

Fundamental Haskell notes

Haskell handbook encyclopedia for learning and undersatanding fundamentals.

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Contents

Part I

Introduction

“Employ your time in improving yourself by other men’s writings so that you shall come easily by what others have labored hard for.”
(Socrates by Plato)

Important notes on Haskell, [category](#) theory & related fields, terms and recommendations.

Resources:

- Web book: <https://blog.latukha.com/haskell-notes>
- GitHub: <https://github.com/Anton-Latukha/haskell-notes>
- GitLab: <https://gitlab.com/Anton.Latukha/haskell-notes>
- View PDF: <https://github.com/Anton-Latukha/haskell-notes/blob/master/README.pdf>
- Download PDF: <https://github.com/Anton-Latukha/haskell-notes/raw/master/README.pdf>
- L^AT_EX: <https://github.com/Anton-Latukha/haskell-notes/raw/master/README.tex>

This is a complex Org markup file with L^AT_EX formulas.
GitHub & GitLab only partially parse Org into HTML.

To get the full view:

- Outline navigation
- L^AT_EX formulas:
$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(\vec{r}, t)\right]\Psi(\vec{r}, t) = i\hbar\frac{\partial}{\partial t}\Psi(\vec{r}, t), \quad \sum_{k,j}\left[-\frac{\hbar^2}{\sqrt{a}}\frac{\partial}{\partial q^k}\left(\sqrt{a}a^{kj}\frac{\partial}{\partial q^j}\right) + V\right]\Psi + \frac{\hbar}{i}\frac{\partial\Psi}{\partial t} = 0$$
- [Interlinks](#): Interlinks

, please refer to Org mode capable viewer/editor, or to the web book.

Note about markup: «<This is a radio target>>» - for dynamic org-mode linking.

To prettify radio targets in Emacs with Elisp snippet to prettify «<Radio targets>>» to Radio targets:

```
;;; 2019-06-12: NOTE: Prettify '<<Radio targets>>'' to be shown as '_Radio_targets_' when `org-
;;; This is improvement of the code from: Tobias@glmorous: https://emacs.stackexchange.com/quest
;;; There exists library created from the sample: https://github.com/talwrii/org-hide-targets
(defcustom org-hidden-links-additional-re "\\(<<\\>\\)[[:print:]]+?\\(>>\\>\\)"
  "Regular expression that matches strings where the invisible-property of the sub-matches 1 and
:type '(choice (const :tag "Off" nil) regexp)
:group 'org-link)
```

```
(make-variable-buffer-local 'org-hidden-links-additional-re)

(defun org-activate-hidden-links-additional (limit)
  "Put invisible-property org-link on strings matching `org-hide-links-additional-re'."
  (if org-hidden-links-additional-re
      (re-search-forward org-hidden-links-additional-re limit t)
      (goto-char limit)
      nil))

(defun org-hidden-links-hook-function ()
  "Add rule for `org-activate-hidden-links-additional' to `org-font-lock-extra-keywords'.
  You can include this function in `org-font-lock-set-keywords-hook'."
  (add-to-list 'org-font-lock-extra-keywords
    '(org-activate-hidden-links-additional
      (1 '(face org-target invisible org-link))
      (2 '(face org-target invisible org-link)))))

(add-hook 'org-font-lock-set-keywords-hook #'org-hidden-links-hook-function)

SCHT: and metadata in :properties: - of my org-drill practices, please just run org-drill-strip-all-data.
```


Part II

Definitions

Chapter 1

Algebra

al-jabr - assemble parts.

A system of parts based on given axioms ([properties](#)).

- a. [Abstract algebra](#) - the study of number systems and operations within them.
- b. [Algebra](#) - vector space over a field with a multiplication.
- c. [Algebra](#) - a [set](#) with its [algebraic structure](#).

1.1 *

Algebras

1.2 Algebraic

Composite from simple parts.

Also: [Algebraic data type](#).

1.3 Algebraic structure

[Algebraic structure](#) on a [set](#) (called carrier [set](#) or underlying [set](#)) is a collection of finitary operations on that [set](#).

The [set](#) with this [structure](#) is also called an [algebra](#).

[Algebraic structures](#) include [groups](#), [rings](#), fields, and lattices. More complex [structures](#) can be defined by introducing multiple operations, different underlying [sets](#), or by altering the defining axioms. Examples of more complex [algebraic structures](#) include vector spaces, modules, and [algebras](#).

Table 1.1: Algebraic structures

	Closure	Associativity	Identity	Invertability	Commutativity
Semigroupoid		✓			
Small Category		✓	✓		
Groupoid		✓	✓	✓	
Magma	✓				
Quasigroup	✓			✓	
Loop	✓		✓	✓	
Semigroup	✓	✓			
Inverse Semigroup	✓	✓		✓	
Monoid	✓	✓	✓		
Group	✓	✓	✓	✓	
Abelian group	✓	✓	✓	✓	✓
Ring	✓	✓	✓	✓	under +

1.3.1 *

Algebraic structures

1.3.2 Fundamental theorem of algebra

Any non-constant single-variable polynomial with complex coefficients has at least one complex root.

From this definition follows property that the field of complex numbers is algebraically closed.

1.4 Modular arithmetic

System for integers where numbers wrap around the certain values (single - modulus, plural - moduli).

Example - 12-hour clock.

1.4.1 *

Clock arithmetic

1.4.2 Modulus

Special numbers where arithmetic wraps around in modular arithmetic.

1.4.2.1 *

Moduli - plural.

Chapter 2

Bind

Establishing equality between two [objects](#).

Most often:

- equating [variable](#) to a value.
- equating [parameter](#) of a [function](#) to an [argument](#) ([variable](#)/value/[function](#)). This term often is equated to [applying argument](#) to a [function](#), which includes [\$\beta\$ -reduction](#).

2.1 *

Binds
Binding
Bindings

Chapter 3

Category theory

Category \mathcal{C} consists of the **basis**:

Primitives:

- a. **Objects** - $a^{\mathcal{C}}$. A **node**. **Object** of some **type**. Often **sets**, than it is **Set category**.
- b. **Arrows** - $(a, b)^{\mathcal{C}}$ (AKA **morphisms** mappings).
- c. **Arrow (morphism) composition** - **binary operation**: $(a, b)^{\mathcal{C}} \circ (b, c)^{\mathcal{C}} \equiv (a, c)^{\mathcal{C}} \mid \forall a, b, c \in \mathcal{C}$.
AKA principle of **compositionality** for **arrows**.

Properties (or axioms):

- a. **Associativity** of **morphisms**: $h \circ (g \circ f) \equiv (h \circ g) \circ f \mid f_{a \rightarrow b}, g_{b \rightarrow c}, h_{c \rightarrow d}$.
- b. Every **object** has (two-sided) **identity morphism** (& in fact - exactly one): $1_x \circ f_{a \rightarrow x} \equiv f_{a \rightarrow x}, g_{x \rightarrow b} \circ 1_x \equiv g_{x \rightarrow b} \mid \forall x \exists 1_x, \forall f_{a \rightarrow x}, \forall g_{x \rightarrow b}$.
- c. Principle of **compositionality**.

From these axioms, can be proven that there is exactly one **identity morphism** for every **object**.

Object and **morphism** are complete **abstractions** for anything.

In majority of cases under **object** is a state and **morphism** is a change.

3.1 *

Category
Categories

3.2 Abelian category

Generalised [category](#) for homological [algebra](#) (having a possibility of basic constructions and techniques for it).

[Category](#) which:

- has a [zero object](#),
- has all [binary](#) biproducts,
- has all [kernel](#)'s and cokernels,
- (it has all [pullbacks](#) and pushouts)
- all [monomorphism](#)'s and [epimorphism](#)'s are normal.

[Abelian category](#) is very stable; for example they are regular and they satisfy the snake lemma. The class of [Abelian categories](#) is [closed](#) under several categorical constructions.

There is notion of [Abelian monoid](#) (AKS [Commutative monoid](#)) and [Abelian group](#) ([Commutative group](#)).

Basic examples of $*$:

- [category](#) of Abelian [groups](#)
- [category](#) of modules over a [ring](#).

$*$ are widely used in [algebra](#), [algebraic](#) geometry, and topology.

$*$ has many constructions like in [categories](#) of modules:

- kernels
- exact [sequences](#)
- [commutative](#) diagrams

$*$ has disadvantage over [category](#) of modules. [Objects](#) do not necessarily have elements that can be manipulated directly, so traditional definitions do not work. Methods must be supplied that allow definition and manipulation of [objects](#) without the use of elements.

3.2.1 $*$

Abelian categories

3.3 Composition

Axiom of [Category](#).

3.3.1 *

Composable
Compositions

3.4 Endofunctor category

From the name, in this [Category](#):

- [objects](#) of *End* are [Endofunctors](#) $E^{C \rightarrow C}$
- [morphisms](#) are [natural transformations](#) between [endofunctors](#)

3.5 Functor

* full translation (map) of one [category](#) into another.

Translating [objects](#) and [morphisms](#) (as input can take [morphism](#) or [object](#)).

* - [forgetful](#) - discards part of the [structure](#).

* - faithful - fully preserves all [morphisms](#) - [injective](#) on Hom-sets.

* - full - translation of [morphisms](#) fully covers all the [morphisms](#) between according objects in the target category.

For [Functor type class](#) or [fmap](#) - see [Power set functor](#).

[Functor properties](#) (axioms):

- $F^{C \rightarrow D}(a) \mid \forall a^C$ - every source [object](#) is mapped to [object](#) in target [category](#)
- $\overrightarrow{(F^{C \rightarrow D}(a), F^{C \rightarrow D}(b))}^D \mid \forall \overrightarrow{(a, b)}^C$ - every source [morphism](#) is mapped to target [category morphism](#) between corresponding [objects](#)
- $F^{C \rightarrow D}(\overrightarrow{g}^C \circ \overrightarrow{f}^C) = F^{C \rightarrow D}(\overrightarrow{g}^C) \circ F^{C \rightarrow D}(\overrightarrow{f}^C) \mid \forall y = \overrightarrow{f}^C(x), \forall \overrightarrow{g}^C(y)$ - [composition](#) of [morphisms](#) translates directly (tautologically goes from other two)

These axioms guarantee that [composition](#) of [functors](#) can be fused into one [functor](#) with [composition](#) of [morphisms](#). This process called [fusion](#).

In Haskell this axioms have form:

```
fmap id = id
fmap (f . g) = fmap f . fmap g
```

Since `*` is 1-1 mapping of initial [objects](#) - it is a memoizable dictionary with [cardinality](#) of initial [objects](#). Also in [Hask category](#) [functors](#) are obviously [endofunctors](#) \therefore they are special [kinds](#) of containers for the parametric values (AKA [product type](#)). In Haskell [product type](#) `*` are [endo-functors](#) from [polymorphic type](#) into a [functor](#) wrapper of a [polymorphic type](#).

`*` translates in one direction, and does not provide algorithm of reversing itself or retrieving the parametric value.

3.5.1 `*`

Functors

3.5.2 Power set functor

$\mathcal{P}^{S \rightarrow \mathcal{P}(S)}$

`*` - [functor](#) from [set](#) S to its [power set](#) $\mathcal{P}(S)$.

[Functor type class](#) in Haskell defines a `*` and allows to do [function application](#) inside [type structure](#) layers (denoted f or m). [IO](#) is also such [structure](#).

[Power set](#) is unique to the [set](#), `*` is unique to the [category](#) ([data type](#)).

`*` embodies in itself any [endofunctor](#). It is easily seen from Haskell definition - that the `*` is the [polymorphic](#) generalization over any [endofunctor](#) in a [category](#). [Application](#) of a [function](#) to `*` gives a particular [endofunctor](#) (see [Hask category](#)).

```
class Functor f where
  fmap :: (a -> b) -> f a -> f b
```

[Functor](#) instance must be of [kind](#) $(* \rightarrow *)$, so instance for [higher-kinded data type](#) must be [applied](#) until this [kind](#).

[Composed](#) `*` can [lift functions](#) through any layers of [structures](#) that belong to [Functor type class](#).

`*` can be used to filter-out [error](#) cases ([Nothing](#) & Left cases) in [Maybe](#), [Either](#) and related [types](#).

3.5.2.1 `*`

fmap
Functor type class

3.5.2.2 Power set functor laws

[Type](#) instance of [functor](#) should abide this laws:

a. `*`

Functor laws

b. Power set functor identity law

```
fmap id == id
```


c. Power set functor composition law

```
fmap (f.g) == fmap f . fmap g
```

In words, it is if several **functions** are **composed** and then **fmap** is **applied** on them - it should be the same as if **functions** was **fmap**ped and then **composed**.

3.5.2.3 Lift

```
fmap :: (a -> b) -> (f a -> f b)
```

Functor takes **function** $a \rightarrow b$ and returns a **function** $f\ a \rightarrow f\ b$ this is called **lifting** a **function**. **Lift** does a **function application** through the **data structure**.

a. *

Lifting

3.5.2.4 Power set functor is a free monad

Since:

- $\forall e \in S : \exists \{e\} \in \mathcal{P}(S) \models \forall e \in S : \exists (e \rightarrow \{e\}) \equiv unit$
- $\forall \mathcal{P}(S) : \mathcal{P}(S) \in \mathcal{P}(S) \models \forall \mathcal{P}(S) : \exists (\mathcal{P}(\mathcal{P}(S)) \rightarrow \mathcal{P}(S)) \equiv join$

3.5.3 Functorial

Corresponds to **functor laws**.

3.5.4 Forgetful functor

Functor that forgets part or all of what defines **structure** in **domain category**. $F^{Grp \rightarrow Set}$ that translates **groups** into their underlying **sets**. **Constant functor** is another example.

3.5.4.1 *

Forgetful

3.5.5 Identity functor

Maps all **category** to itself. All **objects** and **morphisms** to themselves.

Denotation:
 $1^{C \rightarrow C}$

3.5.6 Endofunctor

Is a [functor](#) which source ([domain](#)) and target ([codomain](#)) are the same [category](#).

$$F^{C \rightarrow C}, E^{C \rightarrow C}$$

3.5.6.1 *

Endofunctors

3.5.7 Applicative functor

* - Computer science term. [Category](#) theory name - [lax monoidal functor](#). And in [category](#) *Set*, and so in [category](#) *Hask* all [applicatives](#) and [monads](#) are strong (have [tensorial strength](#)).

* - [sequences functorial](#) computations (plain [functors](#) can't).

```
(<*>) :: f (a -> b) -> f a -> f b
```

Requires [Functor](#) to exist.

Requires [Monoidal structure](#).

Has [monoidal structure](#) rules, separated form [function application](#) inside [structure](#).

[Data type](#) can have several [applicative](#) implementations.

Standard definition:

```
class Functor f => Applicative f
  where
    (<*>) :: f (a -> b) -> f a -> f b
    pure  :: a -> f a
```

`pure` - if a [functor](#), [identity Kleisli arrow](#), [natural transformation](#).

[Composition](#) of `*` always produces `*`, contrary to [monad](#) ([monads](#) are not [closed](#) under [composition](#)).

`Control.Monad` has an old [function](#) `ap` that is old implementation of `<*>`:

```
ap :: Monad m => m (a -> b) -> m a -> m b
```

3.5.7.1 *

[Applicative](#)

[Applicatives](#)

[Applicative functors](#)

3.5.7.2 Applicative law**3.5.7.3 ***

Applicative laws

- a. Applicative identity law

```
pure id <*> v = v
```

- b. Applicative composition law [Function composition](#) works regularly.

```
pure (.) <*> u <*> v <*> w = u <*> (v <*> w)
```

- c. Applicative homomorphism law Internal [function application](#) doesn't change the [structure](#) around values.

```
pure f <*> pure x = pure (f x)
```

- d. Applicative interchange law On condition that internal [order](#) of [evaluation](#) is preserved - [order](#) of operands is not relevant.

```
u <*> pure y = pure ($ y) <*> u
```

3.5.7.4 Applicative function

- a. liftA*

- a. liftA Essentially a [fmap](#).

```
:type liftA
liftA :: Applicative f => (a -> b) -> f a -> f b
```

Lifts [function](#) into [applicative function](#).

- b. liftA2 Lifts [binary function](#) across two [Applicative functors](#).

```
liftA2 :: Applicative f => (a -> b -> c) -> f a -> f b -> f c
```

```
liftA2 f x y == pure f <*> x <*> y
```

- c. «<liftA2 (<*>)»> liftA2 (<*>) is pretty useful. It can [lift binary operation](#) through the two layers:
It is two-layer [Applicative](#).

```
liftA2 :: Applicative f => ( a -> b -> c ) -> f a -> f b -> f c
<*> :: Applicative f => (f (a -> b) -> f a -> f b)
liftA2 (<*>) :: (Applicative f1, Applicative f2) => f1 (f2 (a -> b)) -> f1 (f2 a) -> f1 (f2 b)
```

- d. «<liftA2 (liftA2 (<*>))»> liftA2 (<*>) 3-layer version.

- e. liftA3 liftA2 3-parameter version.

```
liftA3 f x y z == pure f <*> x <*> y <*> z
```

- b. Conditional [applicative](#) computations

```
when :: Applicative f => Bool -> f () -> f ()
```

Only when `True` - perform an `applicative` computation.

```
unless :: Applicative f => Bool -> f () -> f ()
```

Only when `False` - perform an `applicative` computation.

3.5.7.5 Special applicatives

a. Identity applicative

```
-- Applicative f =>
-- f ~ Identity
type Id = Identity
instance Applicative Id
  where
    pure :: a -> Id a
    (<*>) :: Id (a -> b) -> Id a -> Id b

mkId = Identity
xs = [1, 2, 3]

const <$> mkId xs <*> mkId xs'
-- [1,2,3]
```

b. Constant applicative It holds only to one value. The `function` does not exist and last `parameter` is a phantom.

```
-- Applicative f =>
-- f ~ Constant e
type C = Constant
instance Applicative C
  where
    pure :: a -> C e a
    (<*>) :: C e (a -> b) -> C e a -> C e b
```

c. Maybe applicative "There also can be no `function` at all."

If `function` might not exist - embed `f` in `Maybe` structure, and use `Maybe` applicative.

```
-- f ~ Maybe
type M = Maybe
pure :: a -> M a
(<*>) :: M (a -> b) -> M a -> M b
```

d. Either applicative `pure` is `Right`.

Defaults to `Left`.

And if there is two `Left`'s - to `Left` of the first `argument`.

e. Validation applicative The Validation `data type` isomorphic to `Either`, but has accumulative `Applicative` on the `Left` side.

Validation `data type` is not a `monad`. Validation is an example of, "An `applicative functor`

that is not a `monad`."

While `Either monad` on `Left case` just drops computation and returns this first `Left`.

`Monad` needs to process the result of computation - it requires to be able to process all `Left error statement` cases for `Validation`, it is or non-terminating `Monad` or one which is impossible to implement in `polymorphic` way with `Validation`.

3.5.7.6 Monad

$\mu \nu \sigma$ *monos* sole
 $\mu \nu \delta \alpha$ *monáda* `unit`

* - `monoid` in `endofunctor category` with η (`unit`) and μ (`join`) `natural transformations`.

`Monad` on \mathcal{C} is $\{E^{\mathcal{C} \rightarrow \mathcal{C}}, \eta, \mu\}$:

- $E^{\mathcal{C} \rightarrow \mathcal{C}}$ - is an `endofunctor`
- two `natural transformations`, $1^{\mathcal{C}} \rightarrow E$ and $E \circ E \rightarrow E$:
 - $\eta^{1^{\mathcal{C}} \rightarrow E} = \text{unit}^{Identity \rightarrow E}(x) = f^{x \rightarrow E(x)}(x)$
 - $\mu^{(E \circ E) \rightarrow E} = \text{join}^{(E \circ E) \rightarrow (Identity \circ E)}(x) = |y = E(x)| = f^{E(y) \rightarrow y}(y)$

where:

- \mathcal{C} is a `category`
- $1^{\mathcal{C}}$ denotes the \mathcal{C} `identity functor`
- $(E \circ E)$ - `endofunctor` $\mathcal{C} \rightarrow \mathcal{C}$

Definition with $\{E^{\mathcal{C} \rightarrow \mathcal{C}}, \eta, \mu\}$ (in `Hask`: $(\{e :: f a \rightarrow f b, \text{pure}, \text{join}\})$) - is classic categorical, in Haskell minimal complete definition is $\{\text{fmap}, \text{pure}, (>>=)\}$.

If there is a `structure` S , and a way of taking `object` x into S and a way of collapsing $S \circ S$ - there probably a `monad`.

Mostly `monads` used for sequencing actions (computations) (that looks like imperative programming), with ability to depend on previous chains. Note if `monad` is `commutative` - it does not `order` actions.

`Monad` can shorten/terminate `sequence` of computations. It is implemented inside `Monad` instance. For example `Maybe monad` on `Nothing` drops chain of computation and returns `Nothing`.

* inherits the `Applicative` instance methods:

```
import Control.Monad (ap)
return == pure
ap == (<*>) -- + Monad requirement
```

Table 3.1: [Monad](#) in mathematics and Haskell

Math	Meaning	Cat/Fctr	$X \in C$	Type	Haskell
Id	endofunctor "Id"	$C \rightarrow C$	$X \rightarrow Id(X)$	$a \rightarrow a$	id
E	endofunctor "monad"	$C \rightarrow C$	$X \rightarrow E(X)$	$m\ a \rightarrow m\ b$	fmap
η	natural transformation "unit"	$Id \rightarrow E$	$Id(X) \rightarrow E(X)$	$a \rightarrow m\ a$	pure
μ	natural transformation "multiplication"	$E \circ E \rightarrow E$	$E(E(X)) \rightarrow E(X)$	$m\ (m\ a) \rightarrow m\ a$	join

Internals of [Monad](#) are Haskell [data types](#), and as such - they can be consumed any number of times.

[Composition](#) of [monadic types](#) does not always results in [monadic type](#).

a. *

Monads
Monadic

b. Monad law [Monad](#) corresponds to [functor laws](#) & [applicative laws](#) and additionally:

a. * Monad laws

b. Monad left identity law

```
pure x >>= f == f x
```

Explanation:

```
>>= :: Monad f => f a -> (a -> f b) -> f b
      pure x >>= f == f x
```

Rule that `>>=` must get first [argument structure](#) internals and [apply](#) to the [function](#) that is the second [argument](#).

c. Monad right identity law

```
f >>= pure == f
```

Explanation:

```
>>= :: Monad f => f a -> (a -> f b) -> f b
      f >>= pure == f
```

AKA it is a [tacit](#) description of a [monad bind](#) as [endofunctor](#).

d. Monad associativity law

```
(m >>= f) >>= g == m >>= (\ x -> f x >>= g)
```

c. Monad type class

```
class Applicative m => Monad m where
  (>>=) :: m a -> (a -> m b) -> m b
  (>>) :: m a -> m b -> m b
  return :: a -> m a
```

- a. MonadPlus type class is a [monoid](#) over [monad](#), with additional rules. The precise [set](#) of rules ([properties](#)) not agreed upon. Class instances obey [monoid](#) & [left zero](#) rules, some additionally obey [left catch](#) and others [left distribution](#).

Overall there * currently reforms ([MonadPlus](#) reform proposal) in several smaller nad strictly defined [type classes](#).

Subclass of an [Alternative](#).

- a. *
- Monadplus

- d. [Functor](#) -> [Applicative](#) -> [Monad](#) progression

```
<$> :: Functor    f =>    (a -> b)    -> f a -> f b
<*> :: Applicative f => f (a -> b)    -> f a -> f b
=<< :: Monad      f =>    (a -> f b) -> f a -> f b
```

pure & join are [Natural transformations](#) for the `fmap`.

- e. Monad function

- a. Return function

```
return == pure
```

Nonstrict.

- b. Join function

```
join :: Monad m => m (m a) -> m a
```

Generates knowledge of `concat`.

[Kleisli composition](#) that flattens two layers of [structure](#) into one.

The way to express ordering in [lambda calculus](#) is to nest.

- a. *
- join

- b. `join . fmap == (=«)`

```
-- b = f b
```

```
fmap      :: Monad f => (a -> f b) -> f a -> f (f b)
join      :: Monad f =>                      f (f a) -> f a
join . fmap :: Monad f => (a -> f b) -> f a          -> f b
flip      >>= :: Monad f => (a -> f b) -> f a          -> f b
```

- c. Bind function

```
>>=      :: Monad f => f a -> (a -> f b) -> f b
join . fmap :: Monad f => (a -> f b) -> f a -> f b
```

Nonstrict.

The most ubiquitous way to `>>=` something is to use [Lambda function](#):

```
getLine >>= \name -> putStrLn "age pls:"
```

Also very neat way is to bundle and handle `Monad` - is to bundle it with `bind`, and leave `applied` partially.

And use that partial bundle as a `function` - every `evaluation` of the `function` would trigger `evaluation` of internal `Monad structure`. Thumbs up.

```
printOneOf :: Bool -> IO ()
printOneOf False = putStr "1"
printOneOf True = putStr "2"
```

```
quant [U+2237] (Bool -> IO b) -> IO b
quant = (>>=) (randomRIO (False, True))
```

```
recursePrintOneOf [U+2237] Monad m -> (t -> m a) -> t -> m b
recursePrintOneOf f x = (f x) >> (recursePrintOneOf f x)
```

```
main :: IO ()
main = recursePrintOneOf (quant) $ printOneOf
```

- a. *
 - Monadic extend
 - Monadic bind
 - Monad bind
 - Binder
 - Binder function

- a. (\gg)
- b. $\gg=$
- c. ($=\ll$)
- d. $=\ll$

- d. Sequencing operator (\gg) == ($*\gg$): Discard any resulting value of the action and `sequence` next action.

`Applicative` has a similar `operator`.

```
(>>) :: m a -> m b -> m b
(*>) :: f a -> f b -> f b
```

- e. `Monadic` versions of `list functions`

```
sequence :: (Traversable t, Monad m) => t (m a) -> m (t a)
```

`Sequence` gets the traversable of `monadic` computations and swaps it into `monad` computation of traverse. In the result the collection of `monadic` computations turns into one long `monadic` computation on traverse of data.

