

# Fundamental Haskell notes

Encyclopedcal handbook for learning and undersatanding fundamentals

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```

## **Part I**

# **Introduction**

---

*“Employ your time in improving yourself by other men’s writings so that you shall come easily by what others have labored hard for.” (Socrates by Plato)*

Important notes on Haskell, [category](#) theory & related fields, terms and recommendations.

Book comes in forms:

- [Web book](#)
- [PDF](#)
- [Open in web PDF viewer](#)
- [L<sup>A</sup>T<sub>E</sub>X](#)
- [Source code in Org-mode](#)
- [GitHub](#)
- [GitLab](#)

This book is created using complex Org markup file with a lot of L<sup>A</sup>T<sub>E</sub>X and L<sup>A</sup>T<sub>E</sub>X formulas. Be aware - GitHub & GitLab only partially parse Org into HTML.

To get the full view:

- [Outline navigation](#)
- [L<sup>A</sup>T<sub>E</sub>X formulas](#):

$$\left[ -\frac{\hbar^2}{2m} \nabla^2 + V(\vec{r}, t) \right] \Psi(\vec{r}, t) = i\hbar \frac{\partial}{\partial t} \Psi(\vec{r}, t), \quad \sum_{k,j} \left[ -\frac{\hbar^2}{\sqrt{a}} \frac{\partial}{\partial q^k} \left( \sqrt{a} a^{kj} \frac{\partial}{\partial q^j} \right) + V \right] \Psi + \frac{\hbar}{i} \frac{\partial \Psi}{\partial t} = 0$$

- [Interlinks](#): [Interlinks](#)

, please refer to Web book, PDF, L<sup>A</sup>T<sub>E</sub>X, or use Org-mode capable viewer/editor.

Note about the markup: <<<This is a radio target>>> - is the anchor for dynamic linking.

Users of Emacs can prettify radio targets to be shown as hyper-links with this Emacs snippet:

```
;;; 2019-06-12: NOTE:
;;; Prettify '<<<Radio targets>>>' to be shown as '_Radio_targets_',
;;; when `org-descriptive-links` set.
;;; This is improvement of the code from: Tobias&glmorous:
;;; https://emacs.stackexchange.com/questions/19230/how-to-hide-targets
;;; There exists library created from the sample:
;;; https://github.com/talwrii/org-hide-targets
(defun org-hidden-links-additional-re "\\(<<<\\)[[:print:]]+?\\(>>>\\)"
  "Regular expression that matches strings where the invisible-property
  of the sub-matches 1 and 2 is set to org-link."
  :type '(choice (const :tag "Off" nil) regexp)
  :group 'org-link)
(make-variable-buffer-local 'org-hidden-links-additional-re)

(defun org-activate-hidden-links-additional (limit)
  "Put invisible-property org-link on strings matching
  `org-hide-links-additional-re'."
  (if org-hidden-links-additional-re
      (re-search-forward org-hidden-links-additional-re limit t)
      (goto-char limit)
      nil))
```



---

```
(defun org-hidden-links-hook-function ()
  "Add rule for `org-activate-hidden-links-additional'
  to `org-font-lock-extra-keywords'."
  You can include this function in `org-font-lock-set-keywords-hook'."
  (add-to-list 'org-font-lock-extra-keywords
    '(org-activate-hidden-links-additional
      (1 '(face org-target invisible org-link))
      (2 '(face org-target invisible org-link)))))

(add-hook 'org-font-lock-set-keywords-hook #'org-hidden-links-hook-function)
```

SCHT: and metadata in :properties: - of my org-drill practices, please just run org-drill-strip-all-data.

# **Part II**

## **Definitions**

# Chapter 1

## Algebra

↳ *al-jabr* assemble parts

A system of parts based on given axioms ([properties](#)) and operations on them.

=====

Additional meanings:

1. [Algebra](#) - a [set](#) with its [algebraic structure](#).
2. [Abstract algebra](#) - the study of number systems and operations within them.
3. [Algebra](#) - vector space over a field with a multiplication.

