | 74 |
| 4.7 | Alpha equivalence | 74 |
| 4.8 | Ground expression | 74 |
| 4.8.1 | * | 74 |
| 4.9 | Variable | 74 |
| 4.9.1 | * | 74 |
| 4.10 | Phrase | 74 |
| 5 | Function | 75 |
| 5.1 | * | 75 |
| 5.2 | Arity | 75 |
| 5.3 | Bijection | 76 |
| 5.3.1 | * | 76 |
| 5.4 | Combinator | 76 |
| 5.4.1 | Ψ -combinator | 76 |
| 5.4.1.1 | * | 76 |
| 5.5 | Function application | 76 |
| 5.5.1 | * | 77 |
| 5.6 | Function body | 77 |
| 5.7 | Function composition | 77 |
| 5.7.1 | * | 77 |
| 5.8 | Function head | 77 |
| 5.9 | Function range | 77 |
| 5.10 | Higher-order function | 77 |
| 5.10.1 | * | 77 |
| 5.10.2 | Fold | 77 |
| 5.11 | Injection | 78 |
| 5.11.1 | * | 78 |
| 5.12 | Partial function | 78 |
| 5.13 | Purity | 78 |
| 5.13.1 | * | 78 |
| 5.14 | Pure function | 78 |
| 5.15 | Sectioning | 78 |
| 5.16 | Surjection | 78 |
| 5.16.1 | * | 79 |
| 5.17 | Unsafe function | 79 |
| 5.17.1 | * | 79 |
| 5.18 | Variadic | 79 |
| 5.19 | Domain | 79 |
| 5.20 | Codomain | 79 |
| 5.21 | Open formula | 79 |
| 5.22 | Recursion | 79 |
| 5.22.1 | * | 79 |
| 5.22.2 | Base case | 79 |
| 5.22.3 | Tail recursion | 79 |
| 5.22.4 | Polymorphic recursion | 79 |
| 5.22.4.1 | * | 80 |
| 5.23 | Free variable | 80 |
| 5.24 | Closure | 80 |

| | | |
|----------|------------------------------|-----------|
| 5.24.1 | * | 80 |
| 5.25 | Parameter | 80 |
| 5.25.1 | * | 80 |
| 5.26 | Partial application | 80 |
| 5.26.1 | * | 80 |
| 5.27 | Well-formed formula | 80 |
| 5.27.1 | * | 80 |
| 6 | Homotopy | 81 |
| 6.1 | * | 81 |
| 7 | Lambda calculus | 82 |
| 7.1 | * | 82 |
| 7.2 | Lambda cube | 82 |
| 7.2.1 | * | 83 |
| 7.3 | Lambda function | 83 |
| 7.3.1 | * | 83 |
| 7.3.2 | Anonymous lambda function | 84 |
| 7.3.2.1 | * | 84 |
| 7.3.3 | Uncurry | 84 |
| 7.4 | β -reduction | 84 |
| 7.4.1 | * | 84 |
| 7.4.2 | β -normal form | 84 |
| 7.4.2.1 | * | 84 |
| 7.5 | Calculus of constructions | 84 |
| 7.5.1 | * | 84 |
| 7.6 | Curry–Howard correspondence | 84 |
| 7.6.1 | * | 84 |
| 7.7 | Currying | 85 |
| 7.7.1 | * | 85 |
| 7.8 | Hindley–Milner type system | 85 |
| 7.8.1 | * | 85 |
| 7.9 | Reduction | 85 |
| 7.9.1 | * | 85 |
| 7.10 | β - η normal form | 85 |
| 7.10.1 | * | 85 |
| 7.11 | η -abstraction | 85 |
| 7.11.1 | * | 86 |
| 7.12 | Lambda expression | 86 |
| 8 | Operation | 87 |
| 8.1 | Constant | 87 |
| 8.2 | Binary operation | 87 |
| 8.2.1 | * | 87 |
| 8.3 | Operator | 87 |
| 8.3.1 | Shift operator | 87 |
| 8.3.1.1 | * | 87 |
| 8.3.2 | Differential operator | 87 |
| 8.3.2.1 | * | 87 |
| 8.4 | Infix | 88 |
| 8.5 | Fixity | 88 |
| 8.5.1 | * | 88 |
| 8.6 | Zero | 88 |
| 8.7 | Bind | 88 |
| 8.7.1 | * | 88 |

| | |
|--|-----------|
| 8.8 Declaration | 89 |
| 8.9 Dispatch | 89 |
| 8.10 Evaluation | 89 |
| 9 Permutation | 90 |
| 10 Point-free | 91 |
| 10.1 * | 91 |
| 10.2 Blackbird | 91 |
| 10.2.1 * | 91 |
| 10.3 Swing | 91 |
| 10.4 Squish | 92 |
| 11 Polymorphism | 93 |
| 11.1 * | 93 |
| 11.2 Levity polymorphism | 93 |
| 11.3 Parametric polymorphism | 93 |
| 11.3.1 Rank-1 polymorphism | 93 |
| 11.3.1.1 * | 93 |
| 11.3.2 Let-bound polymorphism | 93 |
| 11.3.3 Constrained polymorphism | 94 |
| 11.3.3.1 Ad hoc polymorphism | 94 |
| 11.3.3.1.0.1 * | 94 |
| 11.3.4 Impredicative polymorphism | 94 |
| 11.3.4.1 * | 94 |
| 11.3.5 Higher-rank polymorphism | 94 |
| 11.3.5.1 * | 94 |
| 11.4 Subtype polymorphism | 94 |
| 11.5 Row polymorphism | 95 |
| 11.6 Kind polymorphism | 95 |
| 11.7 Linearity polymorphism | 95 |
| 12 Pragma | 96 |
| 12.1 LANGUAGE pragma | 96 |
| 12.1.1 LANGUAGE option | 96 |
| 12.1.1.1 * | 96 |
| 12.1.1.2 Useful by default | 96 |
| 12.1.1.3 AllowAmbiguousTypes | 96 |
| 12.1.1.4 ApplicativeDo | 96 |
| 12.1.1.5 ConstrainedClassMethods | 97 |
| 12.1.1.6 CPP | 97 |
| 12.1.1.7 DeriveFunctor | 97 |
| 12.1.1.8 ExplicitForAll | 97 |
| 12.1.1.9 FlexibleContexts | 97 |
| 12.1.1.10 FlexibleInstances | 97 |
| 12.1.1.11 GeneralizedNewtypeDeriving | 97 |
| 12.1.1.12 ImplicitParams | 98 |
| 12.1.1.13 LambdaCase | 98 |
| 12.1.1.14 MultiParamTypeClasses | 98 |
| 12.1.1.15 MultiWayIf | 98 |
| 12.1.1.16 OverloadedStrings | 98 |
| 12.1.1.17 PartialTypeSignatures | 98 |
| 12.1.1.18 RankNTypes | 98 |
| 12.1.1.19 ScopedTypeVariables | 99 |
| 12.1.1.20 TupleSections | 99 |
| 12.1.1.21 TypeApplications | 99 |

| | | |
|--------------|--------------------------------------|------------|
| 12.1.1.22 | TypeSynonymInstances | 99 |
| 12.1.1.23 | UndecidableInstances | 100 |
| 12.1.1.24 | ViewPatterns | 100 |
| 12.1.1.25 | DatatypeContexts | 100 |
| 12.1.1.26 | StandaloneKindSignatures | 100 |
| 12.1.1.26.1 | * | 101 |
| 12.1.1.27 | PartialTypeSignatures | 101 |
| 12.1.2 | How to make a GHC LANGUAGE extension | 101 |
| 13 | Compositionality | 102 |
| 13.1 | * | 102 |
| 14 | Referential transparency | 103 |
| 14.1 | * | 103 |
| 15 | Semantics | 104 |
| 15.1 | Operational semantics | 104 |
| 15.1.1 | Argument | 104 |
| 15.1.1.1 | Argument of a function | 104 |
| 15.1.1.1.1 | * | 104 |
| 15.1.1.2 | First-class | 104 |
| 15.1.2 | Relation | 104 |
| 15.1.2.1 | * | 105 |
| 15.2 | Denotational semantics | 105 |
| 15.2.1 | Abstraction | 105 |
| 15.2.1.1 | * | 105 |
| 15.2.1.2 | Leaky abstraction | 105 |
| 15.2.1.2.1 | * | 105 |
| 15.2.1.3 | Object | 106 |
| 15.2.1.3.1 | * | 106 |
| 15.2.1.3.2 | Arrow | 106 |
| 15.2.1.3.2.1 | * | 106 |
| 15.2.1.3.3 | Terminal object | 106 |
| 15.2.1.3.4 | Initial object | 106 |
| 15.2.1.3.5 | Value | 107 |
| 15.2.1.3.5.1 | * | 107 |
| 15.2.1.3.6 | Tensor | 107 |
| 15.2.1.3.6.1 | * | 107 |
| 15.2.2 | Ambigram | 107 |
| 15.2.3 | Binary | 107 |
| 15.2.4 | Arbitrary | 107 |
| 15.2.5 | Refutable | 108 |
| 15.2.6 | Irrefutable | 108 |
| 15.2.7 | Superclass | 108 |
| 15.2.8 | Unit | 108 |
| 15.2.9 | Nullary | 108 |
| 15.2.10 | Syntax tree | 108 |
| 15.2.10.1 | Abstract syntax tree | 108 |
| 15.2.10.1.1 | * | 108 |
| 15.2.10.2 | Concrete syntax tree | 108 |
| 15.2.10.2.1 | * | 108 |
| 15.2.11 | Stream | 109 |
| 15.2.12 | Linear | 109 |
| 15.2.12.1 | * | 109 |
| 15.2.13 | Predicative | 109 |

| | | |
|--------------|----------------------------------|------------|
| 15.2.14 | Quantifier | 109 |
| 15.2.14.1 | * | 109 |
| 15.2.14.2 | Forall quantifier | 109 |
| 15.2.14.2.1 | * | 109 |
| 15.3 | Axiomatic semantics | 110 |
| 15.3.1 | Property | 110 |
| 15.3.1.1 | * | 110 |
| 15.3.1.2 | Associativity | 110 |
| 15.3.1.2.1 | * | 110 |
| 15.3.1.3 | Left associative | 110 |
| 15.3.1.3.1 | * | 110 |
| 15.3.1.4 | Right associative | 110 |
| 15.3.1.5 | Non-associative | 110 |
| 15.3.1.6 | Basis | 111 |
| 15.3.1.6.1 | Contravariant | 111 |
| 15.3.1.6.1.1 | * | 111 |
| 15.3.1.6.2 | Covariant | 111 |
| 15.3.1.6.2.1 | * | 111 |
| 15.3.1.7 | Commutativity | 111 |
| 15.3.1.7.1 | * | 111 |
| 15.3.1.8 | Idempotence | 111 |
| 15.3.1.8.1 | * | 111 |
| 15.3.1.9 | Distributive property | 111 |
| 15.3.1.9.1 | * | 112 |
| 15.3.2 | Effect | 112 |
| 15.3.3 | Bisimulation | 112 |
| 15.3.3.1 | * | 112 |
| 15.4 | Content word | 112 |
| 15.5 | Ancient Greek and Latin prefixes | 112 |
| 15.5.1 | * | 112 |
| 15.6 | Idiom | 112 |
| 15.6.1 | * | 112 |
| 15.7 | Impredicative | 112 |
| 15.8 | Context-free grammar | 112 |
| 15.8.1 | * | 114 |
| 16 | Set | 115 |
| 16.1 | * | 115 |
| 16.2 | Closed set | 115 |
| 16.3 | Power set | 115 |
| 16.4 | Singleton | 115 |
| 16.5 | Russell's paradox | 115 |
| 16.6 | Cartesian product | 115 |
| 16.6.1 | Pullback | 116 |
| 16.6.1.1 | * | 116 |
| 17 | Testing | 117 |
| 17.1 | Property testing | 117 |
| 17.1.1 | Function property | 117 |
| 17.1.2 | Property testing type | 117 |
| 17.1.3 | Generator | 117 |
| 17.1.3.1 | * | 117 |
| 17.1.3.2 | Custom generator | 118 |
| 17.1.4 | Reusing test code | 118 |
| 17.1.4.1 | Test Commutative property | 118 |

| | | |
|--------------|--|------------|
| 17.1.4.2 | Test Symmetry property | 118 |
| 17.1.4.3 | Test Equivalence property | 118 |
| 17.1.4.4 | Test Inverse property | 118 |
| 17.1.5 | QuickCheck | 119 |
| 17.1.5.1 | Manual automation with QuickCheck properties | 119 |
| 17.2 | Write tests algorithm | 119 |
| 17.3 | Shrinking | 120 |
| 18 | Logic | 121 |
| 18.1 | Proposition | 121 |
| 18.1.1 | * | 121 |
| 18.1.2 | Atomic proposition | 121 |
| 18.1.2.1 | * | 121 |
| 18.1.3 | Compound proposition | 121 |
| 18.1.3.1 | * | 121 |
| 18.1.4 | Propositional logic | 121 |
| 18.1.4.1 | * | 121 |
| 18.1.4.2 | First-order logic | 122 |
| 18.1.4.2.1 | * | 122 |
| 18.1.4.2.2 | Second-order logic | 122 |
| 18.1.4.2.2.1 | Higher-order logic | 122 |
| 18.2 | Logical connective | 122 |
| 18.2.1 | * | 122 |
| 18.2.2 | Conjunction | 122 |
| 18.2.3 | Disjunction | 122 |
| 18.3 | Predicate | 122 |
| 18.4 | Statement | 123 |
| 18.4.1 | * | 123 |
| 18.5 | Iff | 123 |
| 19 | Haskell structure | 124 |
| 19.1 | * | 124 |
| 19.2 | Pattern match | 124 |
| 19.2.1 | As-pattern | 124 |
| 19.2.1.1 | * | 124 |
| 19.2.2 | Wild-card | 124 |
| 19.2.2.1 | * | 124 |
| 19.2.3 | Case | 124 |
| 19.2.4 | Guard | 125 |
| 19.2.4.1 | * | 125 |
| 19.2.5 | Pattern guard | 125 |
| 19.2.5.1 | * | 125 |
| 19.2.6 | Lazy pattern | 125 |
| 19.2.6.1 | * | 126 |
| 19.2.7 | Pattern binding | 126 |
| 19.2.7.1 | * | 126 |
| 19.3 | Smart constructor | 126 |
| 19.4 | Level of code | 126 |
| 19.4.1 | * | 126 |
| 19.4.2 | Type level | 126 |
| 19.4.2.1 | Type level declaration | 126 |
| 19.4.2.1.1 | * | 126 |
| 19.4.2.2 | Type check | 127 |
| 19.4.2.2.1 | * | 127 |
| 19.4.2.2.2 | Complete user-specific kind signature | 127 |

| | |
|---|------------|
| 19.4.2.2.2.1 * | 127 |
| 19.4.3 Term level | 127 |
| 19.4.4 Compile level | 127 |
| 19.4.4.1 * | 127 |
| 19.4.5 Runtime level | 127 |
| 19.4.6 Kind level | 127 |
| 19.4.6.1 Kind check | 127 |
| 19.4.6.1.1 * | 128 |
| 19.5 Orphan instance | 128 |
| 19.6 undefined | 128 |
| 19.7 Hierarchical module name | 128 |
| 19.7.1 * | 133 |
| 19.8 Reserved word | 133 |
| 19.8.1 * | 133 |
| 19.8.2 import | 133 |
| 19.8.3 let | 134 |
| 19.8.3.1 * | 134 |
| 19.8.4 where | 134 |
| 19.8.4.1 * | 134 |
| 19.9 Haskell Language Report | 134 |
| 19.9.1 * | 135 |
| 19.10Haskell' | 135 |
| 19.10.1 * | 135 |
| 19.11Lense | 135 |
| 20 Computer science | 136 |
| 20.1 Guerrilla patch | 136 |
| 20.1.1 Monkey patch | 136 |
| 20.2 Interface | 136 |
| 20.3 Module | 136 |
| 20.4 Scope | 136 |
| 20.4.1 Dynamic scope | 136 |
| 20.4.2 Lexical scope | 136 |
| 20.4.2.1 * | 136 |
| 20.4.3 Local scope | 137 |
| 20.4.3.1 * | 137 |
| 20.5 Shadowing | 137 |
| 20.6 Syntactic sugar | 137 |
| 20.7 System F | 137 |
| 20.7.1 * | 137 |
| 20.8 Tail call | 137 |
| 20.9 Thunk | 137 |
| 20.10Application memory | 137 |
| 20.11Turing machine | 138 |
| 20.11.1 Turing complete | 138 |
| 20.11.1.1 * | 138 |
| 20.12REPL | 138 |
| 20.13Domain specific language | 138 |
| 20.13.1 * | 138 |
| 20.13.2 Embedded domain specific language | 138 |
| 20.13.2.1 * | 138 |
| 20.14Data structure | 138 |
| 20.14.1 Cons cell | 138 |
| 20.14.2 Construct | 138 |
| 20.14.2.1 * | 138 |

| | |
|--|------------|
| 20.14.3 Leaf | 139 |
| 20.14.4 Node | 139 |
| 20.14.5 Spine | 139 |
| 21 Graph theory | 140 |
| 21.1 Successor | 140 |
| 21.1.1 Direct successor | 140 |
| 21.2 Predecessor | 140 |
| 21.2.1 Direct predecessor | 140 |
| 21.3 Degree | 140 |
| 21.3.1 Indegree | 140 |
| 21.3.2 Outdegree | 140 |
| 21.4 Adjacency matrix | 140 |
| 21.4.0.1 InstanceSigs | 140 |
| 21.5 Strongly connected | 141 |
| 21.5.1 * | 141 |
| 21.5.2 Strongly connected component | 141 |
| 21.5.2.1 * | 141 |
| 22 Tagless-final | 142 |
| III Give definitions | 143 |
| 23 Identity type | 144 |
| 24 Constant type | 145 |
| 25 Gen | 146 |
| 26 Tensorial strength | 147 |
| 27 Strong monad | 148 |
| 28 Weak head normal form | 149 |
| 28.1 * | 149 |
| 29 Function image | 150 |
| 29.1 * | 150 |
| 30 Invertible | 151 |
| 31 Invertibility | 152 |
| 32 Define LANGUAGE pragma options | 153 |
| 32.1 ExistentialQuantification | 153 |
| 32.2 GADTs | 153 |
| 32.3 * | 153 |
| 32.4 GeneralizedNewTypeClasses | 153 |
| 32.5 FuncitonalDependencies | 153 |
| 33 GHC check keys | 154 |
| 33.1 -Wno-partial-type-signatures | 154 |
| 34 Generalised algebraic data types | 155 |
| 34.1 * | 155 |

| | |
|---|------------|
| 35 Order theory | 156 |
| 35.1 Domain theory | 156 |
| 35.2 Lattice | 156 |
| 35.3 Order | 156 |
| 35.3.1 Preorder | 156 |
| 35.3.1.1 * | 156 |
| 35.3.1.2 Total preorder | 156 |
| 35.3.2 Partial order | 156 |
| 35.3.2.1 * | 157 |
| 35.4 Partial order | 157 |
| 35.5 Total order | 157 |
| 36 Universal algebra | 158 |
| 37 Relation | 159 |
| 37.1 Reflexivity | 159 |
| 37.1.1 * | 159 |
| 37.2 Irreflexivity | 159 |
| 37.2.1 * | 159 |
| 37.3 Transitivity | 159 |
| 37.3.1 * | 159 |
| 37.4 Symmetry | 159 |
| 37.4.1 * | 159 |
| 37.5 Equivalence | 160 |
| 37.5.1 * | 160 |
| 37.6 Antisymmetry | 160 |
| 37.6.1 * | 160 |
| 37.7 Asymmetry | 160 |
| 37.7.1 * | 160 |
| 38 Cryptomorphism | 161 |
| 38.1 * | 161 |
| 39 Lexically scoped type variables | 162 |
| 40 Abstract data type | 163 |
| 40.1 * | 163 |
| 41 Functional dependencies | 164 |
| 42 MonoLocalBinds | 165 |
| 43 KindSignatures | 166 |
| 44 ExplicitNamespaces | 167 |
| 45 Combinator pattern | 168 |
| 46 Symbolic expression | 169 |
| 46.1 * | 169 |
| 47 Polynomial | 170 |
| 47.1 * | 170 |
| 48 Data family | 171 |
| 49 Type synonym family | 172 |

| | |
|---|------------|
| 50 Indexed type family | 173 |
| 50.1 * | 173 |
| 51 TypeFamilies | 174 |
| 52 Error | 175 |
| 52.1 * | 175 |
| 53 Exception | 176 |
| 53.1 * | 176 |
| 54 ConstraintKinds | 177 |
| 55 Specialisation | 178 |
| 55.1 * | 178 |
| 56 Diagram | 179 |
| 57 Cathegory theoretical presheaf | 180 |
| 58 Topological presheaf | 181 |
| 59 Diagonal functor | 182 |
| 60 Limit functor | 183 |
| 61 Dual vector space | 184 |
| 62 Fundamental group | 185 |
| 63 Algebra of continuous function | 186 |
| 64 Tangent and cotangent bundle | 187 |
| 65 Group action / representation | 188 |
| 66 Lie algebra | 189 |
| 67 Tensor product | 190 |
| 68 Forgetful functor | 191 |
| 69 Free functor | 192 |
| 70 Homomorphism group | 193 |
| 71 Representable functor | 194 |
| 72 Corecursion | 195 |
| 73 Coinduction | 196 |
| 74 Initial algebra of an endofunctor | 197 |
| 75 Terminal coalgebra for an endofunctor | 198 |

| | |
|---|------------|
| IV Citations | 199 |
| V Good code | 201 |
| 76 Good: Type aliasing | 202 |
| 77 Good: Type wideness | 203 |
| 78 Good: Print | 204 |
| 79 Good: Fold | 205 |
| 80 Good: Computation model | 206 |
| 81 Good: Make bottoms only local | 207 |
| 82 Good: Newtype wrap is ideally transparent for compiler and does not change performance | 208 |
| 83 Good: Instances of types/type classes must go with code you write | 209 |
| 84 Good: Functions can be abstracted as arguments | 210 |
| 85 Good: Infix operators can be bind to arguments | 211 |
| 86 Good: Arbitrary | 212 |
| 87 Good: Principle of Separation of concerns | 213 |
| 88 Good: Function composition | 214 |
| 89 Good: Point-free | 215 |
| 89.1 Good: Point-free is great in multi-dimentionions | 215 |
| 90 Good: Functor application | 216 |
| 91 Good: Parameter order | 217 |
| 92 Good: Applicative monoid | 218 |
| 93 Good: Creative process | 219 |
| 93.1 Pick phylosophy principles one to three the more - the harder the implementation | 219 |
| 93.2 Draw the most blurred representation | 219 |
| 93.3 Deduce abstractions and write remotely what they are | 219 |
| 93.4 Model of computation | 219 |
| 93.4.1 Model the domain | 219 |
| 93.4.2 Model the types | 219 |
| 93.4.3 Think how to write computations | 219 |
| 93.5 Create | 219 |
| 94 Good: About operators (<\$) (**>) (<*) (>>) | 220 |
| 95 Good: About functions like {mapM, sequence}_ | 221 |
| 96 Good: Guideliles | 222 |
| 96.1 Wiki.haskell | 222 |
| 96.1.1 Documentation | 222 |
| 96.1.1.1 Comments write in application terms, not technical. | 222 |

| | | |
|------------|--|------------|
| 96.1.1.2 | Tell what code needs to do not how it does. | 222 |
| 96.1.2 | Haddock | 222 |
| 96.1.2.1 | Put haddock comments to ever exposed data type and function. | 222 |
| 96.1.2.2 | Haddock header | 222 |
| 96.1.3 | Code | 222 |
| 96.1.3.1 | Try to stay closer to portable (Haskell98) code | 222 |
| 96.1.3.2 | Try make lines no longer 80 chars | 222 |
| 96.1.3.3 | Last char in file should be newline | 222 |
| 96.1.3.4 | Symbolic infix identifiers is only library writer right | 222 |
| 96.1.3.5 | Every function does one thing. | 222 |
| 97 | Good: Use Typed holes to progress the code | 223 |
| 98 | Good: Haskell allows infinite terms but not infinite types | 224 |
| 99 | Good: Use type sysonims to differ the information | 225 |
| 100 | Good: Use <code>Control.Monad.Except</code> instead of <code>Control.Monad.Error</code> | 226 |
| 101 | Good: Monad OR Applicative | 227 |
| 101.0.1 | Start writing monad using 'return', 'ap', 'liftM', 'liftM2', '»' instead of 'do', '»=' | 227 |
| 101.0.2 | Basic case when Applicative can be used | 227 |
| 101.0.3 | Applicative block vs Monad block | 227 |
| 102 | Good: Haskell Package Versioning Policy | 228 |
| 102.1 * | | 229 |
| 103 | Good: Linear type | 230 |
| 104 | Good: Exception vs Error | 231 |
| 105 | Good: Let vs. Where | 232 |
| 106 | Good: RankNTypes | 233 |
| 107 | Good: Orphan instance | 234 |
| 108 | Good: Smart constructor | 235 |
| 109 | Good: Thin category | 236 |
| 110 | Good: Recursion | 237 |
| 111 | Good: Monoid | 238 |
| 112 | Good: Free monad | 239 |
| 113 | Good: Use mostly where clauses | 240 |
| 114 | Good: Where clause is in a scope with function parameters | 241 |
| 115 | Good: Strong preference towards pattern matching over {head, tail, etc.} functions | 242 |
| 116 | Good: Patternmatching is possible on monadic bind in do | 243 |
| 117 | Good: Applicative vs Monad | 244 |

| | | |
|-------------|--|------------|
| 118 | Good: StateT, ReaderT, WriterT | 245 |
| 119 | Good: Working with MonadTrans and lift | 246 |
| 120 | Good: Don't mix Where and Let | 247 |
| 121 | Good: Where vs. Let | 248 |
| 122 | Good: The proper nature algorithm that models behaviour of many objects is computation heavy | 249 |
| 123 | Good: In Haskell parameters bound by lambda declaration instantiate to only one concrete type | 250 |
| 124 | Good: Instance is a good structure to draw a type line | 251 |
| 125 | Good: MTL vs. Transformers | 252 |
| VI | Bad code | 253 |
| 126 | Bad pragma | 254 |
| | 126.1 Bad: Dangerous LANGUAGE pragma option | 254 |
| VII | Useful functions to remember | 255 |
| 127 | Prelude | 256 |
| | 127.1 Ord | 256 |
| | 127.2 Calc | 256 |
| | 127.3 List operations | 256 |
| 128 | Data.List | 257 |
| 129 | Data.Char | 258 |
| 130 | QuickCheck | 259 |
| VIII | Tools | 260 |
| 131 | ghc-pkg | 261 |
| 132 | Integration of NixOS/Nix with Haskell IDE Engine (HIE) and Emacs (Spacemacs) | 262 |
| | 132.11. Install the Cachix | 262 |
| | 132.22. Installation of HIE | 262 |
| | 132.2.1 2.1. Provide cached builds | 262 |
| | 132.2.2 2.2.a. Installation on NixOS distribution: | 262 |
| | 132.2.3 2.2.b. Installation with Nix package manager: | 263 |
| | 132.33. Emacs (Spacemacs) configuration: | 263 |
| | 132.44. Open the Haskell file from a project | 264 |
| | 132.55. Be pleased writing code | 264 |
| | 132.66. (optional) Debugging | 264 |
| 133 | Debugger | 265 |
| 134 | GHCID | 266 |

| | | |
|------------|---|------------|
| 135 | Continuous integration platrorms (CIs) for Open Source Haskell projets | 267 |
| IX | Libs | 268 |
| 136 | Exceptions | 269 |
| 136.1 | Exceptions - optionally pure extensible exceptions that are compatible with the mtl | 269 |
| 136.2 | Safe-exceptions - safe, simple API equivalent to the underlying implementation in terms of power, encourages best practices minimizing the chances of getting the exception handling wrong. | 269 |
| 136.3 | Enclosed-exceptions - capture exceptions from the enclosed computation, while reacting to asynchronous exceptions aimed at the calling thread. | 269 |
| 137 | Memory management | 270 |
| 137.1 | membrain - type-safe memory units | 270 |
| 138 | Parsers - megaparsec | 271 |
| 139 | CLIs - optparse-applicative | 272 |
| 140 | HTML - Lucid | 273 |
| 141 | Web applications - Servant | 274 |
| 142 | IO libraries | 275 |
| 142.1 | Conduit - practical, monolythic, guarantees termination return | 275 |
| 142.2 | Pipes + Pipes Parse - modular, more primitive, theoretically driven | 275 |
| 143 | JSON - aeson | 276 |
| 144 | Backpack | 277 |
| X | Drafts | 278 |
| 145 | Exception handling | 279 |
| 146 | Constraints | 281 |
| 147 | Monad transformers and their type classes | 282 |
| 148 | Layering monad transformers | 283 |
| 149 | Hoogle | 284 |
| 149.1 | Search | 284 |
| 149.2 | Scope | 284 |
| 149.2.1 | Default | 284 |
| 149.2.2 | Hierarchical module name system (from big letter): | 284 |
| 149.2.3 | Packages (lower case): | 284 |
| 150 | ST-Trick monad | 285 |
| 150.1 | * | 285 |
| 151 | Either | 286 |
| 151.1 | * | 286 |
| 152 | Inverse | 287 |

| | | |
|-------------------------------|---|------------|
| 153 | Inversion | 288 |
| 154 | Inverse function | 289 |
| 155 | Inverse morphism | 290 |
| 156 | Partial inverse | 291 |
| 157 | PatternSynonyms | 292 |
| 157.1 * | | 292 |
| 158 | GHC debug keys | 293 |
| 158.1-ddump-ds | | 293 |
| 158.1.1 * | | 293 |
| 159 | GHC optimize keys | 294 |
| 159.1-foptimal-applicative-do | | 294 |
| 160 | Computational trinitarianism | 295 |
| 160.1 * | | 295 |
| 161 | Techniques functional programming deals with the state | 297 |
| 161.1Minimizing | | 297 |
| 161.2Concentrating | | 297 |
| 161.3Deferring | | 297 |
| 162 | Monadic Error handling | 298 |
| 163 | Functions | 299 |
| 164 | Void | 300 |
| 164.1 * | | 300 |
| 165 | Constructive proof | 301 |
| 166 | Intuitionistic logic | 302 |
| 166.1 * | | 302 |
| 167 | Principle of explosion | 303 |
| 167.1 * | | 303 |
| 168 | Universal property | 304 |
| 169 | Yoneda lemma | 305 |
| 170 | Monoidal category, functoriality of ADTs, Profunctors | 306 |
| 171 | Const functor | 307 |
| 172 | Arrow in Haskell | 308 |
| 173 | Contravariant functor | 309 |
| 174 | Profunctor | 310 |
| 175 | Coerce | 311 |
| 175.1 * | | 311 |

| | | |
|------------|---|------------|
| XI | Reference | 312 |
| 176 | Functor-Applicative-Monad Proposal | 313 |
| 176.1 | * | 313 |
| 177 | Haskell-98 | 314 |
| 177.1 | Old instance termination rules | 314 |
| 178 | Performance results and comparisons of types & solutions | 315 |
| 179 | Literature | 316 |
| XII | Giving back | 317 |

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Part I

Introduction

“Employ your time in improving yourself by other men’s writings so that you shall come easily by what others have labored hard for.” (Socrates by Plato)

Important notes on Haskell, [category](#) theory & related fields, terms and recommendations.

Book comes in forms:

- [Web book](#)
- [PDF](#)
- [Open in web PDF viewer](#)
- [L^AT_EX](#)
- [Source code in Org-mode](#)
- [GitHub](#)
- [GitLab](#)

This book is created using complex Org markup file with a lot of L^AT_EX and L^AT_EX formulas. Be aware - GitHub & GitLab only partially parse Org into HTML.

To get the full view:

- [Outline navigation](#)
- [L^AT_EX formulas](#):

$$\left[-\frac{\hbar^2}{2m} \nabla^2 + V(\vec{r}, t) \right] \Psi(\vec{r}, t) = i\hbar \frac{\partial}{\partial t} \Psi(\vec{r}, t), \quad \sum_{k,j} \left[-\frac{\hbar^2}{\sqrt{a}} \frac{\partial}{\partial q^k} \left(\sqrt{a} a^{kj} \frac{\partial}{\partial q^j} \right) + V \right] \Psi + \frac{\hbar}{i} \frac{\partial \Psi}{\partial t} = 0$$

- [Interlinks](#): [Interlinks](#)

, please refer to [Web book](#), [PDF](#), [L^AT_EX](#), or use Org-mode capable viewer/editor.

Note about the markup: <<<This is a radio target>>> - is the anchor for dynamic linking.