If some step of this long computation fails - `monad` fails.

```
mapM :: (Traversable t, Monad m) => (a -> m b) -> t a -> m (t b)
```

`mapM` gets the `AMB function`, then takes traversable data. Then applies `AMB function` to traversable data, and returns converted `monadic` traversable data.

```
foldM :: (Foldable t, Monad m) => (b -> a -> m b) -> b -> t a -> m b
foldl :: Foldable t => (b -> a -> b) -> b -> t a -> b
```


* is a **monadic foldl**.

b is initial cumulative value, **m b** is a cumulative bank.
Right folding achieved by reversing the input **list**.

```
filterM :: Applicative m => (a -> m Bool) -> [a] -> m [a]
filter  ::                (a -> Bool) -> [a] -> [a]
```

Take Boolean **monadic** computation, filter the **list** by it.

```
zipWithM :: Applicative m => (a -> b -> m c) -> [a] -> [b] -> m [c]
zipWith  ::                (a -> b -> c) -> [a] -> [b] -> [c]
```

Take **monadic** combine **function** and combine two lists with it.

```
msum :: (Foldable t, MonadPlus m) => t (m a) -> m a
sum   :: (Foldable t, Num a)       => t a -> a
```

f. liftM*

a. liftM Essentially a **fmap**.

```
liftM :: Monad m => (a -> b) -> m a -> m b
```

Lifts a **function** into **monadic equivalent**.

b. liftM2 **Monadic liftA2**.

```
liftM2 :: Monad m => (a -> b -> c) -> m a -> m a -> m c
```

Lifts **binary function** into **monadic equivalent**.

f. Comonad **Category C** **comonad** is a **monad** of **opposite category** \mathcal{C}^{op} .

g. Kleisli arrow **Morphism** that while doing computation also adds **monadic-able structure**.

a -> m b

a. *

Kleisli arrows
Kleisli morphism
Kleisli morphisms

h. Kleisli composition **Composition** of **Kleisli arrows**.

```
(<=<) :: Monad m => (b -> m c) -> (a -> m b) -> a -> m c infixr 1
;; compare
(.)   ::                (b -> c) -> (a -> b) -> a -> c
```

Often used left-to-right version:

```
(>=>) :: Monad m => (a -> m b) -> (b -> m c) -> a -> m c
;; compare
(>>=) :: Monad m => m a -> (a -> m b) -> m b
```

Which allows to replace `monadic bind` chain with `Kleisli composition`.

```
f1 arg >>= f2 >>= f3
==
f1 >=> f2 >=> f3 $ arg
==
f3 <=< f2 <=< f1 $ arg
```

- i. Kleisli category `Category C`, `[U+2329]E, $\overrightarrow{\eta}, \overrightarrow{\mu}$ [U+232A] monad` over `C`.

Kleisli category `CT` of `C`:

$$\text{Obj}(\mathcal{C}_T) = \text{Obj}(\mathcal{C})$$

$$\text{Hom}_{\mathcal{C}_T}(x, y) = \text{Hom}_{\mathcal{C}}(x, E(y))$$

- j. Special monad

- a. Identity monad Wraps data in the `Identity constructor`.

Useful: Creates `monads` from `monad transformers`.

`Bind`: Applies internal value to the `bound function`.

Code:

```
newtype Identity a = Identity { runIdentity :: a }

-- coerce is a function that directly moves data between type aliases
instance Functor Identity where
    fmap      = coerce

instance Applicative Identity where
    pure      = Identity
    (<*>)     = coerce

instance Monad Identity where
    m >>= k   = k (runIdentity m)
```

Example:

```
-- derive the State monad using the StateT monad transformer
type State s a = StateT s Identity a
```

- b. Maybe monad Something that may not be or not return a result. Any lookups into the real world, database queries.

`Bind`: `Nothing` input gives `Nothing` output, `Just x` input uses `x` as input to the `bound function`.

When some computation results in `Nothing` - drops the chain of computations and returns `Nothing`.

Zero: `Nothing`

Plus: result in first occurrence of `Just` else `Nothing`.

Code:

```
data Maybe a = Nothing | Just a
```

```
instance Monad Maybe where
  return      = Just
  fail        = Nothing
  Nothing >=> _ = Nothing
  (Just x) >=> f = f x
```

```
instance MonadPlus Maybe where
  mzero      = Nothing
  Nothing `mplus` x = x
  x `mplus` _      = x
```

Example:

Given 3 dictionaries:

- a. Full names to email addresses,
- b. Nicknames to email addresses,
- c. Email addresses to email preferences.

Create a `function` that finds a person's email preferences based on `either` a full name or a nickname.

```
data MailPref = HTML | Plain
data MailSystem = ...
```

```
getMailPrefs :: MailSystem -> String -> Maybe MailPref
getMailPrefs sys name =
  do let nameDB = fullNameDB sys
       nickDB   = nickNameDB sys
       prefDB   = prefsDB sys
       addr <- (lookup name nameDB) `mplus` (lookup name nickDB)
       lookup addr prefDB
```

- c. Either monad When computation results in `Left` - drops other computations & returns the received `Left`.
- d. Error monad Something that can fail, throw `exceptions`.

The failure process records the description of a failure. `Bind function` uses successful values as input to the `bound function`, and passes failure information on without executing the `bound function`.

Useful:

Composing `functions` that can fail. Handle `exceptions`, create `error handling structure`.

Zero: empty `error`.

Plus: if first `argument` failed then execute second `argument`.

- e. List monad Computations which may return 0 or more possible results.

Bind: The `bound function` is `applied` to all possible values in the input `list` and the resulting lists are concatenated into `list` of all possible results.

Useful: Building computations from `sequences` of non-deterministic operations.

Zero: `[]`

Plus: `(++)`

a. `*`
`[] monad`

- f. Reader monad Creates a read-only shared environment for computations.

The `pure function` ignores the environment, while `»=` passes the inherited environment to both subcomputations.

Today it is defined though `ReaderT` transformer:

```
type Reader r = ReaderT r Identity    -- equivalent to ((->) e), (e ->)
```

Old definition was:

```
newtype Reader e a = Reader { runReader :: (e -> a) }
```

For `(e ->)`:

- `Functor` is `(.)`

```
fmap :: (b -> c) -> (a -> b) -> a -> c
```

```
fmap = (.)
```

- `Applicative`:

- `pure` is `const`

```
pure :: a -> b -> a
```

```
pure x _ = x
```

- `(<*>)` is:

```
(<*>) :: (a -> b -> c) -> (a -> b) -> a -> c
```

```
(<*>) f g = \a -> f a (g a)
```

- `Monad`:

```
(>>=) :: (a -> b) -> (b -> a -> c) -> a -> c
```

```
(>>=) m k = Reader $ \r ->
```

```
runReader (k (runReader m r)) r
```

```
join :: (e -> e -> a) -> e -> a
```

```
join f x = f x x
```

```

runReader
  :: Reader r a -- the Reader to run
  -> r -- an initial environment
  -> a -- extracted final value

Usage:

data Env = ...

createEnv :: IO Env
createEnv = ...

f :: Reader Env a
f = do
  a <- g
  pure a

g :: Reader Env a
g = do
  env <- ask -- "Open the environment namespace into env"
  a <- h env -- give env to h
  pure a

h :: Env -> a
... -- use env and produce the result

main :: IO ()
main = do
  env <- createEnv
  a = runReader g env
  ...

```

In Haskell under normal circumstances impure [functions](#) should not directly call impure [functions](#).

`h` is an impure [function](#), and `createEnv` is impure [function](#), so they should have intermediary.

- g.* Writer monad Computations which accumulate [monoid](#) data to a shared Haskell storage.

So `*` is parametrized by [monoidal type](#).

Accumulator is maintained separately from the returned values.

Shared value modified through [Writer monad](#) methods.

`*` frees creator and code from manually keeping the track of accumulation.

Bind: The [bound function](#) is [applied](#) to the input value, [bound function](#) allowed to `<>` to the accumulator.

```
type Writer r = WriterT r Identity
```

Example:

```

f :: Monoid b => a -> (a, b)
f a = if _condition_

```

```

        then runWriter $ g a
      else runWriter do
        a1 <- h a
        pure a1

g :: Monoid b => Writer b a
g a = do
  tell _value1_ -- accumulator <> _value1_
  pure a -- observe that accumulator stored inside monad and only a main value needs to

h :: Monoid b => Writer b a
h a = do
  tell _value2_ -- accumulator <> _value_
  pure a

runWriter :: Writer w a -> (a, w) -- Unwrap a writer computation as a (result, accumulator)
-- The inverse of writer.

```

`WriterT`, `Writer` unnecessarily keeps the entire logs in the memory. Use `fast-logger` for logging.

h. State monad Computations that pass-over a state.

The [bound function](#) is [applied](#) to the input value to produce a state transition [function](#) which is [applied](#) to the input state.

Pure functional language cannot update values in place because it violates [referential transparency](#).

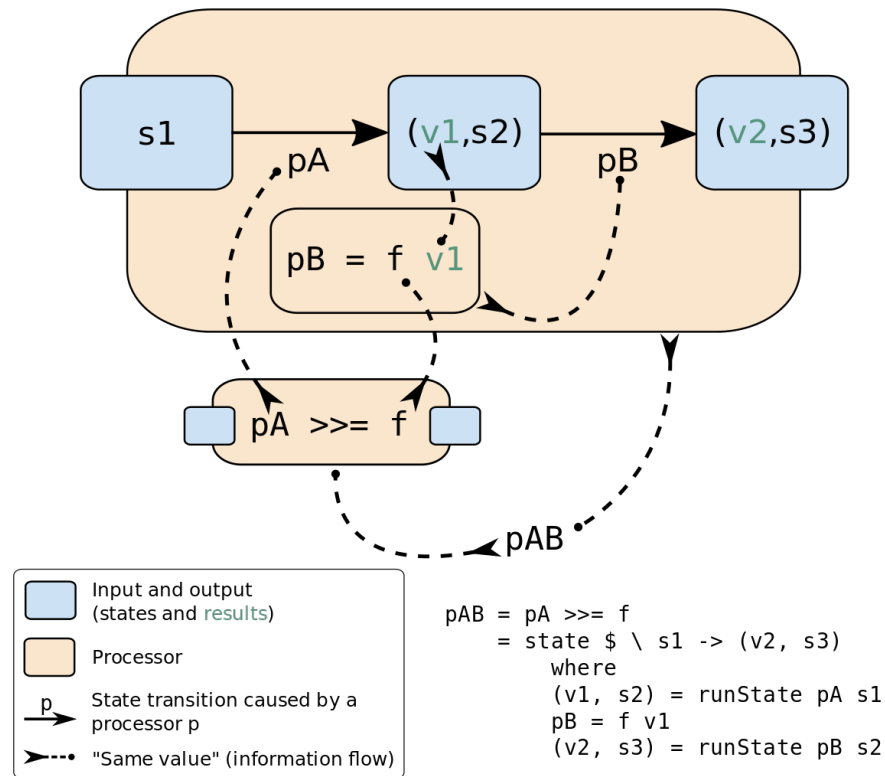
```
type State s = StateT s Identity
```

[Binding](#) copies and transforms the state [parameter](#) through the [sequence](#) of the [bound functions](#) so that the same state storage is never used twice. Overall this gives the illusion of in-place update to the programmer and in the code, while in fact the auto-generated transition [functions](#) handle the state changes.

Example [type](#): `State st a`

`State` describes [functions](#) that consume a state and produce a [tuple](#) of result and an updated state.

[Monad](#) manages the state with the next process:



Where:

- f - processor making function
- pA , pAB , pB - state processors
- sN - states
- vN - values

Bind with a processor making function from state processor (pA) creates a new state processor (pAB).

The wrapping and unwrapping by `State/runState` is implicit.

- k . Monad transformer `*` is a practical solution to the current functional programming problem about composition of monads.

Monad is not closed under composition.

Composition of monadic types does not always results in monadic type.

Basic case: during implementation of monadic type composition, type $m \rightarrow T \rightarrow m \rightarrow a$ arises, which does not allow to `unit`, `join` the m monadic layers.

`*` have desirable properties and can add them to monads. `*` user their implementation to solve the composition type layering and allow to attach desirable property to result.

* solve [monad composition](#) and [type](#) layering by cheating, using own [structure](#) and information about itself. It is often that process involves a [catamorphism](#) of a * [type](#) layer.

In [type](#) signatures of transformers `*T m -> m` is already an extended [monad](#), so `*T` is just a wrapper to point that out.

Transformers have a light wrapper around the data that tags the modification with this transformer.

Main [monadic structure](#) `m` is wrapped around the internal data (core is `a`). The [structure](#) that corresponds to the transformer creation [properties](#) (if it emitted by η of a transformer), goes into `m`. Open [parameters](#) go external to the `m`.

```
newtype ExceptT e m a =
  ExceptT { runExceptT :: m (Either e a) }

newtype MaybeT m a =
  MaybeT { runMaybeT :: m (Maybe a) }

newtype ReaderT r m a =
  ReaderT { runReaderT :: r -> m a }
```

This has an [effect](#) that on stacking [monad](#) transformers, `m` becomes [monad stack](#), and every next transformer injects the transformer creation-specific properties η inside the [stack](#), so out-most transformer has inner-most [structure](#). Base [monad](#) is structurally the outermost.

- a. `MaybeT *` extends [monads](#) by injecting [Maybe](#) layer underneath [monad](#), and processing that [structure](#):

```
newtype MaybeT m a = MaybeT { runMaybeT :: m (Maybe a) }
```

- b. `EitherT *` extends [monads](#) by injecting [Either](#) layer underneath [monad](#), and processing that [structure](#):

```
newtype EitherT e m a = EitherT { runEitherT [U+2237] m (Either e a) }
```

`EitherT` of `either` package gets replaced by `ExceptT` of `transformers` or `mtl` packages.

- a. `* ExceptT`

- c. `ReaderT` Definition:

```
newtype ReaderT r m a = ReaderT { runReaderT :: r -> m a }
* functions: input monad m a, out: m a wrapped it in a free-variable r (partially applied function).
```

That allows to use transformed `m a`, now it requires and can use the `r` passed environment.

To create a [Reader monad](#):

```
type Reader r = ReaderT r Identity
```


- d. `MonadTrans` [type class](#) Allows to [lift monadic](#) actions into a larger [context](#) in a neutral way.

`pure` takes a parametric [type](#) and embodies it into constructed [structure](#) (talking of [monad](#) transformers - [structure](#) of the stacked [monads](#)).

`lift` takes [monad](#) and extends it with a transformer.

In fact, for [monad](#) transformers - `lift` is a last stage of the `pure`, it follows from the [lift](#) law.

Method:

```
lift :: Monad m => m a -> t m a
```

`Lift` a computation from the [argument monad](#) to the constructed [monad](#).

Neutral means:

```
lift . return = return
```

```
lift (m >>= f) = lift m >>= (lift . f)
```

The general pattern with `MonadTrans` instances is that it is usually lifts the [injection](#) of the known [structure](#) of transformer over some `Monad`.

`lift` embeds one [monadic](#) action into [monad transformer](#).

The difference between `pure`, `lift` and `MaybeT` constructor becomes clearer if you look at the [types](#):

Example, for `MaybeT IO a`:

```
pure      ::      a -> MaybeT IO a
lift      ::    IO a -> MaybeT IO a
MaybeT   :: IO (Maybe a) -> MaybeT IO a
```

```
x = (undefined :: IO a)
```

```
:t (pure x)
(pure x) :: Applicative t => t (IO a)  -- t recieves one argument of product type
:t (pure x :: MaybeT IO a)
-- Expected type: MaybeT IO a1
-- Actual type: MaybeT IO (IO a0)
```

```
-- While the real type would be
```

```
:t (pure x :: MaybeT IO (IO a))
(pure x :: MaybeT IO (IO a)) :: MaybeT IO (IO a) -- This goes into a conflict of what ty
```

```
:t (lift x)
(lift x) :: MonadTrans t => t IO a  -- result is a proper expected product type
```

```
-- To belabour
```

```

:t (lift x :: MaybeT IO a)
(lift x) :: MonadTrans t => t IO a -- result is a proper expected product type

```

lift is a [natural transformation](#) η from an [Identity monad](#) ([functor](#)) with other [monad](#) as content into transformer [monad](#) ([functor](#)), with the preservation of the contained [monad](#):

```

-- Abstract monads with content as parameters. Define '~>' as a family of morphisms that
type f ~> g = forall x. f x -> g x
-- follows
lift :: m ~> t m

```

- a. [MonadIO type class](#) $*$ - allows to [lift IO](#) action until reaching the [IO monad](#) layer at the top of the [Monad stack](#) (which is always in the Haskell code that does [IO](#)).

```

class (Monad m) => MonadIO m where
  liftIO :: IO a -> m a

```

liftIO laws:

```

liftIO . pure = pure

```

```

liftIO (m >=> f) = liftIO m >=> (liftIO . f)

```

Which is identical laws to [MonadTrans lift](#).

Since [lift](#) is one step, and [liftIO](#) all steps - all steps defined in terms of one step and all other steps, so the most frequent implementation is self-[recursive lift](#) .

liftIO:

```

liftIO ioa = lift $ liftIO ioa

```

a. $*$

```

liftIO

```

3.5.7.7 Alternative type class

[Monoid](#) over [applicative](#). Has left catch [property](#).

Allows to run simoltaneously several instances of a computation (or computations) and from them yeld one result by law from $\langle|>\rangle :: \text{Type} \rightarrow \text{Type} \rightarrow \text{Type}$.

Minimal complete definition:

```

empty :: f a -- The identity element of <|>
(<|>) :: f a -> f a -> f a -- Associative binary operation

```

a. $*$

Alternative

3.5.7.8 $\langle\langle=* \rangle\rangle$

Do calculation, but ignore the value from the first [argument](#).

$* > \equiv >>$

3.5.8 Monoidal functor

Functors between monoidal categories that preserves monoidal structure.

3.5.9 Fusion

```
fmap f . fmap g = fmap (f . g)
```

* - functor axiom that allows to greatly simplify computations.

3.5.10 «<=\$>=>>

Get & set a value inside Functor.

3.5.11 Multifunctor

Generalizes the concept of functor between categories, canonical morphisms between multicategories.

Works over N type arguments instead of one.

To put simply - accepts multiple arguments, from that information constructs source product category (Cartesian product) of categories, and is a functor from product category to target category.

To put even simpler - functor that takes as an argument the product of types.

In Haskell there is only one category, `Hask`, so in Haskell `*` is still $(Hask \times Hask) \rightarrow Hask \Rightarrow [(Hask \times Hask) \equiv Hask] \Rightarrow Hask \rightarrow Hask$ endofunctor.

Any product or sum in a Cartesian category is a `*`.

Code definition:

```
class Bifunctor f
  where
    bimap :: (a -> a') -> (b -> b') -> f a a' -> f a' a'
    bimap f g = first f . second g
    first :: (a -> a') -> f a b -> f a' b
    first f = bimap f id
    second :: (b -> b') -> f a b -> f a b'
    second = bimap id
```

3.5.11.1 *

Bifunctor

3.5.12 *

«<=<\$>=>>

3.6 Hask category

Category of Haskell where objects are types and morphisms are functions.

It is a hypothetical category at the moment, since undefined and bottom values break the theory, is not Cartesian closed, it does not have sums, products, or initial object, () is not a terminal object, monad identities fail for almost all instances of the Monad class.

That is why Haskell developers think in subset of Haskell where types do not have bottom values. This only includes functions that terminate, and typically only finite values. The corresponding category has the expected initial and terminal objects, sums and products, and instances of Functor and Monad really are endofunctors and monads.

Hask contains subcategories, like Lst containing only list types.

Haskell and Category concepts:

- Things that take a type and return another type are type constructors.
- Things that take a function and return another function are higher-order functions.

3.6.1 *

Hask

3.7 Magma

Set with a binary operation which form a closure.

3.7.1 Mag category

The category of magmas, denoted *Mag*, has as objects - sets with a binary operation, and morphisms given by homomorphisms of operations (in the universal algebra sense).

3.7.1.1 *

MAG

Magma category

Category of magmas

3.7.2 Semigroup

Magma with associative property of operation.

Defined in Haskell as:

```
class Semigroup a where
  (<>) :: a -> a -> a
```

3.7.2.1 *

Semigroups

3.7.2.2 Monoid

Semigroup with [identity](#) element. [Category](#) with a one [object](#).

Ideally fits as an accumulation class.

```
class Monoid m where
  mempty :: m
  mappend :: m -> m -> m
  mappend = (<>)
  mconcat :: [m] -> m
  mconcat = foldr mappend mempty
```

* can be simplified to [category](#) with a single [object](#), remember that [monoid operation](#) is a [composition](#) of [morphisms operation](#) in [category](#).

For example to represent the whole non-negative integers with the one [object](#) and [morphism](#) "1" is absolutely enough, [composition operation](#) is "+".

```
import Data.Monoid
do
  show (mempty :: Num a => Sum a)
  -- "Sum {getSum = 0}"
  show $ Sum 1
  -- "Sum {getSum = 1}"
  show $ (Sum 1) <> (Sum 1) <> (Sum 1)
  -- "Sum {getSum = 3}"
  -- ...
```

Also backwards - any single-object [category](#) is a [monoid](#). [Category](#) has an [identity](#) requirement and [associativity](#) of [composition](#) requirement, which makes it a free [monoid](#).

a. *

Monoidal
Monoids

b. Monoid laws

a. Monoid left identity law

```
mempty <> x = x
```

b. Monoid right identity law

```
x <> mempty = x
```

c. Monoid associativity law

```
x <> mempty = x (y <> z) = (x <> y) <> z
mconcat = foldr (mempty <>)
```

Everything [associative](#) can be mappend.

- c. Commutative monoid [Commutativity property](#):

$$x \circ y = y \circ x$$

Opens a big abilities in concurrent and distributed processing.

- a. *

Abelian monoid

- d. Group [Monoid](#) that has [inverse](#) for every element.

- a. *

Groups

- b. Commutative group [Group operation](#) obeys the axiom of [commutativity](#).

- a. *

Abelian group

- b. Ring [Commutative group](#) under $+$ & [monoid](#) under \times , $+$ \times connected by [distributive property](#).

- and \times are generalized [binary](#) operations of addition and multiplication. \times has no requirement for [commutativity](#).

Example: [set](#) of same size square matrices of numbers with matrix operations form a [ring](#).

- a. *

Rings

3.8 Morphism

$\mu\phi\phi'$ *morphe* form

[Arrow](#) between two [objects](#) in a [category](#).

General description: [Arrow](#) from source to target. Denotes something.

On a level of [objects](#): is probably [structure](#)-preserving map from one mathematical [structure](#) to another of the same [type](#).

[Morphism](#) is a generalization ($f(x * y) \equiv f(x) \diamond f(y)$) of [homomorphism](#) ($f(x * y) \equiv f(x) * f(y)$). Under [morphism](#) almost always is the meaning of [homomorphism](#)-like [properties](#).

[Morphism](#) can be anything.

If [morphism](#) corresponds to [function](#) requirements - then it is a [function](#).

3.8.1 *

Morphisms

Arrow

Arrows

3.8.2 Homomorphism $\mu\acute{\varsigma}$ *homos* same (was chosen because of initial English mistranslation to "similar") $\mu\omicron\rho\varphi'$ *morphe* form

similar form

* map between two [algebraic structures](#) of the same [type](#), [operation](#)-preserving.

$$f_{x \rightarrow y} = f(a \star b) = f(a) \diamond f(b),$$

[where](#) x, y are [sets](#) with additional [algebraic structure](#) that includes \star, \diamond accordingly; a, b are elements of [set](#) x .

* sends [identity morphisms](#) to [identity morphisms](#) and inverses to inverses.

The concept of * has been generalized under the name of [morphism](#) to many [structures](#) that [either](#) do not have an underlying [set](#), or are not [algebraic](#).

3.8.2.1 *

Homomorphic

3.8.3 Identity morphism[Identity morphism](#) - or simply [identity](#): $x \in C : id_x = 1_x : x \rightarrow x$ [Composed](#) with other [morphism](#) gives same [morphism](#).Corresponds to [Reflexivity](#) and [Automorphism](#).**3.8.3.1 Identity**[Identity](#) only possible with [morphism](#). See [Identity morphism](#).There is also distinct [Zero](#) value.

- a. Two-sided identity of a predicate $P(e, a) = P(a, e) = a \mid \exists e \in S, \forall a \in S$
 $P()$ is [commutative](#).

[Predicate](#)

- b. Left identity of a predicate $\exists e \in S, \forall a \in S : P(e, a) = a$

[Predicate](#)

c. Right identity of a predicate $P(a, e) = a \mid \exists e \in S, \forall a \in S$

Predicate

3.8.3.2 Identity function

Return itself.

$(\backslash \ x.x)$

`id :: a -> a`

3.8.4 Monomorphism

$\mu o v o$ *mono* only

$\mu o r \varphi'$ *morphe* form

Maps one to one (uniquely), so invertable (always has [inverse morphism](#)), so preserves the information/[structure](#).

[Domain](#) can be equal or less to the [codomain](#).

$f^{X \rightarrow Y}, \forall x \in X \exists! y = f(x) \models f(x) \equiv f_{mono}(x)$ - from [homomorphism context](#)

$f_{mono} \circ g1 = f_{mono} \circ g2 \models g1 \equiv g2$ - from general [morphism context](#)

Thus $*$ is left cancelable.

If $*$ is a [function](#) - it is [injective](#). Initial [set](#) of f is fully uniquely mapped onto the [image](#) of f .

3.8.4.1 $*$

Monomorphic

Monomorphisms

3.8.5 Epimorphism

$\varepsilon \pi \iota$ *epi* on, over

$\mu o r \varphi'$ *morphe* form

$*$ is right cancelable [morphism](#).

$f^{X \rightarrow Y}, \forall y \in Y \exists f(x) \models f(x) \equiv f_{epi}(x)$ - from [homomorphism context](#)

$g1 \circ f_{epi} = g2 \circ f_{epi} \Rightarrow g1 \equiv g2$ - from general [morphism context](#)

In [Set category](#) if $*$ is a [function](#) - it is [surjective](#) (image of it fully uses [codomain](#))

[Codomain](#) is called a projection of the [domain](#).

$*$ fully maps into the target.

3.8.5.1 $*$

Epimorphic

Epimorphisms

3.8.6 Isomorphism

$\sigma\omicron\varsigma$ *isos* equal
 $\mu\omicron\rho\varphi'$ *morphe* form

Not equal, but equal for current intents and purposes.

[Morphism](#) that has [inverse](#).

Almost equal, but not quite: $(\text{Integer}, \text{Bool})$ & $(\text{Bool}, \text{Integer})$ - but can be transformed losslessly into one another.

[Bijective homomorphism](#) is also [isomorphism](#).

$$f^{-1, b \rightarrow a} \circ f^{a \rightarrow b} \equiv 1^a, \quad f^{a \rightarrow b} \circ f^{-1, b \rightarrow a} \equiv 1^b$$

2 reasons for non-[isomorphism](#):

- [function](#) at least ones collapses a values of [domain](#) into one value in [codomain](#)
- [image](#) (of a [function](#) in [codomain](#)) does not fill-in [codomain](#). Then [isomorphism](#) can exists for [image](#) but not whole [codomain](#).

[Categories](#) are [isomorphic](#) if there $R \circ L = ID$

3.8.6.1 *

Isomorphic
 Isomorphisms

3.8.6.2 Lax

Holds up to [isomorphism](#).
 (upon the transformation can be used as the same)

3.8.7 Endomorphism

$\varepsilon\nu\delta\omicron$ *endo* internal
 $\mu\omicron\rho\varphi'$ *morphe* form

[Arrow](#) from [object](#) to itself.

