Fundamental Haskell notes

Encyclopedcal handbook for learning and undersatanding fundamentals

Anton Latukha

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Part I

Introduction

"Employ your time in improving yourself by other men's writings so that you shall come easily by what others have labored hard for." (Socrates by Plato)

Important notes on Haskell, category theory & related fields, terms and recommendations.

Book comes in forms:

- Web book
- PDF
- Open in web PDF viewer
- LaTeX
- Source code in Org-mode
- GitHub
- GitLab

This book is created using complex Org markup file with a lot of LaTeX and LaTeX formulas. Be aware - GitHub & GitLab only partially parse Org into HTML.

To get the full view:

- Outline navigation
- LaTeX formulas:
  $$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(\vec{r},t)\right]\Psi(\vec{r},t) = i\hbar\frac{\partial}{\partial t}\Psi(\vec{r},t), \quad \sum_{k,j}\left[-\frac{\hbar^2}{\sqrt{a}}\frac{\partial}{\partial q^k}\left(\sqrt{a}a^{kj}\frac{\partial}{\partial q^j}\right) + V\right]\Psi + \frac{\hbar}{i}\frac{\partial\Psi}{\partial t} = 0$$
- Interlinks: Interlinks

, please refere to Web book, PDF, LaTeX, of use Org-mode capable viewer/editor.

Note about the markup: `<<<This is a radio target>>>` - is the ancor for dynamic linking.

Users of `Emacs` can prettify radio targets to be shown as hyper-links with this `Elisp` snippet:

```elisp
;;;;  2019-06-12: NOTE:
;;;;  Prettify '<<<Radio targets>>>' to be shown as '_Radio_targets_',
;;;;  when `org-descriptive-links` set.
;;;;  This is improvement of the code from: Tobias&glmorous:
;;;;  https://emacs.stackexchange.com/questions/19230/how-to-hide-targets
;;;;  There exists library created from the sample:
;;;;  https://github.com/talwrii/org-hide-targets
(defcustom org-hidden-links-additional-re "\\(<<<\\)[[:print:]]+?\\(>>>\\)"
  "Regular expression that matches strings where the invisible-property
   of thesub-matches 1 and 2 is set to org-link."
  :type '(choice (const :tag "Off" nil) regexp)
  :group 'org-link)
(make-variable-buffer-local 'org-hidden-links-additional-re)

(defun org-activate-hidden-links-additional (limit)
  "Put invisible-property org-link on strings matching
   `org-hide-links-additional-re'."
  (if org-hidden-links-additional-re
      (re-search-forward org-hidden-links-additional-re limit t)
    (goto-char limit)
    nil))
```

```
(defun org-hidden-links-hook-function ()
  "Add rule for `org-activate-hidden-links-additional'
    to `org-font-lock-extra-keywords'.
    You can include this function in `org-font-lock-set-keywords-hook'."
  (add-to-list 'org-font-lock-extra-keywords
               '(org-activate-hidden-links-additional
                 (1 '(face org-target invisible org-link))
                 (2 '(face org-target invisible org-link)))))

(add-hook 'org-font-lock-set-keywords-hook #'org-hidden-links-hook-function)
```

`SCHT`: and metadata in `:properties:` - of my `org-drill` practices, please just run `org-drill-strip-all-data`.

Part II

Definitions

Chapter 1

Algebra

ال جبر *al-jabr* assemble parts

A system of parts based on given axioms (properties) and operations on them.

+===

Additional meanings:

1. Algebra - a set with its algebraic structure.
2. Abstract algebra - the study of number systems and operations within them.
3. Algebra - vector space over a field with a multiplication.

1.1 *

Algebras

1.2 Algebraic

Composite from simple parts.

Also: Algebraic data type.

1.3 Algebraic structure

* includes axioms that must be satisfied and operations on the underlying (or "carrier") set.

An underlying set with * on top of it also called "an algebra".

* include groups, rings, fields, and lattices. More complex structures can be defined by introducing multiple operations, different underlying sets, or by altering the defining axioms. Examples of more complex * can be many modules, algebras and other vector spaces, and any variations that the definition includes.

1.3.1 *

Algebraic structures

Table 1.1: Algebraic structures

	Closure	Associativity	Identity	Invertability	Commutativity	Distributive
Semigroupoid		✓				
Small Category		✓	✓			
Groupoid		✓	✓	✓		
Magma	✓					
Quasigroup	✓			✓		
Loop	✓		✓	✓		
Semigroup	✓	✓				
Inverse Semigroup	✓	✓		✓		
Monoid	✓	✓	✓			
Group	✓	✓	✓	✓		
Abelian group	✓	✓	✓	✓	✓	
Non-unital ring (rng)	✓ $+ \times$	✓ $+ \times$	✓ $+$	✓ $+$	✓ $+$	✓
Semiring (rig)	✓ $+ \times$	✓ $+ \times$	✓ $+ \times$	✓ $\times$	✓ $+$	✓
Ring	✓ $+ \times$	✓ $+ \times$	✓ $+ \times$	✓ $+ \times$	✓ $+$	✓

1.3.2 Fundamental theorem of algebra

Any non-constant single-variable polynomial with complex coefficients has at least one complex root.

From this definition follows property that the field of complex numbers is algebraically closed.

1.4 Modular arithmetic

System for integers where numbers wrap around the certain values (single - *modulus*, plural - *moduli*).

Example - 12-hour clock.

1.4.1 *

Clock arithmetic

1.4.2 Modulus

Special numbers where arithmetic wraps around in modular arithmetic.

1.4.2.1 *

Moduli - plural.

Chapter 2

Category theory

Category $\mathcal{C}$ consists of the basis:

Primitives:

1. Objects - $a^{\mathcal{C}}$. A node. Object of some type. Often sets, than it is Set category.
2. Arrows - $(a, b)^{\mathcal{C}}$ (AKA morphisms mappings).
3. Arrow (morphism) composition - binary operation: $(a, b)^{\mathcal{C}} \circ (b, c)^{\mathcal{C}} \equiv (a, c)^{\mathcal{C}} \mid \forall a, b, c \in \mathcal{C}$ AKA principle of compositionality for arrows.

Properties (or axioms):

1. Associativity of morphisms: $h \circ (g \circ f) \equiv (h \circ g) \circ f \mid f_{a \to b}, g_{b \to c}, h_{c \to d}$
2. Every object has (two-sided) identity morphism (& in fact - exactly one): $1_x \circ f_{a \to x} \equiv f_{a \to x}, \ g_{x \to b} \circ 1_x \equiv g_{x \to b} \mid \forall x \ \exists 1_x, \forall f_{a \to x}, \forall g_{x \to b}$
3. Principle of compositionality.

From these axioms, can be proven that there is exactly one identity morphism for every object.

Object and morphism are complete abstractions for anything. In majority of cases under object is a state and morphism is a change.

2.1 *

Category Categories

2.2 Abelian category

Generalised category for homological algebra (having a possibility of basic constructions and techniques for it).

Category which:

- has a zero object,
- has all binary biproducts,
- has all kernel's and cokernels,
- (it has all pullbacks and pushouts)
- all monomorphism's and epimorphism's are normal.

Abelian category is a stable structure; for example it is regular and satisfy the snake lemma. The class of Abelian categories is closed under several categorical constructions.

There is notion of Abelian monoid (AKS Commutative monoid) and Abelian group (Commutative group).

Basic examples of *:

- category of Abelian groups
- category of modules over a ring.

* are widely used in algebra, algebraic geometry, and topology.

* has many constructions like in categories of modules:

- kernels
- exact sequences
- commutative diagrams

* has disadvantage over category of modules. Objects do not necessarily have elements that can be manipulated directly, so traditional definitions do not work. Methods must be supplied that allow definition and manipulation of objects without the use of elements.

2.2.1 *

Abelian categories

2.3 Composition

Axiom of Category.

2.3.1 *

Composable Compositions

2.4 Endofunctor category

From the name, in this Category:

- objects of End are Endofunctors $E^{\mathcal{C} \to \mathcal{C}}$
- morphisms are natural transformations between endofunctors

2.5 Functor

* full translation (map) of one category into another. Translating objects and morphisms (as input can take morphism or object).

* - forgetful - discards part of the structure. * - faithful - fully preserves all morphisms - injective on Hom-sets. * - full - translation of morphisms fully covers all the morphisms between according objecs in the target categoty.

For Functor type class or fmap - see Power set functor.

Functor properties (axioms):

- $F^{\mathcal{C} \to \mathcal{D}}(a) \quad | \quad \forall a^{\mathcal{C}}$ - every source object is mapped to object in target category

- $\overrightarrow{(F^{\mathcal{C}\to\mathcal{D}}(a), F^{\mathcal{C}\to\mathcal{D}}(b))}^{\mathcal{D}} \quad | \quad \forall \overrightarrow{(a,b)}^{\mathcal{C}}$ - every source morphism is mapped to target category morphism between corresponding objects
- $F^{\mathcal{C}\to\mathcal{D}}(\vec{g}^{\mathcal{C}} \circ \vec{f}^{\mathcal{C}}) = F^{\mathcal{C}\to\mathcal{D}}(\vec{g}^{\mathcal{C}}) \circ F^{\mathcal{C}\to\mathcal{D}}(\vec{f}^{\mathcal{C}}) \quad | \quad \forall y = \vec{f}^{\mathcal{C}}(x), \forall \vec{g}^{\mathcal{C}}(y)$ - composition of morphisms translates directly (tautologically goes from other two)

These axioms guarantee that composition of functors can be fused into one functor with composition of morphisms. This process called fusion.

In Haskell this axioms have form:

```
fmap id = id
fmap (f . g) = fmap f . fmap g
```

Since * is 1-1 mapping of initial objects - it is a memoizable dictionary with cardinality of initial objects. Also in Hask category functors are obviously endofunctors ∴ they are special kinds of containers for the parametric values (AKA product type). In Haskell product type * are endofunctors from polymorphic type into a functor wrapper of a polymorphic type.

* translates in one direction, and does not provide algorythm of reversing itself or retriving the parametric value.

2.5.1 *

Functors

2.5.2 Power set functor

$\mathcal{P}^{S\to\mathcal{P}(S)}$

* - functor from set S to its power set $\mathcal{P}(S)$.

Functor type class in Haskell defines a * and allows to do function application inside type structure layers (denoted f or m). IO is also such structure. Power set is unique to the set, * is unique to the category (data type). * embodies in itself any endofunctor. It is easily seen from Haskell definition - that the * is the polymorphic generalization over any endofunctor in a category. Application of a function to * gives a particular endofunctor (see Hask category).

```
class Functor f where
  fmap :: (a -> b) -> f a -> f b
```

Functor instance must be of kind `( * -> * )`, so instance for higher-kinded data type must be applied until this kind.

Composed * can lift functions through any layers of structures that belong to Functor type class.

* can be used to filter-out error cases (Nothing & Left cases) in Maybe, Either and related types.

2.5.2.1 *

fmap Functor type class

2.5.2.2 Power set functor laws

Type instance of functor should abide this laws:

2.5.2.2.1 *

Functor laws

2.5.2.2.2 Power set functor identity law

```
fmap id == id
```

2.5.2.2.3 Power set functor composition law

```
fmap (f.g) == fmap f . fmap g
```

In words, it is if several functions are composed and then fmap is applied on them - it should be the same as if functions was fmapped and then composed.

2.5.2.3 Lift

```
fmap :: (a -> b) -> (f a -> f b)
```

Functor takes function `a -> b` and returns a function `f a -> f b` this is called lifting a function. Lift does a function application through the data structure.

2.5.2.3.1 *

Lifting

2.5.2.4 Power set functor is a free monad

Since:

- $\forall e \in S : \exists \{e\} \in \mathcal{P}(S) \vDash \forall e \in S : \exists (e \to \{e\}) \equiv unit$
- $\forall \mathcal{P}(S) : \mathcal{P}(S) \in \mathcal{P}(S) \vDash \forall \mathcal{P}(S) : \exists (\mathcal{P}(\mathcal{P}(S)) \to \mathcal{P}(S)) \equiv join$

2.5.3 Functorial

Corresponds to functor laws.

2.5.4 Forgetful functor

Functor that forgets part or all of what defines structure in domain category. $F^{\mathbf{Grp} \to \mathbf{Set}}$ that translates groups into their underlying sets. Constant functor is another example.

2.5.4.1 *

Forgetful

2.5.5 Identity functor

Maps all category to itself. All objects and morphisms to themselves.

Denotation: $1^{\mathcal{C} \to \mathcal{C}}$

2.5.6 Endofunctor

Is a functor which source (domain) and target (codomain) are the same category.

$F^{\mathcal{C} \to \mathcal{C}}, E^{\mathcal{C} \to \mathcal{C}}$

2.5.6.1 *

Endofunctors

2.5.7 Applicative functor

* - Computer science term. Category theory name - lax monoidal functor. And in category *Set*, and so in category *Hask* all applicatives and monads are strong (have tensorial strength).

* - sequences functorial computations (plain functors can't).

```
(<*>) :: f (a -> b) -> f a -> f b
```

Requires Functor to exist. Requires Monoidal structure.

Has monoidal structure rules, separated form function application inside structure.

Data type can have several applicative implementations.

Standard definition:

```
class Functor f => Applicative f
 where
  (<*>) :: f (a -> b) -> f a -> f b
  pure :: a -> f a
```

`pure` - if a functor, identity Kleisli arrow, natural transformation.

Composition of * always produces *, contrary to monad (monads are not closed under composition).

`Control.Monad` has an old function `ap` that is old implementation of `<*>`:

```
ap :: Monad m => m (a -> b) -> m a -> m b
```

2.5.7.1 *

Applicative Applicatives Applicative functors

2.5.7.2 Applicative law

2.5.7.3 *

Applicative laws

2.5.7.3.1 Applicative identity law

```
pure id <*> v = v
```

2.5.7.3.2 Applicative composition law

Function composition works regularly.

```
pure (.) <*> u <*> v <*> w = u <*> (v <*> w)
```

2.5.7.3.3 Applicative homomorphism law

Internal function application doesn't change the structure around values.

```
pure f <*> pure x = pure (f x)
```

2.5.7.3.4 Applicative interchange law

On condition that internal order of evaluation is preserved - order of operands is not relevant.

```
u <*> pure y = pure ($ y) <*> u
```

2.5.7.4 Applicative function

2.5.7.4.1 liftA*

2.5.7.4.1.1 liftA

Essentially a fmap.

```
:type liftA
liftA :: Applicative f => (a -> b) -> f a -> f b
```

Lifts function into applicative function.

2.5.7.4.1.2 liftA2

Lifts binary function across two Applicative functors.

```
liftA2 :: Applicative f => (a -> b -> c) -> f a -> f b -> f c
```

```
liftA2 f x y == pure f <*> x <*> y
```

2.5.7.4.1.3 «<liftA2 (<*>)»>

liftA2 (<*>) is pretty useful. It can lift binary operation through the two layers: It is two-layer Applicative.

```
liftA2 :: Applicative f => (     a          ->  b  ->  c ) -> f       a          ->  f    b    ->  f    c
<*> :: Applicative f =>      (f   (a -> b) -> f a -> f b)
liftA2 (<*>) :: (Applicative f1, Applicative f2) =>      f1 (f2 (a -> b)) -> f1 (f2 a) -> f1 (f2
```

2.5.7.4.1.4 liftA2 (liftA2 (<*>))

liftA2 (<*>) 3-layer version.

2.5.7.4.1.5 liftA3

liftA2 3-parameter version.

```
liftA3 f x y z == pure f <*> x <*> y <*> z
```

2.5.7.4.2 Conditional applicative computations

```
when :: Applicative f => Bool -> f () -> f ()
```

Only when `True` - perform an applicative computation.

```
unless :: Applicative f => Bool -> f () -> f ()
```

Only when `False` - perform an applicative computation.

2.5.7.5 Special applicatives

2.5.7.5.1 Identity applicative

```
-- Applicative f =>
-- f ~ Identity
type Id = Identity
instance Applicative Id
  where
    pure :: a -> Id a
    (<*>) :: Id (a -> b) -> Id a -> Id b

mkId = Identity
```

```
xs = [1, 2, 3]

const <$> mkId xs <*> mkId xs'
-- [1,2,3]
```

2.5.7.5.2 Constant applicative

It holds only to one value. The function does not exist and last parameter is a phantom.

```
-- Applicative f =>
-- f ~ Constant e
type C = Constant
instance Applicative C
 where
  pure :: a -> C e a
  (<*>) :: C e (a -> b) -> C e a -> C e b
```

2.5.7.5.3 Maybe applicative

"There also can be no function at all."

If function might not exist - embed `f` in Maybe structure, and use Maybe applicative.

```
-- f ~ Maybe
type M = Maybe
pure :: a -> M a
(<*>) :: M (a -> b) -> M a -> M b
```

2.5.7.5.4 Either applicative

`pure` is `Right`. Defaults to `Left`. And if there is two `Left`'s - to Left of the first argument.

2.5.7.5.5 Validation applicative

The Validation data type isomorphic to Either, but has accumulative Applicative on the Left side. Validation data type is not a monad. Validation is an example of, "An applicative functor that is not a monad." While Either monad on `Left` case just drops computation and returns this first `Left`. Monad needs to process the result of computation - it requires to be able to process all Left error statement cases for Validation, it is or non-terminaring Monad or one which is impossible to implement in polymorphic way with Validation.

2.5.7.6 Monad

μόνος *monos* sole μονάδα *monáda* unit

* - monoid in endofunctor category with η (unit) and μ (join) natural transformations.

Monad on $\mathcal{C}$ is $\{E^{\mathcal{C}\to\mathcal{C}}, \eta, \mu\}$:

- $E^{\mathcal{C}\to\mathcal{C}}$ - is an endofunctor
- two natural transformations, $1^c \to E$ and $E \circ E \to E$:
  - $\eta^{1^{\mathcal{C}}\to E} = unit^{Identity\to E}(x) = f^{x\to E(x)}(x)$
  - $\mu^{(E\circ E)\to E} = join^{(E\circ E)\to(Identity\circ E)}(x) = |y = E(x)| = f^{E(y)\to y}(y)$

where:

- $\mathcal{C}$ is a category
- $1^{\mathcal{C}}$ denotes the $\mathcal{C}$ identity functor

- $(E \circ E)$ - endofunctor $\mathcal{C} \to \mathcal{C}$

Definition with $\{E^{\mathcal{C} \to \mathcal{C}}, \eta, \mu\}$ (in Hask: $(\{e :: fa \to fb, pure, join\})$) - is classic categorical, in Haskell minimal complete definition is $\{fmap, pure, (\gg=)\}$.

If there is a structure S, and a way of taking object x into S and a way of collapsing $S \circ S$ - there probably a monad.

Mostly monads used for sequencing actions (computations) (that looks like imperative programming), with ability to dependend on previous chains. Note if monad is commutative - it does not order actions.

Monad can shorten/terminate sequence of computations. It is implemented inside Monad instance. For example Maybe monad on Nothing drops chain of computation and returns Nothing.

* inherits the Applicative instance methods:

```
import Control.Monad (ap)
return == pure
ap == (<*>) -- + Monad requirement
```

Table 2.1: Monad in mathematics and Haskell

Math	Meaning	Cat/Fctr	$X \in C$	Type	Haskell
Id	endofunctor "Id"	$C \to C$	$X \to Id(X)$	$a \to a$	id
E	endofunctor "monad"	$C \to C$	$X \to E(X)$	$m\ a \to m\ b$	fmap
η	natural transformation "unit"	$Id \to E$	$Id(X) \to E(X)$	$a \to m\ a$	pure
μ	natural transformation "multiplication"	$E \circ E \to E$	$E(E(X)) \to E(X)$	$m\ (m\ a) \to m\ a$	join

Internals of Monad are Haskell data types, and as such - they can be consumed any number of times.

Composition of monadic types does not always results in monadic type.

2.5.7.6.1 *

Monads Monadic

2.5.7.6.2 Monad law

Monad corresponds to functor laws & applicative laws and additionally:

2.5.7.6.2.1 *

Monad laws

2.5.7.6.2.2 Monad left identity law

```
pure x >>= f == f x
```

Explanation:

```
>>= :: Monad f =>     f a  -> (a -> f b) -> f b
                 pure x >>=       f      == f x
```

Rule that »= must get first argument structure internals and apply to the function that is the second argument.

2.5.7.6.2.3 Monad right identity law

```
f >>= pure == f
```

Explanation:

```
>>= :: Monad f => f a  -> (a -> f b) -> f b
                  f   >>=     pure     == f
```

AKA it is a tacit description of a monad bind as endofunctor.

2.5.7.6.2.4 Monad associativity law

```
(m >>= f) >>= g == m >>= (\ x -> f x >>= g)
```

2.5.7.6.3 Monad type class

```
class Applicative m => Monad m where
  (>>=) :: m a -> (a -> m b) -> m b
  (>>) :: m a -> m b -> m b
  return :: a -> m a
```

2.5.7.6.3.1 MonadPlus type class

Is a monoid over monad, with additional rules. The precise set of rules (properties) not agreed upon. Class instances obey *monoid* & *left zero* rules, some additionally obey *left catch* and others *left distribution*.

Overall there * currently reforms (MonadPlus reform proposal) in several smaller nad strictly defined type classes.

Subclass of an Alternative.

*

Monadplus

2.5.7.6.4 Functor -> Applicative -> Monad progression

```
<$> :: Functor     f =>   (a -> b)   -> f a -> f b
<*> :: Applicative f => f (a -> b)   -> f a -> f b
=<< :: Monad       f =>   (a -> f b) -> f a -> f b
```

`pure` & `join` are Natural transformations for the `fmap`.

2.5.7.6.5 Monad function

2.5.7.6.5.1 Return function

```
return == pure
```

Nonstrict.

2.5.7.6.5.2 Join function

```
join :: Monad m => m (m a) -> m a
```

Generales knowledge of `concat`.

Kleisli composition that flattens two layers of structure into one.

The way to express ordering in lambda calculus is to nest.

*

join

join . fmap == (=<<)

```
-- b = f b
fmap        :: Monad f => (a -> f b) -> f a -> f (f b)
join        :: Monad f =>                      f (f a) -> f a
join . fmap :: Monad f => (a -> f b) -> f a           -> f b
flip    >>= :: Monad f => (a -> f b) -> f a           -> f b
```

2.5.7.6.5.3 Bind function

```
>>=         :: Monad f => f a -> (a -> f b) -> f b
join . fmap :: Monad f => (a -> f b) -> f a -> f b
```

Nonstrict.

The most ubiqutous way to >>= something is to use Lambda function:

```
getLine >>= \name -> putStrLn "age pls:"
```

Also a neet way is to bundle and handle Monad - is to bundle it with bind, and leave applied partially. And use that partial bundle as a function - every evaluation of the function would trigger evaluation of internal Monad structure. Thumbs up.

```
printOneOf :: Bool -> IO ()
printOneOf False = putStr "1"
printOneOf  True = putStr "2"

quant :: (Bool -> IO b) -> IO b
quant = (>>=) (randomRIO (False, True))

recursePrintOneOf :: Monad m => (t -> m a) -> t -> m b
recursePrintOneOf f x = (f x) >> (recursePrintOneOf f x)

main :: IO ()
main = recursePrintOneOf (quant) $ printOneOf
```

*

Monadic extend Monadic bind Monad bind Binder

```
(>>=)
>>=
(=<<)
=<<
```

2.5.7.6.5.4 Sequencing operator (>>) \equiv (*>):

Discard any resulting value of the action and sequence next action. Applicative has a similar operator.

```
(>>) :: m a -> m b -> m b
(*>) :: f a -> f b -> f b
```

2.5.7.6.5.5 Monadic versions of list functions

```
sequence :: (Traversable t, Monad m) => t (m a) -> m (t a)
```

Sequence gets the traversable of monadic computations and swaps it into monad computation of taverse. In the result the collection of monadic computations turns into one long monadic computation on traverse of data.

If some step of this long computation fails - monad fails.

```
mapM :: (Traversable t, Monad m) => (a -> m b) -> t a -> m (t b)
```

mapM gets the AMB function, then takes traversable data. Then applies AMB function to traversable data, and returns converted monadic traversable data.

```
foldM :: (Foldable t, Monad m) => (b -> a -> m b) -> b -> t a -> m b
foldl ::  Foldable t            => (b -> a ->   b) -> b -> t a ->   b
```

* is a monadic `foldl`.

`b` is initial comulative value, `m b` is a comulative bank. Right folding achieved by reversing the input list.

```
filterM :: Applicative m => (a -> m Bool) -> [a] -> m [a]
filter ::                   (a ->   Bool) -> [a] ->   [a]
```

Take Boolean monadic computation, filter the list by it.

```
zipWithM :: Applicative m => (a -> b -> m c) -> [a] -> [b] -> m [c]
zipWith  ::                  (a -> b ->   c) -> [a] -> [b] ->   [c]
```

Take monadic combine function and combine two lists with it.

```
msum :: (Foldable t, MonadPlus m) => t (m a) -> m a
sum  :: (Foldable t, Num a)       => t    a  ->   a
```

2.5.7.6.5.6 liftM*

liftM Essentially a fmap.

```
liftM :: Monad m => (a -> b) -> m a -> m b
```

Lifts a function into monadic equivalent.

liftM2 Monadic liftA2.

```
liftM2 :: Monad m => (a -> b -> c) -> m a -> m a -> m c
```

Lifts binary function into monadic equivalent.

2.5.7.6.6 Comonad

Category $\mathcal{C}$ comonad is a monad of opposite category $\mathcal{C}^{op}$.