### 1.1 \*

Algebras

### 1.2 Algebraic

Composite from simple parts.

Also: [Algebraic data type](#).

### 1.3 Algebraic structure

\* includes axioms that must be satisfied and operations on the underlying (or "carrier") [set](#).

An underlying [set](#) with \* on top of it also called "an [algebra](#)".

\* include [groups](#), [rings](#), fields, and lattices. More complex [structures](#) can be defined by introducing multiple operations, different underlying [sets](#), or by altering the defining axioms. Examples of more complex \* can be many modules, [algebras](#) and other vector spaces, and any variations that the definition includes.

#### 1.3.1 \*

Algebraic structures

Table 1.1: Algebraic structures

	Closure	Associativity	Identity	Invertability	Commutativity	Distributive
Semigroupoid		✓				
Small Category		✓	✓			
Groupoid		✓	✓	✓		
Magma	✓					
Quasigroup	✓			✓		
Loop	✓		✓	✓		
Semigroup	✓	✓				
Inverse Semigroup	✓	✓		✓		
Monoid	✓	✓	✓			
Group	✓	✓	✓	✓		
Abelian group	✓	✓	✓	✓	✓	
Non-unital ring (rng)	✓ + ×	✓ + ×	✓ +	✓ +	✓ +	✓
Semiring (rig)	✓ + ×	✓ + ×	✓ + ×	✓ ×	✓ +	✓
Ring	✓ + ×	✓ + ×	✓ + ×	✓ + ×	✓ +	✓

### 1.3.2 Fundamental theorem of algebra

Any non-constant single-variable polynomial with complex coefficients has at least one complex root.

From this definition follows property that the field of complex numbers is algebraically closed.

## 1.4 Modular arithmetic

System for integers where numbers wrap around the certain values (single - modulus, plural - moduli).

Example - 12-hour clock.

### 1.4.1 \*

Clock arithmetic

### 1.4.2 Modulus

Special numbers where arithmetic wraps around in modular arithmetic.

#### 1.4.2.1 \*

Moduli - plural.

# Chapter 2

## Category theory

Category  $\mathcal{C}$  consists of the **basis**:

Primitives:

1. **Objects** -  $a^{\mathcal{C}}$ . A **node**. **Object** of some **type**. Often **sets**, than it is **Set category**.
2. **Arrows** -  $(a, b)^{\mathcal{C}}$  (AKA **morphisms** mappings).
3. **Arrow (morphism) composition** - **binary operation**:  $(a, b)^{\mathcal{C}} \circ (b, c)^{\mathcal{C}} \equiv (a, c)^{\mathcal{C}} \mid \forall a, b, c \in \mathcal{C}$  AKA principle of **compositionality** for **arrows**.

**Properties** (or axioms):

1. **Associativity** of **morphisms**:  $h \circ (g \circ f) \equiv (h \circ g) \circ f \mid f_{a \rightarrow b}, g_{b \rightarrow c}, h_{c \rightarrow d}$
2. Every **object** has (two-sided) **identity morphism** (& in fact - exactly one):  $1_x \circ f_{a \rightarrow x} \equiv f_{a \rightarrow x}, g_{x \rightarrow b} \circ 1_x \equiv g_{x \rightarrow b} \mid \forall x \exists 1_x, \forall f_{a \rightarrow x}, \forall g_{x \rightarrow b}$
3. Principle of **compositionality**.

From these axioms, can be proven that there is exactly one **identity morphism** for every **object**.

**Object** and **morphism** are complete **abstractions** for anything. In majority of cases under **object** is a state and **morphism** is a change.

### 2.1 \*

Category Categories

### 2.2 Abelian category

Generalised **category** for homological **algebra** (having a possibility of basic constructions and techniques for it).

**Category** which:

- has a **zero object**,
- has all **binary** biproducts,
- has all **kernel**'s and cokernels,
- (it has all **pullbacks** and pushouts)
- all **monomorphism**'s and **epimorphism**'s are normal.