[Endomorphism](#) forms a [monoid](#) ([object](#) exists and [category](#) requirements already in place).

3.8.7.1 Automorphism

$\alpha\nu\tau\omicron$ *auto* self
 $\mu\omicron\rho\varphi'$ *form* form

[Isomorphic endomorphism](#).

Corresponds to [identity](#), [reflexivity](#), [permutation](#).

a. *

Automorphic
Automorphisms

3.8.7.2 *

Endomorphic
Endomorphisms

3.8.8 Catamorphism

$\kappa\alpha\tau$ *kata* downward
 $\mu\omicron\rho\varphi$ *morphe* form

Unique [arrow](#) from an initial [algebra structure](#) into different [algebra structure](#).

* in FP is a generalization folding, deconstruction of a [data structure](#) into more primitive [data structure](#) using a [functor](#) [F-algebra structure](#).

* reduces the [structure](#) to a lower level [structure](#).

* creates a projection of a [structure](#) to a lower level [structure](#).

3.8.8.1 *

Catamorphic
Catamorphisms

3.8.8.2 Catamorphism law

Table 3.2: [Catamorphism](#) laws in Haskell

Rule name	Haskell
cata-cancel	<code>cata phi . InF = phi . fmap (cata phi)</code>
cata-refl	<code>cata InF = id</code>
cata-fusion	<code>f . phi = phi . fmap f => f . cata phi = cata phi</code>
cata-compose	<code>eps :: f ~> g => cata phi . cata (In . eps) = cata (phi . eps)</code>

a. Hylomorphism [catamorphism](#) \circ [anamorphism](#)

Expanding and collapsing the [structure](#).

a. *

Hylomorphic
Hylomorphisms

3.8.8.3 Anamorphism

Generalizes unfold.

Dual concept to [catamorphism](#).

Increases the [structure](#).

[Morphism](#) from a [coalgebra](#) to the final [coalgebra](#) for that [endofunctor](#).

Is a [function](#) that generates a [sequence](#) by repeated [application](#) of the [function](#) to its previous result.

a. *
 Anamorphic
 Anamorphisms

3.8.9 Kernel

[Kernel](#) of a [homomorphism](#) is a number that measures the [degree homomorphism](#) fails to meet [injectivity](#) (AKA be [monomorphic](#)).

It is a number of [domain](#) elements that fail [injectivity](#):

- elements not included into [morphism](#)
- elements that collapse into one element in [codomain](#)

thou [Kernel](#) $[x|x \leftarrow 0 || x \geq 2]$.

Denotation:

$$\ker T = \{\mathbf{v} \in V : T(\mathbf{v}) = \mathbf{0}_W\}.$$

3.8.9.1 Kernel homomorphism

[Morphism](#) of elements from the [kernel](#).

Complementary [morphism](#) of elements that make main [morphism](#) not [monomorphic](#).

3.9 Set category

[Category](#) in which [objects](#) are [sets](#), [morphisms](#) are [functions](#).

Denotation:

Set

3.10 Natural transformation

Roughly * is:

`trans :: F a -> G a`

, while `a` is [polymorphic variable](#).

[Naturality](#) condition: $\forall a \exists (F a \rightarrow G a)$, or , analogous to [parametric polymorphism](#) in [functions](#). Since $*$ in a [category](#), stating $\forall (F a \rightarrow G a)$

[Naturality](#) condition means that all [morphisms](#) that take part in [homotopy](#) of source [functor](#) to target [functor](#) must exist, and that is the same, diagrams that take part in transformation, should commute, and different paths brings same result: if α - [natural transformation](#), α_a [natural transformation component](#) - $G f \circ \alpha_a = \alpha_b \circ F f$.

Since $*$ are just a [type](#) of parametric [polymorphic function](#) - they can [compose](#).

$*$ ($\overrightarrow{\eta}^{\mathcal{D}}$) is transforming : $\overrightarrow{\eta}^{\mathcal{D}} \circ F^{C \rightarrow \mathcal{D}} = G^{C \rightarrow \mathcal{D}}$.

$*$ [abstraction](#) creates higher-language of [Category](#) theory, allowing to talk about the [composition](#) and transformation of complex entities.

It is a process of transforming $F^{C \rightarrow \mathcal{D}}$ into $G^{C \rightarrow \mathcal{D}}$ using existing [morphisms](#) in target [category](#) \mathcal{D} .

Since it uses [morphisms](#) - it is [structure](#)-preserving transformation of one [functor](#) into another. It is mostly a lossy transformation. Only existing [morphisms](#) can make it exist.

Existence of $*$ between two [functors](#) can be seen as some [relation](#).

Can be observed to be a "[morphism of functors](#)", especially in [functor category](#).

$*$ by $\overrightarrow{\eta}_{y^c}^{\mathcal{D}}((x, y)^c) \circ F^{C \rightarrow \mathcal{D}}((x, y)^c) = G^{C \rightarrow \mathcal{D}}((x, y)^c) \circ \overrightarrow{\eta}_{x^c}^{\mathcal{D}}((x, y)^c)$, often written short $\overrightarrow{\eta}_b \circ F(\overrightarrow{f}) = G(\overrightarrow{f}) \circ \overrightarrow{\eta}_a$.

Notice that the $\overrightarrow{\eta}_{x^c}^{\mathcal{D}}((x, y)^c)$ depends on [objects&morphisms](#) of \mathcal{C} .

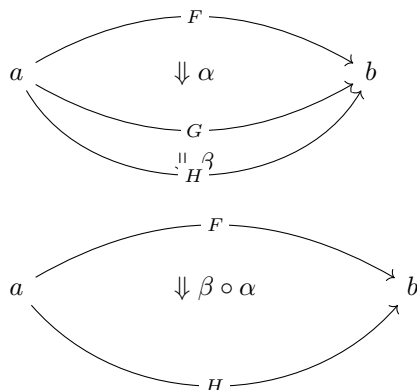
In words: $*$ depends on F and G [functors](#), ability of \mathcal{D} [morphisms](#) to do a [homotopy](#) of F to G , and $*$:

- for every [object](#) in \mathcal{C} picks [natural transformation component](#) in \mathcal{D} .
- for every [morphism](#) in \mathcal{C} picks the [commuting diagram](#) in \mathcal{D} , called naturality square.

Also see: [Natural transformation in Haskell](#)

Knowledge of $*$ forms a [2-category](#).

Can be [composed](#) "vertically":



And horizontally, aka "Godement [product](#)":

$$\begin{array}{ccccc}
 a & \xrightarrow{F_1} & b & \xrightarrow{F_2} & c \\
 & \Downarrow \alpha & & \Downarrow \beta & \\
 a & \xrightarrow{G_1} & b & \xrightarrow{G_2} & c \\
 \\
 a & \xrightarrow{F_2 \circ F_1} & c & & \\
 & \Downarrow \beta * \alpha & & & \\
 a & \xrightarrow{G_2 \circ G_1} & c & &
 \end{array}$$

[Compositions](#) can be done in any right [order](#), they abide the exchange law.

3.10.1 *

Natural transformations

Naturality condition

Naturality

3.10.2 Natural transformation component

$$\vec{\eta}^{\mathcal{D}}(x) = F^{\mathcal{D}}(x) \rightarrow G^{\mathcal{D}}(x) \mid x \in \mathcal{C}$$

3.10.2.1 *

Component of natural transformation

3.10.3 Natural transformation in Haskell

Family of [parametric polymorphism functions](#) between [endofunctors](#).

In [Hask](#) is $F\ a \rightarrow G\ a$. Can be analogued to repackaging data into another container, never modifies the [object](#) content, it only if - can delete it, because [operation](#) is lossy.

Can be sees as ortogonal to [functors](#).

3.10.4 Cat category

[Category](#) where:

	Part	Is	#
*	object	category	0-cell
\Rightarrow	morphism	functor	1-cell
\Rightarrow	2-morphism	natural transformation , morphisms homotopy	2-cell

$$\begin{array}{ccc}
 & F & \\
 a & \xrightarrow{\quad} & b \\
 & \Downarrow nt & \\
 & G &
 \end{array}$$

Is Cartesian [closed category](#).

3.10.4.1 *

Cat
2-category

3.10.4.2 Bicategory

2-category that is enriched and lax.

For handling relaxed associativity - introduces associator, and for identity 1 - left/right unit.

Forming from bicategories higher categories by stacking levels of abstraction of such categories - leads to explosion of special cases, differences of every level, and so overall difficulties.

Stacking groupoids (category in which are morphisms are invertable) is much more homogenous up to infinity, and forms base of the homotopy type theory.

3.11 Category dual

Category duality behaves like a logical inverse.

Inverse $\mathcal{C} = \mathcal{C}^{op}$ - inverts the direction of morphisms.

Composition accordingly changes to the morphisms: $(g \circ f)^{op} = f^{op} \circ g^{op}$

Any statement in the terms of \mathcal{C} in \mathcal{C}^{op} has the dual - the logical inverse that is true in \mathcal{C}^{op} terms.

Opposite preserves properties:

- products: $(\mathcal{C} \times \mathcal{D})^{op} \cong \mathcal{C}^{op} \times \mathcal{D}^{op}$
- functors: $(F^{\mathcal{C} \rightarrow \mathcal{D}})^{op} \cong F^{\mathcal{C}^{op} \rightarrow \mathcal{D}^{op}}$
- slices: $(\mathcal{F} \downarrow \mathcal{G})^{op} \cong (\mathcal{G}^{op} \downarrow \mathcal{F}^{op})$

a. *

Opposite category
Opposite categories
Category duality
Duality
Dual category
Dual

3.11.1 Coalgebra

Structures that are dual (in the category-theoretic sense of reversing arrows) to unital associative algebras.

Every coalgebra, by vector space duality, reversing arrows - gives rise to an algebra. In finite dimensions, this duality goes in both directions. In infinite - it should be determined.

3.12 Thin category

\forall [Hom sets](#) contain [zero](#) or one [morphism](#).

$$f \equiv g \mid \forall x, y \forall f, g : x \rightarrow y$$

A proset ([preordered set](#)).

3.12.1 *

Proset category
 Prosetal category
 Poset category
 Posetal category

3.13 Commuting diagram

Establishes equality in [morphisms](#) that have same source and target.

Draws the [morphisms](#) that are:

$$f = g \Rightarrow \{f, g\} : X \rightarrow Y$$

3.13.1 *

Diagram commutes
 Commutes

3.14 Universal construction

Algorithm of constructing definitions in [Category](#) theory.

Specially good to translate [properties](#)/definitions from other theories ([Set theory](#)) to [Categories](#).

Method:

- a. Define a pattern that you defining.
- b. Establish ranking for pattern matches.
- c. The top of ranking, the best match or [set](#) of matches - is the thing you was looking for.
 Matches are [isomorphic](#) for defined rules.

* uses Yoneda lemma, and as such constructions are defined until [isomorphism](#), and so [isomorphic](#) between each-other.

3.14.1 *

Universal constructions

3.15 Product

Universal construction:

$$\begin{array}{ccccc} & & c' & & \\ & p \swarrow & \downarrow ! & \searrow q & \\ a & \xleftarrow{\pi_a} & c & \xrightarrow{\pi_b} & b \end{array}$$

Pattern: $p : c \rightarrow a, q : c \rightarrow b$

Ranking: $\max \sum^{\forall} (! : c' \rightarrow c \mid p' = p \circ !, q' = q \circ !)$

c' is another candidate.

For [sets](#) - Cartesian product.

$*$ is a pair. Corresponds to [product data type](#) in [Hask](#) (inhabited with all elements of the [Cartesian product](#)).

Dual is [Coproduct](#).

3.15.1 *

Products

3.16 Coproduct

Universal constructuon:

$$\begin{array}{ccccc} & & c' & & \\ & p \nearrow & \uparrow ! & \nwarrow q & \\ a & \xrightarrow{\iota_a} & c & \xleftarrow{\iota_b} & b \end{array}$$

Pattern: $i : a \rightarrow c, j : b \rightarrow c$

Ranking: $\max \sum^{\forall} (! : c \rightarrow c' \mid i' = ! \circ i, j' = ! \circ j)$

c' is another candidate.

For [sets](#) - Disjoint union.

$*$ is a [set](#) assembled from other two [sets](#), in Haskell it is a tagged [set](#) (analogous to disjoint union).

Dual is [Product](#).

3.16.1 *

Coproducts

3.17 Free object

General particular [structure](#).

In which [structure](#), [properties](#) autofollows from definition, axioms.

Also uses as a term when surcomstances of [structures](#), rules, [properties](#), axioms used coincide with the definition of a particular [object](#) \therefore form [object](#) of this [type](#) with the according [properties](#)

and possibilities.

3.18 Internal category

[Category](#) which is included into a bigger [category](#).

3.19 Hom set

All [morphisms](#) from source [object](#) to target [object](#).

Denotation:

$$hom_C(X, Y) = (\forall f : X \rightarrow Y) = hom(X, Y) = C(X, Y)$$

Denotation was not standartized.

[Hom sets](#) belong to [Set category](#).

In [Set category](#): $\exists!(a, b) \iff \exists!Hom, \forall Hom \in Set$. [Set category](#) is special, [Hom sets](#) are also [objects](#) of it.

[Category](#) can include [Set](#), and [hom sets](#), or not.

3.19.1 *

Hom-set

Hom sets

3.19.2 Hom-functor

$$hom : \mathcal{C}^{op} \times \mathcal{C} \rightarrow Set$$

[Functor](#) from the [product](#) of \mathcal{C} with its [opposite category](#) to the [category](#) of [sets](#).

Denotation variants:

$$H_A = Hom(-, A)$$

$$h_A = \mathcal{C}(-, A)$$

$$Hom(A, -) : \mathcal{C} \rightarrow Set$$

Hom-[bifunctor](#):

$$Hom(-, -) : \mathcal{C}^{op} \times \mathcal{C} \rightarrow Set$$

3.19.3 Exponential object

Generalises the notion of [function set](#) to internal [object](#).

As also [hom set](#) to [internal hom objects](#).

Cartesian [closed](#) ([monoidal](#)) [category](#) strictly required, as $*$ multiplicaton holds [composition](#) requirement:

$$\circ : hom(y, z) \otimes hom(x, y) \rightarrow hom(x, z)$$

Denotation:

b^a

Universal construction:

$$\begin{array}{ccc} c & c \times a & \\ \vdots & \vdots & \searrow \\ u & u \times 1^a & \\ \Downarrow & \Downarrow & \\ b^a & b^a \times a \xrightarrow{\text{eval}} b & \end{array}, \text{ where in Category: } b^a - \text{exponential object, } \times - \text{product bifunctor,}$$

a - argument of $*$, b - result, c - candidate, $b^a \equiv (a \Rightarrow b) - *$.

$*$ b^a (also as $(a \Rightarrow b)$) represent exponentiation of cardinality of $\forall b^a$ possibilities.

3.19.3.1 $*$

Function object

Internal hom

Exponential objects

Hom object

Hom objects

3.19.3.2 Enriched category

Uses Hom objects (exponential objects), which do not belong into Set category. Category is no longer small, now may be called large.

$$\text{hom}(x, y) \in K.$$

Called: $*$ over K (which holds hom objects).

- a. $*$ Enriched
Large category

Chapter 4

Data type

Set of values.

For **type** to have sense the values must share some sense, **properties**.

4.1 *

Type

Types

Data types

4.2 Actual type

Data type recieved by ->**inferring**->compiling->execution.

4.3 Algebraic data type

Composite **type** formed by combining other **types**.

4.3.1 *

AlgDT

4.4 Cardinality

Number of possible implementations for a given **type** signature.

Disjunction, **sum** - adds **cardinalities**.

Conjunction, **product** - multiplies **cardinalities**.

4.4.1 *

Cardinalities

4.5 Data constant

* - [constant](#) value; [nullary data constructor](#).

4.6 Data constructor

One instance that [inhabit data type](#).

4.7 data declaration

[Data type declaration](#) is the most general and versatile form to create a new [data type](#).
Form:

```
data [context =>] type typeVars1..n
  = con1 c1t1..i
  | ...
  | conm cmt1..q
  [deriving]
```

4.8 Dependent type

When [type](#) and values have [relation](#) between them. [Type](#) has restrictions for values, value of a [type variable](#) has a result on the [type](#).

4.8.1 *

Dependent types

4.9 Gen type

[Generator](#). [Gen type](#) is to generate pseudo-random values for parent [type](#). Produces a [list](#) of values that gets infinitely cycled.

4.10 Higher-kinded data type

Any combination of * and ->

[Type](#) that take more [types](#) as arguments.

Humblly really a [function](#)

4.10.1 *

Higher-kinded data types

4.11 newtype declaration

Create a new [type](#) from old [type](#) by attaching a new [constructor](#), allowing [type class instance declaration](#).

```
newtype FirstName = FirstName String
```

Data will have exactly the same representation at runtime, as the [type](#) that is wrapped.

```
newtype Book = Book (Int, Int)
```

```
    (,)
    / \
Integer Integer
```

4.12 Principal type

The most generic [data type](#) that still [typechecks](#).

4.13 Product data type

Is an [algebraic data type](#) representation of a [product](#) construction.
Formed by logical [conjunction](#) (AND, '[*](#)').

Haskell forms:

```
-- 1. As a tuple (the uncurried & most true-form)
(T1, T2)

-- 2. Curried form, data constructor that takes two types
C T1 T2

-- 3. Using record syntax. =r# <inhabitant>= would return the respective =T#=.
C { r1 :: T1
    , r2 :: T2
    }
```

4.13.1 *

Product type

4.13.2 Sequence

Enumerated (ordered) [set](#).

Denotation:

```
()
( , )
( , , )
( , , ... )
```

More general mathematical denotation was not established, variants:

$$(n)_{n \in \mathbb{N}}$$

$$\omega \rightarrow X$$

$$\{i : Ord \mid i < \alpha\}$$

In Haskell: [Data type](#) that stores multiple ordered values withing a single value.

Table 4.1: [Sequence constructor](#) naming by [arity](#)

Name	Arity	Denotation
Unit , empty	0	()
Singleton	1	(_)
Tuple, pair, two-tuple	2	(_ , _)
Triple, three-tuple	3	(_ , _ , _)
Sequence	N	(_ , _ , ...)

4.13.2.1 *

Sequences
Tuples
Ordered pair
Ordered triple

4.13.2.2 List

The same [type objects sequence](#).

Denotation:

[\[\]](#)
[[_](#), [_](#)]
[[_](#), [_](#), [_](#)]
[[_](#), [_](#), ...]

Haskell definition:

```
data [] a = [] | a : [a]
```

Definition is self-referential (self-[recursive](#)), can be seen as [anamorphism](#) (unfold) of the [\[\]](#) (empty [list](#), memory cell which is container of particular [type](#)) and [:](#) ([cons operation](#), pointer). As such - can create non-terminating [data type](#) (and computation), in other words - infinite.

4.14 Proxy type

[Proxy type](#) holds no data, but has a phantom [parameter](#) of [arbitrary type](#) (or even [kind](#)). Able to provide [type](#) information, even though has no value of that [type](#) (or it can be may too costly to create one).

```
data Proxy a = ProxyValue
```

```
let proxy1 = (ProxyValue :: Proxy Int) -- a has kind `Type`
let proxy2 = (ProxyValue :: Proxy List) -- a has kind `Type -> Type`
```

4.15 Static typing

Typechecking takes place at compile level.

4.16 Structural type

Mathematical type. They form into structural type system.

4.16.1 *

Structural

4.17 Structural type system

Strict global hierarchy and relationships of types and their properties.

Haskell type system is *.

In most languages typing is name-based, not structural.

4.17.1 *

Structural typing

4.18 Sum data type

Algebraic data type formed by logical disjunction (OR '|').

4.19 Type alias

Create new type constructor, and use all data structure of the base type.

4.20 Type class

Type system construct that adds a support of ad hoc polymorphism.

Type class makes a nice way for defining behaviour, properties over many types/objects at once.

4.20.1 *

Type classes

Typeclass

Typeclasses

4.20.2 Arbitrary type class

Type class of `QuickCheck.Arbitrary` (that is reexported by `QuickCheck`) for creating a generator/distribution of values.

Useful function is `arbitrary` - that autogenerates values.

4.20.2.1 Arbitrary function

Depends on `type` and generates values of that `type`.

4.20.3 CoArbitrary type class

Pseudogenerates a function basing on resulting `type`.

```
coarbitrary :: CoArbitrary a => a -> Gen b -> Gen b
```

4.20.3.1 *

CoArbitrary

4.20.4 Typeable type class

Allows dynamic `type` checking in Haskell for a `type`.

Shift a `typechecking` of `type` from compile time to runtime.

* `type` gets wrapped in the universal `type`, that shifts the `type` checks to runtime.

Also allows:

- Get the `type` of something at runtime (ex. print the `type` of something `typeOf`).
- Compare the `types`.
- Reifying `functions` from `polymorphic type` to concrete (for `functions` like `:: Typeable a => a -> String`).

4.20.4.1 *

Typeable

4.20.5 Type class inheritance

Type class has a `superclass`.

4.20.6 Derived instance

Type class instances sometimes can be automatically [derived](#) from the parent [types](#).

Type classes such as Eq, Enum, Ord, Show can have instances generated based on definition of data type.

P.S.

Language options:

- DeriveAnyClass
- DeriveDataTypeable
- DeriveFoldable
- [DeriveFunctor](#)
- DeriveGeneric
- DeriveLift
- DeriveTraversable
- DerivingStrategies
- DerivingVia
- GeneralisedNewtypeDeriving
- StandaloneDeriving

4.20.6.1 *

Derived
Deriving

4.21 Type constant

[Nullary type constructor](#).

4.22 Type constructor

Name of the [data type](#).

[Constructor](#) that takes [type](#) as an [argument](#) and produces new [type](#).

4.23 type declaration

Synonym for existing [type](#). Uses the same [data constructor](#).

```
type FirstName = String
```

Used to distinct one entities from other entities, while they have the same [type](#).
Also main [type functions](#) can operate on a new [type](#).

4.24 Typed hole

`*` - is a `_` or `_name` in the [expression](#). On [evaluation](#) GHC would show the [derived type](#) information which should be in place of the `*`. That information helps to fill in the gap.

4.24.1 *

Typed holes

4.25 Type inference

Automatic [data type](#) detection for [expression](#).

4.25.1 *

Inferring
Infer
Infers
Inferred

4.26 Type class instance

Block of implementations of [functions](#), based on unique [type class](#)->[type](#) pairing.

4.27 Type rank

Weak ordering of [types](#).

The rank of [polymorphic type](#) shows at what level of nesting `forall` [quantifier](#) appears.
Count-in only [quantifiers](#) that appear to the left of [arrows](#).

```
f1 :: forall a b. a -> b -> a == fi :: a -> b -> c
g1 :: forall a b. (Ord a, Eq b) => a -> b -> a == g1 :: (Ord a, Eq b) => a -> b -> a
```

f1, g1 - [rank-1 types](#). Haskell itself implicitly adds universal [quantification](#).

```
f2 :: (forall a. a->a) -> Int -> Int
g2 :: (forall a. Eq a => [a] -> a -> Bool) -> Int -> Int
```

f2, g2 - [rank-2 types](#). Quantifier is on the left side of a \rightarrow . Quantifier shows that [type](#) on the left can be overloaded.

[Type inference](#) in Rank-2 is possible, but not higher.

```
f3 :: ((forall a. a->a) -> Int) -> Bool -> Bool
```

f3 - [rank3-type](#). Has [rank-2 types](#) on the left of a \rightarrow .

```
f :: Int -> (forall a. a -> a)
g :: Int -> Ord a => a -> a
```

f, g are rank 1. [Quantifier](#) appears to the right of an [arrow](#), not to the left. These [types](#) are not Haskell-98. They are supported in [RankNTypes](#).

4.27.1 *

Type ranks
Rank type
Rank types
Rank-1 type
Rank-1 types
Rank-2 type
Rank-2 types
Rank-3 type
Rank-3 types

4.28 Type variable

Refer to an unspecified [type](#) in Haskell [type](#) signature.

4.29 Unlifted type

[Type](#) that directly exist on the hardware. The [type abstraction](#) can be completely removed. With [unlifted types](#) Haskell [type](#) system directly manages data in the hardware.

4.29.1 *

Unlifted types

4.30 Data structure

4.30.1 Cons cell

Cell that values may [inhabit](#).

4.30.2 Construct

```
(:) :: a -> [a] -> [a]
```

4.30.2.1 *

Cons

4.30.3 Leaf

-

4.30.4 Node

```
*
/ \
```

4.31 Linear type

[Type](#) system and [algebra](#) that also track the multiplicity of data.
There are 3 general [linear type](#) groups:

- 0 - exists only at [type level](#) and is not allowed to be used at value level. Aka **s** in [ST-Trick](#).
- 1 - data that is not duplicated
- 1< - all other data, that can be duplicated multiple times.

4.31.1 *

Linear types

4.32 NonEmpty list data type

Data.[List](#).NonEmpty

Has a [Semigroup](#) instance but can't have a [Monoid](#) instance. It never can be an empty [list](#).

```
data NonEmpty a = a :| [a]
  deriving (Eq, Ord, Show)
```

`:|` - an [infix](#) data constructor that takes two ([type](#)) arguments. In other words `:|` returns a [product type](#) of left and right

4.33 Session type

* - allows to check that behaviour conforms to the protocol.

So far very complex, not very productive (or well-established) topic.

4.34 Binary tree

```
data BinaryTree a
  = [[Leaf]]
  | [[Node]] (BinaryTree a) a (BinaryTree a)
```

4.35 Bottom value

A `_` non-value in the [type](#) or [pattern match expression](#). Placeholder for anything.

```
-- _ fits *.
```

4.35.1 *

Bottom
Bottom values

4.36 Bound

Haskell `*` [type class](#) means to have lowest value & highest value, so a [bounded](#) range of values.

4.36.1 *

Bounded

4.37 Constructor

a. [Type constructor](#)

b. [Data constructor](#)

Also see: [Constant](#)

4.37.1 *

Constructors

4.38 Context

Type constraints for polymorphic variables.

Written before the main `type` signature, denoted:

```
TypeClass a => ...
```

4.38.1 *

Contexts

4.39 Inhabit

Values that is a component of `data type set`.

4.40 Maybe

```
data Maybe
  = Nothing
  | Just a
```

Does not represent the information why `Nothing` happened.

For `error` - use `Either`.

Do not propagate `*`.

Handle `*` locally to `where` it is produced. `Nothing` does not hold useful info for debugging & short-circuits the processes. Do not expect code `type` being bug-free, do not return `Maybe` to end user since it would be impossible to debug, return something that preserves `error` information.

4.40.0.1 *

Nodes

4.41 Expected type

`Data type inferred` from the text of the code.

4.42 ADT

a. Abstract data type

b. Algebraic data type

4.43 Concrete type

Fully defined `type`. Non-polymorphic type.