Users of Emacs can prettify radio targets to be shown as hyper-links with this Elisp snippet:

```
;;; 2019-06-12: NOTE:
;;; Prettify '<<<Radio targets>>>' to be shown as '_Radio_targets_',
;;; when `org-descriptive-links` set.
;;; This is improvement of the code from: Tobias&glmorous:
;;; https://emacs.stackexchange.com/questions/19230/how-to-hide-targets
;;; There exists library created from the sample:
;;; https://github.com/talwrii/org-hide-targets
(defun org-hidden-links-additional-re "\\(<<<\\)[[:print:]]+?\\(>>>\\)"
  "Regular expression that matches strings where the invisible-property
  of the sub-matches 1 and 2 is set to org-link."
  :type '(choice (const :tag "Off" nil) regexp)
  :group 'org-link)
(make-variable-buffer-local 'org-hidden-links-additional-re)

(defun org-activate-hidden-links-additional (limit)
  "Put invisible-property org-link on strings matching
  `org-hide-links-additional-re'."
  (if org-hidden-links-additional-re
      (re-search-forward org-hidden-links-additional-re limit t)
      (goto-char limit)
      nil))
```

```
(defun org-hidden-links-hook-function ()
  "Add rule for `org-activate-hidden-links-additional'
  to `org-font-lock-extra-keywords'."
  You can include this function in `org-font-lock-set-keywords-hook'."
  (add-to-list 'org-font-lock-extra-keywords
    '(org-activate-hidden-links-additional
      (1 '(face org-target invisible org-link))
      (2 '(face org-target invisible org-link)))))

(add-hook 'org-font-lock-set-keywords-hook #'org-hidden-links-hook-function)
```

SCHT: and metadata in :properties: - of my org-drill practices, please just run org-drill-strip-all-data.

Part II

Definitions

Chapter 1

Algebra

↳ *al-jabr* assemble parts

A system of parts based on given axioms ([properties](#)) and operations on them.

====

Additional meanings:

1. [Algebra](#) - a [set](#) with its [algebraic structure](#).
2. [Abstract algebra](#) - the study of number systems and operations within them.
3. [Algebra](#) - vector space over a field with a multiplication.

1.1 *

Algebras

1.2 Algebraic

Composite from simple parts.

Also: [Algebraic data type](#).

1.3 Algebraic structure

* includes axioms that must be satisfied and operations on the underlying (or "carrier") [set](#).

An underlying [set](#) with * on top of it also called "an [algebra](#)".

* include [groups](#), [rings](#), fields, and lattices. More complex [structures](#) can be defined by introducing multiple operations, different underlying [sets](#), or by altering the defining axioms. Examples of more complex * can be many modules, [algebras](#) and other vector spaces, and any variations that the definition includes.

1.3.1 *

Algebraic structures

Table 1.1: Algebraic structures

| | Closure | Associativity | Identity | Invertability | Commutativity | Distributive |
|-----------------------|---------|---------------|----------|---------------|---------------|--------------|
| Semigroupoid | | ✓ | | | | |
| Small Category | | ✓ | ✓ | | | |
| Groupoid | | ✓ | ✓ | ✓ | | |
| Magma | ✓ | | | | | |
| Quasigroup | ✓ | | | ✓ | | |
| Loop | ✓ | | ✓ | ✓ | | |
| Semigroup | ✓ | ✓ | | | | |
| Inverse Semigroup | ✓ | ✓ | | ✓ | | |
| Monoid | ✓ | ✓ | ✓ | | | |
| Group | ✓ | ✓ | ✓ | ✓ | | |
| Abelian group | ✓ | ✓ | ✓ | ✓ | ✓ | |
| Non-unital ring (rng) | ✓ + × | ✓ + × | ✓ + | ✓ + | ✓ + | ✓ |
| Semiring (rig) | ✓ + × | ✓ + × | ✓ + × | ✓ × | ✓ + | ✓ |
| Ring | ✓ + × | ✓ + × | ✓ + × | ✓ + × | ✓ + | ✓ |

1.3.2 Fundamental theorem of algebra

Any non-constant single-variable polynomial with complex coefficients has at least one complex root.

From this definition follows property that the field of complex numbers is algebraically closed.

1.4 Modular arithmetic

System for integers where numbers wrap around the certain values (single - modulus, plural - moduli).

Example - 12-hour clock.

1.4.1 *

Clock arithmetic

1.4.2 Modulus

Special numbers where arithmetic wraps around in modular arithmetic.

1.4.2.1 *

Moduli - plural.

Chapter 2

Category theory

Category \mathcal{C} consists of the basis:

Primitives:

1. **Objects** - $a^{\mathcal{C}}$. A **node**. **Object** of some **type**. Often **sets**, than it is **Set category**.
2. **Arrows** - $(a, b)^{\mathcal{C}}$ (AKA **morphisms** mappings).
3. **Arrow (morphism) composition** - **binary operation**: $(a, b)^{\mathcal{C}} \circ (b, c)^{\mathcal{C}} \equiv (a, c)^{\mathcal{C}} \mid \forall a, b, c \in \mathcal{C}$ AKA principle of **compositionality** for **arrows**.

Properties (or axioms):

1. **Associativity** of **morphisms**: $h \circ (g \circ f) \equiv (h \circ g) \circ f \mid f_{a \rightarrow b}, g_{b \rightarrow c}, h_{c \rightarrow d}$
2. Every **object** has (two-sided) **identity morphism** (& in fact - exactly one): $1_x \circ f_{a \rightarrow x} \equiv f_{a \rightarrow x}, g_{x \rightarrow b} \circ 1_x \equiv g_{x \rightarrow b} \mid \forall x \exists 1_x, \forall f_{a \rightarrow x}, \forall g_{x \rightarrow b}$
3. Principle of **compositionality**.

From these axioms, can be proven that there is exactly one **identity morphism** for every **object**.

Object and **morphism** are complete **abstractions** for anything. In majority of cases under **object** is a state and **morphism** is a change.

2.1 *

Category Categories

2.2 Abelian category

Generalised **category** for homological **algebra** (having a possibility of basic constructions and techniques for it).

Category which:

- has a **zero object**,
- has all **binary** biproducts,
- has all **kernel**'s and cokernels,
- (it has all **pullbacks** and pushouts)
- all **monomorphism**'s and **epimorphism**'s are normal.

Abelian category is a stable **structure**; for example it is regular and satisfy the snake lemma. The class of **Abelian categories** is **closed** under several categorical constructions.

There is notion of **Abelian monoid** (AKS **Commutative monoid**) and **Abelian group** (**Commutative group**).

Basic examples of $*$:

- **category** of Abelian **groups**
- **category** of modules over a **ring**.

$*$ are widely used in **algebra**, **algebraic** geometry, and topology.

$*$ has many constructions like in **categories** of modules:

- kernels
- exact **sequences**
- **commutative** diagrams

$*$ has disadvantage over **category** of modules. **Objects** do not necessarily have elements that can be manipulated directly, so traditional definitions do not work. Methods must be supplied that allow definition and manipulation of **objects** without the use of elements.

2.2.1 $*$

Abelian categories

2.3 Composition

Axiom of **Category**.

2.3.1 $*$

Composable Compositions

2.4 Endofunctor category

From the name, in this **Category**:

- **objects** of End are **Endofunctors** $E^{C \rightarrow C}$
- **morphisms** are **natural transformations** between **endofunctors**

2.5 Functor

$*$ full translation (map) of one **category** into another. Translating **objects** and **morphisms** (as input can take **morphism** or **object**).

$*$ - **forgetful** - discards part of the **structure**. $*$ - faithful - fully preserves all **morphisms** - **injective** on Hom-sets. $*$ - full - translation of **morphisms** fully covers all the **morphisms** between according objects in the target category.

For **Functor type class** or **fmap** - see **Power set functor**.

Functor properties (axioms):

- $F^{C \rightarrow D}(a) \mid \forall a^C$ - every source **object** is mapped to **object** in target **category**

- $\overline{(F^{C \rightarrow \mathcal{D}}(a), F^{C \rightarrow \mathcal{D}}(b))}^{\mathcal{D}} \mid \forall (a, b)^C$ - every source **morphism** is mapped to target **category morphism** between corresponding **objects**
- $F^{C \rightarrow \mathcal{D}}(\vec{g}^C \circ \vec{f}^C) = F^{C \rightarrow \mathcal{D}}(\vec{g}^C) \circ F^{C \rightarrow \mathcal{D}}(\vec{f}^C) \mid \forall y = \vec{f}^C(x), \forall \vec{g}^C(y)$ - **composition** of **morphisms** translates directly (tautologically goes from other two)

These axioms guarantee that **composition** of **functors** can be fused into one **functor** with **composition** of **morphisms**. This **process** called **fusion**.

In Haskell this axioms have form:

```
fmap id = id
fmap (f . g) = fmap f . fmap g
```

Since $*$ is 1-1 mapping of initial **objects** - it is a memoizable dictionary with **cardinality** of initial **objects**. Also in **Hask category functors** are obviously **endofunctors** \therefore they are special **kinds** of containers for the parametric values (AKA **product type**). In Haskell **product type** $*$ are **endofunctors** from **polymorphic type** into a **functor** wrapper of a **polymorphic type**.

$*$ translates in one direction, and does not provide algorithm of reversing itself or retrieving the parametric value.

2.5.1 $*$

Functors

2.5.2 Power set functor

$\mathcal{P}^{\mathcal{S} \rightarrow \mathcal{P}(\mathcal{S})}$

$*$ - **functor** from **set** S to its **power set** $\mathcal{P}(S)$.

Functor type class in Haskell defines a $*$ and allows to do **function application** inside **type structure** layers (denoted f or m). **IO** is also such **structure**. **Power set** is unique to the **set**, $*$ is unique to the **category (data type)**. $*$ embodies in itself any **endofunctor**. It is easily seen from Haskell definition - that the $*$ is the **polymorphic** generalization over any **endofunctor** in a **category**. **Application** of a **function** to $*$ gives a particular **endofunctor** (see **Hask category**).

```
class Functor f where
  fmap :: (a -> b) -> f a -> f b
```

Functor instance must be of **kind** $(* \rightarrow *)$, so instance for **higher-kinded data type** must be **applied** until this **kind**.

Composed $*$ can **lift functions** through any layers of **structures** that belong to **Functor type class**.

$*$ can be used to filter-out **error** cases (**Nothing** & **Left** cases) in **Maybe**, **Either** and related **types**.

2.5.2.1 $*$

fmap Functor type class

2.5.2.2 Power set functor laws

Type instance of **functor** should abide this laws:

2.5.2.2.1 $*$

Functor laws

2.5.2.2.2 Power set functor identity law

```
fmap id == id
```

2.5.2.2.3 Power set functor composition law

```
fmap (f.g) == fmap f . fmap g
```

In words, it is if several **functions** are **composed** and then **fmap** is **applied** on them - it should be the same as if **functions** was **fmapped** and then **composed**.

2.5.2.3 Lift

```
fmap :: (a -> b) -> (f a -> f b)
```

Functor takes **function** $a \rightarrow b$ and returns a **function** $f\ a \rightarrow f\ b$ this is called **lifting** a **function**. **Lift** does a **function application** through the **data structure**.

2.5.2.3.1 *

Lifting

2.5.2.4 Power set functor is a free monad

Since:

- $\forall e \in S : \exists \{e\} \in \mathcal{P}(S) \models \forall e \in S : \exists (e \rightarrow \{e\}) \equiv \text{unit}$
- $\forall \mathcal{P}(S) : \mathcal{P}(S) \in \mathcal{P}(S) \models \forall \mathcal{P}(S) : \exists (\mathcal{P}(\mathcal{P}(S)) \rightarrow \mathcal{P}(S)) \equiv \text{join}$

2.5.3 Functorial

Corresponds to **functor laws**.

2.5.4 Forgetful functor

Functor that forgets part or all of what defines **structure** in **domain category**. $F^{\text{Grp} \rightarrow \text{Set}}$ that translates **groups** into their underlying **sets**. **Constant functor** is another example.

2.5.4.1 *

Forgetful

2.5.5 Identity functor

Maps all **category** to itself. All **objects** and **morphisms** to themselves.

Denotation: $1^{\mathcal{C} \rightarrow \mathcal{C}}$

2.5.6 Endofunctor

Is a **functor** which source (**domain**) and target (**codomain**) are the same **category**.

$F^{\mathcal{C} \rightarrow \mathcal{C}}, E^{\mathcal{C} \rightarrow \mathcal{C}}$

2.5.6.1 *

Endofunctors

2.5.7 Applicative functor

* - Computer science term. Category theory name - **lax monoidal functor**. And in category *Set*, and so in category *Hask* all **applicatives** and **monads** are strong (have **tensorial strength**).

* - **sequences functorial** computations (plain **functors** can't).

```
(<*>) :: f (a -> b) -> f a -> f b
```

Requires **Functor** to exist. Requires **Monoidal structure**.

Has **monoidal structure** rules, separated from **function application** inside **structure**.

Data type can have several **applicative** implementations.

Standard definition:

```
class Functor f => Applicative f
  where
    (<*>) :: f (a -> b) -> f a -> f b
    pure  :: a -> f a
```

pure - if a **functor**, **identity Kleisli arrow**, **natural transformation**.

Composition of * always produces *, contrary to **monad** (**monads** are not **closed** under **composition**).

`Control.Monad` has an old **function ap** that is old implementation of **<*>**:

```
ap :: Monad m => m (a -> b) -> m a -> m b
```

2.5.7.1 *

Applicative Applicatives Applicative functors

2.5.7.2 Applicative law

2.5.7.3 *

Applicative laws

2.5.7.3.1 Applicative identity law

```
pure id <*> v = v
```

2.5.7.3.2 Applicative composition law

Function composition works regularly.

```
pure (.) <*> u <*> v <*> w = u <*> (v <*> w)
```

2.5.7.3.3 Applicative homomorphism law

Internal **function application** doesn't change the **structure** around values.

```
pure f <*> pure x = pure (f x)
```

2.5.7.3.4 Applicative interchange law

On condition that internal **order** of **evaluation** is preserved - **order** of operands is not relevant.

```
u <*> pure y = pure ($ y) <*> u
```

2.5.7.4 Applicative function

2.5.7.4.1 liftA*

2.5.7.4.1.1 liftA

Essentially a `fmap`.

```
:type liftA
liftA :: Applicative f => (a -> b) -> f a -> f b
```

Lifts `function` into `applicative function`.

2.5.7.4.1.2 liftA2

Lifts `binary function` across two `Applicative functors`.

```
liftA2 :: Applicative f => (a -> b -> c) -> f a -> f b -> f c
liftA2 f x y == pure f <*> x <*> y
```

2.5.7.4.1.3 «<liftA2 (<*>)»>

`liftA2 (<*>)` is pretty useful. It can `lift binary operation` through the two layers: It is two-layer `Applicative`.

```
liftA2 :: Applicative f => (a -> b -> c) -> f a -> f b -> f c
<*> :: Applicative f => (f (a -> b) -> f a -> f b)
liftA2 (<*>) :: (Applicative f1, Applicative f2) => f1 (f2 (a -> b)) -> f1 (f2 a) -> f1 (f2 b)
```

2.5.7.4.1.4 liftA2 (liftA2 (<*>))

`liftA2 (<*>)` 3-layer version.

2.5.7.4.1.5 liftA3

`liftA2` 3-parameter version.

```
liftA3 f x y z == pure f <*> x <*> y <*> z
```

2.5.7.4.2 Conditional `applicative` computations

```
when :: Applicative f => Bool -> f () -> f ()
```

Only when `True` - perform an `applicative` computation.

```
unless :: Applicative f => Bool -> f () -> f ()
```

Only when `False` - perform an `applicative` computation.

2.5.7.5 Special applicatives

2.5.7.5.1 Identity applicative

```
-- Applicative f =>
-- f ~ Identity
type Id = Identity
instance Applicative Id
  where
    pure :: a -> Id a
    (<*>) :: Id (a -> b) -> Id a -> Id b
```

```
mkId = Identity
```

```
xs = [1, 2, 3]

const <$> mkId xs <*> mkId xs'
-- [1,2,3]
```

2.5.7.5.2 Constant applicative

It holds only to one value. The `function` does not exist and last `parameter` is a phantom.

```
-- Applicative f =>
-- f ~ Constant e
type C = Constant
instance Applicative C
where
  pure :: a -> C e a
  (<*>) :: C e (a -> b) -> C e a -> C e b
```

2.5.7.5.3 Maybe applicative

"There also can be no `function` at all."

If `function` might not exist - embed `f` in `Maybe structure`, and use `Maybe applicative`.

```
-- f ~ Maybe
type M = Maybe
pure :: a -> M a
(<*>) :: M (a -> b) -> M a -> M b
```

2.5.7.5.4 Either applicative

`pure` is `Right`. Defaults to `Left`. And if there is two `Left`'s - to `Left` of the first `argument`.

2.5.7.5.5 Validation applicative

The `Validation data type` isomorphic to `Either`, but has accumulative `Applicative` on the `Left` side. `Validation data type` is not a `monad`. `Validation` is an example of, "An `applicative functor` that is not a `monad`." While `Either monad` on `Left case` just drops computation and returns this first `Left`. `Monad` needs to `process` the result of computation - it requires to be able to `process` all `Left error statement` cases for `Validation`, it is or non-terminating `Monad` or one which is impossible to implement in `polymorphic` way with `Validation`.

2.5.7.6 Monad

μόνος *monos* sole

μονάδα *monáda* `unit`

In loose terms, `*` - is an ability built over `structures` that allows to `compose functions` that produce that `structures`.

Since it is possible to express unpure `functions` with `equivalent pure functions` that produce a `structure`, `*` become widely used in Haskell for those cases also. `*` with lazy `evaluation` also allows controll over the continuation of calculations by early terminations.

`*` - `lax monoid` in `endofunctor category`, that relies on η (`unit`) and μ (`join`) `natural transformations` to form an `equivalent` of `identity`.

`Monad` on \mathcal{C} is $\{E^{\mathcal{C} \rightarrow \mathcal{C}}, \eta, \mu\}$:

- $E^{\mathcal{C} \rightarrow \mathcal{C}}$ - is an `endofunctor`
- two `natural transformations`, $1^{\mathcal{C}} \rightarrow E$ and $E \circ E \rightarrow E$:

$$\begin{aligned}
- \eta^{1^{\mathcal{C}} \rightarrow E} &= \text{unit}^{Identity \rightarrow E}(x) = f^{x \rightarrow E(x)}(x) \\
- \mu^{(E \circ E) \rightarrow E} &= \text{join}^{(E \circ E) \rightarrow (Identity \circ E)}(x) = |y = E(x)| = f^{E(y) \rightarrow y}(y)
\end{aligned}$$

where:

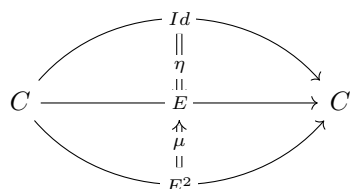
- \mathcal{C} is a **category**
- $1^{\mathcal{C}}$ denotes the \mathcal{C} **identity functor**
- $(E \circ E)$ - **endofunctor** $\mathcal{C} \rightarrow \mathcal{C}$

Definition with $\{E^{\mathcal{C} \rightarrow \mathcal{C}}, \eta, \mu\}$ (in **Hask**: $(\{e :: fa \rightarrow fb, \text{pure}, \text{join}\})$) - is classic categorical, in Haskell minimal complete definition is $\{\text{fmap}, \text{pure}, (\gg=)\}$.

While T is mode classical **Category** theory notation, we used the $E \equiv T$ substitution for purposes of notation being more understandable.

If there is a **structure** S , and a way of taking **object** x into S and a way of collapsing $S \circ S$ - there probably a **monad**.

Monad structure:



;

Mostly **monads** used for sequencing actions (computations) (that looks like imperative programming), with ability to depend on previous chains. Note if **monad** is **commutative** - it does not **order** actions.

Monad can shorten/terminate **sequence** of computations. It is implemented inside **Monad** instance. For example **Maybe monad** on **Nothing** drops chain of computation and returns **Nothing**.

* inherits the **Applicative** instance methods:

```
import Control.Monad (ap)
return == pure
ap == (<*>) -- + Monad requirement
```

Table 2.1: **Monad** in mathematics and Haskell

| Math | Meaning | Cat/Fctr | $X \in \mathcal{C}$ | Type | Haskell |
|--------|--|---------------------------------------|----------------------------|------------------------------|---------|
| Id | endofunctor "Id" | $\mathcal{C} \rightarrow \mathcal{C}$ | $X \rightarrow Id(X)$ | $a \rightarrow a$ | id |
| E | endofunctor "monad" | $\mathcal{C} \rightarrow \mathcal{C}$ | $X \rightarrow E(X)$ | $m\ a \rightarrow m\ b$ | fmap |
| η | natural transformation "unit" | $Id \rightarrow E$ | $Id(X) \rightarrow E(X)$ | $a \rightarrow m\ a$ | pure |
| μ | natural transformation "multiplication" | $E \circ E \rightarrow E$ | $E(E(X)) \rightarrow E(X)$ | $m\ (m\ a) \rightarrow m\ a$ | join |

Internals of **Monad** are Haskell **data types**, and as such - they can be consumed any number of times.

Composition of **monadic types** does not always results in **monadic type**.

2.5.7.6.1 *

Monads Monadic

2.5.7.6.2 Monad law

Monad corresponds to **functor laws** & **applicative laws** and additionally:

2.5.7.6.2.1 *

Monad laws

2.5.7.6.2.2 Monad left identity law

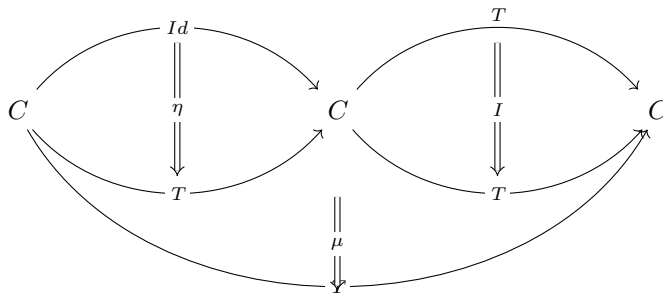
`pure x >>= f == f x`

Explanation:

```
>>= :: Monad f =>      f a  -> (a -> f b) -> f b
      pure x >>=      f      == f x
```

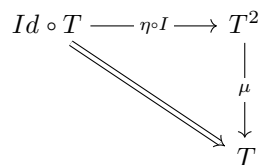
Rule that `>>=` must get first **argument structure** internals and **apply** to the **function** that is the second **argument**.

Diagram on **category** level:



;

Diagram on **endomorphism** level:



;

2.5.7.6.2.3 Monad right identity law

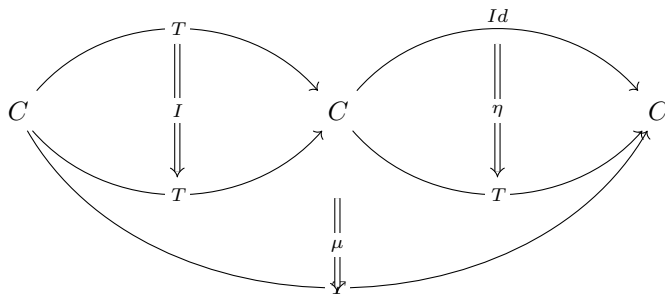
`f >>= pure == f`

Explanation:

```
>>= :: Monad f => f a  -> (a -> f b) -> f b
      f    >>=    pure    == f
```

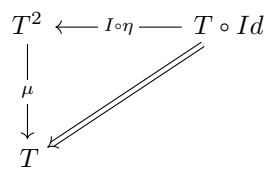
AKA it is a **tacit** description of a **monad bind** as **endofunctor**.

Diagram on **category** level:



;

Diagram on endomorphism level:



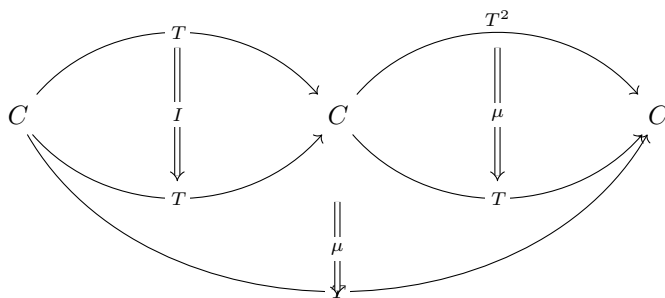
;

2.5.7.6.2.4 Monad associativity law

$$(m \gg= f) \gg= g == m \gg= (\backslash x \rightarrow f\ x \gg= g)$$

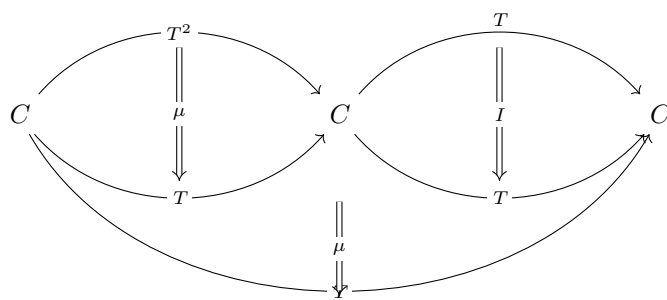
In diagram form:

Category level:



;

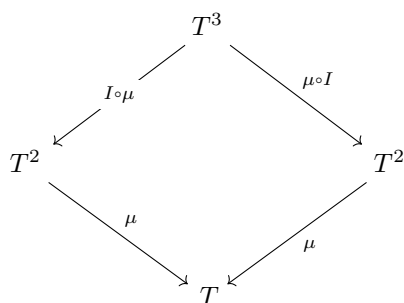
= to:



;

So, $\mu \circ (\mu \circ I) = \mu \circ (I \circ \mu)$

Endomorphism level:



;

2.5.7.6.3 Monad type class

```
class Applicative m => Monad m where
  (>>=) :: m a -> (a -> m b) -> m b
  (>>)  :: m a -> m b -> m b
  return :: a -> m a
```

2.5.7.6.3.1 MonadPlus type class

Is a **monoid** over **monad**, with additional rules. The precise **set** of rules (**properties**) not agreed upon. Class instances obey **monoid** & **left zero** rules, some additionally obey **left catch** and others **left distribution**.

Overall there * currently reforms (**MonadPlus** reform proposal) in several smaller nad strictly defined **type classes**.

Subclass of an **Alternative**.

*

Monadplus

2.5.7.6.4 Functor -> Applicative -> Monad progression

```
<$> :: Functor    f => (a -> b) -> f a -> f b
<*> :: Applicative f => f (a -> b) -> f a -> f b
=<< :: Monad      f => (a -> f b) -> f a -> f b
```

pure & join are **Natural transformations** for the fmap.

2.5.7.6.5 Monad function

2.5.7.6.5.1 Return function

```
return == pure
```

Nonstrict.

2.5.7.6.5.2 Join function

```
join :: Monad m => m (m a) -> m a
```

Generates knowledge of concat.

[Kleisli composition](#) that flattens two layers of [structure](#) into one.

The way to express ordering in [lambda calculus](#) is to nest.

*

join

```
join . fmap == (=«)
```

```
-- b = f b
```

```
fmap      :: Monad f => (a -> f b) -> f a -> f (f b)
join      :: Monad f =>                      f (f a) -> f a
join . fmap :: Monad f => (a -> f b) -> f a          -> f b
flip      >>= :: Monad f => (a -> f b) -> f a          -> f b
```

2.5.7.6.5.3 Bind function

```
>>=      :: Monad f => f a -> (a -> f b) -> f b
join . fmap :: Monad f => (a -> f b) -> f a -> f b
```

Nonstrict.

The most ubiquitous way to `>>=` something is to use [Lambda function](#):

```
getLine >>= \name -> putStrLn "age pls:"
```

Also a neat way is to bundle and handle [Monad](#) - is to bundle it with [bind](#), and leave [applied](#) partially. And use that partial bundle as a [function](#) - every [evaluation](#) of the [function](#) would trigger [evaluation](#) of internal [Monad structure](#). Thumbs up.

```
printOneOf :: Bool -> IO ()
printOneOf False = putStr "1"
printOneOf True  = putStr "2"
```

```
quant :: (Bool -> IO b) -> IO b
quant = (>>=) (randomRIO (False, True))
```

```
recursePrintOneOf :: Monad m => (t -> m a) -> t -> m b
recursePrintOneOf f x = (f x) >> (recursePrintOneOf f x)
```

```
main :: IO ()
main = recursePrintOneOf (quant) $ printOneOf
```

*

Monadic extend Monadic bind Monad bind Binder

```
(>>=)
>>=
```



```
(=<<)
=<<
```

2.5.7.6.5.4 Sequencing operator (>>) \equiv (*>):

Discard any resulting value of the action and [sequence](#) next action. [Applicative](#) has a similar [operator](#).

```
(>>) :: m a -> m b -> m b
(*>) :: f a -> f b -> f b
```

2.5.7.6.5.5 Monadic versions of list functions

```
sequence :: (Traversable t, Monad m) => t (m a) -> m (t a)
```

[Sequence](#) gets the traversable of [monadic](#) computations and swaps it into [monad](#) computation of traverse. In the result the collection of [monadic](#) computations turns into one long [monadic](#) computation on traverse of data.

If some step of this long computation fails - [monad](#) fails.

```
mapM :: (Traversable t, Monad m) => (a -> m b) -> t a -> m (t b)
```

[mapM](#) gets the [AMB function](#), then takes traversable data. Then applies [AMB function](#) to traversable data, and returns converted [monadic](#) traversable data.

```
foldM :: (Foldable t, Monad m) => (b -> a -> m b) -> b -> t a -> m b
foldl :: Foldable t          => (b -> a -> b) -> b -> t a -> b
```

[*](#) is a [monadic foldl](#).

[b](#) is initial cumulative value, [m b](#) is a cumulative bank. Right folding achieved by reversing the input [list](#).

```
filterM :: Applicative m => (a -> m Bool) -> [a] -> m [a]
filter :: (a -> Bool) -> [a] -> [a]
```

Take Boolean [monadic](#) computation, filter the [list](#) by it.

```
zipWithM :: Applicative m => (a -> b -> m c) -> [a] -> [b] -> m [c]
zipWith :: (a -> b -> c) -> [a] -> [b] -> [c]
```

Take [monadic](#) combine [function](#) and combine two lists with it.

```
msum :: (Foldable t, MonadPlus m) => t (m a) -> m a
sum :: (Foldable t, Num a)          => t a -> a
```

2.5.7.6.5.6 liftM*

[liftM](#) Essentially a [fmap](#).

```
liftM :: Monad m => (a -> b) -> m a -> m b
```

Lifts a [function](#) into [monadic](#) equivalent.

[liftM2](#) [Monadic liftA2](#).

```
liftM2 :: Monad m => (a -> b -> c) -> m a -> m a -> m c
```

Lifts [binary function](#) into [monadic](#) equivalent.

2.5.7.6.6 Comonad

Category \mathcal{C} [comonad](#) is a [monad](#) of [opposite category](#) \mathcal{C}^{op} .

2.5.7.6.7 Kleisli arrow

Morphism that while doing computation also adds **monadic**-able **structure**.

```
a -> m b
```

2.5.7.6.7.1 *

Kleisli arrows Kleisli morphism Kleisli morphisms

2.5.7.6.8 Kleisli composition

Composition of **Kleisli arrows**.

```
(<=<) :: Monad m => (b -> m c) -> (a -> m b) -> a -> m c infixr 1
;; compare
(.)   ::          (b -> c) -> (a -> b) -> a -> c
```

Often used left-to-right version:

```
(>=>) :: Monad m => (a -> m b) -> (b -> m c) -> a -> m c
;; compare
(>=>) :: Monad m =>          m a -> (a -> m b)          -> m b
```

Which allows to replace **monadic bind** chain with **Kleisli composition**.

```
f1 arg >=> f2 >=> f3
==
f1 >=> f2 >=> f3 $ arg
==
f3 <=< f2 <=< f1 $ arg
```

2.5.7.6.9 Kleisli category

Category \mathcal{C} , $\langle E, \vec{\eta}, \vec{\mu} \rangle$ **monad** over \mathcal{C} .

Kleisli category \mathcal{C}_T of \mathcal{C} :

$$\text{Obj}(\mathcal{C}_T) = \text{Obj}(\mathcal{C}) \quad \text{Hom}_{\mathcal{C}_T}(x, y) = \text{Hom}_{\mathcal{C}}(x, E(y))$$

2.5.7.6.10 Special monad

2.5.7.6.10.1 Identity monad

Wraps data in the **Identity constructor**.

Useful: Creates **monads** from **monad** transformers.

Bind: Applies internal value to the **bound function**.

Code: (see: **coerce**)

```
newtype Identity a = Identity { runIdentity :: a }
```

```
instance Functor Identity where
    fmap      = coerce
```

```
instance Applicative Identity where
    pure      = Identity
    (<*>)     = coerce
```

```
instance Monad Identity where
    m >=> k   = k (runIdentity m)
```

Example:

```
-- derive the State monad using the StateT monad transformer
type State s a = StateT s Identity a
```

2.5.7.6.10.2 Maybe monad

Something that may or not return a result. Any lookups into the real world, database queries.