**Abelian category** is a stable **structure**; for example it is regular and satisfy the snake lemma. The class of **Abelian categories** is **closed** under several categorical constructions.

There is notion of **Abelian monoid** (AKS **Commutative monoid**) and **Abelian group** (**Commutative group**).

Basic examples of  $*$ :

- **category** of Abelian **groups**
- **category** of modules over a **ring**.

$*$  are widely used in **algebra**, **algebraic** geometry, and topology.

$*$  has many constructions like in **categories** of modules:

- kernels
- exact **sequences**
- **commutative** diagrams

$*$  has disadvantage over **category** of modules. **Objects** do not necessarily have elements that can be manipulated directly, so traditional definitions do not work. Methods must be supplied that allow definition and manipulation of **objects** without the use of elements.

### 2.2.1 $*$

Abelian categories

## 2.3 Composition

Axiom of **Category**.

### 2.3.1 $*$

Composable Compositions

## 2.4 Endofunctor category

From the name, in this **Category**:

- **objects** of  $End$  are **Endofunctors**  $E^{C \rightarrow C}$
- **morphisms** are **natural transformations** between **endofunctors**

## 2.5 Functor

$*$  full translation (map) of one **category** into another. Translating **objects** and **morphisms** (as input can take **morphism** or **object**).

$*$  - **forgetful** - discards part of the **structure**.  $*$  - **faithful** - fully preserves all **morphisms** - **injective** on **Hom-sets**.  $*$  - **full** - translation of **morphisms** fully covers all the **morphisms** between according objects in the target category.

For **Functor type class** or **fmap** - see **Power set functor**.

**Functor properties** (axioms):

- $F^{C \rightarrow D}(a) \mid \forall a^C$  - every source **object** is mapped to **object** in target **category**

- $\overrightarrow{(F^{C \rightarrow D}(a), F^{C \rightarrow D}(b))}^D \mid \forall \overrightarrow{(a, b)}^C$  - every source **morphism** is mapped to target **category morphism** between corresponding **objects**
- $F^{C \rightarrow D}(\overrightarrow{g}^C \circ \overrightarrow{f}^C) = F^{C \rightarrow D}(\overrightarrow{g}^C) \circ F^{C \rightarrow D}(\overrightarrow{f}^C) \mid \forall y = \overrightarrow{f}^C(x), \forall \overrightarrow{g}^C(y)$  - **composition** of **morphisms** translates directly (tautologically goes from other two)

These axioms guarantee that **composition** of **functors** can be fused into one **functor** with **composition** of **morphisms**. This **process** called fusion.

In Haskell this axioms have form:

```
fmap id = id
fmap (f . g) = fmap f . fmap g
```

Since  $*$  is 1-1 mapping of initial **objects** - it is a memoizable dictionary with **cardinality** of initial **objects**. Also in **Hask category functors** are obviously **endofunctors**  $\therefore$  they are special **kinds** of containers for the parametric values (AKA **product type**). In Haskell **product type**  $*$  are **endofunctors** from **polymorphic type** into a **functor** wrapper of a **polymorphic type**.

$*$  translates in one direction, and does not provide algorithm of reversing itself or retrieving the parametric value.

### 2.5.1 $*$

Functors Functorial - something that has **functor properties**, and so also is a **functor**.

### 2.5.2 Power set functor

$\mathcal{P}^{S \rightarrow \mathcal{P}(S)}$

$*$  - **functor** from **set**  $S$  to its **power set**  $\mathcal{P}(S)$ .

**Functor type class** in Haskell defines a  $*$  and allows to do **function application** inside **type structure** layers (denoted  $f$  or  $m$ ). **IO** is also such **structure**. **Power set** is unique to the **set**,  $*$  is unique to the **category (data type)**.  $*$  embodies in itself any **endofunctor**. It is easily seen from Haskell definition - that the  $*$  is the **polymorphic** generalization over any **endofunctor** in a **category**. **Application** of a **function** to  $*$  gives a particular **endofunctor** (see **Hask category**).

```
class Functor f where
  fmap :: (a -> b) -> f a -> f b
```

**Functor** instance must be of **kind**  $( * \rightarrow * )$ , so instance for **higher-kinded data type** must be **applied** until this **kind**.