2.5.7.6.7 Kleisli arrow

Morphism that while doing computation also adds monadic-able structure.

```
a -> m b
```

2.5.7.6.7.1 *

Kleisli arrows Kleisli morphism Kleisli morphisms

2.5.7.6.8 Kleisli composition

Composition of Kleisli arrows.

```
(<=<) :: Monad m => (b -> m c) -> (a -> m b) -> a -> m c infixr 1
;; compare
(.)   ::            (b ->   c ) -> (a ->   b ) -> a ->   c
```

Often used left-to-right version:

```
(>=>) :: Monad m => (a -> m b) -> (b -> m c) -> a -> m c
;; compare
(>>=) :: Monad m =>       m a  -> (a -> m b)       -> m b
```

Which allows to replace monadic bind chain with Kleisli composition.

```
f1 arg >>= f2 >>= f3
==
f1 >=> f2 >=> f3 $ arg
==
f3 <=< f2 <=< f1 $ arg
```

2.5.7.6.9 Kleisli category

Category $\mathcal{C}$, $\langle E, \vec{\eta}, \vec{\mu} \rangle$ monad over $\mathcal{C}$.

Kleisli category $\mathcal{C}_T$ of $\mathcal{C}$:

$\mathrm{Obj}(\mathcal{C}_T) = \mathrm{Obj}(\mathcal{C})$ $\mathrm{Hom}_{\mathcal{C}_T}(x, y) = \mathrm{Hom}_{\mathcal{C}}(x, E(y))$

2.5.7.6.10 Special monad

2.5.7.6.10.1 Identity monad

Wraps data in the Identity constructor.

Useful: Creates monads from monad transformers.

Bind: Applies internal value to the bound function.

Code: (see: coerce)

```
newtype Identity a = Identity { runIdentity :: a }

instance Functor Identity where
  fmap     = coerce

instance Applicative Identity where
  pure     = Identity
  (<*>)    = coerce

instance Monad Identity where
  m >>= k  = k (runIdentity m)
```

Example:

```
-- derive the State monad using the StateT monad transformer
type State s a = StateT s Identity a
```

2.5.7.6.10.2 Maybe monad

Something that may not be or not return a result. Any lookups into the real world, database queries.

Bind: `Nothing` input gives `Nothing` output, `Just x` input uses `x` as input to the bound function.

When some computation results in Nothing - drops the chain of computations and returns Nothing.

Zero: Nothing Plus: result in first occurence of Just else Nothing.

Code:

```
data Maybe a = Nothing | Just a

instance Monad Maybe where
  return         = Just
  fail           = Nothing
  Nothing  >>= _ = Nothing
  (Just x) >>= f = f x

instance MonadPlus Maybe where
  mzero               = Nothing
  Nothing `mplus` x = x
  x `mplus` _         = x
```

Example: Given 3 dictionaries:

1. Full names to email addresses,
2. Nicknames to email addresses,
3. Email addresses to email preferences.

Create a function that finds a person's email preferences based on either a full name or a nickname.

```
data MailPref = HTML | Plain
data MailSystem = ...

getMailPrefs :: MailSystem -> String -> Maybe MailPref
getMailPrefs sys name =
  do let nameDB = fullNameDB sys
         nickDB = nickNameDB sys
         prefDB = prefsDB sys
  addr <- (lookup name nameDB) `mplus` (lookup name nickDB)
  lookup addr prefDB
```

2.5.7.6.10.3 Either monad

When computation results in `Left` - drops other computations & returns the recieved `Left`.

2.5.7.6.10.4 Error monad

Someting that can fail, throw exceptions.

The failure process records the description of a failure. Bind function uses successful values as input to the bound function, and passes failure information on without executing the bound function.

Useful: Composing functions that can fail. Handle exceptions, crate error handling structure.

Zero: empty error. Plus: if first argument failed then execute second argument.

2.5.7.6.10.5 List monad

Computations which may return 0 or more possible results.

Bind: The bound function is applied to all possible values in the input list and the resulting lists are concatenated into list of all possible results.

Useful: Building computations from sequences of non-deterministic operations.

Zero: [] Plus: (++)

*

[] monad

2.5.7.6.10.6 Reader monad

Creates a read-only shared environment for computations.

The `pure` function ignores the environment, while »= passes the inherited environment to both subcomputations.

Today it is defined though ReaderT transformer:

```
type Reader r = ReaderT r Identity   -- equivalent to ((->) e), (e ->)
```

Old definition was:

```
newtype Reader e a = Reader { runReader :: (e -> a) }
```

For `(e ->)`:

- Functor is `(.)`

```
fmap :: (b -> c) -> (a -> b) -> a -> c
fmap = (.)
```

- Applicative:
  - `pure` is `const`

```
pure :: a -> b -> a
pure x _ = x
```

- `(<*>)` is:

```
(<*>) :: (a -> b -> c) -> (a -> b) -> a -> c
(<*>) f g = \a -> f a (g a)
```

- Monad:

```
(>>=) :: (a -> b) -> (b -> a -> c) -> a -> c
(>>=) m k = Reader $ \r ->
  runReader (k (runReader m r)) r

join :: (e -> e -> a) -> e -> a
join f x = f x x
```

```
runReader
  :: Reader r a  -- the Reader to run
  -> r   -- an initial environment
  -> a   -- extracted final value
```

Usage:

```
data Env = ...

createEnv :: IO Env
createEnv = ...

f :: Reader Env a
f = do
  a <- g
  pure a

g :: Reader Env a
g = do
  env <- ask   -- "Open the environment namespace into env"
  a <- h env   -- give env to h
```

```haskell
  pure a

h :: Env -> a
...  -- use env and produce the result

main :: IO ()
main = do
  env <- createEnv
  a = runReader g env
  ...
```

In Haskell under normal circumstances impure functions should not directy call impure functions. `h` is an impure function, and `createEnv` is impure function, so they should have intermediary.

2.5.7.6.10.7 Writer monad

Computations which accumulate monoid data to a shared Haskell storage. So * is parametrized by monoidal type.

Accumulator is maintained separately from the returned values.

Shared value modified through Writer monad methods.

* frees creator and code from manually keeping the track of accumulation.

Bind: The bound function is applied to the input value, bound function allowed to <> to the accumulator.

```haskell
type Writer r = WriterT r Identity
```

Example:

```haskell
f :: Monoid b => a -> (a, b)
f a = if _condition_
         then runWriter $ g a
         else runWriter do
           a1 <- h a
           pure a1

g :: Monoid b => Writer b a
g a = do
  tell _value1_  -- accumulator <> _value1_
  pure a  -- observe that accumulator stored inside monad
          -- and only a main value needs to be returned.

h :: Monoid b => Writer b a
h a = do
  tell _value2_  -- accumulator <> _value_
  pure a

runWriter :: Writer w a -> (a, w)  -- Unwrap a writer computation
                                   -- as a (result, accumulator) pair.
                                   -- The inverse of writer.
```

`WriterT`, `Writer` unnecessarily keeps the entire logs in the memory. Use `fast-logger` for logging.

2.5.7.6.10.8 State monad

Computations that pass-over a state.

The bound function is applied to the input value to produce a state transition function which is applied to the input state.

Pure functional language cannot update values in place because it violates referential transparency.

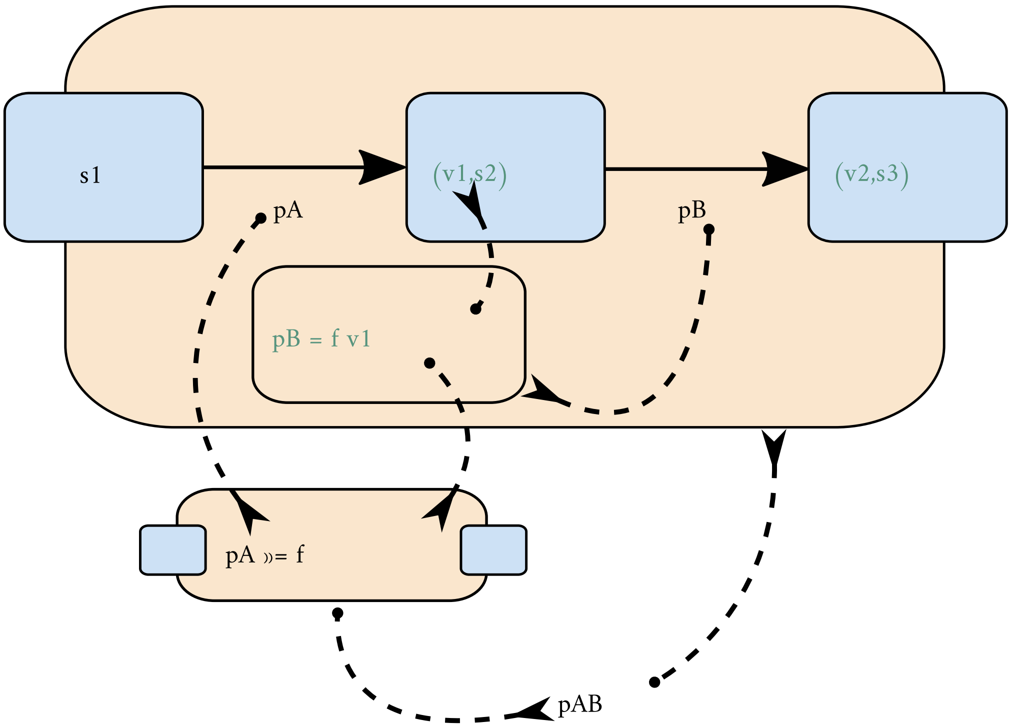
```
type State s = StateT s Identity
```

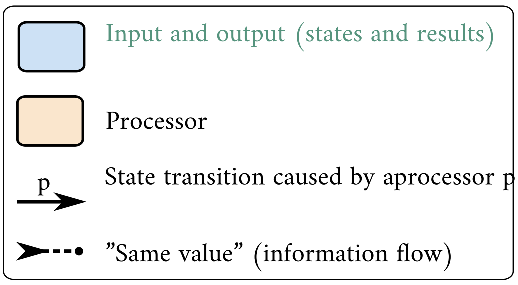
Binding copies and transforms the state parameter through the sequence of the bound functions so that the same state storage is never used twice. Overall this gives the illusion of in-place update to the programmer and in the code, while in fact the autogenerated transition functions handle the state changes.

Example type: `State st a`

`State` describes functions that consume a state and produce a tuple of result and an updated state.

Monad manages the state with the next process:





pAB = pA »= f = state $s1-> (v2, s3) where (v1, s2) = runStatepAs1p$

Where:

- f - processsor making function
- pA, pAB, pB - state processors
- sN - states
- vN - values

Bind with a processor making function from state procesor (pA) creates a new state processor (pAB). The wrapping and unwrapping by State/runState is implicit.

2.5.7.6.11 Monad transformer

* is a practical solution to the current functional programming problem about composition of monads.

Monad is not closed under compostion. Composition of monadic types does not always results in monadic type.

Basic case: during implementation of monadic type composition, type `m T m a` arises, which does not allow to `unit`, `join` the `m` monadic layers.

* have desirable properies and can add them to monads. * user their implementation to solve the composition type layering and allow to attach desirable property to result.

* solve monad composition and type layering by cheating, using own structure and information about itself. It is often that process involves a catamorphism of a * type layer.

In type signatures of transformers `*T m` - `m` is already an extended monad, so `*T` is just a wrapper to point that out.

Transformers have a light wrapper around the data that tags the modification with this transformer.

Main monadic structure `m` is wrapped around the internal data (core is `a`). The structure that corresponds to the transformer creation properties (if it emitted by η of a transformer), goes into `m` . Open parameters go external to the `m`.

```
newtype ExceptT e m a =
  ExceptT { runExceptT :: m (Either e a) }

newtype MaybeT m a =
  MaybeT { runMaybeT :: m (Maybe a) }

newtype ReaderT r m a =
  ReaderT { runReaderT :: r -> m a }
```

This has an effect that on stacking monad transformers, `m` becomes monad stack, and every next transformer injects the transformer creation-specific properies η inside the stack, so out-most transformer has inner-most structure. Base monad is structurally the outermost.

2.5.7.6.11.1 MaybeT

* extends monads by injecting Maybe layer underneath monad, and processing that structure:

```
newtype MaybeT m a = MaybeT { runMaybeT :: m (Maybe a) }
```

2.5.7.6.11.2 EitherT

* extends monads by injecting Either layer underneath monad, and processing that structure:

```
newtype EitherT e m a = EitherT { runEitherT :: m (Either e a) }
```

`EitherT` of `either` package gets replaced by `ExceptT` of `transformers` or `mtl` packages.

* ExceptT

2.5.7.6.11.3 ReaderT

Definition:

```
newtype ReaderT r m a = ReaderT { runReaderT :: r -> m a }
```

* functions: input monad `m a`, out: `m a` wrapped it in a free-variable `r` (partially applied function). That allows to use transformed `m a`, now it requires and can use the `r` passed environment.

To create a Reader monad:

```
type Reader r = ReaderT r Identity
```

2.5.7.6.11.4 MonadTrans type class

Allows to lift monadic actions into a larger context in a neutral way.

`pure` takes a parametric type and embodies it into constructed structure (talking of monad transformers - structure of the stacked monads).

`lift` takes monad and extends it with a transformer.

In fact, for monad transformers - `lift` is a last stage of the `pure`, it follows from the lift law.

Method:

```
lift :: Monad m => m a -> t m a
```

Lift a computation from the argument monad to the constructed monad.

Neutral means:

```
lift . return = return

lift (m >>= f) = lift m >>= (lift . f)
```

The general pattern with MonadTrans instances is that it is usually lifts the injection of the known structure of transformer over some Monad.

lift embeds one monadic action into monad transformer.

The difference between pure, lift and MaybeT contructor becomes clearer if you look at the types:

Example, for `MaybeT IO a`:

```
pure    ::      a  -> MaybeT IO a
lift   ::    IO a  -> MaybeT IO a
MaybeT :: IO (Maybe a) -> MaybeT IO a

x = (undefined :: IO a)

:t (pure x)
(pure x) :: Applicative t => t (IO a)  -- t recieves one argument of product type
:t (pure x :: MaybeT IO a)
-- Expected type: MaybeT IO a1
--   Actual type: MaybeT IO (IO a0)

-- While the real type would be
:t (pure x :: MaybeT IO (IO a))
(pure x :: MaybeT IO (IO a)) :: MaybeT IO (IO a) -- This goes into a conflict of what type&kind (


:t (lift x)
(lift x) :: MonadTrans t => t IO a  -- result is a proper expected product type

-- To belabour
:t (lift x :: MaybeT IO a)
(lift x) :: MonadTrans t => t IO a  -- result is a proper expected product type
```

`lift` is a natural transformation η from an Identity monad (functor) with other monad as content into transformer monad (functor), with the preservation of the conteined monad:

```
-- Abstract monads with content as parameters. Define '~>' as a family of morphisms that translat
type f ~> g = forall x. f x -> g x
-- follows
lift :: m ~> t m
```

MonadIO type class * - allows to lift IO action until reaching the IO monad layer at the top of the `Monad` stack (which is allways in the Haskell code that does IO).

```
class (Monad m) => MonadIO m where
  liftIO :: IO a -> m a
```

`liftIO` laws:

```
liftIO . pure = pure
```

```
liftIO (m >>= f) = liftIO m >>= (liftIO . f)
```

Which is identical laws to MonadTrans `lift`.

Since `lift` is one step, and liftIO all steps - all steps defined in terms of one step and all other steps, so the most frequent implementation is self-recursive `lift . liftIO`:

```
liftIO ioa = lift $ liftIO ioa
```

*

liftIO

2.5.7.7 Alternative type class

Monoid over applicative. Has left catch property.

Allows to run simolteniously several instances of a computation (or computations) and from them yeld one result by law from `(<|>) :: Type -> Type -> Type`.

Minimal complete definition:

```
empty :: f a     -- The identity element of <|>
(<|>) :: f a -> f a -> f a     -- Associative binary operation
```

2.5.7.7.1 *

Alternative

2.5.8 Monoidal functor

Functors between monoidal categories that preserves monoidal structure.

2.5.9 Fusion

```
fmap f . fmap g = fmap (f . g)
```

* - functor axiom that allows to greatly simplify computations.

2.5.10 $>

Get & set a value inside Functor.

2.5.10.1 *

<$

2.5.11 Multifunctor

Generalizes the concept of functor between categories, canonical morphisms between multicategories.

Works over N type arguments instead of one.

To put simply - accepts multiple argumets, from that informatioin constructs source product category (Cartesian product) of categories, and is a functor from product category to target category.

To put even simplier - functor that takes as an argument the product of types.

In Haskell there is only one category, Hask, so in Haskell * is still $(Hask \times Hask) \rightarrow Hask \Rightarrow |(Hask \times Hask) \equiv Hask| \Rightarrow Hask \rightarrow Hask$ endofunctor.

Any product or sum in a Cartesian category is a *.

Code definition:

```
class Bifunctor f
 where
  bimap :: (a -> a') -> (b -> b') -> f a a' -> f a a'
  bimap f g = first f . second g
  first :: (a -> a') -> f a b -> f a' b
  first f = bimap f id
  second :: (b -> b') -> f a b -> f a b'
  second = bimap id
```

2.5.11.1 *

Bifunctor

2.6 Hask category

Category of Haskell where objects are types and morphisms are functions.

It is a hypothetical category at the moment, since undefined and bottom values break the theory, is not Cartesian closed, it does not have sums, products, or initial object, () is not a terminal object, monad identities fail for almost all instances of the Monad class.

That is why Haskell developers think in subset of Haskell where types do not have bottom values. This only includes functions that terminate, and typically only finite values. The corresponding category has the expected initial and terminal objects, sums and products, and instances of Functor and Monad really are endofunctors and monads.

Hask contains subcategories, like Lst containing only list types.

Haskell and Category concepts:

- Things that take a type and return another type are type constructors.
- Things that take a function and return another function are higher-order functions.

2.6.1 *

Hask

2.7 Magma

Set with a binary operation which form a closure.

2.7.1 Mag category

The category of magmas, denoted Mag, has as objects - sets with a binary operation, and morphisms given by homomorphisms of operations (in the universal algebra sense).

2.7.1.1 *

MAG Magma category Category of magmas

2.7.2 Semigroup

Magma with associative property of operation.

Defined in Haskell as:

```
class Semigroup a where
(<>) :: a -> a -> a
```

2.7.2.1 *

Semigroups

2.7.2.2 Monoid

Semigroup with identity element. Category with a one object.

Ideally fits as an accumulation class.

```
class Monoid m where
mempty :: m
mappend :: m -> m -> m
mappend = (<>)
mconcat :: [m] -> m
mconcat = foldr mappend mempty
```

* can be simplified to category with a single object, remember that monoid operation is a composition of morphisms operation in category. For example to represent the whole non-negative integers with the one object and morphism "1" is absolutely enough, composition operation is "+".

```
import Data.Monoid
do
  show (mempty :: Num a => Sum a)
  -- "Sum {getSum = 0}"
  show $ Sum 1
  -- "Sum {getSum = 1}"
  show $ (Sum 1) <> (Sum 1) <> (Sum 1)
  -- "Sum {getSum = 3}"
  -- ...
```

Also backwards - any single-object category is a monoid. Category has an identity requirement and associativity of composition requirement, which makes it a free monoid.

2.7.2.2.1 *

Monoidal Monoids

2.7.2.2.2 Monoid laws

2.7.2.2.2.1 Monoid left identity law

```
mempty <> x = x
```

2.7.2.2.2.2 Monoid right identity law

```
x <> mempty = x
```

2.7.2.2.2.3 Monoid associativity law

```
x <> mempty = x (y <> z) = (x <> y) <> z
mconcat = foldr (mempty <>)
```

Everything associative can be mappend.

2.7.2.2.3 Commutative monoid

Operation that forms structure has commutativity property: $x \circ y = y \circ x$

Opens a big abilities in concurrent and distributed processing.

2.7.2.2.3.1 *

Abelian monoid

2.7.2.2.4 Group

Monoid that has inverse for every element.

2.7.2.2.4.1 *

Groups

2.7.2.2.4.2 Commutative group

Commutative monoid that is a group.

*

Abelian group

Ring Commutative group under + & monoid under ×, + × connected by distributive property.

- and × are generalized binary operations of addition and multiplication. × has no requirement for commutativity.

Example: set of same size square matricies of numbers with matrix operations form a ring.

*

Rings

2.8 Morphism

μορφή *morphe* form

Arrow between two objects inside a category.

Morphism can be anything.

Morphism is a generalization $(f(x * y) \equiv f(x) \diamond f(y))$ of homomorphism $(f(x * y) \equiv f(x) * f(y))$. Since general morphisms not so much often ment and discussed - under morphism people almost always really mean the meaning of homomorphism-like properties, hense they discuss the algebraic structures (types) and homomorphisms between them.

In most usage, on a level under uder the objects: * is most often means a map (relation) that translates from one mathematical structure (that source object represents) to another (that target object represents) (that is called (somewhat, somehow) "structure-preserving", but that phrase still means that translation can be lossy and irrevertable, so it is only bear reassemblence of preservation), and in the end the morphism can be anything and not hold to this conditions.

Morphism needs to correspond to function requirements to be it.

2.8.1 *

Morphisms Arrow Arrows

2.8.2 Homomorphism

ὁμός *homos* same (was chosen becouse of initial Anglish mistranslation to "similar")

μορφή *morphe* form

similar form

* map between two algebraic structures of the same type, operation-preserving.

$f_{x \to y} = f(a \star b) = f(a) \diamond f(b)$, where $x,\ y$ are sets with additonal algebraic structure that includes $\star, \diamond$ accordingly; $a,\ b$ are elements of set x.

* sends identity morphisms to identity morphisms and inverses to inverses.

The concept of * has been generalized under the name of morphism to many structures that either do not have an underlying set, or are not algebraic.

2.8.2.1 *

Homomorphic

2.8.3 Identity morphism

Identity morphism - or simply identity: $x \in C: \ id_x = 1_x : x \to x$ Composed with other morphism gives same morphism.

Corresponds to Reflexivity and Automorphism.

2.8.3.1 Identity

Identity only possible with morphism. See Identity morphism.

There is also distinct Zero value.

2.8.3.1.1 Two-sided identity of a predicate

$P(e, a) = P(a, e) = a \mid \exists e \in S, \forall a \in S\ P()$ is commutative.

Predicate

2.8.3.1.2 Left identity of a predicate

$\exists e \in S, \forall a \in S: \ P(e, a) = a$

Predicate

2.8.3.1.3 Right identity of a predicate

$P(a, e) = a \mid \exists e \in S, \forall a \in S$

Predicate

2.8.3.2 Identity function

Return itself. (\ x.x)

```
id :: a -> a
```

2.8.4 Monomorphism

μονο *mono* only

μορφή *morphe* form

Maps one to one (uniquely), so invertable (always has inverse morphism), so preserves the information/structure. Domain can be equal or less to the codomain.

$f^{X \to Y}$, $\forall x \in X \, \exists! y = f(x) \vDash f(x) \equiv f_{mono}(x)$ - from homomorphism context $f_{mono} \circ g1 = f_{mono} \circ g2 \ \vDash \ g1 \equiv g2$ - from general morphism context Thus * is left canselable.

If * is a function - it is injective. Initial set of *f* is fully uniquely mapped onto the image of *f*.

2.8.4.1 *

Monomorphic Monomorphisms

2.8.5 Epimorphism

επι *epi* on, over

μορφή *morphe* form

* is right canselable morphism. $f^{X \to Y}, \forall y \in Y \exists f(x) \vDash f(x) \equiv f_{epi}(x)$ - from homomorphism context $g_1 \circ f_{epi} = g_2 \circ f_{epi} \Rightarrow \ g_1 \equiv g_2$ - from general morphism context

In Set category if * is a function - it is surjective (image of it fully uses codomain) Codomain is a called a projection of the domain.

* fully maps into the target.

2.8.5.1 *

Epimorphic Epimorphisms

2.8.6 Isomorphism

ἴσος *isos* equal

μορφή *morphe* form

Not equal, but equal for current intents and purposes. Morphism that has inverse. Almost equal, but not quite: `(Integer, Bool)` & `(Bool, Integer)` - but can be transformed losslessly into one another.

Bijective homomorphism is also isomorphism.

$f^{-1,b \to a} \circ f^{a \to b} \equiv 1^a, \ f^{a \to b} \circ f^{-1,b \to a} \equiv 1^b$

2 reasons for non-isomorphism:

- function at least ones collapses a values of domain into one value in codomain
- image (of a function in codomain) does not fill-in codomain. Then isomorphism can exists for image but not whole codomain.

Categories are isomorphic if there $R \circ L = ID$

2.8.6.1 *

Isomorphic Isomorphisms

2.8.6.2 Lax

Holds up to isomorphism. (upon the transformation can be used as the same)

2.8.7 Endomorphism

ενδο *endo* internal

μορφή *morphe* form

Arrow from object to itself. Endomorphism forms a monoid (object exists and category requirements already in place).

2.8.7.1 Automorphism

αυτο *auto* self

μορφή *form* form

Isomorphic endomorphism.

Corresponds to identity, reflexivity, permutation.

2.8.7.1.1 *

Automorphic Automorphisms

2.8.7.2 *

Endomorphic Endomorphisms

2.8.8 Catamorphism

κατά *kata* downward

μορφή *morphe* form

Unique arrow from an initial algebra structure into different algebra structure.

* in FP is a generalization folding, deconstruction of a data structure into more primitive data structure using a functor F-algebra structure.

* reduces the structure to a lower level structure. * creates a projection of a structure to a lower level structure.

2.8.8.1 *

Catamorphic Catamorphisms

2.8.8.2 Catamorphism law

2.8.8.2.1 Hylomorphism

catamorphism ∘ anamorphism

Expanding and collapsing the structure.