4.44 Type punning

When `type constructor` and `data constructor` have the same name.

Theoretically if person knows the rules - * can be solved, because in Haskell `type` and `data declaration` have different places of use.

4.45 Kind

`Kind -> Type -> Data`

4.45.1 *

Kinds

4.46 IO

`Type` for values whose evaluations has a possibility to cause side effects or return unpredictable result.

Haskell standard uses `monad` for constructing and transforming `IO` actions.

`IO` action can be evaluated multiple times.

`IO data type` has unpure imperative actions inside. Haskell is `pure Lambda calculus`, and un-pure `IO` integrates in the Haskell purely (`type` system abstracts `IO` unpurity inside `IO data type`).

`IO sequences effect` computation one after another in `order` of needed computation, or occurrence:

```
twoBinds :: IO ()
twoBinds =
  putStrLn "First:" >>
  getLine >>=
  \a ->
  putStrLn "Second:" >>
  getLine >>=
  \b ->
  putStrLn ("\nFirst: "
    ++ a ++ ".\nSecond "
    ++ b ++ ".")
main = twoBinds
```

Sequencing is achieved by compilation of effects performing only while they receive the sugared-in & passed around the `RealWorld` fake `type` value, that value in the every computation gets the new "value" and then passed to the next request computation. But special thing is about this `parameter`, this `RealWorld type` value passed, but never looked at. GHC realizes, since value is never used, - it means value and `type` can be equated to `()` and moreover reduced from the code, and sequencing stays.

Chapter 5

Declaration

[Binding](#) name to [expression](#).

Chapter 6

Differential operator

Denotation.

$\frac{d}{dx}$, D , D_x , ∂_x .

Last one is partial.

$e^{t\frac{d}{dx}}$ - Shift operator.

6.1 *

Differential

Chapter 7

Dispatch

Send, transmission, reference.

Chapter 8

Effect

Observable action.

Chapter 9

Evaluation

For FP see [Bind](#).

Chapter 10

Expression

Finite combination of symbols that is well-formed according to rules that depend on the [context](#).

10.1 *

Expressions

10.2 Closed-form expression

* - mathematical [expression](#) that can be evaluated in a finite number of operations.

May contain:

- constants
- [variables](#)
- operations (e.g., $+$ $-$ \times \div)
- [functions](#) (e.g., nth root, exponent, logarithm, trigonometric [functions](#), and [inverse](#) hyperbolic [functions](#)), but usually no limit.

10.3 RHS

Right-hand side of the [expression](#).

10.4 LHS

Left-hand side of the [expression](#).

10.5 Redex

Reducible expression.

10.6 Concatenate

Link together sequences, expressions.

10.7 Alpha equivalence

Equivalence of a processes in expressions. If expressions have according parameters different, but the internal processes are literally the same process.

10.8 Ground expression

Expression that does not contain any free variables.

10.8.1 *

Ground formula

10.9 Variable

A name for expression.

Haskell has immutable variables.

Except when you hack it with explicit funtions.

10.9.1 *

Variables

Chapter 11

First-class

Means *it*:

- Can be used as value.
- Passed as an [argument](#).

From 1&2 -> *it* can include itself.

Chapter 12

Function

Full dependency of one quantity from another quantity.

Denotation:

$$y = f(x)$$

$$f : X \rightarrow Y,$$

where X is domain, Y is codomain.

Directionality and property of invariability emerge from one another.

-- domain func codomain
* -> *

$$y(x) = (zx^2 + bx + 3 \mid b = 5)$$

\Name of the function
\Parameter
\Free variable
\Bound variable
\Var \Constant

Lambda abstraction is a function.
Function is a mathematical operation.

Function = Total function = Pure function. Function theoretically can be to memoized.

Also see:

Partial function

Inverse function - often partially exists (partial function).

12.1 *

Functions

Bound variable

12.2 Arity

Number of [parameters](#) of the [function](#).

- [nullary](#) - $f()$
- [unary](#) - $f(x)$
- [binary](#) - $f(x,y)$
- [ternary](#) - $f(x,y,z)$
- [n-ary](#) - $f(x,y,z..)$

12.3 Bijection

[Function](#) is a complete one-to-one pairing of elements of [domain](#) and [codomain](#) ([image](#)).
It means [function](#) both [surjective](#) (so [image](#) == [codomain](#)) and [injective](#) (every [domain](#) element has unique correspondence to the [image](#) element).

For [bijection inverse](#) always exists.

[Bijection operation](#) holds the [equivalence](#) of [domain](#) and [codomain](#).

Denotation:

[U+2916]
>->>
 $f : X \text{ [U+2916] } Y$

L^AT_EX needed to combine symbols:
 $f : X \mapsto Y$

Corresponds to [isomorphism](#).

12.3.1 *

Bijjective
Bijjective function

12.4 Combinator

[Function](#) without free [variables](#).
[Higher-order function](#) that uses only [function application](#) and other combinators.

```

\a -> a
\ a b -> a b
\f g x -> f (g x)
\f g x y -> f (g x y)

```

Not combinators:

```

\ xs -> sum xs

```

Informal broad meaning: referring to the style of organizing libraries centered around the idea of combining things.

12.5 Function application

* - [bind](#) the [argument](#) to the [parameter](#) of a [function](#), and do a [beta-reduction](#).

12.5.1 *

Apply
Applied
Applying
Application

12.6 Function body

[Expression](#) that haracterizes the process.

12.7 Function composition

```

(.) :: (b -> c) -> (a -> b) -> a -> c

```

```

a -> (a -> b) -> (b -> c) -> c

```

In Haskell inline [composition](#) requires:

```

h.g.f $ i

```

12.7.1 *

Composition
Compose
Composed

12.8 Function head

Is a part with name of the [function](#) and it's paramenteres.
AKA: $f(x)$

12.9 Function range

The range of a [function](#) refers to [either](#) the [codomain](#) or the [image](#) of the [function](#), depending upon usage. Modern usage almost always uses range to mean [image](#).

So, see [Function image](#).

12.10 Higher-order function

[Function arity](#) > 1.

—

Has [function](#) as a [parameter](#).

Evaluates to [function](#).

12.10.1 *

HOF

12.10.2 Fold

[Catamorphism](#) of a [structure](#) to a lower [type](#) of [structure](#). Often to a single value.

[*](#) is a [higher-order function](#) that takes a [function](#) which operates with both main [structure](#) and accumulator [structure](#), [*](#) applies units of [data structure](#) to a [function](#) which works with accumulator. Upon traversing the whole [structure](#) - the accumulator is returned.

12.11 Injection

[Function](#) one-to-one injects from [domain](#) into [codomain](#).

Keeps distinct pairing of elements of [domain](#) and [image](#).

Every element in [image](#) corresponds to one element in [domain](#).

$$\forall a, b \in X, f(a) = f(b) \Rightarrow a = b$$

$$\exists(\text{inverse function}) \mid \forall(\text{injective function})$$

Denotion:

$$\mapsto$$

$$\mapsto$$

$$f : X \mapsto Y$$

$$f : X \mapsto Y$$

Corresponds to [Monomorphism](#).

12.11.1 *

Injective
Injective function
Injectivity

12.12 Partial function

One that does not cover all [domain](#).
[Unsafe](#) and causes trouble.

12.13 Purity

* means properly abstracted.

If the contrary - [abstraction](#) is unpure.

Also see: [pure function](#).

12.13.1 *

Pure

12.14 Pure function

[Function](#) that is [pure](#) \equiv [referentially transparent function](#).

12.15 Sectioning

Writing [function](#) in a parentheses. Allows to pass around [partially applied functions](#).

12.16 Surjection

[Function](#) uses [codomain](#) fully.

$$\forall y \in Y, \exists x \in X$$

Denotation:
 $f : X \twoheadrightarrow Y$

Corresponds to [Epimorphism](#).

12.16.1 *

Surjective
Surjective function

12.17 Unsafe function

Function that does not cover at least one edge **case**.

12.17.1 *

Unsafe

12.18 Variadic

* - having **variable arity** (often up to indefinite).

12.19 Domain

Source **set** of a **function**.
 X in $X \rightarrow Y$.

12.20 Codomain

Y in $X \rightarrow Y$.
Codomain - target **set** of a **function**.

12.21 Open formula

Logical **function** that has **arity** and produces **proposition**.

12.22 Recursion

Repeated **function application** when sometimes same **function** gets called.

Allows computation that may require indefinite amount of work.

12.22.1 *

Recursive

12.22.2 Base case

A part of a [recursive function](#) that trivially produces result.

12.22.3 Tail recursion

Tail calls are [recursive](#) invocantions of itself.

12.22.4 Polymorphic recursion

[Type](#) of the [parameter](#) changes in [recursive](#) invocations of [function](#).

Is always a higher-ranked [type](#).

12.22.4.1 *

Milner–Mycroft typability

Milner–Mycroft calculus

12.23 Free variable

[Variable](#) in the fuction that is not [bound](#) by the head.

Until there are * - [function](#) stays [partially applied](#).

12.24 Closure

$f(x) = f^{\mathcal{X} \rightarrow \mathcal{X}} \mid \forall x \in \mathcal{X}, \mathcal{X}$ is [closed](#) under f , it is a trivial [case](#) when [operation](#) is legitimate for all values of the [domain](#).

[Operation](#) on members of the [domain](#) always produces a members of the [domain](#). The [domain](#) is [closed](#) under the [operation](#).

In the [case](#) when there is a [domain](#) values for which [operation](#) is not legitimate/not exists:

$f(x) = f^{\mathcal{V} \rightarrow \mathcal{X}} \mid \mathcal{V} \in \mathcal{X}, \forall x \in \mathcal{V}, \mathcal{X}$ is [closed](#) under f .

12.24.1 *

Closed

12.25 Parameter

$\pi\alpha\rho'$ *para* subsidiary

$\mu'\tau\rho\nu$ *metron* measure

Named variable of a [function](#).

[Argument](#) is a supplied value to a [function parameter](#).

[Parameter](#) ([formal parameter](#)) is an [irrefutable](#) pattern, and implemented that way in Haskell.

12.25.1 *

Parameters

Formal parameter

Formal parameters

12.26 Partial application

Part of [function parameters](#) applied.

12.26.1 *

Partially applied

12.27 Well-formed formula

[Expression](#), logical [function](#) that is/can produce a [proposition](#).

12.27.1 *

Well formed formula

WFF

wff

WFFS

wffs

Chapter 13

Homotopy

$\mu' \zeta$ homós same

One can be "continuously deformed" into the other.

For example - [functions](#), [functors](#).

[Natural transformation](#) is a [homotopy](#) of [functors](#).

13.1 *

Homotopies

Homotopic

Chapter 14

Lambda calculus

Universal model of computation. Which means λ can implement any [Turing machine](#).
Based on [function abstraction](#) and [application](#) by substituting [variables](#) and [binding](#) values.

λ has [lambda terms](#):

- [variable](#) (x)
- [application](#) ($((ts))$)
- [abstraction](#) ([lambda function](#)) ($((\lambda x.t))$)

14.1 λ

Lambda term
Lambda terms
Lambda variable
Lambda variables

14.2 Lambda cube

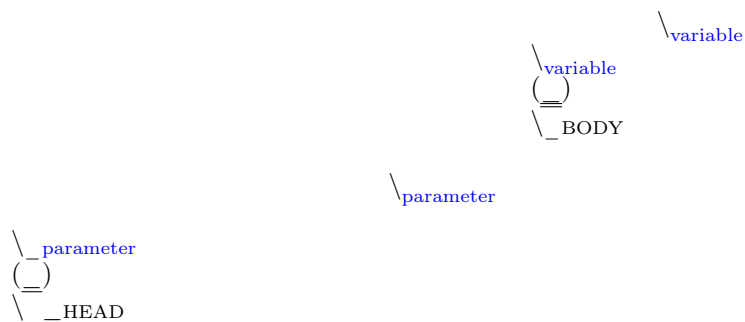
λ -cube shows the 3 dimensions of generalizations from simply typed [Lambda calculus](#) to [Calculus of constructions](#).

Each dimension of the cube corresponds to a new way of making [objects](#) depend on other [objects](#):

- ([First-class polymorphism](#)) - terms allowed to depend on [types](#), corresponding to [polymorphism](#).
- ([Higher-rank polymorphism](#)) - [types](#) depending on terms, corresponding to [dependent types](#).
- ([Type class](#)) - [types](#) depending on [types](#), corresponding to [type](#) operators.

14.2.1 * λ -cube λ -cube**14.3 Lambda function**

Function of Lambda calculus.

 $\lambda xy.x^2 + y^3$
^ ^ ^ ^**14.3.1 ***

Lambda abstraction

14.3.2 Anonymous lambda function

Lambda function that is not binded to any name.

14.3.2.1 *

Anonymous lambda function

14.4 β -reduction

Equation of a parameter to a bound variable, then reducing parameter from the head.

14.4.1 * β reduction

Beta-reduction

Beta reduction

14.4.2 β -normal form

No beta reduction is possible.

14.4.2.1 *

β normal form
 Beta normal form
 Beta-normal form

14.5 Calculus of constructions

Extends the Curry–Howard correspondence to the proofs in the full intuitionistic [predicate](#) calculus (includes proofs of [quantified statements](#)).

[Type](#) theory, typed programming language, and constructivism (philosophy) foundation for mathematics.

Directly relates to Coq programming language.

14.5.1 *

«<CoC>»

14.6 Curry–Howard correspondence

[Equivalence](#) of {[First-order logic](#), computer programming, [Category](#) theory}. They represent each-other, possible in one - possible in the other, so all the definitions and theorems have analogues in other two.

Gives a ground to the [equivalence](#) of computer programs and mathematical proofs.

Lambek added analogue to Cartesian [closed category](#), which can be used to model logic and [type](#) theory.

Table 14.1: Table of basic correspondence

Logic	Type	Category
True	() (any inhabited type)	Terminal
False	Void	Initial
$a \wedge b$	(a, b)	$a \times b$
$a \vee b$	Either a b	a / b
$a \Rightarrow b$	$a \rightarrow b$	b^a

14.6.1 *

Curry–Howard isomorphism
 Curry–Howard–Lambek

14.7 Currying

Translating the [evaluation](#) of a multiple [argument function](#) (or a [tuple](#) of arguments) into evaluating a [sequence](#) of [functions](#), each with a single [argument](#).

14.7.1 *

Curry

14.8 Hindley–Milner type system

Classical [type](#) system for the [Lambda calculus](#) with [Parametric polymorphism](#) and [Type inference](#). [Types](#) marked as [polymorphic variables](#), which enables [type inference](#) over the code.

14.8.1 *

Hindley–Milner

Damas–Milner

Damas–Hindley–Milner

14.9 Reduction

Take out something from a [structure](#), make simpler.

See [Beta reduction](#)

14.9.1 *

Reducible

14.10 β - η normal form

All [\$\beta\$ -reduction](#) and [\$\eta\$ -abstraction](#) are done in the [expression](#).

14.10.1 *

beta-eta normal form

beta eta normal form

14.11 η -abstraction

$$(\lambda x.Mx) \xleftarrow{\eta} M$$

```

\ x -> g . f $ x
\ x -> g . f      --eta-abstraction

```

14.11.1 *

η -reduction
 η -conversion
 η abstraction
 η reduction
 η conversion
eta-abstraction
eta-reduction
eta-conversion
eta abstraction
eta reduction
eta conversion

14.12 Lambda expression

See [Lambda calculus](#) ([Lambda terms](#)) and [Expression](#). In majority cases meaning some [Lambda function](#).

Chapter 15

Lense

Library of combinators to provide Haskell (functional language without mutation) with the emulation of **get**-ters and **set**-ters of imperative language.

Chapter 16

Operation

Calculation into output value. Can have [zero](#) & more inputs.

16.1 Constant

[Nullary operation](#).

Also see: [Type constant](#).

16.2 Binary operation

$\forall(a, b) \in S, \exists P(a, b) = f(a, b) : S \times S \rightarrow S$

16.2.1 *

Binary operations

16.3 Operator

Denotation symbol/name for the [operation](#).

16.3.1 Shift operator

[Shift operator](#) defined by Lagrange through [Differential operator](#).

$$T^t = e^{t \frac{d}{dx}}$$

16.3.1.1 *

Shift

16.4 Infix

Form of writing of **operator** or **function** in-between **variables** for **application**.

For priorities see **Fixity**.

16.5 Fixity

Declares the presedence of action of a **function/operator**.

Funciton **application** has presedence higher then all **infix** operators/**functions** (virtually giving it a **priority** 10).

Table 16.1: Haskell operators **priority** and **fixity** association

P	L	Non	R
10			F.A.
9	!!		.
8			^ ^ ^ **
7	*/ div		
6	+-		
5			: , ++
4		<comparison> elem	
3			&&
2			OR
1			
0			\$ \$! seq

16.5.1 *

Infixl

Infixr

Priority

Precedence

16.6 Zero

* is the value with which **operation** always yelds **Zero** value.

$zero, n \in C : \forall n, zero * n = zero$

* is distinct from **Identity** value.

Chapter 17

Permutation

Bijjective function from domain to itself.

Domain & permutation functions & function composition form a group.

Chapter 18

Phrase

Composable expression.

Chapter 19

Point-free

Paradigm [where function](#) only describes the [morphism](#) itself.

Process of converting [function](#) to [point-free](#).

If brackets `()` can be changed to `$` then `$` equal to [composition](#):

```
\ x -> g (f x)
\ x -> g $ f x
\ x -> g . f $ x
\ x -> g . f      --eta-abstraction
```

```
\ x1 x2 -> g (f x1 x2)
\ x1 x2 -> g $ f x1 x2
\ x1 x2 -> g . f x1 $ x2
\ x1      -> g . f x1
```

19.1 *

Pointfree
Tacit
Tacit programming

19.2 Blackbird

`(.) . (.) :: (b -> c) -> (a1 -> a2 -> b) -> a1 -> a2 -> c`

[Composition of compositions](#) `(.) . (.)`. Allows to [compose](#)-in a [binary function](#) `f1(c) (.) . (.) f2(a,b)`.

```
\ f g x y -> f (g x y)
```

19.2.1 *

`.) .`
`(.) . (.)`
Composition of compositions

19.3 Swing

```
swing :: ((a -> b) -> b) -> c -> d) -> c -> a -> d
swing = flip . (. flip id)
swing f = flip (f . runCont . return)
swing f c a = f ($) a c
```

19.4 Squish

```
f >>= a . b . c =<< g
```

Chapter 20

Polymorphism

$\pi o\lambda'\varsigma$ *polús* many

At once several forms.

In Haskell - [abstract](#) over [data types](#).

* [types](#):

20.1 *

Polymorphic

20.2 Levity polymorphism

Extending [polymorphism](#) to work with unlifted and lifted [types](#).

20.3 Parametric polymorphism

[Abstracting](#) over [data types](#) by [parameter](#).

In most languages named as 'Generics' (generic programming).

[Types](#):

20.3.1 Rank-1 polymorphism

[Parametric polymorphism](#) in [rank-1 types](#) by [type variables](#).

20.3.1.1 *

Prenex

Prenex polymorphism

20.3.2 Let-bound polymorphism

It is [property](#) chosen for Haskell [type](#) system.

Haskell is based on [Hindley-Milner type](#) system, it is [let-bound](#).

To have strict [type inference](#) with `*` - if `let` and `where` declarations are [polymorphic](#) - λ declarations - should be not.

See: [Good: In Haskell parameters bound by lambda declaration instantiate to only one concrete type](#).

20.3.3 Constrained polymorphism

Constrained [Parametric polymorphism](#).

20.3.3.1 Ad hoc polymorphism

Artificial [constrained polymorphism](#) dependent on incoming [data type](#).

It is [interface dispatch](#) mechanism of [data types](#).

Achieved by creating a [type class instance functions](#).

Commonly known as overloading.

a. `*`

Ad-hoc polymorphism

Ad hoc polymorphic

Ad-hoc polymorphic

Constraint

Constraints

20.3.4 Impredicative polymorphism

`*` allows [type](#) τ entities with [polymorphic types](#) that can contain [type](#) τ itself.

$T = \forall X. X \rightarrow X : T \in X \models T \in T$

The most powerful form of [parametric polymorphism](#).

See: [Impredicative](#).

This approach has Girard's paradox ([type systems](#) [Russell's paradox](#)).

20.3.4.1 `*`

First-class polymorphism

20.3.5 Higher-rank polymorphism

Means that [polymorphic types](#) can appear within other [types](#) ([types of function](#)).

There is a case where [higher-rank polymorphism](#) than the Ad hoc - is needed. For example [where ad hoc polymorphism](#) is used in [constraints](#) of several different implementations of [functions](#), and you want to build a [function](#) on top - and use the [abstract interface](#) over these [functions](#).

```

-- ad-hoc polymorphism
f1 :: forall a. MyType Class a => a -> String    ==    f1 :: MyType Class a => a -> String
f1 = -- ...

-- higher-rank polymorphism
f2 :: Int -> (forall a. MyType Class a => a -> String) -> Int
f2 = -- ...

```

By moving forall inside the function - we can achieve higher-rank polymorphism.

From: <https://news.ycombinator.com/item?id=8130861>

Higher-rank polymorphism is formalized using System F, and there are a few implementations of (in
Useful example aslo a [ST-Trick monad](#).

20.3.5.1 *

Rank-n polymorphism

20.4 Subtype polymorphism

Allows to declare usage of a Type and all of its Subtypes.

T - Type

S - Subtype of Type

<: - subtype of

$S <: T = S \leq T$

Subtyping is:

If it can be done to T, and there is subtype S - then it also can be done to S.

$S <: T : f^{T \rightarrow X} \Rightarrow f^{S \rightarrow X}$

20.5 Row polymorphism

Is a lot like Subtype polymorphism, but aligns itself on allowance (with | r) of subtypes and types with requested properties.

```

printX :: { x :: Int | r } -> String
printX rec = show rec.x

printY :: { y :: Int | r } -> String
printY rec = show rec.y

-- type is inferred as `{x :: Int, y :: Int | r } -> String`
printBoth rec = printX rec ++ printY rec

```

20.6 Kind polymorphism

Achieved using a phantom type argument in the data type declaration.

```
;;          * -> *
data Proxy a = ProxyValue
```

Then, by default the `data type` can be inhabited and fully work being partially defined.
But multiple instances of `kind polymorphic type` can be distinguished by their particular `type`.

Example is the `Proxy type`:

```
data Proxy a = ProxyValue

let proxy1 = (ProxyValue :: Proxy Int) -- * :: Proxy Int
let proxy2 = (ProxyValue :: Proxy a)   -- * -> * :: Proxy a
```

20.7 Linearity polymorphism

Leverages `linear types`.

For example - if `fold` over a dynamic array:

- a. In basic Haskell - array would be copied at every step.
- b. Use low-level `unsafe functions`.
- c. With `Linear type function` we guarantee that the array would be used only at one place at a time.

So, if we use a `function` `(* -o * -o -o *)` in `foldr` - the `fold` will use the initial value only once.

Chapter 21

Pragma

Pragma - instruction to the compiler that specifies how a compiler should process the code. **Pragma** in Haskell have form:

```
{-# PRAGMA options #-}
```

21.1 LANGUAGE pragma

Controls what variations of the language are permitted.

It has a **set** of allowed options: https://downloads.haskell.org/~ghc/latest/docs/html/users_guide/glasgow_exts.html, which can be supplied.

21.1.1 LANGUAGE option

21.1.1.1 *

Language options

21.1.1.2 Useful by default

```
import EmptyCase
import FlexibleContexts
import FlexibleInstances
import InstanceSigs
import MultiParamTypeClasses
```

21.1.1.3 AllowAmbiguousTypes

Allow **type** signatures which appear that they would result in an unusable **binding**. However GHC will still check and complain about a **functions** that can never be called.

21.1.1.4 ApplicativeDo

Enables an **alternative** in-depth **reduction** that translates the do-notation to the operators `<$>`, `<*>`, `join` as far as possible.

For GHC to pickup the patterns, the final **statement** must match one of these patterns exactly:

```

pure E
pure $ E
return E
return $ E

```

When the `statements` of `do expression` have dependencies between them, and `ApplicativeDo` cannot `infer` an `Applicative type` - GHC uses a heuristic $O(n^2)$ algorithm to try to use `<*>` as much as possible. This algorithm usually finds the best solution, but in rare complex cases it might miss an opportunity. There is also $O(n^3)$ algorithm that finds the optimal solution: `-foptimal-applicative-do`.

Requires `ap = <*>`, `return = pure`, which is true for the most `monadic types`.

- Allows use of `do`-notation with `types` that are an instance of `Applicative` and `Functor`
- In some `monads`, using the `applicative` operators is more efficient than `monadic bind`. For example, it may enable more parallelism.

The only way it shows up at the source level is that you can have a `do expression` with only `Applicative` or `Functor` constraint.

It is possible to see the actual translation by using `-ddump-ds`.

21.1.1.5 ConstrainedClassMethods

Enable the definition of further `constraints` on individual class methods.

21.1.1.6 CPP

Enable `C preprocessor`.

21.1.1.7 DeriveFunctor

Automatic `deriving` of instances for the `Functor type class`.

For `type power set functor` is unique, its derivation implementation can be autochecked.

21.1.1.8 ExplicitForAll

Allow explicit `forall` quantifier in places `where` it is implicit by Haskell.

21.1.1.9 FlexibleContexts

Ability to use complex `constraints` in class `declaration contexts`.

The only restriction on the `context` in a class `declaration` is that the class hierarchy must be acyclic.

```

class C a where
  op :: D b => a -> b -> b

class C a => D a where ...

```

$C \rightarrow D$, so in `C` we can talk about `D`.

Synergizes with `ConstraintKinds`.

21.1.1.10 FlexibleInstances

Allow `type class` instances `types` contain nested `types`.

```
instance C (Maybe Int) where ...
```

Implies `TypeSynonymInstances`.

21.1.1.11 GeneralizedNewtypeDeriving

Enable GHC's `newtype` cunning generalised `deriving` mechanism.

```
newtype Dollars = Dollars Int
  deriving (Eq, Ord, Show, Read, Enum, Num, Real, Bounded, Integral)
```

(In old Haskell-98 only `Eq`, `Ord`, `Enum` could be inherited.)

21.1.1.12 ImplicitParams

Allow definition of `functions` expecting implicit `parameters`. In the Haskell that has static scoping of `variables` allows the dynamic scoping, such as in classic Lisp or ELisp. Sure thing this one can be puzzling as hell inside Haskell.

21.1.1.13 LambdaCase

Enables `expressions` of the form:

```
\case { p1 -> e1; ...; pN -> eN }
```

-- OR

```
\case
  p1 -> e1
  ...
  pN -> eN
```

21.1.1.14 MultiParamTypeClasses

Implies: `ConstrainedClassMethods`

Enable the definitions of `typeclasses` with more than one `parameter`.

```
class Collection c a where
```

21.1.1.15 MultiWayIf

Enable multi-way-if syntax.

```
if | guard1 -> code1
   | ...
   | guardN -> codeN
```

21.1.1.16 OverloadedStrings

Enable overloaded string literals (string literals become desugared via the `IsString` class).