**Composed**  $*$  can **lift functions** through any layers of **structures** that belong to **Functor type class**.

$*$  can be used to filter-out **error** cases (**Nothing** & Left cases) in **Maybe**, **Either** and related **types**.

#### 2.5.2.1 $*$

fmap Functor type class

#### 2.5.2.2 Power set functor properties

**Type** instance of **functor** should abide this **properties**:

##### 2.5.2.2.1 $*$

Functor properties

### 2.5.2.2.2 Power set functor identity property

Functor translates **object** & its **identity morphism** to target **object** & its **identity morphism**.

```
fmap id == id
```

### 2.5.2.2.3 Power set functor composition property

Full transparency of **composition** translation. So **order** of **composition** and translation does not matter, the result is always the same.

```
fmap (f . g) == fmap f . fmap g
```

Including cases: a) translate everything one-by-one and assemble at destination **category**. b) assemble everything in source category and translate in one go once.

Composing in source **category** and translating at once - is a much-much more effective computation (known as "**functor fusion**").

### 2.5.2.3 Lift

```
fmap :: (a -> b) -> (f a -> f b)
```

Functor takes **function**  $a \rightarrow b$  and returns a **function**  $f\ a \rightarrow f\ b$  this is called **lifting** a **function**. Lift does a **function application** through the **data structure**.

#### 2.5.2.3.1 \*

Lifting

### 2.5.2.4 Power set functor is a free monad

Since:

- $\forall e \in S : \exists \{e\} \in \mathcal{P}(S) \models \forall e \in S : \exists (e \rightarrow \{e\}) \equiv \text{unit}$
- $\forall \mathcal{P}(S) : \mathcal{P}(S) \in \mathcal{P}(S) \models \forall \mathcal{P}(S) : \exists (\mathcal{P}(\mathcal{P}(S)) \rightarrow \mathcal{P}(S)) \equiv \text{join}$

## 2.5.3 Forgetful functor

Functor that forgets part or all of what defines **structure** in **domain category**.  $F^{\text{Grp} \rightarrow \text{Set}}$  that translates **groups** into their underlying **sets**. **Constant functor** is another example.

#### 2.5.3.1 \*

Forgetful

## 2.5.4 Identity functor

Maps all **category** to itself. All **objects** and **morphisms** to themselves.

Denotation:  $1^{C \rightarrow C}$

## 2.5.5 Endofunctor

Is a **functor** which source (**domain**) and target (**codomain**) are the same **category**.

$F^{C \rightarrow C}, E^{C \rightarrow C}$

#### 2.5.5.1 \*

Endofunctors



### 2.5.6 Applicative functor

\* - Computer science term. Category theory name - **lax monoidal functor**. And in category *Set*, and so in category *Hask* all **applicatives** and **monads** are strong (have **tensorial strength**).

\* - **sequences functorial** computations (plain **functors** can't).

```
(<*>) :: f (a -> b) -> f a -> f b
```

Requires **Functor** to exist. Requires **Monoidal structure**.

Has **monoidal structure** rules, separated from **function application** inside **structure**.