2.8.8.2.1.1 *

Hylomorphic Hylomorphisms

Table 2.2: Catamorphism laws in Haskell

Rule name	Haskell
cata-cancel	`cata phi . InF = phi . fmap (cata phi)`
cata-refl	`cata InF = id`
cata-fusion	`f . phi = phi . fmap f => f . cata phi = cata phi`
cata-compose	`eps :: f :~> g => cata phi . cata (In . eps) = cata (phi . eps)`

2.8.8.3 Anamorphism

Generalizes unfold.

Dual concept to catamorphism.

Increases the structure.

Morphism from a coalgebra to the final coalgebra for that endofunctor.

Is a function that generates a sequence by repeated application of the function to its previous result.

2.8.8.3.1 *

Anamorphic Anamorphisms

2.8.9 Kernel

Kernel of a homomorphism is a number that measures the degree homomorphism fails to meet injectivity (AKA be monomorphic). It is a number of domain elements that fail injectivity:

- elements not included into morphism
- elements that collapse into one element in codomain

thou Kernel $[x|x \leftarrow 0||x \geq 2]$.

Denotation: $\ker T = \{\mathbf{v} \in V : T(\mathbf{v}) = \mathbf{0}_W\}$.

2.8.9.1 Kernel homomorphism

Morphism of elements from the kernel. Complementary morphism of elements that make main morphism not monomorphic.

2.9 Set category

Category in which objects are sets, morphisms are functions.

Denotation: Set

2.10 Natural transformation

Roughly * is:

```
trans :: F a -> G a
```

, while `a` is polymorphic variable.

Naturality condition: $\forall\, a\, \exists\, (F\, a \rightarrow G\, a)$, or , analogous to parametric polymorphism in functions. Since * in a category, stating $\forall(F\ a \rightarrow G\ a)$ Naturality condition means that all morphisms that take part in homotopy of source functor to target functor must exist, and that is the same, diagrams that take part in transformation, should commute, and different paths brins same result: if α - natural transformation, α_a natural transformation component - $G\, f \circ \alpha_a = \alpha_b \circ F\, f$. Since * are just a type of parametric polymorphic function - they can compose.

* $(\vec{\eta}^{\mathcal{D}})$ is transforming : $\vec{\eta}^{\mathcal{D}} \circ F^{\mathcal{C}\to\mathcal{D}} = G^{\mathcal{C}\to\mathcal{D}}$. * abstraction creates higher-language of Category theory, allowing to talk about the composition and transformation of complex entities.

It is a process of transforming $F^{\mathcal{C}\to\mathcal{D}}$ into $G^{\mathcal{C}\to\mathcal{D}}$ using existing morphisms in target category $\mathcal{D}$.

Since it uses morphisms - it is structure-preserving transformation of one functor into another. Iy mostly a lossy transformation. Only existing morphisms cab make it exist.

Existence of * between two functors can be seen as some relation.

Can be observed to be a "morphism of functors", especially in functor category. * by $\vec{\eta}^{\mathcal{D}}_{y^{\mathcal{C}}}(\overrightarrow{(x,y)}^{\mathcal{C}}) \circ F^{\mathcal{C}\to\mathcal{D}}(\overrightarrow{(x,y)}^{\mathcal{C}}) = G^{\mathcal{C}\to\mathcal{D}}(\overrightarrow{(x,y)}^{\mathcal{C}}) \circ \vec{\eta}^{\mathcal{D}}_{x^{\mathcal{C}}}(\overrightarrow{(x,y)}^{\mathcal{C}})$, often written short $\vec{\eta}_b \circ F(\vec{f}) = G(\vec{f}) \circ \vec{\eta}_a$. Notice that the $\vec{\eta}^{\mathcal{D}}_{x^{\mathcal{C}}}(\overrightarrow{(x,y)}^{\mathcal{C}})$ depends on objects&morphisms of $\mathcal{C}$.

In words: * depends on F and G functors, ability of D morphisms to do a homotopy of F to G, and *:

- for every object in $\mathcal{C}$ picks natural transformation component in $\mathcal{D}$.
- for every morphism in $\mathcal{C}$ picks the commuting diagram in $\mathcal{D}$, called naturality square.

Also see: Natural transformation in Haskell

Knowledge of * forms a 2-category.

Can be composed

- "vertically":

$$\begin{tikzcd} a \arrow[r, bend left=40, "F"] \arrow[r, "G" description] \arrow[r, bend right=50, "H"'] \arrow[r, phantom, shift left=3, "\Downarrow \alpha"] \arrow[r, phantom, shift right=4, "\Downarrow \beta"] & b \end{tikzcd}$$

$$\begin{tikzcd} a \arrow[r, bend left=40, "F"] \arrow[r, bend right=40, "H"'] \arrow[r, phantom, "\Downarrow \beta \circ \alpha"] & b \end{tikzcd}$$

- "horizontally" ("Godement product"):

$$\begin{tikzcd} a \arrow[r, bend left, "F_1"] \arrow[r, bend right, "G_1"'] \arrow[r, phantom, "\Downarrow \alpha"] & b \arrow[r, bend left, "F_2"] \arrow[r, bend right, "G_2"'] \arrow[r, phantom, "\Downarrow \beta"] & c \end{tikzcd}$$

$$\begin{tikzcd} a \arrow[r, bend left, "F_2 \circ F_1"] \arrow[r, bend right, "G_2 \circ G_1"'] \arrow[r, phantom, "\Downarrow \beta * \alpha"] & c \end{tikzcd}$$

Vertical and horizontal compositions can be done in any order, they abide the exchange law.

2.10.1 *

Natural transformations Naturality condition Naturality

2.10.2 Natural transformation component

$\vec{\eta}^{\mathcal{D}}(x) = F^{\mathcal{D}}(x) \to G^{\mathcal{D}}(x) \mid x \in \mathcal{C}$

2.10.2.1 *

Component of natural transformation

2.10.3 Natural transformation in Haskell

Family of parametric polymorphism functions between endofunctors.

In Hask is $F\ a \to G\ a$. Can be analogued to repackaging data into another container, never modifies the object content, it only if - can delete it, because operation is lossy.

Can be sees as ortogonal to functors.

2.10.4 Cat category

Category where:

	Part	Is	#
*	object	category	0-cell
⇒	morphism	functor	1-cell
⇛	2-morphism	natural transformation, morphisms homotopy	2-cell

$$\begin{tikzcd} a \arrow[r, bend left, "F"] \arrow[r, bend right, "G"'] \arrow[r, phantom, "\Downarrow nt"] & b \end{tikzcd}$$

Is Cartesian closed category.

2.10.4.1 *

Cat 2-category

2.10.4.2 Bicategory

2-category that is enriched and lax.

For handling relaxed associativity - introduces associator, and for identity l -eft/right unitor.

Forming from bicategories higher categories by stacking levels of abstraction of such categories - leads to explosion of special cases, differences of every level, and so overall difficulties.

Stacking groupoids (category in which are morphisms are invertable) is much more homogenous up to infinity, and forms base of the homotopy type theory.

2.11 Category dual

Category duality behaves like a logical inverse.

Inverse $\mathcal{C} = \mathcal{C}^{op}$ - inverts the direction of morphisms.

Composition accordingly changes to the morphisms: $(g \circ f)^{op} = f^{op} \circ g^{op}$

Any statement in the terms of $\mathcal{C}$ in $\mathcal{C}^{op}$ has the dual - the logical inverse that is true in $\mathcal{C}^{op}$ terms.

Opposite preserves properties:

- products: $(\mathcal{C} \times \mathcal{D})^{op} \cong \mathcal{C}^{op} \times \mathcal{D}^{op}$
- functors: $(F^{\mathcal{C} \to \mathcal{D}})^{op} \cong F^{\mathcal{C}^{op} \to \mathcal{D}^{op}}$
- slices: $(\mathcal{F} \downarrow \mathcal{G})^{op} \cong (\mathcal{G}^{op} \downarrow \mathcal{F}^{op})$

2.11.0.0.1 *

Opposite category Opposite categories Category duality Duality Dual category Dual

2.11.1 Coalgebra

Structures that are dual (in the category-theoretic sense of reversing arrows) to unital associative algebras. Every coalgebra, by vector space duality, reversing arrows - gives rise to an algebra. In finite dimensions, this duality goes in both directions. In infinite - it should be determined.

2.12 Thin category

$\forall$ Hom sets contain zero or one morphism.

$f \equiv g \mid \forall x, y \; \forall f, g : x \to y$

A proset (preordered set).

2.12.1 *

Proset category Prosetal category Poset category Posetal category

2.13 Commuting diagram

Establishes equality in morphisms that have same source and target.

Draws the morphisms that are: $f = g \Rightarrow \{f, y\} : X \to Y$

2.13.1 *

Diagram commutes Commutes

2.14 Universal construction

Algorythm of constructing definitions in Category theory. Specially good to translate properties/definitions from other theories (Set theory) to Categories.

Method:

1. Define a pattern that you defining.
2. Establish ranking for pattern matches.
3. The top of ranking, the best match or set of matches - is the thing you was looking for. Matches are isomorphic for defined rules.

* uses Yoneda lemma, and as such constructions are defined until isomorphism, and so isomorphic betweem each-other.

2.14.1 *

Universal constructions

2.15 Product

Universal construction:

$$\begin{tikzcd} & c' \arrow[dl, "p"'] \arrow[d, "!"] \arrow[dr, "q"] & \\ a & c \arrow[l, "\pi_a"] \arrow[r, "\pi_b"'] & b \end{tikzcd}$$

Pattern: $p : c \to a, \ q : c \to b$

Ranking: $\max \sum^{\forall} (! : c' \to c \mid p' = p \circ !, \ q' = q \circ !)$

c' is another candidate.

For sets - Cartesian product.

* is a pair. Corresponds to product data type in Hask (inhabited with all elements of the Cartesian product).

Dual is Coproduct.

2.15.1 *

Products

2.16 Coproduct

Universal construction:

$$\begin{tikzcd} & c' & \\ a \arrow[ur, "p"] \arrow[r, "\iota_a"'] & c \arrow[u, "!"] & b \arrow[ul, "q"'] \arrow[l, "\iota_b"] \end{tikzcd}$$

Pattern: $i : a \to c, \ j : b \to c$

Ranking: $\max \sum^{\forall} (! : c \to c' \mid i' = ! \circ i, \ j' = ! \circ j)$

c' is another candidate.

For sets - Disjoint union.

* is a set assembled from other two sets, in Haskell it is a tagged set (analogous to disjoint union).

Dual is Product.

2.16.1 *

Coproducts

2.17 Free object

General particular structure. In which structure, properties autofollows from definition, axioms.

Also uses as a term when surcomstances of structures, rules, properties, axioms used coinside with the definition of a particular object $\therefore$ form object of this type with the according properties and possibilities.

2.18 Internal category

Category which is includded into a bigger category.

2.19 Hom set

All morphisms from source object to target object.

Denotation: $hom_C(X, Y) = (\forall f : X \to Y) = hom(X, Y) = C(X, Y)$ Denotation was not standartized.

Hom sets belong to Set category.

In Set category: $\exists!(a, b) \iff \exists! Hom, \forall Hom \in\ Set$. Set category is special, Hom sets are also objects of it.

Category can include Set, and hom sets, or not.

2.19.1 *

Hom-set Hom sets

2.19.2 Hom-functor

$hom : \mathcal{C}^{op} \times \mathcal{C} \to Set$ Functor from the product of $\mathcal{C}$ with its opposite category to the category of sets.

Denotation variants: $H_A = \mathrm{Hom}(-, A)$ $h_A = \mathcal{C}(-, A)$ $Hom(A, -) : \mathcal{C} \to Set$

Hom-bifunctor: $Hom(-, -) : \mathcal{C}^{op} \times \mathcal{C} \to Set$

2.19.3 Exponential object

Generalises the notion of function set to internal object. As also hom set to internal hom objects.

Cartesian closed (monoidal) category strictly required, as * multiplicaton holds composition requirement:

$\circ : hom(y, z) \otimes hom(x, y) \to hom(x, z)$

Denotation: b^a

Universal construction:

$$
\begin{tikzcd}
c \arrow[d, dotted, "u"] & c \times a \arrow[d, dotted, "u \times 1^a"] \arrow[dr] & \\
b^a & b^a \times a \arrow[r, "eval"] & b
\end{tikzcd}
$$

, where in Category: b^a - exponential object, $\times$ - product bifunctor, a - argument of *, b - result, c - candidate, $b^a \equiv (a \Rightarrow b)$ - *.

* b^a (also as $(a \Rightarrow b)$) represent exponentiation of cardinality of $\forall b^a$ possiblities.

2.19.3.1 *

Function object Internal hom Exponential objects Hom object Hom objects

2.19.3.2 Enriched category

Uses Hom objects (exponential objects), which do not belong into Set category. Category is no longer small, now may be called large.

$hom(x, y) \in K$.

Called: * over `K` (whick holds hom objects).

2.19.3.2.1 *

Enriched Large category

Chapter 3

Data type

Set of values. For type to have sence the values must share some sence, properties.

3.1 *

Type Types Data types

3.2 Actual type

Data type recieved by ->inferring->compiling->execution.

3.3 Algebraic data type

Composite type formed by combining other types.

3.3.1 *

AlgDT

3.4 Cardinality

Number of possible implementations for a given type signature.

Disjunction, sum - adds cardinalities. Conjunction, product - multiplies cardinalities.

3.4.1 *

Cardinalities

3.5 Data constant

* - constant value; nullary data constructor.

3.6 Data constructor

One instance that inhabit data type.

3.7 data declaration

Data type declaration is the most general and versatile form to create a new data type. Form:

```
data [context =>] type typeVars1..n
  = con1  c1t1..i
  | ...
  | conm  cmt1..q
  [deriving]
```

3.8 Dependent type

When type and values have relation between them. Type has restrictions for values, value of a type variable has a result on the type.

3.8.1 *

Dependent types

3.9 Gen type

Generator. Gen type is to generate pseudo-random values for parent type. Produces a list of values that gets infinitely cycled.

3.10 Higher-kinded data type

Any combination of * and ->

Type that take more types as arguments.

Humbly really a function

3.10.1 *

Higher-kinded data types

3.11 newtype declaration

Create a new type from old type by attaching a new constructor, allowing type class instance declaration.

```
newtype FirstName = FirstName String
```

Data will have exactly the same representation at runtime, as the type that is wrapped.

```
newtype Book = Book (Int, Int)
```

```
      (,)
      / \
Integer Integer
```

3.12 Principal type

The most generic data type that still typechecks.

3.13 Product data type

Is an algebraic data type respesentation of a product construction. Formed by logical conjunction (AND, '* *').

Haskell forms:

```
-- 1. As a tuple (the uncurried & most true-form)
(T1, T2)

-- 2. Curried form, data constructor that takes two types
C T1 T2

-- 3. Using record syntax. =r# <inhabitant>= would return the respective =T#=.
C { r1 :: T1
  , r2 :: T2
  }
```

3.13.1 *

Product type

3.13.2 Sequence

Enumerated (ordered) set.

Denotation:

```
()
( , )
( , , )
( , , ... )
```

More general mathematical denotation was not established, variants: $(n)_{n \in \mathbb{N}}$ $\omega \rightarrow X$ $\{i : Ord \mid i < \alpha\}$

In Haskell: Data type that stores multiple ordered values withing a single value.

Table 3.1: Sequence constructor naming by arity

Name	Arity	Denotation
Unit, empty	0	()
Singleton	1	(_)
Tuple, pair, two-tuple	2	(,)
Triple, three-tuple	3	(, ,)
Sequence	N	(, , ...)

3.13.2.1 *

Sequences Tuples Ordered pair Ordered triple

3.13.2.2 List

Sequence of one type objects.

Denotation:

```
[]
[ , ]
[ , , ]
[ , , ... ]
```

Haskell definition:

```
data [] a = [] | a : [a]
```

Definition is self-referential (self-recursive), can be seen as anamorphism (unfold) of the [] (empty list, memory cell which is container of particular type) and : (cons operation, pointer). As such - can create non-terminating data type (and computation), in other words - infinite.

3.14 Proxy type

Proxy type holds no data, but has a phantom parameter of arbitrary type (or even kind). Able to provide type information, even though has no value of that type (or it can be may too costly to create one).

```
data Proxy a = ProxyValue

let proxy1 = (ProxyValue :: Proxy Int) -- a has kind `Type`
let proxy2 = (ProxyValue :: Proxy List) -- a has kind `Type -> Type`
```

3.15 Static typing

Typechecking takes place at compile level.

3.16 Structural type

Mathematical type. They form into structural type system.

3.16.1 *

Structural

3.17 Structural type system

Strict global hierarchy and relationships of types and their properties. Haskell type system is *. In most languages typing is name-based, not structural.

3.17.1 *

Structural typing

3.18 Sum data type

Algebraic data type formed by logical disjunction (OR '|').

3.19 Type alias

Create new type constructor, and use all data structure of the base type.

3.20 Type class

Type system construct that adds a support of ad hoc polymorphism.

Type class makes a nice way for defining behaviour, properties over many types/objects at once.

3.20.1 *

Type classes

3.20.2 Arbitrary type class

Type class of QuickCheck.Arbitrary (that is reexported by QuickCheck) for creating a generator/distribution of values. Useful function is arbitrary - that autogenerates values.

3.20.2.1 Arbitrary function

Depends on type and generates values of that type.

3.20.3 CoArbitrary type class

Pseudogenerates a function basing on resulting type.

```
coarbitrary :: CoArbitrary a => a -> Gen b -> Gen b
```

3.20.3.1 *

CoArbitrary

3.20.4 Typeable type class

Allows dynamic type checking in Haskell for a type. Shift a typechecking of type from compile time to runtime. * type gets wrapped in the universal type, that shifts the type checks to runtime.

Also allows:

- Get the type of something at runtime (ex. print the type of something `typeOf`).
- Compare the types.
- Reifying functions from polymorphic type to conrete (for functions like `:: Typeable a => a -> String`).

3.20.4.1 *

Typeable

3.20.5 Type class inheritance

Type class has a superclass.

3.20.6 Derived instance

Type class instances sometimes can be automatically derived from the parent types.

Type classes such as Eq, Enum, Ord, Show can have instances generated based on definition of data type.

P.S.

Language options:

- DeriveAnyClass
- DeriveDataTypeable
- DeriveFoldable
- DeriveFunctor
- DeriveGeneric
- DeriveLift
- DeriveTraversable
- DerivingStrategies
- DerivingVia
- GeneralisedNewtypeDeriving
- StandaloneDeriving

3.20.6.1 *

Derived Deriving

3.21 Type constant

Nullary type constructor.

3.22 Type constructor

Name of the data type.

Constructor that takes type as an argument and produces new type.

3.23 type declaration

Synonim for existing type. Uses the same data constructor.

```
type FirstName = String
```

Used to distinct one entities from other entities, while they have the same type. Also main type functions can operate on a new type.

3.24 Typed hole

* - is a _ or `_name` in the expression. On evaluation GHC would show the derived type information which should be in place of the *. That information helps to fill in the gap.

3.24.1 *

Typed holes

3.25 Type inference

Automatic data type detection for expression.

3.25.1 *

Inferring Infer Infers Inferred

3.26 Type class instance

Block of implementations of functions, based on unique type class->type pairing.

3.27 Type rank

Weak ordering of types.

The rank of polymorphic type shows at what level of nesting forall quantifier appears. Count-in only quantifiers that appear to the left of arrows.

```
f1 :: forall a b. a -> b -> a     ==     fi :: a -> b -> c
g1 :: forall a b. (Ord a, Eq b) => a -> b -> a     ==     g1 :: (Ord a, Eq b) => a -> b -> a
```

f1, g1 - rank-1 types. Haskell itself implicitly adds universal quantification.

```
f2 :: (forall a. a->a) -> Int -> Int
g2 :: (forall a. Eq a => [a] -> a -> Bool) -> Int -> Int
```

f2, g2 - rank-2 types. Quantificator is on the left side of a →. Quantificator shows that type on the left can be overloaded.

Type inference in Rank-2 is possible, but not higher.

```
f3 :: ((forall a. a->a) -> Int) -> Bool -> Bool
```

f3 - rannk3-type. Has rank-2 types on the left of a →.

```
f :: Int -> (forall a. a -> a)
g :: Int -> Ord a => a -> a
```

f, g are rank 1. Quantifier appears to the right of an arrow, not to the left. These types are not Haskell-98. They are supported in RankNTypes.

3.27.1 *

Type ranks Rank type Rank types Rank-1 type Rank-1 types Rank-2 type Rank-2 types Rank-3 type Rank-3 types

3.28 Type variable

Refer to an unspecified type in Haskell type signature.

3.29 Unlifted type

Type that directly exist on the hardware. The type abstraction can be completely removed. With unlifted types Haskel type system directly manages data in the hardware.

3.29.1 *

Unlifted types

3.30 Linear type

Type system and algebra that also track the multiplicity of data. There are 3 general linear type groups:

- 0 - exists only at type level and is not allowed to be used at value level. Aka `s` in ST-Trick.
- 1 - data that is not duplicated
- 1< - all other data, that can be duplicated multiple times.

3.30.1 *

Linear types

3.31 NonEmpty list data type

Data.List.NonEmpty Has a Semigroup instance but can't have a Monoid instance. It never can be an empty list.

```
data NonEmpty a = a :| [a]
  deriving (Eq, Ord, Show)
```

:| - an infix data costructor that takes two (type) arguments. In other words :| returns a product type of left and right

3.32 Session type

* - allows to check that behaviour conforms to the protocol.

So far very complex, not very productive (or well-established) topic.

3.33 Binary tree

```
data BinaryTree a
  = [[Leaf]]
  | [[Node]] (BinaryTree a) a (BinaryTree a)
```

3.34 Bottom value

A _ non-value in the type or pattern match expression. Placeholder for enything.

```
-- _ fits *.
```

3.34.1 *

Bottom Bottom values

3.35 Bound

Haskell * type class means to have lowest value & highest value, so a bounded range of values.

3.35.1 *

Bounded

3.36 Constructor

1. Type constructor
2. Data constructor

Also see: Constant

3.36.1 *

Constructors

3.37 Context

Type constraints for polymorphic variables. Written before the main type signature, denoted:

```
TypeClass a => ...
```

3.37.1 *

Contexts

3.38 Inhabit

Values that is a component of data type set.

3.39 Maybe

```
data Maybe
  = Nothing
  | Just a
```

Does not represent the information why `Nothing` happened. For error - use Either. Do not propagate *.

Handle * locally to where it is produced. `Nothing` does not hold useful info for debugging & short-circuits the processes. Do not expect code type being bug-free, do not return `Maybe` to end user since it would be impossible to debug, return something that preserves error information.

3.39.0.1 *

Nodes

3.40 Expected type

Data type inferred from the text of the code.

3.41 ADT

1. Abstract data type
2. Algebraic data type

3.42 Concrete type

Fully resolved, definitive, non-polymorphic type.

3.43 Type punning

When type constructor and data constructor have the same name.

Theoretically if person knows the rules - * can be solved, because in Haskell type and data declaration have different places of use.

3.44 Kind

Kind -> Type -> Data

3.44.1 *

Kinds

3.45 IO

Type for values whose evaluations has a posibility to cause side effects or return unpredictable result. Haskell standard uses monad for constructing and transforming IO actions. IO action can be evaluated multiple times.

IO data type has unpure imperative actions inside. Haskell is pure Lambda calculus, and unpure IO integrates in the Haskell purely (type system abstracts IO unpurity inside IO data type).

IO sequences effect computation one after another in order of needed computation, or occurence:

```
twoBinds :: IO ()
twoBinds =
  putStrLn "First:" >>
  getLine >>=
  \a ->
  putStrLn "Second:" >>
  getLine >>=
  \b ->
  putStrLn ("\nFirst: "
    ++ a ++ ".\nSecond "
    ++ b ++ ".")
main = twoBinds
```

Sequencing is achieved by compilation of effects performing only while they recieve the sugared-in & passed around the `RealWorld` fake type value, that value in the every computation gets the new ”value” and then passed to the next requestes computation. But special thing is about this parameter, this `RealWorld` type value passed, but never looked at. GHC realizes, since value is never used, - it means value and type can be equated to `()` and moreover reduced from the code, and sequencing stays.

Chapter 4

Expression

Finite combination of symbols that is well-formed according to context-free grammar.

Generally meaningless. Meaning gets derived from an * & context (and/or content words) by congruency with knowledge & expirience.

4.1 *

Expressions

4.2 Closed-form expression

* - mathematical expression that can be evaluated in a finite number of operations.

May contain:

- constants
- variables
- operations (e.g., $+ - \times \div$)
- functions (e.g., nth root, exponent, logarithm, trigonometric functions, and inverse hyperbolic functions), but usually no limit.

4.3 RHS

Right-hand side of the expression.

4.4 LHS

Left-hand side of the expression.

4.5 Redex

Reducible expression.

4.6 Concatenate

Link together sequences, expressions.

4.7 Alpha equivalence

Equivalence of a processes in expressions. If expressions have according parameters different, but the internal processes are literally the same process.

4.8 Ground expression

Expression that does not contain any free variables.

4.8.1 *

Ground formula

4.9 Variable

A name for expression.

+===

There fequently can be heard: one of most notable Haskell properties is `Haskell has immutable "variables"` (and term here used in the sence that imperative programmers frequently use). Logically we see statement is contradictory with itself: "variables" - something that has change as a defining propery - are not changing; it is a nonsencical statement. Please, read the saying as: `Haskell has immutable values`, due to following the value semantics: see "Value". And Haskell expressions are functions (that are referentially transparent - meaning itself immutable) - and they are also values (hense term "functional programming" means - functions are first-class values). Since values bind to variables - people are wrongly mix-up terms and say their names (according "*") are immutable.