With overload, string literals has [type](#):

```
(IsString a) => a
```

The usual string syntax can be used, e.g. `ByteString`, `Text`, and other variations of string-like [types](#).

Now they can be used in pattern matches as `char->integer` translations. To [pattern match](#) `Eq` must be [derived](#).

To use class `IsString` - [import](#) it from `GHC.Ext`.

21.1.1.17 PartialTypeSignatures

Partial [type](#) signature contains [wildcards](#), placeholders (`_`, `_name`).

Allows programmer to which parts of a [type](#) to annotate and which to [infer](#). Also applies to [constraint](#) part.

As untuped [expression](#), partly typed can not polymorphically recurse.

[-Wno-partial-type-signatures](#) supresses [infer](#) warnings.

21.1.1.18 RankNTypes

Enable [types](#) of [arbitrary](#) rank.

See [Type rank](#).

Implies [ExplicitForAll](#).

Allows `forall` [quantifier](#):

- Left side of \rightarrow
- Right side of \rightarrow
- as [argument](#) of a [constructor](#)
- as [type](#) of a field

- as `type` of an implicit `parameter`
- used in pattern `type` signature of `lexically scoped type variables`

21.1.1.19 ScopedTypeVariables

By default `type variables` do not have a `scope` except inside `type` signatures `where` they are used.

When there are internal `type` signatures provided in the code block (`where`, `let`, etc.) they (main `type` description of a `function` and internal `type` descriptions) restrain one-another and become not truly `polymorphic`, which creates a bounding interdependency of `types` that GHC would complain about.

* option provides the `lexical scope` inside the code block for `type variables` that have `forall quantifier`. Because they are now lexically scoped - those `type variables` are used across internal `type` signatures.

For details see: <https://ocharles.org.uk/guest-posts/2014-12-20-scoped-type-variables.html>

Implies `ExplicitForAll`.

21.1.1.20 TupleSections

Allow `tuple` section syntax:

```
(, True)
(, "I", , , "Love", , 1337)
```

21.1.1.21 TypeApplications

Allow `type application` syntax:

```
read @Int 5

:type pure @[]
pure @[] :: a -> [a]

:type (<*>) @[]
(<*>) @[] :: [a -> b] -> [a] -> [b]

--

instance (CoArbitrary a, Arbitrary b) => Arbitrary (a -> b)

λ> ($ 0) <$> generate (arbitrary @(Int -> Int))
```

21.1.1.22 TypeSynonymInstances

Now `type` synonym can have its own `type class` instances.

21.1.1.23 UndecidableInstances

Permit instances which may lead to [type-checker](#) non-termination.

GHC has Instance termination rules regardless of [FlexibleInstances](#) [FlexibleContexts](#).

21.1.1.24 ViewPatterns

```
foo (f1 -> Pattern1) = c1
foo (fn -> Pattern2 a b) = g1 a b
```

(*expression* → *pattern*): take what is came to match - [apply](#) the *expression*, then do *pattern*-match, and return what originally came to match.

[Semantics](#):

- *expression* & *pattern* share the [scope](#), so also [variables](#).

n (*expression* → *pattern*) t1.

- * are like [pattern guards](#) that can be nested inside of other patterns.
- * are a convenient way to pattern-match [algebraic data type](#).

Additional possible usage:

```
foo a (f2 a -> Pattern3 b c) = g2 b c -- only for function definitions
foo ((f,_), f -> Pattern4) = c2 -- variables can be bount to the left in data constructors and t
```

21.1.1.25 DatatypeContexts

Allow [contexts](#) in [data types](#).

```
data Eq a => Set a = NilSet | ConsSet a (Set a)

-- NilSet :: Set a
-- ConsSet :: Eq a => a -> Set a -> Set a
```

Considered misfeature. Deprecated. Going to be removed.

21.1.1.26 StandaloneKindSignatures

[Type](#) signatures for [type-level declarations](#).

```
type <name_1> , ... , <name_n> :: <kind>

type MonoTagged :: Type -> Type -> Type
data MonoTagged t x = MonoTagged x

type Id :: forall k. k -> k
type family Id x where
  Id x = x
```

```

type C :: (k -> Type) -> k -> Constraint
class C a b where
  f :: a b

type TypeRep :: forall k. k -> Type
data TypeRep a where
  TyInt    :: TypeRep Int
  TyMaybe :: TypeRep Maybe
  TyApp    :: TypeRep a -> TypeRep b -> TypeRep (a b)

```

< GHC 8.10.1 - [type](#) signatures were only for [term level](#) declarations.

Extension makes signatures feature more uniformal.

Allows to [set](#) the [order](#) of [quantification](#), [order](#) of [variables](#) in a [kind](#). For example when using [TypeApplications](#).

Allows to [set](#) full [kind](#) of derivable class, solving situations with [GADT](#) return [kind](#).

```

a. *
   SAKS
   Standalone kind signatures

```

21.1.1.27 PartialTypeSignatures

Very helpful. Helps to solve [type level](#), helps to establish [type](#) signatures and [constraints](#).

Allow to provide `_` in the [type](#) signatures to automatically infer in the [type](#) information there.

Wild cards:

- [Type](#)

```
f :: _ -> _ -> a
```

- [Constraint](#)

```
f :: _ => a -> b -> c
```

- [Named](#)

```
f :: _x -> _x -> a
```

allows to identify the same [wildcard](#).

21.1.2 How to make a GHC LANGUAGE extension

In `libraries/ghc-boot-th/GHC/LanguageExtensions/Type.hs` add new [constructor](#) to the Extension [type](#)

```
data Extension
  = Cpp
  | OverlappingInstances
  ...
  | Foo
```

/main/DynFlags.hs extend xFlagsDeps:

```
xFlagsDeps = [
  flagSpec "AllowAmbiguousTypes" LangExt.AllowAmbiguousTypes,
  ...
  flagSpec "Foo"                  LangExt.Foo
]
```

It is for basic [case](#). For [testing](#), parser see further: <https://blog.shaynefletcher.org/2019/02/adding-ghc-language-extension.html>

Chapter 22

Predicative

Non-self-referencing definition.

—

Antonym - [Impredicative](#).

Chapter 23

Compositionality

Complex [expression](#) is determined by the constituent [expressions](#) and the rules used to combine them.

If the meaning fully obtainable from the parts and [composition](#) - it is full, [pure compositionality](#).

If there exists [composed idiomatic expression](#) - it is unfull, unpure [compositionality](#), because meaning leaks-in from the sources that are not in the [composition](#).

23.1 *

Principle of compositionality

Composition

Compositional

Chapter 24

Ψ -combinator

Transforms two of the same [type](#), [applying](#) same mediate transformation, and then transforming those into the result.

```
import Data.Function (on)
on :: (b -> b -> c) -> (a -> b) -> a -> a -> c

--\
  * ---
--/
```

24.1 *

Psi-combinator
On-combinator

Chapter 25

Quantifier

Specifies the quantity of specimens.

Two most common [quantifiers](#) \forall ([Forall](#)) and \exists (Exists).
 $\exists!$ - one and only one (exists only unique).

25.1 *

Quantification
Quantifiers
Quantified

25.2 Forall quantifier

Permits to not [infer](#) the [type](#), but to use any that fits. The variant depends on the [LANGUAGE option](#) used:
[ScopedTypeVariables](#)
[RankNTypes](#)
[ExistentialQuantification](#)

25.2.1 *

Forall

Chapter 26

Referential transparency

Given the same input return the same output.

So:

- * **expression** can be replaced with its corresponding resulting value without change for program's behavior.

- * **functions** are **pure**.

26.1 *

Referentially transparent

Chapter 27

Relation

[Relationship](#) between two [objects](#).
Subset of a [Cartesian product](#) between [sets](#) of [objects](#).
Is not directed and not limited.

27.1 *

Relations
Relationship

Chapter 28

REPL

Read-eval-print loop, aka interactive shell.

Chapter 29

Semantics

Philosophical study of meaning.
Meaning of symbols, words.

29.1 Operational semantics

Constructing proofs from logical [assertions](#) and verifying/checking/asserting things about execution and procedures their [properties](#), such as correctness, safety or security.

Good to solve in-point localized tasks.

Process of [abstraction](#).

29.2 Denotational semantics

Construction of [objects](#), that describe/tag the meanings. In Haskell often [abstractions](#) that are ment (denotations), implemented directly in the code, sometimes exist over the code - allowing to reason and implement.

* are [composable](#).

Good to achieve more broad approach/meaning.

Also see [Abstraction](#).

29.2.1 Abstraction

abs away from, off (in absentia)
tractus draw, haul, drag

Purified generalization of process.

Forgetting the details ([axiomatic semantics](#)). Simplified approach. Out of sight - out of mind.

* creates a new semantic level in which one can be absolutely precise ([operational semantics](#)).

It is a great did to name an [abstraction](#) ([denotational semantics](#)).

29.2.1.1 *

Abstractions

Abstracting

Abstract

29.2.1.2 Leaky abstraction

[Abstraction](#) that leaks details that it is supposed to [abstract](#) away.

a. *

Leaky abstractions

29.2.1.3 Object

Absolute [abstraction](#).

Point.

Can have [properties](#).

Often abstracts sometging, for example some [structure](#), [maybe](#) mathematical.

a. *

Structure

Structures

Objects

b. Terminal object One that recieves unique [arrow](#) from every [object](#).

$$\exists ! : x \rightarrow 1 \mid \exists 1 \in \mathcal{C}, \forall x \in \mathcal{C}$$

* is an empty [sequence](#) () in Haskell.

Called a [unit](#), so recieves *terminal* or [unit](#) arrow.

[Dual](#) of [initial object](#).

Denotation:

[Category](#) theory

1

Haskell

()

- c. Initial object One that emits unique **arrow** into every **object**.

$$\exists! : \emptyset \rightarrow x \mid \exists \emptyset \in \mathcal{C}, \forall x \in \mathcal{C}$$

If **initial object** is **Void** (most frequently) - emitted **arrows** called absurd, because they can not be called.

Dual of **terminal object**.

Denotation:

Category theory:

\emptyset

Haskell:

Void

29.2.2 Ambigram

ambi both

$\gamma\rho'\mu\mu\alpha$ *grámma* written character

Object that from different points of view has the same meaning.

While this word has two contradictory diametrically opposite usages, one was chosen (more frequent).

But it has... Both.

*TODO: For merit of differentiating the meaning about different meaning referring to **Tensor** as **object** with many meanings.*

29.2.3 Binary

Two of something.

29.2.4 Arbitrary

arbitrarius uncertain

Random, any one of.

Used as: Any one with *this* **set** of **properties**. (**constraints**, **type**, etc.).

When there is a talk about any **arbitrary** value - in fact it is a talk about the generalization of computations over the **set** of **properties**.

29.2.5 Refutable

One that has an option to fail.

29.2.6 Irrefutable

One that can not fail.

29.3 Axiomatic semantics

Empirical process of studying something complex by finding and analyzing true [statements](#) about it.

Good for examining interconnections.

29.3.1 Property

Something has a [property](#) in the real world, and in theory its [property](#) corresponds to the law/laws, axioms.

In Haskell under [property](#)/law most often [properties](#) of [algebraic structures](#).

There [property testing](#) wich does what it says.

29.3.1.1 *

Properties

29.3.1.2 Associativity

Joined with common purpose.

$$P(a, P(b, c)) \equiv P(P(a, b), c) \mid \forall (a, b, c) \in S,$$

* - the operations can be grouped arbitrarily.

[Property](#) that determines how operators of the same [precedence](#) are grouped, (in computer science also in the absence of parentheses).

Etymology:

Latin *associatus* past participle of *associare* "[join](#) with", from assimilated form of *ad* "to" + *sociare* "unite with", from *socius* "companion, ally" from PIE **sokw-yo-*, suffixed form of root **sekw-* "to follow".

In Haskell * has influence on parsing when compounds have same [fixity](#).

a. *

Associative

Associative law

Associativity law

29.3.1.3 Left associative

* - the operations are grouped from the left.

Example:

In lambda [expressions](#) same level parts follow grouping from left to right.

$(\lambda x.x)(\lambda y.y)z \equiv ((\lambda x.x)(\lambda y.y))z$

a. *

Left associativity

Left-associative

29.3.1.4 Right associative

* - the operations are grouped from the right.

29.3.1.5 Non-associative

Operations can't be chained.

Often is the [case](#) when the output [type](#) is incompatible with the input [type](#).

29.3.1.6 Basis

$\beta\alpha\sigma\iota\varsigma$ - stepping

The initial point, unreducible axioms and terms that spawn a theory.

AKA see [Category](#) theory, or Euclidian geometry [basis](#).

- a. Contravariant The [property](#) of [basis](#), in which if new [basis](#) is a [linear](#) combination of the prior [basis](#), and the change of [basis](#) inverse-proportional for the description of a [Tensors](#) in this basis.

Denotation:

Components for [contravariant basis](#) denoted in the upper indices:

$V^i = x$

The [inverse](#) of a [covariant](#) transformation is a [contravariant](#) transformation. Whenever a vector should be invariant under a change of [basis](#), that is to say it should represent the same geometrical or physical [object](#) having the same magnitude and direction as before, its components must transform according to the [contravariant](#) rule.

a. *

Contravariant cofunctor

Contravariant functor - More inline term is [Contravariant cofunctor](#)

- b. Covariant The [property](#) of [basis](#), in which if new [basis](#) is a [linear](#) combination of the prior [basis](#), and the change of [basis](#) proportional for a descriptions of [tensors](#) in basis.

Denotation:

Components for [covariant basis](#) denoted in the upper indices:

$$V_i = x$$

a. *

Covariant functor

Covariant cofunctor

29.3.1.7 Commutativity

$$\forall (a, b) \in S : P(a, b) \equiv P(b, a)$$

a. *

Commutative

Commutative law

29.3.1.8 Idempotence

First [application](#) gives a result. Then same [operation](#) can be [applied](#) multiple times without changing the result.

Example: Start and Stop buttons on machines.

a. *

Idempotent

Idempotency

29.3.1.9 Distributive property

Set S and two [binary](#) operators $+$ \times :

- $x \times (y + z) = (x \times y) + (x \times z)$ - \times is left-[distributive](#) over $+$
- $(y + z) \times x = (y \times x) + (z \times x)$ - \times is right-[distributive](#) over $+$
- left-&right-[distributive](#) - \times is [distributive](#) over $+$

a. *

Distributive rule

Distributive axiom

Distributive law

Distributive

29.4 Argument

arguere to make clear, to shine

* - evidence, proof, [statement](#) that results in system consequences.

29.4.1 Argument of a function

A value binded to the [function parameter](#). Value/topic that the fuction would process/deal with.

Also see Argument.

29.4.1.1 *

Function argument

29.5 Content word

Words that name [objects](#) of reality and their qualities.

29.6 Ancient Greek and Latin prefixes

29.6.1 *

Greek prefix

Latin prefix

29.7 Idiom

* - something having a meaning that can not be [derived](#) from the conjoined meanings.
Meaning can be special for language speakers or human with particular knowledge.

* can also mean [applicative functor](#).

29.7.1 *

Idioms

Idiomatic

29.8 Impredicative

Self-referencing definition.

—

Antonym - *[Predicative](#)*.

Table 29.1: Ancient Greek and Latin prefixes

Meaning	Greek prefix	Latin prefix
above, excess	hyper-	super-, ultra-
across, beyond, through	dia-	trans-
after		post-
again, back		re-
against	anti-	contra-, (in-, ob-)
all	pan	omni-
around	peri-	circum-
away or from	apo-, ap-	ab- (or de-)
bad, difficult, wrong	dys-	mal-
before	pro-	ante-, pre-
between, among		inter-
both	amphi-	ambi-
completely or very		de-, ob-
down		de-, ob-
four	tetra-	quad-
good	eu-	ben-, bene-
half, partially	hemi-	semi-
in, into	en-	il-, im-, in-, ir-
in front of	pro-	pro-
inside	endo-	intra-
large	macro-	(macro-, from Greek)
many	poly-	multi-
not*	a-, an-	de-, dis-, in-, ob-
on	epi-	
one	mono-	uni-
out of	ek-	ex-, e-
outside	ecto-, exo-	extra-, extro-
over	epi-	ob- (sometimes)
self	auto-, aut-, auth-	ego-
small	micro-	
three	tri-	tri-
through	dia-	trans-
to or toward	epi-	ad-, a-, ac-, as-
two	di-	bi-
under, insufficient	hypo-	sub-
with	sym-, syn-	co-, com-, con-
within, inside	endo-	intra-
without	a-, an-	dis- (sometimes)

29.9 Context-free grammar

Type of formal grammar that is: a **set** of production rules that describe all possible string is a given formal language.

Term is invented by Noam Chomsky.

29.9.1 *

CFG

Chapter 30

Set

Well-defined collection of distinct [objects](#).

30.1 *

Sets
Set theory

30.2 Closed set

- a. [Set](#) which complements an open [set](#).
- b. Is form of [Closed-form expression](#). [Set](#) can be [closed](#) in under a [set](#) of operations.

30.3 Power set

For some [set](#) \mathcal{S} , the [power set](#) ($\mathcal{P}(\mathcal{S})$) is a [set](#) of all subsets of \mathcal{S} , including $\{\}$ & \mathcal{S} itself.

Denotation:
 $\mathcal{P}(\mathcal{S})$

30.4 Singleton

[Singleton](#) - [unit set](#) - [set](#) with exactly one element.
Also 1-[sequence](#).

30.5 Russell's paradox

If there exists normal [set](#) of all [sets](#) - it should contain itself, which makes it abnormal.

30.6 Cartesian product

$\mathcal{A} \times \mathcal{B} \equiv \sum^{\forall} (a, b) \mid \forall a \in \mathcal{A}, \forall b \in \mathcal{B}.$

Operation, returns a **set** of all ordered pairs (a, b)

Any **function**, **functor** is a subset of **Cartesian product**.

$$\sum (elem \in (\mathcal{A} \times \mathcal{B})) = cardinality^{A \times B}$$

Properties:

- not **associative**
- not **commutative**

30.6.1 Pullback

Subset of the **cartesian product** of two **sets**.

30.6.1.1 *

Pullbacks

Chapter 31

Shrinking

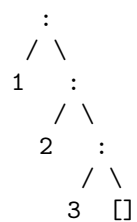
Process of reducing complexity in the test [case](#) - re-run with smaller values and make sure that the test still fails.

Chapter 32

Spine

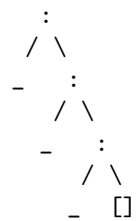
Is a chain of memory cells, each points to the both value of element and to the next memory cell.

Array:



1:2:3: []

Spine:



Chapter 33

Superclass

Broader parent class.

Chapter 34

Tensor

Object existing out of planes, thus it can translate **objects** from one plane into another. They can be tried to be described with knowledge existing inside planes, but representation would always be partial.

Tensor of rank 1 is a vector.

Translatioin with **tensor** can be seen as **functors**.

34.1 *

Tensors
Tensorial

Chapter 35

Testing

35.1 Property testing

Since [property](#) has a law, then family of that [unit](#) tests can be abstracted into the [lambda function](#).

And tests cases come from [generator](#).

35.1.1 Function property

[Property](#) corresponds to the according law.

In [property testing](#) you need to think additionally about [generator](#) and [shrinking](#).

35.1.2 Property testing type

Table 35.1: [Property testing types](#)

	Exhaustive	Randomized	Unit test
Whole set of values	Exhaustive property test	Randomised property test	One element
Special subset of values	Exhaustive specialised property test	Randomised specialised property test	One element

35.1.3 Generator

Seed
|
v
Gen A -> A
^
|
Size

Seed allows reproducibility.

There is anyway a need to have some seed.

Size allows setting upper [bound](#) on size of generated value. Think about infinity of [list](#).

After failed test - [shrinking](#) tests value parts of contrexample, finds a part that still fails, and recurses [shrinking](#).

35.1.3.1 *

Generators

35.1.3.2 Custom generator

When certain theorem only works for a specific [set](#) of values - the according [generator](#) needs to be produced.

```
arbitrary :: Arbitrary a => Gen a
suchThat :: Gen a -> (a -> Bool) -> Gen a
elements :: [a] -> Gen a
```

35.1.4 Reusing test code

Often it is convinient to [abstract testing](#) of same [function properties](#):

It can be done with (aka TestSuite [combinator](#)):

```
-- Definition
{-# LANGUAGE ScopedTypeVariables #-}
{-# LANGUAGE AllowAmbiguousTypes #-}
eqSpec :: forall a. Arbitrary a => Spec

-- Usage
{-# LANGUAGE TypeApplications #-}
spec :: Spec
spec = do
  eqSpec @Int

Eq Int
(==) :: Int -> Int -> Bool
  is reflexive
  is symetric
  is transitive
  is equivalent to (\ a b -> not $ a /= b)
(/=) :: Int -> Int -> Bool
  is antireflexive
  is equivalent to (\ a b -> not $ a == b)
```

35.1.4.1 Test Commutative property

[Commutativity](#)

```
:: Arbitrary a => (a -> a -> a) -> Property
```

35.1.4.2 Test Symmetry property

[Symmetry](#)

```
:: Arbitrary a => (a -> a -> Bool) -> Property
```


35.1.4.3 Test Equivalence property

Equivalence

```
:: (Arbitrary a, Eq b) => (a -> b) -> (a -> b) -> Property
```

35.1.4.4 Test Inverse property

```
:: (Arbitrary a, Eq b) => (a -> b) -> (b -> a) -> Property
```

35.1.5 QuickCheck

Target is a member of the [Arbitrary type class](#).

Target -> Bool is something Testable. This [properties](#) can be complex.

[Generator](#) arbitrary gets the seed, and produces values of Target.

[Function](#) quickCheck runs the loop and tests that generated Target values always comply the property.

35.1.5.1 Manual automation with QuickCheck properties

```
import Test.QuickCheck
import Test.QuickCheck.Function
import Test.QuickCheck.Property.Common
import Test.QuickCheck.Property.Functor
import Test.QuickCheck.Property.Common.Internal
```

```
data Four' a b = Four' a a a b
  deriving (Eq, Show)
```

```
instance Functor (Four' a) where
  fmap f (Four' a b c d) = Four' a b c (f d)
```

```
instance (Arbitrary a, Arbitrary b) => Arbitrary (Four' a b) where
  arbitrary = do
    a1 <- arbitrary
    a2 <- arbitrary
    a3 <- arbitrary
    b  <- arbitrary
    return (Four' a1 a2 a3 b)
```

```
-- Wrapper around `prop_FunctorId`
prop_AutoFunctorId [U+2237] Functor f => f a => Equal (f a)
prop_AutoFunctorId = prop_FunctorId T
```

```
type Prop_AutoFunctorId f a
  = f a
  => Equal (f a)
```

```
-- Wrapper around `prop_AutoFunctorCompose`
prop_AutoFunctorCompose [U+2237] Functor f => Fun a1 a2 => Fun a2 c => f a1 => Equal (f c)
prop_AutoFunctorCompose f1 f2 = prop_FunctorCompose (applyFun f1) (applyFun f2) T
```

```
type Prop_AutoFunctorCompose structureType origType midType resultType
  = Fun origType midType
  => Fun midType resultType
```

```
→ structureType origType
→ Equal (structureType resultType)

main = do
  quickCheck $ eq $ (prop_AutoFunctorId [U+2237] Prop_AutoFunctorId (Four' ()))Integer
  quickCheck $ eq $ (prop_AutoFunctorId [U+2237] Prop_AutoFunctorId (Four' ())) (Either Bool String Integer)
  quickCheck $ eq $ (prop_AutoFunctorCompose [U+2237] Prop_AutoFunctorCompose (Four' ())) String Integer
  quickCheck $ eq $ (prop_AutoFunctorCompose [U+2237] Prop_AutoFunctorCompose (Four' ())) Integer Integer
```

35.2 Write tests algorithm

- a. Pick the right language/[stack](#) to implement features.
- b. How expensive breakage can be.
- c. Pick the right tools to test this.

Chapter 36

Uncurry

Replace sequenced lambda [functions](#) into single [function](#) taking [sequence/product](#) of values as [argument](#).

Chapter 37

Unit

Represents existence. Denoted as empty [sequence](#).

[\(\)](#)

Type [\(\)](#) holds only self-representation [constructor](#) [\(\)](#), & [constructor](#) holds [nothing](#).

Haskell code always should receive something back, hence [nothing](#), emptiness, [void](#) can not be theoretically addressed, practically constructed or received - [unit](#) in Haskell also has a role of a stub in place of emptiness, like in IO [\(\)](#).

Chapter 38

Nullary

Takes no entries; has the [arity](#) of [zero](#).
Has the trivial [domain](#).

Chapter 39

Logic

39.1 Proposition

Purely abstract & theoretical logical [object](#) (idea) that has a Boolean value.

* is expressed by a [statement](#).

39.1.1 *

Propositions

39.1.2 Atomic proposition

Logically undividable [unit](#). Does not contain [logical connectives](#).

39.1.2.1 *

Atomic propositions

39.1.3 Compound proposition

Formed by connecting [propositions](#) by [logical connectives](#).

39.1.3.1 *

Compound propositions

39.1.4 Propositional logic

Studies [propositions](#) and [argument](#) flow.

Refers to logically indivisible units ([atomic propositions](#)) as such, for theory - they are [abstractions](#) with Boolean [properties](#).

Not Turing-complete, impossible to [construct](#) an [arbitrary](#) loop.

39.1.4.1 *

Proposition logic
 Proposition calculus
 Propositional calculus
 Statement logic
 Sentential logic
 Sentential calculus
 Zeroth-order logic

39.1.4.2 First-order logic

Notation systems that use [quantifiers](#), [relations](#), [variables](#) over non-logical [objects](#), allows the use of [expressions](#) that contain [variables](#).

Turing-complete.

Extension of a [propositional logic](#).

a. *

Predicate logic
 First-order predicate logic
 First-order predicate calculus

b. Second-order logic Extension over [first-order logic](#) that quantifies over [relations](#).*a.* Higher-order logic Extension over [second-order logic](#) that uses additional [quantifiers](#), stronger [semantics](#).

Is more expressive, but model-theoretic [properties](#) are less well-behaved.

39.2 Logical connective

Logical [operation](#).

39.2.1 *

Logical connectives

39.2.2 Conjunction

Logical AND.

Denotation:

\wedge

Multiplies [cardinalities](#).

Haskell [kind](#):

* *

39.2.3 Disjunction

Logical *OR*

Denotation:

\vee

Summs [cardinalities](#).

39.3 Predicate

[Function](#) with Boolean [codomain](#).

$P : X \rightarrow \{true, false\}$ - * on X .

Notation: $P(x)$

Almost always can include [relations](#), [quantifiers](#).