Bind: Nothing input gives Nothing output, Just x input uses x as input to the **bound function**.

When some computation results in **Nothing** - drops the chain of computations and returns **Nothing**.

Zero: **Nothing** Plus: result in first occurrence of Just else **Nothing**.

Code:

```
data Maybe a = Nothing | Just a

instance Monad Maybe where
    return      = Just
    fail        = Nothing
    Nothing >>= _ = Nothing
    (Just x) >>= f = f x

instance MonadPlus Maybe where
    mzero      = Nothing
    Nothing `mplus` x = x
    x `mplus` _      = x
```

Example: Given 3 dictionaries:

1. Full names to email addresses,
2. Nicknames to email addresses,
3. Email addresses to email preferences.

Create a **function** that finds a person's email preferences based on **either** a full name or a nickname.

```
data MailPref = HTML | Plain
data MailSystem = ...

getMailPrefs :: MailSystem -> String -> Maybe MailPref
getMailPrefs sys name =
    do let nameDB = fullNameDB sys
        nickDB = nickNameDB sys
        prefDB = prefsDB sys
        addr <- (lookup name nameDB) `mplus` (lookup name nickDB)
        lookup addr prefDB
```

2.5.7.6.10.3 Either monad

When computation results in Left - drops other computations & returns the received Left.

2.5.7.6.10.4 Error monad

Something that can fail, throw **exceptions**.

The failure **process** records the description of a failure. **Bind function** uses successful values as input to the **bound function**, and passes failure information on without executing the **bound function**.

Useful: Composing [functions](#) that can fail. Handle [exceptions](#), crate [error](#) handling [structure](#).

[Zero](#): empty [error](#). Plus: if first [argument](#) failed then execute second [argument](#).

2.5.7.6.10.5 List monad

Computations which may return 0 or more possible results.

[Bind](#): The [bound function](#) is [applied](#) to all possible values in the input [list](#) and the resulting lists are concatenated into [list](#) of all possible results.

Useful: Building computations from [sequences](#) of non-deterministic operations.

[Zero](#): [] Plus: (++)

*

[] monad

2.5.7.6.10.6 Reader monad

Creates a read-only shared environment for computations.

The pure [function](#) ignores the environment, while `>=>` passes the inherited environment to both subcomputations.

Today it is defined though [ReaderT](#) transformer:

```
type Reader r = ReaderT r Identity -- equivalent to ((->) e), (e ->)
```

Old definition was:

```
newtype Reader e a = Reader { runReader :: (e -> a) }
```

For `(e ->)`:

- [Functor](#) is `(.)`

```
fmap :: (b -> c) -> (a -> b) -> a -> c
```

```
fmap = (.)
```

- [Applicative](#):

— pure is `const`

```
pure :: a -> b -> a
```

```
pure x _ = x
```

- `(<*>)` is:

```
(<*>) :: (a -> b -> c) -> (a -> b) -> a -> c
```

```
(<*>) f g = \a -> f a (g a)
```

- [Monad](#):

```
(>>=) :: (a -> b) -> (b -> a -> c) -> a -> c
```

```
(>>=) m k = Reader $ \r ->
```

```
runReader (k (runReader m r)) r
```

```
join :: (e -> e -> a) -> e -> a
```

```
join f x = f x x
```

```
runReader
```

```
  :: Reader r a -- the Reader to run
```

```
  -> r -- an initial environment
```

```
  -> a -- extracted final value
```

Usage:

```
data Env = ...

createEnv :: IO Env
createEnv = ...

f :: Reader Env a
f = do
  a <- g
  pure a

g :: Reader Env a
g = do
  env <- ask  -- "Open the environment namespace into env"
  a <- h env  -- give env to h
  pure a

h :: Env -> a
...  -- use env and produce the result

main :: IO ()
main = do
  env <- createEnv
  a = runReader g env
  ...
```

In Haskell under normal circumstances impure [functions](#) should not directly call impure [functions](#). `h` is an impure [function](#), and `createEnv` is impure [function](#), so they should have intermediary.

2.5.7.6.10.7 Writer monad

Computations which accumulate [monoid](#) data to a shared Haskell storage. So `*` is parametrized by [monoidal type](#).

Accumulator is maintained separately from the returned values.

Shared value modified through [Writer monad](#) methods.

`*` frees creator and code from manually keeping the track of accumulation.

Bind: The [bound function](#) is [applied](#) to the input value, [bound function](#) allowed to `<>` to the accumulator.

```
type Writer r = WriterT r Identity
```

Example:

```
f :: Monoid b => a -> (a, b)
f a = if _condition_
      then runWriter $ g a
      else runWriter do
        a1 <- h a
        pure a1

g :: Monoid b => Writer b a
g a = do
  tell _value1_  -- accumulator <> _value1_
  pure a  -- observe that accumulator stored inside monad
          -- and only a main value needs to be returned.
```

```

h :: Monoid b => Writer b a
h a = do
  tell _value2_ -- accumulator <> _value_
  pure a
runWriter :: Writer w a -> (a, w) -- Unwrap a writer computation
                                   -- as a (result, accumulator) pair.
                                   -- The inverse of writer.

```

WriterT, Writer unnecessarily keeps the entire logs in the memory. Use fast-logger for logging.

2.5.7.6.10.8 State monad

Computations that pass-over a state.

The [bound function](#) is [applied](#) to the input value to produce a state transition [function](#) which is [applied](#) to the input state.

[Pure](#) functional language cannot update values in place because it violates [referential transparency](#).

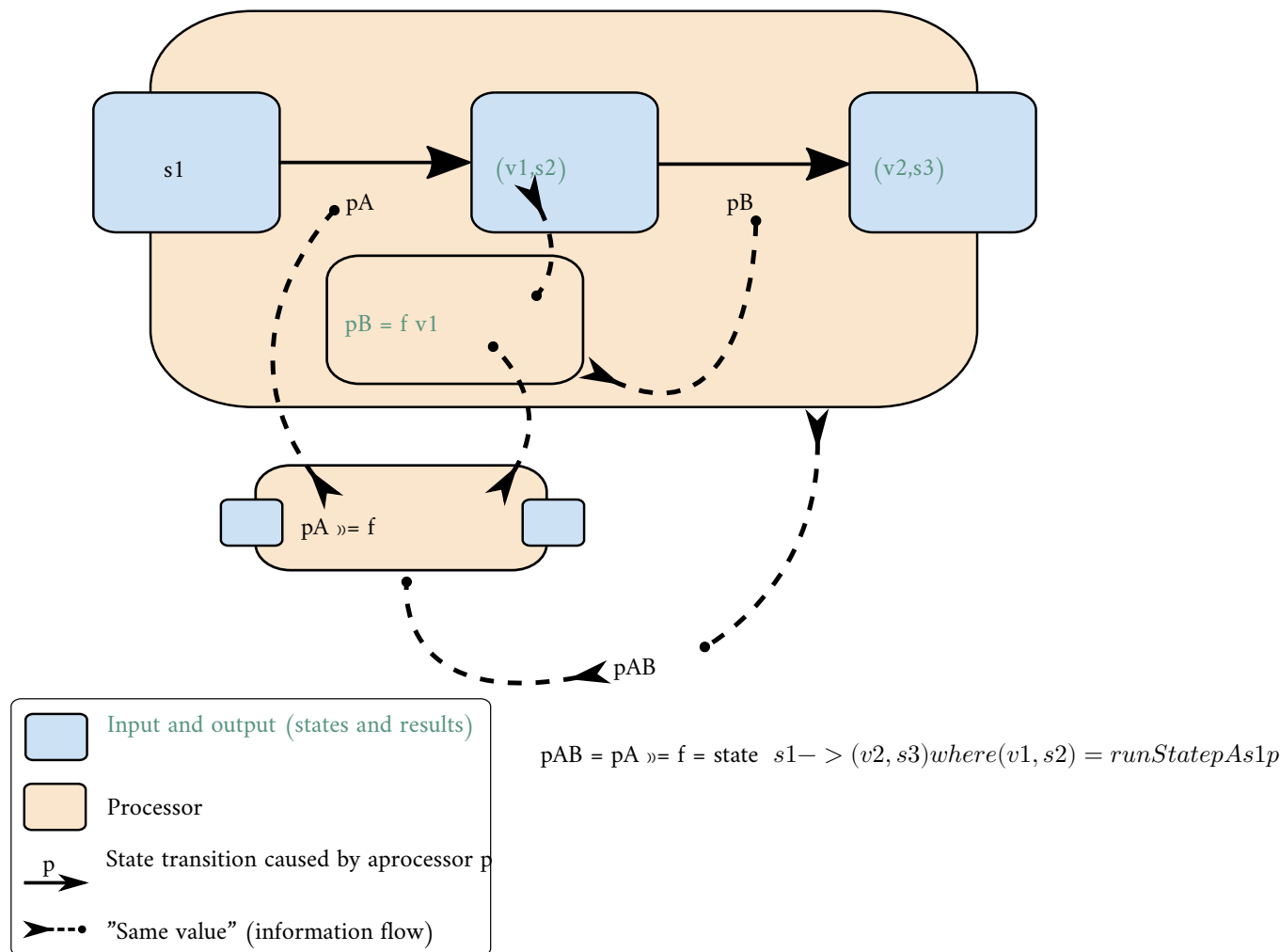
```
type State s = StateT s Identity
```

[Binding](#) copies and transforms the state [parameter](#) through the [sequence](#) of the [bound functions](#) so that the same state storage is never used twice. Overall this gives the illusion of in-place update to the programmer and in the code, while in fact the autogenerated transition [functions](#) handle the state changes.

Example [type](#): State st a

State describes [functions](#) that consume a state and produce a [tuple](#) of result and an updated state.

[Monad](#) manages the state with the next [process](#):



Where:

- f - processor making **function**
- pA, pAB, pB - state processors
- sN - states
- vN - values

Bind with a processor making **function** from state processor (pA) creates a new state processor (pAB). The wrapping and unwrapping by `State/runState` is implicit.

2.5.7.6.11 Monad transformer

* is a practical solution to the current functional programming problem about **composition** of **monads**.

Monad is not **closed** under composition. **Composition** of **monadic types** does not always results in **monadic type**.

Basic **case**: during implementation of **monadic type composition**, **type** $m \rightarrow T \rightarrow m$ arises, which does not allow to `unit`, `join` the m **monadic** layers.

* have desirable properties and can add them to **monads**. * user their implementation to solve the composition **type** layering and allow to attach desirable **property** to result.

* solve **monad composition** and **type** layering by cheating, using own **structure** and information about itself. It is often that **process** involves a **catamorphism** of a * **type** layer.

In **type** signatures of transformers $*T \ m \rightarrow m$ is already an extended **monad**, so $*T$ is just a wrapper to point that out.

Transformers have a light wrapper around the data that tags the modification with this transformer.

Main **monadic structure** m is wrapped around the internal data (core is a). The **structure** that corresponds to the transformer creation **properties** (if it emitted by η of a transformer), goes into m . Open **parameters** go external to the m .

```
newtype ExceptT e m a =
  ExceptT { runExceptT :: m (Either e a) }
```

```
newtype MaybeT m a =
  MaybeT { runMaybeT :: m (Maybe a) }
```

```
newtype ReaderT r m a =
  ReaderT { runReaderT :: r -> m a }
```

This has an **effect** that on stacking **monad** transformers, m becomes **monad stack**, and every next transformer injects the transformer creation-specific properties η inside the **stack**, so out-most transformer has inner-most **structure**. Base **monad** is structurally the outermost.

2.5.7.6.11.1 MaybeT

* extends **monads** by injecting **Maybe** layer underneath **monad**, and processing that **structure**:

```
newtype MaybeT m a = MaybeT { runMaybeT :: m (Maybe a) }
```

2.5.7.6.11.2 EitherT

* extends **monads** by injecting **Either** layer underneath **monad**, and processing that **structure**:

```
newtype EitherT e m a = EitherT { runEitherT :: m (Either e a) }
```

EitherT of either package gets replaced by **ExceptT** of transformers or **mtl** packages.

* **ExceptT**

2.5.7.6.11.3 ReaderT

Definition:

```
newtype ReaderT r m a = ReaderT { runReaderT :: r -> m a }
```

* **functions**: input **monad** $m \ a$, out: $m \ a$ wrapped it in a free-variable r (**partially applied function**). That allows to use transformed $m \ a$, now it requires and can use the r passed environment.

To create a **Reader monad**:

```
type Reader r = ReaderT r Identity
```

2.5.7.6.11.4 MonadTrans type class

Allows to **lift monadic** actions into a larger **context** in a neutral way.

pure takes a parametric **type** and embodies it into constructed **structure** (talking of **monad** transformers - **structure** of the stacked **monads**).

lift takes **monad** and extends it with a transformer.

In fact, for **monad** transformers - **lift** is a last stage of the **pure**, it follows from the **lift** law.

Method:

```
lift :: Monad m => m a -> t m a
```

Lift a computation from the **argument monad** to the constructed **monad**.

Neutral means:

```
lift . return = return
```

```
lift (m >=> f) = lift m >=> (lift . f)
```

The general pattern with **MonadTrans** instances is that it usually lifts the **injection** of the known **structure** of transformer over some **Monad**.

lift embeds one **monadic** action into **monad transformer**.

The difference between **pure**, **lift** and **MaybeT** constructor becomes clearer if you look at the **types**:

Example, for **MaybeT IO a**:

```
pure      ::      a -> MaybeT IO a
lift      ::      IO a -> MaybeT IO a
MaybeT   :: IO (Maybe a) -> MaybeT IO a
```

```
x = (undefined :: IO a)
```

```
:t (pure x)
(pure x) :: Applicative t => t (IO a)  -- t recieves one argument of product type
:t (pure x :: MaybeT IO a)
-- Expected type: MaybeT IO a1
-- Actual type: MaybeT IO (IO a0)
```

```
-- While the real type would be
```

```
:t (pure x :: MaybeT IO (IO a))
```

```
(pure x :: MaybeT IO (IO a)) :: MaybeT IO (IO a) -- This goes into a conflict of what type&kind (
```

```
:t (lift x)
```

```
(lift x) :: MonadTrans t => t IO a  -- result is a proper expected product type
```

```
-- To belabour
```

```
:t (lift x :: MaybeT IO a)
```

```
(lift x) :: MonadTrans t => t IO a  -- result is a proper expected product type
```

lift is a **natural transformation** η from an **Identity monad** (**functor**) with other **monad** as content into transformer **monad** (**functor**), with the preservation of the contained **monad**:

```
-- Abstract monads with content as parameters. Define '~>' as a family of morphisms that translate
type f ~> g = forall x. f x -> g x
```

```
-- follows
```

```
lift :: m ~> t m
```

MonadIO type class * - allows to **lift IO** action until reaching the **IO monad** layer at the top of the **Monad stack** (which is allways in the Haskell code that does **IO**).

```
class (Monad m) => MonadIO m where
```

```
    liftIO :: IO a -> m a
```

liftIO laws:

```
liftIO . pure = pure
```

```
liftIO (m >>= f) = liftIO m >>= (liftIO . f)
```

Which is identical laws to `MonadTrans lift`.

Since `lift` is one step, and `liftIO` all steps - all steps defined in terms of one step and all other steps, so the most frequent implementation is self-recursive `lift . liftIO`:

```
liftIO ioa = lift $ liftIO ioa
```

```
*
```

```
liftIO
```

2.5.7.7 Alternative type class

`Monoid` over `applicative`. Has left catch `property`.

Allows to run simlteniously several instances of a computation (or computations) and from them yield one result by law from `(<|>) :: Type -> Type -> Type`.

Minimal complete definition:

```
empty :: f a    -- The identity element of </>
(<|>) :: f a -> f a -> f a    -- Associative binary operation
```

2.5.7.7.1 *

Alternative

2.5.8 Monoidal functor

`Functors` between `monoidal categories` that preserves `monoidal structure`.

2.5.9 Fusion

```
fmap f . fmap g = fmap (f . g)
```

* - `functor` axiom that allows to greatly simplify computations.

2.5.10 \$>

Get & `set` a value inside `Functor`.

2.5.10.1 *

```
<$
```

2.5.11 Multifunctor

Generalizes the concept of `functor` between `categories`, canonical `morphisms` between multicategories.

Works over N `type` arguments instead of one.

To put simply - accepts multiple argumets, from that informatioin constructs source `product category` (`Cartesian product`) of `categories`, and is a `functor` from `product category` to target `category`.

To put even simpler - `functor` that takes as an `argument` the `product` of `types`.

In Haskell there is only one `category`, `Hask`, so in Haskell $*$ is still $(Hask \times Hask) \rightarrow Hask \Rightarrow |(Hask \times Hask) \equiv Hask| \Rightarrow Hask \rightarrow Hask$ `endofunctor`.

Any `product` or sum in a Cartesian `category` is a $*$.

Code definition:

```
class Bifunctor f
  where
    bimap :: (a -> a') -> (b -> b') -> f a a' -> f a a'
    bimap f g = first f . second g
    first :: (a -> a') -> f a b -> f a' b
    first f = bimap f id
    second :: (b -> b') -> f a b -> f a b'
    second = bimap id
```

2.5.11.1 *

Bifunctor

2.6 Hask category

Category of Haskell where objects are types and morphisms are functions.

It is a hypothetical category at the moment, since undefined and bottom values break the theory, is not Cartesian closed, it does not have sums, products, or initial object, () is not a terminal object, monad identities fail for almost all instances of the Monad class.

That is why Haskell developers think in subset of Haskell where types do not have bottom values. This only includes functions that terminate, and typically only finite values. The corresponding category has the expected initial and terminal objects, sums and products, and instances of Functor and Monad really are endofunctors and monads.

Hask contains subcategories, like Lst containing only list types.

Haskell and Category concepts:

- Things that take a type and return another type are type constructors.
- Things that take a function and return another function are higher-order functions.

2.6.1 *

Hask

2.7 Magma

Set with a binary operation which form a closure.

2.7.1 Mag category

The category of magmas, denoted Mag , has as objects - sets with a binary operation, and morphisms given by homomorphisms of operations (in the universal algebra sense).

2.7.1.1 *

MAG Magma category Category of magmas

2.7.2 Semigroup

Magma with associative property of operation.

Defined in Haskell as:

```
class Semigroup a where
  (<>) :: a -> a -> a
```

2.7.2.1 *

Semigroups

2.7.2.2 Monoid

`Semigroup` with `identity` element, `Category` with a one `object`.

Ideally fits as an accumulation class.

```
class Monoid m where
  mempty :: m
  mappend :: m -> m -> m
  mappend = (<>)
  mconcat :: [m] -> m
  mconcat = foldr mappend mempty
```

* can be simplified to `category` with a single `object`, remember that `monoid operation` is a `composition` of `morphisms operation` in `category`.

For example to represent the whole non-negative integers with the one `object` and `morphism` "1" is absolutely enough, `composition operation` is "+".

```
import Data.Monoid
do
  show (mempty :: Num a => Sum a)
  -- "Sum {getSum = 0}"
  show $ Sum 1
  -- "Sum {getSum = 1}"
  show $ (Sum 1) <> (Sum 1) <> (Sum 1)
  -- "Sum {getSum = 3}"
  -- ...
```

Also backwards - any single-object `category` is a `monoid`. `Category` has an `identity` requirement and `associativity` of `composition` requirement, which makes it a free `monoid`.

2.7.2.2.1 *

Monoidal Monoids

2.7.2.2.2 Monoid laws

2.7.2.2.2.1 Monoid left identity law

```
mempty <> x = x
```

2.7.2.2.2.2 Monoid right identity law

```
x <> mempty = x
```

2.7.2.2.2.3 Monoid associativity law

```
x <> mempty = x (y <> z) = (x <> y) <> z
mconcat = foldr (mempty <>)
```

Everything `associative` can be `mappend`.

2.7.2.2.3 Commutative monoid

Operation that forms **structure** has **commutativity property**: $x \circ y = y \circ x$

Opens a big abilities in concurrent and distributed processing.

2.7.2.2.3.1 *

Abelian monoid

2.7.2.2.4 Group

Monoid that has **inverse** for every element.

2.7.2.2.4.1 *

Groups

2.7.2.2.4.2 Commutative group

Commutative monoid that is a **group**.

*

Abelian group

Ring **Commutative group** under $+$ & **monoid** under \times , $+$ & \times connected by **distributive property**.

- and \times are generalized **binary** operations of addition and multiplication. \times has no requirement for **commutativity**.

Example: **set** of same size square matrices of numbers with matrix operations form a **ring**.

*

Rings

2.8 Morphism

μορφή *morphe* form

Arrow between two **objects** inside a **category**.

Morphism can be anything.

Morphism is a generalization ($f(x * y) \equiv f(x) \diamond f(y)$) of **homomorphism** ($f(x * y) \equiv f(x) * f(y)$). Since general **morphisms** not so much often ment and discussed - under **morphism** people almost always really mean the meaning of **homomorphism**-like **properties**, hence they discuss the **algebraic structures (types)** and homomorphisms between them.

In most usage, on a level under the **objects**: $*$ is most often means a map (**relation**) that translates from one mathematical **structure** (that source **object** represents) to another (that target **object** represents) (that is called (somewhat, somehow) "**structure-preserving**", but that **phrase** still means that translation can be lossy and irrevertable, so it is only bear reassemblance of preservation), and in the end the **morphism** can be anything and not hold to this conditions.

Morphism needs to correspond to **function** requirements to be it.

2.8.1 *

Morphisms Arrow Arrows

2.8.2 Homomorphism

ὁμός *homos* same (was chosen because of initial English mistranslation to "similar")

μορφή *morphe* form

similar form

* map between two [algebraic structures](#) of the same [type](#), [operation](#)-preserving.

$f_{x \rightarrow y} = f(a \star b) = f(a) \diamond f(b)$, where x, y are [sets](#) with additional [algebraic structure](#) that includes \star, \diamond accordingly; a, b are elements of [set](#) x .

* sends [identity morphisms](#) to [identity morphisms](#) and inverses to inverses.

The concept of * has been generalized under the name of [morphism](#) to many [structures](#) that [either](#) do not have an underlying [set](#), or are not [algebraic](#).

2.8.2.1 *

Homomorphic

2.8.3 Identity morphism

[Identity morphism](#) - or simply [identity](#): $x \in C : id_x = 1_x : x \rightarrow x$ [Composed](#) with other [morphism](#) gives same [morphism](#).

Corresponds to [Reflexivity](#) and [Automorphism](#).

2.8.3.1 Identity

[Identity](#) only possible with [morphism](#). See [Identity morphism](#).

There is also distinct [Zero](#) value.

2.8.3.1.1 Two-sided identity of a predicate

$P(e, a) = P(a, e) = a \mid \exists e \in S, \forall a \in S P()$ is [commutative](#).

[Predicate](#)

2.8.3.1.2 Left identity of a predicate

$\exists e \in S, \forall a \in S : P(e, a) = a$

[Predicate](#)

2.8.3.1.3 Right identity of a predicate

$P(a, e) = a \mid \exists e \in S, \forall a \in S$

[Predicate](#)

2.8.3.2 Identity function

Return itself. $(\backslash x.x)$

`id :: a -> a`

2.8.4 Monomorphism

$\mu\omicron\nu\omicron$ *mono* only

$\mu\omicron\rho\phi\eta$ *morphe* form

Maps one to one (uniquely), so invertable (always has [inverse morphism](#)), so preserves the information/[structure](#). [Domain](#) can be equal or less to the [codomain](#).

$f^{X \rightarrow Y}, \forall x \in X \exists! y = f(x) \models f(x) \equiv f_{mono}(x)$ - from [homomorphism context](#) $f_{mono} \circ g1 = f_{mono} \circ g2 \models g1 \equiv g2$ - from general [morphisms context](#) Thus $*$ is left cancelable.

If $*$ is a [function](#) - it is [injective](#). Initial [set](#) of f is fully uniquely mapped onto the [image](#) of f .

2.8.4.1 $*$

Monomorphic Monomorphisms

2.8.5 Epimorphism

$\epsilon\pi\iota$ *epi* on, over

$\mu\omicron\rho\phi\eta$ *morphe* form

$*$ is right cancelable [morphisms](#). $f^{X \rightarrow Y}, \forall y \in Y \exists f(x) \models f(x) \equiv f_{epi}(y)$ - from [homomorphism context](#) $g1 \circ f_{epi} = g2 \circ f_{epi} \Rightarrow g1 \equiv g2$ - from general [morphisms context](#)

In [Set category](#) if $*$ is a [function](#) - it is [surjective](#) ([image](#) of it fully uses [codomain](#)) [Codomain](#) is a called a projection of the [domain](#).

$*$ fully maps into the target.

2.8.5.1 $*$

Epimorphic Epimorphisms

2.8.6 Isomorphism

$\iota\sigma\omicron\varsigma$ *isos* equal

$\mu\omicron\rho\phi\eta$ *morphe* form

Not equal, but equal for current intents and purposes. [Morphisms](#) that has [inverse](#). Almost equal, but not quite: ([Integer](#), [Bool](#)) & ([Bool](#), [Integer](#)) - but can be transformed losslessly into one another.

[Bijective homomorphism](#) is also [isomorphism](#).

$$f^{-1, b \rightarrow a} \circ f^{a \rightarrow b} \equiv 1^a, f^{a \rightarrow b} \circ f^{-1, b \rightarrow a} \equiv 1^b$$

2 reasons for non-[isomorphism](#):

- [function](#) at least ones collapses a values of [domain](#) into one value in [codomain](#)
- [image](#) (of a [function](#) in [codomain](#)) does not fill-in [codomain](#). Then [isomorphism](#) can exists for [image](#) but not whole [codomain](#).

[Categories](#) are [isomorphic](#) if there $R \circ L = ID$

2.8.6.1 $*$

Isomorphic Isomorphisms

2.8.6.2 Lax

Holds up to [isomorphism](#). (upon the transformation can be used as the same)

2.8.7 Endomorphism

ενδο *endo* internal

μορφή *morphe* form

[Arrow](#) from [object](#) to itself. [Endomorphism](#) forms a [monoid](#) ([object](#) exists and [category](#) requirements already in place).

2.8.7.1 Automorphism

αυτο *auto* self

μορφή *form* form

[Isomorphic endomorphism](#).

Corresponds to [identity](#), [reflexivity](#), [permutation](#).

2.8.7.1.1 *

Automorphic Automorphisms

2.8.7.2 *

Endomorphic Endomorphisms

2.8.8 Catamorphism

κατά *kata* downward

μορφή *morphe* form

Unique [arrow](#) from an initial [algebra structure](#) into different [algebra structure](#).

* in FP is a generalization folding, deconstruction of a [data structure](#) into more primitive [data structure](#) using a [functor F-algebra structure](#).

* reduces the [structure](#) to a lower level [structure](#). * creates a projection of a [structure](#) to a lower level [structure](#).

2.8.8.1 *

Catamorphic Catamorphisms

2.8.8.2 Catamorphism law

Table 2.2: [Catamorphism](#) laws in Haskell

| Rule name | Haskell |
|--------------|--|
| cata-cancel | <code>cata phi . InF = phi . fmap (cata phi)</code> |
| cata-refl | <code>cata InF = id</code> |
| cata-fusion | <code>f . phi = phi . fmap f => f . cata phi = cata phi</code> |
| cata-compose | <code>eps :: f :~> g => cata phi . cata (In . eps) = cata (phi . eps)</code> |

2.8.8.2.1 Hylomorphism

catamorphism ◦ anamorphism

Expanding and collapsing the structure.

2.8.8.2.1.1 *

Hylomorphic Hylomorphisms

2.8.8.3 Anamorphism

Generalizes unfold.

Dual concept to catamorphism.

Increases the structure.

Morphism from a coalgebra to the final coalgebra for that endofunctor.

Is a function that generates a sequence by repeated application of the function to its previous result.

2.8.8.3.1 *

Anamorphic Anamorphisms

2.8.9 Kernel

Kernel of a homomorphism is a number that measures the degree homomorphism fails to meet injectivity (AKA be monomorphic). It is a number of domain elements that fail injectivity:

- elements not included into morphism
- elements that collapse into one element in codomain

thou Kernel $[x|x \leftarrow 0 || x \geq 2]$.

Denotation: $\ker T = \{\mathbf{v} \in V : T(\mathbf{v}) = \mathbf{0}_W\}$.

2.8.9.1 Kernel homomorphism

Morphism of elements from the kernel. Complementary morphism of elements that make main morphism not monomorphic.

2.9 Set category

Category in which objects are sets, morphisms are functions.

Denotation: *Set*

2.10 Natural transformation

Roughly * is:

trans :: F a -> G a

, while a is polymorphic variable.

Naturality condition: $\forall a \exists (F a \rightarrow G a)$, or, analogous to parametric polymorphism in functions. Since * in a category, stating $\forall (F a \rightarrow G a)$ Naturality condition means that all morphisms that take part in homotopy of source functor to target functor must exist, and that is the same, diagrams that take part in transformation, should commute, and different paths bring same result: if α - natural transformation, α_a

natural transformation component - $G f \circ \alpha_a = \alpha_b \circ F f$. Since $*$ are just a **type** of parametric **polymorphic function** - they can **compose**.

$*$ ($\vec{\eta}^{\mathcal{D}}$) is transforming: $\vec{\eta}^{\mathcal{D}} \circ F^{\mathcal{C} \rightarrow \mathcal{D}} = G^{\mathcal{C} \rightarrow \mathcal{D}}$. $*$ **abstraction** creates higher-language of **Category** theory, allowing to talk about the **composition** and transformation of complex entities.

It is a **process** of transforming $F^{\mathcal{C} \rightarrow \mathcal{D}}$ into $G^{\mathcal{C} \rightarrow \mathcal{D}}$ using existing **morphisms** in target **category** \mathcal{D} .

Since it uses **morphisms** - it is **structure-preserving** transformation of one **functor** into another. It is mostly a lossy transformation. Only existing **morphisms** can make it exist.

Existence of $*$ between two **functors** can be seen as some **relation**.

Can be observed to be a "**morphism of functors**", especially in **functor category**. $*$ by $\vec{\eta}_{y^{\mathcal{C}}}^{\mathcal{D}}((x, y)^{\mathcal{C}}) \circ F^{\mathcal{C} \rightarrow \mathcal{D}}((x, y)^{\mathcal{C}}) = G^{\mathcal{C} \rightarrow \mathcal{D}}((x, y)^{\mathcal{C}}) \circ \vec{\eta}_{x^{\mathcal{C}}}^{\mathcal{D}}((x, y)^{\mathcal{C}})$, often written short $\vec{\eta}_b \circ F(\vec{f}) = G(\vec{f}) \circ \vec{\eta}_a$. Notice that the $\vec{\eta}_{x^{\mathcal{C}}}^{\mathcal{D}}((x, y)^{\mathcal{C}})$ depends on **objects&morphisms** of \mathcal{C} .

In words: $*$ depends on F and G **functors**, ability of \mathcal{D} **morphisms** to do a **homotopy** of F to G , and $*$.

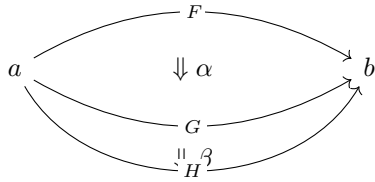
- for every **object** in \mathcal{C} picks **natural transformation component** in \mathcal{D} .
- for every **morphism** in \mathcal{C} picks the **commuting diagram** in \mathcal{D} , called naturality square.

Also see: **Natural transformation in Haskell**

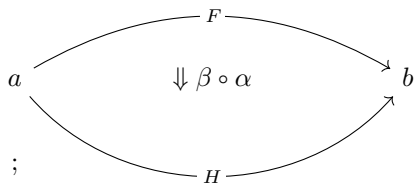
Knowledge of $*$ forms a **2-category**.

Can be **composed**

- "vertically":

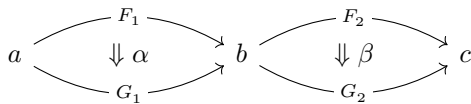


;

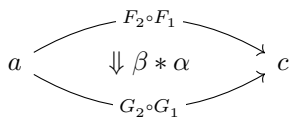


;

- "horizontally" ("Godement **product**"):



;



;

Vertical and horizontal [compositions](#) can be done in any [order](#), they abide the exchange law.

2.10.1 *

Natural transformations Naturality condition Naturality

2.10.2 Natural transformation component

$$\vec{\eta}^{\mathcal{D}}(x) = F^{\mathcal{D}}(x) \rightarrow G^{\mathcal{D}}(x) \mid x \in \mathcal{C}$$

2.10.2.1 *

Component of natural transformation

2.10.3 Natural transformation in Haskell

Family of [parametric polymorphism functions](#) between [endofunctors](#).

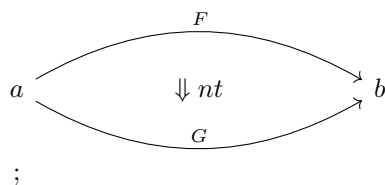
In [Hask](#) is $F a \rightarrow G a$. Can be analogued to repackaging data into another container, never modifies the [object](#) content, it only if - can delete it, because [operation](#) is lossy.