As you see in the code - Haskell variables (same names) hold different values at different times. Variables are reused, meaning "names are reused" - binded to different values on scope changes. But all values that Haskell holds - are, by the design of the language, are treated immutable, any transformations Haskell resolves by creating new values, and frees the space by freeing-up from no longer needed values.

4.9.1 *

Variables

4.10 Phrase

Composable expression.

Chapter 5

Function

Full dependency of one quantity from another quantity.

Denotation: $y = f(x)$ $f : X \to Y$, where X is domain, Y is codomain.

Directionality and property of invariability emerge from one another.

```
-- domain func codomain
   *       ->   *
```

$y(x) = (zx^2 + bx + 3 \mid b = 5)$ ^ ^ ^^ ^ ^

\Var \Constant

\Bound variable

\Free variable

\Parameter

\Name of the function

Lambda abstraction is a function. Function is a mathematical operation.

Function = Total function = Pure function. Function theoretically can be to memoized.

Also see: Partial function Inverse function - often partially exists (partial function).

5.1 *

Functions Bound variable

5.2 Arity

Number of parameters of the function.

- nullary - f()
- unary - f(x)
- binary - f(x,y)
- ternary - f(x,y,z)
- n-ary - f(x,y,z..)

5.3 Bijection

Function is a complete one-to-one pairing of elements of domain and codomain (image). It means function both surjective (so image == codomain) and injective (every domain element has unique correspondence to the image element).

For bijection inverse always exists.

Bijective operation holds the equivalence of domain and codomain.

Denotation:

```
⤖
>->>
f : X ⤖ Y
```

LaTeX needed to combine symbols: $f : X \rightarrowtail\!\!\!\!\twoheadrightarrow Y$

Corersponds to isomorphism.

5.3.1 *

Bijective Bijective function

5.4 Combinator

Function without free variables. Higher-order function that uses only function application and other combinators.

```
\a -> a
\ a b -> a b
\f g x -> f (g x)
\f g x y -> f (g x y)
```

Not combinators:

```
\ xs -> sum xs
```

Informal broad meaning: referring to the style of organizing libraries centered around the idea of combining things.

5.4.1 Ψ-combinator

Transforms two of the same type, applying same mediate transformation, and then transforming those into the result.

```
import Data.Function (on)
on :: (b -> b -> c) -> (a -> b) -> a -> a -> c
```

```
a--\b
    * ---c
a--/b
```

5.4.1.1 *

Psi-combinator On-combinator

5.5 Function application

* - bind the argument to the parameter of a function, and do a beta-reduction.

5.5.1 *

Apply Applied Applying Application

5.6 Function body

Expression that haracterizes the process.

5.7 Function composition

```
(.) :: (b -> c) -> (a -> b) -> a -> c
```

```
a -> (a -> b) -> (b -> c) -> c
```

In Haskell inline composition requires:

```
h.g.f $ i
```

5.7.1 *

Composition Compose Composed

5.8 Function head

Is a part with name of the function and it's paramenters. AKA: $f(x)$

5.9 Function range

The range of a function refers to either the codomain or the image of the function, depending upon usage. Modern usage almost always uses range to mean image. So, see Function image.

5.10 Higher-order function

Function that has arity > 1.

+===

HOF is:

- function that accepts function as a parameter
- function that has more then one parameter.

Application of an argument to * produces a function that has `arity - 1`.

5.10.1 *

HOF

5.10.2 Fold

Catamorphism of a structure to a lower type of structure. Often to a single value.

* is a higher-order function that takes a function which operates with both main structure and accumulator structure, * applies units of data structure to a function wich works with accumulator. Upoun traversing the whole structure - the accumulator is returned.

5.11 Injection

Function one-to-one injects from domain into codomain.

Keeps distinct pairing of elements of domain and image. Every element in image coresponds to one element in domain.

$\forall a, b \in X,\ f(a) = f(b) \Rightarrow a = b$

$\exists (inverse\ function) \mid \forall (injective\ function)$

Denotation:

```
↣
>->
f : X ↣ Y
```

$f : X \rightarrowtail Y$

Corresponds to Monomorphism.

5.11.1 *

Injective Injective function Injectivity

5.12 Partial function

One that does not cover all domain. Unsafe and causes trouble.

5.13 Purity

* means properly abstracted.

If the contrary - abstraction is unpure.

Also see: pure function.

5.13.1 *

Pure

5.14 Pure function

Function that is pure $\equiv$ referentially transparent function.

5.15 Sectioning

Writing function in a parentheses. Allows to pass around partially applied functions.

5.16 Surjection

Function uses codomain fully.

$\forall y \in Y, \exists x \in X$

Denotation: $f : X \twoheadrightarrow Y$

Corresponds to Epimorphism.

5.16.1 *

Surjective Surjective function

5.17 Unsafe function

Function that does not cover at least one edge case.

5.17.1 *

Unsafe

5.18 Variadic

* - having variable arity (often up to indefinite).

5.19 Domain

Source set of a function. X in $X \to Y$.

5.20 Codomain

Y in $X \to Y$. Codomain - target set of a function.

5.21 Open formula

Logical function that has arity and produces proposition.

5.22 Recursion

Repeated function application when sometimes same function gets called.

Allows computation that may require indefinite amount of work.

5.22.1 *

Recursive

5.22.2 Base case

A part of a recursive function that trivially produces result.

5.22.3 Tail recursion

Tail calls are recursive invocantions of itself.

5.22.4 Polymorphic recursion

Type of the parameter changes in recursive invocations of function.

Is always a higher-ranked type.

5.22.4.1 *

Milner–Mycroft typability Milner–Mycroft calculus

5.23 Free variable

Variable in the fuction that is not bound by the head. Until there are * - function stays partially applied.

5.24 Closure

$f(x) = f^{\mathcal{X} \to \mathcal{X}} \mid \forall x \in \mathcal{X}$, $\mathcal{X}$ is closed under f, it is a trivial case when operation is legitimate for all values of the domain.

Operation on members of the domain always produces a members of the domain. The domain is closed under the operation.

In the case when there is a domain values for which operation is not legitimate/not exists:

$f(x) = f^{\mathcal{V} \to \mathcal{X}} \mid \mathcal{V} \in \mathcal{X}, \forall x \in \mathcal{V}$, $\mathcal{X}$ is closed under f.

5.24.1 *

Closed

5.25 Parameter

□□□□ *para* subsidiary □□□□□□ *metron* measure

Named varible of a function.

Argument is a supplied value to a function parameter.

Parameter (formal parameter) is an irrefutable pattern, and implemeted that way in Haskell.

5.25.1 *

Parameters Formal parameter Formal parameters

5.26 Partial application

Part of function parameters applied.

5.26.1 *

Partially applied

5.27 Well-formed formula

Expression, logical function that is/can produce a proposition.

5.27.1 *

Well formed formula WFF wff WFFS wffs

Chapter 6

Homotopy

□□□□ homós same

One can be "continuously deformed" into the other.

For example - functions, functors. Natural transformation is a homotopy of functors.

6.1 *

Homotopies Homotopic

Chapter 7

Lambda calculus

Universal model of computation. Which means * can implement any Turing machine. Based on function abstraction and application by substituting variables and binding values.

* has lambda terms:

- variable (x)
- application ((ts))
- abstraction (lambda function) ($(\lambda x.t)$)

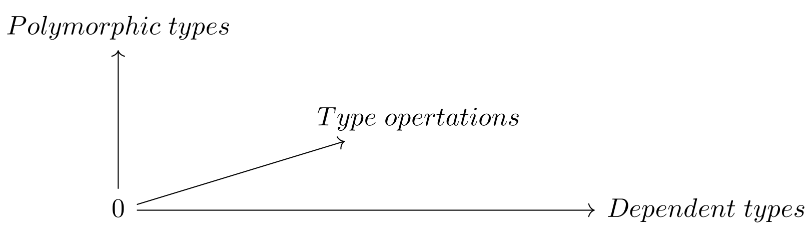
7.1 *

Lambda term Lambda terms

7.2 Lambda cube

-cube shows the 3 dimentions of generalizations from simply typed Lambda calculus to Calculus of constructions.

+===



$$\begin{tikzcd}
 & \lambda\omega \arrow[rr] & & \lambda C \\
\lambda 2 \arrow[rr] \arrow[ur] & & \lambda P2 \arrow[ur] & \\
 & \lambda\underline{\omega} \arrow[rr] \arrow[uu] & & \lambda P\underline{\omega} \arrow[uu] \\
\lambda \arrow[rr] \arrow[ur] \arrow[uu] & & \lambda P \arrow[ur] \arrow[uu] &
\end{tikzcd}$$

Each dimension of the cube corresponds to extensions (a new type of relation of objects depending on objects):

Table 7.1: Three degrees of type systems generalizations

Denotation	Name	Programming	New type of relations
2	Polymorphic types	First-class polymorphism of types	Terms depending on types
ω	Type operation	Type class, type families	Types depending on types
P	Dependent types	Higher-rank polymorphism, dependent types	Types depending on terms

Table 7.2: λ-cube: Names of the type systems

Denotation	Logical system
$\lambda \rightarrow$	(First Order) Propositional Calculus
$\lambda 2$	Second Order Propositional Caculus
$\lambda\omega$	Weakly Higher Order Propositional Calculus
$\lambda\underline{\omega}$	Higher Order Propositional Calculus
λP	(First Order) Predicate Logic
$\lambda P2$	Second Order Predicate Calculus
$\lambda P\omega$	Weak Higher Order Predicate Calculus
λC	Calculus of Constructions

7.2.1 *

□-cube λ-cube

7.3 Lambda function

Function of Lambda calculus. $\lambda xy.x^2 + y^3$ ^^ ^ ^

\variable

\variable
(_)
_BODY

\parameter

parameter()_−HEAD

7.3.1 *

Lambda abstraction

7.3.2 Anonymous lambda function

Lambda function that is not binded to any name.

7.3.2.1 *

Anonymous lambda function

7.3.3 Uncurry

Replace sequenced lambda functions into single function taking sequence/product of values as argument.

7.4 β-reduction

Equation of a parameter to a bound variable, then reducing parameter from the head.

7.4.1 *

β reduction Beta-reduction Beta reduction

7.4.2 β-normal form

No beta reduction is possible.

7.4.2.1 *

β normal from Beta normal form Beta-normal form

7.5 Calculus of constructions

Extends the Curry–Howard correspondence to the proofs in the full intuitionistic predicate calculus (includes proofs of quantified statements). Type theory, typed programming language, and constructivism (phylosophy) foundation for mathematics. Directly relates to Coq programming language.

7.5.1 *

«<CoC»>

7.6 Curry–Howard correspondence

Equivalence of {First-order logic, computer programming, Category theory}. They represent each-other, possible in one - possible in the other, so all the definitions and theorems have analogues in other two.

Gives a ground to the equivalence of computer programs and mathematical proofs.

Lambek added analogue to Cartesian closed category, which can be used to model logic and type theory.

7.6.1 *

Curry–Howard isomorphism Curry-Howard-Lambek

Table 7.3: Table of basic correspondence

Logic	Type	Category
True	() (any inhabited type)	Terminal
False	Void	Initial
a ∧ b	(a, b)	a × b
a ∨ b	Either a b	a / b
a ⇒ b	a → b	b^a

7.7 Currying

Translating the evaluation of a multiple argument function (or a tuple of arguments) into evaluating a sequence of functions, each with a single argument.

7.7.1 *

Curry

7.8 Hindley–Milner type system

Classical type system for the Lambda calculus with Parametric polymorphism and Type inference. Types marked as polymorphic variables, which enables type inference over the code.

7.8.1 *

Hindley-Milner Damas-Milner Damas–Hindley–Milner

7.9 Reduction

Take out something from a structure, make simplier.

See Beta reduction

7.9.1 *

Reducible

7.10 β-η normal form

All β-reduction and η-abstraction are done in the expression.

7.10.1 *

beta-eta normal form beta eta normal form

7.11 η-abstraction

$(\lambda x.Mx) \underset{\eta}{\leftarrow} M$

```
\ x -> g . f $ x
\ x -> g . f      --eta-abstraction
```

7.11.1 *

η-reduction η-conversion η abstraction η reduction η conversion eta-abstraction eta-reduction eta-conversion eta abstraction eta reduction eta conversion

7.12 Lambda expression

See Lambda calculus (Lambda terms) and Expression. In majority cases meaning some Lambda function.

Chapter 8

Operation

Calculation into output value. Can have zero & more inputs.

8.1 Constant

Nullary operation.

Also see: Type constant.

8.2 Binary operation

$\forall(a,b) \in S, \exists P(a,b) = f(a,b) : S \times S \to S$

8.2.1 *

Binary operations

8.3 Operator

Denotation symbol/name for the operation.

8.3.1 Shift operator

Shift operator defined by Lagrange through Differential operator. $T^t = e^{t\frac{d}{dx}}$

8.3.1.1 *

Shift

8.3.2 Differential operator

Denotation. $\frac{d}{dx}$, D, D_x, ∂_x. Last one is partial.

$e^{t\frac{d}{dx}}$ - Shift operator.

8.3.2.1 *

Differential

8.4 Infix

Form of writing of operator or function in-between variables for application.

For priorities see Fixity.

8.5 Fixity

Declares the presedence of action of a function/operator.

Funciton application has presedence higher then all infix operators/functions (virtually giving it a priority 10).

Table 8.1: Haskell operators priority and fixity association

P	L	Non	R
10			F.A.
9	!!		.
8			^ ^^ **
7	*/ div		
6	+-		
5			: , ++
4		<comparison> elem	
3			&&
2			OR
1			
0			$ $! seq

8.5.1 *

Infixl Infixr Priority Precedence

8.6 Zero

* is the value with which operation always yelds Zero value. $zero, n \in C : \forall n, zero * n = zero$

* is distinct from Identity value.

8.7 Bind

Establishing equality between two objects.

Most often:

- equating variable to a value.
- equating parameter of a function to an argument (variable/value/function). This term often is equated to applying argument to a function, which includes β-reduction.

8.7.1 *

Binds Binding Bindings

8.8 Declaration

Binding name to expression.

8.9 Dispatch

Sort-out & send.

8.10 Evaluation

For FP see Bind.

Chapter 9

Permutation

Bijective function from domain to itself.

Domain & permutation functions & function composition form a group.

Chapter 10

Point-free

Paradigm where function only describes the morphism itself.

Process of converting function to point-free. If brackets () can be changed to $ then $ equal to composition:

```
\ x -> g (f x)
\ x -> g $ f x
\ x -> g . f $ x
\ x -> g . f      --eta-abstraction

\ x1 x2 -> g (f x1 x2)
\ x1 x2 -> g $ f x1 x2
\ x1 x2 -> g . f x1 $ x2
\ x1    -> g . f x1
```

10.1 *

Pointfree Tacit Tacit programming

10.2 Blackbird

```
(.).(.) :: (b -> c) -> (a1 -> a2 -> b) -> a1 -> a2 -> c
```

Composition of compositions `(.).(.)`. Allows to compose-in a binary function `f1(c) (.).(.) f2(a,b)`.

```
\ f g x y -> f (g x y)
```

10.2.1 *

.) . (.).(.) Composition of compositions

10.3 Swing

```
swing :: (((a -> b) -> b) -> c -> d) -> c -> a -> d
swing = flip . (. flip id)
swing f = flip (f . runCont . return)
swing f c a = f ($ a) c
```

10.4 Squish

```
f >>= a . b . c =<< g
```

Chapter 11

Polymorphism

πολύς *polús* many

At once several forms.

In Haskell - abstract over data types.

* types:

11.1 *

Polymorphic

11.2 Levity polymorphism

Extending polymorphism to work with unlifted and lifted types.

11.3 Parametric polymorphism

Abstracting over data types by parameter.

In most languages named as 'Generics' (generic programming).

Types:

11.3.1 Rank-1 polymorphism

Parametric polymorphism in rank-1 types by type variables.

11.3.1.1 *

Prenex Prenex polymorpism

11.3.2 Let-bound polymorphism

It is property chosen for Haskell type system. Haskell is based on Hindley-Milner type system, it is let-bound. To have strict type inference with * - if `let` and `where` declarations are polymorphic - λ declarations - should be not.

See: Good: In Haskell parameters bound by lambda declaration instantiate to only one concrete type.

11.3.3 Constrained polymorphism

Constrained Parametric polymorphism.

11.3.3.1 Ad hoc polymorphism

Artificial constrained polymorphism dependent on incoming data type. It is interface dispatch mechanism of data types. Achieved by creating a type class instance functions.

Commonly known as overloading.

11.3.3.1.0.1 *

Ad-hoc polymorphism Ad hoc polymorphic Ad-hoc polymorphic Constraint Constraints

11.3.4 Impredicative polymorphism

* allows type ▯ entities with polymorphic types that can contain type ▯ itself. $T = \forall X.X \to X : T \in X \vDash T \in T$

The most powerful form of parametric polymorphism. See: Impredicative.

This approach has Girard's paradox (type systems Russell's paradox).

11.3.4.1 *

First-class polymorphism

11.3.5 Higher-rank polymorphism

Means that polymorphic types can apper within other types (types of function). There is a cases where higher-rank polymorphism than the a Ad hoc - is needed. For example where ad hoc polymorphism is used in constraints of several different implementations of functions, and you want to build a function on top - and use the abstract interface over these functions.

```
-- ad-hoc polymorphism
f1 :: forall a. MyType Class a => a -> String    ==    f1 :: MyType Class a => a  -> String
f1 = -- ...

-- higher-rank polymorphism
f2 :: Int -> (forall a. MyType Class a => a -> String) -> Int
f2 = -- ...
```

By moving `forall` inside the function - we can achive higher-rank polymorphism.

From: `https://news.ycombinator.com/item?id=8130861`

`Higher-rank polymorphism is formalized using System F, and there are a few implementations of (in`

Useful example aslo a ST-Trick monad.

11.3.5.1 *

Rank-n polymorphism

11.4 Subtype polymorphism

Allows to declare usage of a Type and all of its Subtypes. T - Type S - Subtype of Type <: - subtype of $S <: T = S \leq T$

Subtyping is: If it can be done to T, and there is subtype S - then it also can be done to S. $S <: T : f^{T \to X} \Rightarrow f^{S \to X}$

11.5 Row polymorphism

Is a lot like Subtype polymorphism, but alings itself on allowence (with | r) of subtypes and types with requested properties.

```
printX :: { x :: Int | r } -> String
printX rec = show rec.x

printY :: { y :: Int | r } -> String
printY rec = show rec.y

-- type is inferred as `{x :: Int, y :: Int | r } -> String`
printBoth rec = printX rec ++ printY rec
```

11.6 Kind polymorphism

Achieved using a phantom type argument in the data type declaration.

```
;;          * -> *
data Proxy a = ProxyValue
```

Then, by default the data type can be inhabited and fully work being partially defined. But multiple instances of kind polymorphic type can be distinguished by their particular type.

Example is the Proxy type:

```
data Proxy a = ProxyValue

let proxy1 = (ProxyValue :: Proxy Int) -- * :: Proxy Int
let proxy2 = (ProxyValue :: Proxy a)   -- * -> * :: Proxy a
```

11.7 Linearity polymorphism

Leverages linear types. For exampe - if fold over a dynamic array:

1. In basic Haskell - array would be copied at every step.
2. Use low-level unsafe functions.
3. With Linear type function we guarantee that the array would be used only at one place at a time.

So, if we use a function (* -o * -o -o *) in foldr - the fold will use the initial value only once.

Chapter 12

Pragma

Pragma - instruction to the compiler that specifies how a compiler should process the code. Pragma in Haskell have form:

```
{-# PRAGMA options #-}
```

12.1 LANGUAGE pragma

Controls what variations of the language are permitted. It has a set of allowed options: https://downloads.haskell.org/~ghc/latest/docs/html/users_guide/glasgow_exts.html, which can be supplied.

12.1.1 LANGUAGE option

12.1.1.1 *

Language options

12.1.1.2 Useful by default

```
import EmptyCase
import FlexibleContexts
import FlexibleInstances
import InstanceSigs
import MultiParamTypeClasses
```

12.1.1.3 AllowAmbiguousTypes

Allow type signatures which appear that they would result in an unusable binding. However GHC will still check and complain about a functions that can never be called.

12.1.1.4 ApplicativeDo

Enables an alternative in-depth reduction that translates the do-notation to the operators `<$>`, `<*>`, `join` as far as possible.

For GHC to pickup the patterns, the final statement must match one of these patterns exactly:

```
pure E
pure $ E
return E
return $ E
```

When the statements of do expression have dependencies between them, and ApplicativeDo cannot infer an Applicative type - GHC uses a heuristic $O(n^2)$ algorithm to try to use <*> as much as possible. This algorithm usually finds the best solution, but in rare complex cases it might miss an opportunity. There is aslo $O(n^3)$ algorithm that finds the optimal solution: `-foptimal-applicative-do`.

Requires `ap = <*>`, `return = pure`, which is true for the most monadic types.

- Allows use of do-notation with types that are an instance of Applicative and Functor
- In some monads, using the applicative operators is more efficient than monadic bind. For example, it may enable more parallelism.

The only way it shows up at the source level is that you can have a `do` expression with only Applicative or Functor constaint.

It is possible to see the actual translation by using `-ddump-ds`.

12.1.1.5 ConstrainedClassMethods

Enable the definition of further constraints on individual class methods.

12.1.1.6 CPP

Enable C preprocessor.

12.1.1.7 DeriveFunctor

Automatic deriving of instances for the Functor type class. For type power set functor is unique, its derivation inplementation can be autochecked.

12.1.1.8 ExplicitForAll

Allow explicit forall quantificator in places where it is implicit by Haskell.

12.1.1.9 FlexibleContexts

Ability to use complex constraints in class declaration contexts. The only restriction on the context in a class declaration is that the class hierarchy must be acyclic.

```
class C a where
  op :: D b => a -> b -> b

class C a => D a where ...
```

$C :> D$, so in C we can talk about D.

Synergizes with ConstraintKinds.

12.1.1.10 FlexibleInstances

Allow type class instances types contain nested types.

```
instance C (Maybe Int) where ...
```

Implies TypeSynonymInstances.

12.1.1.11 GeneralizedNewtypeDeriving

Enable GHC's `newtype` cunning generalised deriving mechanism.

```
newtype Dollars = Dollars Int
  deriving (Eq, Ord, Show, Read, Enum, Num, Real, Bounded, Integral)
```

(In old Haskell-98 only Eq, Ord, Enum could been inherited.)

12.1.1.12 ImplicitParams

Allow definition of functions expecting implicit parameters. In the Haskell that has static scoping of variables allows the dynamic scoping, such as in classic Lisp or ELisp. Sure thing this one can be puzzling as hell inside Haskell.

12.1.1.13 LambdaCase

Enables expressions of the form:

```
\case { p1 -> e1; ...; pN -> eN }

-- OR

\case
  p1 -> e1
  ...
  pN -> eN
```

12.1.1.14 MultiParamTypeClasses

Implies: ConstrainedClassMethods Enable the definitions of typeclasses with more than one parameter.

```
class Collection c a where
```

12.1.1.15 MultiWayIf

Enable multi-way-if syntax.

```
if | guard1 -> code1
   | ...
   | guardN -> codeN
```

12.1.1.16 OverloadedStrings

Enable overloaded string literals (string literals become desugared via the `IsString` class).

With overload, string literals has type:

```
(IsString a) => a
```

The usual string syntax can be used, e.g. `ByteString`, `Text`, and other variations of string-like types. Now they can be used in pattern matches as `char->integer` translations. To pattern match `Eq` must be derived.

To use class `IsString` - import it from `GHC.Ext`.

12.1.1.17 PartialTypeSignatures

Partial type signature containins wildcards, placeholders (`_`, `_name`). Allows programmer to which parts of a type to annotate and which to infer. Also applies to constraint part.

As untuped expression, partly typed can not polymorphicly recurse.

-Wno-partial-type-signatures supresses infer warnings.

12.1.1.18 RankNTypes

Enable types of arbitrary rank. See Type rank.

Implies ExplicitForAll.

Allows `forall` quantifier:

- Left side of →
- Right side of →
- as argument of a constructor
- as type of a field
- as type of an implicit parameter
- used in pattern type signature of lexically scoped type variables

12.1.1.19 ScopedTypeVariables

By default type variables do not have a scope except inside type signatures where they are used.

When there are internall type signatures provided in the code block (`where`, `let`, etc.) they (main type description of a function and internal type descriptions) restrain one-another and become not trully polymorphic, which creates a bounding interdependency of types that GHC would complain about.

* option provides the lexical scope inside the code block for type variables that have forall quantifier. Because they are now lexiacally scoped - those type variables are used across internal type signatures.

For details see: `https://ocharles.org.uk/guest-posts/2014-12-20-scoped-type-variables.html`

Implies ExplicitForAll.

12.1.1.20 TupleSections

Allow tuple section syntax:

```
(, True)
(, "I", , , "Love", , 1337)
```

12.1.1.21 TypeApplications

Allow type application syntax:

```
read @Int 5

:type pure @[]
pure @[] :: a -> [a]

:type (<*>) @[]
(<*>) @[] :: [a -> b] -> [a] -> [b]

--

instance (CoArbitrary a, Arbitrary b) => Arbitrary (a -> b)

λ> ($ 0) <$> generate (arbitrary @(Int -> Int))
```

12.1.1.22 TypeSynonymInstances

Now type synonim can have it's own type class instances.