39.4 Statement

Declarative [expression](#) that is a bearer of a [proposition](#).

When we talk about [expression](#) or [statement](#) being true/false - in fact we refer to the [proposition](#) that they represent.

Difference between [proposition](#), [statement](#), [expression](#):

a. "2 + 3 = 5"

b. "two plus three equals five"

- 1 & 2 are [statements](#). Each of them is a collection of transmission symbols (linguistic [objects](#)) from a symbol systems \equiv [expression](#). Each of them is [expression](#) that bears [proposition](#) (an idea resulting in a Boolean value) \equiv [statement](#).
- 1 & 2 represent the same [proposition](#). [Proposition](#) from 1 \equiv [proposition](#) from 2.
- [Statement](#) 1 \neq [statement](#) 2. They are two different [statements](#), written in different systems. And [statement](#) "2 + 3 = 5" \neq [statement](#) "3 + 2 = 5".

39.4.1 *

Assertion

Assertions

Statements

39.5 Iff

If and only if, exactly when, just.

Denotation:

\iff

Chapter 40

Haskell structures

40.1 Pattern match

Are not [first-class](#). It is a [set](#) of patten match semantic notations.

Must be [linear](#).

* [precedence](#) (especially with more then one [parameter](#), especially with `_` used) often changes the [function](#).

40.1.1 As-pattern

```
f list@(x, xs) = ...
```

```
f (x:xs)    = x:x:xs -- Can be compiled with reconstruction of x:xs
```

```
f a@(x:_) = x:a -- Reuses structure without reconstruction
```

40.1.1.1 *

As-patterns

As pattern

As patterns

40.1.2 Wild-card

Matches anything and can not be binded. For matching someting that should pass not checked and is not used.

```
head (x:_)      = x
tail (_:xs)     = xs
```

40.1.2.1 *

Wild-cards

Wildcard

Wildcards

40.1.3 Case

```
case x of
  pattern1 -> ex1
  pattern2 -> ex2
  pattern3 -> ex3
  otherwise -> exDefault
```

Bolting [guards](#) & [expressions](#) with syntactic sugar on `case`:

```
case () of _
  | expr1      -> ex1
  | expr2      -> ex2
  | expr3      -> ex3
  | otherwise -> exDefault
```

Pattern matching in [function](#) definitions is realized with [case expressions](#).

40.1.4 Guard

Check values against the [predicate](#) and use the first match definition:

```
f x
  | predicate1 = definition1
  | predicate2 = definition2
  ...
  | x < 0      = definitionN
  ...
  | otherwise  = definitionZ
```

40.1.4.1 *

Guards

40.1.5 Pattern guard

Allows check a [list](#) of pattern matches against [functions](#), and then proceed.

$$(patternMatch1) <- (funcCheck1)$$

```
, (patternMatch2) <- (funcCheck2)
= RHS
```

```
lookup :: Eq a => a -> [(a, b)] -> Maybe b
```

```
addLookup l a1 a2
  | Just b1 <- lookup a1 l
  , Just b2 <- lookup a2 l
  = b1 + b2
{-...other equations...-}
```

Run [functions](#), they must succeed. Then [pattern match](#) results to `b1`, `b2`. Only if successful - execute the equation.

Default in Haskell 2010.

40.1.5.1 *

Pattern guards

40.1.6 Lazy pattern

Defers the [pattern match](#) directly to the last moment of need during execution of the code.

```
f (a, b) = g a b -- It would be checked that the pattern of the pair constructor
-- is present, and that parameters are present in the constructor.
-- Only after that success - work would start on the RHS, aka then construction
-- g would start only then.
```

```
f ~(a, b) = g a b -- Pattern match of (a, b) deferred to the last moment,
-- RHS starts, construction of g starts.
-- For this lazy pattern the equivalent implementation would be:
-- f p = g (fst p) (snd p) -- RHS starts, during construction of g
-- the arguments would be computed and found, or error would be thrown.
```

Due to full laziness deferring everything to the runtime execution - the [lazy pattern](#) is one-size-fits all ([irrefutable](#)), analogous to `_`, and so it does not produce any checks during compilation, and raises [errors](#) during runtime.

`*` is very useful during [recursive](#) construction of [recursive structure](#)/process, especially infinite.

40.1.6.1 *

Lazy-pattern
Lazy patterns

40.1.7 Pattern binding

Entire [LHS](#) is a pattern, is a [lazy pattern](#).

```
fib@(1:tfib) = 1 : 1 : [ a+b | (a,b) <- zip fib tfib ]
```

40.1.7.1 *

Pattern bindings

40.2 Smart constructor

Process/code placing extra rules & [constraints](#) on the construction of values.

40.3 Level of code

There are these levels of Haskell code:

40.3.1 *

Code level

40.3.2 Type level

Level of code that works with data types.

40.3.2.1 Type level declaration

```
type ...
newtype ...
data ...
class ...
instance ...
```

a. *

Type level declarations
Type-level declaration
Type-level declarations

40.3.2.2 Type check

if The type level information is complete (strongly connected graph)

then

Generalize the types and check if type level consistent to term level.

else

Infer the missing type level part from the term level. There are certain situations and structures where ambiguity arises and is unsolvable from the information of the term level (most basic example is polymorphic recursion).

a. *

Typecheck
Typechecking
Typechecks

- b. Complete user-specific kind signature Type level declaration is considered to "have a CUSK" is it has enough syntactic information to warrant completeness (strongly connected graph) and start checking type level correspondence to term level, it is a ad-hock state of type inferring.

In the future GHC would use other algorithm over/instead of CUSK.

- a.* *
 - CUSK
 - CUSKs
 - Complete user-specific kind signatures
 - Complete, user-specific kind signature

40.3.3 Term level

[Level of code](#) that does logical execution.

40.3.4 Compile level

[Level of code](#), about compilation processes/results.

40.3.4.1 *

Compilation level

40.3.5 Runtime level

[Level of code](#) of main program [operation](#), when machine does computations with compiled [binary](#) code.

40.3.6 Kind level

[Level of code](#) where [kinds](#) & [kind](#) declarations are situated, inferred and checked.

40.3.6.1 Kind check

Applying the [type check](#) to [kind](#) check:

if The [kind](#) level information is complete ([strongly connected](#) graph)

then

Check if [kind](#) level consistent to [term level](#).

else

Infer the missing [kind](#) level parts from the [type level](#). There are certain situations and [structures](#) where ambiguity arises and is unsolvable from the information of the [kind](#) level.

With `StandaloneKindSignatures` [kind](#) completeness happens against found (standalone) [kind](#) signature.

With `CUSKs` extension kind completeness happens against "[complete user-specific kind signature](#)"

- a. *
Kindcheck
Kind checks

40.4 Orphan type instance

Hanging [type](#) instance from inconsistent code base.

- a. Supporting [structure](#) not fully present.
- b. Several implementations of instance present.

40.5 Undefined

Placeholder value that helps to do [typechecking](#).

40.6 Hierarchical module name

Hierarchical naming scheme:

```
Algebra                -- Was this ever used?
  DomainConstructor    -- formerly DoCon
  Geometric             -- formerly BasGeomAlg

Codec                  -- Coders/Decoders for various data formats
  Audio
    Wav
    MP3
    ...
  Compression
    Gzip
    Bzip2
    ...
  Encryption
    DES
    RSA
    BlowFish
    ...
  Image
    GIF
    PNG
    JPEG
    TIFF
    ...
  Text
    UTF8
    UTF16
    ISO8859
```

```

    ...
Video
  Mpeg
  QuickTime
  Avi
  ...
Binary          -- these are for encoding binary data into text
  Base64
  Yenc

Control
  Applicative
  Arrow
  Exception      -- (opt, inc. error & undefined)
  Concurrent     -- as hslibs/concurrent
    Chan         -- these could all be moved under Data
    MVar
    Merge
    QSem
    QSemN
    SampleVar
    Semaphore
  Parallel       -- as hslibs/concurrent/Parallel
  Strategies
  Monad          -- Haskell 98 Monad library
    ST           -- ST defaults to Strict variant?
      Strict     -- renaming for ST
      Lazy       -- renaming for LazyST
    State        -- defaults to Lazy
      Strict
      Lazy
    Error
    Identity
    Monoid
    Reader
    Writer
    Cont
    Fix          -- to be renamed to Rec?
    List
    RWS

Data
  Binary         -- Binary I/O
  Bits
  Bool           -- &&, ||, not, otherwise
  Tuple         -- fst, snd
  Char           -- H98
  Complex        -- H98
  Dynamic
  Either
  Int
  Maybe          -- H98
  List           -- H98
  PackedString
  Ratio          -- H98

```



```

Word
IORef
STRef          -- Same as Data.STRef.Strict
    Strict
    Lazy        -- The lazy version (for Control.Monad.ST.Lazy)
Binary          -- Haskell binary I/O
Digest
    MD5
    ...         -- others (CRC ?)
Array           -- Haskell 98 Array library
    Unboxed
    IArray
    MArray
    IO          -- mutable arrays in the IO/ST monads
    ST
Trees
    AVL
    RedBlack
    BTree
Queue
    Bankers
    FIFO
Collection
Graph           -- start with GHC's DiGraph?
FiniteMap
Set
Memo            -- (opt)
Unique

Edison          -- (opt, uses multi-param type classes)
    Prelude     -- large self-contained packages should have
    Collection  -- their own hierarchy? Like a vendor branch.
    Queue       -- Or should the whole Edison tree be placed

Database
    MySQL
    PostgreSQL
    ODBC

Dotnet
    ...         -- Mirrors the MS .NET class hierarchy

Debug           -- see also: Test
    Trace
    Observe     -- choose a default amongst the variants
        Textual    -- Andy Gill's release 1
        ToXmlFile  -- Andy Gill's XML browser variant
        GHood      -- Claus Reinke's animated variant

Foreign
    Ptr
    StablePtr
    ForeignPtr  -- rename to FinalisedPtr? to void confusion with Foreign.Ptr
    Storable
    Marshal

```

```
    Alloc
    Array
    Errors
    Utils
C
    Types
    Errors
    Strings

GHC
    Exts          -- hslibs/lang/GlaExts
    ...

Graphics
    HGL
    Rendering
        Direct3D
        FRAN
        Metapost
        Inventor
        Haven
        OpenGL
            GL
            GLU
    Pan
UI
    FranTk
    Fudgets
    GLUT
    Gtk
    Motif
    ObjectIO
    TkHaskell
X11
    Xt
    Xlib
    Xmu
    Xaw

Hugs
    ...

Language
    Haskell      -- hslibs/hssource
        Syntax
        Lexer
        Parser
        Pretty
    HaskellCore
    Python
    C
    ...

Nhc
    ...
```

```

Numeric          -- exports std. H98 numeric type classes
  Statistics

Network          -- (== hslibs/net/Socket), depends on FFI only
  BER            -- Basic Encoding Rules
  Socket         -- or rename to Posix?
  URI            -- general URI parsing
  CGI            -- one in hslibs is ok?
  Protocol
    HTTP
    FTP
    SMTP

Prelude          -- Haskell98 Prelude (mostly just re-exports
                  other parts of the tree).

Sound            -- Sound, Music, Digital Signal Processing
  ALSA
  JACK
  MIDI
  OpenAL
  SC3            -- SuperCollider

System           -- Interaction with the "system"
  Cmd            -- ( system )
  CPUTime        -- H98
  Directory      -- H98
  Exit           -- ( ExitCode(..), exitWith, exitFailure )
  Environment    -- ( getArgs, getProgName, getEnv ... )
  Info           -- info about the host system
  IO             -- H98 + IOExts - IOArray - IORef
    Select
    Unsafe       -- unsafePerformIO, unsafeInterleaveIO
  Console
    GetOpt
    Readline
  Locale         -- H98
  Posix
    Console
    Directory
    DynamicLinker
      Prim
      Module
    IO
    Process
    Time
  Mem            -- rename from cryptic 'GC'
    Weak         -- (opt)
    StableName   -- (opt)
  Time          -- H98 + extensions
  Win32         -- the full win32 operating system API

Test
  HUnit

```

```

QuickCheck

Text
  Encoding
    QuotedPrintable
    Rot13
  Read
    Lex          -- cut down lexer for "read"
  Show
    Functions    -- optional instance of Show for functions.
  Regex         -- previously RegexString
  Posix         -- Posix regular expression interface
  PrettyPrint   -- default (HughesPJ?)
    HughesPJ
    Wadler
    Chitil
    ...
  HTML          -- HTML combinator lib
  XML
    Combinators
    Parse
    Pretty
    Types
  ParserCombinators -- no default
    ReadP       -- a more efficient "ReadS"
    Parsec
    Hutton_Meijer
    ...

Training          -- Collect study and learning materials
  <name of the tutor>

```

40.6.1 *

Top-level module name
 Top-level module names

40.7 import

`import` [statement](#) by default imports identifiers from the other [module](#), using [hierarchical module name](#), brings into [scope](#) the identifiers to the global [scope](#) both into unqualified and qualifies by the [hierarchical module name](#) forms.

This possibilities can mix and match:

- `<modName> ()` - `import` only instances of [type classes](#).
- `<modName> (x, y)` - `import` only declared indentifiers.
- `qualified <modName>` - discards unqualified names, forse obligatory namespace for the imports.

- `hiding (x, y)` - skip `import` of declared identifies.
- `<modName> as <modName>` - renames `module` namespace.
- `<type/class> (..)` - `import` class & it's methods, or `type`, all its data `constructors` & field names.

40.8 Let

* `expression` is a `set` of cross-recursive lazy pattern bindings.

Declarations permitted:

- `type` signatures
- `function bindings`
- `pattern bindings`

It is an `expression` (macro) and that integrates in external `lexical scope expression` it `applied` in.

Form:

```
let
  b1
  bn
in
  c
```

40.8.1 *

Let expression
Let expressions

40.9 Where

Part of the syntax of the whole `function declaration`, has according `scope`.

As part of whole `declaration` - can extend over definitions of the function (pattern matches, `guards`).

Form:

```
f match1 = y
f match2 = y
f x =
  | cond1 x = y
```

```

| cond2 x = y
| otherwise = y
where
y = ... x ...

```

40.9.1 *

Where clause

40.10 Reserved word

Haskell has special meaning for:

`case`, `class`, `data`, `deriving`, `do`, `else`, `if`, `import`, `in`, `infix`, `infixl`, `infixr`, `instance`, `let`, `of`, `module`

40.10.1 *

Reserved words

40.11 Haskell Language Report

Document that is a standart of language.

40.11.1 *

Report
Haskell Report
Haskell 98 Language Report
Haskell 98 Report
Haskell 1998 Language Report
Haskell 2010 Language Report
Haskell 2010 Report

40.12 Haskell'

Current language development mod.

<https://prime.haskell.org/>

40.12.1 *

Haskell prime

Chapter 41

Computer science

41.1 Guerrilla patch

* changing code/[applying](#) patch sneakily - and possibility incompatibility with other at runtime. [Monkey patch](#) is derivative term.

41.1.1 Monkey patch

From [Guerrilla patch](#).

* is a way for program to modify supporting system software affecting only the running instance of the program.

41.2 Interface

Point of mutual meeting. Code behind [interface](#) determines how data is consumed.

41.3 Module

Importable organizational [unit](#).

41.4 Scope

Area [where binds](#) are accessible.

41.4.1 Dynamic scope

The name resolution depends upon the program state when the name is encountered, which is determined by the execution [context](#) or calling [context](#).

41.4.2 Lexical scope

[Scope bound](#) by the [structure](#) of source code [where](#) the named entity is defined.

41.4.2.1 *

Static scope

41.4.3 Local scope

Scope applies only in (current) area.

41.4.3.1 *

Local

41.5 Shadowing

When in the local scope bigger scope variable overridden by same name variable from the local scope.

41.6 Syntactic sugar

Artificial way to make language easier to read and write.

41.7 System F

Is parametric polymorphism in programming.

Extends the Lambda calculus by introducing \forall (universal quantifier) over types.

41.7.1 *

Girard–Reynolds polymorphic lambda calculus
Girard-Raynolds

41.8 Tail call

Final evaluation inside the function. Produces the function result.

41.9 Thunk

Not evaluated calculation. Can be dragged around, until be lazily evaluated.

Table 41.1: Application memory structural parts

Storage of	Block name
All not currently processing data	Heap
Function call, local variables	Stack
Static and global variables	Static/Global
Instructions	Binary code

41.10 Application memory

When even Main invoked - it work in [Stack](#), and called [Stack](#) frame. [Stack](#) frame size for [function](#) calculated when it is compiled.

When stacked [Stack](#) frames exceed the [Stack](#) size - [stack](#) overflow happens.

41.11 Turing machine

Mathematical model of computation that defines [abstract Turing machine](#). [Abstract](#) machine which manipulates symbols on a strip of tape, according to a table of rules.

41.11.1 Turing complete

[Set](#) of action rules that can simulate any [Turing machine](#).

41.11.1.1 *

Turing incomplete

Turing incompleteness

Turing completeness

Computationally universal

Chapter 42

Graph theory

42.1 Successor

Object that receives the **arrow**.

42.1.1 Direct successor

Immediate **successor**.

42.2 Predecessor

Object that sends **arrow**.

42.2.1 Direct predecessor

Immediate **predecessor**.

42.3 Degree

Number of **arrows** of **object**.

42.3.1 Indegree

Number of ingoing **arrows**.

42.3.2 Outdegree

Number of outgoing **arrows**.

42.4 Adjacency matrix

Matrix of connection of objects $\{-1, 0, 1\}$.

42.4.0.1 InstanceSigs

Allow adding [type](#) signatures to [type class function](#) instance [declaration](#).

42.5 Strongly connected

If every vertex in a graph is reachable from every other vertex.

It is possible to find all [strongly connected components](#) (and that way also test graph for strong connectivity), in [linear](#) time ($\Theta(V+E)$).

[Binary relation](#) of being [strongly connected](#) is an [equivalence relation](#).

42.5.1 *

Strongly-connected

42.5.2 Strongly connected component

Full [strongly connected](#) subgraph of some graph.

* of a directed graph G is a subgraph that is [strongly connected](#), and has [property](#): no additional edges or vertices from G can be included in the subgraph without breaking its [property](#) of being [strongly connected](#).

42.5.2.1 *

SCC

Strongly connected components

Strongly-connected component

Strongly-connected components

Chapter 43

Linear

Values consumed once or not used.

x^2 consumes x two times.

43.1 *

Linearity

Chapter 44

Stream

* an infinite **sequence** that forgets previous **objects**, and remembers only currently relevant **objects**.

$E \mid X \rightarrow (X \times A + 1)$, the **set** (or **object**) of streams on A (final **coalgebra** A_* of E).

cycle is one of **stream functions**.

```
a = (cycle [Nothing, Nothing, Just "Fizz"])
b = (cycle [Nothing, Nothing, Nothing, Nothing, Just "Buzz"])
```

Can be:

- indexed, timeless, with current **object**
- timed:

```
* [(timescale, event)]
* [(realtime, event)]
```

Has amalgamation with Functional Reactive Programming.

Chapter 45

Bisimulation

When systems have exact external behaviour so for observer they are the same.

[Binary relation](#) between state transition systems that match each other's moves.

45.1 *

Bisimilar

Chapter 46

Syntax tree

Tree of syntactic elements (each **node** denotes **construct** occurring in the source code) that represent the source code (or human language).

46.1 Abstract syntax tree

"**Abstract**" since does not represent every detail of the syntax (ex. parentheses), but rather concentrates on **structure** and content.

Widely used in compilers to check the code **structure** for accuracy and coherence.

46.1.1 *

AST

46.2 Concrete syntax tree

An ordered, rooted **syntax tree** that represents the syntactic **structure** of a string according to some **context-free grammar**.

"Concrete" since (in contrast to "**abstract**") - concretely reflects the syntax of the input language.

46.2.1 *

Parse tree

Derivation tree

Chapter 47

Domain specific language

Language design/fitted for particular [domain](#) of [application](#). Mainly should be [Turing incomplete](#), since general-purpose language implies [Turing completeness](#).

47.1 *

Domain-specific language
DSL

47.2 Embedded domain specific language

[DSL](#) used inside outer language.

Two levels of embedding:

- Shallow: [DSL](#) translates into Haskell directly
- Deep: Between [DSL](#) and Haskell there is a [data structure](#) that reflects the [expression tree](#), AKA stores the [syntax tree](#).

47.2.1 *

eDSL

Chapter 48

Tagless-final

Method of embedding **eDSL** in a typed functional host language (Haskell). **Alternative** to the embedding as a (generalized) **algebraic data type**. For parsers of DLS **expressions**: (1/partial) evaluator, compiler, pretty printer, multi-pass optimizer.

* embedding is writing **denotational semantics** for the **DSL** in the host language.

Approach can be used **iff eDSL** is typed. Only well-typed terms become embeddable, and host language can implemen also a **eDSL type** system. Approach that **eDSL** code interpretations are **type-preserving**.

One of main pros of * - extensibility: implementation of **DSL** can be used to analyze/evaluate/transform/pretty-print/compile and interpreters can be extended to more passes, optimizations, and new versions of **DSL** while keeping/using/reusing the old versions.

Example fields of **application**: language-integrated queries, non-deterministic & probabilistic programming, delimiter continuation, computability theory, **stream** processsing, hardware description languages, generation of specialized numerical kernels, **semantics** of natural language.

Part III

Give definitions

Chapter 49

Identity type

Chapter 50

Constant type

Chapter 51

Gen

Chapter 52

Tensorial strength

Chapter 53

Strong monad

Chapter 54

Weak head normal form

54.1 * WHNF

Chapter 55

Function image

55.1 *

Image

Chapter 56

Invertible

Chapter 57

Invertibility

Chapter 58

Define LANGUAGE pragma options

58.1 ExistentialQuantification

58.2 GADTs

GADT is a generalization over parametric [algebraic data types](#) which allow explicitly denote the [types](#) ([type matching](#)) of the [constructors](#) and define [data types](#) using pattern matching on the left side of "data" [statements](#).

58.3 *

GADT

Generalized algebraic data type

First-class phantom data type

Guarded recursive data type

Equality-qualified data type

58.4 GeneralizedNewTypeClasses

58.5 FuncitonalDependencies

Chapter 59

GHC check keys

59.1 -Wno-partial-type-signatures

Supresses [PartialTypeSignatures wildcard infer](#) warning.

Chapter 60

Generalised algebraic data types

LANGUAGE [GADTs](#)

60.1 *

GADT

Chapter 61

Order theory

Investigates in thepht the intuitive notion of [order](#) using [binary relations](#).

61.1 Domain theory

Formalizes approximation and convergense.
Has close [relation](#) to Topology.

61.2 Lattice

[Abstract structure](#) that consists of [partially ordered set](#), [where](#) every two elements have unique supremum and infimum. \Rightarrow * [algebraic structure](#) satisfying certain axiomatic identities.
* [order-theory](#) & [algebraic](#).

61.3 Order

61.3.1 Preorder

$R^X \rightarrow X : \text{Reflexive} \ \& \ \text{Transitive}:$
 aRa
 $aRb, bRc \Rightarrow aRc$

Generalization of [equivalence relations](#) [partial orders](#).

* [Antisymmetric](#) \Rightarrow Partial ordering.
* [Symmetric](#) \Rightarrow [Equivalence](#).

61.3.1.1 *

Preordered

61.3.1.2 Total preorder

$\forall a, b : a \leq b \vee b \leq a \Rightarrow \text{Total Preorder}.$

61.3.2 Partial order

A [binary relation](#) must be [reflexive](#), [antisymmetric](#) and [transitive](#).

Partial - not every elements between them need to be comparable.

Good example of $*$ is a genealogical descendancy. Only related people produce [relation](#), not related do not.

61.3.2.1 $*$

Partial orders

Partially ordered set

Partially ordered sets

Poset

Posets

61.4 Partial order

61.5 Total order

Chapter 62

Universal algebra

Studies [algebraic structures](#).

Chapter 63

Relation

63.1 Reflexivity

$$R^{X \rightarrow X}, \forall x \in X : xRx$$

[Order](#) theory: $a \leq a$

* - each element is comparable to itself.

Corresponds to [Identity](#) and [Automorphism](#).

63.1.1 *

Reflexive

Reflexive relation

63.2 Irreflexivity

$$R^{X \rightarrow X}, \forall x \in X : \nexists R(x, x)$$

63.2.1 *

Anti-reflexive

Anti-reflexive relation

Irreflexive

Irreflexive relation

63.3 Transitivity

$$\forall a, b, c \in X, \forall R^{X \rightarrow X} : (aRb \wedge bRc) \Rightarrow aRc$$

* - the start of a chain of [precedence relations](#) must precede the end of the chain.

63.3.1 *

Transitive
Transitive relation

63.4 Symmetry

$$\forall a, b \in X : (aRb \iff bRa)$$

63.4.1 *

Symmetric
Symmetric relation

63.5 Equivalence

Reflexive	Symmetric	Transitive
$\forall x \in X, \exists R : xRx$ $a = a$	$\forall a, b \in X : (aRb \iff bRa)$ $a = b \iff b = a$	$\forall a, b, c \in X, \forall R^{X \rightarrow X} : (aRb \wedge bRc) \Rightarrow aRc$ $a = b, b = c \Rightarrow a = c$

63.5.1 *

Equivalent
Equivalent relation

63.6 Antisymmetry

$\forall a, b \in X : aRb, bRa \Rightarrow a = b \sim aRb, a \neq b \Rightarrow \nexists bRa.$
[Antisymmetry](#) does not say anything about $R(a, a)$.

* - no two different elements precede each other.

63.6.1 *

Antisymmetric
Antisymmetric relation

63.7 Asymmetry

$\forall a, b \in X (aRb \Rightarrow \neg(bRa))$
* \iff [Antisymmetric](#) \wedge [Irreflexive](#).
[Asymmetry](#) \neq "not symmetric"
[Symmetric](#) \wedge [Asymmetric](#) is only empty relation.

63.7.1 *

Asymmetric

Asymmetric relation

Chapter 64

Cryptomorphism

[Equivalent](#), interconvertable with no loss of information.

64.1 *

Crypromorphic

Chapter 65

Lexically scoped type variables

Enable [lexical scope](#) for [forall quantifier](#) defined [type variables](#)

Implemented in [ScopedTypeVariables](#)

Chapter 66

Abstract data type

Several definitions here, reduce them.

Data type mathematical model, defined by its **semantics** from the user point of view, listing possible values, operations on the data of the **type**, and behaviour of these operations.

* class of **objects** whose logical behaviour is defined by a **set** of values and **set** of operations (analogue to **algebraic structure** in mathematics).

A specification of a **data type** like a **stack** or queue **where** the specification does not contain any implementation details at all, only the operations for that **data type**. This can be thought of as the contract of the **data type**.

66.1 *

AbsDT

Chapter 67

Concrete type

Fully defined [type](#). Non-[polymorphic type](#).