Can be sees as ortogonal to [functors](#).

2.10.4 Cat category

[Category](#) where:

| | Part | Is | # |
|---------------|----------------------------|---|--------|
| * | object | category | 0-cell |
| \Rightarrow | morphism | functor | 1-cell |
| \Rightarrow | 2-morphism | natural transformation , morphisms homotopy | 2-cell |



Is Cartesian [closed category](#).

2.10.4.1 *

Cat 2-category

2.10.4.2 Bicategory

[2-category](#) that is [enriched](#) and [lax](#).

For handling relaxed [associativity](#) - introduces associator, and for [identity](#) l -eft/right unitor.

Forming from bicategories higher [categories](#) by stacking levels of [abstraction](#) of such [categories](#) - leads to explosion of special cases, differences of every level, and so overall difficulties.

Stacking groupoids ([category](#) in which are [morphisms](#) are invertable) is much more homogenous up to infinity, and forms base of the [homotopy type](#) theory.

2.11 Category dual

Category duality behaves like a logical inverse.

Inverse $\mathcal{C} = \mathcal{C}^{op}$ - inverts the direction of morphisms.

Composition accordingly changes to the morphisms: $(g \circ f)^{op} = f^{op} \circ g^{op}$

Any statement in the terms of \mathcal{C} in \mathcal{C}^{op} has the dual - the logical inverse that is true in \mathcal{C}^{op} terms.

Opposite preserves properties:

- products: $(\mathcal{C} \times \mathcal{D})^{op} \cong \mathcal{C}^{op} \times \mathcal{D}^{op}$
- functors: $(F^{\mathcal{C} \rightarrow \mathcal{D}})^{op} \cong F^{\mathcal{C}^{op} \rightarrow \mathcal{D}^{op}}$
- slices: $(\mathcal{F} \downarrow \mathcal{G})^{op} \cong (\mathcal{G}^{op} \downarrow \mathcal{F}^{op})$

2.11.0.0.1 *

Opposite category Opposite categories Category duality Duality Dual category Dual

2.11.1 Coalgebra

Structures that are dual (in the category-theoretic sense of reversing arrows) to unital associative algebras. Every coalgebra, by vector space duality, reversing arrows - gives rise to an algebra. In finite dimensions, this duality goes in both directions. In infinite - it should be determined.

2.12 Thin category

\forall Hom sets contain zero or one morphism.

$$f \equiv g \mid \forall x, y \forall f, g : x \rightarrow y$$

A proset (preordered set).

2.12.1 *

Proset category Prosetal category Poset category Posetal category

2.13 Commuting diagram

Establishes equality in morphisms that have same source and target.

Draws the morphisms that are: $f = g \Rightarrow \{f, g\} : X \rightarrow Y$

2.13.1 *

Diagram commutes Commutes

2.14 Universal construction

Algorithm of constructing definitions in Category theory. Specially good to translate properties/definitions from other theories (Set theory) to Categories.

Method:

1. Define a pattern that you defining.
2. Establish ranking for pattern matches.

3. The top of ranking, the best match or [set](#) of matches - is the thing you was looking for. Matches are [isomorphic](#) for defined rules.

* uses Yoneda lemma, and as such constructions are defined until [isomorphism](#), and so [isomorphic](#) between each-other.

2.14.1 *

Universal constructions

2.15 Product

Universal construction:

$$\begin{array}{ccccc} & & c' & & \\ & p \swarrow & \downarrow ! & \searrow q & \\ a & \xleftarrow{\pi_a} & c & \xrightarrow{\pi_b} & b \end{array}$$

Pattern: $p : c \rightarrow a, q : c \rightarrow b$

Ranking: $\max \sum^{\forall} (! : c' \rightarrow c \mid p' = p \circ !, q' = q \circ !)$

c' is another candidate.

For [sets](#) - Cartesian product.

* is a pair. Corresponds to [product data type](#) in [Hask](#) (inhabited with all elements of the [Cartesian product](#)).

Dual is [Coproduct](#).

2.15.1 *

Products

2.16 Coproduct

Universal construction:

$$\begin{array}{ccccc} & & c' & & \\ & p \nearrow & \uparrow ! & \nwarrow q & \\ a & \xrightarrow{\iota_a} & c & \xleftarrow{\iota_b} & b \end{array}$$

Pattern: $i : a \rightarrow c, j : b \rightarrow c$

Ranking: $\max \sum^{\forall} (! : c \rightarrow c' \mid i' = ! \circ i, j' = ! \circ j)$

c' is another candidate.

For [sets](#) - Disjoint union.

* is a [set](#) assembled from other two [sets](#), in Haskell it is a tagged [set](#) (analogous to disjoint union).

Dual is [Product](#).

2.16.1 *

Coproducts

2.17 Free object

General particular **structure**. In which **structure**, **properties** autofollows from definition, axioms.

Also uses as a term when surcomstances of **structures**, rules, **properties**, axioms used coincide with the definition of a particular **object** \therefore form **object** of this **type** with the according **properties** and possibilities.

2.18 Internal category

Category which is included into a bigger **category**.

2.19 Hom set

All **morphisms** from source **object** to target **object**.

Denotation: $hom_C(X, Y) = (\forall f : X \rightarrow Y) = hom(X, Y) = C(X, Y)$ Denotation was not standardized.

Hom sets belong to **Set category**.

In **Set category**: $\exists!(a, b) \iff \exists! Hom, \forall Hom \in Set$. **Set category** is special, **Hom sets** are also **objects** of it.

Category can include **Set**, and **hom sets**, or not.

2.19.1 *

Hom-set Hom sets

2.19.2 Hom-functor

$hom : \mathcal{C}^{op} \times \mathcal{C} \rightarrow Set$ **Functor** from the **product** of \mathcal{C} with its **opposite category** to the **category** of **sets**.

Denotation variants: $H_A = Hom(-, A)$ $h_A = \mathcal{C}(-, A)$ $Hom(A, -) : \mathcal{C} \rightarrow Set$

Hom-**bifunctor**: $Hom(-, -) : \mathcal{C}^{op} \times \mathcal{C} \rightarrow Set$

2.19.3 Exponential object

Generalises the notion of **function set** to internal **object**. As also **hom set** to **internal hom objects**.

Cartesian **closed (monoidal) category** strictly required, as * multiplication holds **composition** requirement:

$$\circ : hom(y, z) \otimes hom(x, y) \rightarrow hom(x, z)$$

Denotation: b^a

Universal construction:

$$\begin{array}{ccc} \mathcal{C} & \mathcal{C} \times a & \\ \vdots & \vdots & \searrow \\ u & u \times 1^a & \\ \vee & \vee & \\ b^a & b^a \times a \cdot eval & b \end{array}$$

;

, where in Category: b^a - exponential object, \times - product bifunctor, a - argument of $*$, b - result, c - candidate, $b^a \equiv (a \Rightarrow b) - *$.

$*$ b^a (also as $(a \Rightarrow b)$) represent exponentiation of cardinality of $\forall b^a$ possibilities.

2.19.3.1 $*$

Function object Internal hom Exponential objects Hom object Hom objects

2.19.3.2 Enriched category

Uses Hom objects (exponential objects), which do not belong into Set category. Category is no longer small, now may be called large.

$hom(x, y) \in K$.

Called: $*$ over K (which holds hom objects).

2.19.3.2.1 $*$

Enriched Large category

Chapter 3

Data type

Set of values. For [type](#) to have sense the values must share some sense, [properties](#).

3.1 *

Type Types Data types

3.2 Actual type

[Data type](#) recieved by [->inferring->compiling->execution](#).

3.3 Algebraic data type

Composite [type](#) formed by combining other [types](#).

3.3.1 *

AlgDT

3.4 Cardinality

Number of possible implementations for a given [type](#) signature.

[Disjunction](#), sum - adds [cardinalities](#). [Conjunction](#), [product](#) - multiplies [cardinalities](#).

3.4.1 *

Cardinalities

3.5 Data constant

* - [constant](#) value; [nullary data constructor](#).

3.6 Data constructor

One instance that [inhabit data type](#).

3.7 data declaration

Data type declaration is the most general and versatile form to create a new **data type**. Form:

```
data [context =>] type typeVars1..n
  = con1 c1t1..i
  | ...
  | conm cmt1..q
[deriving]
```

3.8 Dependent type

When **type** and values have **relation** between them. **Type** has restrictions for values, value of a **type variable** has a result on the **type**.

3.8.1 *

Dependent types

3.9 Gen type

Generator. **Gen type** is to generate pseudo-random values for parent **type**. Produces a **list** of values that gets infinitely cycled.

3.10 Higher-kinded data type

Any combination of * and ->

Type that take more **types** as arguments.

*Humbly really a **function***

3.10.1 *

Higher-kinded data types

3.11 newtype declaration

Create a new **type** from old **type** by attaching a new **constructor**, allowing **type class instance declaration**.

```
newtype FirstName = FirstName String
```

Data will have exactly the same representation at runtime, as the **type** that is wrapped.

```
newtype Book = Book (Int, Int)
```

```
    (,)
    /\
Integer Integer
```

3.12 Principal type

The most generic **data type** that still **typechecks**.

3.13 Product data type

Is an [algebraic data type](#) representation of a [product](#) construction. Formed by logical [conjunction](#) (AND, '[* *](#)').

Haskell forms:

```
-- 1. As a tuple (the uncurried & most true-form)
(T1, T2)

-- 2. Curried form, data constructor that takes two types
C T1 T2

-- 3. Using record syntax. =r# <inhabitant>= would return the respective =T#=
C { r1 :: T1
  , r2 :: T2
  }
```

3.13.1 *

Product type

3.13.2 Sequence

Enumerated (ordered) [set](#).

Denotation:

```
()
( , )
( , , )
( , , ... )
```

More general mathematical denotation was not established, variants: $(n)_{n \in \mathbb{N}} \omega \rightarrow X \{i : Ord \mid i < \alpha\}$

In Haskell: [Data type](#) that stores multiple ordered values withing a single value.

Table 3.1: [Sequence constructor](#) naming by [arity](#)

| Name | Arity | Denotation |
|------------------------------|-------|-------------|
| Unit , empty | 0 | () |
| Singleton | 1 | (_) |
| Tuple, pair, two-tuple | 2 | (,) |
| Triple, three-tuple | 3 | (, ,) |
| Sequence | N | (, , ...) |

3.13.2.1 *

Sequences Tuples Ordered pair Ordered triple

3.13.2.2 List

[Sequence](#) of one [type](#) objects.

Denotation:

```
[]
[ , ]
[ , , ]
[ , , ... ]
```

Haskell definition:

```
data [] a = [] | a : [a]
```

Definition is self-referential (self-recursive), can be seen as [anamorphism](#) (unfold) of the [] (empty list, memory cell which is container of particular type) and : (cons operation, pointer). As such - can create non-terminating data type (and computation), in other words - infinite.

3.14 Proxy type

Proxy type holds no data, but has a phantom parameter of arbitrary type (or even kind). Able to provide type information, even though has no value of that type (or it can be may too costly to create one).

```
data Proxy a = ProxyValue
```

```
let proxy1 = (ProxyValue :: Proxy Int) -- a has kind `Type`
let proxy2 = (ProxyValue :: Proxy List) -- a has kind `Type -> Type`
```

3.15 Static typing

Typechecking takes place at compile level.

3.16 Structural type

Mathematical type. They form into structural type system.

3.16.1 *

Structural

3.17 Structural type system

Strict global hierarchy and relationships of types and their properties. Haskell type system is *. In most languages typing is name-based, not structural.

3.17.1 *

Structural typing

3.18 Sum data type

Algebraic data type formed by logical disjunction (OR '|').

3.19 Type alias

Create new type constructor, and use all data structure of the base type.

3.20 Type class

Type system [construct](#) that adds a support of [ad hoc polymorphism](#).

Type class makes a nice way for defining behaviour, [properties](#) over many [types/objects](#) at once.

3.20.1 *

Type classes

3.20.2 Arbitrary type class

Type class of [QuickCheck.Arbitrary](#) (that is reexported by [QuickCheck](#)) for creating a [generator](#)/distribution of values. Useful [function](#) is [arbitrary](#) - that autogenerates values.

3.20.2.1 Arbitrary function

Depends on [type](#) and generates values of that [type](#).

3.20.3 CoArbitrary type class

Pseudogenerates a [function](#) basing on resulting [type](#).

```
coarbitrary :: CoArbitrary a => a -> Gen b -> Gen b
```

3.20.3.1 *

CoArbitrary

3.20.4 Typeable type class

Allows dynamic [type](#) checking in Haskell for a [type](#). Shift a [typechecking](#) of [type](#) from compile time to runtime. * [type](#) gets wrapped in the universal [type](#), that shifts the [type](#) checks to runtime.

Also allows:

- Get the [type](#) of something at runtime (ex. print the [type](#) of something [typeof](#)).
- Compare the [types](#).
- Reifying [functions](#) from [polymorphic type](#) to concrete (for [functions](#) like `:: Typeable a => a -> String`).

3.20.4.1 *

Typeable

3.20.5 Type class inheritance

Type class has a [superclass](#).

3.20.6 Derived instance

Type class instances sometimes can be automatically [derived](#) from the parent [types](#).

Type classes such as [Eq](#), [Enum](#), [Ord](#), [Show](#) can have instances generated based on definition of [data type](#).

P.S.

Language options:

- `DeriveAnyClass`
- `DeriveDataTypeable`
- `DeriveFoldable`
- `DeriveFunctor`
- `DeriveGeneric`
- `DeriveLift`
- `DeriveTraversable`
- `DerivingStrategies`
- `DerivingVia`
- `GeneralisedNewtypeDeriving`
- `StandaloneDeriving`

3.20.6.1 *

Derived Deriving

3.21 Type constant

[Nullary type constructor](#).

3.22 Type constructor

Name of the [data type](#).

[Constructor](#) that takes [type](#) as an [argument](#) and produces new [type](#).

3.23 type declaration

Synonym for existing [type](#). Uses the same [data constructor](#).

`type FirstName = String`

Used to distinct one entities from other entities, while they have the same [type](#). Also main [type functions](#) can operate on a new [type](#).

3.24 Typed hole

* - is a `_` or `_name` in the [expression](#). On [evaluation](#) GHC would show the [derived type](#) information which should be in place of the *. That information helps to fill in the gap.

3.24.1 *

Typed holes

3.25 Type inference

Automatic [data type](#) detection for [expression](#).

3.25.1 *

Inferring Infer Infers Inferred

3.26 Type class instance

Block of implementations of [functions](#), based on unique [type class](#)->[type](#) pairing.

3.27 Type rank

Weak ordering of [types](#).

The rank of [polymorphic type](#) shows at what level of nesting forall [quantifier](#) appears. Count-in only [quantifiers](#) that appear to the left of [arrows](#).

```
f1 :: forall a b. a -> b -> a == fi :: a -> b -> c
g1 :: forall a b. (Ord a, Eq b) => a -> b -> a == g1 :: (Ord a, Eq b) => a -> b -> a
```

f1, g1 - [rank-1 types](#). Haskell itself implicitly adds universal [quantification](#).

```
f2 :: (forall a. a->a) -> Int -> Int
g2 :: (forall a. Eq a => [a] -> a -> Bool) -> Int -> Int
```

f2, g2 - [rank-2 types](#). Quantificator is on the left side of a \rightarrow . Quantificator shows that [type](#) on the left can be overloaded.

[Type inference](#) in Rank-2 is possible, but not higher.

```
f3 :: ((forall a. a->a) -> Int) -> Bool -> Bool
```

f3 - [rank-3-type](#). Has [rank-2 types](#) on the left of a \rightarrow .

```
f :: Int -> (forall a. a -> a)
g :: Int -> Ord a => a -> a
```

f, g are rank 1. [Quantifier](#) appears to the right of an [arrow](#), not to the left. These [types](#) are not Haskell-98. They are supported in [RankNTypes](#).

3.27.1 *

Type ranks Rank type Rank types Rank-1 type Rank-1 types Rank-2 type Rank-2 types Rank-3 type Rank-3 types

3.28 Type variable

Refer to an unspecified [type](#) in Haskell [type](#) signature.

3.29 Unlifted type

[Type](#) that directly exist on the hardware. The [type abstraction](#) can be completely removed. With [unlifted types](#) Haskell [type](#) system directly manages data in the hardware.

3.29.1 *

Unlifted types

3.30 Linear type

Type system and algebra that also track the multiplicity of data. There are 3 general linear type groups:

- 0 - exists only at type level and is not allowed to be used at value level. Aka s in ST-Trick.
- 1 - data that is not duplicated
- 1< - all other data, that can be duplicated multiple times.

3.30.1 *

Linear types

3.31 NonEmpty list data type

Data.List.NonEmpty Has a Semigroup instance but can't have a Monoid instance. It never can be an empty list.

```
data NonEmpty a = a :| [a]
    deriving (Eq, Ord, Show)
```

:| - an infix data constructor that takes two (type) arguments. In other words :| returns a product type of left and right

3.32 Session type

* - allows to check that behaviour conforms to the protocol.

So far very complex, not very productive (or well-established) topic.

3.33 Binary tree

```
data BinaryTree a
    = [[Leaf]]
    | [[Node]] (BinaryTree a) a (BinaryTree a)
```

3.34 Bottom value

A _ non-value in the type or pattern match expression. Placeholder for anything.

```
-- _ fits *.
```

3.34.1 *

Bottom Bottom values

3.35 Bound

Haskell * type class means to have lowest value & highest value, so a bounded range of values.

3.35.1 *

Bounded

3.36 Constructor

1. [Type constructor](#)
2. [Data constructor](#)

Also see: [Constant](#)

3.36.1 *

Constructors

3.37 Context

[Type constraints](#) for [polymorphic variables](#). Written before the main [type](#) signature, denoted:

[TypeClass](#) a => ...

3.37.1 *

Contexts

3.38 Inhabit

Values that is a component of [data type set](#).

3.39 Maybe

```
data Maybe
  = Nothing
  | Just a
```

Does not represent the information why Nothing happened. For [error](#) - use [Either](#). Do not propagate *.

Handle * locally to [where](#) it is produced. Nothing does not hold useful info for debugging & short-circuits the processes. Do not expect code [type](#) being bug-free, do not return Maybe to end user since it would be impossible to debug, return something that preserves [error](#) information.

3.39.0.1 *

Nodes

3.40 Expected type

[Data type inferred](#) from the text of the code.

3.41 ADT

1. [Abstract data type](#)
2. [Algebraic data type](#)

3.42 Concrete type

Fully resolved, definitive, non-polymorphic type.

3.43 Type punning

When [type constructor](#) and [data constructor](#) have the same name.

Theoretically if person knows the rules - * can be solved, because in Haskell [type](#) and [data declaration](#) have different places of use.

3.44 Kind

[Kind](#) -> [Type](#) -> Data

3.44.1 *

Kinds

3.45 IO

[Type](#) for values whose evaluations has a possibility to cause side effects or return unpredictable result. Haskell standard uses [monad](#) for constructing and transforming [IO](#) actions. [IO](#) action can be evaluated multiple times.

[IO data type](#) has unpure imperative actions inside. Haskell is [pure Lambda calculus](#), and unpure [IO](#) integrates in the Haskell purely ([type](#) system abstracts [IO](#) unpurity inside [IO data type](#)).

[IO sequences effect](#) computation one after another in [order](#) of needed computation, or occurrence:

```
twoBinds :: IO ()
twoBinds =
  putStrLn "First:" >>
  getLine >>=
  \a ->
  putStrLn "Second:" >>
  getLine >>=
  \b ->
  putStrLn ("\nFirst: "
    ++ a ++ ".\nSecond "
    ++ b ++ ".")
main = twoBinds
```

Sequencing is achieved by compilation of effects performing only while they receive the sugared-in & passed around the RealWorld fake [type](#) value, that value in the every computation gets the new "value" and then passed to the next requested computation. But special thing is about this [parameter](#), this RealWorld [type](#) value passed, but never looked at. GHC realizes, since value is never used, - it means value and [type](#) can be equated to () and moreover reduced from the code, and sequencing stays.

Chapter 4

Expression

Finite combination of symbols that is well-formed according to [context-free grammar](#).

Generally meaningless. Meaning gets [derived](#) from an * & [context](#) (and/or content words) by congruency with knowledge & experience.

4.1 *

Expressions

4.2 Closed-form expression

* - mathematical [expression](#) that can be evaluated in a finite number of operations.

May contain:

- constants
- [variables](#)
- operations (e.g., $+$ $-$ \times \div)
- [functions](#) (e.g., nth root, exponent, logarithm, trigonometric [functions](#), and [inverse](#) hyperbolic [functions](#)), but usually no limit.

4.3 RHS

Right-hand side of the [expression](#).

4.4 LHS

Left-hand side of the [expression](#).

4.5 Redex

[Reducible expression](#).

4.6 Concatenate

Link together [sequences](#), [expressions](#).

4.7 Alpha equivalence

[Equivalence](#) of a processes in [expressions](#). If [expressions](#) have according [parameters](#) different, but the internal processes are literally the same [process](#).

4.8 Ground expression

[Expression](#) that does not contain any free [variables](#).

4.8.1 *

Ground formula

4.9 Variable

A name for [expression](#).

++++

There fequently can be heard: one of most notable Haskell [properties](#) is Haskell has immutable "variables" (and term here used in the sence that imperative programmers frequently use). Logically we see [statement](#) is contradictory with itself: "[variables](#)" - something that has change as a defining property - are not changing; it is a nonsencical [statement](#). Please, read the saying as: Haskell has immutable values, due to following the value [semantics](#): see "Value". And Haskell [expressions](#) are [functions](#) (that are [referentially transparent](#) - meaning itself immutable) - and they are also values (hense term "functional programming" means - [functions](#) are [first-class](#) values). Since values [bind](#) to [variables](#) - people are wrongly mix-up terms and say their names (according "*) are immutable.

As you see in the code - Haskell [variables](#) (same names) hold different values at different times. [Variables](#) are reused, meaning "names are reused" - binded to different values on [scope](#) changes. But all values that Haskell holds - are, by the design of the language, are treated immutable, any transformations Haskell resolves by creating new values, and frees the space by freeing-up from no longer needed values.

4.9.1 *

Variables

4.10 Phrase

[Composable expression](#).

Chapter 5

Function

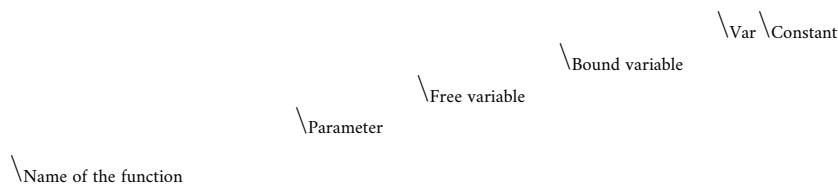
Full dependency of one quantity from another quantity.

Denotation: $y = f(x)$ $f : X \rightarrow Y$, where X is domain, Y is codomain.

Directionality and property of invariability emerge from one another.

-- domain func codomain
* -> *

$$y(x) = (zx^2 + bx + 3 \mid b = 5) \wedge \wedge \wedge \wedge \wedge$$



Lambda abstraction is a function. Function is a mathematical operation.

Function = Total function = Pure function. Function theoretically can be to memoized.

Also see: Partial function Inverse function - often partially exists (partial function).

5.1 *

Functions Bound variable

5.2 Arity

Number of parameters of the function.

- nullary - $f()$
- unary - $f(x)$
- binary - $f(x,y)$
- ternary - $f(x,y,z)$
- n-ary - $f(x,y,z,\dots)$

5.3 Bijection

Function is a complete one-to-one pairing of elements of **domain** and **codomain** (**image**). It means **function** both **surjective** (so **image** == **codomain**) and **injective** (every **domain** element has unique correspondence to the **image** element).

For **bijection inverse** always exists.

Bijection operation holds the **equivalence** of **domain** and **codomain**.

Denotation:

\boxtimes

$\succ \rightarrow \gg$

$f : X \boxtimes Y$

LaTeX needed to combine symbols: $f : X \succ \rightarrow Y$

Corresponds to **isomorphism**.

5.3.1 *

Bijection Bijection function

5.4 Combinator

Function without free **variables**. **Higher-order function** that uses only **function application** and other combinators.

```
\a -> a
\ a b -> a b
\f g x -> f (g x)
\f g x y -> f (g x y)
```

Not combinators:

```
\ xs -> sum xs
```

Informal broad meaning: referring to the style of organizing libraries centered around the idea of combining things.

5.4.1 Ψ -combinator

Transforms two of the same **type**, **applying** same mediate transformation, and then transforming those into the result.

```
import Data.Function (on)
on :: (b -> b -> c) -> (a -> b) -> a -> a -> c

a--\b
  * ---c
a--/b
```

5.4.1.1 *

Psi-combinator On-combinator

5.5 Function application

* - **bind** the **argument** to the **parameter** of a **function**, and do a **beta-reduction**.

5.5.1 *

Apply Applied Applying Application

5.6 Function body

Expression that haracterizes the process.

5.7 Function composition

$(.) :: (b \rightarrow c) \rightarrow (a \rightarrow b) \rightarrow a \rightarrow c$

$a \rightarrow (a \rightarrow b) \rightarrow (b \rightarrow c) \rightarrow c$

In Haskell inline composition requires:

$h.g.f \$ i$

5.7.1 *

Composition Compose Composed

5.8 Function head

Is a part with name of the function and it's paramenters. AKA: $f(x)$

5.9 Function range

The range of a function refers to either the codomain or the image of the function, depending upon usage. Modern usage almost always uses range to mean image. So, see Function image.

5.10 Higher-order function

Function that has $\text{arity} > 1$.

=====

HOF is:

- function that accepts function as a parameter
- function that has more then one parameter.

Application of an argument to * produces a function that has $\text{arity} - 1$.

5.10.1 *

HOF

5.10.2 Fold

Catamorphism of a structure to a lower type of structure. Often to a single value.

* is a higher-order function that takes a function which operates with both main structure and accumulator structure, * applies units of data structure to a function wich works with accumulator. Upoun traversing the whole structure - the accumulator is returned.

5.11 Injection

Function one-to-one injects from **domain** into **codomain**.

Keeps distinct pairing of elements of **domain** and **image**. Every element in **image** corresponds to one element in **domain**.

$$\forall a, b \in X, f(a) = f(b) \Rightarrow a = b$$

$$\exists(\text{inverse function}) \mid \forall(\text{injective function})$$

Denotation:

☒

>->

f : X ☒ Y

f : X ↗→ Y

Corresponds to **Monomorphism**.

5.11.1 *

Injective Injective function Injectivity

5.12 Partial function

One that does not cover all **domain**. **Unsafe** and causes trouble.

5.13 Purity

* means properly abstracted.

If the contrary - **abstraction** is unpure.

Also see: **pure function**.

5.13.1 *

Pure

5.14 Pure function

Function that is **pure** \equiv **referentially transparent function**.

5.15 Sectioning

Writing **function** in a parentheses. Allows to pass around **partially applied functions**.

5.16 Surjection

Function uses **codomain** fully.

$$\forall y \in Y, \exists x \in X$$

Denotation: $f : X \twoheadrightarrow Y$

Corresponds to **Epimorphism**.

5.16.1 *

Surjective Surjective function

5.17 Unsafe function

Function that does not cover at least one edge case.

5.17.1 *

Unsafe

5.18 Variadic

* - having variable arity (often up to indefinite).

5.19 Domain

Source set of a function. X in $X \rightarrow Y$.

5.20 Codomain

Y in $X \rightarrow Y$. Codomain - target set of a function.

5.21 Open formula

Logical function that has arity and produces proposition.

5.22 Recursion

Repeated function application when sometimes same function gets called.

Allows computation that may require indefinite amount of work.

5.22.1 *

Recursive

5.22.2 Base case

A part of a recursive function that trivially produces result.

5.22.3 Tail recursion

Tail calls are recursive invocations of itself.

5.22.4 Polymorphic recursion

Type of the parameter changes in recursive invocations of function.

Is always a higher-ranked type.

5.22.4.1 *

Milner–Mycroft typability Milner–Mycroft calculus

5.23 Free variable

Variable in the function that is not **bound** by the head. Until there are * - **function** stays **partially applied**.

5.24 Closure

$f(x) = f^{X \rightarrow X} \mid \forall x \in X, X$ is **closed** under f , it is a trivial **case** when **operation** is legitimate for all values of the **domain**.

Operation on members of the **domain** always produces a members of the **domain**. The **domain** is **closed** under the **operation**.

In the **case** when there is a **domain** values for which **operation** is not legitimate/not exists:

$f(x) = f^{V \rightarrow X} \mid V \in X, \forall x \in V, X$ is **closed** under f .

5.24.1 *

Closed

5.25 Parameter

para subsidiary metron measure

Named variable of a **function**.

Argument is a supplied value to a **function parameter**.

Parameter (**formal parameter**) is an **irrefutable** pattern, and implemeted that way in Haskell.

5.25.1 *

Parameters Formal parameter Formal parameters

5.26 Partial application

Part of **function parameters applied**.

5.26.1 *

Partially applied

5.27 Well-formed formula

Expression, logical **function** that is/can produce a **proposition**.

5.27.1 *

Well formed formula WFF wff WFFS wffs

Chapter 6

Homotopy

Two things are homotopic

One can be "continuously deformed" into the other.

For example - [functions](#), [functors](#). [Natural transformation](#) is a [homotopy](#) of [functors](#).

6.1 *

Homotopies Homotopic

Chapter 7

Lambda calculus

Universal model of computation. Which means * can implement any [Turing machine](#). Based on [function abstraction](#) and [application](#) by substituting [variables](#) and [binding](#) values.

* has [lambda terms](#):

- [variable](#) (x)
- [application](#) ($((ts))$)
- [abstraction](#) ([lambda function](#)) ($((\lambda x.t))$)

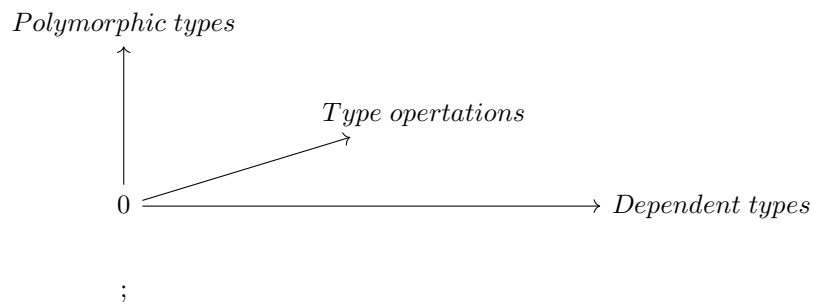
7.1 *

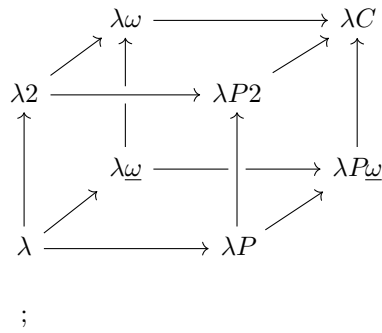
Lambda term Lambda terms

7.2 Lambda cube

[λ-cube](#) shows the 3 dimention of generalizations from simply typed [Lambda calculus](#) to [Calculus of constructions](#).

+===





Each dimension of the cube corresponds to extensions (a new **type** of **relation** of **objects** depending on **objects**):

Table 7.1: Three degrees of **type** systems generalizations

| Denotation | Name | Programming | New type of relations |
|------------|-------------------|---|--|
| 2 | Polymorphic types | First-class polymorphism of types | Terms depending on types |
| ω | Type operation | Type class, type families | Types depending on types |
| P | Dependent types | Higher-rank polymorphism, dependent types | Types depending on terms |

Table 7.2: λ -cube: Names of the **type** systems

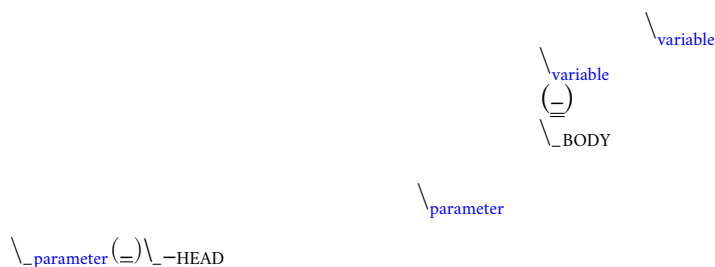
| Denotation | Logical system |
|------------------------------|--|
| $\lambda \rightarrow$ | (First Order) Propositional Calculus |
| $\lambda 2$ | Second Order Propositional Calculus |
| $\lambda \omega$ | Weakly Higher Order Propositional Calculus |
| $\lambda \underline{\omega}$ | Higher Order Propositional Calculus |
| λP | (First Order) Predicate Logic |
| $\lambda P2$ | Second Order Predicate Calculus |
| $\lambda P\omega$ | Weak Higher Order Predicate Calculus |
| λC | Calculus of Constructions |

7.2.1 *

λ -cube λ -cube

7.3 Lambda function

Function of Lambda calculus. $\lambda xy.x^2 + y^3 \wedge \wedge \wedge$



7.3.1 *

Lambda abstraction

7.3.2 Anonymous lambda function

[Lambda function](#) that is not binded to any name.