**Data type** can have several **applicative** implementations.

Standard definition:

```
class Functor f => Applicative f
  where
    (<*>) :: f (a -> b) -> f a -> f b
    pure  :: a -> f a
```

**pure** - if a **functor**, **identity Kleisli arrow**, **natural transformation**.

**Composition** of \* always produces \*, contrary to **monad** (**monads** are not **closed** under **composition**).

`Control.Monad` has an old **function ap** that is old implementation of **<\*>**:

```
ap :: Monad m => m (a -> b) -> m a -> m b
```

#### 2.5.6.1 \*

Applicative Applicatives Applicative functors

#### 2.5.6.2 Applicative property

#### 2.5.6.3 \*

Applicative properties

##### 2.5.6.3.1 Applicative identity property

```
pure id <*> v = v
```

##### 2.5.6.3.2 Applicative composition property

**Function composition** works regularly.

```
pure (.) <*> u <*> v <*> w = u <*> (v <*> w)
```

##### 2.5.6.3.3 Applicative homomorphism property

Internal **function application** doesn't change the **structure** around values.

```
pure f <*> pure x = pure (f x)
```

##### 2.5.6.3.4 Applicative interchange property

On condition that internal **order** of **evaluation** is preserved - **order** of operands is not relevant.

```
u <*> pure y = pure ($ y) <*> u
```

### 2.5.6.4 Applicative function

#### 2.5.6.4.1 liftA\*

##### 2.5.6.4.1.1 liftA

Essentially a `fmap`.

```
:type liftA
liftA :: Applicative f => (a -> b) -> f a -> f b
```

Lifts `function` into `applicative function`.

##### 2.5.6.4.1.2 liftA2

Lifts `binary function` across two `Applicative functors`.

```
liftA2 :: Applicative f => (a -> b -> c) -> f a -> f b -> f c
liftA2 f x y == pure f <*> x <*> y
```

##### 2.5.6.4.1.3 «<liftA2 (<\*>)»>

`liftA2 (<*>)` is pretty useful. It can `lift binary operation` through the two layers: It is two-layer `Applicative`.

```
liftA2 :: Applicative f => (a -> b -> c) -> f a -> f b -> f c
<*> :: Applicative f => (f (a -> b) -> f a -> f b)
liftA2 (<*>) :: (Applicative f1, Applicative f2) => f1 (f2 (a -> b)) -> f1 (f2 a) -> f1 (f2 b)
```

##### 2.5.6.4.1.4 liftA2 (liftA2 (<\*>))

`liftA2 (<*>)` 3-layer version.

##### 2.5.6.4.1.5 liftA3

`liftA2` 3-parameter version.

```
liftA3 f x y z == pure f <*> x <*> y <*> z
```

#### 2.5.6.4.2 Conditional `applicative` computations

```
when :: Applicative f => Bool -> f () -> f ()
```

Only when `True` - perform an `applicative` computation.

```
unless :: Applicative f => Bool -> f () -> f ()
```

Only when `False` - perform an `applicative` computation.

### 2.5.6.5 Special applicatives

#### 2.5.6.5.1 Identity applicative

```
-- Applicative f =>
-- f ~ Identity
type Id = Identity
instance Applicative Id
  where
    pure :: a -> Id a
    (<*>) :: Id (a -> b) -> Id a -> Id b
```

```
mkId = Identity
```

```
xs = [1, 2, 3]

const <$> mkId xs <*> mkId xs'
-- [1,2,3]
```

#### 2.5.6.5.2 Constant applicative

It holds only to one value. The `function` does not exist and last `parameter` is a phantom.

```
-- Applicative f =>
-- f ~ Constant e
type C = Constant
instance Applicative C
where
  pure :: a -> C e a
  (<*>) :: C e (a -> b) -> C e a -> C e b
```

#### 2.5.6.5.3 Maybe applicative

"There also can be no `function` at all."

If `function` might not exist - embed `f` in `Maybe structure`, and use `Maybe applicative`.

```
-- f ~ Maybe
type M = Maybe
pure :: a -> M a
(<*>) :: M (a -> b) -> M a -> M b
```

#### 2.5.6.5.4 Either applicative

`pure` is `Right`. Defaults to `Left`. And if there is two `Left`'s - to `Left` of the first `argument`.

#### 2.5.6.5.5 Validation applicative

The `Validation` `data type` isomorphic to `Either`, but has accumulative `Applicative` on the `Left` side. `Validation data type` is not a `monad`. `Validation` is an example of, "An `applicative functor` that is not a `monad`." While `Either monad` on `Left case` just drops computation and returns this first `Left`. `Monad` needs to `process` the result of computation - it requires to be able to `process` all `Left error statement` cases for `Validation`, it is or non-terminating `Monad` or one which is impossible to implement in `polymorphic` way with `Validation`.