Chapter 68

Functional dependencies

Chapter 69

MonoLocalBinds

Chapter 70

KindSignatures

Chapter 71

ExplicitNamespaces

Chapter 72

Combinator pattern

Chapter 73

Symbolic expression

Nested tree [data structure](#).

Introduced & used in Lisp. Lisp code and data are $*$.

$*$ in Lisp: Atom or [expression](#) of the form $(x \ . \ y)$, x and y are $*$.

Modern abbreviated notation of $*$: $(x \ y)$.

73.1 $*$

S-expression
S-expressions
Sexpression
Sexpressions
Sexp
Sexprs
Sexpr
Sexprs

Chapter 74

Polynomial

[Expression](#) consisting of:

- [variables](#)
- coefficients
- addition
- subtraction
- multiplication (including positive integer [variable](#) exponentiation)

[Polynomials](#) form a [ring](#). [Polynomial ring](#).

74.1 *

Polynomials

Chapter 75

Data family

Indexed form of data and newtype definitions.

Chapter 76

Type synonym family

Indexed form of [type](#) synonyms.

Chapter 77

Indexed type family

* additional structure in language that allows ad-hoc overloading of [data types](#). AKA are to [types](#) as [type class](#) to methods.

Varieties:

- [data family](#)
- [type](#) synonym families

Defined by pattern matching the partial [functions](#) between [types](#).

Associates [data types](#) by [type](#)-level [function](#) defined by open-ended collection of valid instances of input [types](#) and corresponding output [types](#).

Normal [type classes](#) define partial [functions](#) from [types](#) to a collection of named values by pattern matching on the input [types](#), while [type](#) families define partial [functions](#) from [types](#) to [types](#) by pattern matching on the input [types](#). In fact, in many uses of [type](#) families there is a single [type class](#) which logically contains both values and [types](#) associated with each instance. A [type family](#) declared inside a [type class](#) is called an associated [type](#).

77.1 *

Type family

Chapter 78

TypeFamilies

Allow use and definition of indexed [type](#) families and data families.

- * are [type](#)-level programming.
- * are overload [data types](#) in the same way that [type classes](#) overload [functions](#).
- * allow handling of [dependent types](#). Before it [Functional dependencies](#) and [GADTs](#) were used to solve that.
- * useful for generic programming, creating highly parametrised interfaces for libraries, and creating interfaces with enhanced static information (much like [dependent types](#)).

Implies: [MonoLocalBinds](#), [KindSignatures](#), [ExplicitNamespaces](#)

Two [types](#) of * are:

Chapter 79

Error

Mistake in the program that can be resolved only by fixing the program.

`error` is a sugar for `undefined`.

Distinct from [Exception](#).

79.1 *

Errors

Chapter 80

Exception

Expected but irregular situation.

Distinct from [Error](#). Also see Exception vs Error

80.1 *

Exceptions

Chapter 81

ConstraintKinds

`Constraints` are just handled as `types` of a particular `kind` (`Constraint`). Any `type` of the `kind` `Constraints` can be used as a `constraint`.

- Anything which is already allowed in code as a `constraint` without `*`. Saturated applications to `type classes`, implicit `parameter` and equality `constraints`.
- `Tuples`, all of whose component `types` have `kind` `Constraint`.

```
type Some a = (Show a, Ord a, Arbitrary a) -- is of kind Constraint.
```

- Anything form of which is not yet known, but the user has declared for it to have `kind` `Constraint` (for which they need to `import` it from `GHC.Exts`):

```
Foo (f :: Type -> Constraint) = forall b. f b => b -> b -- is allowed
-- as well as examples involving type families:
type family Typ a b :: Constraint
type instance Typ Int b = Show b
type instance Typ Bool b = Num b

func :: Typ a b => a -> b -> b
func = ...
```

Chapter 82

Specialisation

Turns [ad hoc polymorphic function](#) into compiled [type](#)-specific implementations.

82.1 *

Specialise
Specialize
Specialization

Chapter 83

Diagram

For [categories](#) C and J , a [diagram](#) of [type](#) J in C is a [covariant functor](#) $D : J \rightarrow C$.

Chapter 84

Category theoretical presheaf

For [categories](#) C and J , a J -[presheaf](#) on C is a [contravariant functor](#) $D : C \rightarrow J$.

Chapter 85

Topological presheaf

If X is a topological space, then the open sets in X form a partially ordered set $\text{Open}(X)$ under inclusion. Like every partially ordered set, $\text{Open}(X)$ forms a small category by adding a single arrow $U \rightarrow V$ if and only if $U \subseteq V$. Contravariant functors on $\text{Open}(X)$ are called presheaves on X . For instance, by assigning to every open set U the associative algebra of real-valued continuous functions on U , one obtains a presheaf of algebras on X .

Chapter 86

Diagonal functor

The [diagonal functor](#) is defined as the [functor](#) from D to the [functor category](#) D^C which sends each [object](#) in D to the [constant functor](#) at that [object](#).

Chapter 87

Limit functor

For a fixed index [category](#) J , if every [functor](#) $J \rightarrow C$ has a limit (for instance if C is complete), then the [limit functor](#) $C^J \rightarrow C$ assigns to each [functor](#) its limit. The existence of this [functor](#) can be proved by realizing that it is the right-adjoint to the [diagonal functor](#) and invoking the Freyd adjoint [functor](#) theorem. This requires a suitable version of the axiom of choice. Similar remarks [apply](#) to the colimit [functor](#) (which is [covariant](#)).

Chapter 88

Dual vector space

The map which assigns to every vector space its [dual](#) space and to every [linear](#) map its [dual](#) or transpose is a [contravariant functor](#) from the [category](#) of all vector spaces over a fixed field to itself.

Chapter 89

Fundamental group

Consider the [category](#) of pointed topological spaces, i.e. topological spaces with distinguished points. The [objects](#) are pairs (X, x_0) , [where](#) X is a topological space and x_0 is a point in X . A [morphism](#) from (X, x_0) to (Y, y_0) is given by a continuous map $f : X \rightarrow Y$ with $f(x_0) = y_0$.

To every topological space X with distinguished point x_0 , one can define the [fundamental group](#) based at x_0 , denoted $\pi_1(X, x_0)$. This is the [group](#) of [homotopy](#) classes of loops based at x_0 . If $f : X \rightarrow Y$ is a [morphism](#) of pointed spaces, then every loop in X with base point x_0 can be [composed](#) with f to yield a loop in Y with base point y_0 . This [operation](#) is compatible with the [homotopy equivalence relation](#) and the [composition](#) of loops, and we get a [group homomorphism](#) from $\pi_1(X, x_0)$ to $\pi_1(Y, y_0)$. We thus obtain a [functor](#) from the [category](#) of pointed topological spaces to the [category](#) of [groups](#).

In the [category](#) of topological spaces (without distinguished point), one considers [homotopy](#) classes of generic curves, but they cannot be [composed](#) unless they share an endpoint. Thus one has the fundamental groupoid instead of the [fundamental group](#), and this construction is [functorial](#).

Chapter 90

Algebra of continuous function

A [contravariant functor](#) from the [category](#) of topological spaces (with continuous maps as [morphisms](#)) to the [category](#) of real [associative algebras](#) is given by assigning to every topological space X the [algebra](#) $C(X)$ of all real-valued continuous [functions](#) on that space. Every continuous map $f : X \rightarrow Y$ induces an [algebra homomorphism](#) $C(f) : C(Y) \rightarrow C(X)$ by the rule $C(f)(\varphi) = \varphi \circ f$ for every φ in $C(Y)$.

Chapter 91

Tangent and cotangent bundle

The map which sends every differentiable manifold to its tangent bundle and every smooth map to its derivative is a [covariant functor](#) from the [category](#) of differentiable manifolds to the [category](#) of vector bundles.

Doing this constructions pointwise gives the tangent space, a [covariant functor](#) from the [category](#) of pointed differentiable manifolds to the [category](#) of real vector spaces. Likewise, cotangent space is a [contravariant functor](#), essentially the [composition](#) of the tangent space with the [dual](#) space above.

Chapter 92

Group action / representation

Every [group](#) G can be considered as a [category](#) with a single [object](#) whose [morphisms](#) are the elements of G . A [functor](#) from G to [Set](#) is then [nothing](#) but a [group](#) action of G on a particular [set](#), i.e. a G -[set](#). Likewise, a [functor](#) from G to the [category](#) of vector spaces, Vect_K , is a [linear](#) representation of G . In general, a [functor](#) $G \rightarrow C$ can be considered as an "action" of G on an [object](#) in the [category](#) C . If C is a [group](#), then this action is a [group homomorphism](#).

Chapter 93

Lie algebra

Assigning to every real (complex) Lie [group](#) its real (complex) [Lie algebra](#) defines a [functor](#).

Chapter 94

Tensor product

If \mathcal{C} denotes the [category](#) of vector spaces over a fixed field, with [linear](#) maps as [morphisms](#), then the [tensor product](#) $V \otimes W$ defines a [functor](#) $\mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}$ which is [covariant](#) in both arguments.

Chapter 95

Forgetful functor

The functor $U : \mathbf{Grp} \rightarrow \mathbf{Set}$ which maps a group to its underlying set and a group homomorphism to its underlying function of sets is a functor.[8] Functors like these, which "forget" some structure, are termed forgetful functors. Another example is the functor $\mathbf{Rng} \rightarrow \mathbf{Ab}$ which maps a ring to its underlying additive abelian group. Morphisms in \mathbf{Rng} (ring homomorphisms) become morphisms in \mathbf{Ab} (abelian group homomorphisms).

Chapter 96

Free functor

Going in the opposite direction of [forgetful functors](#) are free [functors](#). The free functor $F : \mathbf{Set} \rightarrow \mathbf{Grp}$ sends every [set](#) X to the free [group](#) generated by X . [Functions](#) get mapped to [group](#) homomorphisms between free [groups](#). Free constructions exist for many [categories](#) based on structured [sets](#). See [free object](#).

Chapter 97

Homomorphism group

To every pair A, B of abelian groups one can assign the abelian group $\text{Hom}(A, B)$ consisting of all group homomorphisms from A to B . This is a functor which is contravariant in the first and covariant in the second argument, i.e. it is a functor $\text{Abop} \times \text{Ab} \rightarrow \text{Ab}$ (where Ab denotes the category of abelian groups with group homomorphisms). If $f : A_1 \rightarrow A_2$ and $g : B_1 \rightarrow B_2$ are morphisms in Ab , then the group homomorphism $\text{Hom}(f, g) : \text{Hom}(A_2, B_1) \rightarrow \text{Hom}(A_1, B_2)$ is given by $\varphi \mapsto g \circ \varphi \circ f$. See Hom functor.

Chapter 98

Representable functor

We can generalize the previous example to any [category](#) C . To every pair X, Y of [objects](#) in C one can assign the [set](#) $\text{Hom}(X, Y)$ of [morphisms](#) from X to Y . This defines a [functor](#) to [Set](#) which is [contravariant](#) in the first [argument](#) and [covariant](#) in the second, i.e. it is a [functor](#) $C^{\text{op}} \times C \rightarrow \text{Set}$. If $f : X_1 \rightarrow X_2$ and $g : Y_1 \rightarrow Y_2$ are [morphisms](#) in C , then the [group homomorphism](#) $\text{Hom}(f, g) : \text{Hom}(X_2, Y_1) \rightarrow \text{Hom}(X_1, Y_2)$ is given by $\varphi \mapsto g \circ \varphi \circ f$.

[Functors](#) like these are called representable [functors](#). An important goal in many settings is to determine whether a given [functor](#) is representable.

Chapter 99

Corecursion

Chapter 100

Coinduction

proper definition

* [dual](#) to induction.

Generalises to [corecursion](#).

Chapter 101

Initial algebra of an endofunctor

Chapter 102

Terminal coalgebra for an endofunctor

Part IV

Citations

"One of the finer points of the Haskell community has been its propensity for recognizing [abstract](#) patterns in code which have well-defined, lawful representations in mathematics." (Chris Allen, Julie Moronuki - "Haskell Programming from First Principles" (2017))

Part V

Good code

Chapter 103

Good: Type aliasing

Use [data type](#) aliases to deferentiate logic of values.

Chapter 104

Good: Type wideness

Wider the [type](#) the more it is [polymorphic](#), means it has broader [application](#) and fits more [types](#).

The more constrained system has more usefulness.

Unconstrained means most flexible, but also most useless.

Chapter 105

Good: Print

```
print :: Show a => a -> IO ()  
print a = putStrLn (show a)
```

Chapter 106

Good: Fold

`foldr` [spine recursion](#) intermediated by the folding. Can terminate at any point.

`foldl` [spine recursion](#) is unconditional, then folding starts. Unconditionally recurses across the whole [spine](#), if it infinite - infinitely.

Chapter 107

Good: Computation model

Model the [domain](#) and [types](#) before thinking about how to write computations.

Chapter 108

Good: Make bottoms only local

Chapter 109

Good: Newtype wrap is ideally transparent for compiler and does not change performance

Chapter 110

Good: Instances of types/type classes must go with code you write

Chapter 111

Good: Functions can be abstracted
as arguments

Chapter 112

Good: Infix operators can be bind to arguments

Chapter 113

Good: Arbitrary

Product types can be tested as a product of random generators.

Sum types require to implement generators with separate constructors, and picking one of them, use oneof or frequency to pick generators.

Chapter 114

Good: Principle of Separation of concerns

Chapter 115

Good: Function composition

In Haskell inline [composition](#) requires:

```
h.g.f $ i
```

[Function application](#) has a higher [priority](#) than [composition](#). That is why parentheses over [argument](#) are needed.

This [precedence](#) allows idiomatically [compose partially applied functions](#).

But it is a way better then:

```
h (g (f i))
```

Chapter 116

Good: Point-free

Use `Tacit` very carefully - it hides `types` and harder to change code `where` it is used.
Use just enough `Tacit` to communicate a bit better. Mostly only partial `point-free` communicates better.

116.1 Good: Point-free is great in multi-dimensions

BigData and OLAP analysis.

Chapter 117

Good: Functor application

Function application on n levels beneath:

```
(fmap . fmap) function twoLevelStructure
```

How `fmap . fmap` typechecks:

```
(.)      :: (b -> c) -> (a -> b) -> a -> c
fmap     :: Functor f => (m -> n) -> f m -> f n
fmap     :: Functor g => (x -> y) -> g x -> g y

fmap . fmap :: (Functor f, Functor g)
            => ((g x -> g y) -> f . g x -> f . g y)
            -> (( x -> y) -> g x -> g y)
            -> ( x -> y) -> f . g x -> f . g y
fmap . fmap :: (x -> y) -> f . g x -> f . g y
```

Chapter 118

Good: Parameter order

In [functions parameter order](#) is important.

It is best to use first the most reusable [parameters](#).

And as last one the one that can be the most [variable](#), that is important to chain.

Chapter 119

Good: Applicative monoid

There can be more than one valid [Monoid](#) for a [data type](#). &&

There can be more than one valid [Applicative](#) instance for a [data type](#). ->

There can be different [Applicatives](#) with different [Monoid](#) implementations.

Chapter 120

Good: Creative process

- 120.1 Pick philosophy principles one to three the more - the harder the implementation
- 120.2 Draw the most blurred representation
- 120.3 Deduce **abstractions** and write remotely what they are
- 120.4 Model of computation
 - 120.4.1 Model the **domain**
 - 120.4.2 Model the **types**
 - 120.4.3 Think how to write computations
- 120.5 Create

Chapter 121

«<Good: About operators (<\$) (**>) (<*) (») »>

[Where](#) character is not present - discard the according processing of a [parameter](#).
(») is an [exception](#), it does the reverse. ignores the first [parameter](#).

Chapter 122

Good: About functions like `{mapM, sequence}_`

Trailing `_` means ignoring the result.

Chapter 123

Good: Guideliles

123.1 Wiki.haskell

123.1.1 Documentation

123.1.1.1 Comments write in **application** terms, not technical.

123.1.1.2 Tell what code needs to do not how it does.

123.1.2 Haddock

123.1.2.1 Put haddock comments to ever exposed **data type** and **function**.

123.1.2.2 Haddock header

```
{- |  
Module      : <File name or $Header$ to be replaced automatically>  
Description : <optional short text displayed on contents page>  
Copyright   : (c) <Authors or Affiliations>  
License     : <license>  
  
Maintainer  : <email>  
Stability   : unstable / experimental / provisional / stable / frozen  
Portability : portable / non-portable (<reason>  
  
<module description starting at first column>  
-}
```

123.1.3 Code

123.1.3.1 Try to stay closer to portable (Haskell98) code

123.1.3.2 Try make lines no longer 80 chars

123.1.3.3 Last char in file should be newline

123.1.3.4 Symbolic **infix** identifiers is only library writer right

123.1.3.5 Every **function** does one thing.

Chapter 124

Good: Use Typed holes to progress the code

[Typed holes](#) help build code in complex situations.

Chapter 125

Good: Haskell allows infinite terms
but not infinite types

That is why infinite [types](#) throw infinite [type error](#).

Chapter 126

Good: Use type synonyms to differ the information

Even if there is `types` - define `type` synonyms. They are free.

That distinction with synonyms, would allow `TypeSynonymInstances`, which would allow to create a different `type class` instances and behaviour for different information.

Chapter 127

«<Good: Control.[Monad.Error](#) ->
Control.[Monad.Except](#)» >

Chapter 128

Good: Monad OR Applicative

128.0.1 Start writing `monad` using `'return'`, `'ap'`, `'liftM'`, `'liftM2'`, `'>>'` instead of `'do'`, `'>=>'`

If you wrote code and really needed only those - move that code to [Applicative](#).

```
return -> pure
ap -> <*>
liftM -> liftA -> <$>
>> -> *>
```

128.0.2 Basic `case` when [Applicative](#) can be used

Can be rewritten in [Applicative](#):

```
func = do
  a <- f
  b <- g
pure (a, b)
```

Can't be rewritten in [Applicative](#):

```
somethingdoSomething' n = do
  a <- f n
  b <- g a
pure (a, b)
```

`(f n)` creates [monadic structure](#), [binds](#) it to `a` which is consumed then by `g`.

128.0.3 [Applicative](#) block vs [Monad](#) block

With [Type Applicative](#) every condition fails/succeeds independently. It needs a boilerplate [data constructor](#)/value pattern matching code to work. And code you can write only for so many cases and [types](#), so boilerplate can not be so flexible as [Monad](#) that allows [polymorphism](#).

With [Type Monad](#) computation can return value that dependent from the previous computation result. So abort or dependent processing can happen.

Chapter 129

Good: Haskell Package Versioning Policy

Version policy and dependency management.

[width=.9]Good_{code}/pvp - decision - tree₂019 - 06 - 17₁5 - 49 - 21

129.1 *

PVP

Good: PVP

Chapter 130

Good: Linear type

Linear types are great to control/minimize resource usage.

Chapter 131

Good: Exception vs Error

Many languages and Haskell have it all mixup. Here is table showing what belongs to one or other in standard libraries:

Exception	Prelude.catch, Control. Exception .catch, Control. Exception .try, IOError, Control. Monad . Error
Error	error , assert, Control. Exception .catch, Debug.Trace.trace

Chapter 132

Good: Let vs. Where

`let ... in ...` is a separate [expression](#). In contrast, `where` is [bound](#) to a surrounding syntactic [construct](#) (namespace).

Chapter 133

Good: RankNTypes

Can powerfully synergyze with [ScopedTypeVariables](#).

Chapter 134

Good: Orphan type instance

Practice to address orphan instances:

Does [type class](#) or [type](#) defined by you:

Type class	Type	Recommendation
✓	✓	{ Type , instance} in the same module { Typeclass & instance} in the same module {Define newtype wrap, its instances} in the same module

Chapter 135

Good: Smart constructor

Only proper smart [constructors](#) should be exported. Do not export [data type constructor](#), only a [type](#).

Chapter 136

Good: Thin category

In * all [morphisms](#) are [epimorphisms](#) and [monomorphisms](#).

Chapter 137

Good: Recursion

Writing/thinking about [recursion](#):

- a.* Find the base cases, on input of which the answer can be provided right away. There is mostly one [base case](#), but sometimes there can be several of them. Typical base cases are: [zero](#), the empty [list](#), the empty tree, null, etc.
- b.* Do inductive [case](#). The [recursive](#) invocation. The [argument](#) of a [recursive](#) call needs to be smaller than the current [argument](#). So it would be gradually closer to the [base case](#). The idea is that processes eventually hits the [base case](#).

Simple functional [application](#) is used in the [recursion](#).

Assume that the [functions](#) would return the right result.

Chapter 138

Good: Monoid

<>:

[Sets](#) - union.

Maps - left-biased union.

Number - `Sum`, `Product` form separate [monoid categories](#).

Chapter 139

Good: Free monad

The main [case](#) of usage of Free [monads](#) in Haskell:

Start implementation of the [monad](#) from a Free [monad](#), drafting the base [monadic](#) operations, then add custom operations.

Gradually build on top of Free [monad](#) and try to find homomorphisms from [monad](#) to [objects](#), and if only [objects](#) are needed - get rid of the free [monad](#).

Chapter 140

Good: Use mostly where clauses

Chapter 141

Good: Where clause is in a scope
with function parameters

Chapter 142

Good: Strong preference towards pattern matching over {head, tail, etc.} functions

head and tail and alike [functions](#) are often partial ([unsafe](#)) functions.

Chapter 143

Good: Patternmatching is possible on monadic bind in do

Example:

```
instance (Monad m) => Functor (StateT s m) where
  fmap f m = StateT $ \s -> do
    (x, s') <- runStateT m s  -- Here is a pattern matching bind
    return (f x, s')
```

Chapter 144

Good: Applicative vs Monad

Giving not `Monad` but `Applicative` requirement allows parallel computation, but if there should be a chaining of the intermediate state - it must be [monadic](#).

Chapter 145

Good: StateT, ReaderT, WriterT

Reader trait: `(r ->)`.

Writer trait: `(a, w)`.

State trait is combination of both:

```
newtype StateT s m a =  
  StateT { runStateT :: s -> m (a, s) }
```

```
newtype ReaderT r m a =  
  ReaderT { runReaderT :: r -> m a }
```

```
newtype WriterT w m a =  
  WriterT { runWriterT :: m (a, w) }
```

State trait fully replaces writer.

Chapter 146

Good: Working with MonadTrans and lift

From the `lift . pure = pure` follows that `MonadTrans` [type](#) can have a `pure` defined with `lift`.

Stacking of `MonadTrans` [monads](#) can result in a lot of chained `lift` and `unwraps`. There is many ways to cope with that but the most robust and common is to [abstract](#) representation with `newtype` on the `Monad` [stack](#). This can reduce caining or remove the manual [lifting](#) withing the [Monad](#).

For perfect combination for contributors to be able to extend the code - keep the `Internal` [module](#) that has a raw representation.

Chapter 147

Good: Don't mix Where and Let

`let` and `where` create a [recursive set](#) of definitions with can explode, don't mix them together in code.

Chapter 148

Good: Where vs. Let

`Let` is self-recursive lazy pattern. It is checked and errors only at execution time. `Binds` only inside `expression` it is binded to.

`Where` is a part of definition, scoped over definition implemetations and `guards`, not self-recursive.

Chapter 149

Good: The proper nature algorithm that models behaviour of many objects is computation heavy

God does not care about our mathematical difficulties. He integrates empirically.

One who is found of mathematical meaning loves to [apply](#) it. But if we implement the "real" algorithms behind nature processes, we face the need to go through the computations of laws of all particles.

Computation of nature is always a middle way between ideal theory behaviour and computation simplification.

Chapter 150

Good: In Haskell parameters bound by lambda declaration instantiate to only one concrete type

Because of [let-bound polymorphism](#):

This is illegal in Haskell:

```
foo :: (Int, Char)
foo = (\f -> (f 1, f 'a')) id
```

Lambda-bound function (i.e., one passed as [argument](#) to another [function](#)) cannot be instantiated in two different ways, if there is a [let-bound polymorphism](#).

Chapter 151

Good: Instance is a good structure to draw a type line

Instances for [data type](#) can differentiate by [constraints](#) & [types](#) of arguments. So instance can preserve [type](#) boundary, and [data type declaration](#) can stay very [polymorphic](#). If the need to extend the [type](#) boundaries arrive - the instances may extend, or new instances are created, while used [data type](#) still the same and unchanged.

Chapter 152

Good: MTL vs. Transformers

Default of `mtl`.

`Transformers` is Haskell-98, doesn't have functional dependencies, lacks the `monad` classes, has manual `lift` of operations to the composite `monad`.

MTL extends `transformers`, providing more instances, features and possibilities, may include `alternative` packages features as `mtl-tf`.

Part VI

Bad code

Chapter 153

Bad pragma

153.1 Bad: Dangerous **LANGUAGE pragma** option

- [DatatypeContexts](#)
- `OverlappingInstances`
- `IncoherentInstances`
- `ImpredicativeTypes`
- `AllowAmbiguousTypes`
- [UndecidableInstances](#) - often

Part VII

Useful **functions** to remember

Chapter 154

Prelude

```
enumFromTo
enumFromThenTo
reverse
show :: Show a => a -> String
flip
sequence - Evaluate each monadic action in the structure from left to right, and collect the results
:sprint - show variables to see what has been evaluated already.
minBound - smaller bound
maxBound - larger bound
cycle :: [a] -> [a] - indefinitely cycle s list
repeat - indefinitely list from value
elemIndex e l - return first index, returns Maybe
fromMaybe (default if Nothing) e :: Maybe a -> a
lookup :: Eq a => a -> [(a, b)] -> Maybe b
```

154.1 Ord

compare

154.2 Calc

div - always makes rounding down, to infinity

divMod - returns a tuple containing the result of integral division and modulo

154.3 List operations

```
concat - [ [a] ] -> [a]
elem x xs - is element a part of a list
zip :: [a] -> [b] -> [(a, b)] - zips two lists together. Zip stops when one list runs out.
zipWith :: (a -> b -> c) -> [a] -> [b] -> [c] - do the action on corresponding elements of list a
```

Chapter 155

Data.List

`intersperse :: a -> [a] -> [a]` - gets the value and incerts it between values `in` array
`nub` - remove duplicates from the list

Chapter 156

Data.Char

```
ord (Char -> Int)
chr (Int -> Char)
isUpper (Char -> Bool)
toUpper (Char -> Char)
```

Chapter 157

QuickCheck

```
quickCheck :: Testable prop => prop -> IO ()
```

```
quickCheck . verbose - run verbose mode
```

Part VIII

Tools

Chapter 158

ghc-pkg

[List](#) installed packages:

```
ghc-pkg list
```

Chapter 159

Search over the Haskell packages code: Codesearch from Aelve

<https://codesearch.aelve.com/>

Chapter 160

Integration of NixOS/Nix with Haskell IDE Engine (HIE) and Emacs (Spacemacs)

160.1 1. Install the Cachix: <https://github.com/cachix/cachix>

160.2 2. Installation of HIE: <https://github.com/infinisil/all-hies/#cached-builds>

160.2.1 2.1. Provide cached builds

```
cachix use all-hies
```

160.2.2 2.2.a. Installation on NixOS distribution:

```
{ config, pkgs, ... }:
```

```
let
```

```
    all-hies = import (fetchTarball "https://github.com/infinisil/all-hies/tarball/master") {};
```

```
in {
```

```
    environment.systemPackages = with pkgs; [
```

```
        (all-hies.selection { selector = p: { inherit (p) ghc865 ghc864; }; })
```

```
    ];
```

```
}
```

Insert your GHC versions.