7.3.2.1 *

Anonymous lambda function

7.3.3 Uncurry

Replace sequenced lambda [functions](#) into single [function](#) taking [sequence/product](#) of values as [argument](#).

7.4 β -reduction

Equation of a [parameter](#) to a [bound variable](#), then reducing [parameter](#) from the head.

7.4.1 *

β reduction Beta-reduction Beta reduction

7.4.2 β -normal form

No [beta reduction](#) is possible.

7.4.2.1 *

β normal form Beta normal form Beta-normal form

7.5 Calculus of constructions

Extends the [Curry](#)–Howard correspondence to the proofs in the full intuitionistic [predicate](#) calculus (includes proofs of [quantified statements](#)). [Type](#) theory, typed programming language, and constructivism (philosophy) foundation for mathematics. Directly relates to Coq programming language.

7.5.1 *

«CoC»

7.6 Curry–Howard correspondence

[Equivalence](#) of {[First-order logic](#), computer programming, [Category](#) theory}. They represent each-other, possible in one - possible in the other, so all the definitions and theorems have analogues in other two.

Gives a ground to the [equivalence](#) of computer programs and mathematical proofs.

Lambek added analogue to Cartesian [closed category](#), which can be used to model logic and [type](#) theory.

7.6.1 *

Curry–Howard isomorphism Curry-Howard-Lambek

Table 7.3: Table of basic correspondence

| Logic | Type | Category |
|-------------------|--|--------------|
| True | () (any inhabited type) | Terminal |
| False | Void | Initial |
| $a \wedge b$ | (a, b) | $a \times b$ |
| $a \vee b$ | Either $a\ b$ | a / b |
| $a \Rightarrow b$ | $a \rightarrow b$ | b^a |

7.7 Currying

Translating the [evaluation](#) of a multiple [argument function](#) (or a [tuple](#) of arguments) into evaluating a [sequence](#) of [functions](#), each with a single [argument](#).

7.7.1 *

Curry

7.8 Hindley–Milner type system

Classical [type](#) system for the [Lambda calculus](#) with [Parametric polymorphism](#) and [Type inference](#). [Types](#) marked as [polymorphic variables](#), which enables [type inference](#) over the code.

7.8.1 *

Hindley-Milner Damas-Milner Damas-Hindley-Milner

7.9 Reduction

Take out something from a [structure](#), make simpler.

See [Beta reduction](#)

7.9.1 *

Reducible

7.10 β - η normal form

All [\$\beta\$ -reduction](#) and [\$\eta\$ -abstraction](#) are done in the [expression](#).

7.10.1 *

beta-eta normal form beta eta normal form

7.11 η -abstraction

$$(\lambda x.Mx) \xrightarrow{\eta} M$$

```
\ x -> g . f $ x
\ x -> g . f      --eta-abstraction
```

7.11.1 *

η -reduction η -conversion η abstraction η reduction η conversion eta-abstraction eta-reduction eta-conversion eta abstraction eta reduction eta conversion

7.12 Lambda expression

See [Lambda calculus](#) ([Lambda terms](#)) and [Expression](#). In majority cases meaning some [Lambda function](#).

Chapter 8

Operation

Calculation into output value. Can have [zero](#) & more inputs.

8.1 Constant

[Nullary operation](#).

Also see: [Type constant](#).

8.2 Binary operation

$\forall(a, b) \in S, \exists P(a, b) = f(a, b) : S \times S \rightarrow S$

8.2.1 *

Binary operations

8.3 Operator

Denotation symbol/name for the [operation](#).

8.3.1 Shift operator

[Shift operator](#) defined by Lagrange through [Differential operator](#). $T^t = e^{t \frac{d}{dx}}$

8.3.1.1 *

Shift

8.3.2 Differential operator

Denotation. $\frac{d}{dx}$, D , D_x , ∂_x . Last one is partial.

$e^{t \frac{d}{dx}}$ - [Shift operator](#).

8.3.2.1 *

Differential

8.4 Infix

Form of writing of [operator](#) or [function](#) in-between [variables](#) for [application](#).

For priorities see [Fixity](#).

8.5 Fixity

Declares the presedence of action of a [function/operator](#).

Funciton [application](#) has presedence higher then all [infix](#) operators/[functions](#) (virtually giving it a [priority](#) 10).

Table 8.1: Haskell operators [priority](#) and [fixity](#) association

| P | L | Non | R |
|----|---------|-------------------|------------|
| 10 | | | F.A. |
| 9 | !! | | . |
| 8 | | | ^ ^ ^ ** |
| 7 | * / div | | |
| 6 | + - | | |
| 5 | | | : , ++ |
| 4 | | <comparison> elem | |
| 3 | | | && |
| 2 | | | OR |
| 1 | | | |
| 0 | | | \$ \$! seq |

8.5.1 *

Infixl Infixr Priority Precedence

8.6 Zero

* is the value with which [operation](#) always yields [Zero](#) value. $zero, n \in C : \forall n, zero * n = zero$

* is distinct from [Identity](#) value.

8.7 Bind

Establishing equality between two [objects](#).

Most often:

- equating [variable](#) to a value.
- equating [parameter](#) of a [function](#) to an [argument](#) ([variable](#)/value/[function](#)). This term often is equated to [applying argument](#) to a [function](#), which includes [β-reduction](#).

8.7.1 *

Binds Binding Bindings

8.8 Declaration

[Binding](#) name to [expression](#).

8.9 Dispatch

Sort-out & send.

8.10 Evaluation

For FP see [Bind](#).

Chapter 9

Permutation

Bijjective function from domain to itself.

Domain & permutation functions & function composition form a group.

Chapter 10

Point-free

Paradigm [where function](#) only describes the [morphism](#) itself.

[Process](#) of converting [function](#) to [point-free](#). If brackets `()` can be changed to `$` then `$` equal to [composition](#):

```
\ x -> g (f x)
\ x -> g $ f x
\ x -> g . f $ x
\ x -> g . f      --eta-abstraction
```

```
\ x1 x2 -> g (f x1 x2)
\ x1 x2 -> g $ f x1 x2
\ x1 x2 -> g . f x1 $ x2
\ x1      -> g . f x1
```

10.1 *

Pointfree Tacit Tacit programming

10.2 Blackbird

```
(.).(.) :: (b -> c) -> (a1 -> a2 -> b) -> a1 -> a2 -> c
```

[Composition of compositions](#) `(.).(.)`. Allows to [compose-in](#) a [binary function](#) `f1(c) (.).(.) f2(a,b)`.

```
\ f g x y -> f (g x y)
```

10.2.1 *

`.) . (.) . (.)` Composition of compositions

10.3 Swing

```
swing :: ((a -> b) -> b) -> c -> d -> c -> a -> d
swing = flip . (. flip id)
swing f = flip (f . runCont . return)
swing f c a = f ($ a) c
```

10.4 Squish

`f >>= a . b . c =<< g`

Chapter 11

Polymorphism

πολύς *polús* many

At once several forms.

In Haskell - [abstract](#) over [data types](#).

* [types](#):

11.1 *

Polymorphic

11.2 Levity polymorphism

Extending [polymorphism](#) to work with unlifted and lifted [types](#).

11.3 Parametric polymorphism

[Abstracting](#) over [data types](#) by [parameter](#).

In most languages named as 'Generics' (generic programming).

[Types](#):

11.3.1 Rank-1 polymorphism

[Parametric polymorphism](#) in [rank-1 types](#) by [type variables](#).

11.3.1.1 *

Prenex Prenex polymorphism

11.3.2 Let-bound polymorphism

It is [property](#) chosen for Haskell [type](#) system. Haskell is based on [Hindley-Milner type](#) system, it is [let-bound](#). To have strict [type inference](#) with * - if [let](#) and [where](#) declarations are [polymorphic](#) - λ declarations - should be not.

See: [Good](#): In Haskell parameters bound by lambda declaration instantiate to only one concrete type.

11.3.3 Constrained polymorphism

Constrained [Parametric polymorphism](#).

11.3.3.1 Ad hoc polymorphism

Artificial [constrained polymorphism](#) dependent on incoming [data type](#). It is [interface dispatch](#) mechanism of [data types](#). Achieved by creating a [type class instance functions](#).

Commonly known as overloading.

11.3.3.1.0.1 *

Ad-hoc polymorphism Ad hoc polymorphic Ad-hoc polymorphic Constraint Constraints

11.3.4 Impredicative polymorphism

* allows [type](#) λ entities with [polymorphic types](#) that can contain [type](#) λ itself. $T = \forall X. X \rightarrow X : T \in X \models T \in T$

The most powerful form of [parametric polymorphism](#). See: [Impredicative](#).

This approach has Girard's paradox ([type systems](#) [Russell's paradox](#)).

11.3.4.1 *

First-class polymorphism

11.3.5 Higher-rank polymorphism

Means that [polymorphic types](#) can appear within other [types](#) ([types of function](#)). There is a cases [where higher-rank polymorphism](#) than the a Ad hoc - is needed. For example [where ad hoc polymorphism](#) is used in [constraints](#) of several different implementations of [functions](#), and you want to build a [function](#) on top - and use the [abstract interface](#) over these [functions](#).

```
-- ad-hoc polymorphism
f1 :: forall a. MyType Class a => a -> String    ==    f1 :: MyType Class a => a -> String
f1 = -- ...

-- higher-rank polymorphism
f2 :: Int -> (forall a. MyType Class a => a -> String) -> Int
f2 = -- ...
```

By moving forall inside the [function](#) - we can achieve [higher-rank polymorphism](#).

From: <https://news.ycombinator.com/item?id=8130861>

Higher-rank polymorphism is formalized using System F, and there are a few implementations of (in

Useful example also a [ST-Trick monad](#).

11.3.5.1 *

Rank-n polymorphism

11.4 Subtype polymorphism

Allows to declare usage of a [Type](#) and all of its Subtypes. T - [Type](#) S - Subtype of [Type](#) $<:$ - subtype of $S <: T = S \leq T$

Subtyping is: If it can be done to T, and there is subtype S - then it also can be done to S. $S <: T : f^{T \rightarrow X} \Rightarrow f^{S \rightarrow X}$

11.5 Row polymorphism

Is a lot like [Subtype polymorphism](#), but aligns itself on allowance (with | r) of subtypes and [types](#) with requested [properties](#).

```
printX :: { x :: Int | r } -> String
printX rec = show rec.x

printY :: { y :: Int | r } -> String
printY rec = show rec.y

-- type is inferred as `{x :: Int, y :: Int | r } -> String`
printBoth rec = printX rec ++ printY rec
```

11.6 Kind polymorphism

Achieved using a phantom [type argument](#) in the [data type declaration](#).

```
;;          * -> *
data Proxy a = ProxyValue
```

Then, by default the [data type](#) can be inhabited and fully work being partially defined. But multiple instances of [kind polymorphic type](#) can be distinguished by their particular [type](#).

Example is the [Proxy type](#):

```
data Proxy a = ProxyValue

let proxy1 = (ProxyValue :: Proxy Int) -- * :: Proxy Int
let proxy2 = (ProxyValue :: Proxy a)   -- * -> * :: Proxy a
```

11.7 Linearity polymorphism

Leverages [linear types](#). For example - if [fold](#) over a dynamic array:

1. In basic Haskell - array would be copied at every step.
2. Use low-level [unsafe functions](#).
3. With [Linear type function](#) we guarantee that the array would be used only at one place at a time.

So, if we use a [function](#) `(* -o * -o -o *)` in `foldr` - the [fold](#) will use the initial value only once.

Chapter 12

Pragma

Pragma - instruction to the compiler that specifies how a compiler should **process** the code. **Pragma** in Haskell have form:

```
{-# PRAGMA options #-}
```

12.1 LANGUAGE pragma

Controls what variations of the language are permitted. It has a **set** of allowed options: https://downloads.haskell.org/~ghc/latest/docs/html/users_guide/ghc_exts.html, which can be supplied.

12.1.1 LANGUAGE option

12.1.1.1 *

Language options

12.1.1.2 Useful by default

```
import EmptyCase
import FlexibleContexts
import FlexibleInstances
import InstanceSigs
import MultiParamTypeClasses
```

12.1.1.3 AllowAmbiguousTypes

Allow **type** signatures which appear that they would result in an unusable **binding**. However GHC will still check and complain about a **functions** that can never be called.

12.1.1.4 ApplicativeDo

Enables an **alternative** in-depth **reduction** that translates the do-notation to the operators `<$>`, `<*>`, `join` as far as possible.

For GHC to pickup the patterns, the final **statement** must match one of these patterns exactly:

```
pure E
pure $ E
return E
return $ E
```

When the `statements` of `do expression` have dependencies between them, and `ApplicativeDo` cannot `infer` an `Applicative type` - GHC uses a heuristic $O(n^2)$ algorithm to try to use `<*>` as much as possible. This algorithm usually finds the best solution, but in rare complex cases it might miss an opportunity. There is also $O(n^3)$ algorithm that finds the optimal solution: `-foptimal-applicative-do`.

Requires `ap = <*>`, `return = pure`, which is true for the most `monadic types`.

- Allows use of `do`-notation with `types` that are an instance of `Applicative` and `Functor`
- In some `monads`, using the `applicative` operators is more efficient than `monadic bind`. For example, it may enable more parallelism.

The only way it shows up at the source level is that you can have a `do expression` with only `Applicative` or `Functor` constraint.

It is possible to see the actual translation by using `-ddump-ds`.

12.1.1.5 ConstrainedClassMethods

Enable the definition of further `constraints` on individual class methods.

12.1.1.6 CPP

Enable `C preprocessor`.

12.1.1.7 DeriveFunctor

Automatic `deriving` of instances for the `Functor type class`. For `type power set functor` is unique, its derivation implementation can be autochecked.

12.1.1.8 ExplicitForAll

Allow explicit `forall` quantifier in places `where` it is implicit by Haskell.

12.1.1.9 FlexibleContexts

Ability to use complex `constraints` in class `declaration contexts`. The only restriction on the `context` in a class `declaration` is that the class hierarchy must be acyclic.

```
class C a where
  op :: D b => a -> b -> b
```

```
class C a => D a where ...
```

$C :> D$, so in `C` we can talk about `D`.

Synergizes with `ConstraintKinds`.

12.1.1.10 FlexibleInstances

Allow `type class` instances `types` contain nested `types`.

```
instance C (Maybe Int) where ...
```

Implies `TypeSynonymInstances`.

12.1.1.11 GeneralizedNewtypeDeriving

Enable GHC's newtype cunning generalised `deriving` mechanism.

```
newtype Dollars = Dollars Int
  deriving (Eq, Ord, Show, Read, Enum, Num, Real, Bounded, Integral)
```

(In old Haskell-98 only `Eq`, `Ord`, `Enum` could be inherited.)

12.1.1.12 ImplicitParams

Allow definition of [functions](#) expecting implicit [parameters](#). In the Haskell that has static scoping of [variables](#) allows the dynamic scoping, such as in classic Lisp or ELisp. Sure thing this one can be puzzling as hell inside Haskell.

12.1.1.13 LambdaCase

Enables [expressions](#) of the form:

```
\case { p1 -> e1; ...; pN -> eN }
```

-- OR

```
\case
  p1 -> e1
  ...
  pN -> eN
```

12.1.1.14 MultiParamTypeClasses

Implies: [ConstrainedClassMethods](#) Enable the definitions of typeclasses with more than one [parameter](#).

```
class Collection c a where
```

12.1.1.15 MultiWayIf

Enable multi-way-if syntax.

```
if | guard1 -> code1
   | ...
   | guardN -> codeN
```

12.1.1.16 OverloadedStrings

Enable overloaded string literals (string literals become desugared via the `IsString` class).

With overload, string literals has [type](#):

```
(IsString a) => a
```

The usual string syntax can be used, e.g. `ByteString`, `Text`, and other variations of string-like [types](#). Now they can be used in pattern matches as `char->integer` translations. To [pattern match](#) `Eq` must be [derived](#).

To use class `IsString` - [import](#) it from `GHC.Ext`.

12.1.1.17 PartialTypeSignatures

Partial [type](#) signature contains [wildcards](#), placeholders (`_`, `_name`). Allows programmer to which parts of a [type](#) to annotate and which to [infer](#). Also applies to [constraint](#) part.

As untuped [expression](#), partly typed can not polymorphically recurse.

[-Wno-partial-type-signatures](#) supresses [infer](#) warnings.

12.1.1.18 RankNTypes

Enable [types](#) of arbitrary rank. See [Type rank](#).

Implies [ExplicitForAll](#).

Allows forall quantifier:

- Left side of \rightarrow
- Right side of \rightarrow
- as argument of a constructor
- as type of a field
- as type of an implicit parameter
- used in pattern type signature of lexically scoped type variables

12.1.1.19 ScopedTypeVariables

By default type variables do not have a scope except inside type signatures where they are used.

When there are internal type signatures provided in the code block (where, let, etc.) they (main type description of a function and internal type descriptions) restrain one-another and become not trully polymorphic, which creates a bounding interdependency of types that GHC would complain about.

* option provides the lexical scope inside the code block for type variables that have forall quantifier. Because they are now lexiacally scoped - those type variables are used across internal type signatures.

For details see: <https://ocharles.org.uk/guest-posts/2014-12-20-scoped-type-variables.html>

Implies ExplicitForAll.

12.1.1.20 TupleSections

Allow tuple section syntax:

```
(, True)
(, "I", , , "Love", , 1337)
```

12.1.1.21 TypeApplications

Allow type application syntax:

```
read @Int 5

:type pure @[]
pure @[] :: a -> [a]

:type (<*>) @[]
(<*>) @[] :: [a -> b] -> [a] -> [b]

--

instance (CoArbitrary a, Arbitrary b) => Arbitrary (a -> b)

λ> ($ 0) <$> generate (arbitrary @(Int -> Int))
```

12.1.1.22 TypeSynonymInstances

Now type synonym can have it's own type class instances.

12.1.1.23 UndecidableInstances

Permit instances which may lead to [type-checker](#) non-termination.

GHC has Instance termination rules regardless of [FlexibleInstances](#) [FlexibleContexts](#).

12.1.1.24 ViewPatterns

```
foo (f1 -> Pattern1) = c1
foo (fn -> Pattern2 a b) = g1 a b
```

(*expression* → *pattern*): take what is came to match - [apply](#) the *expression*, then do *pattern*-match, and return what originally came to match.

[Semantics](#):

- *expression* & *pattern* share the [scope](#), so also [variables](#).

ion :: t1 -> t2) && (pattern t2)=) then (ViewPattern (/expression/ -> /pattern/) :: t1) (return what originally was recieved into [pattern match](#)) else skip

* are like [pattern guards](#) that can be nested inside of other patterns. * are a convenient way to pattern-match [algebraic data type](#).

Additional possible usage:

```
foo a (f2 a -> Pattern3 b c) = g2 b c -- only for function definitions
foo ((f, _), f -> Pattern4) = c2 -- variables can be bount to the left in data constructors and t
```

12.1.1.25 DatatypeContexts

Allow [contexts](#) in [data types](#).

```
data Eq a => Set a = NilSet | ConsSet a (Set a)
```

```
-- NilSet :: Set a
-- ConsSet :: Eq a => a -> Set a -> Set a
```

Considered misfeature. Deprecated. Going to be removed.

12.1.1.26 StandaloneKindSignatures

[Type](#) signatures for [type-level declarations](#).

```
type <name_1> , ... , <name_n> :: <kind>
```

```
type MonoTagged :: Type -> Type -> Type
```

```
data MonoTagged t x = MonoTagged x
```

```
type Id :: forall k. k -> k
```

```
type family Id x where
  Id x = x
```

```
type C :: (k -> Type) -> k -> Constraint
```

```
class C a b where
  f :: a b
```

```
type TypeRep :: forall k. k -> Type
```

```
data TypeRep a where
```

```
  TyInt    :: TypeRep Int
```

```
  TyMaybe :: TypeRep Maybe
```

```
  TyApp    :: TypeRep a -> TypeRep b -> TypeRep (a b)
```

< GHC 8.10.1 - [type](#) signatures were only for [term level](#) declarations.

Extension makes signatures feature more uniformal.

Allows to [set](#) the [order](#) of [quantification](#), [order](#) of [variables](#) in a [kind](#). For example when using [TypeApplications](#).

Allows to [set](#) full [kind](#) of derivable class, solving situations with [GADT](#) return [kind](#).

12.1.1.26.1 *

SAKS Standalone kind signatures

12.1.1.27 PartialTypeSignatures

Very helpful. Helps to solve [type level](#), helps to establish [type](#) signatures and [constraints](#). Allow to provide [_](#) in the [type](#) signatures to automatically infer-in the [type](#) information there.

Wild cards:

- [Type](#)

```
f :: _ -> _ -> a
```

- [Constraint](#)

```
f :: _ => a -> b -> c
```

- [Named](#)

```
f :: _x -> _x -> a
```

allows to identify the same [wildcard](#).

12.1.2 How to make a GHC LANGUAGE extension

In `libraries/ghc-boot-th/GHC/LanguageExtensions/Type.hs` add new [constructor](#) to the `Extension` [type](#)

```
data Extension
  = Cpp
  | OverlappingInstances
  ...
  | Foo
```

`/main/DynFlags.hs` extend `xFlagsDeps`:

```
xFlagsDeps = [
  flagSpec "AllowAmbiguousTypes" LangExt.AllowAmbiguousTypes,
  ...
  flagSpec "Foo" LangExt.Foo
]
```

It is for basic [case](#). For [testing](#), parser see further: <https://blog.shaynefletcher.org/2019/02/adding-ghc-language-extension.html>

Chapter 13

Compositionality

Complex [expression](#) is determined by the constituent [expressions](#) and the rules used to combine them.

If the meaning fully obtainable from the parts and [composition](#) - it is full, [pure compositionality](#).

If there exists [composed idiomatic expression](#) - it is unfull, unpure [compositionality](#), because meaning leaks-in from the sources that are not in the [composition](#).

13.1 *

Principle of compositionality Composition Compositional

Chapter 14

Referential transparency

Given the same input return the same output. So: * [expression](#) can be replaced with its corresponding resulting value without change for program's behavior. * [functions](#) are [pure](#).

14.1 *

Referentially transparent

Chapter 15

Semantics

Philosophical study of meaning. Meaning of symbols, words.

15.1 Operational semantics

Constructing proofs from logical [assertions](#) and verifying/checking/asserting things about execution and procedures their [properties](#), such as correctness, safety or security.

Good to solve in-point localized tasks.

Process of [abstraction](#).

15.1.1 Argument

arguere make clear/make known, to prove, to shine

* - evidence, proof, [statement](#) that results in consequences in the system.

15.1.1.1 Argument of a function

A value binded to the [function parameter](#). Value/topic that the function would [process](#)/deal with.

Also see Argument.

15.1.1.1.1 *

Function argument

15.1.1.2 First-class

Means *it*:

- Can be used as value.
- Passed as an [argument](#).

From 1&2 -> *it* can include itself.

15.1.2 Relation

[Relationship](#) between two [objects](#). By default it is not directed and not limited. In [Set theory](#): some subset of a [Cartesian product](#) between [sets](#) of [objects](#).

15.1.2.1 *

Relations Relationship

15.2 Denotational semantics

Construction of [objects](#), that describe/tag the meanings. In Haskell often [abstractions](#) that are ment (denotations), implemented directly in the code, sometimes exist over the code - allowing to reason and implement.

* are [composable](#).

Good to achive more broad approach/meaning.

Also see [Abstraction](#).

15.2.1 Abstraction

abs away from, off (in absentia)

tractus draw, haul, drag

Purified generalization.

Forgetting the details ([axiomatic semantics](#)). Simplified approach. Out of sight - out of mind.

* creates a new semantic level in which one can be absolutely precise ([operational semantics](#)).

It is a great did to name an [abstraction](#) ([denotational semantics](#)).

The ideal [abstractions](#) are:

- integrative (global):
 - [nothing](#), [void](#), emptiness - "none", [initial object](#)
 - everything - "all", "existance", [terminal object](#)
- [differential](#) (local):
 - point - "this", "is", "one", stasis
 - chaos - "any", "of", "many", [process](#)

They are ideal - because they are the [basis](#), the beginning. Because you can not express any other obstrac-tions without these.

+===

This is personal idea & the thought of autor of the book regarding basic [abstractions](#) particularly. Other definitions in the book basing on this are the proof that [statement](#) has some ground truth in it. There is ongoing philosophical discussion on the topics like these.

15.2.1.1 *

Abstractions Abstracting Abstract

15.2.1.2 Leaky abstraction

[Abstraction](#) that leaks details that it is supposed to [abstract](#) away.

15.2.1.2.1 *

Leaky abstractions

15.2.1.3 Object

Absolute **abstraction**.

Point that additionally can have **properties**.

Often abstracts something, that is why it exposes external **properties** on **abstracting** something, for example some **structure**, maybe mathematical. In this book **objects** represent **algebraic structures**, as we are talking about Haskell and **Category** theory.

Objects without **process** are in **constant** state.

15.2.1.3.1 *

Structure Structures Objects

15.2.1.3.2 Arrow

Second level of absolute **abstraction**.

Arrow.

Can have target, can have source. Both often are **objects**.

Often abstracts **process**.

Can have **properties**.

Also alias in **Category** Theory for "**morphism**", thou theory imposes **properties**.

15.2.1.3.2.1 *

Arrows Process

15.2.1.3.3 Terminal object

One that receives unique **arrow** from every **object**.

$$\exists ! : x \rightarrow 1 \mid \exists 1 \in \mathcal{C}, \forall x \in \mathcal{C}$$

* is an empty **sequence** **()** in Haskell.

Called a **unit**, so receives **terminal** or **unit arrow**.

Dual of **initial object**.

Denotation:

Category theory 1

Haskell

()

15.2.1.3.4 Initial object

One that emits unique **arrow** into every **object**.

$$\exists ! : \emptyset \rightarrow x \mid \exists \emptyset \in \mathcal{C}, \forall x \in \mathcal{C}$$

If **initial object** is **Void** (most frequently) - emitted **arrows** called absurd, because they can not be called.

Dual of **terminal object**.

Denotation:

Category theory: \emptyset

Haskell:

Void

15.2.1.3.5 Value

What **object** abstracts. Without any **object** external **structure** (aka **identity** in **Category Theory**). So ***** is immutable. Such heresy is called "Value **semantics**" and leads such things as **referential transparency**, functional programming and Haskell.

(Except, when you hack Haskell with explicit low-level functions, and start to directly mute values - then you are on your own, Haskell paradigm does not expect that.)

15.2.1.3.5.1 *

Value **semantics** Values

15.2.1.3.6 Tensor

Object existing out of planes, thus it can translate **objects** from one plane into another. ***** can be tried to be described with knowledge existing inside planes (from projection on the plane), but representation would always be partial.

Tensor of rank 1 is a vector.

Translations with **tensor** can be seen as **functors**.

15.2.1.3.6.1 *

Tensors Tensorial

15.2.2 Ambigram

ambi both

γράμμα *grámma* written character

Object that from different points of view has the same meaning.

While this word has two contradictory diametrically opposite usages, one was chosen (more frequent).

But it has... Both.

*TODO: For merit of differentiating the meaning about different meaning referring to **Tensor** as **object** with many meanings.*

15.2.3 Binary

Two of something.

15.2.4 Arbitrary

arbitrarius uncertain

Random, any one of.

Used as: Any one with *this* **set** of **properties**. (**constraints**, **type**, etc.).

When there is a talk about any **arbitrary** value - in fact it is a talk about the generalization of computations over the **set** of **properties**.

15.2.5 Refutable

One that has an option to fail.

15.2.6 Irrefutable

One that can not fail.

15.2.7 Superclass

Broader parent class.

15.2.8 Unit

Represents existence. Denoted as empty [sequence](#).

()

Type () holds only self-representation [constructor](#) (), & [constructor](#) holds [nothing](#).

Haskell code always should receive something back, hence [nothing](#), emptiness, [void](#) can not be theoretically addressed, practically constructed or received - [unit](#) in Haskell also has a role of a stub in place of emptiness, like in IO ().

15.2.9 Nullary

Takes no entries (for example has the [arity](#) of [zero](#)). Has the trivial [domain](#).

15.2.10 Syntax tree

Tree of syntactic elements (each [node](#) denotes [construct](#) occurring in the language/source code) that represent the full particular [expression](#)/implementation (or said).

15.2.10.1 Abstract syntax tree

"Abstract" since does not represent every detail of the syntax (ex. parentheses), but rather concentrates on [structure](#) and content.

Widely used in compilers to check the code [structure](#) for accuracy and coherence.

15.2.10.1.1 *

AST

15.2.10.2 Concrete syntax tree

An ordered, rooted [syntax tree](#) that represents the syntactic [structure](#) of a string according to some [context-free grammar](#).

"Concrete" since (in contrast to "abstract") - concretely reflects the syntax of the input language.

15.2.10.2.1 *

Parse tree Derivation tree

15.2.11 Stream

* an infinite [sequence](#) that forgets previous [objects](#), and remembers only currently relevant [objects](#).

$E \mid X \rightarrow (X \times A + 1)$, the [set](#) (or [object](#)) of streams on A (final [coalgebra](#) A_* of E).

[cycle](#) is one of [stream functions](#).

[a](#) = (cycle [[Nothing](#), [Nothing](#), [Just](#) "Fizz"])

[b](#) = (cycle [[Nothing](#), [Nothing](#), [Nothing](#), [Nothing](#), [Just](#) "Buzz"])

Can be:

- indexed, timeless, with current [object](#)
- timed:

* [([timescale](#), [event](#))] * [([realtime](#), [event](#))]

Has amalgamation with Functional Reactive Programming.

15.2.12 Linear

Values consumed once or not used.

x^2 consumes/uses x two times ($x*x$).

15.2.12.1 *

Linearity

15.2.13 Predicative

Non-self-referencing definition.

+===

Antonym - [Impredicative](#).

15.2.14 Quantifier

Specifies the quantity of specimens.

Two most common [quantifiers](#) \forall ([Forall](#)) and \exists (Exists). $\exists!$ - one and only one (exists only unique).

15.2.14.1 *

Quantification Quantifiers Quantified

15.2.14.2 Forall quantifier

Permits to not [infer](#) the [type](#), but to use any that fits. The variant depends on the [LANGUAGE option](#) used:

- [ScopedTypeVariables](#)
- [RankNTypes](#)
- [ExistentialQuantification](#)

15.2.14.2.1 *

Forall

15.3 Axiomatic semantics

Empirical [process](#) of studying something complex by finding and analyzing true [statements](#) about it.

Good for examining interconnections.

15.3.1 Property

Something has a [property](#) in the real world, and in theory its [property](#) corresponds to the law/laws, axioms.

In Haskell under [property](#)/law most often [properties](#) of [algebraic structures](#).

There [property testing](#) which does what it says.

15.3.1.1 *

Properties

15.3.1.2 Associativity

Joined with common purpose.

$$P(a, P(b, c)) \equiv P(P(a, b), c) \mid \forall (a, b, c) \in S,$$

* - the operations can be grouped arbitrarily.

[Property](#) that determines how operators of the same [precedence](#) are grouped, (in computer science also in the absence of parentheses).

Etymology: Latin *associatus* past participle of *associare* "[join with](#)", from assimilated form of *ad* "to" + *sociare* "unite with", from *socius* "companion, ally" from PIE **sokw-yo-*, suffixed form of root **sekw-* "to follow".

In Haskell * has influence on parsing when compounds have same [fixity](#).

15.3.1.2.1 *

Associative Associative law Associativity law

15.3.1.3 Left associative

* - the operations are grouped from the left.

Example: In lambda [expressions](#) same level parts follow grouping from left to right. $(\lambda x.x)(\lambda y.y)z \equiv ((\lambda x.x)(\lambda y.y))z$

15.3.1.3.1 *

Left associativity Left-associative

15.3.1.4 Right associative

* - the operations are grouped from the right.

15.3.1.5 Non-associative

Operations can't be chained.

Often is the [case](#) when the output [type](#) is incompatible with the input [type](#).

15.3.1.6 Basis*βασις* - stepping

The initial point, unreducible axioms and terms that spawn a theory. AKA see [Category](#) theory, or Euclidian geometry [basis](#).

15.3.1.6.1 Contravariant

The [property](#) of [basis](#), in which if new [basis](#) is a [linear](#) combination of the prior [basis](#), and the change of [basis](#) [inverse](#)-proportional for the description of a [Tensors](#) in this basis.

Denotation: Components for [contravariant basis](#) denoted in the upper indices: $V^i = x$

The [inverse](#) of a [covariant](#) transformation is a [contravariant](#) transformation. Whenever a vector should be invariant under a change of [basis](#), that is to say it should represent the same geometrical or physical [object](#) having the same magnitude and direction as before, its components must transform according to the [contravariant](#) rule.