#### 2.5.6.6 Monad

μόνος *monos* sole

μονάδα *monáda* `unit`

In loose terms, `*` - is an ability built over `structures` that allows to `compose functions` that produce that `structures`.

Since it is possible to express unpure `functions` with `equivalent pure functions` that produce a `structure`, `*` become widely used in Haskell for those cases also. `*` with lazy `evaluation` also allows controll over the continuation of calculations by early terminations.

`*` - `lax monoid` in `endofunctor category`, that relies on  $\eta$  (`unit`) and  $\mu$  (`join`) `natural transformations` to form an `equivalent` of `identity`.

`Monad` on  $\mathcal{C}$  is  $\{E^{\mathcal{C} \rightarrow \mathcal{C}}, \eta, \mu\}$ :

- $E^{\mathcal{C} \rightarrow \mathcal{C}}$  - is an `endofunctor`
- two `natural transformations`,  $1^{\mathcal{C}} \rightarrow E$  and  $E \circ E \rightarrow E$ :

$$\begin{aligned}
- \eta^{1^C \rightarrow E} &= \text{unit}^{Identity \rightarrow E}(x) = f^{x \rightarrow E(x)}(x) \\
- \mu^{(E \circ E) \rightarrow E} &= \text{join}^{(E \circ E) \rightarrow (Identity \circ E)}(x) = |y = E(x)| = f^{E(y) \rightarrow y}(y)
\end{aligned}$$

where:

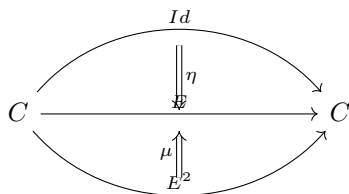
- $\mathcal{C}$  is a **category**
- $1^{\mathcal{C}}$  denotes the  $\mathcal{C}$  **identity functor**
- $(E \circ E)$  - **endofunctor**  $\mathcal{C} \rightarrow \mathcal{C}$

Definition with  $\{E^{\mathcal{C} \rightarrow \mathcal{C}}, \eta, \mu\}$  (in **Hask**:  $(\{e :: f a \rightarrow f b, \text{pure}, \text{join}\})$ ) - is classic categorical, in Haskell minimal complete definition is  $\{\text{fmap}, \text{pure}, (>=>)\}$ .

While  $T$  is mode classical **Category** theory notation, we used the  $E \equiv T$  substitution for purposes of notation being more understandable.

If there is a **structure**  $S$ , and a way of taking **object**  $x$  into  $S$  and a way of collapsing  $S \circ S$  - there probably a **monad**.

**Monad structure:**



Mostly **monads** used for sequencing actions (computations) (that looks like imperative programming), with ability to depend on previous chains. Note if **monad** is **commutative** - it does not **order** actions.

**Monad** can shorten/terminate **sequence** of computations. It is implemented inside **Monad** instance. For example **Maybe monad** on **Nothing** drops chain of computation and returns **Nothing**.

\* inherits the **Applicative** instance methods:

```
import Control.Monad (ap)
return == pure
ap == (<*>) -- + Monad requirement
```

Table 2.1: **Monad** in mathematics and Haskell

Math	Meaning	Cat/Fctr	$X \in \mathcal{C}$	Type	Haskell
$Id$	endofunctor "Id"	$\mathcal{C} \rightarrow \mathcal{C}$	$X \rightarrow Id(X)$	$a \rightarrow a$	id
$E$	endofunctor "monad"	$\mathcal{C} \rightarrow \mathcal{C}$	$X \rightarrow E(X)$	$m a \rightarrow m b$	fmap
$\eta$	natural transformation "unit"	$Id \rightarrow E$	$Id(X) \rightarrow E(X)$	$a \rightarrow m a$	pure
$\mu$	natural transformation "multiplication"	$E \circ E \rightarrow E$	$E(E(X)) \rightarrow E(X)$	$m (m a) \rightarrow m a$	join

Internals of **Monad** are Haskell **data types**, and as such - they can be consumed any number of times.