12.1.1.23 UndecidableInstances

Permit instances which may lead to type-checker non-termination.

GHC has Instance termination rules regardless of FlexibleInstances FlexibleContexts.

12.1.1.24 ViewPatterns

```
foo (f1 -> Pattern1) = c1
foo (fn -> Pattern2 a b) = g1 a b
```

(*expression* → *pattern*): take what is came to match - apply the *expression*, then do *pattern*-match, and return what originally came to match.

Semantics:

- *expression* & *pattern* share the scope, so also variables.

sion :: t1 -> t2) && (*pattern* t2)=) then (ViewPattern (/expression/ -> /pattern/) :: t1) (return what originally was recieved into pattern match) else skip

* are like pattern guards that can be nested inside of other patterns. * are a convenient way to pattern-match algebraic data type.

Additional possible usage:

```
foo a (f2 a -> Pattern3 b c) = g2 b c  -- only for function definitions
foo ((f,_), f -> Pattern4) = c2  -- variables can be bount to the left in data constructors and t
```

12.1.1.25 DatatypeContexts

Allow contexts in data types.

```
data Eq a => Set a = NilSet | ConsSet a (Set a)

-- NilSet :: Set a
-- ConsSet :: Eq a => a -> Set a -> Set a
```

Considered misfeature. Deprecated. Going to be removed.

12.1.1.26 StandaloneKindSignatures

Type signatures for type-level declarations.

```
type <name_1> , ... , <name_n> :: <kind>
```

```
type MonoTagged :: Type -> Type -> Type
data MonoTagged t x = MonoTagged x

type Id :: forall k. k -> k
type family Id x where
  Id x = x

type C :: (k -> Type) -> k -> Constraint
class C a b where
  f :: a b

type TypeRep :: forall k. k -> Type
data TypeRep a where
  TyInt   :: TypeRep Int
  TyMaybe :: TypeRep Maybe
  TyApp   :: TypeRep a -> TypeRep b -> TypeRep (a b)
```

< GHC 8.10.1 - type signatures were only for term level declarations.

Extension makes signatures feature more uniformal.

Allows to set the order of quantification, order of variables in a kind. For example when using TypeApplications.

Allows to set full kind of derivable class, solving situations with GADT return kind.

12.1.1.26.1 *

SAKS Standalone kind signatures

12.1.1.27 PartialTypeSignatures

Very healpful. Helps to solve type level, helps to establish type signatures and constraints. Allow to provide _ in the type signatures to automatically infere-in the type information there.

Wild cards:

- Type

```
f :: _ -> _ -> a
```

- Constraint

```
f :: _ => a -> b -> c
```

- Named

```
f :: _x -> _x -> a
```

allows to identify the same wildcard.

12.1.2 How to make a GHC LANGUAGE extension

In `libraries/ghc-boot-th/GHC/LanguageExtensions/Type.hs` add new constructor to the `Extension` type

```
data Extension
  = Cpp
  | OverlappingInstances
  ...
  | Foo
```

`/main/DynFlags.hs` extend `xFlagsDeps`:

```
xFlagsDeps = [
  flagSpec "AllowAmbiguousTypes"  LangExt.AllowAmbiguousTypes,
  ...
  flagSpec "Foo"                  LangExt.Foo
]
```

It is for basic case. For testing, parser see further: https://blog.shaynefletcher.org/2019/02/adding-ghc-language-extension.html

Chapter 13

Compositionality

Complex expression is determined by the constituent expressions and the rules used to combine them.

If the meaning fully obtainable form the parts and composition - it is full, pure compositionality.

If there exists composed idiomatic expression - it is unfull, unpure compositionality, because meaning leaks-in from the sources that are not in the composition.

13.1 *

Principe of compositionality Composition Compositional

Chapter 14

Referential transparency

Given the same input return the same output. So: * expression can be replaced with its corresponding resulting value without change for program's behavior. * functions are pure.

14.1 *

Referentially transparent

Chapter 15

Semantics

Philosophical study of meaning. Meaning of symbols, words.

15.1 Operational semantics

Constructing proofs from logical assertions and verifying/checking/asserting things about execution and procedures their properties, such as correctness, safety or security.

Good to solve in-point localized tasks.

Process of abstraction.

15.1.1 Argument

arguere make clearmake known, to prove, to shine

* - evidence, proof, statement that results in consequencesin the system.

15.1.1.1 Argument of a function

A value binded to the function parameter. Value/topic that the fuction would process/deal with.

Also see Argument.

15.1.1.1.1 *

Function argument

15.1.1.2 First-class

Means *it*:

- Can be used as value.
- Passed as an argument.

From 1&2 -> *it* can include itself.

15.1.2 Relation

Relationship between two objects. By default it is not directed and not limited. In Set theory: some subset of a Cartesian product between sets of objects.

15.1.2.1 *

Relations Relationship

15.2 Denotational semantics

Construction of objects, that describe/tag the meanings. In Haskell often abstractions that are ment (denotations), implemented directly in the code, sometimes exist over the code - allowing to reason and implement.

* are composable.

Good to achive more broad approach/meaning.

Also see Abstraction.

15.2.1 Abstraction

abs away from, off (in absentia)

tractus draw, haul, drag

Purified generalization.

Forgeting the details (axiomatic semantics). Simplified approach. Out of sight - out of mind.

* creates a new semantic level in which one can be absolutely precise (operational semantics).

It is a great did to name an abstraction (denotational semantics).

The ideal abstractions are:

- integrative (global):
  - nothing, void, emptiness - "none", initial object
  - everything - "all", "existance", terminal object
- differential (local):
  - point - "this", "is", "one", stasis
  - chaos - "any", "of", "many", process

They are ideal - because they are the basis, the beginning. Because you can not express any other obstractions without these.

+===

This is personal idea & the thought of autor of the book regarding basic abstractions particularly. Other definitions in the book basing on this are the proof that statement has some ground truth in it. There is ongoing philosophical discussion on the topics like these.

15.2.1.1 *

Abstractions Abstracting Abstract

15.2.1.2 Leaky abstraction

Abstraction that leaks details that it is supposed to abstract away.

15.2.1.2.1 *

Leaky abstractions

15.2.1.3 Object

Absolute abstraction.

Point that additionally can have properties.

Often abstracts something, that is why it exposes external properties on abstracting something, for example some structure, maybe mathematical. In this book objects represent algebraic structures, as we are talking about Haskell and Category theory.

Objects without process are in constant state.

15.2.1.3.1 *

Structure Structures Objects

15.2.1.3.2 Arrow

Second level of absolute abstraction.

Arrow.

Can have target, can have source. Both often are objects.

Often abstracts process.

Can have properties.

Also alias in Category Theory for "morphism", thou theory emposes properties.

15.2.1.3.2.1 *

Arrows Process

15.2.1.3.3 Terminal object

One that recieves unique arrow from every object.

$\exists ! : x \rightarrow 1 \mid \exists 1 \in \mathcal{C}, \ \forall x \in \mathcal{C}$

* is an empty sequence `()` in Haskell.

Called a *unit*, so recieves *terminal* or *unit* arrow.

Dual of initial object.

Denotation:

Category theory 1

Haskell

`()`

15.2.1.3.4 Initial object

One that emits unique arrow into every object.

$\exists ! : \emptyset \rightarrow x \mid \exists \emptyset \in \mathcal{C}, \ \forall x \mathcal{C}$

If initial object is `Void` (most frequently) - emitted arrows called absurd, because they can not be called.

Dual of terminal object.

Denotation:

Category theory: ∅

Haskell:

`Void`

15.2.1.3.5 Value

What object abstracts. Without any object external structure (aka identity in Category Theory). So * is immutable. Such herecy is called "Value semantics" and leads such things as referential transparency, functional programming and Haskell.

(Except, when you hack Haskell with explicit low-level fun�tions, and start to directly mute values - then you are on your own, Haskell paradigm does not expect that.)

15.2.1.3.5.1 *

Value semantics Values

15.2.1.3.6 Tensor

Object existing out of planes, thus it can translate objects from one plane into another. * can be tried to be described with knowledge existing inside planes (from projection on the plane), but representation would always be partial.

Tensor of rank 1 is a vector.

Translations with tensor can be seen as functors.

15.2.1.3.6.1 *

Tensors Tensorial

15.2.2 Ambigram

ambi both

γράμμα *grámma* written character

Object that from different points of view has the same meaning.

While this word has two contradictory diametrically opposite usages, one was chosen (more frequent).

But it has... Both.

TODO: For merit of differentiating the meaning about different meaning referring to Tensor as object with many meanings.

15.2.3 Binary

Two of something.

15.2.4 Arbitrary

arbitrarius uncertain

Random, any one of.

Used as: Any one with *this* set of properties. (constraints, type, etc.).

When there is a talk about any arbitrary value - in fact it is a talk about the generalization of computations over the set of properties.

15.2.5 Refutable

One that has an option to fail.

15.2.6 Irrefutable

One that can not fail.

15.2.7 Superclass

Broader parent class.

15.2.8 Unit

Represents existence. Denoted as empty sequence.

`()`

Type `()` holds only self-representation constructor `()`, & constructor holds nothing.

Haskell code always should recieve something back, hense nothing, emptiness, void can not be theoretically addressed, practically constructed or recieved - unit in Haskell also has a role of a stub in place of emptiness, like in `IO ()`.

15.2.9 Nullary

Takes no entries (for example has the arity of zero). Has the trivial domain.

15.2.10 Syntax tree

Tree of syntactic elements (each node denotes construct occurring in the language/source code) that represent the full particular expression/implementation (or said).

15.2.10.1 Abstract syntax tree

"Abstract" since does not represent every detail of the syntax (ex. parentheses), but rather concentrates on structure and content.

Widely used in compilers to check the code structure for accuracy and coherence.

15.2.10.1.1 *

AST

15.2.10.2 Concrete syntax tree

An ordered, rooted syntax tree that represents the syntactic structure of a string according to some context-free grammar.

"Concrete" since (in contrast to "abstract") - concretely reflects the syntax of the input language.

15.2.10.2.1 *

Parse tree Derivation tree

15.2.11 Stream

* an infinite sequence that forgets previous objects, and remembers only currently relevant objects.

$E \mid X \to (X \times A + 1)$, the set (or object) of streams on A (final coalgebra A_* of E).

`cycle` is one of stream functions.

```
a = (cycle [Nothing, Nothing, Just "Fizz"])
b = (cycle [Nothing, Nothing, Nothing, Nothing, Just "Buzz"])
```

Can be:

- indexed, timeless, with current object
- timed:

* `[(timescale, event)]` * `[(realtime, event)]`

Has amalgamation with Functional Reactive Programming.

15.2.12 Linear

Values consumed once or not used.

`x^2` consumes/uses `x` two times (`x*x`).

15.2.12.1 *

Linearity

15.2.13 Predicative

Non-self-referencing definition.

+===

Antonym - *Impredicative*.

15.2.14 Quantifier

Specifies the quantity of specimens.

Two most common quantifiers $\forall$ (Forall) and $\exists$ (Exists). $\exists!$ - one and only one (exists only unique).

15.2.14.1 *

Quantification Quantifiers Quantified

15.2.14.2 Forall quantifier

Permits to not infer the type, but to use any that fits. The variant depends on the LANGUAGE option used:

- ScopedTypeVariables
- RankNTypes
- ExistentialQuantification

15.2.14.2.1 *

Forall

15.3 Axiomatic semantics

Empirical process of studying something complex by finding and analyzing true statements about it.

Good for examining interconnections.

15.3.1 Property

Something has a property in the real world, and in theory its property corresponds to the law/laws, axioms.

In Haskell under property/law most often properties of algebraic structures.

There property testing wich does what it says.

15.3.1.1 *

Properties

15.3.1.2 Associativity

Joined with common purpose.

$P(a, P(b, c)) \equiv P(P(a, b), c) \mid \forall (a, b, c) \in S$,

* - the operations can be grouped arbitrarily.

Property that determines how operators of the same precedence are grouped, (in computer science also in the absence of parentheses).

Etymology: Latin *associatus* past participle of *associare "join with"*, from assimilated form of *ad "to"* + *sociare "unite with"*, from *socius "companion, ally"* from PIE **sokw-yo-*, suffixed form of root **sekw- "to follow"*.

In Haskell * has influence on parsing when compounds have same fixity.

15.3.1.2.1 *

Associative Associative law Associativity law

15.3.1.3 Left associative

* - the operations are grouped from the left.

Example: In lambda expressions same level parts follow grouping from left to right. $(\lambda x.x)(\lambda y.y)z \equiv ((\lambda x.x)(\lambda y.y))z$

15.3.1.3.1 *

Left associativity Left-associative

15.3.1.4 Right associative

* - the operations are grouped from the right.

15.3.1.5 Non-associative

Operations can't be chained.

Often is the case when the output type is incompatible with the input type.

15.3.1.6 Basis

$\beta\alpha\sigma\iota\varsigma$ - stepping

The initial point, unreducible axioms and terms that spawn a theory. AKA see Category theory, or Euclidian geometry basis.

15.3.1.6.1 Contravariant

The property of basis, in which if new basis is a linear combination of the prior basis, and the change of basis inverse-proportional for the description of a Tensors in this basisis.

Denotation: Components for contravariant basis denoted in the upper indices: $V^i = x$

The inverse of a covariant transformation is a contravariant transformation. Whenever a vector should be invariant under a change of basis, that is to say it should represent the same geometrical or physical object having the same magnitude and direction as before, its components must transform according to the contravariant rule.

15.3.1.6.1.1 *

Contravariant cofunctor Contravariant functor - More inline term is Contravariant cofunctor

15.3.1.6.2 Covariant

The property of basis, in which if new basis is a linear combination of the prior basis, and the change of basis proportional for a descriptions of tensors in basisis.

Denotation: Components for covariant basis denoted in the upper indices: $V_i = x$

15.3.1.6.2.1 *

Covariant functor Covariant cofunctor

15.3.1.7 Commutativity

$\forall (a, b) \in S : \; P(a, b) \equiv P(b, a)$

15.3.1.7.1 *

Commutative Commutative law

15.3.1.8 Idempotence

First application gives a result. Then same operation can be applied multiple times without changing the result. Example: Start and Stop buttons on machines.

15.3.1.8.1 *

Idempotent Idempotency

15.3.1.9 Distributive property

Set S and two binary operators + ×:

- $x \times (y + z) = (x \times y) + (x \times z)$ - × is left-distributive over +
- $(y + z) \times x = (y \times x) + (z \times x)$ - × is right-distributive over +
- left-&right-distributive - × is distributive over +

15.3.1.9.1 *

Distributive rule Distributive axiom Distributive law Distributive

15.3.2 Effect

Observable action.

15.3.3 Bisimulation

When systems have exact external behaviour so for observer they are the same.

Binary relation between state transition systems that match each other's moves.

15.3.3.1 *

Bisimilar

15.4 Content word

Words that name objects of reality and their qualities.

15.5 Ancient Greek and Latin prefixes

15.5.1 *

Greek prefix Latin prefix

15.6 Idiom

* - something having a meaning that can not be derived from the conjoined meanings of * constituents. Meaning can be special for language speakers or human with particular knowledge.

* can also mean applicative functor, people better stop making idiom from the term "idiom".

15.6.1 *

Idioms Idiomatic

15.7 Impredicative

Self-referencing definition.

+===

Antonym - Predicative.

15.8 Context-free grammar

Type of formal grammar that is: a set of production rules that describe all possible string is a given formal language.

Term is invented by Noam Chomsky.

Table 15.1: Ancient Greek and Latin prefixes

Meaning	Greek prefix	Latin prefix
above, excess	hyper-	super-, ultra-
across, beyond, through	dia-	trans-
after		post-
again, back		re-
against	anti-	contra-, (in-, ob-)
all	pan	omni-
around	peri-	circum-
away or from	apo-, ap-	ab- (or de-)
bad, difficult, wrong	dys-	mal-
before	pro-	ante-, pre-
between, among		inter-
both	amphi-	ambi-
completely or very		de-, ob-
down		de-, ob-
four	tetra-	quad-
good	eu-	ben-, bene-
half, partially	hemi-	semi-
in, into	en-	il-, im-, in-, ir-
in front of	pro-	pro-
inside	endo-	intra-
large	macro-	(macro-, from Greek)
many	poly-	multi-
not*	a-, an-	de-, dis-, in-, ob-
on	epi-	
one	mono-	uni-
out of	ek-	ex-, e-
outside	ecto-, exo-	extra-, extro-
over	epi-	ob- (sometimes)
self	auto-, aut-,auth-	ego-
small	micro-	
three	tri-	tri-
through	dia-	trans-
to or toward	epi-	ad-, a-, ac-, as-
two	di-	bi-
under, insufficient	hypo-	sub-
with	sym-, syn-	co-. com-, con-
within, inside	endo-	intra-
without	a-, an-	dis- (sometimes)

15.8.1 *

CFG

Chapter 16

Set

Well-defined collection of distinct objects.

16.1 *

Sets Set theory

16.2 Closed set

1. Set which complements an open set.
2. Is form of Closed-form expression. Set can be closed in under a set of operations.

16.3 Power set

For some set $\mathcal{S}$, the power set $(\mathcal{P}(\mathcal{S}))$ is a set of all subsets of $\mathcal{S}$, including $\{\}$ & $\mathcal{S}$ itself.

Denotation: $\mathcal{P}(\mathcal{S})$

16.4 Singleton

Singleton - unit set - set with exactly one element. Also 1-sequence.

16.5 Russell's paradox

If there exists normal set of all sets - it should contain itself, which makes it abnormal.

16.6 Cartesian product

$\mathcal{A} \times \mathcal{B} \equiv \sum^{\forall} (a, b) \mid \forall a \in \mathcal{A}, \forall b \in \mathcal{B}$. Operation, returns a set of all ordered pairs (a, b)

Any function, functor is a subset of Cartesian product.

$\sum (elem \in (\mathcal{A} \times \mathcal{B})) = cardinality^{A \times B}$

Properties:

- not associative

- not commutative

16.6.1 Pullback

Subset of the cartesian product of two sets.

16.6.1.1 *

Pullbacks

Chapter 17

Testing

17.1 Property testing

Since property has a law, then family of that unit tests can be abstracted into the lambda function. And tests cases come from generator.

17.1.1 Function property

Property corresponds to the according law. In property testing you need to think additionally about generator and shrinking.

17.1.2 Property testing type

Table 17.1: Property testing types

	Exhaustive	Randomized	Unit test
Whole set of values	Exhaustive property test	Randomised property test	One eleme
Special subset of values	Exhaustive specialised property test	Randomised specialised property test	One eleme

17.1.3 Generator

```
Seed
|
v
Gen A -> A
^
|
Size
```

Seed allows reproducibility. There is anyway a need to have some seed. Size allows setting upper bound on size of generated value. Think about infinity of list.

After failed test - shrinking tests value parts of contrexample, finds a part that still fails, and recurses shrinking.

17.1.3.1 *

Generators

17.1.3.2 Custom generator

When sertain theorem only works for a specific set of values - the according generator needs to be produced.

```
arbitrary :: Arbitrary a => Gen a
suchThat :: Gen a -> (a -> Bool) -> Gen a
elements :: [a] -> Gen a
```

17.1.4 Reusing test code

Often it is convinient to abstract testing of same function properties:

It can be done with (aka TestSuite combinator):

```
-- Definition
{-# LANGUAGE ScopedTypeVariables #-}
{-# LANGUAGE AllowAmbiguousTypes #-}
eqSpec :: forall a. Arbitrary a => Spec

-- Usage
{-# LANGUAGE TypeApplications #-}
spec :: Spec
spec = do
  eqSpec @Int
```

```
Eq Int
  (==) :: Int -> Int -> Bool
    is reflexive
    is symetric
    is transitive
    is equivalent to (\ a b -> not $ a /= b)
  (/=) :: Int -> Int -> Bool
    is antireflexive
    is equivalent to (\ a b -> not $ a == b)
```

17.1.4.1 Test Commutative property

Commutativity

```
:: Arbitrary a => (a -> a -> a) -> Property
```

17.1.4.2 Test Symmetry property

Symmetry

```
:: Arbitrary a => (a -> a -> Bool) -> Property
```

17.1.4.3 Test Equivalence property

Equivalence

```
:: (Arbitrary a, Eq b) => (a -> b) -> (a -> b) -> Property
```

17.1.4.4 Test Inverse property

```
:: (Arbitrary a, Eq b) => (a -> b) -> (b -> a) -> Property
```

17.1.5 QuickCheck

Target is a member of the Arbitrary type class. Target -> Bool is something Testable. This properties can be complex. Generator arbitrary gets the seed, and produces values of Target. Function quickCheck runs the loop and tests that generated Target values always comply the property.

17.1.5.1 Manual automation with QuickCheck properties

```
import Test.QuickCheck
import Test.QuickCheck.Function
import Test.QuickCheck.Property.Common
import Test.QuickCheck.Property.Functor
import Test.QuickCheck.Property.Common.Internal

data Four' a b = Four' a a a b
  deriving (Eq, Show)

instance Functor (Four' a) where
  fmap f (Four' a b c d) = Four' a b c (f d)

instance (Arbitrary a, Arbitrary b) => Arbitrary (Four' a b) where
  arbitrary = do
    a1 <- arbitrary
    a2 <- arbitrary
    a3 <- arbitrary
    b <- arbitrary
    return (Four' a1 a2 a3 b)

-- Wrapper around `prop_FunctorId`
prop_AutoFunctorId :: Functor f => f a -> Equal (f a)
prop_AutoFunctorId = prop_FunctorId T

type Prop_AutoFunctorId f a
  = f a
  -> Equal (f a)

-- Wrapper around `prop_AutoFunctorCompose`
prop_AutoFunctorCompose :: Functor f => Fun a1 a2 -> Fun a2 c -> f a1 -> Equal (f c)
prop_AutoFunctorCompose f1 f2 = prop_FunctorCompose (applyFun f1) (applyFun f2) T

type Prop_AutoFunctorCompose structureType origType midType resultType
  = Fun origType midType
  -> Fun midType resultType
  -> structureType origType
  -> Equal (structureType resultType)

main = do
  quickCheck $ eq $ (prop_AutoFunctorId :: Prop_AutoFunctorId (Four' ())Integer)
  quickCheck $ eq $ (prop_AutoFunctorId :: Prop_AutoFunctorId (Four' ()) (Either Bool String))
  quickCheck $ eq $ (prop_AutoFunctorCompose :: Prop_AutoFunctorCompose (Four' ()) String Integer
  quickCheck $ eq $ (prop_AutoFunctorCompose :: Prop_AutoFunctorCompose (Four' ()) Integer String
```

17.2 Write tests algorithm

1. Pick the right language/stack to implement features.

2. How expensive breakage can be.
3. Pick the right tools to test this.

17.3 Shrinking

Process of reducing coplexity in the test case - re-run with smaller values and make sure that the test still fails.

Chapter 18

Logic

18.1 Proposition

Purely abtract & theoretical logical object (idea) that has a Boolean value.

* is expressed by a statement.

18.1.1 *

Propositions

18.1.2 Atomic proposition

Logically undividable unit. Does not contain logical connectives.

18.1.2.1 *

Atomic propositions

18.1.3 Compound proposition

Formed by connecting propositions by logical connectives.

18.1.3.1 *

Compound propositions

18.1.4 Propositional logic

Studies propositions and argument flow.

Refers to logically indivisible units (atomic propositions) as such, for theory - they are abstractions with Boolean properties.

Not Turing-complete, impossible to construct an arbitrary loop.

18.1.4.1 *

Proposition logic Proposition calculus Propositional calculus Statement logic Sentential logic Sentential calculus Zeroth-order logic

18.1.4.2 First-order logic

Notation systems that use quantifiers, relations, variables over non-logical objects, allows the use of expressions that contain variables.

Turing-complete.

Extension of a propositional logic.

18.1.4.2.1 *

Predicate logic First-order predicate logic First-order predicate calculus

18.1.4.2.2 Second-order logic

Extension over first-order logic that quantifies over relations.

18.1.4.2.2.1 Higher-order logic

Extension over second-order logic that uses additional quantifiers, stronger semantics.

Is more expressive, but model-theoretic properties are less well-behaved.

18.2 Logical connective

Logical operation.

18.2.1 *

Logical connectives

18.2.2 Conjunction

Logical AND.

Denotation: $\wedge$

Multiplies cardinalities.

Haskell kind:

* *

18.2.3 Disjunction

Logical OR Denotation: $\vee$

Summs cardinalities.

18.3 Predicate

Function with Boolean codomain. $P : X \to \{true, \ false\}$ - * on X.

Notation: $P(x)$

Im many cases includes relations, quantifiers.

18.4 Statement

Declarative expression that is a bearer of a proposition.

When we talk about expression or statement being true/false - in fact we refer to the proposition that they represent.

Difference between proposition, statement, expression:

1. "2 + 3 = 5"
2. "two plus three equals five"
    - 1 & 2 are statements. Each of them is a collection of transmission symbols (linguistic objects) from a symbol systems $\equiv$ expression. Each of them is expression that bears proposition (an idea resulting in a Boolean value) $\equiv$ statement.
    - 1 & 2 represent the same proposition. Proposition from 1 $\equiv$ proposition from 2.
    - Statement 1 $\neq$ statement 2. They are two different statements, written in different systems. And statement "2 + 3 = 5" $\neq$ statement "3 + 2 = 5".

18.4.1 *

Assertion Assertions Statements

18.5 Iff

If and only if, exectly when, just. Denotation: $\iff$

Chapter 19

Haskell structure

19.1 *

Haskell structures

19.2 Pattern match

Are not first-class. It is a set of patter match semantic notations.

Must be linear.