Switch to new configuration:

```
sudo -i nixos-rebuild switch
```

160.2.3 2.2.b. Installation with Nix package manager:

```
nix-env -iA selection --arg selector 'p: { inherit (p) ghc865 ghc864; }' -f 'https://github.com/i
```

Insert your GHC versions.

160.3 3. Emacs (Spacemacs) configuration:

```
dotspacemacs-configuration-layers
'(

  auto-completion

  (lsp :variables
    default-nix-wrapper (lambda (args)
      (append
        (append (list "nix-shell" "-I" "." "--command" )
          (list (mapconcat 'identity args " ")))
        )
      (list (nix-current-sandbox))
      )
    )

    lsp-haskell-process-wrapper-function default-nix-wrapper
  )

  (haskell :variables
    haskell-enable-hindent t
    haskell-completion-backend 'lsp
    haskell-process-type 'cabal-new-repl
  )

)

dotspacemacs-additional-packages '(
  direnv
  nix-sandbox
)

(defun dotspacemacs/user-config ()

  (add-hook 'haskell-mode-hook 'direnv-update-environment) ;; If direnv configured

)
```

Where:

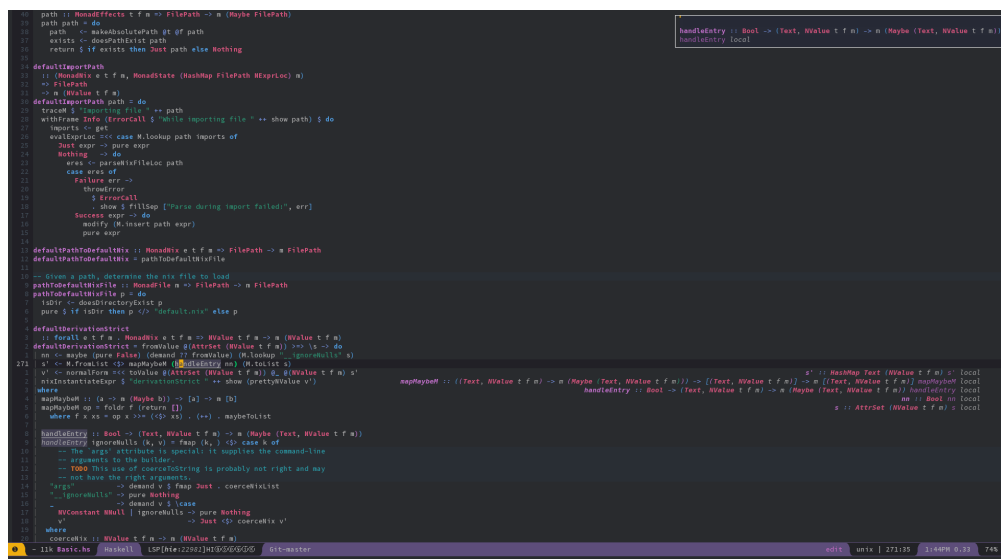
auto-completion configures YASnippet.

nix-sandbox (<https://github.com/travisbhartwell/nix-emacs>) has a great helper [functions](#). Using [nix-current-sandbox function](#) in default-nix-wrapper that used to properly configure lsp-haskell-process-wrapper-function.

Configuration was reassembled from: <https://github.com/emacs-lsp/lsp-haskell/blob/8f2dbb6e827b1adce6360c56f795f29ecff1d7f6/lsp-haskell.el#L57> & its authors config: [\[\[https://github.com/sevanspowell/dotfiles/blob/master/spacemacs\]\]](https://github.com/sevanspowell/dotfiles/blob/master/spacemacs)/

160.4 4. Open the Haskell file from a project

160.5 5. Be pleased writing code



160.6 6. (optional) Debugging

- ```
readCreateProcess : cabal-helper-wrapper failure
```

300

- a.* If HIE keeps getting ready, failing & restarting - check that the projects `ghc --version` is declared in your `all-hie` NixOS configuration.

# Chapter 161

## Debugger

Provides:

- [set](#) a breakpoints
- observe step-by-step [evaluation](#)
- tracing mode

Breakpoints

```
:break 2
:show breaks
:delete 0
:continue
```

Step-by-step

```
:step main
```

[List](#) information at the breakpoint

```
:list
```

What been evaluated already

```
:sprint name
```

## Chapter 162

# GHCID

Commands to run the compile/check loop:

cabal > 3.0 command:

```
ghcid --command='cabal v2-repl --repl-options=-fno-code --repl-options=-fno-break-on-exception --g
```

cabal < 3.0 command:

```
ghcid --command='cabal new-repl --ghc-options=-fno-code --ghc-options=-fno-break-on-exception --g
```

nix-shell cabal > 3.0 command:

```
nix-shell --command 'ghcid --command="cabal v2-repl --repl-options=-fno-code --repl-options=-fno-
```

nix-shell cabal < 3.0 command:

```
nix-shell --command 'ghcid --command="cabal new-repl --ghc-options=-fno-code --ghc-options=-fno-b
```



## Part IX

### Libs

## Chapter 163

# Exceptions

- 163.1 **Exceptions** - optionally **pure** extensible **exceptions** that are compatible with the mtl
- 163.2 **Safe-exceptions** - safe, simple API **equivalent** to the underlying implementation in terms of power, encourages best practices minimizing the chances of getting the **exception** handling wrong.
- 163.3 **Enclosed-exceptions** - capture **exceptions** from the enclosed computation, while reacting to asynchronous **exceptions** aimed at the calling thread.

## Chapter 164

# Memory management

### 164.1 membrain - [type](#)-safe memory units

## Chapter 165

# Parsers - megaparsec

## Chapter 166

### CLIs - `optparse-`[applicative](#)

## Chapter 167

### HTML - Lucid

## Chapter 168

# Web applications - Servant

## Chapter 169

### IO libraries

- 169.1 Conduit - practical, monolythic, guarantees termination return
- 169.2 Pipes + Pipes Parse - modular, more primitive, theoretically driven



## Chapter 170

### JSON - aeson

## Chapter 171

# Backpack

On 1-st compilation - `*` analyzes the `abstract` signatures without loading side modules, doing the `type check` with assumption that modules provide right `type` signatures, the process does not emit any `binary` code and stores the intermediate code in a special form that allows flexibly connect modules provided. Which allows later to compile project with particular instantiations of the modules. Major work of this process being done by internal Cabal `*` support and `*` system that modifies the intermediate code to fit the `module`.

Part X

Drafts

## Chapter 172

# Exception handling

**Exception** must include all **context** information that may be useful.

Store information in a form for further probable deeper automatic diagnostic.

Sensitive data/dummies for it - can be useful during development.

Sensitive data should be stripped from a program logging & **exceptions**.

**Exception** system should be extendable, data storage & representation should be easily extendable.

**Exception** system should allow easy exhaustive checking of **errors**, since the different **errors** can happen.

**Exception** system should be automatically well-documented and transparent.

**Exception** system should have controllable breaking changes downstream.

**Exception** system should allow complex composite (**sets**) **exceptions**.

**Exception** system should be lightweight on the **type** signatures of other **functions**.

**Exception** system should automate the collection of **context** for a **exception**.

**Exception** system should have **properties** and according **functions** for particular **types** of **errors**.

**String** is simple and convenient to throw **exception**, but really a mistake because it the most cumbersome choice:

- Any **Exception** instance can be converted to a **String** with **either** **show** or **displayException**.
- Does not include key debugging information in the **error** message.
- Does not allow developer to access/manage the **Exception** information.
- **Exception** messages need to be constructed ahead of time, it can not be internationalized, converted to some data/file format.
- **Exception** can have a sensitive information that can be useful for developer during work, but should not be logged/shown to end-user. Stripping it from **Strings** in the changing project is a hard task.
- Impossible to rely on this representation for further/deeper inspection.
- Impossible to have exhaustive checking - no knowledge no check, no warning if some cases are not handled.

Universal [exception type](#):

- Able to inspect every possible [error case](#) with [pattern match](#).
- Self-documenting. Shows the hierarchical system of all [exceptions](#).
- Transparent. Ability to discern in current situation what [exceptions](#) can happen
- New [exception constructor](#) causes breaking change to downstream.
- Wrongly implies completeness. Untreated [Errors](#) can happen, different [exception](#) can arrive from the outside code.

Sum [type](#) must be separate, and [product type structure](#) over it.

Separate [exception type](#) of

Individual [exception types](#):

- Writing & seing & working with exactly what will go wrong because there is only one possible [error](#) for this [type](#) of [exception](#). [Pattern match](#) happens only onconditions, [constructors](#) that should happen.
- Knowledge what exactly goes wrong allows wide usage of [Either](#).
- It is hard to handle complex [exceptions](#) in the unitary system. Real wrorld can return not a particular [case](#), but a [set](#) of cases {[object](#) not found, path is unreachable, access is denied}.
- [Type](#) signatures grow, and even can become complex, since every [case](#) of [exception](#) has its own [type](#).
- Impure [throw](#) that users can/should use for your code must account for all your [exception types](#).

[Abstract exception type](#):

[Exception type](#) entirely opaque and inspectable only by accessor [functions](#).

- Updating the internals without breaking the API
- Semi-automates the [context](#) of [exception](#) with passing it to accessors.
- Predicates can be [applied](#) to more than one [constructor](#). Which are [properties](#) that allows to make complex [exceptions](#) much easier to handle.
- Not self-documenting.

- Possible options by design are hidden from the downstream, documentation must be kept.
- When you change the `exception` handling/throwing `errors` it does not show to the downstream.

Composit approach:

Provide the `set` of `constructors` and also a `set` of predicates and `set` of accessors.

Use `pattern synonyms` to provide a documented accessor `set` without exposing internal `data type`.

In GHC 8.8 the change was made:

The `fail` method of `Monad` has been removed in favor of the method of the same name in the `MonadFail` class.

`MonadFail(..)` is now exported from the `Prelude` and `Control.Monad` modules.

The `MonadFailDesugaring` language extension is now deprecated, as its effects are always enabled.

So:

```
import Control.Monad.Fail
...
class MonadFail m => MonadFile m
...
-- use error instead of fail
Nothing -> error ("Message " <> show x)
```

# Chapter 173

## Constraints

Very strong Haskell [type](#) system makes possible to work with code from the top down, an [axiomatic semantics](#) approach, from [constraints](#) into [types](#).

- Helps to form the [type level](#) code (aka [join](#) points of the code).
- Uses the piling up of [constraints/types](#) information. At some point pick and satisfy [constraints](#), can be done one at a time.
- Provides hints through [type level](#) formulation for [term level](#) calculations, does not formulate the [term level](#).
- Tedious method (a lot of boilerplate and rewriting it) but pretty simple and relaxing.
- [Set](#) of [constraints](#).
- When it is needed or convenient, single [constraint](#) gets a little more realistically concrete/abstracted.

Main [type](#) detail annotation thread can happen in [main](#) or special wrapper [function](#), localization is inside [functions](#).

a. Rest of [constraints set](#) shifts to source [type](#).

3.a. For the class handled or known how to handle - write a [base case](#) instance description.

```
instance (Monad m) => MonadReader r (ReaderT r m)
```

3.b. For others write [recursive](#) instance descriptions:

All other unsolved [constraints](#) move into the source [polymorphic variable](#).

```
instance (MonadError e m) => MonadError e (ReaderT r m)
instance (MonadState s m) => MonadState s (ReaderT r m)
```

- a.* Repeat from 1 until considered done.
- b.* Code condensed into terse form.

`MonadError constraints` is `IOException`, not for the `String`. `IOException` vs `String`.

Reverse pluck `MonadReader constraint` with `runReader` on the `object`.

`MonadState - StateT`



## Chapter 174

# Monad transformers and their type classes

## Chapter 175

# Layering **monad** transformers

Different layering of the same **monad** transformers is functionality is the same, but the form is different. Surrounding handling **functions** would need to be different.

# Chapter 176

## Hoogle

### 176.1 Search

Text search (case insensitive):

- `a`
- `map`
- `con map`

Type search:

- `:: a`
- `:: a -> a`

Text & type:

`=id a -> a=`

### 176.2 Scope

#### 176.2.1 Default

Scope is Haskell Platform (and Haskell keywords).

All Hackage packages are available to search with:

#### 176.2.2 Hierarchical module name system (from big letter):

- `fold +Data.Map` finds results in the `Data.Map` module

- `file -System` excludes results from modules such as `System.IO`, `System.FilePath.Windows` and `Distribution.System`

### 176.2.3 Packages (lower **case**):

- `mode +platform`
- `mode +cmdargs` (only)
- `mode +platform +cmdargs`
- `file -base` (Haskell Platform, excluding the "base" package)

## Chapter 177

# ST-Trick monad

ST is like a [lexical scope](https://wiki.haskell.org/Lexical_scope), where all the [variables](https://wiki.haskell.org/Variables)/state disappear when the [function](https://wiki.haskell.org/Function) returns  
[https://wiki.haskell.ohttps://www.schoolofhaskell.com/school/to-infinity-and-beyond/older-but-still-interesting/deamortized-strg/Monad/ST](https://wiki.haskell.org/https://www.schoolofhaskell.com/school/to-infinity-and-beyond/older-but-still-interesting/deamortized-strg/Monad/ST)  
<https://dev.to/jvanbruegge/what-the-heck-is-polymorphism-nmh>

### 177.1 \*

ST-Trick

## Chapter 178

# Either

Allows to separate and preserve information about happened, ex. [error](#) handling.

### 178.1 \*

Either data type

## Chapter 179

# Inverse

- a.* [Inverse function](#)
- b.* In logic:  $P \rightarrow Q \Rightarrow \neg P \rightarrow \neg Q$ , & same for [category duality](#).
- c.* For [operation](#): element that allows reversing [operation](#), having an element that with the [dual](#) produces the [identity](#) element.
- d.* See [Inversion](#).

## Chapter 180

# Inversion

- a.* Is a [permutation where](#) two elements are out of [order](#).
- b.* See [Inverse](#)



## Chapter 181

# Inverse function

$$f_{x \rightarrow y} \circ (f_{x \rightarrow y})^{-1} = 1_x$$

\*  $\iff$  function is bijective.

Otherwise - partial inverse

## Chapter 182

# Inverse morphism

For  $f : x \rightarrow y$ :

$\exists g : g \circ f = 1^x$  -  $g$  is left [inverse](#) of  $f$ .

$\exists g : f \circ g = 1^y$  -  $g$  is right [inverse](#) of  $f$ .

## Chapter 183

# Partial inverse

\* when [function](#) is now [bijective](#). When [bijective](#) see [inverse function](#).

## Chapter 184

# PatternSynonyms

Enables [pattern synonym declaration](#), which always begins with the `pattern` word.  
Allows to [abstract](#)-away the [structures](#) of pattern matching.

### 184.1   \*

Pattern synonym  
Pattern synonyms

## Chapter 185

# GHC debug keys

### 185.1 `-ddump-ds`

Dump desugarer output.

#### 185.1.1 `*`

Desugar  
GHC desugar

## Chapter 186

# GHC optimize keys

### 186.1 -foptimal-applicative-do

$O(n^3)$

Always finds optimal [reduction](#) into  $\langle * \rangle$  for [ApplicativeDo](#) do notation.

## Chapter 187

# Computational trinitarianism

Taken from: <https://ncatlab.org/nlab/show/computational+trinitarianism>

Under the [statements](#):

- [propositions](#) as [types](#)
- programs as proofs
- [relation](#) between [type](#) theory and [category](#) theory

the following notions are [equivalent](#):

== [proposition](#) proof (Logic)

== generalized element of an [object](#) ([Category](#) theory)

== typed program with output ([Type](#) theory & Computer science)

Table 187.1: [Computational trinitarianism](#)

| Logic                             | Category theory                                                 | Type theory       |
|-----------------------------------|-----------------------------------------------------------------|-------------------|
| true                              | terminal object / (-2)-truncated object                         | h-level 0-type    |
| false                             | initial object                                                  | empty type        |
| proposition                       | (-1)-truncated object                                           | h-proposition     |
| proof                             | generalized element                                             | program           |
| cut rule                          | composition of classifying morphisms / pullback of display maps | substitution      |
| cut elimination for implication   | counit for hom-tensor adjunction                                | beta reduction    |
| introduction rule for implication | unit for hom-tensor adjunction                                  | eta conversion    |
| logical conjunction               | product                                                         | product type      |
| disjunction                       | coproduct ((-1)-truncation of)                                  | sum type (br)     |
| implication                       | internal hom                                                    | function type     |
| negation                          | internal hom into initial object                                | function type     |
| universal quantification          | dependent product                                               | dependent product |
| existential quantification        | dependent sum ((-1)-truncation of)                              | dependent sum     |
| equivalence                       | path space object                                               | identity type     |
| equivalence class                 | quotient                                                        | quotient type     |

| Continued from previous page  |                                       |                       |
|-------------------------------|---------------------------------------|-----------------------|
| Logic                         | Category theory                       | Type theory           |
| induction                     | colimit                               | inductive type        |
| higher induction              | higher colimit                        | higher inductive type |
| completely presented set      | discrete object/0-truncated object    | h-level 2-type        |
| set                           | internal 0-groupoid                   | Bishop set/set        |
| universe                      | object classifier                     | type of types         |
| modality                      | closure operator, (idempotent) monad  | modal type theory     |
| linear logic                  | (symmetric, closed) monoidal category | linear type theory    |
| proof net                     | string diagram                        | quantum circuit       |
| (absence of) contraction rule | (absence of) diagonal                 | no-cloning theorem    |
|                               | synthetic mathematics                 | domain specification  |

187.1 \*

Trinitarism



## Chapter 188

# Techniques functional programming deals with the state

### 188.1 Minimizing

Do not rely on state, try not to change the state. Use it only when it is very necessary.

### 188.2 Concentrating

Concentrate the state in one place.

### 188.3 Deferring

Defer state to the last step of the program, or to external system.

## Chapter 189

# Monadic Error handling

```
(>>=) :: m a -> (a -> m b) -> m b -- $\lambda A.E \vee A$ - computes and drops if error value happens.
catch :: c a -> (e -> c a) -> c a -- $\lambda E.E \vee A$ - handles "errors" as "normal" values and stops wh
```

## Chapter 190

# Functions

Total [function](#) uses [domain](#) fully, but takes only part of the [codomain](#).

[Function](#) allows to collapse [domain](#) values into [codomain](#) value. Meaning the [function](#) allows to loose the information.

So total [function](#) is a computation that loses the information or into bigger codomains.

That is why the [function](#) has a directionality, and [inverse](#) total process is partially possible.

Directionality and invertability are terms.

# Chapter 191

## Void

Emptiness.

Can not be grasped, touched.

A logically uninhabited [data type](#).

(Since [basis](#) of logic is tautologically True and [Void](#) value can not be addressed - there is a logical paradox with the [Void](#)).

Is an [object](#) included into the [Hask category](#), since:

```
:t (id :: Void -> Void)
(id :: Void -> Void) :: Void -> Void
```

id for it exists.

[Type](#) system corresponds to [constructive logic](#) and not to the classical logic.

Classical logic answers the question "Is this actually true".

Constructive (Intuitionistic) logic answers the question "Is this provable".

Also has [functions](#):

```
-- Represents logical principle of explosion: from falsehood, anything follows.
```

```
absurd :: Void -> a
```

```
-- If Functor holds only Void - it holds no values.
```

```
vacuous :: Functor f => f Void -> f a
```

```
-- If Monad holds only Void - it holds no values.
```

```
vacuousM :: Monad m => m Void -> m a
```

Design pattern: use [polymorphic data types](#) and [Void](#) to get rid of possibilities when you need to.

### 191.1 \*

Nothing, Haskell [expressions](#) can't return [Void](#).

Also see: [Maybe](#).

## Chapter 192

# Constructive proof

Method of proof that demonstrates the existence of a mathematical [object](#) by creating or providing a method for creating the [object](#).

## Chapter 193

# Intuitionistic logic

[Proposition](#) considered **True** due to direct evidence of existence through constructive proof using [Curry-Howard isomorphism](#).

\* does not include classic logic fundamental axioms of the excluded middle and double negation elimination. Hence \* is weaker then classical logic. Classical logic includes \*, all theorems of \* are also in classical logic.

### 193.1 \*

Constructive logic

## Chapter 194

# Principle of explosion

From asserted [statement](#) that contains contradiction - anything can be proven.  
Ancient principle of logic. Both in classical & intuitionistic logic.

### 194.1 \*

Ex falso quodlibet

Ex falso sequitur quodlibet

EFG

Ex contradictione quodlibet

Ex contradictione sequitur quodlibet

ECQ

Deductive explosion

Pseudo-Scotus

## Chapter 195

# Universal **property**

A **property** of some construction which boils down to (is manifestly **equivalent** to) the **property** that an associated **object** is a universal **initial object** of some (auxiliary) **category**.



## Chapter 196

# Yoneda lemma

Allows the embedding of any [category](#) into a [category](#) of [functors](#) ([contravariant set-valued functors](#)) defined on that [category](#). It also clarifies how the embedded [category](#), of representable [functors](#) and their [natural transformations](#), relates to the other [objects](#) in the larger [functor category](#).

The Yoneda lemma suggests that instead of studying the (locally small) [category](#)  $C \{\{\{C\}\}\}\mathcal{C}$ , *one should study the [category](#)*

## Chapter 197

# Monoidal category, functoriality of ADTs, Profunctors

Category equipped with tensor product.

<>

wich is a functor for  $*$ .

Set category can be monoidal under both product (having terminal object) or coproduct (having initial object) operations, if according operation exist for all objects.

Any one-object category is  $*$ .

$(a, ()) \sim a$  up to unique isomorphism, which is called Lax monoidal functor.

Product and coproduct are functorial, so, since:  
Algebraic data type construction can use:

- Type constructor
- Data constructor
- Const functor
- Identity functor
- Product
- Coproduct

Any algebraic data type is functorial.

## Chapter 198

# Const functor

Maps all [objects](#) of source [category](#) into one (fixed) [object](#) of target [category](#), and all [morphisms](#) to [identity morphism](#) of that fixed [object](#).

```
instance Functor (Const c)
 where
 fmap :: (a -> b) -> Const c a -> Const c b
 fmap _ (Const c) = Const c
```

In [Category](#) theory denoted:

$\Delta$

Last [type parameter](#) that bears the target [type](#) of lifted [function](#) ([b](#)) and is a [proxy type](#).

Analogy: the container that allways has an [object](#) attached to it, and everything that is put inside - changes the container [type](#) accordingly, and dissapears.

## Chapter 199

# Arrow in Haskell

```
(->) a b = a -> b
```

`Functorial` in the last `argument` & called Reader `functor`.

```
newtype Reader c a = Reader (c -> a)
```

```
fmap = (.)
```

## Chapter 200

# Contravariant functor

```
fmap :: (a -> b) -> Op c a -> Op c b
 (a -> c) -> (b -> c)
```

$$\begin{array}{ccc} a & \longrightarrow & b \\ & \searrow & \vdots \\ & & c \end{array}$$
$$(a \rightarrow b)^C = (a \leftarrow b)^{C^{op}}$$

```
class Contravariant f
 where
 contramap :: (b -> a) -> (f a -> f b)
```

$$\begin{array}{ccc} a & \longrightarrow & b \\ & \searrow & \vdots \text{contravariant} \\ & & c \end{array}$$

If [arrows](#) does not commute Contravariant functor anyway allows to [construct](#) transformation between these such [arrows](#) to other [arrow](#).

## Chapter 201

# Profunctor

$(\multimap) \ a \ b$

$C^{op} \times C \rightarrow C$

It is called profunctor.

`dimap :: (a' -> a) -> (b -> b') -> p a b -> p a' b'`

So, profunctor in `case` of `arrow`:

$$\begin{array}{ccc} a & \xleftarrow{f} & a' \\ \downarrow h & & \downarrow \text{profunctor} \\ b & \xrightarrow{g} & b' \end{array}$$

```
dimap :: (a' -> a) -> (b -> b') -> p a b -> p a' b'
dimap :: f g -> (a -> b) -> (a' -> b')
dimap :: f g -> h -> (a' -> b')
dimap = g . h . f
```

It is `contravariant functor` in the first `argument`, and `covariant functor` in the second `argument`.

```
dimap id <==> fmap
(flip dimap) id <==> contramap
```

## Part XI

# Reference

## Chapter 202

# Functor-Applicative-Monad Proposal

Well known event in Haskell history: [https://github.com/quchen/articles/blob/master/applicative\\_monad.md](https://github.com/quchen/articles/blob/master/applicative_monad.md).

Math justice was restored with a RETroactive CONTinuity. Invented in computer science term [Applicative](#) (lax monoidal functor) become a [superclass](#) of [Monad](#).

& that is why:

- `return = pure`
- `ap = <*>`
- `>= = *>`
- `liftM = liftA = fmap`
- `liftM* = liftA*`

Also, a side-kick - [Alternative](#) became a [superclass](#) of [MonadPlus](#). Hence:

- `mzero = empty`
- `mplus = (<|>)`

### 202.1 \*

Applicative-Monad proposal  
AMP



# Chapter 203

## Haskell-98

### 203.1 Old instance termination rules

- a.  $\forall$  class **constraint** ( $C\ t_1 \dots t_n$ ):
  - 1.1. **type variables** have occurrences  $\leq$  head
  - 1.2. **constructors+variables+repetitions**  $<$  head
  - 1.3.  $\neg$  **type functions** (**type func application** can expand to **arbitrary** size)
- b.  $\forall$  **functional dependencies**,  $\langle \text{tvs} \rangle_{\text{left}} \rightarrow \langle \text{tvs} \rangle_{\text{right}}$ , of the class, every **type variable** in  $S(\langle \text{tvs} \rangle_{\text{right}})$  must appear in  $S(\langle \text{tvs} \rangle_{\text{left}})$ , where  $S$  is the substitution mapping each **type variable** in the class **declaration** to the corresponding **type** in the instance head.

## Chapter 204

# Performance results and comparisons of **types** & solutions

Haskell performance

## Part XII

# Liturgy

---

*λειτ* <- *λα΄ς* *Laos* the people  
*ουργ΄ς* <- *ργο* *ergon* work  
*λειτουργ΄α* *leitourgia* giving back to the community

The life is beautiful.  
For all humans that make the life have more uniqueness.

This study would not be possible without mathematicians, Haskellers, scientists, creators, contributors. These people are the most fascinating in my life.

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