15.3.1.6.1.1 *

Contravariant cofunctor Contravariant functor - More inline term is [Contravariant cofunctor](#)

15.3.1.6.2 Covariant

The [property](#) of [basis](#), in which if new [basis](#) is a [linear](#) combination of the prior [basis](#), and the change of [basis](#) proportional for a descriptions of [tensors](#) in basis.

Denotation: Components for [covariant basis](#) denoted in the upper indices: $V_i = x$

15.3.1.6.2.1 *

Covariant functor Covariant cofunctor

15.3.1.7 Commutativity

$\forall(a, b) \in S : P(a, b) \equiv P(b, a)$

15.3.1.7.1 *

Commutative Commutative law

15.3.1.8 Idempotence

First [application](#) gives a result. Then same [operation](#) can be [applied](#) multiple times without changing the result. Example: Start and Stop buttons on machines.

15.3.1.8.1 *

Idempotent Idempotency

15.3.1.9 Distributive property

[Set](#) S and two [binary](#) operators $+$ \times :

- $x \times (y + z) = (x \times y) + (x \times z)$ - \times is left-[distributive](#) over $+$
- $(y + z) \times x = (y \times x) + (z \times x)$ - \times is right-[distributive](#) over $+$
- left-&right-[distributive](#) - \times is [distributive](#) over $+$

15.3.1.9.1 *

Distributive rule Distributive axiom Distributive law Distributive

15.3.2 Effect

Observable action.

15.3.3 Bisimulation

When systems have exact external behaviour so for observer they are the same.

Binary relation between state transition systems that match each other's moves.

15.3.3.1 *

Bisimilar

15.4 Content word

Words that name objects of reality and their qualities.

15.5 Ancient Greek and Latin prefixes**15.5.1 ***

Greek prefix Latin prefix

15.6 Idiom

* - something having a meaning that can not be derived from the conjoined meanings of * constituents. Meaning can be special for language speakers or human with particular knowledge.

* can also mean applicative functor, people better stop making idiom from the term "idiom".

15.6.1 *

Idioms Idiomatic

15.7 Impredicative

Self-referencing definition.

+=

Antonym - Predicative.

15.8 Context-free grammar

Type of formal grammar that is: a set of production rules that describe all possible string in a given formal language.

Term is invented by Noam Chomsky.

Table 15.1: Ancient Greek and Latin prefixes

| Meaning | Greek prefix | Latin prefix |
|-------------------------|--------------------|----------------------|
| above, excess | hyper- | super-, ultra- |
| across, beyond, through | dia- | trans- |
| after | | post- |
| again, back | | re- |
| against | anti- | contra-, (in-, ob-) |
| all | pan | omni- |
| around | peri- | circum- |
| away or from | apo-, ap- | ab- (or de-) |
| bad, difficult, wrong | dys- | mal- |
| before | pro- | ante-, pre- |
| between, among | | inter- |
| both | amphi- | ambi- |
| completely or very | | de-, ob- |
| down | | de-, ob- |
| four | tetra- | quad- |
| good | eu- | ben-, bene- |
| half, partially | hemi- | semi- |
| in, into | en- | il-, im-, in-, ir- |
| in front of | pro- | pro- |
| inside | endo- | intra- |
| large | macro- | (macro-, from Greek) |
| many | poly- | multi- |
| not* | a-, an- | de-, dis-, in-, ob- |
| on | epi- | |
| one | mono- | uni- |
| out of | ek- | ex-, e- |
| outside | ecto-, exo- | extra-, extro- |
| over | epi- | ob- (sometimes) |
| self | auto-, aut-, auth- | ego- |
| small | micro- | |
| three | tri- | tri- |
| through | dia- | trans- |
| to or toward | epi- | ad-, a-, ac-, as- |
| two | di- | bi- |
| under, insufficient | hypo- | sub- |
| with | sym-, syn- | co-. com-, con- |
| within, inside | endo- | intra- |
| without | a-, an- | dis- (sometimes) |

15.8.1 *

CFG

Chapter 16

Set

Well-defined collection of distinct [objects](#).

16.1 *

Sets Set theory

16.2 Closed set

1. [Set](#) which complements an open [set](#).
2. Is form of [Closed-form expression](#). [Set](#) can be [closed](#) in under a [set](#) of operations.

16.3 Power set

For some [set](#) \mathcal{S} , the [power set](#) ($\mathcal{P}(\mathcal{S})$) is a [set](#) of all subsets of \mathcal{S} , including $\{\}$ & \mathcal{S} itself.

Denotation: $\mathcal{P}(\mathcal{S})$

16.4 Singleton

[Singleton](#) - [unit set](#) - [set](#) with exactly one element. Also 1-[sequence](#).

16.5 Russell's paradox

If there exists normal [set](#) of all [sets](#) - it should contain itself, which makes it abnormal.

16.6 Cartesian product

$\mathcal{A} \times \mathcal{B} \equiv \sum^{\vee} (a, b) \mid \forall a \in \mathcal{A}, \forall b \in \mathcal{B}$. [Operation](#), returns a [set](#) of all ordered pairs (a, b)

Any [function](#), [functor](#) is a subset of [Cartesian product](#).

$\sum (elem \in (\mathcal{A} \times \mathcal{B})) = cardinality^{\mathcal{A} \times \mathcal{B}}$

[Properties](#):

- not [associative](#)

- not commutative

16.6.1 Pullback

Subset of the cartesian product of two sets.

16.6.1.1 *

Pullbacks

Chapter 17

Testing

17.1 Property testing

Since [property](#) has a law, then family of that [unit](#) tests can be abstracted into the [lambda function](#). And tests cases come from [generator](#).

17.1.1 Function property

[Property](#) corresponds to the according law. In [property testing](#) you need to think additionally about [generator](#) and [shrinking](#).

17.1.2 Property testing type

Table 17.1: [Property testing types](#)

| | Exhaustive | Randomized | Unit test |
|-------------------------------------|--|--|---------------------------|
| Whole set of values | Exhaustive property test | Randomised property test | One element |
| Special subset of values | Exhaustive specialised property test | Randomised specialised property test | One element |

17.1.3 Generator

Seed
|
v
Gen A -> A
^
|
Size

Seed allows reproducibility. There is anyway a need to have some seed. Size allows setting upper [bound](#) on size of generated value. Think about infinity of [list](#).

After failed test - [shrinking](#) tests value parts of contrexample, finds a part that still fails, and recurses [shrinking](#).

17.1.3.1 *

Generators

17.1.3.2 Custom generator

When certain theorem only works for a specific [set](#) of values - the according [generator](#) needs to be produced.

```
arbitrary :: Arbitrary a => Gen a
suchThat :: Gen a -> (a -> Bool) -> Gen a
elements :: [a] -> Gen a
```

17.1.4 Reusing test code

Often it is convinient to [abstract testing](#) of same [function properties](#):

It can be done with (aka TestSuite [combinator](#)):

```
-- Definition
{-# LANGUAGE ScopedTypeVariables #-}
{-# LANGUAGE AllowAmbiguousTypes #-}
eqSpec :: forall a. Arbitrary a => Spec

-- Usage
{-# LANGUAGE TypeApplications #-}
spec :: Spec
spec = do
  eqSpec @Int

Eq Int
(==) :: Int -> Int -> Bool
  is reflexive
  is symetric
  is transitive
  is equivalent to (\ a b -> not $ a /= b)
(/=) :: Int -> Int -> Bool
  is antireflexive
  is equivalent to (\ a b -> not $ a == b)
```

17.1.4.1 Test Commutative property

[Commutativity](#)

```
:: Arbitrary a => (a -> a -> a) -> Property
```

17.1.4.2 Test Symmetry property

[Symmetry](#)

```
:: Arbitrary a => (a -> a -> Bool) -> Property
```

17.1.4.3 Test Equivalence property

[Equivalence](#)

```
:: (Arbitrary a, Eq b) => (a -> b) -> (a -> b) -> Property
```

17.1.4.4 Test Inverse property

```
:: (Arbitrary a, Eq b) => (a -> b) -> (b -> a) -> Property
```

17.1.5 QuickCheck

Target is a member of the [Arbitrary type class](#). Target -> Bool is something Testable. This [properties](#) can be complex. [Generator](#) arbitrary gets the seed, and produces values of Target. [Function](#) quickCheck runs the loop and tests that generated Target values always comply the [property](#).

17.1.5.1 Manual automation with QuickCheck properties

```
import Test.QuickCheck
import Test.QuickCheck.Function
import Test.QuickCheck.Property.Common
import Test.QuickCheck.Property.Functor
import Test.QuickCheck.Property.Common.Internal

data Four' a b = Four' a a a b
    deriving (Eq, Show)

instance Functor (Four' a) where
    fmap f (Four' a b c d) = Four' a b c (f d)

instance (Arbitrary a, Arbitrary b) => Arbitrary (Four' a b) where
    arbitrary = do
        a1 <- arbitrary
        a2 <- arbitrary
        a3 <- arbitrary
        b <- arbitrary
        return (Four' a1 a2 a3 b)

-- Wrapper around `prop_FunctorId`
prop_AutoFunctorId :: Functor f => f a -> Equal (f a)
prop_AutoFunctorId = prop_FunctorId T

type Prop_AutoFunctorId f a
    = f a
    -> Equal (f a)

-- Wrapper around `prop_AutoFunctorCompose`
prop_AutoFunctorCompose :: Functor f => Fun a1 a2 -> Fun a2 c -> f a1 -> Equal (f c)
prop_AutoFunctorCompose f1 f2 = prop_FunctorCompose (applyFun f1) (applyFun f2) T

type Prop_AutoFunctorCompose structureType origType midType resultType
    = Fun origType midType
    -> Fun midType resultType
    -> structureType origType
    -> Equal (structureType resultType)

main = do
    quickCheck $ eq $ (prop_AutoFunctorId :: Prop_AutoFunctorId (Four' ()))Integer
    quickCheck $ eq $ (prop_AutoFunctorId :: Prop_AutoFunctorId (Four' ())) (Either Bool String)
    quickCheck $ eq $ (prop_AutoFunctorCompose :: Prop_AutoFunctorCompose (Four' ())) String Integer
    quickCheck $ eq $ (prop_AutoFunctorCompose :: Prop_AutoFunctorCompose (Four' ())) Integer String
```

17.2 Write tests algorithm

1. Pick the right language/[stack](#) to implement features.

2. How expensive breakage can be.
3. Pick the right tools to test this.

17.3 Shrinking

Process of reducing complexity in the test **case** - re-run with smaller values and make sure that the test still fails.

Chapter 18

Logic

18.1 Proposition

Purely abstract & theoretical logical [object](#) (idea) that has a Boolean value.

* is expressed by a [statement](#).

18.1.1 *

Propositions

18.1.2 Atomic proposition

Logically undividable [unit](#). Does not contain [logical connectives](#).

18.1.2.1 *

Atomic propositions

18.1.3 Compound proposition

Formed by connecting [propositions](#) by [logical connectives](#).

18.1.3.1 *

Compound propositions

18.1.4 Propositional logic

Studies [propositions](#) and [argument](#) flow.

Refers to logically indivisible units ([atomic propositions](#)) as such, for theory - they are [abstractions](#) with Boolean [properties](#).

Not Turing-complete, impossible to [construct](#) an [arbitrary](#) loop.

18.1.4.1 *

Proposition logic Proposition calculus Propositional calculus Statement logic Sentential logic Sentential calculus Zeroth-order logic

18.1.4.2 First-order logic

Notation systems that use [quantifiers](#), [relations](#), [variables](#) over non-logical [objects](#), allows the use of [expressions](#) that contain [variables](#).

Turing-complete.

Extension of a [propositional logic](#).

18.1.4.2.1 *

Predicate logic First-order predicate logic First-order predicate calculus

18.1.4.2.2 Second-order logic

Extension over [first-order logic](#) that quantifies over [relations](#).

18.1.4.2.2.1 Higher-order logic

Extension over [second-order logic](#) that uses additional [quantifiers](#), stronger [semantics](#).

Is more expressive, but model-theoretic [properties](#) are less well-behaved.

18.2 Logical connective

Logical [operation](#).

18.2.1 *

Logical connectives

18.2.2 Conjunction

Logical AND.

Denotation: \wedge

Multiplies [cardinalities](#).

Haskell [kind](#):

* *

18.2.3 Disjunction

Logical OR Denotation: \vee

Summs [cardinalities](#).

18.3 Predicate

[Function](#) with Boolean [codomain](#). $P : X \rightarrow \{true, false\}$ - * on X .

Notation: $P(x)$

Im many cases includes [relations](#), [quantifiers](#).

18.4 Statement

Declarative **expression** that is a bearer of a **proposition**.

When we talk about **expression** or **statement** being true/false - in fact we refer to the **proposition** that they represent.

Difference between **proposition**, **statement**, **expression**:

1. " $2 + 3 = 5$ "
2. "two plus three equals five"
 - 1 & 2 are **statements**. Each of them is a collection of transmission symbols (linguistic **objects**) from a symbol systems \equiv **expression**. Each of them is **expression** that bears **proposition** (an idea resulting in a Boolean value) \equiv **statement**.
 - 1 & 2 represent the same **proposition**. **Proposition** from 1 \equiv **proposition** from 2.
 - **Statement** 1 \neq **statement** 2. They are two different **statements**, written in different systems. And **statement** " $2 + 3 = 5$ " \neq **statement** " $3 + 2 = 5$ ".

18.4.1 *

Assertion Assertions Statements

18.5 Iff

If and only if, exactly when, just. Denotation: \iff

Chapter 19

Haskell **structure**

19.1 *

Haskell **structures**

19.2 Pattern match

Are not **first-class**. It is a **set** of patter match semantic notations.

Must be **linear**.

* **precedence** (especially with more then one **parameter**, especially with **_** used) often changes the **function**.

19.2.1 As-pattern

```
f list@(x, xs) = ...  
f (x:xs)      = x:x:xs -- Can be compiled with reconstruction of x:xs  
f a@(x:_)    = x:a    -- Reuses structure without reconstruction
```

19.2.1.1 *

As-patterns As pattern As patterns

19.2.2 Wild-card

Matches anything and can not be binded. For matching someting that should pass not checked and is not used.

```
head (x:_)      = x  
tail (_:xs)     = xs
```

19.2.2.1 *

Wild-cards Wildcard Wildcards

19.2.3 Case

```
case x of  
  pattern1 -> ex1  
  pattern2 -> ex2
```

```

pattern3 -> ex3
otherwise -> exDefault

```

Bolting [guards](#) & [expressions](#) with [syntactic sugar](#) on [case](#):

```

case () of _
| expr1      -> ex1
| expr2      -> ex2
| expr3      -> ex3
| otherwise  -> exDefault

```

Pattern matching in [function](#) definitions is realized with [case expressions](#).

19.2.4 Guard

Check values against the [predicate](#) and use the first match definition:

```

f x
| predicate1 = definition1
| predicate2 = definition2
...
| x < 0      = definitionN
...
| otherwise  = definitionZ

```

19.2.4.1 *

Guards

19.2.5 Pattern guard

Allows check a [list](#) of pattern matches against [functions](#), and then proceed.

```
(patternMatch1) <- (funcCheck1)
```

```
, (patternMatch2) <- (funcCheck2) = RHS
```

```
lookup :: Eq a => a -> [(a, b)] -> Maybe b
```

```

addLookup l a1 a2
| Just b1 <- lookup a1 l
, Just b2 <- lookup a2 l
= b1 + b2
{-...other equations...-}

```

Run [functions](#), they must succeed. Then [pattern match](#) results to b1, b2. Only if successful - execute the equation.

Default in Haskell 2010.

19.2.5.1 *

Pattern guards

19.2.6 Lazy pattern

Defers the [pattern match](#) directly to the last moment of need during execution of the code.

```

f (a, b) = g a b -- It would be checked that the pattern of the pair constructor
-- is present, and that parameters are present in the constructor.
-- Only after that success - work would start on the RHS, aka then construction

```

```
-- g would start only then.
```

```
f ~(a, b) = g a b -- Pattern match of (a, b) deferred to the last moment,
-- RHS starts, construction of g starts.
-- For this lazy pattern the equivalent implementation would be:
-- f p = g (fst p) (snd p) -- RHS starts, during construction of g
-- the arguments would be computed and found, or error would be thrown.
```

Due to full laziness deferring everything to the runtime execution - the [lazy pattern](#) is one-size-fits all ([irrefutable](#)), analogous to `_`, and so it does not produce any checks during compilation, and raises [errors](#) during runtime.

`*` is very useful during [recursive](#) construction of [recursive structure/process](#), especially infinite.

19.2.6.1 *

Lazy-pattern Lazy patterns

19.2.7 Pattern binding

Entire [LHS](#) is a pattern, is a [lazy pattern](#).

```
fib@(1:tfib) = 1 : 1 : [ a+b | (a,b) <- zip fib tfib ]
```

19.2.7.1 *

Pattern bindings

19.3 Smart constructor

[Process](#)/code placing extra rules & [constraints](#) on the construction of values.

19.4 Level of code

There are these levels of Haskell code:

19.4.1 *

Code level

19.4.2 Type level

[Level of code](#) that works with [data types](#).

19.4.2.1 Type level declaration

```
type ...
newtype ...
data ...
class ...
instance ...
```

19.4.2.1.1 *

Type level declarations Type-level declaration Type-level declarations

19.4.2.2 Type check

if The [type level](#) information is complete ([strongly connected](#) graph)

then

Generalize the [types](#) and check if [type level](#) consistent to [term level](#).

else

[Infer](#) the missing [type level](#) part from the [term level](#). There are certain situations and [structures where](#) ambiguity arises and is unsolvable from the information of the [term level](#) (most basic example is [polymorphic recursion](#)).

19.4.2.2.1 *

Typecheck Typechecking Typechecks

19.4.2.2.2 Complete user-specific kind signature

[Type level declaration](#) is considered to "have a [CUSK](#)" if it has enough syntactic information to warrant completeness ([strongly connected](#) graph) and start checking [type level](#) correspondence to [term level](#), it is a ad-hock state of [type inferring](#).

In the future GHC would use other algorithm over/instead of [CUSK](#).

19.4.2.2.2.1 *

CUSK CUSKs Complete user-specific kind signatures Complete, user-specific kind signature

19.4.3 Term level

[Level of code](#) that does logical execution.

19.4.4 Compile level

[Level of code](#), about compilation processes/results.

19.4.4.1 *

Compilation level

19.4.5 Runtime level

[Level of code](#) of main program [operation](#), when machine does computations with compiled [binary](#) code.

19.4.6 Kind level

[Level of code where kinds & kind](#) declarations are situated, inferred and checked.

19.4.6.1 Kind check

[Applying](#) the [type check](#) to [kind](#) check:

if The [kind](#) level information is complete ([strongly connected](#) graph)

then

Check if [kind](#) level consistent to [term level](#).

else

Infer the missing [kind](#) level parts from the [type level](#). There are certain situations and [structures where](#) ambiguity arises and is unsolvable from the information of the [kind](#) level.

With StandaloneKindSignatures [kind](#) completeness happens against found (standalone) [kind](#) signature.

With CUSKs extension kind completeness happens against "[complete user-specific kind signature](#)"

19.4.6.1.1 *

Kindcheck Kind checks

19.5 Orphan instance

Hanging instance from inconsistent code base.

1. Supporting [structure](#) not fully present.
2. Several implementations of instance present.

19.6 undefined

Placeholder value that helps to do [typechecking](#).

19.7 Hierarchical module name

Hierarchical naming scheme:

```
Algebra                -- Was this ever used?
  DomainConstructor    -- formerly DoCon
  Geometric            -- formerly BasGeomAlg

Codec                  -- Coders/Decoders for various data formats
  Audio
    Wav
    MP3
    ...
  Compression
    Gzip
    Bzip2
    ...
  Encryption
    DES
    RSA
    BlowFish
    ...
  Image
    GIF
    PNG
    JPEG
    TIFF
    ...
  Text
    UTF8
    UTF16
    ISO8859
```

```

    ...
Video
  Mpeg
  QuickTime
  Avi
  ...
Binary          -- these are for encoding binary data into text
  Base64
  Yenc

Control
  Applicative
  Arrow
  Exception      -- (opt, inc. error & undefined)
  Concurrent      -- as hslibs/concurrent
    Chan          -- these could all be moved under Data
    MVar
    Merge
    QSem
    QSemN
    SampleVar
    Semaphore
  Parallel        -- as hslibs/concurrent/Parallel
  Strategies
  Monad           -- Haskell 98 Monad library
    ST            -- ST defaults to Strict variant?
      Strict      -- renaming for ST
      Lazy        -- renaming for LazyST
    State         -- defaults to Lazy
      Strict
      Lazy
    Error
    Identity
    Monoid
    Reader
    Writer
    Cont
    Fix           -- to be renamed to Rec?
    List
    RWS

Data
  Binary          -- Binary I/O
  Bits
  Bool            -- &&, ||, not, otherwise
  Tuple          -- fst, snd
  Char           -- H98
  Complex        -- H98
  Dynamic
  Either
  Int
  Maybe          -- H98
  List           -- H98
  PackedString
  Ratio          -- H98

```

```

Word
IORef
STRef          -- Same as Data.STRef.Strict
    Strict
    Lazy        -- The lazy version (for Control.Monad.ST.Lazy)
Binary         -- Haskell binary I/O
Digest
    MD5
    ...         -- others (CRC ?)
Array          -- Haskell 98 Array library
    Unboxed
    IArray
    MArray
    IO          -- mutable arrays in the IO/ST monads
    ST
Trees
    AVL
    RedBlack
    BTree
Queue
    Bankers
    FIFO
Collection
Graph          -- start with GHC's DiGraph?
FiniteMap
Set
Memo           -- (opt)
Unique

Edison         -- (opt, uses multi-param type classes)
    Prelude    -- large self-contained packages should have
    Collection -- their own hierarchy? Like a vendor branch.
    Queue      -- Or should the whole Edison tree be placed

Database
    MySQL
    PostgreSQL
    ODBC

Dotnet
    ...         -- Mirrors the MS .NET class hierarchy

Debug          -- see also: Test
    Trace
    Observe     -- choose a default amongst the variants
        Textual    -- Andy Gill's release 1
        ToXmlFile  -- Andy Gill's XML browser variant
        GHood      -- Claus Reinke's animated variant

Foreign
    Ptr
    StablePtr
    ForeignPtr  -- rename to FinalisedPtr? to void confusion with Foreign.Ptr
    Storable
    Marshal

```

```
    Alloc
    Array
    Errors
    Utils
C
    Types
    Errors
    Strings

GHC
    Exts                -- hslibs/lang/GlaExts
    ...

Graphics
    HGL
    Rendering
        Direct3D
        FRAN
        Metapost
        Inventor
        Haven
        OpenGL
            GL
            GLU
    Pan
UI
    FranTk
    Fudgets
    GLUT
    Gtk
    Motif
    ObjectIO
    TkHaskell
X11
    Xt
    Xlib
    Xmu
    Xaw

Hugs
    ...

Language
    Haskell            -- hslibs/hssource
        Syntax
        Lexer
        Parser
        Pretty
    HaskellCore
    Python
    C
    ...

Nhc
    ...
```

```

Numeric          -- exports std. H98 numeric type classes
  Statistics

Network          -- (== hslibs/net/Socket), depends on FFI only
  BER            -- Basic Encoding Rules
  Socket         -- or rename to Posix?
  URI            -- general URI parsing
  CGI            -- one in hslibs is ok?
  Protocol
    HTTP
    FTP
    SMTP

Prelude          -- Haskell98 Prelude (mostly just re-exports
                  other parts of the tree).

Sound            -- Sound, Music, Digital Signal Processing
  ALSA
  JACK
  MIDI
  OpenAL
  SC3            -- SuperCollider

System           -- Interaction with the "system"
  Cmd            -- ( system )
  CPUTime        -- H98
  Directory      -- H98
  Exit           -- ( ExitCode(..), exitWith, exitFailure )
  Environment    -- ( getArgs, getProgName, getEnv ... )
  Info           -- info about the host system
  IO             -- H98 + IOExts - IOArray - IORef
    Select
    Unsafe      -- unsafePerformIO, unsafeInterleaveIO
  Console
    GetOpt
    Readline
  Locale        -- H98
  Posix
    Console
    Directory
    DynamicLinker
      Prim
      Module
    IO
    Process
    Time
  Mem           -- rename from cryptic 'GC'
    Weak        -- (opt)
    StableName  -- (opt)
  Time          -- H98 + extensions
  Win32         -- the full win32 operating system API

Test
  HUnit

```

```

QuickCheck

Text
  Encoding
    QuotedPrintable
    Rot13
  Read
    Lex          -- cut down lexer for "read"
  Show
    Functions    -- optional instance of Show for functions.
  Regex         -- previously RegexString
  Posix         -- Posix regular expression interface
  PrettyPrint   -- default (HughesPJ?)
  HughesPJ
  Wadler
  Chitil
  ...
  HTML          -- HTML combinator lib
  XML
    Combinators
    Parse
    Pretty
    Types
  ParserCombinators -- no default
  ReadP         -- a more efficient "ReadS"
  Parsec
  Hutton_Meijer
  ...

Training          -- Collect study and learning materials
  <name of the tutor>

```

19.7.1 *

Top-level module name Top-level module names

19.8 Reserved word

Haskell has special meaning for:

case, class, data, deriving, do, else, if, import,
 in, infix, infixl, infixr, instance, let,
 of, module, newtype, then, type, where

19.8.1 *

Reserved words

19.8.2 import

`import` [statement](#) by default imports identifiers from the other [module](#), using [hierarchical module name](#), brings into [scope](#) the identifiers to the global [scope](#) both into unqualified and qualifies by the [hierarchical module name](#) forms.

This possibilities can mix and match:

- `<modName> ()` - [import](#) only instances of [type classes](#).
- `<modName> (x, y)` - [import](#) only declared identifiers.
- `qualified <modName>` - discards unqualified names, forces obligatory namespace for the imports.
- `hiding (x, y)` - skip [import](#) of declared identifiers.
- `<modName> as <modName>` - renames [module](#) namespace.
- `<type/class> (..)` - [import](#) class & its methods, or [type](#), all its data [constructors](#) & field names.

19.8.3 let

* [expression](#) is a [set](#) of cross-[recursive lazy pattern bindings](#).

Declarations permitted:

- [type](#) signatures
- [function bindings](#)
- [pattern bindings](#)

It is an [expression](#) (macro) and that integrates in external [lexical scope expression](#) it applied in.

Form:

```
let
  b1
  ..
  bn
in
  c
```

19.8.3.1 *

Let expression Let expressions

19.8.4 where

Part of the syntax of the whole [function declaration](#), has according [scope](#).

As part of whole [declaration](#) - can extend over definitions of the function (pattern matches, [guards](#)).

Form:

```
f match1 = y
f match2 = y
f x =
  | cond1 x = y
  | cond2 x = y
  | otherwise = y
where
  y = ... x ...
```

19.8.4.1 *

Where clause

19.9 Haskell Language Report

Document that is a standard of language.

19.9.1 *

Report Haskell Report Haskell 98 Language Report Haskell 98 Report Haskell 1998 Language Report Haskell 2010 Language Report Haskell 2010 Report

19.10 Haskell'

Current language development mod.

<https://prime.haskell.org/>

19.10.1 *

Haskell prime

19.11 Lense

Library of combinators to provide Haskell (functional language without mutation) with the emulation of get-ters and set-ters of imperative language.

Chapter 20

Computer science

20.1 Guerrilla patch

* changing code/[applying](#) patch sneakily - and possibility incompatibility with other at runtime. [Monkey patch](#) is derivative term.

20.1.1 Monkey patch

From [Guerrilla patch](#).

* is a way for program to modify supporting system software affecting only the running instance of the program.

20.2 Interface

Point of mutual meeting. Code behind [interface](#) determines how data is consumed.

20.3 Module

Importable organizational [unit](#).

20.4 Scope

Area [where binds](#) are accessible.

20.4.1 Dynamic scope

The name resolution depends upon the program state when the name is encountered, which is determined by the execution [context](#) or calling [context](#).

20.4.2 Lexical scope

[Scope bound](#) by the [structure](#) of source code [where](#) the named entity is defined.

20.4.2.1 *

Static scope

20.4.3 Local scope

Scope applies only in (current) area.

20.4.3.1 *

Local

20.5 Shadowing

When in the local scope bigger scope variable overridden by same name variable from the local scope.

20.6 Syntactic sugar

Artificial way to make language easier to read and write.

20.7 System F

Is parametric polymorphism in programming.

Extends the Lambda calculus by introducing \forall (universal quantifier) over types.

20.7.1 *

Girard–Reynolds polymorphic lambda calculus Girard–Reynolds

20.8 Tail call

Final evaluation inside the function. Produces the function result.

20.9 Thunk

Not evaluated calculation. Can be dragged around, until be lazily evaluated.

20.10 Application memory

Table 20.1: Application memory structural parts

| Storage of | Block name |
|-----------------------------------|---------------|
| All not currently processing data | Heap |
| Function call, local variables | Stack |
| Static and global variables | Static/Global |
| Instructions | Binary code |

When even Main invoked - it work in Stack, and called Stack frame. Stack frame size for function calculated when it is compiled. When stacked Stack frames exceed the Stack size - stack overflow happens.

20.11 Turing machine

Mathematical model of computation that defines [abstract Turing machine](#). [Abstract](#) machine which manipulates symbols on a strip of tape, according to a table of rules.

20.11.1 Turing complete

[Set](#) of action rules that can simulate any [Turing machine](#).

20.11.1.1 *

Turing incomplete Turing incompleteness Turing completeness Computationally universal

20.12 REPL

Read-eval-print loop, aka interactive shell.

20.13 Domain specific language

Language design/fitted for particular [domain](#) of [application](#). Mainly should be [Turing incomplete](#), since general-purpose language implies [Turing completeness](#).

20.13.1 *

Domain-specific language DSL

20.13.2 Embedded domain specific language

[DSL](#) used inside outer language.

Two levels of embedding:

- Shallow: [DSL](#) translates into Haskell directly
- Deep: Between [DSL](#) and Haskell there is a [data structure](#) that reflects the [expression](#) tree, AKA stores the [syntax tree](#).

20.13.2.1 *

eDSL

20.14 Data structure

20.14.1 Cons cell

Cell that values may [inhabit](#).

20.14.2 Construct

`(:) :: a -> [a] -> [a]`

20.14.2.1 *

Cons

20.14.3 Leaf

-

20.14.4 Node

*
/ \

20.14.5 Spine

Is a chain of memory cells, each points to the both value of element and to the next memory cell.

Array:

```

      :
     / \
1    :
    / \
  2    :
   / \
 3  []

```

1:2:3: []

Spine:

```

      :
     / \
-    :
   / \
  -   :
   / \
  -  []

```

Chapter 21

Graph theory

21.1 Successor

[Object](#) that receives the [arrow](#).

21.1.1 Direct successor

Immediate [successor](#).

21.2 Predecessor

[Object](#) that sends [arrow](#).

21.2.1 Direct predecessor

Immediate [predecessor](#).

21.3 Degree

Number of [arrows](#) of [object](#).

21.3.1 Indegree

Number of ingoing [arrows](#).

21.3.2 Outdegree

Number of outgoing [arrows](#).

21.4 Adjacency matrix

Matrix of connection of objects $\{-1, 0, 1\}$.

21.4.0.1 InstanceSigs

Allow adding [type](#) signatures to [type class function](#) instance [declaration](#).

21.5 Strongly connected

If every vertex in a graph is reachable from every other vertex.

It is possible to find all **strongly connected components** (and that way also test graph for strong connectivity), in **linear** time ($\Theta(V+E)$).

Binary relation of being **strongly connected** is an **equivalence relation**.

21.5.1 *

Strongly-connected

21.5.2 Strongly connected component

Full **strongly connected** subgraph of some graph.

* of a directed graph G is a subgraph that is **strongly connected**, and has **property**: no additional edges or vertices from G can be included in the subgraph without breaking its **property** of being **strongly connected**.

21.5.2.1 *

SCC Strongly connected components Strongly-connected component Strongly-connected components

Chapter 22

Tagless-final

Method of embedding [eDSL](#) in a typed functional host language (Haskell). [Alternative](#) to the embedding as a (generalized) [algebraic data type](#). For parsers of DLS [expressions](#): (1/partial) evaluator, compiler, pretty printer, multi-pass optimizer.

* embedding is writing [denotational semantics](#) for the [DSL](#) in the host language.

Approach can be used [iff eDSL](#) is typed. Only well-typed terms become embeddable, and host language can implemen also a [eDSL type](#) system. Approach that [eDSL](#) code interpretations are [type](#)-preserving.

One of main pros of * - extensibility: implementation of [DSL](#) can be used to analyze/evaluate/transform/pretty-print/compile and interpreters can be extended to more passes, optimizations, and new versions of [DSL](#) while keeping/using/reusing the old versions.