* precedence (especially with more then one parameter, especially with _ used) often changes the function.

19.2.1 As-pattern

```
f list@(x, xs) = ...
```

```
f (x:xs)    = x:x:xs -- Can be compiled with reconstruction of x:xs
f a@(x:_) = x:a -- Reuses structure without reconstruction
```

19.2.1.1 *

As-patterns As pattern As patterns

19.2.2 Wild-card

Matches anything and can not be binded. For matching someting that should pass not checked and is not used.

```
head (x:_)              = x
tail (_:xs)             = xs
```

19.2.2.1 *

Wild-cards Wildcard Wildcards

19.2.3 Case

```
case x of
  pattern1  -> ex1
  pattern2  -> ex2
```

```
  pattern3  -> ex3
  otherwise -> exDefault
```

Bolting guards & expressions with syntatic sugar on case:

```
case () of _
  | expr1     -> ex1
  | expr2     -> ex2
  | expr3     -> ex3
  | otherwise -> exDefault
```

Pattern matching in function definitions is realized with case expressions.

19.2.4 Guard

Check values against the predicate and use the first match definition:

```
f x
  | predicate1 = definition1
  | predicate2 = definition2
  ...
  | x < 0      = definitionN
  ...
  | otherwise  = definitionZ
```

19.2.4.1 *

Guards

19.2.5 Pattern guard

Allows check a list of pattern matches against functions, and then proceed.

$$(patternMatch1) \text{ <- } (funcCheck1)$$

, $(patternMatch2)$ <- $(funcCheck2)$ = RHS

```
lookup :: Eq a => a -> [(a, b)] -> Maybe b

addLookup l a1 a2
   | Just b1 <- lookup a1 l
   , Just b2 <- lookup a2 l
   = b1 + b2
{-...other equations...-}
```

Run functions, they must succeed. Then pattern match results to b1, b2. Only if successful - execute the equation.

Default in Haskell 2010.

19.2.5.1 *

Pattern guards

19.2.6 Lazy pattern

Defers the pattern match directly to the last moment of need during execution of the code.

```
f (a, b) = g a b -- It would be checked that the pattern of the pair constructor
-- is present, and that parameters are present in the constructor.
-- Only after that success - work would start on the RHS, aka then construction
```

```
-- g would start only then.

f ~(a, b) = g a b -- Pattern match of (a, b) deferred to the last moment,
-- RHS starts, construction of g starts.
-- For this lazy pattern the  equivalent implementation would be:
-- f p = g (fst p) (snd p)  -- RHS starts, during construction of g
-- the arguments would be computed and found, or error would be thrown.
```

Due to full laziness deferring everything to the runtime execution - the lazy pattern is one-size-fits all (irrefutable), analogous to _, and so it does not produce any checks during compilation, and raises errors during runtime.

* is very useful during recursive construction of recursive structure/process, especially infinite.

19.2.6.1 *

Lazy-pattern Lazy patterns

19.2.7 Pattern binding

Entire LHS is a pattern, is a lazy pattern.

```
fib@(1:tfib)    = 1 : 1 : [ a+b | (a,b) <- zip fib tfib ]
```

19.2.7.1 *

Pattern bindings

19.3 Smart constructor

Process/code placing extra rules & constraints on the construction of values.

19.4 Level of code

There are these levels of Haskell code:

19.4.1 *

Code level

19.4.2 Type level

Level of code that works with data types.

19.4.2.1 Type level declaration

```
type ...
newtype ...
data ...
class ...
instance ...
```

19.4.2.1.1 *

Type level declarations Type-level declaration Type-level declarations

19.4.2.2 Type check

`if` The type level information is complete (strongly connected graph)

`then`

Generalize the types and check if type level consistent to term level.

`else`

Infer the missing type level part from the term level. There are certain situations and structures where ambiguity arises and is unsolvable from the information of the term level (most basic example is polymorphic recursion).

19.4.2.2.1 *

Typecheck Typechecking Typechecks

19.4.2.2.2 Complete user-specific kind signature

Type level declaration is considered to "have a CUSK" is it has enough syntatic information to warrant completeness (strongly connected graph) and start checking type level correspondence to term level, it is a ad-hock state ot type inferring.

In the future GHC would use other algorythm over/instead of CUSK.

19.4.2.2.2.1 *

CUSK CUSKs Complete user-specific kind signatures Complete, user-specific kind signature

19.4.3 Term level

Level of code that does logical execution.

19.4.4 Compile level

Level of code, about compilation processes/results.

19.4.4.1 *

Compilation level

19.4.5 Runtime level

Level of code of main program operation, when machine does computations with compiled binary code.

19.4.6 Kind level

Level of code where kinds & kind declarations are situated, infered and checked.

19.4.6.1 Kind check

Applying the type check to kind check:

`if` The kind level information is complete (strongly connected graph)

`then`

Check if kind level consistent to term level.

`else`

Infer the missing kind level parts from the type level. There are certain situations and structures where ambiguity arises and is unsolvable from the information of the kind level.

With `StandaloneKindSignatures` kind completeness happens against found (standalone) kind signature.

With `CUSKs` extension kind completeness happens agains "complete user-specific kind signature"

19.4.6.1.1 *

Kindcheck Kind checks

19.5 Orphan instance

Hanging instance from inconsistent code base.

1. Supporting structure not fully present.
2. Several implementations of instance present.

19.6 undefined

Placeholder value that helps to do typechecking.

19.7 Hierarchical module name

Hierarchical naming scheme:

```
Algebra                 -- Was this ever used?
    DomainConstructor   -- formerly DoCon
    Geometric           -- formerly BasGeomAlg

Codec                   -- Coders/Decoders for various data formats
    Audio
       Wav
       MP3
       ...
    Compression
       Gzip
       Bzip2
       ...
    Encryption
       DES
       RSA
       BlowFish
       ...
    Image
       GIF
       PNG
       JPEG
       TIFF
       ...
    Text
       UTF8
       UTF16
       ISO8859
```

```
      ...
   Video
      Mpeg
      QuickTime
      Avi
      ...
   Binary                 -- these are for encoding binary data into text
      Base64
      Yenc

Control
    Applicative
    Arrow
    Exception          -- (opt, inc. error & undefined)
    Concurrent         -- as hslibs/concurrent
        Chan           -- these could all be moved under Data
        MVar
        Merge
        QSem
        QSemN
        SampleVar
        Semaphore
    Parallel           -- as hslibs/concurrent/Parallel
        Strategies
    Monad              -- Haskell 98 Monad library
        ST             -- ST defaults to Strict variant?
            Strict     -- renaming for ST
            Lazy       -- renaming for LazyST
        State          -- defaults to Lazy
            Strict
            Lazy
        Error
        Identity
        Monoid
        Reader
        Writer
        Cont
        Fix             -- to be renamed to Rec?
        List
        RWS

Data
    Binary             -- Binary I/O
    Bits
    Bool               -- &&, ||, not, otherwise
    Tuple              -- fst, snd
    Char               -- H98
    Complex            -- H98
    Dynamic
    Either
    Int
    Maybe              -- H98
    List               -- H98
    PackedString
    Ratio              -- H98
```

```
    Word
    IORef
    STRef               -- Same as Data.STRef.Strict
        Strict
        Lazy            -- The lazy version (for Control.Monad.ST.Lazy)
    Binary              -- Haskell binary I/O
    Digest
        MD5
        ...             -- others (CRC ?)
    Array               -- Haskell 98 Array library
        Unboxed
        IArray
        MArray
        IO              -- mutable arrays in the IO/ST monads
        ST
    Trees
        AVL
        RedBlack
        BTree
    Queue
        Bankers
        FIFO
    Collection
    Graph               -- start with GHC's DiGraph?
    FiniteMap
    Set
    Memo                -- (opt)
    Unique

    Edison              -- (opt, uses multi-param type classes)
        Prelude         -- large self-contained packages should have
        Collection      -- their own hierarchy?  Like a vendor branch.
        Queue           -- Or should the whole Edison tree be placed

Database
    MySQL
    PostgreSQL
    ODBC

Dotnet
    ...                 -- Mirrors the MS .NET class hierarchy

Debug                   -- see also: Test
    Trace
    Observe             -- choose a default amongst the variants
        Textual            -- Andy Gill's release 1
        ToXmlFile          -- Andy Gill's XML browser variant
        GHood              -- Claus Reinke's animated variant

Foreign
    Ptr
    StablePtr
    ForeignPtr  -- rename to FinalisedPtr?  to void confusion with Foreign.Ptr
    Storable
    Marshal
```

```
        Alloc
        Array
        Errors
        Utils
    C
        Types
        Errors
        Strings

GHC
    Exts                -- hslibs/lang/GlaExts
    ...

Graphics
    HGL
    Rendering
       Direct3D
       FRAN
       Metapost
       Inventor
       Haven
       OpenGL
          GL
          GLU
       Pan
    UI
       FranTk
       Fudgets
       GLUT
       Gtk
       Motif
       ObjectIO
       TkHaskell
    X11
       Xt
       Xlib
       Xmu
       Xaw

Hugs
    ...

Language
    Haskell             -- hslibs/hssource
        Syntax
        Lexer
        Parser
        Pretty
    HaskellCore
    Python
    C
    ...

Nhc
    ...
```

```
Numeric                 -- exports std. H98 numeric type classes
    Statistics

Network                 -- (== hslibs/net/Socket), depends on FFI only
    BER                 -- Basic Encoding Rules
    Socket              -- or rename to Posix?
    URI                 -- general URI parsing
    CGI                 -- one in hslibs is ok?
    Protocol
        HTTP
        FTP
        SMTP

Prelude                 -- Haskell98 Prelude (mostly just re-exports
                           other parts of the tree).

Sound                   -- Sound, Music, Digital Signal Processing
    ALSA
    JACK
    MIDI
    OpenAL
    SC3                 -- SuperCollider

System                  -- Interaction with the "system"
    Cmd                 -- ( system )
    CPUTime             -- H98
    Directory           -- H98
    Exit                -- ( ExitCode(..), exitWith, exitFailure )
    Environment         -- ( getArgs, getProgName, getEnv ... )
    Info                -- info about the host system
    IO                  -- H98 + IOExts - IOArray - IORef
        Select
        Unsafe          -- unsafePerformIO, unsafeInterleaveIO
    Console
        GetOpt
        Readline
    Locale              -- H98
    Posix
        Console
        Directory
        DynamicLinker
            Prim
            Module
        IO
        Process
        Time
    Mem                 -- rename from cryptic 'GC'
        Weak            -- (opt)
        StableName      -- (opt)
    Time                -- H98 + extensions
    Win32               -- the full win32 operating system API

Test
    HUnit
```

```
    QuickCheck

Text
    Encoding
        QuotedPrintable
        Rot13
    Read
        Lex                -- cut down lexer for "read"
    Show
        Functions          -- optional instance of Show for functions.
    Regex                  -- previously RegexString
        Posix              -- Posix regular expression interface
    PrettyPrint            -- default (HughesPJ?)
        HughesPJ
        Wadler
        Chitil
        ...
    HTML                   -- HTML combinator lib
    XML
        Combinators
        Parse
        Pretty
        Types
    ParserCombinators      -- no default
        ReadP              -- a more efficient "ReadS"
        Parsec
        Hutton_Meijer
        ...

Training                   --  Collect study and learning materials
    <name of the tutor>
```

19.7.1 *

Top-level module name Top-level module names

19.8 Reserved word

Haskell has special meaning for:

```
case, class, data, deriving, do,else, if, import,
in, infix, infixl, infixr, instance, let,
of, module, newtype, then, type, where
```

19.8.1 *

Reserved words

19.8.2 import

`import` statement by default imports identifiers from the other module, using hierarchical module name, brings into scope the identifiers to the global scope both into unqualified and qualifies by the hierarchical module name forms.

This possibilities can mix and match:

- `<modName> ()` - import only instances of type classes.
- `<modName> (x, y)` - import only declared indentifiers.
- `qualified <modName>` - discards unqialified names, forse obligatory namespace for the imports.
- `hiding (x, y)` - skip import of declared identifies.
- `<modName> as <modName>` - renames module namespace.
- `<type/class> (..)` - import class & it's methods, or type, all its data constructors & field names.

19.8.3 let

* expression is a set of cross-recursive lazy pattern bindings.

Declarations permitted:

- type signatures
- function bindings
- pattern bindings

It is an expression (macro) and that integrates in external lexical scope expression it applied in.

Form:

```
let
  b1
  bn
in
  c
```

19.8.3.1 *

Let expression Let expressions

19.8.4 where

Part of the syntax of the whole function declaration, has according scope.

As part of whole declaration - can extend over definitions of the funtion (pattern matches, guards).

Form:

```
f match1 = y
f match2 = y
f x =
  | cond1 x = y
  | cond2 x = y
  | otherwise = y
 where
  y = ... x ...
```

19.8.4.1 *

Where clause

19.9 Haskell Language Report

Document that is a standart of language.

19.9.1 *

Report Haskell Report Haskell 98 Language Report Haskell 98 Report Haskell 1998 Language Report Haskell 2010 Language Report Haskell 2010 Report

19.10 Haskell'

Current language development mod.

https://prime.haskell.org/

19.10.1 *

Haskell prime

19.11 Lense

Library of combinators to provide Haskell (functional language without mutation) with the emulation of `get`-ters and `set`-ters of imperative language.

Chapter 20

Computer science

20.1 Guerrilla patch

* changing code/applying patch sneakily - and possibility incompatibility with other at runtime. Monkey patch is derivative term.

20.1.1 Monkey patch

From Guerrilla patch.

* is a way for program to modify supporting system software affecting only the running instance of the program.

20.2 Interface

Point of mutual meeting. Code behind interface determines how data is consumed.

20.3 Module

Importable organizational unit.

20.4 Scope

Area where binds are accessible.

20.4.1 Dynamic scope

The name resolution depends upon the program state when the name is encountered, which is determined by the execution context or calling context.

20.4.2 Lexical scope

Scope bound by the structure of source code where the named entity is defined.

20.4.2.1 *

Static scope

20.4.3 Local scope

Scope applies only in (current) area.

20.4.3.1 *

Local

20.5 Shadowing

When in the local scope bigger scope variable overriden by same name variable from the local scope.

20.6 Syntatic sugar

Artificial way to make language easier to read and write.

20.7 System F

Is parametric polymorphism in programming.

Extends the Lambda calculus by introducing $\forall$ (universal quantifier) over types.

20.7.1 *

Girard–Reynolds polymorphic lambda calculus Girard-Raynolds

20.8 Tail call

Final evaluation inside the function. Produces the function result.

20.9 Thunk

Not evaluated calculation. Can be dragged around, until be lazily evaluated.

20.10 Application memory

Table 20.1: Application memory structural parts

Storage of	Block name
All not currently processing data	Heap
Function call, local variables	Stack
Static and global variables	Static/Global
Instructions	Binary code

When even Main invoked - it work in Stack, and called Stack frame. Stack frame size for function calculated when it is compiled. When stacked Stack frames exceed the Stack size - stack overflow happens.

20.11 Turing machine

Mathematical model of computation that defines abstract Turing machine. Abstract machine which manipulates symbols on a strip of tape, according to a table of rules.

20.11.1 Turing complete

Set of action rules that can simulate any Turing machine.

20.11.1.1 *

Turing incomplete Turing incompleteness Turing completeness Computationally universal

20.12 REPL

Read-eval-print loop, aka interactive shell.

20.13 Domain specific language

Language design/fitted for particular domain of application. Mainly should be Turing incomplete, since general-purpose language implies Turing completeness.

20.13.1 *

Domain-specific language DSL

20.13.2 Embedded domain specific language

DSL used inside outer language.

Two levels of embedding:

- Shallow: DSL translates into Haskell directly
- Deep: Between DSL and Haskell there is a data structure that reflects the expression tree, AKA stores the syntax tree.

20.13.2.1 *

eDSL

20.14 Data structure

20.14.1 Cons cell

Cell that values may inhabit.

20.14.2 Construct

```
(:) :: a -> [a] -> [a]
```

20.14.2.1 *

Cons

20.14.3 Leaf

```
-
```

20.14.4 Node

```
 *
/ \
```

20.14.5 Spine

Is a chain of memory cells, each points to the both value of element and to the next memory cell.

```
Array:

  :
 / \
1   :
   / \
  2   :
     / \
    3  []

1:2:3:[]

Spine:
  :
 / \
_   :
   / \
  _   :
     / \
    _  []
```

Chapter 21

Graph theory

21.1 Successor

Object that recieves the arrow.

21.1.1 Direct successor

Immidiate successor.

21.2 Predecessor

Object that sends arrow.

21.2.1 Direct predecessor

Immidiate predecessor.

21.3 Degree

Number of arrows of object.

21.3.1 Indegree

Number of ingoing arrows.

21.3.2 Outdegree

Number of outgoinf arrows.

21.4 Adjacency matrix

Matrix of connection of odjects `{-1,0,1}`.

21.4.0.1 InstanceSigs

Allow adding type signatures to type class function instance declaration.

21.5 Strongly connected

If every vertex in a graph is reachable from every other vertex.

It is possible to find all strongly connected components (and that way also test graph for strong connectivity), in linear time (`Θ(V+E)`).

Binary relation of being strongly connected is an equivalence relation.

21.5.1 *

Strongly-connected

21.5.2 Strongly connected component

Full strongly connected subgraph of some graph.

* of a directed graph G is a subgraph that is strongly connected, and has property: no additional edges or vertices from G can be included in the subgraph without breaking its property of being strongly connected.

21.5.2.1 *

SCC Strongly connected components Strongly-connected component Strongly-connected components

Chapter 22

Tagless-final

Method of embedding eDSL in a typed functional host language (Haskell). Alternative to the embedding as a (generalized) algebraic data type. For parsers of DLS expressions: (1/partial) evaluator, compiler, pretty printer, multi-pass optimizer.

* embedding is writing denotational semantics for the DSL in the host language.

Approach can be used iff eDSL is typed. Only well-typed terms become embeddable, and host language can implemen also a eDSL type system. Approach that eDSL code interpretations are type-preserving.

One of main pros of * - extensibility: implementation of DSL can be used to analyze/evaluate/transform/pretty-print/compile and interpreters can be extended to more passes, optimizations, and new versions of DSL while keeping/using/reusing the old versions.

Example fields of application: language-integrated queries, non-deterministic & probabilistic programming, delimiter continuation, computability theory, stream processsing, hardware description languages, generation of specialized numerical kernels, semantics of natural language.

Part III

Give definitions

Chapter 23

Identity type

Chapter 24

Constant type

Chapter 25

Gen

Chapter 26

Tensorial strength

Chapter 27

Strong monad

Chapter 28

Weak head normal form

28.1 *

WHNF

Chapter 29

Function image

29.1 *

Image

Chapter 30

Invertible

Chapter 31

Invertibility

Chapter 32

Define LANGUAGE pragma options

32.1 ExistentialQuantification

32.2 GADTs

GADT is a generalization over parametric algebraic data types which allow explicitly denote the types (type matching) of the constructors and define data types using pattern matching on the left side of ”data” statements.

32.3 *

GADT Generalized algebraic data type First-class phantom data type Guarded recursive data type Equality-qualified data type

32.4 GeneralizedNewTypeClasses

32.5 FuncitonalDependencies

Chapter 33

GHC check keys

33.1 -Wno-partial-type-signatures

Supresses PartialTypeSignatures wildcard infer warning.

Chapter 34

Generalised algebraic data types

LANGUAGE GADTs

34.1 *

GADT

Chapter 35

Order theory

Investigates in thepth the intuitive notion of order using binary relations.

35.1 Domain theory

Formalizes approximation and convergense. Has close relation to Topology.

35.2 Lattice

Abstract structure that consists of partially ordered set, where every two elements have unique supremum and infinum. == * algebraic structure satisfying certain axiomatic identities. * order-theory & algebraic.

35.3 Order

35.3.1 Preorder

$\mathrm{R}^{\mathrm{X} \to \mathrm{X}}$: Reflexive & Transitive: aRa $aRb, bRc \Rightarrow aRc$

Generalization of equivalence relations partial orders.

* Antisymmetric $\Rightarrow$ Partial ordering. * Symmetric $\Rightarrow$ Equivalence.

35.3.1.1 *

Preordered

35.3.1.2 Total preorder

$\forall a, b : a \leq b \vee b \leq a \Rightarrow$ Total Preorder.

35.3.2 Partial order

A binary relation must be reflexive, antisymmetric and transitive.

Partial - not every elempents between them need to be comparable.

Good example of * is a genealogical descendancy. Only related people produce relation, not related do not.

35.3.2.1 *

Partial orders Partially ordered set Partially ordered sets Poset Posets

35.4 Partial order

35.5 Total order

Chapter 36

Universal algebra

Studies algebraic structures.

Chapter 37

Relation

37.1 Reflexivity

$R^{X \to X}, \forall x \in X : xRx$ Order theory: $a \leq a$

* - each element is comparable to itself.

Corresponds to Identity and Automorphism.

37.1.1 *

Reflexive Reflexive relation

37.2 Irreflexivity

$R^{X \to X}, \forall x \in X : \nexists R(x, x)$

37.2.1 *

Anti-reflexive Anti-reflexive relation Irreflexive Irreflexive relation

37.3 Transitivity

$\forall a, b, c \in X, \forall R^{X \to X} : (aRb \wedge bRc) \Rightarrow aRc$

* - the start of a chain of precedence relations must precede the end of the chain.

37.3.1 *

Transitive Transitive relation

37.4 Symmetry

$\forall a, b \in X : (aRb \iff bRa)$

37.4.1 *

Symmetric Symmetric relation

37.5 Equivalence

Reflexive	Symmetric	Transitive
$\forall x \in X, \exists R : xRx$	$\forall a, b \in X : (aRb \iff bRa)$	$\forall a, b, c \in X, \forall R^{X \to X} : (aRb \wedge bRc) \Rightarrow aRc$
$a = a$	$a = b \iff b = a$	$a = b, b = c \Rightarrow a = c$

37.5.1 *

Equivalent Equivalent relation

37.6 Antisymmetry

$\forall a, b \in X : aRb, bRa \Rightarrow a = b \sim aRb, a \neq b \Rightarrow \nexists bRa$. Antisymmetry does not say anything about $R(a, a)$.

* - no two different elements precede each other.

37.6.1 *

Antisymmetric Antisymmetric relation

37.7 Asymmetry

$\forall a, b \in X(aRb \Rightarrow \neg(bRa))$ * $\iff$ Antisymmetric $\wedge$ Irreflexive. Asymmetry $\neq$ "not symmetric" Symmetric $\wedge$ Asymmetric is only empty relation.

37.7.1 *

Asymmetric Asymmetric relation

Chapter 38

Cryptomorphism

Equivalent, interconvertable with no loss of information.

38.1 *

Crypromorphic

Chapter 39

Lexically scoped type variables

Enable lexical scope for forall quantifier defined type variables

Implemented in ScopedTypeVariables

Chapter 40

Abstract data type

Several definitions here, reduce them.

Data type mathematical model, defined by its semantics from the user point of view, listing possible values, operations on the data of the type, and behaviour of these operations.

* class of objects whose logical behaviour is defined by a set of values and set of operations (analogue to algebraic structure in mathematics).

A specification of a data type like a stack or queue where the specification does not contain any implementation details at all, only the operations for that data type. This can be thought of as the contract of the data type.

40.1 *

AbsDT

Chapter 41

Functional dependencies

Chapter 42

MonoLocalBinds

Chapter 43

KindSignatures

Chapter 44

ExplicitNamespaces

Chapter 45

Combinator pattern

Chapter 46

Symbolic expression

Nested tree data structure.

Introduced & used in Lisp. Lisp code and data are *.

* in Lisp: Atom or expression of the form `(x . y)`, x and y are *.

Modern abbriviated notation of *: `(x y)`.

46.1 *

S-expression S-expressions Sexpression Sexpressions Sexp Sexps Sexpr Sexprs

Chapter 47

Polynomial

Expression consisting of:

- variables
- coefficients
- addition
- substraction
- multiplication (including positive integer variable exponentiation)

Polynomials form a ring. Polynomial ring.

47.1 *

Polynomials

Chapter 48

Data family

Indexed form of data and newtype definitions.

Chapter 49

Type synonym family

Indexed form of type synonyms.

Chapter 50

Indexed type family

* additional stucture in language that allows ad-hoc overloading of data types. AKA are to types as type class to methods.

Variaties:

- data family
- type synonym families

Defined by pattern matching the partial functions between types. Associates data types by type-level function defined by open-ended collection of valid instances of input types and corresponding output types.

Normal type classes define partial functions from types to a collection of named values by pattern matching on the input types, while type families define partial functions from types to types by pattern matching on the input types. In fact, in many uses of type families there is a single type class which logically contains both values and types associated with each instance. A type family declared inside a type class is called an associated type.

50.1 *

Type family

Chapter 51

TypeFamilies

Allow use and definition of indexed type families and data families.

* are type-level programming. * are overload data types in the same way that type classes overload functions. * allow handling of dependent types. Before it Functional dependencies and GADTs were used to solve that. * useful for generic programming, creating highly parametrised interfaces for libraries, and creating interfaces with enhanced static iformation (much like dependent types).

Implies: MonoLocalBinds, KindSignatures, ExplicitNamespaces

Two types of * are:

Chapter 52

Error

Mistake in the program that can be resolved only by fixing the program.

`error` is a sugar for `undefined`.

Distinct from Exception.

52.1 *

Errors

Chapter 53

Exception

Expected but irregular situation.

Distinct from Error. Also see Exception vs Error

53.1 *

Exceptions

Chapter 54

ConstraintKinds

Constraints are just handled as types of a particular kind (Constraint). Any type of the kind Constraints can be used as a constraint.

- Anything which is already allowed in code as a constraint without *. Saturated applications to type classes, implicit parameter and equality constraints.
- Tuples, all of whose component types have kind Constraint.

```
type Some a = (Show a, Ord a, Arbitrary a) -- is of kind Constraint.
```

- Anything form of which is not yet known, but the user has declared for it to have kind Constraint (for which they need to import it from GHC.Exts):

```
Foo (f :: Type -> Constraint) = forall b. f b => b -> b -- is allowed
-- as well as examples involving type families:
type family Typ a b :: Constraint
type instance Typ Int  b = Show b
type instance Typ Bool b = Num b

func :: Typ a b => a -> b -> b
func = ...
```

Chapter 55

Specialisation

Turns ad hoc polymorphic function into compiled type-specific inmpementations.

55.1 *

Specialise Specialize Specialization

Chapter 56

Diagram

For categories C and J, a diagram of type J in C is a covariant functor D : J → C.

Chapter 57

Cathegory theoretical presheaf

For categories C and J, a J-presheaf on C is a contravariant functor D : C → J.

Chapter 58

Topological presheaf

If X is a topological space, then the open sets in X form a partially ordered set $\mathrm{Open}(X)$ under inclusion. Like every partially ordered set, $\mathrm{Open}(X)$ forms a small category by adding a single arrow $U \to V$ if and only if $U \subseteq V$. Contravariant functors on $\mathrm{Open}(X)$ are called presheaves on X. For instance, by assigning to every open set U the associative algebra of real-valued continuous functions on U, one obtains a presheaf of algebras on X.

Chapter 59

Diagonal functor

The diagonal functor is defined as the functor from D to the functor category D^C which sends each object in D to the constant functor at that object.

Chapter 60

Limit functor

For a fixed index category J, if every functor $J \to C$ has a limit (for instance if C is complete), then the limit functor $C^J \to C$ assigns to each functor its limit. The existence of this functor can be proved by realizing that it is the right-adjoint to the diagonal functor and invoking the Freyd adjoint functor theorem. This requires a suitable version of the axiom of choice. Similar remarks apply to the colimit functor (which is covariant).

Chapter 61

Dual vector space

The map which assigns to every vector space its dual space and to every linear map its dual or transpose is a contravariant functor from the category of all vector spaces over a fixed field to itself.

Chapter 62

Fundamental group

Consider the category of pointed topological spaces, i.e. topological spaces with distinguished points. The objects are pairs (X, x_0), where X is a topological space and x_0 is a point in X. A morphism from (X, x_0) to (Y, y_0) is given by a continuous map $f : X \to Y$ with $f(x_0) = y_0$.

To every topological space X with distinguished point x_0, one can define the fundamental group based at x_0, denoted $\pi_1(X, x_0)$. This is the group of homotopy classes of loops based at x_0. If $f : X \to Y$ is a morphism of pointed spaces, then every loop in X with base point x_0 can be composed with f to yield a loop in Y with base point y_0. This operation is compatible with the homotopy equivalence relation and the composition of loops, and we get a group homomorphism from $\pi(X, x_0)$ to $\pi(Y, y_0)$. We thus obtain a functor from the category of pointed topological spaces to the category of groups.

In the category of topological spaces (without distinguished point), one considers homotopy classes of generic curves, but they cannot be composed unless they share an endpoint. Thus one has the fundamental groupoid instead of the fundamental group, and this construction is functorial.

Chapter 63

Algebra of continuous function

A contravariant functor from the category of topological spaces (with continuous maps as morphisms) to the category of real associative algebras is given by assigning to every topological space X the algebra $C(X)$ of all real-valued continuous functions on that space. Every continuous map $f : X \to Y$ induces an algebra homomorphism $C(f) : C(Y) \to C(X)$ by the rule $C(f)(\varphi) = \varphi \circ f$ for every φ in $C(Y)$.

Chapter 64

Tangent and cotangent bundle

The map which sends every differentiable manifold to its tangent bundle and every smooth map to its derivative is a covariant functor from the category of differentiable manifolds to the category of vector bundles.

Doing this constructions pointwise gives the tangent space, a covariant functor from the category of pointed differentiable manifolds to the category of real vector spaces. Likewise, cotangent space is a contravariant functor, essentially the composition of the tangent space with the dual space above.

Chapter 65

Group action / representation

Every group G can be considered as a category with a single object whose morphisms are the elements of G. A functor from G to Set is then nothing but a group action of G on a particular set, i.e. a G-set. Likewise, a functor from G to the category of vector spaces, Vect_K, is a linear representation of G. In general, a functor G C can be considered as an "action" of G on an object in the category C. If C is a group, then this action is a group homomorphism.

Chapter 66

Lie algebra

Assigning to every real (complex) Lie group its real (complex) Lie algebra defines a functor.

Chapter 67

Tensor product

If C denotes the category of vector spaces over a fixed field, with linear maps as morphisms, then the tensor product $V \otimes W$ defines a functor $C \times C \to C$ which is covariant in both arguments.

Chapter 68

Forgetful functor

The functor $U : \mathrm{Grp} \to \mathrm{Set}$ which maps a group to its underlying set and a group homomorphism to its underlying function of sets is a functor.[8] Functors like these, which ”forget” some structure, are termed forgetful functors. Another example is the functor $\mathrm{Rng} \to \mathrm{Ab}$ which maps a ring to its underlying additive abelian group. Morphisms in Rng (ring homomorphisms) become morphisms in Ab (abelian group homomorphisms).

Chapter 69

Free functor

Going in the opposite direction of forgetful functors are free functors. The free functor $F : \mathrm{Set} \to \mathrm{Grp}$ sends every set X to the free group generated by X. Functions get mapped to group homomorphisms between free groups. Free constructions exist for many categories based on structured sets. See free object.

Chapter 70

Homomorphism group

To every pair A, B of abelian groups one can assign the abelian group Hom(A, B) consisting of all group homomorphisms from A to B. This is a functor which is contravariant in the first and covariant in the second argument, i.e. it is a functor $Ab^{op} \times Ab \to Ab$ (where Ab denotes the category of abelian groups with group homomorphisms). If $f : A_1 \to A_2$ and $g : B_1 \to B_2$ are morphisms in Ab, then the group homomorphism Hom(f, g): $Hom(A_2, B_1) \to Hom(A_1, B_2)$ is given by $\varphi \mapsto g \circ \varphi \circ f$. See Hom functor.

Chapter 71

Representable functor

We can generalize the previous example to any category C. To every pair X, Y of objects in C one can assign the set Hom(X, Y) of morphisms from X to Y. This defines a functor to Set which is contravariant in the first argument and covariant in the second, i.e. it is a functor Cop × C Set. If f : X1 X2 and g : Y1 Y2 are morphisms in C, then the group homomorphism Hom(f, g) : Hom(X2, Y1) Hom(X1, Y2) is given by   g   f.

Functors like these are called representable functors. An important goal in many settings is to determine whether a given functor is representable.

Chapter 72

Corecursion

Chapter 73

Coinduction

proper definition

* dual to induction. Generalises to corecursion.

Chapter 74

Initial algebra of an endofunctor

Chapter 75

Terminal coalgebra for an endofunctor

Part IV

Citations

"One of the finer points of the Haskell community has been its propensity for recognizing abstract patterns in code which have well-defined, lawful representations in mathematics." (Chris Allen, Julie Moronuki - "Haskell Programming from First Principles" (2017))

Part V

Good code

Chapter 76

Good: Type aliasing

Use data type aliases to deferentiate logic of values.

Chapter 77

Good: Type wideness

Wider the type the more it is polymorphic, means it has broader application and fits more types.

The more constrained system has more usefulness.

Unconstrained means most flexible, but also most useless.

Chapter 78

Good: Print

```
print :: Show a => a -> IO ()
print a = putStrLn (show a)
```

Chapter 79

Good: Fold

`foldr` spine recursion intermediated by the folding. Can terminate at any point. `foldl` spine recursion is unconditional, then folding starts. Unconditionally recurses across the whole spine, if it infinite - infinitely.

Chapter 80

Good: Computation model

Model the domain and types before thinking about how to write computations.

Chapter 81

Good: Make bottoms only local

Chapter 82

Good: Newtype wrap is ideally transparent for compiler and does not change performance

Chapter 83

Good: Instances of types/type classes must go with code you write

Chapter 84

Good: Functions can be abstracted as arguments

Chapter 85

Good: Infix operators can be bind to arguments

Chapter 86

Good: Arbitrary

Product types can be tested as a product of random generators. Sum types require to implement generators with separate constructors, and picking one of them, use `oneof` or `frequency` to pick generators.

Chapter 87

Good: Principle of Separation of concerns

Chapter 88

Good: Function composition

In Haskell inline composition requires:

```
h.g.f $ i
```

Function application has a higher priority than composition. That is why parentheses over argument are needed. This precedence allows idiomatically compose partially applied functions.

But it is a way better then:

```
h (g (f i))
```

Chapter 89

Good: Point-free

Use Tacit very carefully - it hides types and harder to change code where it is used. Use just enough Tacit to communicate a bit better. Mostly only partial point-free communicates better.

89.1 Good: Point-free is great in multi-dimentions

BigData and OLAP analysis.

Chapter 90

Good: Functor application

Function application on n levels beneath:

```
(fmap . fmap) function twoLevelStructure
```

How `fmap . fmap` typechecks:

```
(.)         :: (b -> c) -> (a -> b) -> a -> c
fmap        :: Functor f => (m -> n) -> f m -> f n
fmap        :: Functor g => (x -> y) -> g x -> g y

fmap . fmap :: (Functor f, Functor g)
              => ((g x -> g y) -> f . g x -> f . g y)
              -> ((  x ->   y) ->     g x  ->    g y)
              -> (   x ->   y) -> f . g x -> f . g y
fmap . fmap ::   (x    ->   y) -> f . g x -> f . g y
```

Chapter 91

Good: Parameter order

In functions parameter order is important. It is best to use first the most reusable parameters. And as last one the one that can be the most variable, that is important to chain.

Chapter 92

Good: Applicative monoid

There can be more than one valid Monoid for a data type. && There can be more than one valid Applicative instance for a data type. -> There can be differnt Applicatives with different Monoid implementations.

Chapter 93

Good: Creative process

93.1 Pick phylosophy principles one to three the more - the harder the implementation

93.2 Draw the most blurred representation

93.3 Deduce abstractions and write remotely what they are

93.4 Model of computation

93.4.1 Model the domain

93.4.2 Model the types

93.4.3 Think how to write computations

93.5 Create

Chapter 94

Good: About operators (<$) (**>) (<*) (>>)

Where character is not present - discard the according processing of a parameter. `(>> )` is an exception, it does the reverse. ignores the first parameter, in fact `>> \equiv *>`.

= *>= does the proper action: does calculation, but ignores the value from the first argument.

Chapter 95

Good: About functions like {mapM, sequence}_

Trailing _ means ignoring the result.

Chapter 96

Good: Guideliles

96.1 Wiki.haskell

96.1.1 Documentation

96.1.1.1 Comments write in application terms, not technical.

96.1.1.2 Tell what code needs to do not how it does.

96.1.2 Haddoc

96.1.2.1 Put haddock comments to ever exposed data type and function.

96.1.2.2 Haddock header

```
{- |
Module      :  <File name or $Header$ to be replaced automatically>
Description :  <optional short text displayed on contents page>
Copyright   :  (c) <Authors or Affiliations>
License     :  <license>

Maintainer  :  <email>
Stability   :  unstable | experimental | provisional | stable | frozen
Portability :  portable | non-portable (<reason>)

<module description starting at first column>
-}
```

96.1.3 Code

96.1.3.1 Try to stay closer to portable (Haskell98) code

96.1.3.2 Try make lines no longer 80 chars

96.1.3.3 Last char in file should be newline

96.1.3.4 Symbolic infix identifiers is only library writer right

96.1.3.5 Every function does one thing.

Chapter 97

Good: Use Typed holes to progress the code

Typed holes help build code in complex situations.

Chapter 98

Good: Haskell allows infinite terms but not infinite types

That is why infinite types throw infinite type error.

Chapter 99

Good: Use type sysnonims to differ the information

Even if there is types - define type synonims. They are free. That distinction with synonims, would allow TypeSynonymInstances, which would allow to create a diffrent type class instances and behaviour for different information.

Chapter 100

Good: Use `Control.Monad.Except` instead of `Control.Monad.Error`

Chapter 101

Good: Monad OR Applicative

101.0.1 Start writing monad using 'return', 'ap', 'liftM', 'liftM2', '»' instead of 'do','»='

If you wrote code and really needed only those - move that code to Applicative.

```
return -> pure
ap -> <*>
liftM -> liftA -> <$>
>> -> *>
```

101.0.2 Basic case when Applicative can be used

Can be rewriten in Applicative:

```
func = do
  a <- f
  b <- g
pure (a, b)
```

Can't be rewritten in Applicative:

```
somethingdoSomething' n = do
a <- f n
b <- g a
pure (a, b)
```

(f n) creates monadic structure, binds ot to *a* wich is consumed then by g.

101.0.3 Applicative block vs Monad block

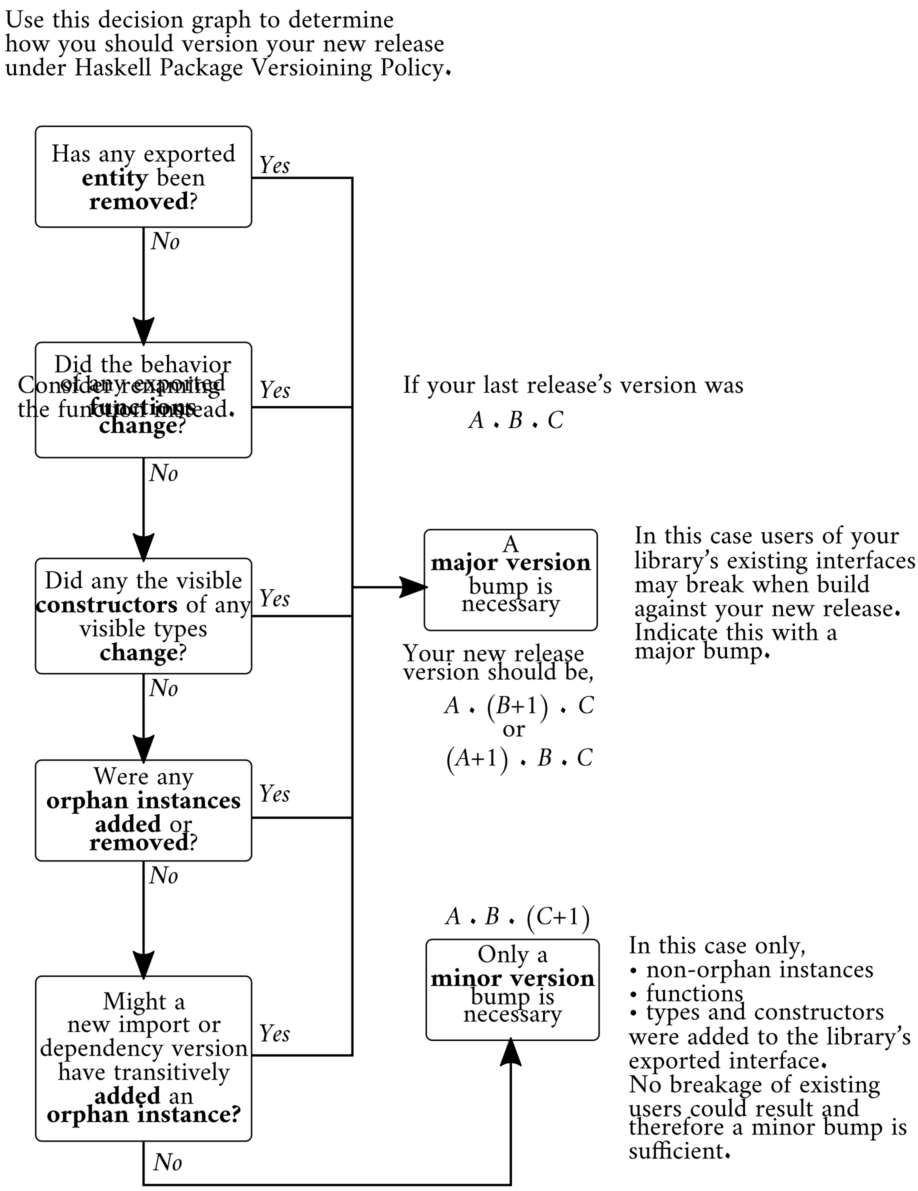
With Type Applicative every condition fails/succseeds independently. It needs a boilerplate data constructor/value pattern matching code to work. And code you can write only for so many cases and types, so boilerplate can not be so flexible as Monad that allows polymorphism. With Type Monad computation can return value that dependent from the previous computation result. So abort or dependent processing can happen.

Chapter 102

Good: Haskell Package Versioning Policy

Version policy and dependency management.

So you are releasing a new package version?



102.1 *

PVP Good: PVP

Chapter 103

Good: Linear type

Linear types are great to control/minimize resource usage.

Chapter 104

Good: Exception vs Error

Many languages and Haskell have it all mixup. Here is table showing what belongs to one or other in standard libraries:

Exception	Prelude.catch, Control.Exception.catch, Control.Exception.try, IOError, Control.Monad.Error
Error	error, assert, Control.Exception.catch, Debug.Trace.trace

Chapter 105

Good: Let vs. Where

`let ... in ...` is a separate expression. In contrast, `where` is bound to a surrounding syntactic construct (namespace).

Chapter 106

Good: RankNTypes

Can powerfully synergyze with ScopedTypeVariables.

Chapter 107

Good: Orphan instance

Practice to address orphan instances:

Does type class or type defined by you:

Type class	Type	Recommendation
	✓	{Type, instance} in the same module
✓		{Type class & instance} in the same module
		{Define newtype wrap, its instances} in the same module

Chapter 108

Good: Smart constructor

Only proper smart constructors should be exported. Do not export data type constructor, only a type.

Chapter 109

Good: Thin category

In * all morphisms are epimorphisms and monomorphisms.

Chapter 110

Good: Recursion

Writing/thinking about recursion:

1. Find the base cases, om imput of which the answer can be provided right away. There is mosly one base case, but sometimes there can be several of them. Typical base cases are: zero, the empty list, the empty tree, null, etc.
2. Do inductive case. The recursive invocation. The argument of a recursive call needs to be smaller then the current argument. So it would be gradually closer to the base case. The idea is that processes eventually hits the base case.

Simple functional application is used in the recursion. Assume that the functions would return the right result.

Chapter 111

Good: Monoid

<>: Sets - union. Maps - left-biased union. Number - `Sum`, `Product` form separate monoid categories.

Chapter 112

Good: Free monad

The main case of usage of Free monads in Haskell:

Start implementation of the monad from a Free monad, drafting the base monadic operations, then add custom operations.

Gradually build on top of Free monad and try to find homomorphisms from monad to objects, and if only objects are needed - get rid of the free monad.

Chapter 113

Good: Use mostly where clauses

Chapter 114

Good: Where clause is in a scope with function parameters

Chapter 115

Good: Strong preference towards pattern matching over {head, tail, etc.} functions

`head` and `tail` and alike functions are often partial (unsafe) funcitons.

Chapter 116

Good: Patternmatching is possible on monadic bind in do

Example:

```
instance (Monad m) => Functor (StateT s m) where
  fmap f m = StateT $ \s -> do
    (x, s') <- runStateT m s  -- Here is a pattern matching bind
    return (f x, s')
```

Chapter 117

Good: Applicative vs Monad

Giving not `Monad` but `Applicative` requirement allows parralel computation, but if there should be a chaining of the intemidiate state - it must be monadic.

Chapter 118

Good: StateT, ReaderT, WriterT

Reader trait: `(r ->)`.

Writer trait: `(a, w)`.

State trait is combination of both:

```
newtype StateT s m a =
  StateT  { runStateT  :: s -> m (a, s) }

newtype ReaderT r m a =
  ReaderT { runReaderT :: r -> m   a    }

newtype WriterT w m a =
  WriterT { runWriterT ::      m (a, w) }
```

State trait fully replaces `writer`.

Chapter 119

Good: Working with MonadTrans and lift

From the `lift . pure = pure` follows that `MonadTrans` type can have a `pure` defined with `lift`.

Stacking of `MonadTrans` monads can result in a lot of chained `lift` and unwraps. There is many ways to cope with that but the most robust and common is to abstract representation with `newtype` on the `Monad` stack. This can reduce caining or remove the manual lifting withing the Monad. For perfect combination for contributors to be able to extend the code - keep the `Internal` module that has a raw representation.

Chapter 120

Good: Don't mix Where and Let

`let` and `where` create a recursive set of definitions with can explode, don't mix them togather in code.

Chapter 121

Good: Where vs. Let

Let is self-recursive lazy pattern. It is checked and errors only at execution time. Binds only inside expression it is binded to.

Where is a part of definition, scoped over definition implemetations and guards, not self-recursive.

Chapter 122

Good: The proper nature algorithm that models behaviour of many objects is computation heavy

God does not care about our mathematical difficulties. He integrates empirically.

One who is found of mathematical meaning loves to apply it. But if we implement the ”real” algorithms behind nature processes, we face the need to go through the computations of laws of all particles.

Computation of nature is always a middle way between ideal theory behaviour and computation simplification.

Chapter 123

Good: In Haskell parameters bound by lambda declaration instantiate to only one concrete type

Because of let-bound polymorphism:

This is illegal in Haskell:

```haskell
foo :: (Int, Char)
foo = (\f -> (f 1, f 'a')) id
```

Lambda-bound function (i.e., one passed as argument to another function) cannot be instantiated in two different ways, if there is a let-bound polymorphism.

Chapter 124

Good: Instance is a good structure to drew a type line

Instances for data type can differentiate by constraints & types of arguments. So instance can preserve type boundary, and data type declaration can stay very polymorphic. If the need to extend the type boundaries arrive - the instances may extend, or new instances are created, while used data type still the same and unchanged.

Chapter 125

Good: MTL vs. Transformers

Default ot `mtl`.

`Transformers` is Haskell-98, doesn't have funcitonal dependencies, lacks the monad classes, has manual lift of operations to the composite monad.

`MTL` extends `trasformers`, providing more instances, features and possibilities, may include alternative packages features as `mtl-tf`.

Part VI

Bad code

Chapter 126

Bad pragma

126.1 Bad: Dangerous LANGUAGE pragma option

- DatatypeContexts
- OverlappingInstances
- IncoherentInstances
- ImpredicativeTypes
- AllowAmbigiousTypes
- UndecidableInstances - often

Part VII

Useful functions to remember

Chapter 127

Prelude

```
enumFromTo
enumFromThenTo
reverse
show :: Show a => a -> String
flip
sequence - Evaluate each monadic action in the structure from left to right, and collect the resu
:sprint - show variables to see what has been evaluated already.
minBound - smaller bound
maxBound - larger bound
cycle :: [a] -> [a] - indefinitely cycle s list
repeat - indefinit lis from value
elemIndex e l - return first index, returns Maybe
fromMaybe (default if Nothing) e ::Maybe a -> a
lookup :: Eq a => a -> [(a, b)] -> Maybe b
```

127.1 Ord

compare

127.2 Calc

div - always makes rounding down, to infinity divMod - returns a tuple containing the result of integral division and modulo

127.3 List operations

```
concat - [ [a] ] -> [a]
elem x xs - is element a part of a list
zip :: [a] -> [b] -> [(a, b)] - zips two lists together. Zip stops when one list runs out.
zipWith :: (a -> b -> c) -> [a] -> [b] -> [c] - do the action on corresponding elements of list a
```

Chapter 128

Data.List

```
intersperse :: a -> [a] -> [a]  -  gets the value and incerts it between values in array
nub - remove duplicates from the list
```

Chapter 129

Data.Char

```
ord (Char -> Int)
chr (Int -> Char)
isUpper (Char -> Bool)
toUpper (Char -> Char)
```

Chapter 130

QuickCheck

```haskell
quickCheck :: Testable prop => prop -> IO ()
```

```haskell
quickCheck . verbose - run verbose mode
```

Part VIII

Tools

Chapter 131

ghc-pkg

List installed packages:

```
ghc-pkg list
```

Chapter 132

Integration of NixOS/Nix with Haskell IDE Engine (HIE) and Emacs (Spacemacs)

132.1 1. Install the Cachix

Upstream doc: https://github.com/cachix/cachix

132.2 2. Installation of HIE

Upstream doc: https://github.com/infinisil/all-hies/#cached-builds

132.2.1 2.1. Provide cached builds

```
cachix use all-hies
```

132.2.2 2.2.a. Installation on NixOS distribution:

```
{ config, pkgs, ... }:

let

  all-hies = import (fetchTarball "https://github.com/infinisil/all-hies/tarball/master") {};

in {
  environment.systemPackages = with pkgs; [

    (all-hies.selection { selector = p: { inherit (p) ghc865 ghc864; }; })

  ];
}
```

Insert your GHC versions.

Switch to new configuration:

```
sudo -i nixos-rebuild switch
```

132.2.3 2.2.b. Installation with Nix package manager:

```
nix-env -iA selection --arg selector 'p: { inherit (p) ghc865 ghc864; }' -f 'https://github.com/i
```

Insert your GHC versions.