Example fields of [application](#): language-integrated queries, non-deterministic & probabilistic programming, delimiter continuation, computability theory, [stream](#) processing, hardware description languages, generation of specialized numerical kernels, [semantics](#) of natural language.

Part III

Give definitions

Chapter 23

Identity type

Chapter 24

Constant type

Chapter 25

Gen

Chapter 26

Tensorial strength

Chapter 27

Strong monad

Chapter 28

Weak head normal form

28.1 *

WHNF

Chapter 29

Function image

29.1 *

Image

Chapter 30

Invertible

Chapter 31

Invertibility

Chapter 32

Define LANGUAGE pragma options

32.1 ExistentialQuantification

32.2 GADTs

GADT is a generalization over parametric [algebraic data types](#) which allow explicitly denote the [types](#) ([type matching](#)) of the [constructors](#) and define [data types](#) using pattern matching on the left side of "data" [statements](#).

32.3 *

GADT Generalized algebraic data type First-class phantom data type Guarded recursive data type Equality-qualified data type

32.4 GeneralizedNewTypeClasses

32.5 FuncitonalDependencies

Chapter 33

GHC check keys

33.1 -Wno-partial-type-signatures

Supresses [PartialTypeSignatures wildcard infer](#) warning.

Chapter 34

Generalised algebraic data types

LANGUAGE [GADTs](#)

34.1 *

GADT

Chapter 35

Order theory

Investigates in the depth the intuitive notion of [order](#) using [binary relations](#).

35.1 Domain theory

Formalizes approximation and convergence. Has close [relation](#) to Topology.

35.2 Lattice

[Abstract structure](#) that consists of [partially ordered set](#), where every two elements have unique supremum and infimum. \equiv * [algebraic structure](#) satisfying certain axiomatic identities. * [order-theory](#) & [algebraic](#).

35.3 Order

35.3.1 Preorder

$R^X \rightarrow X$: [Reflexive](#) & [Transitive](#): $aRa, aRb, bRc \Rightarrow aRc$

Generalization of [equivalence relations](#) [partial orders](#).

* [Antisymmetric](#) \Rightarrow Partial ordering, * [Symmetric](#) \Rightarrow [Equivalence](#).

35.3.1.1 *

Preordered

35.3.1.2 Total preorder

$\forall a, b : a \leq b \vee b \leq a \Rightarrow$ [Total Preorder](#).

35.3.2 Partial order

A [binary relation](#) must be [reflexive](#), [antisymmetric](#) and [transitive](#).

Partial - not every elements between them need to be comparable.

Good example of * is a genealogical descendancy. Only related people produce [relation](#), not related do not.

35.3.2.1 *

Partial orders Partially ordered set Partially ordered sets Poset Posets

35.4 Partial order

35.5 Total order

Chapter 36

Universal algebra

Studies [algebraic structures](#).

Chapter 37

Relation

37.1 Reflexivity

$R^{X \rightarrow X}, \forall x \in X : xRx$ **Order** theory: $a \leq a$

* - each element is comparable to itself.

Corresponds to **Identity** and **Automorphism**.

37.1.1 *

Reflexive Reflexive relation

37.2 Irreflexivity

$R^{X \rightarrow X}, \forall x \in X : \neg R(x, x)$

37.2.1 *

Anti-reflexive Anti-reflexive relation Irreflexive Irreflexive relation

37.3 Transitivity

$\forall a, b, c \in X, \forall R^{X \rightarrow X} : (aRb \wedge bRc) \Rightarrow aRc$

* - the start of a chain of **precedence relations** must precede the end of the chain.

37.3.1 *

Transitive Transitive relation

37.4 Symmetry

$\forall a, b \in X : (aRb \iff bRa)$

37.4.1 *

Symmetric Symmetric relation

37.5 Equivalence

| Reflexive | Symmetric | Transitive |
|---|---|---|
| $\forall x \in X, \exists R : xRx$ $a = a$ | $\forall a, b \in X : (aRb \iff bRa)$ $a = b \iff b = a$ | $\forall a, b, c \in X, \forall R^{X \rightarrow X} : (aRb \wedge bRc) \Rightarrow aRc$ $a = b, b = c \Rightarrow a = c$ |

37.5.1 *

Equivalent Equivalent relation

37.6 Antisymmetry

$\forall a, b \in X : aRb, bRa \Rightarrow a = b \sim aRb, a \neq b \Rightarrow \nexists bRa$. **Antisymmetry** does not say anything about $R(a, a)$.

* - no two different elements precede each other.

37.6.1 *

Antisymmetric Antisymmetric relation

37.7 Asymmetry

$\forall a, b \in X (aRb \Rightarrow \neg(bRa)) \iff \text{Antisymmetric} \wedge \text{Irreflexive}$. **Asymmetry** \neq "not symmetric"
Symmetric \wedge **Asymmetric** is only empty relation.

37.7.1 *

Asymmetric Asymmetric relation

Chapter 38

Cryptomorphism

[Equivalent](#), interconvertable with no loss of information.

38.1 *

Crypromorphic

Chapter 39

Lexically scoped type variables

Enable [lexical scope](#) for [forall quantifier](#) defined [type variables](#)

Implemented in [ScopedTypeVariables](#)

Chapter 40

Abstract data type

Several definitions here, reduce them.

Data type mathematical model, defined by its **semantics** from the user point of view, listing possible values, operations on the data of the **type**, and behaviour of these operations.

* class of **objects** whose logical behaviour is defined by a **set** of values and **set** of operations (analogue to **algebraic structure** in mathematics).

A specification of a **data type** like a **stack** or queue **where** the specification does not contain any implementation details at all, only the operations for that **data type**. This can be thought of as the contract of the **data type**.

40.1 *

AbsDT

Chapter 41

Functional dependencies

Chapter 42

MonoLocalBinds

Chapter 43

KindSignatures

Chapter 44

ExplicitNamespaces

Chapter 45

Combinator pattern

Chapter 46

Symbolic expression

Nested tree [data structure](#).

Introduced & used in Lisp. Lisp code and data are *.

* in Lisp: Atom or [expression](#) of the form (x . y), x and y are *.

Modern abbreviated notation of *: (x y).

46.1 *

S-expression S-expressions Sexpression Sexpressions Sexp Sexps Sexpr Sexprs

Chapter 47

Polynomial

Expression consisting of:

- **variables**
- coefficients
- addition
- subtraction
- multiplication (including positive integer **variable** exponentiation)

Polynomials form a **ring**. **Polynomial ring**.

47.1 *

Polynomials

Chapter 48

Data family

Indexed form of data and newtype definitions.

Chapter 49

Type synonym family

Indexed form of [type](#) synonyms.

Chapter 50

Indexed type family

* additional structure in language that allows ad-hoc overloading of [data types](#). AKA are to [types](#) as [type class](#) to methods.

Varieties:

- [data family](#)
- [type](#) synonym families

Defined by pattern matching the partial [functions](#) between [types](#). Associates [data types](#) by [type-level function](#) defined by open-ended collection of valid instances of input [types](#) and corresponding output [types](#).

Normal [type classes](#) define partial [functions](#) from [types](#) to a collection of named values by pattern matching on the input [types](#), while [type](#) families define partial [functions](#) from [types](#) to [types](#) by pattern matching on the input [types](#). In fact, in many uses of [type](#) families there is a single [type class](#) which logically contains both values and [types](#) associated with each instance. A [type family](#) declared inside a [type class](#) is called an associated [type](#).

50.1 *

Type family

Chapter 51

TypeFamilies

Allow use and definition of indexed [type](#) families and data families.

* are [type](#)-level programming. * are overload [data types](#) in the same way that [type classes](#) overload [functions](#). * allow handling of [dependent types](#). Before it [Functional dependencies](#) and [GADTs](#) were used to solve that. * useful for generic programming, creating highly parametrised interfaces for libraries, and creating interfaces with enhanced static information (much like [dependent types](#)).

Implies: [MonoLocalBinds](#), [KindSignatures](#), [ExplicitNamespaces](#)

Two [types](#) of * are:

Chapter 52

Error

Mistake in the program that can be resolved only by fixing the program.

`error` is a sugar for `undefined`.

Distinct from [Exception](#).

52.1 *

Errors

Chapter 53

Exception

Expected but irregular situation.

Distinct from [Error](#). Also see Exception vs Error

53.1 *

Exceptions

Chapter 54

ConstraintKinds

`Constraints` are just handled as `types` of a particular `kind` (`Constraint`). Any `type` of the `kind` `Constraints` can be used as a `constraint`.

- Anything which is already allowed in code as a `constraint` without `*`. Saturated applications to `type` `classes`, implicit `parameter` and equality `constraints`.
- `Tuples`, all of whose component `types` have `kind` `Constraint`.

```
type Some a = (Show a, Ord a, Arbitrary a) -- is of kind Constraint.
```

- Anything form of which is not yet known, but the user has declared for it to have `kind` `Constraint` (for which they need to `import` it from `GHC.Exts`):

```
Foo (f :: Type -> Constraint) = forall b. f b => b -> b -- is allowed
-- as well as examples involving type families:
type family Typ a b :: Constraint
type instance Typ Int b = Show b
type instance Typ Bool b = Num b

func :: Typ a b => a -> b -> b
func = ...
```

Chapter 55

Specialisation

Turns [ad hoc polymorphic function](#) into compiled [type](#)-specific innpementations.

55.1 *

Specialise Specialize Specialization

Chapter 56

Diagram

For categories \mathcal{C} and \mathcal{J} , a diagram of type \mathcal{J} in \mathcal{C} is a covariant functor $D : \mathcal{J} \rightarrow \mathcal{C}$.

Chapter 57

Category theoretical presheaf

For categories C and J , a J -presheaf on C is a contravariant functor $D : C \rightarrow J$.

Chapter 58

Topological presheaf

If X is a topological space, then the open sets in X form a partially ordered set $\text{Open}(X)$ under inclusion. Like every partially ordered set, $\text{Open}(X)$ forms a small category by adding a single arrow $U \rightarrow V$ if and only if $U \subseteq V$. Contravariant functors on $\text{Open}(X)$ are called presheaves on X . For instance, by assigning to every open set U the associative algebra of real-valued continuous functions on U , one obtains a presheaf of algebras on X .

Chapter 59

Diagonal functor

The **diagonal functor** is defined as the **functor** from D to the **functor category** D^C which sends each **object** in D to the **constant functor** at that **object**.

Chapter 60

Limit functor

For a fixed index [category](#) J , if every [functor](#) $J \rightarrow C$ has a limit (for instance if C is complete), then the [limit functor](#) $C^J \rightarrow C$ assigns to each [functor](#) its limit. The existence of this [functor](#) can be proved by realizing that it is the right-adjoint to the [diagonal functor](#) and invoking the Freyd adjoint [functor](#) theorem. This requires a suitable version of the axiom of choice. Similar remarks [apply](#) to the colimit [functor](#) (which is [covariant](#)).

Chapter 61

Dual vector space

The map which assigns to every vector space its [dual](#) space and to every [linear](#) map its [dual](#) or transpose is a [contravariant functor](#) from the [category](#) of all vector spaces over a fixed field to itself.

Chapter 62

Fundamental group

Consider the [category](#) of pointed topological spaces, i.e. topological spaces with distinguished points. The [objects](#) are pairs (X, x_0) , [where](#) X is a topological space and x_0 is a point in X . A [morphism](#) from (X, x_0) to (Y, y_0) is given by a continuous map $f : X \rightarrow Y$ with $f(x_0) = y_0$.

To every topological space X with distinguished point x_0 , one can define the [fundamental group](#) based at x_0 , denoted $\pi_1(X, x_0)$. This is the [group](#) of [homotopy](#) classes of loops based at x_0 . If $f : X \rightarrow Y$ is a [morphism](#) of pointed spaces, then every loop in X with base point x_0 can be [composed](#) with f to yield a loop in Y with base point y_0 . This [operation](#) is compatible with the [homotopy equivalence relation](#) and the [composition](#) of loops, and we get a [group homomorphism](#) from $\pi_1(X, x_0)$ to $\pi_1(Y, y_0)$. We thus obtain a [functor](#) from the [category](#) of pointed topological spaces to the [category](#) of [groups](#).

In the [category](#) of topological spaces (without distinguished point), one considers [homotopy](#) classes of generic curves, but they cannot be [composed](#) unless they share an endpoint. Thus one has the fundamental groupoid instead of the [fundamental group](#), and this construction is [functorial](#).

Chapter 63

Algebra of continuous function

A [contravariant functor](#) from the [category](#) of topological spaces (with continuous maps as [morphisms](#)) to the [category](#) of real [associative algebras](#) is given by assigning to every topological space X the [algebra](#) $C(X)$ of all real-valued continuous [functions](#) on that space. Every continuous map $f : X \rightarrow Y$ induces an [algebra homomorphism](#) $C(f) : C(Y) \rightarrow C(X)$ by the rule $C(f)(\varphi) = \varphi \circ f$ for every φ in $C(Y)$.

Chapter 64

Tangent and cotangent bundle

The map which sends every differentiable manifold to its tangent bundle and every smooth map to its derivative is a [covariant functor](#) from the [category](#) of differentiable manifolds to the [category](#) of vector bundles.

Doing this constructions pointwise gives the tangent space, a [covariant functor](#) from the [category](#) of pointed differentiable manifolds to the [category](#) of real vector spaces. Likewise, cotangent space is a [contravariant functor](#), essentially the [composition](#) of the tangent space with the [dual](#) space above.

Chapter 65

Group action / representation

Every **group** G can be considered as a **category** with a single **object** whose **morphisms** are the elements of G . A **functor** from G to **Set** is then **nothing** but a **group** action of G on a particular **set**, i.e. a **G-set**. Likewise, a **functor** from G to the **category** of vector spaces, Vect_K , is a **linear** representation of G . In general, a **functor** $G \rightarrow C$ can be considered as an "action" of G on an **object** in the **category** C . If C is a **group**, then this action is a **group homomorphism**.

Chapter 66

Lie algebra

Assigning to every real (complex) Lie [group](#) its real (complex) [Lie algebra](#) defines a [functor](#).

Chapter 67

Tensor product

If \mathcal{C} denotes the [category](#) of vector spaces over a fixed field, with [linear](#) maps as [morphisms](#), then the [tensor product](#) $V \otimes W$ defines a [functor](#) $\mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}$ which is [covariant](#) in both arguments.

Chapter 68

Forgetful functor

The functor $U : \mathbf{Grp} \rightarrow \mathbf{Set}$ which maps a group to its underlying set and a group homomorphism to its underlying function of sets is a functor.^[8] Functors like these, which "forget" some structure, are termed forgetful functors. Another example is the functor $\mathbf{Rng} \rightarrow \mathbf{Ab}$ which maps a ring to its underlying additive abelian group. Morphisms in \mathbf{Rng} (ring homomorphisms) become morphisms in \mathbf{Ab} (abelian group homomorphisms).

Chapter 69

Free functor

Going in the opposite direction of [forgetful functors](#) are free [functors](#). The [free functor](#) $F : \mathbf{Set} \rightarrow \mathbf{Grp}$ sends every [set](#) X to the free [group](#) generated by X . [Functions](#) get mapped to [group](#) homomorphisms between free [groups](#). Free constructions exist for many [categories](#) based on structured [sets](#). See [free object](#).

Chapter 70

Homomorphism group

To every pair A, B of abelian groups one can assign the abelian group $\text{Hom}(A, B)$ consisting of all group homomorphisms from A to B . This is a functor which is contravariant in the first and covariant in the second argument, i.e. it is a functor $\text{Ab}^{\text{op}} \times \text{Ab} \rightarrow \text{Ab}$ (where Ab denotes the category of abelian groups with group homomorphisms). If $f : A_1 \rightarrow A_2$ and $g : B_1 \rightarrow B_2$ are morphisms in Ab , then the group homomorphism $\text{Hom}(f, g) : \text{Hom}(A_2, B_1) \rightarrow \text{Hom}(A_1, B_2)$ is given by $\varphi \mapsto g \circ \varphi \circ f$. See Hom functor.

Chapter 71

Representable functor

We can generalize the previous example to any category C . To every pair X, Y of objects in C one can assign the set $\text{Hom}(X, Y)$ of morphisms from X to Y . This defines a functor to \mathbf{Set} which is contravariant in the first argument and covariant in the second, i.e. it is a functor $C^{\text{op}} \times C \rightarrow \mathbf{Set}$. If $f : X_1 \rightarrow X_2$ and $g : Y_1 \rightarrow Y_2$ are morphisms in C , then the group homomorphism $\text{Hom}(f, g) : \text{Hom}(X_2, Y_1) \rightarrow \text{Hom}(X_1, Y_2)$ is given by $\varphi \mapsto g \circ \varphi \circ f$.

Functors like these are called representable functors. An important goal in many settings is to determine whether a given functor is representable.

Chapter 72

Corecursion

Chapter 73

Coinduction

proper definition

* [dual](#) to induction. Generalises to [corecursion](#).

Chapter 74

Initial algebra of an endofunctor

Chapter 75

Terminal coalgebra for an endofunctor

Part IV

Citations

"One of the finer points of the Haskell community has been its propensity for recognizing [abstract](#) patterns in code which have well-defined, lawful representations in mathematics." (Chris Allen, Julie Moronuki - "Haskell Programming from First Principles" (2017))

Part V

Good code

Chapter 76

Good: Type aliasing

Use [data type](#) aliases to deferentiate logic of values.

Chapter 77

Good: Type wideness

Wider the [type](#) the more it is [polymorphic](#), means it has broader [application](#) and fits more [types](#).

The more constrained system has more usefulness.

Unconstrained means most flexible, but also most useless.

Chapter 78

Good: Print

```
print :: Show a => a -> IO ()  
print a = putStrLn (show a)
```

Chapter 79

Good: Fold

`foldr spine recursion` intermediated by the folding. Can terminate at any point. `foldl spine recursion` is unconditional, then folding starts. Unconditionally recurses across the whole `spine`, if it infinite - infinitely.

Chapter 80

Good: Computation model

Model the [domain](#) and [types](#) before thinking about how to write computations.

Chapter 81

**Good: Make bottoms only
local**

Chapter 82

Good: Newtype wrap is ideally transparent for compiler and does not change performance

Chapter 83

Good: Instances of types/type classes must go with code you write

Chapter 84

**Good: Functions can be
abstracted as arguments**

Chapter 85

**Good: Infix operators can be
bind to arguments**

Chapter 86

Good: Arbitrary

Product types can be tested as a product of random generators. Sum types require to implement generators with separate constructors, and picking one of them, use oneof or frequency to pick generators.

Chapter 87

Good: Principle of Separation of concerns

Chapter 88

Good: Function composition

In Haskell inline [composition](#) requires:

```
h.g.f $ i
```

[Function application](#) has a higher [priority](#) than [composition](#). That is why parentheses over [argument](#) are needed. This [precedence](#) allows idiomatically [compose partially applied functions](#).

But it is a way better then:

```
h (g (f i))
```

Chapter 89

Good: Point-free

Use [Tacit](#) very carefully - it hides [types](#) and harder to change code [where](#) it is used. Use just enough [Tacit](#) to communicate a bit better. Mostly only partial [point-free](#) communicates better.

89.1 Good: Point-free is great in multi-dimensions

BigData and OLAP analysis.

Chapter 90

Good: Functor application

Function application on n levels beneath:

```
(fmap . fmap) function twoLevelStructure
```

How `fmap . fmap` typechecks:

```
(.)      :: (b -> c) -> (a -> b) -> a -> c
fmap     :: Functor f => (m -> n) -> f m -> f n
fmap     :: Functor g => (x -> y) -> g x -> g y

fmap . fmap :: (Functor f, Functor g)
            => ((g x -> g y) -> f . g x -> f . g y)
            -> (( x -> y) -> g x -> g y)
            -> ( x -> y) -> f . g x -> f . g y
fmap . fmap :: (x -> y) -> f . g x -> f . g y
```

Chapter 91

Good: Parameter order

In [functions parameter order](#) is important. It is best to use first the most reusable [parameters](#). And as last one the one that can be the most [variable](#), that is important to chain.

Chapter 92

Good: Applicative monoid

There can be more than one valid [Monoid](#) for a [data type](#). && There can be more than one valid [Applicative](#) instance for a [data type](#). -> There can be different [Applicatives](#) with different [Monoid](#) implementations.

Chapter 93

Good: Creative process

- 93.1 Pick philosophy principles one to three the more - the harder the implementation
- 93.2 Draw the most blurred representation
- 93.3 Deduce **abstractions** and write remotely what they are
- 93.4 Model of computation
 - 93.4.1 Model the **domain**
 - 93.4.2 Model the **types**
 - 93.4.3 Think how to write computations
- 93.5 Create

Chapter 94

Good: About operators ($\langle \$ \rangle$) ($\langle * \rangle$) ($\langle * \rangle$) ($\langle * \rangle$)

Where character is not present - discard the according processing of a [parameter](#). ($\langle * \rangle$) is an [exception](#), it does the reverse. ignores the first [parameter](#), in fact $\langle * \rangle \equiv \langle * \rangle$.

$\langle * \rangle$ does the proper action: does calculation, but ignores the value from the first [argument](#).

Chapter 95

**Good: About functions like
{mapM, sequence}_**

Trailing _ means ignoring the result.

Chapter 96

Good: Guideliles

96.1 Wiki.haskell

96.1.1 Documentation

96.1.1.1 Comments write in **application** terms, not technical.

96.1.1.2 Tell what code needs to do not how it does.

96.1.2 Haddock

96.1.2.1 Put haddock comments to ever exposed **data type** and **function**.

96.1.2.2 Haddock header

```
{- |
Module      : <File name or $Header$ to be replaced automatically>
Description : <optional short text displayed on contents page>
Copyright   : (c) <Authors or Affiliations>
License     : <license>

Maintainer  : <email>
Stability   : unstable | experimental | provisional | stable | frozen
Portability : portable | non-portable (<reason>)

<module description starting at first column>
-}
```

96.1.3 Code

96.1.3.1 Try to stay closer to portable (Haskell98) code

96.1.3.2 Try make lines no longer 80 chars

96.1.3.3 Last char in file should be newline

96.1.3.4 Symbolic **infix** identifiers is only library writer right

96.1.3.5 Every **function** does one thing.

Chapter 97

Good: Use Typed holes to progress the code

[Typed holes](#) help build code in complex situations.

Chapter 98

Good: Haskell allows infinite terms but not infinite types

That is why infinite `types` throw infinite `type error`.

Chapter 99

Good: Use type synonyms to differ the information

Even if there is `types` - define `type` synonyms. They are free. That distinction with synonyms, would allow `TypeSynonymInstances`, which would allow to create a different `type class` instances and behaviour for different information.

Chapter 100

**Good: Use `Control.Monad.Except`
instead of `Control.Monad.Error`**

Chapter 101

Good: Monad OR Applicative

101.0.1 Start writing **monad** using 'return', 'ap', 'liftM', 'liftM2', '»' instead of 'do', '»='

If you wrote code and really needed only those - move that code to **Applicative**.

```
return -> pure
ap -> <*>
liftM -> liftA -> <$>
>> -> *>
```

101.0.2 Basic **case** when **Applicative** can be used

Can be rewritten in **Applicative**:

```
func = do
  a <- f
  b <- g
pure (a, b)
```

Can't be rewritten in **Applicative**:

```
somethingdoSomething' n = do
  a <- f n
  b <- g a
pure (a, b)
```

(f n) creates **monadic structure**, **binds** ot to a wich is consumed then by g.

101.0.3 **Applicative** block vs **Monad** block

With **Type Applicative** every condition fails/succseeds independently. It needs a boilerplate **data constructor**/value pattern matching code to work. And code you can write only for so many cases and **types**, so boilerplate can not be so flexible as **Monad** that allows **polymorphism**. With **Type Monad** computation can return value that dependent from the previous computation result. So abort or dependent processing can happen.

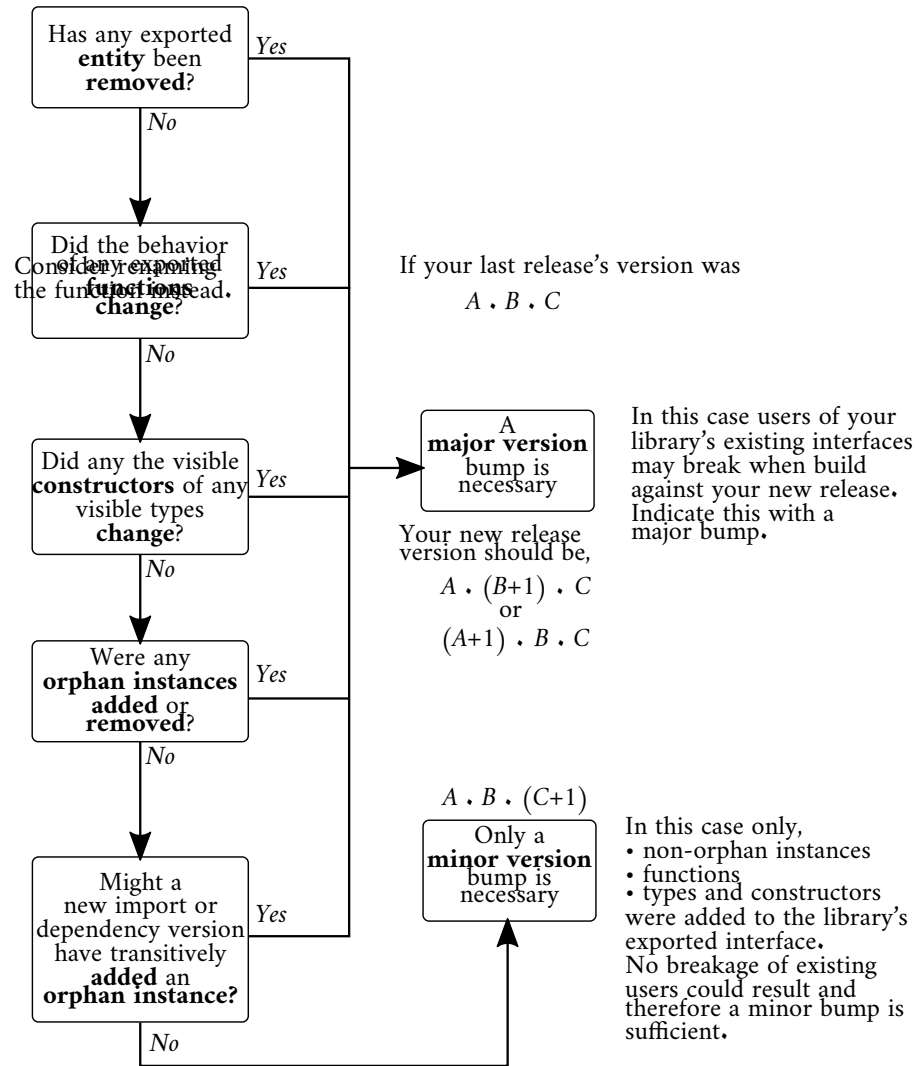
Chapter 102

Good: Haskell Package Versioning Policy

Version policy and dependency management.

So you are releasing a new package version?

Use this decision graph to determine how you should version your new release under Haskell Package Versioning Policy.



102.1 *

PVP Good: PVP

Chapter 103

Good: Linear type

[Linear types](#) are great to control/minimize resource usage.

Chapter 104

Good: Exception vs Error

Many languages and Haskell have it all mixup. Here is table showing what belongs to one or other in standard libraries:

| | |
|---------------------------|--|
| Exception | Prelude.catch, Control. Exception .catch, Control. Exception .try, IOError, Control. Monad . Error |
| Error | error , assert, Control. Exception .catch, Debug.Trace.trace |

Chapter 105

Good: Let vs. Where

`let ... in ...` is a separate [expression](#). In contrast, `where` is [bound](#) to a surrounding syntactic [construct](#) (namespace).

Chapter 106

Good: RankNTypes

Can powerfully synergyze with [ScopedTypeVariables](#).

Chapter 107

Good: Orphan instance

Practice to address orphan instances:

Does `type class` or `type` defined by you:

| Type class | Type | Recommendation |
|------------|------|---|
| | ✓ | { <code>Type</code> , instance} in the same <code>module</code> |
| ✓ | | { <code>Type class</code> & instance} in the same <code>module</code> |
| | | {Define newtype wrap, its instances} in the same <code>module</code> |

Chapter 108

Good: Smart constructor

Only proper smart [constructors](#) should be exported. Do not export [data type constructor](#), only a [type](#).

Chapter 109

Good: Thin category

In * all [morphisms](#) are [epimorphisms](#) and [monomorphisms](#).

Chapter 110

Good: Recursion

Writing/thinking about [recursion](#):

1. Find the base cases, on input of which the answer can be provided right away. There is mostly one [base case](#), but sometimes there can be several of them. Typical base cases are: [zero](#), the empty [list](#), the empty tree, null, etc.
2. Do inductive [case](#). The [recursive](#) invocation. The [argument](#) of a [recursive](#) call needs to be smaller than the current [argument](#). So it would be gradually closer to the [base case](#). The idea is that processes eventually hit the [base case](#).

Simple functional [application](#) is used in the [recursion](#). Assume that the [functions](#) would return the right result.

Chapter 111

Good: Monoid

<>: [Sets](#) - union. Maps - left-biased union. Number - Sum, Product form separate [monoid categories](#).

Chapter 112

Good: Free monad

The main [case](#) of usage of Free [monads](#) in Haskell:

Start implementation of the [monad](#) from a Free [monad](#), drafting the base [monadic](#) operations, then add custom operations.

Gradually build on top of Free [monad](#) and try to find homomorphisms from [monad](#) to [objects](#), and if only [objects](#) are needed - get rid of the free [monad](#).

Chapter 113

Good: Use mostly where clauses

Chapter 114

**Good: Where clause is in a scope
with function parameters**

Chapter 115

Good: Strong preference towards pattern matching over {head, tail, etc.} functions

head and tail and alike [functions](#) are often partial ([unsafe](#)) functions.

Chapter 116

Good: Patternmatching is possible on monadic bind in do

Example:

```
instance (Monad m) => Functor (StateT s m) where
  fmap f m = StateT $ \s -> do
    (x, s') <- runStateT m s  -- Here is a pattern matching bind
    return (f x, s')
```

Chapter 117

Good: Applicative vs Monad

Giving not Monad but Applicative requirement allows parallel computation, but if there should be a chaining of the intermediate state - it must be [monadic](#).

Chapter 118

Good: StateT, ReaderT, WriterT

Reader trait: (r ->).

Writer trait: (a, w).

State trait is combination of both:

```
newtype StateT s m a =  
  StateT { runStateT :: s -> m (a, s) }
```

```
newtype ReaderT r m a =  
  ReaderT { runReaderT :: r -> m a }
```

```
newtype WriterT w m a =  
  WriterT { runWriterT :: m (a, w) }
```

State trait fully replaces writer.

Chapter 119

Good: Working with MonadTrans and lift

From the `lift . pure = pure` follows that `MonadTrans` [type](#) can have a `pure` defined with `lift`.

Stacking of `MonadTrans` [monads](#) can result in a lot of chained `lift` and `unwraps`. There is many ways to cope with that but the most robust and common is to [abstract](#) representation with `newtype` on the `Monad` [stack](#). This can reduce caining or remove the manual [lifting](#) withing the [Monad](#). For perfect combination for contributors to be able to extend the code - keep the `Internal` [module](#) that has a raw representation.

Chapter 120

Good: Don't mix Where and Let

let and where create a [recursive set](#) of definitions with can explode, don't mix them together in code.

Chapter 121

Good: Where vs. Let

Let is self-[recursive lazy pattern](#). It is checked and [errors](#) only at execution time. [Binds](#) only inside [expression](#) it is binded to.

Where is a part of definition, scoped over definition implemetations and [guards](#), not self-[recursive](#).

Chapter 122

Good: The proper nature algorithm that models behaviour of many objects is computation heavy

God does not care about our mathematical difficulties. He integrates empirically.

One who is found of mathematical meaning loves to [apply](#) it. But if we implement the "real" algorithms behind nature processes, we face the need to go through the computations of laws of all particles.

Computation of nature is always a middle way between ideal theory behaviour and computation simplification.

Chapter 123

Good: In Haskell parameters bound by lambda declaration instantiate to only one concrete type

Because of [let-bound polymorphism](#):

This is illegal in Haskell:

```
foo :: (Int, Char)
foo = (\f -> (f 1, f 'a')) id
```

Lambda-bound function (i.e., one passed as [argument](#) to another [function](#)) cannot be instantiated in two different ways, if there is a [let-bound polymorphism](#).

Chapter 124

Good: Instance is a good structure to draw a type line

Instances for [data type](#) can differentiate by [constraints](#) & [types](#) of arguments. So instance can preserve [type](#) boundary, and [data type declaration](#) can stay very [polymorphic](#). If the need to extend the [type](#) boundaries arrive - the instances may extend, or new instances are created, while used [data type](#) still the same and unchanged.

Chapter 125

Good: MTL vs. Transformers

Default of mtl.

Transformers is Haskell-98, doesn't have functional dependencies, lacks the [monad](#) classes, has manual [lift](#) of operations to the composite [monad](#).