132.3 3. Emacs (Spacemacs) configuration:

```
  dotspacemacs-configuration-layers
  '(

    auto-completion

    (lsp :variables
         default-nix-wrapper (lambda (args)
                               (append
                                (append (list "nix-shell" "-I" "." "--command" )
                                        (list (mapconcat 'identity args " "))
                                        )
                                (list (nix-current-sandbox))
                                )
                               )

         lsp-haskell-process-wrapper-function default-nix-wrapper
         )

    (haskell :variables
             haskell-enable-hindent t
             haskell-completion-backend 'lsp
             haskell-process-type 'cabal-new-repl
             )

  )

   dotspacemacs-additional-packages '(
                                      direnv
                                      nix-sandbox
                                      )

(defun dotspacemacs/user-config ()

  (add-hook 'haskell-mode-hook 'direnv-update-environment) ;; If direnv configured

  )
```

Where:

`auto-complettion` configures YASnippet.

`nix-sandbox` (https://github.com/travisbhartwell/nix-emacs) has a great helper functions. Using `nix-current-sandbox` function in `default-nix-wrapper` that used to properly configure `lsp-haskell-process-wrapper-function`.

Configuration of the `lsp-haskell-process-wrapper-function default-nix-wrapper` is a key for HIE to work in `nix-shell`

Inside `nix-shell` the `haskell-process-type 'cabal-new-repl` is required.

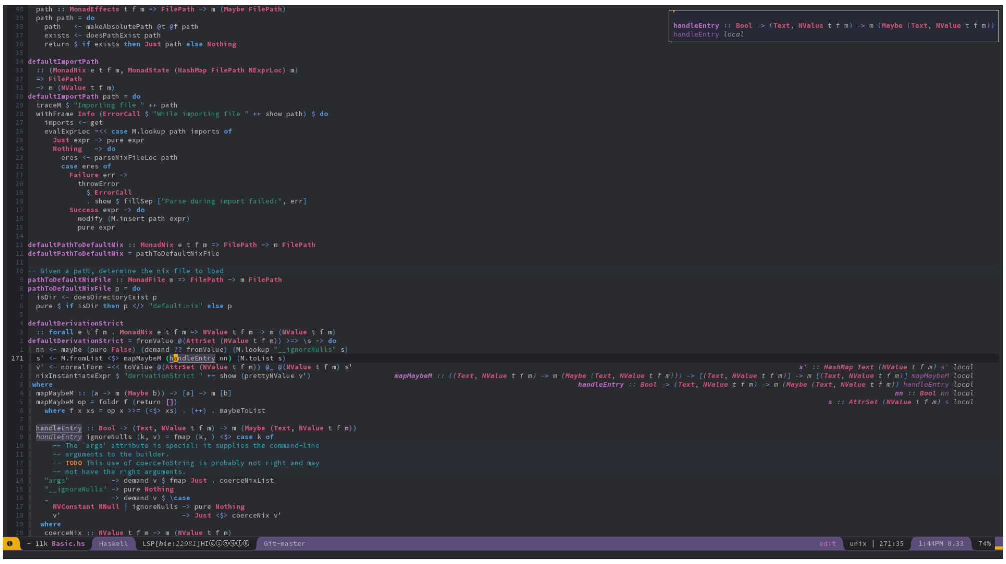
Configuration was reassembled from: `https://github.com/emacs-lsp/lsp-haskell/blob/8f2dbb6e827b1adce6360c56 lsp-haskell.el#L57` *& its authors config*: [[`https://github.com/sevanspowell/dotfiles/blob/master`.spacemacs]]/

Refresh Emasc.

132.4 4. Open the Haskell file from a project

Open system monitor, observe the process of environment establishing, packages loading & compiling.

132.5 5. Be pleased writing code



Now, the powers of the Haskell, Nix & Emacs combined. It's fully in your hands now. Be cautious - you can change the world.

132.6 6. (optional) Debugging

1. If recieving sort-of:

```
readCreateProcess : cabal-helper-wrapper failure
```

HIE tries to run `cabal` operations like on the non-Nix system. So it is a problem with detection of `nix-shell` environment, running inside it.

1. If HIE keeps getting ready, failing & restarting - check that the projects `ghc --version` is declared in your `all-hie` NixOS configuration.

Chapter 133

Debugger

Provides:

- set a breakpoints
- observe step-by-step evaluation
- tracing mode

Breakpoints

```
:break 2
  :show breaks
  :delete 0
:continue
```

Step-by-step

```
:step main
```

List information at the breakpoint

```
:list
```

What been evaluated already

```
:sprint name
```

Chapter 134

GHCID

Commands to run the compile/check loop:

`cabal` > `3.0` command:

```
ghcid --command='cabal v2-repl --repl-options=-fno-code --repl-options=-fno-break-on-exception --
```

`cabal` < `3.0` command:

```
ghcid --command='cabal new-repl --ghc-options=-fno-code --ghc-options=-fno-break-on-exception --g
```

`nix-shell cabal` > `3.0` command:

```
nix-shell --command 'ghcid --command="cabal v2-repl --repl-options=-fno-code --repl-options=-fno-
```

`nix-shell cabal` < `3.0` command:

```
nix-shell --command 'ghcid --command="cabal new-repl --ghc-options=-fno-code --ghc-options=-fno-b
```

Chapter 135

Continuous integration platrorms (CIs) for Open Source Haskell projets

Since Open Source projects mostly use free tiers of CIs, and different CIs have different features - there is a constant flux of how to construct the best possible integration pipeline for Haskell projects.

The current state of affairs is best put in this quote:

> Probably the biggest constraint is whether or not CI needs to test Windows or OS X, since build machines for those are harder to come by. We currently use AppVeyor for Windows builds and Travis for OS X builds since they are free. For Linux you can basically use any CI provider, but in this case I pay for a Linode VM which I use to host all Dhall-related infrastructure (i.e. all of the *.dhall-lang.org domains), so I reuse that to host Hydra for Nix-related CI so that I can use more parallelism and more efficient caching to test a wider range of GHC versions on a budget.
>
> For testing OS X and Windows platforms we use stack. The main reason we don't use Nix for either platform is that Nix only supports building release binaries on Linux (and even then it's still experimental).
>
> So the basic summary I can give is:
>
> For testing everything other than cross-platform support: Nix + Linux is best in my opinion
>
> … because you get much more control and intelligent build caching, which is usually where most CI solutions fall short
>
> For cross-platform support: stack + whatever CI provider provides free builds for that platform
>
> Also, if you ever can pay for your own NixOS VM and you want to reuse the setup I built, you can find the NixOS configuration for dhall-lang.org here:
>
> https://github.com/dhall-lang/dhall-lang/tree/master/nixops

Part IX

Libs

Chapter 136

Exceptions

136.1 Exceptions - optionally pure extensible exceptions that are compatible with the mtl

136.2 Safe-exceptions - safe, simple API equivalent to the underlying implementation in terms of power, encourages best practices minimizing the chances of getting the exception handling wrong.

136.3 Enclosed-exceptions - capture exceptions from the enclosed computation, while reacting to asynchronous exceptions aimed at the calling thread.

Chapter 137

Memory management

137.1 membrain - type-safe memory units

Chapter 138

Parsers - megaparsec

Chapter 139

CLIs - optparse-applicative

Chapter 140

HTML - Lucid

Chapter 141

Web applications - Servant

Chapter 142

IO libraries

142.1 Conduit - practical, monolythic, guarantees termination return

142.2 Pipes + Pipes Parse - modular, more primitive, theoretically driven

Chapter 143

JSON - aeson

Chapter 144

Backpack

On 1-st compilation - * analyzes the abstract signatures without loading side modules, doing the type check with assumption that modules provide right type signatures, the process does not emitt any binary code and stores the intermediate code in a special form that allows flexibily connect modules provided. Which allows later to compile project with particular instanciations of the modules. Major work of this process being done by internal Cabal * support and * system that modifies the intermediate code to fit the module.

Part X

Drafts

Chapter 145

Exception handling

Ideal model:

- ☒ Exception must include all context information that may be useful.
- ☒ Store information in a form for further probable deeper automatic diagnostic.
- ☒ Sensitive data/dummies for it - can be useful during development.
- ☒ Sensitive data should be stripped from a program logging & exceptions.
- ☒ Exception system should be extendable, data storage & representation should be easily extendable.
- ☒ Exception system should allow easy exaustive checking of errors, since the different errors can happen.
- ☒ Exception system should be automatically well-documented and transparent.
- ☒ Exception system should have controllable breaking changes downstream.
- ☒ Exception system should allow complex composite (sets) exceptions.
- ☒ Exception system should be lightweight on the type signatures of other functions.
- ☒ Exception system should automate the collection of context for a exception.
- ☒ Exception system should have properties and according functions for particular types of errors.

`String` is simple and convinient to throw exception, but really a mistake because it the most cumbersome choise:

- ☒ Any Exception instance can be converted to a `String` with either `show` or `displayException`.
- ☐ Does not include key debugging information in the error message.
- ☐ Does not allow developer to access/manage the Exception information.
- ☐ Exception messages need to be constructed ahead of time, it can not be internationalized, converted to some data/file format.
- ☐ Exception can have a sensitive information that can be useful for developer during work, but should not be logged/shown to end-user. Stripping it from `Strings` in the changing project is a hard task.
- ☐ Impossible to rely on this representation for further/deeper inspection.
- ☐ Impossible to have exhaustive checking - no knowledge no check, no warning if some cases are not handled.

Universal exception type:

- ☒ Able to inspect every possible error case with pattern match.

- ☒ Self-documenting. Shows the hierarchical system of all exceptions.
- ☒ Transparent. Ability to discern in current situation what exceptions can happen
- ☐ New exception constructor causes breaking change to downstream.
- ☐ Wrongly implies completeness. Untreated Errors can happen, different exception can arrive from the outside code.

Sum type must be separate, and product type structure over it. Separate exception type of

Individual exception types:

- ☒ Writing & seing & working with exactly what will go wrong because there is only one possible error for this type of exception. Pattern match happens only onconditions, constructors that should happen.
- ☒ Knowledge what exectly goes wrong allows wide usage of Either.
- ☐ It is hard to handle complex exceptions in the unitary system. Real wrorld can return not a particular case, but a set of cases {object not found, path is unreachable, access is denied}.
- ☐ Type signatures grow, and even can become complex, since every case of exception has its own type.
- ☐ Impure `throw` that users can/should use for your code must account for all your exception types.

Abstract exception type:

Exception type entirely opague and inspectable only by accessor functions.

- ☒ Updating the internals without breaking the API
- ☒ Semi-automates the context of exception with passing it to accessors.
- ☒ Predicates can be applied to more than one constructor. Which are properties that allows to make complex exceptions much easier to handle.
- ☐ Not self-documenting.
- ☐ Possible options by design are hidden from the downstream, documentation must be kept.
- ☐ When you change the exception handling/throwing errors it does not shows to the downstream.

Composit approach: Provide the set of constructors and also a set of predicates and set of accessors. Use pattern synonyms to provide a documented accessor set without exposing internal data type.

In GHC 8.8 the change was made:

> The fail method of Monad has been removed in favor of the method of the same name in the MonadFail class.
>
> MonadFail(..) is now exported from the Prelude and Control.Monad modules. The MonadFailDesugaring language extension is now deprecated, as its effects are always enabled.

So:

```
import           Control.Monad.Fail
...
class MonadFail m => MonadFile m
...
-- use error instead of fail
Nothing     -> error ("Message " <> show x)
```

Chapter 146

Constraints

Very strong Haskell type system makes possible to work with code from the top down, an axiomatic semantics approach, from constraints into types.

- Helps to form the type level code (aka join points of the code).
- Uses the piling up of constraints/types information. At some point pick and satisfy constraints, can be done one at a time.
- Provides hints through type level formulation for term level calculations, does not formulate the term level.
- Tedious method (a lot of boilerplate and rewriting it) but pretty simple and relaxing.
- Set of constraints.
- When it is needed or convenient, single constraint gets a little more realistically concrete/abstracted.

Main type detail annotation thread can happen in `main` or special wrapper function, localization is inside functions.

1. Rest of constraints set shifts to source type.

3.a. For the class handled or known how to handle - writte a base case instance description.

```
instance (Monad m) => MonadReader r (ReaderT r m)
```

3.b. For others write recursive instance descriptions:

All other unsolved constraints move into the source polymorphic variable.

```
instance (MonadError e m) => MonadError e (ReaderT r m)
instance (MonadState s m) => MonadState s (ReaderT r m)
```

1. Repeat from 1 until considered done.
2. Code condensed into terse form.

`MonadError` constraints is `IOException`, not for the `String`. `IOException` vs `String`.

Reverse pluck `MonadReader` constraint with `runReader` on the object.

`MonadState` - `StateT`

Chapter 147

Monad transformers and their type classes

Chapter 148

Layering monad transformers

Different layering of the same monad transformers is functionality is the same, but the form is different. Surrounding handling functions would need to be different.

Chapter 149

Hoogle

149.1 Search

Text search (case insensitive):

- `a`
- `map`
- `con map`

Type search:

- `:: a`
- `:: a -> a`

Text & type:

=id a -> a=

149.2 Scope

149.2.1 Default

Scope is Haskell Platform (and Haskell keywords).

All Hackage packages are available to search with:

149.2.2 Hierarchical module name system (from big letter):

- `fold +Data.Map` finds results in the `Data.Map` module
- `file -System` excludes results from modules such as `System.IO`, `System.FilePath.Windows` and `Distribution.System`

149.2.3 Packages (lower case):

- `mode +platform`
- `mode +cmdargs` (only)
- `mode +platform +cmdargs`
- `file -base` (Haskell Platform, excluding the ”base” package)

Chapter 150

ST-Trick monad

ST is like a lexical scope, where all the variables/state disappear when the function returns https://wiki.haskell.ohttps://www.schoolofhaskell.com/school/to-infinity-and-beyond/older-but-still-interesti deamortized-strg/Monad/ST https://dev.to/jvanbruegge/what-the-heck-is-polymorphism-nmh

150.1 *

ST-Trick

Chapter 151

Either

Allows to separate and preserve information about happened, ex. error handling.

151.1 *

Either data type

Chapter 152

Inverse

1. Inverse function
2. In logic: $P \rightarrow Q \Rightarrow \neg P \rightarrow \neg Q$, & same for category duality.
3. For operation: element that allows reversing operation, having an element that with the dual produces the identity element.
4. See Inversion.

Chapter 153

Inversion

1. Is a permutation where two elements are out of order.
2. See Inverse

Chapter 154

Inverse function

$f_{x \to y} \circ (f_{x \to y})^{-1} = 1_x$

* $\iff$ function is bijective. Otherwise - partial inverse

Chapter 155

Inverse morphism

For $f : x \to y$: $\exists g \ : \ g \circ f = 1^x$ - g is left inverse of f. $\exists g \ : \ f \circ g = 1^y$ - g is right inverse of f.

Chapter 156

Partial inverse

* when function is now bijective. When bijective see inverse function.

Chapter 157

PatternSynonyms

Enables pattern synonym declaration, which always begins with the `pattern` word. Allows to abstract-away the structures of pattern matching.

157.1 *

Pattern synonym Pattern synonyms

Chapter 158

GHC debug keys

158.1 -ddump-ds

Dump desugarer output.

158.1.1 *

Desugar GHC desugar

Chapter 159

GHC optimize keys

159.1 -foptimal-applicative-do

$O(n^3)$ Always finds optimal reduction into <*> for ApplicativeDo do notation.

Chapter 160

Computational trinitarianism

Taken from: https://ncatlab.org/nlab/show/computational+trinitarianism

Under the statements:

- propositions as types
- programs as proofs
- relation between type theory and category theory

the following notions are equivalent:

== proposition proof (Logic)

== generalized element of an object (Category theory)

== typed program with output (Type theory & Computer science)

160.1 *

Trinitarism

Table 160.1: Computational trinitarianism

Logic	Category theory	Type theory
true	terminal object/(-2)-truncated object	h-level 0-type/unit type
false	initial object	empty type
proposition	(-1)-truncated object	h-proposition, mere proposition
proof	generalized element	program
cut rule	composition of classifying morphisms / pullback of display maps	substitution
cut elimination for implication	counit for hom-tensor adjunction	beta reduction
introduction rule for implication	unit for hom-tensor adjunction	eta conversion
logical conjunction	product	product type
disjunction	coproduct ((-1)-truncation of)	sum type (bracket type of)
implication	internal hom	function type
negation	internal hom into initial object	function type into empty type
universal quantification	dependent product	dependent product type
existential quantification	dependent sum ((-1)-truncation of)	dependent sum type (bracket type of)
equivalence	path space object	identity type
equivalence class	quotient	quotient type
induction	colimit	inductive type, W-type, M-type
higher induction	higher colimit	higher inductive type
completely presented set	discrete object/0-truncated object	h-level 2-type/preset/h-set
set	internal 0-groupoid	Bishop set/setoid
universe	object classifier	type of types
modality	closure operator, (idemponent) monad	modal type theory, monad (in computer science)
linear logic	(symmetric, closed) monoidal category	linear type theory/quantum computation
proof net	string diagram	quantum circuit
(absence of) contraction rule	(absence of) diagonal	no-cloning theorem
	synthetic mathematics	domain specific embedded programming language

Chapter 161

Techniques functional programming deals with the state

161.1 Minimizing

Do not rely on state, try not to change the state. Use it only when it is very necessary.

161.2 Concentrating

Concentrate the state in one place.

161.3 Deferring

Defer state to the last step of the program, or to external system.

Chapter 162

Monadic Error handling

```
(>>=) :: m a -> (a -> m b) -> m b -- λA.E ⊠ A - computes and drops if error value happens.
catch :: c a -> (e -> c a) -> c a -- λE.E ⊠ A - handles "errors" as "normal" values and stops whe
```

Chapter 163

Functions

Total function uses domain fully, but takes only part of the codomain. Function allows to collapse domain values into codomain value. Meaning the function allows to loose the information. So total function is a computation that looses the information or into bigger codomains. That is why the function has a directionality, and inverse total process is partially possible.

Directionality and invertability are terms.

Chapter 164

Void

Emptiness.

Can not be grasped, touched.

A logically uninhabited data type.

(Since basis of logic is tautologically True and Void value can not be addressed - there is a logical paradox with the Void).

Is an object includded into the Hask category, since:

```
:t (id :: Void -> Void)
(id :: Void -> Void) :: Void -> Void
```

`id` for it exists.

Type system corresponds to constructive logic and not to the classical logic. Classical logic answers the question "Is this actually true". Constuctive (Intuitionistic) logic answers the question "Is this provable".

Also has functions:

```
-- Represents logical principle of explosion: from falsehood, anything follows.
absurd :: Void -> a

-- If Functor holds only Void - it holds no values.
vacuous :: Functor f => f Void -> f a

-- If Monad holds only Void - it holds no values.
vacuousM :: Monad m => m Void -> m a
```

Design pattern: use polymorphic data types and Void to get rid of possibilities when you need to.

164.1 *

Nothing, Haskell expressions can't return Void.

Also see: Maybe.

Chapter 165

Constructive proof

Method of proof that demonstrates the existence of a mathematical object by creating or providing a method for creating the object.

Chapter 166

Intuitionistic logic

Proposition considered `True` due to direct evidence of existence through constructive proof using Curry-Howard isomorphism.

* does not include classic logic fundamental axioms of the excluded middle and double negation elimination. Hense * is weaker then classical logic. Classical logic includes *, all theorems of * are also in classical logic.

166.1 *

Constructive logic

Chapter 167

Principle of explosion

From asserted statement that contains contradiction - anything can be proven. Ancient principle of logic. Both in classical & intuitionistic logic.

167.1 *

Ex falso quodlibet Ex falso sequitur quodlibet EFG Ex contradictione quodlibet Ex contradictione sequitur quodlibet ECQ Deductive explosion Pseudo-Scotus

Chapter 168

Universal property

A property of some construction which boils down to (is manifestly equivalent to) the property that an associated object is a universal initial object of some (auxiliary) category.

Chapter 169

Yoneda lemma

Allows the embedding of any category into a category of functors (contravariant set-valued functors) defined on that category. It also clarifies how the embedded category, of representable functors and their natural transformations, relates to the other objects in the larger functor category.

The Yoneda lemma suggests that instead of studying the (locally small) category C $\{\{\{C\}\}\}\mathcal{C}, oneshouldstudythecategoryofal$

Chapter 170

Monoidal category, functoriality of ADTs, Profunctors

Category equipped with tensor product.

<>

wich is a functor for *.

Set category can be monoidal under both product (having terminal object) or coproduct (having initial object) operations, if according operation exist for all objects.

Any one-object category is *.

$(a, ()) \sim a$ up to unique isomorphism, which is called Lax monoidal functor.

Product and coproduct are functorial, so, since: Algebraic data type construction can use:

- Type constructor
- Data constructor
- Const functor
- Identity functor
- Product
- Coproduct

Any algebraic data type is functorial.

Chapter 171

Const functor

Maps all objects of source category into one (fixed) object of target category, and all morphisms to identity morphism of that fixed object.

```
instance Functor (Const c)
 where
  fmap :: (a -> b) -> Const c a -> Const c b
  fmap _ (Const c) = Const c
```

In Category theory denoted:

Δ

Last type parameter that bears the target type of lifted function (`b`) and is a proxy type.

Analogy: the container that allways has an object attached to it, and everything that is put inside - changes the container type accordingly, and dissapears.

Chapter 172

Arrow in Haskell

```
(->) a b = a -> b
```

Functorial in the last argument & called Reader functor.

```
newtype Reader c a = Reader (c -> a)
```

```
  fmap = ( . )
```

Chapter 173

Contravariant functor

```
fmap :: (a -> b) -> Op c a -> Op c b
                 (a -> c) -> (b -> c)
```

$$\begin{tikzcd} a \arrow[r] \arrow[dr] & b \arrow[d, dashed] \\ & c \end{tikzcd}$$

$(a \to b)^C = (a \leftarrow b)^{C^{op}}$

```
class Contravariant f
 where
  contramap :: (b -> a) -> (f a -> f b)
```

$$\begin{tikzcd} a \arrow[r] \arrow[dr] & b \arrow[d, dashed, "contravariant"] \\ & c \end{tikzcd}$$

If arrows does not commute Contravatiant funtor anyway allows to construct transformation between these such arrows to other arrow.

Chapter 174

Profunctor

```
(->) a b
```

$C^{op} \times C \to C$

It is called profunctor.

```
dimap :: (a' -> a) -> (b -> b') -> p a b -> p a' b'
```

So, profunctor in case of arrow:

$$\begin{tikzcd} a \arrow[d, "h"] & a' \arrow[l, "f"'] \arrow[d, dashed, "profunctor"] \\ b \arrow[r, "g"] & b' \end{tikzcd}$$

```
dimap :: (a' -> a) -> (b -> b') -> p a b -> p a' b'
dimap ::    f            g      -> (a -> b) -> (a' -> b')
dimap ::    f            g      ->     h     -> (a' -> b')
dimap = g . h . f
```

It is contravariant functor in the first argument, and covariant functor in the second argument.

```
dimap id <==> fmap
(flip dimap) id <==> contramap
```

Chapter 175

Coerce

Operates under condition that source and target types have same representation. Same representation means they are type aliases, or it the compiler can infer that they have the same representation. Directly shares the values from the source type to the target type. Conversion is free, there is no run-time computations.

The function implementing the transition:

```
coerce :: Coercible a b => a -> b
```

Type class implementing the instances for transitions:

```
class a ~R# b => Coercible (a :: k0) (b :: k0)
```

When compiler detects types have same structure, type class instances coerse implementation for this pairs of types. This type class does not have regular instances; instead they are created on-the-fly during type-checking. Trying to manually declare an instance of Coercible is an error.

175.1 *

Coercible

Part XI

Reference

Chapter 176

Functor-Applicative-Monad Proposal

Well known event in Haskell history: https://github.com/quchen/articles/blob/master/applicative_monad.md.

Math justice was restored with a RETroactive CONtinuity. Invented in computer science term Applicative (lax monoidal functor) become a superclass of Monad.

& that is why:

- `return = pure`
- `ap = <*>`
- `>> = *>`
- `liftM = liftA = fmap`
- `liftM* = liftA*`

Also, a side-kick - Alternative became a superclass of MonadPlus. Hense:

- `mzero = empty`
- `mplus = (<|>)`

Work of unification continues under: https://gitlab.haskell.org/ghc/ghc/wikis/proposal/monad-of-no-return

176.1 *

Applicative-Monad proposal AMP

Chapter 177

Haskell-98

177.1 Old instance termination rules

1. $\forall$ class constraint $(C\ t1\ ..\ tn)$: 1.1. type variables have occurances $\leq$ head 1.2. constructors+variables+repetitions < head 1.3. $\neg$ type functions (type func application can expand to arbitrary size)
2. $\forall$ functional dependencies, $\square tvs\square_{left} \to \square tvs\square_{right}$, of the class, every type variable in $S(\square tvs\square_{right})$ must appear in $S(\square tvs\square_{left})$, where S is the substitution mapping each type variable in the class declaration to the corresponding type in the instance head.

Chapter 178

Performance results and comparisons of types & solutions

Haskell performance

Chapter 179

Literature

- GHC Team ”GHC User’s Guide Documentation”: https://downloads.haskell.org/~ghc/latest/docs/users_guide.pdf
- Stephen Diehl & contributors ”What I Wish I Knew When Learning Haskell”: http://dev.stephendiehl.com/hask/tutorial.pdf

Part XII

Giving back

λειτ <- λαός *Laos* the people ουργός <- ἔργο *ergon* work λειτουργία *leitourgia* public work

Moral value of people developed from the community to give back, improving the community.

The life is beautiful. For all humans that make the life have more magic.

This study and work would not be possible without the community: tearchers, mathematicians, Haskellers, scientists, creators, contributors. These sides of people are fascinating.

Special accolades for the guys at Serokell. They were the force that got me inspired & gave resources to seriously learn Haskell and create this pocket guide.