MTL extends transformers, providing more instances, features and possibilities, may include [alternative](#) packages features as mtl-tf.

Part VI

Bad code

Chapter 126

Bad pragma

126.1 Bad: Dangerous **LANGUAGE pragma** option

- [DatatypeContexts](#)
- `OverlappingInstances`
- `IncoherentInstances`
- `ImpredicativeTypes`
- `AllowAmbiguousTypes`
- [UndecidableInstances](#) - often

Part VII

Useful **functions** to remember

Chapter 127

Prelude

```
enumFromTo
enumFromThenTo
reverse
show :: Show a => a -> String
flip
sequence - Evaluate each monadic action in the structure from left to right, and collect the results
:sprint - show variables to see what has been evaluated already.
minBound - smaller bound
maxBound - larger bound
cycle :: [a] -> [a] - indefinitely cycle s list
repeat - indefinitely list from value
elemIndex e l - return first index, returns Maybe
fromMaybe (default if Nothing) e :: Maybe a -> a
lookup :: Eq a => a -> [(a, b)] -> Maybe b
```

127.1 Ord

compare

127.2 Calc

div - always makes rounding down, to infinity divMod - returns a tuple containing the result of integral division and modulo

127.3 List operations

```
concat - [ [a] ] -> [a]
elem x xs - is element a part of a list
zip :: [a] -> [b] -> [(a, b)] - zips two lists together. Zip stops when one list runs out.
zipWith :: (a -> b -> c) -> [a] -> [b] -> [c] - do the action on corresponding elements of list a
```

Chapter 128

Data.List

`intersperse :: a -> [a] -> [a]` - gets the value and incerts it between values `in` array
`nub` - remove duplicates from the list

Chapter 129

Data.Char

```
ord (Char -> Int)
chr (Int -> Char)
isUpper (Char -> Bool)
toUpper (Char -> Char)
```

Chapter 130

QuickCheck

```
quickCheck :: Testable prop => prop -> IO ()  
quickCheck . verbose - run verbose mode
```

Part VIII

Tools

Chapter 131

ghc-pkg

[List](#) installed packages:

```
ghc-pkg list
```

Chapter 132

Integration of NixOS/Nix with Haskell IDE Engine (HIE) and Emacs (Spacemacs)

132.1 1. Install the Cachix

Upstream doc: <https://github.com/cachix/cachix>

132.2 2. Installation of HIE

Upstream doc: <https://github.com/infinisil/all-hies/#cached-builds>

132.2.1 2.1. Provide cached builds

```
cachix use all-hies
```

132.2.2 2.2.a. Installation on NixOS distribution:

```
{ config, pkgs, ... }:
```

```
let
```

```
    all-hies = import (fetchTarball "https://github.com/infinisil/all-hies/tarball/master") {};
```

```
in {
```

```
    environment.systemPackages = with pkgs; [
```

```
        (all-hies.selection { selector = p: { inherit (p) ghc865 ghc864; }; })
```

```
    ];
```

```
}
```

Insert your GHC versions.

Switch to new configuration:

```
sudo -i nixos-rebuild switch
```

132.2.3 2.2.b. Installation with Nix package manager:

```
nix-env -iA selection --arg selector 'p: { inherit (p) ghc865 ghc864; }' -f 'https://github.com/i
```

Insert your GHC versions.

132.3 3. Emacs (Spacemacs) configuration:

```
dotspacemacs-configuration-layers
'(
  auto-completion

  (lsp :variables
    default-nix-wrapper (lambda (args)
      (append
        (append (list "nix-shell" "-I" "." "--command" )
          (list (mapconcat 'identity args " ")))
        )
      (list (nix-current-sandbox))
      )
    )

    lsp-haskell-process-wrapper-function default-nix-wrapper
  )

  (haskell :variables
    haskell-enable-hindent t
    haskell-completion-backend 'lsp
    haskell-process-type 'cabal-new-repl
  )
)

dotspacemacs-additional-packages '(
  direnv
  nix-sandbox
)

(defun dotspacemacs/user-config ()

  (add-hook 'haskell-mode-hook 'direnv-update-environment) ;; If direnv configured

)
```

Where:

auto-completion configures YASnippet.

nix-sandbox (<https://github.com/travisbhartwell/nix-emacs>) has a great helper functions. Using nix-current-sandbox function in default-nix-wrapper that used to properly configure lsp-haskell-process-wrapper-function.

Configuration of the lsp-haskell-process-wrapper-function default-nix-wrapper is a key for HIE to work in nix-shell

Inside nix-shell the haskell-process-type 'cabal-new-repl is required.

CHAPTER 132. INTEGRATION OF NIXOS/NIX WITH HASKELL IDE ENGINE (HIE)

132.4. 4. OPEN THE HASKELL FILE FROM A PROJECT AND EMACS (SPACEMACS)

Configuration was reassembled from: <https://github.com/emacs-lsp/lsp-haskell/blob/8f2dbb6e827b1adce6360c56lsp-haskell.el#L57> & its authors config: [\[\[https://github.com/sevanspowell/dotfiles/blob/master/spacemacs\]\]](https://github.com/sevanspowell/dotfiles/blob/master/spacemacs)/

Refresh Emacs.

132.4 4. Open the Haskell file from a project

Open system monitor, observe the [process](#) of environment establishing, packages loading & compiling.

132.5 5. Be pleased writing code

```
11 path :: MonadEffects t f m => FilePath -> m (Maybe FilePath)
12 path path = do
13   path <- makeAbsolutePath $ if path
14   exists <- doesPathExist path
15   return $ if exists then Just path else Nothing
16
17 defaultImportPath
18   :: (MonadIO m => f m, MonadState (HashMap FilePath HExprLoc) m)
19   => FilePath
20   -> m (Maybe t f m)
21 defaultImportPath path = do
22   traceM $ "Importing file:" ++ path
23   withFrameInfo (throwError $ "While importing file:" ++ show path) $ do
24     imports <- get
25     evaluateLoc <- case HashMap path imports of
26       Nothing -> do
27         pure expr
28       Just expr -> pure expr
29     area <- parseWithFile path
30     case area of
31       Failure err ->
32         $ throwError
33           $ fromError
34             $ show $ fillSep ["Parse during import failed:", err]
35     success expr -> do
36       modify (M.insert path expr)
37       pure expr
38
39 defaultPathDefaultThis :: MonadIO m => f m => FilePath -> m FilePath
40 defaultPathDefaultThis = pathDefaultThisFile
41
42 -- Given a path, determine the file to load
43 pathDefaultThisFile :: MonadIO m => FilePath -> m FilePath
44 pathDefaultThisFile p = do
45   idir <- doesDirectoryExist p
46   pure $ if idir then p <|> "default.nix" else p
47
48 defaultDerivationStrict
49   :: forall t f m. MonadIO m => t f m -> m (Maybe t f m)
50 defaultDerivationStrict = fromValue $ \attrSet (Maybe t f m) -> do
51   m <- maybe pure $ case demand of
52     Demand D (fromValue) M (Just m) -> ignoreM m
53     s' <- M.fromList <$> mapMaybe (M.fromList m) (M.toList s')
54     v' <- normalize <$> toValue $ attrSet (Maybe t f m) <$> s'
55     m <- instantiateExpr $ "derivationStrict" ++ show (prettyValue v')
56     where
57       mapMaybe :: (a -> m (Maybe b)) -> [a] -> m [b]
58       mapMaybe op = foldr f (return [])
59       where f x xs = op x >>> (D $ xs) <|> (++) . maybeToList
60
61 handleEntry :: Bool -> (Text, WValue t f m) -> m (Maybe (Text, WValue t f m))
62 handleEntry ignoreNulls (x, v) = case x of
63   -- The 'args' attribute is special: it supplies the command-line
64   -- arguments to the builder.
65   -- This use of coercion is probably not right and may
66   -- not have the right arguments.
67   "args" -> demand v $ Pure Just . coerceToList
68   -- ignoreNulls -> pure Nothing
69   -- demand v $ case
70   -- WConstant Null | ignoreNulls -> pure Nothing
71   -- v
72   --> Just $> coerce v'
73   where
74     coerce :: (t f m -> m (Maybe t f m))
75     coerce = \v -> do
76       v' <- fromValue v
77       pure v'
```

Now, the powers of the Haskell, Nix & Emacs combined. It's fully in your hands now. Be cautious - you can change the world.

132.6 6. (optional) Debugging

1. If receiving sort-of:

readCreateProcess : cabal-helper-wrapper failure

HIE tries to run cabal operations like on the non-Nix system. So it is a problem with detection of nix-shell environment, running inside it.

1. If HIE keeps getting ready, failing & restarting - check that the projects ghc --version is declared in your all-hie NixOS configuration.

Chapter 133

Debugger

Provides:

- [set](#) a breakpoints
- observe step-by-step [evaluation](#)
- tracing mode

Breakpoints

```
:break 2
:show breaks
:delete 0
:continue
```

Step-by-step

```
:step main
```

[List](#) information at the breakpoint

```
:list
```

What been evaluated already

```
:sprint name
```

Chapter 134

GHCID

Commands to run the compile/check loop:

cabal > 3.0 command:

```
ghcid --command='cabal v2-repl --repl-options=-fno-code --repl-options=-fno-break-on-exception --
```

cabal < 3.0 command:

```
ghcid --command='cabal new-repl --ghc-options=-fno-code --ghc-options=-fno-break-on-exception --g
```

nix-shell cabal > 3.0 command:

```
nix-shell --command 'ghcid --command="cabal v2-repl --repl-options=-fno-code --repl-options=-fno-
```

nix-shell cabal < 3.0 command:

```
nix-shell --command 'ghcid --command="cabal new-repl --ghc-options=-fno-code --ghc-options=-fno-b
```

Chapter 135

Continuous integration platrorms (CIs) for Open Source Haskell projects

Since Open Source projects mostly use free tiers of CIs, and different CIs have different features - there is a [constant](#) flux of how to [construct](#) the best possible integration pipeline for Haskell projects.

The current state of affairs is best put in this quote:

Probably the biggest [constraint](#) is whether or not CI needs to test Windows or OS X, since build machines for those are harder to come by. We currently use AppVeyor for Windows builds and Travis for OS X builds since they are free. For Linux you can basically use any CI provider, but in this [case](#) I pay for a Linode VM which I use to host all Dhall-related infrastructure (i.e. all of the *.dhall-lang.org domains), so I reuse that to host Hydra for Nix-related CI so that I can use more parallelism and more efficient caching to test a wider range of GHC versions on a budget.

For [testing](#) OS X and Windows platforms we use [stack](#). The main reason we don't use Nix for [either](#) platform is that Nix only supports building release binaries on Linux (and even then it's still experimental).

So the basic summary I can give is:

For [testing](#) everything other than cross-platform support: Nix + Linux is best in my opinion ... because you get much more control and intelligent build caching, which is usually [where](#) most CI solutions fall short

For cross-platform support: [stack](#) + whatever CI provider provides free builds for that platform

Also, if you ever can pay for your own NixOS VM and you want to reuse the setup I built, you can find the NixOS configuration for dhall-lang.org here:

<https://github.com/dhall-lang/dhall-lang/tree/master/nixops>

Part IX

Libs

Chapter 136

Exceptions

- 136.1 **Exceptions** - optionally **pure** extensible **exceptions** that are compatible with the mtl
- 136.2 **Safe-exceptions** - safe, simple API **equivalent** to the underlying implementation in terms of power, encourages best practices minimizing the chances of getting the **exception** handling wrong.
- 136.3 **Enclosed-exceptions** - capture **exceptions** from the enclosed computation, while reacting to asynchronous **exceptions** aimed at the calling thread.

Chapter 137

Memory management

137.1 membrain - [type](#)-safe memory units

Chapter 138

Parsers - megaparsec

Chapter 139

CLIs - `optparse-`[applicative](#)

Chapter 140

HTML - Lucid

Chapter 141

Web applications - Servant

Chapter 142

IO libraries

142.1 Conduit - practical, monolythic, guarantees termination return

142.2 Pipes + Pipes Parse - modular, more primitive, theoretically driven

Chapter 143

JSON - aeson

Chapter 144

Backpack

On 1-st compilation - * analyzes the [abstract](#) signatures without loading side modules, doing the [type check](#) with assumption that modules provide right [type](#) signatures, the [process](#) does not emit any [binary](#) code and stores the intermediate code in a special form that allows flexibly connect modules provided. Which allows later to compile project with particular instantiations of the modules. Major work of this [process](#) being done by internal Cabal * support and * system that modifies the intermediate code to fit the [module](#).

Part X

Drafts

Chapter 145

Exception handling

Ideal model:

- ☒ **Exception** must include all **context** information that may be useful.
- ☒ Store information in a form for further probable deeper automatic diagnostic.
- ☒ Sensitive data/dummies for it - can be useful during development.
- ☒ Sensitive data should be stripped from a program logging & **exceptions**.
- ☒ **Exception** system should be extendable, data storage & representation should be easily extendable.
- ☒ **Exception** system should allow easy exhaustive checking of **errors**, since the different **errors** can happen.
- ☒ **Exception** system should be automatically well-documented and transparent.
- ☒ **Exception** system should have controllable breaking changes downstream.
- ☒ **Exception** system should allow complex composite (**sets**) **exceptions**.
- ☒ **Exception** system should be lightweight on the **type** signatures of other **functions**.
- ☒ **Exception** system should automate the collection of **context** for a **exception**.
- ☒ **Exception** system should have **properties** and according **functions** for particular **types** of **errors**.

String is simple and convinient to throw **exception**, but really a mistake because it the most cumbersome choise:

- ☒ Any **Exception** instance can be converted to a **String** with **either** **show** or **displayException**.
- ☐ Does not include key debugging information in the **error** message.
- ☐ Does not allow developer to access/manage the **Exception** information.
- ☐ **Exception** messages need to be constructed ahead of time, it can not be internationalized, converted to some data/file format.
- ☐ **Exception** can have a sensitive information that can be useful for developer during work, but should not be logged/shown to end-user. Stripping it from **Strings** in the changing project is a hard task.
- ☐ Impossible to rely on this representation for further/deeper inspection.
- ☐ Impossible to have exhaustive checking - no knowledge no check, no warning if some cases are not handled.

Universal **exception type**:

- ☒ Able to inspect every possible **error case** with **pattern match**.

- ☒ Self-documenting. Shows the hierarchical system of all [exceptions](#).
- ☒ Transparent. Ability to discern in current situation what [exceptions](#) can happen
- ☐ New [exception constructor](#) causes breaking change to downstream.
- ☐ Wrongly implies completeness. Untreated [Errors](#) can happen, different [exception](#) can arrive from the outside code.

Sum [type](#) must be separate, and [product type structure](#) over it. Separate [exception type](#) of

Individual [exception types](#):

- ☒ Writing & seeing & working with exactly what will go wrong because there is only one possible [error](#) for this [type of exception](#). [Pattern match](#) happens only on conditions, [constructors](#) that should happen.
- ☒ Knowledge what exactly goes wrong allows wide usage of [Either](#).
- ☐ It is hard to handle complex [exceptions](#) in the unitary system. Real world can return not a particular [case](#), but a [set](#) of cases {[object](#) not found, path is unreachable, access is denied}.
- ☐ [Type](#) signatures grow, and even can become complex, since every [case of exception](#) has its own [type](#).
- ☐ Impure [throw](#) that users can/should use for your code must account for all your [exception types](#).

[Abstract exception type](#):

[Exception type](#) entirely opaque and inspectable only by accessor [functions](#).

- ☒ Updating the internals without breaking the API
- ☒ Semi-automates the [context of exception](#) with passing it to accessors.
- ☒ Predicates can be [applied](#) to more than one [constructor](#). Which are [properties](#) that allows to make complex [exceptions](#) much easier to handle.
- ☐ Not self-documenting.
- ☐ Possible options by design are hidden from the downstream, documentation must be kept.
- ☐ When you change the [exception handling/throwing errors](#) it does not show to the downstream.

Composit approach: Provide the [set of constructors](#) and also a [set of predicates](#) and [set of accessors](#). Use [pattern synonyms](#) to provide a documented accessor [set](#) without exposing internal [data type](#).

In GHC 8.8 the change was made:

The `fail` method of [Monad](#) has been removed in favor of the method of the same name in the `MonadFail` class.

`MonadFail(..)` is now exported from the `Prelude` and `Control.Monad` modules. The `Monad-FailDesugaring` language extension is now deprecated, as its effects are always enabled.

So:

```
import           Control.Monad.Fail
...
class MonadFail m => MonadFile m
...
-- use error instead of fail
Nothing      -> error ("Message " <> show x)
```

Chapter 146

Constraints

Very strong Haskell [type](#) system makes possible to work with code from the top down, an [axiomatic semantics](#) approach, from [constraints](#) into [types](#).

- Helps to form the [type level](#) code (aka [join](#) points of the code).
- Uses the piling up of [constraints/types](#) information. At some point pick and satisfy [constraints](#), can be done one at a time.
- Provides hints through [type level](#) formulation for [term level](#) calculations, does not formulate the [term level](#).
- Tedious method (a lot of boilerplate and rewriting it) but pretty simple and relaxing.
- [Set](#) of [constraints](#).
- When it is needed or convenient, single [constraint](#) gets a little more realistically concrete/abstracted.

Main [type](#) detail annotation thread can happen in [main](#) or special wrapper [function](#), localization is inside [functions](#).

1. Rest of [constraints set](#) shifts to source [type](#).

3.a. For the class handled or known how to handle - write a [base case](#) instance description.

```
instance (Monad m) => MonadReader r (ReaderT r m)
```

3.b. For others write [recursive](#) instance descriptions:

All other unsolved [constraints](#) move into the source [polymorphic variable](#).

```
instance (MonadError e m) => MonadError e (ReaderT r m)
instance (MonadState s m) => MonadState s (ReaderT r m)
```

1. Repeat from 1 until considered done.
2. Code condensed into terse form.

MonadError [constraints](#) is [IOException](#), not for the [String](#). [IOException](#) vs [String](#).

Reverse pluck MonadReader [constraint](#) with runReader on the [object](#).

MonadState - StateT

Chapter 147

Monad transformers and their type classes

Chapter 148

Layering **monad** transformers

Different layering of the same **monad** transformers is functionality is the same, but the form is different. Surrounding handling **functions** would need to be different.

Chapter 149

Hoogle

149.1 Search

Text search ([case](#) insensitive):

- `a`
- `map`
- `con map`

[Type](#) search:

- `:: a`
- `:: a -> a`

Text & [type](#):

`=id a -> a =`

149.2 Scope

149.2.1 Default

[Scope](#) is [Haskell Platform](#) (and [Haskell keywords](#)).

All [Package](#) packages are available to search with:

149.2.2 Hierarchical module name system (from big letter):

- `fold +Data.Map` finds results in the `Data.Map` [module](#)
- `file -System` excludes results from modules such as `System.IO`, `System.FilePath.Windows` and `Distribution.System`

149.2.3 Packages (lower [case](#)):

- `mode +platform`
- `mode +cmdargs` (only)
- `mode +platform +cmdargs`
- `file -base` (Haskell Platform, excluding the "base" package)

Chapter 150

ST-Trick monad

ST is like a [lexical scope](#), where all the [variables](#)/state disappear when the [function](#) returns <https://wiki.haskell.org/ST> <https://www.schoolofhaskell.com/school/to-infinity-and-beyond/older-but-still-interesting-monads/deamortized-strg/Monad/ST> <https://dev.to/jvanbruegge/what-the-heck-is-polymorphism-nmh>

150.1 *

ST-Trick

Chapter 151

Either

Allows to separate and preserve information about happened, ex. [error](#) handling.

151.1 *

Either data type

Chapter 152

Inverse

1. [Inverse function](#)
2. In logic: $P \rightarrow Q \Rightarrow \neg P \rightarrow \neg Q$, & same for [category duality](#).
3. For [operation](#): element that allows reversing [operation](#), having an element that with the [dual](#) produces the [identity](#) element.
4. See [Inversion](#).

Chapter 153

Inversion

1. Is a [permutation where](#) two elements are out of [order](#).
2. See [Inverse](#)

Chapter 154

Inverse function

$$f_{x \rightarrow y} \circ (f_{x \rightarrow y})^{-1} = 1_x$$

* \iff function is **bijective**. Otherwise - **partial inverse**

Chapter 155

Inverse morphism

For $f : x \rightarrow y$: $\exists g : g \circ f = 1^x$ - g is left [inverse](#) of f , $\exists g : f \circ g = 1^y$ - g is right [inverse](#) of f .

Chapter 156

Partial inverse

* when [function](#) is now [bijective](#). When [bijective](#) see [inverse function](#).

Chapter 157

PatternSynonyms

Enables [pattern synonym declaration](#), which always begins with the `pattern` word. Allows to [abstract](#) away the [structures](#) of pattern matching.

157.1 *

Pattern synonym Pattern synonyms

Chapter 158

GHC debug keys

158.1 -ddump-ds

Dump desugarer output.

158.1.1 *

Desugar GHC desugar

Chapter 159

GHC optimize keys

159.1 -foptimal-applicative-do

$O(n^3)$ Always finds optimal [reduction](#) into `<*>` for [ApplicativeDo](#) do notation.

Chapter 160

Computational trinitarianism

Taken from: <https://ncatlab.org/nlab/show/computational+trinitarianism>

Under the [statements](#):

- [propositions](#) as [types](#)
- programs as proofs
- [relation](#) between [type](#) theory and [category](#) theory

the following notions are [equivalent](#):

== [proposition](#) proof (Logic)

== generalized element of an [object](#) ([Category](#) theory)

== typed program with output ([Type](#) theory & Computer science)

160.1 *

Trinitarism

Table 160.1: Computational trinitarianism

| Logic | Category theory | Type theory |
|-----------------------------------|---|--|
| true | terminal object / (-2)-truncated object | h-level 0-type/unit type |
| false | initial object | empty type |
| proposition | (-1)-truncated object | h-proposition, mere proposition |
| proof | generalized element | program |
| cut rule | composition of classifying morphisms / pullback of display maps | substitution |
| cut elimination for implication | countit for hom-tensor adjunction | beta reduction |
| introduction rule for implication | unit for hom-tensor adjunction | eta conversion |
| logical conjunction | product | product type |
| disjunction | coproduct ((-1)-truncation of) | sum type (bracket type of) |
| implication | internal hom | function type |
| negation | internal hom into initial object | function type into empty type |
| universal quantification | dependent product | dependent product type |
| existential quantification | dependent sum ((-1)-truncation of) | dependent sum type (bracket type of) |
| equivalence | path space object | identity type |
| equivalence class | quotient | quotient type |
| induction | colimit | inductive type, W-type, M-type |
| higher induction | higher colimit | higher inductive type |
| completely presented set | discrete object/0-truncated object | h-level 2-type/preset/h-set |
| set | internal 0-groupoid | Bishop set/setoid |
| universe | object classifier | type of types |
| modality | closure operator, (idempotent) monad | modal type theory, monad (in computer science) |
| linear logic | (symmetric, closed) monoidal category | linear type theory/quantum computation |
| proof net | string diagram | quantum circuit |
| (absence of) contraction rule | (absence of) diagonal | no-cloning theorem |
| | synthetic mathematics | domain specific embedded programming language |

Chapter 161

Techniques functional programming deals with the state

161.1 Minimizing

Do not rely on state, try not to change the state. Use it only when it is very necessary.

161.2 Concentrating

Concentrate the state in one place.

161.3 Deferring

Defer state to the last step of the program, or to external system.

Chapter 162

Monadic Error handling

```
(>>=) :: m a -> (a -> m b) -> m b --  $\lambda A.E \boxtimes A$  - computes and drops if error value happens.  
catch :: c a -> (e -> c a) -> c a --  $\lambda E.E \boxtimes A$  - handles "errors" as "normal" values and stops when
```

Chapter 163

Functions

Total **function** uses **domain** fully, but takes only part of the **codomain**. **Function** allows to collapse **domain** values into **codomain** value. Meaning the **function** allows to loose the information. So total **function** is a computation that looses the information or into bigger codomains. That is why the **function** has a directionality, and **inverse** total **process** is partially possible.

Directionality and invertability are terms.

Chapter 164

Void

Emptiness.

Can not be grasped, touched.

A logically uninhabited [data type](#).

(Since [basis](#) of logic is tautologically True and [Void](#) value can not be addressed - there is a logical paradox with the [Void](#)).

Is an [object](#) included into the [Hask category](#), since:

```
:t (id :: Void -> Void)
(id :: Void -> Void) :: Void -> Void
```

id for it exists.

[Type](#) system corresponds to [constructive logic](#) and not to the classical logic. Classical logic answers the question "Is this actually true". Constructive (Intuitionistic) logic answers the question "Is this provable".

Also has [functions](#):

```
-- Represents logical principle of explosion: from falsehood, anything follows.
absurd :: Void -> a
```

```
-- If Functor holds only Void - it holds no values.
vacuous :: Functor f => f Void -> f a
```

```
-- If Monad holds only Void - it holds no values.
vacuousM :: Monad m => m Void -> m a
```

Design pattern: use [polymorphic data types](#) and [Void](#) to get rid of possibilities when you need to.

164.1 *

Nothing, Haskell [expressions](#) can't return [Void](#).

Also see: [Maybe](#).

Chapter 165

Constructive proof

Method of proof that demonstrates the existence of a mathematical [object](#) by creating or providing a method for creating the [object](#).

Chapter 166

Intuitionistic logic

[Proposition](#) considered True due to direct evidence of existence through constructive proof using [Curry-Howard isomorphism](#).

* does not include classic logic fundamental axioms of the excluded middle and double negation elimination. Hence * is weaker than classical logic. Classical logic includes *, all theorems of * are also in classical logic.

166.1 *

Constructive logic

Chapter 167

Principle of explosion

From asserted [statement](#) that contains contradiction - anything can be proven. Ancient principle of logic.
Both in classical & intuitionistic logic.

167.1 *

Ex falso quodlibet Ex falso sequitur quodlibet EFG Ex contradictione quodlibet Ex contradictione sequitur quodlibet ECQ Deductive explosion Pseudo-Scotus

Chapter 168

Universal **property**

A **property** of some construction which boils down to (is manifestly **equivalent** to) the **property** that an associated **object** is a universal **initial object** of some (auxiliary) **category**.

Chapter 169

Yoneda lemma

Allows the embedding of any [category](#) into a [category](#) of [functors](#) ([contravariant set-valued functors](#)) defined on that [category](#). It also clarifies how the embedded [category](#), of representable [functors](#) and their [natural transformations](#), relates to the other [objects](#) in the larger [functor category](#).

The Yoneda lemma suggests that instead of studying the (locally small) [category](#) $C \{\{\{C\}\}\}^{\mathcal{C}}$, *one should study the [category](#) of al*

Chapter 170

Monoidal category, functoriality of ADTs, Profunctors

Category equipped with [tensor product](#).

<>

wich is a [functor](#) for $*$.

Set category can be [monoidal](#) under both [product](#) (having [terminal object](#)) or [coproduct](#) (having [initial object](#)) operations, if according [operation](#) exist for all [objects](#).

Any one-object category is $*$.

$(a, ()) \sim a$ up to unique [isomorphism](#), which is called [Lax monoidal functor](#).

[Product](#) and [coproduct](#) are [functorial](#), so, since: [Algebraic data type](#) construction can use:

- [Type constructor](#)
- [Data constructor](#)
- [Const functor](#)
- [Identity functor](#)
- [Product](#)
- [Coproduct](#)

Any [algebraic data type](#) is [functorial](#).

Chapter 171

Const functor

Maps all **objects** of source **category** into one (fixed) **object** of target **category**, and all **morphisms** to **identity morphism** of that fixed **object**.

```
instance Functor (Const c)
  where
    fmap :: (a -> b) -> Const c a -> Const c b
    fmap _ (Const c) = Const c
```

In **Category** theory denoted:

Δ

Last **type parameter** that bears the target **type** of lifted **function** (b) and is a **proxy type**.

Analogy: the container that always has an **object** attached to it, and everything that is put inside - changes the container **type** accordingly, and dissapears.

Chapter 172

Arrow in Haskell

```
(->) a b = a -> b
```

`Functorial` in the last `argument` & called Reader `functor`.

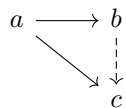
```
newtype Reader c a = Reader (c -> a)
```

```
fmap = ( . )
```

Chapter 173

Contravariant functor

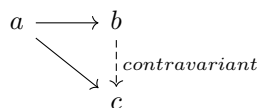
```
fmap :: (a -> b) -> Op c a -> Op c b
      (a -> c) -> (b -> c)
```



;

$$(a \rightarrow b)^C = (a \leftarrow b)^{C^{op}}$$

```
class Contravariant f
  where
    contramap :: (b -> a) -> (f a -> f b)
```



;

If [arrows](#) does not commute Contravariant functor anyway allows to [construct](#) transformation between these such [arrows](#) to other [arrow](#).

Chapter 174

Profunctor

$(\multimap) \ a \ b$

$C^{op} \times C \rightarrow C$

It is called profunctor.

`dimap :: (a' -> a) -> (b -> b') -> p a b -> p a' b'`

So, profunctor in [case](#) of [arrow](#):

$$\begin{array}{ccc} a & \xleftarrow{f} & a' \\ \downarrow h & & \downarrow \text{profunctor} \\ b & \xrightarrow{g} & b' \end{array}$$

;

```
dimap :: (a' -> a) -> (b -> b') -> p a b -> p a' b'
dimap ::      f          g      -> (a -> b) -> (a' -> b')
dimap ::      f          g      ->      h      -> (a' -> b')
dimap = g . h . f
```

It is [contravariant functor](#) in the first [argument](#), and [covariant functor](#) in the second [argument](#).

```
dimap id <==> fmap
(flip dimap) id <==> contramap
```

Chapter 175

Coerce

Operates under condition that source and target [types](#) have same representation. Same representation means they are [type](#) aliases, or it the compiler can [infer](#) that they have the same representation. Directly shares the values from the source [type](#) to the target [type](#). Conversion is free, there is no run-time computations.

The [function](#) implementing the transition:

```
coerce :: Coercible a b => a -> b
```

[Type class](#) implementing the instances for transitions:

```
class a ~R# b => Coercible (a :: k0) (b :: k0)
```

When compiler detects [types](#) have same [structure](#), [type class](#) instances coerse implementation for this pairs of [types](#). This [type class](#) does not have regular instances; instead they are created on-the-fly during [type](#)-checking. Trying to manually declare an instance of Coercible is an [error](#).

175.1 *

Coercible

Part XI

Reference

Chapter 176

Functor-Applicative-Monad Proposal

Well known event in Haskell history: https://github.com/quchen/articles/blob/master/applicative_monad.md.

Math justice was restored with a RETroactive CONtinuity. Invented in computer science term [Applicative](#) ([lax monoidal functor](#)) become a [superclass](#) of [Monad](#).

& that is why:

- `return = pure`
- `ap = <*>`
- `>> = *>`
- `liftM = liftA = fmap`
- `liftM* = liftA*`

Also, a side-kick - [Alternative](#) became a [superclass](#) of [MonadPlus](#). Hence:

- `mzero = empty`
- `mplus = (<|>)`

Work of unification continues under: <https://gitlab.haskell.org/ghc/ghc/wikis/proposal/monad-of-no-return>

176.1 *

Applicative-Monad proposal AMP

Chapter 177

Haskell-98

177.1 Old instance termination rules

1. \forall class **constraint** $(C \ t1 \ \dots \ tn)$: 1.1. **type variables** have occurrences \leq head 1.2. **constructors+variables+repetitions** $<$ head 1.3. \neg **type functions** (**type** func **application** can expand to **arbitrary** size)
2. \forall **functional dependencies**, $\text{[]} \text{ tvs } \square_{\text{left}} \rightarrow \text{[]} \text{ tvs } \square_{\text{right}}$, of the class, every **type variable** in $S(\text{[]} \text{ tvs } \square_{\text{right}})$ must appear in $S(\text{[]} \text{ tvs } \square_{\text{left}})$, **where** S is the substitution mapping each **type variable** in the class **declaration** to the corresponding **type** in the instance head.

Chapter 178

Performance results and comparisons of **types** & solutions

Haskell performance

Chapter 179

Literature

- GHC Team "GHC User's Guide Documentation": https://downloads.haskell.org/~ghc/latest/docs/users_guide.pdf
- Stephen Diehl & contributors "What I Wish I Knew When Learning Haskell": <http://dev.stephendiehl.com/hask/tutorial.pdf>

Part XII

Giving back

λειτ <- λαός *Laos* the people ουργός <- ἔργο *ergon* work λειτουργία *leitourgia* public work

Moral value of people developed from the community to give back, improving the community.

The life is beautiful. For all humans that make the life have more magic.

This study and work would not be possible without the community: teachers, mathematicians, Haskellers, scientists, creators, contributors. These sides of people are fascinating.

Special accolades for the guys at Serokell. They were the force that got me inspired & gave resources to seriously learn Haskell and create this pocket guide.