**Composition** of **monadic types** does not always results in **monadic type**.

#### 2.5.6.6.1 \*

Monads Monadic

#### 2.5.6.6.2 Monad property

**Monad** corresponds to **functor properties** & **applicative properties** and additionally:

## 2.5.6.6.2.1 \*

Monad properties

## 2.5.6.6.2.2 Monad left identity property

```
pure x >>= f == f x
```

Explanation:

```
>>= :: Monad f =>    f a  -> (a -> f b) -> f b
                pure x >>=      f          == f x
```

Rule that `>>=` must get first **argument structure** internals and **apply** to the **function** that is the second **argument**.

Diagram on **category** level:

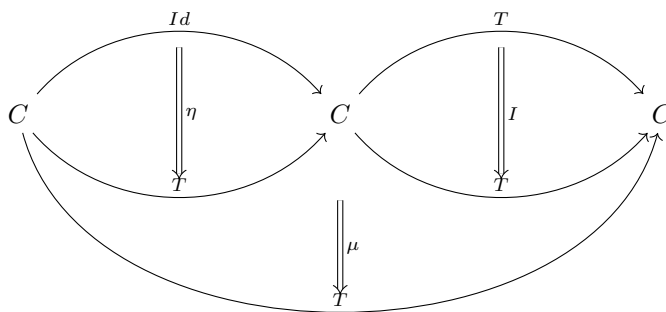
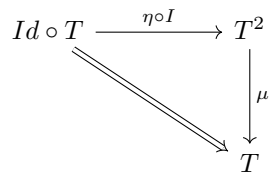


Diagram on **endomorphism** level:



## 2.5.6.6.2.3 Monad right identity property

```
f >>= pure == f
```

Explanation:

```
>>= :: Monad f => f a  -> (a -> f b) -> f b
                f    >>=   pure    == f
```

AKA it is a **tacit** description of a **monad bind** as **endofunctor**.

Diagram on **category** level:

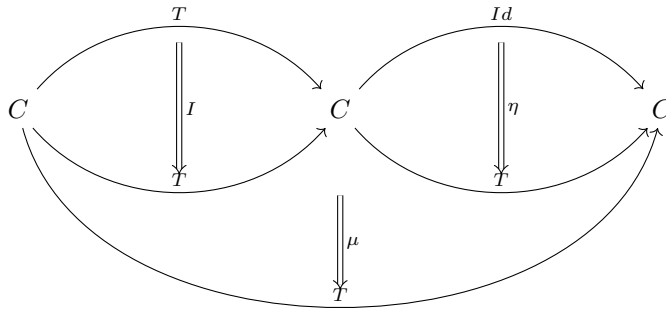
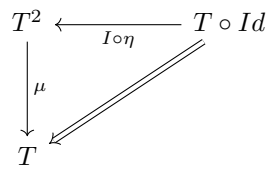


Diagram on endomorphism level:

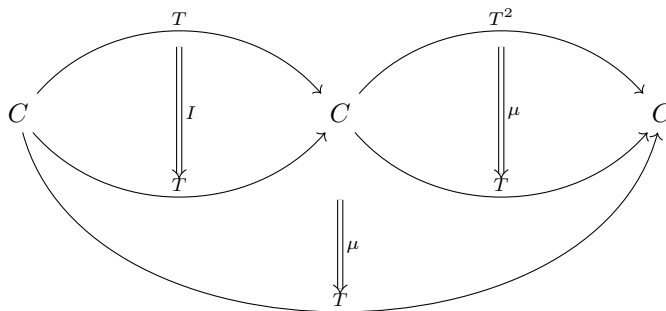


#### 2.5.6.6.2.4 Monad associativity property

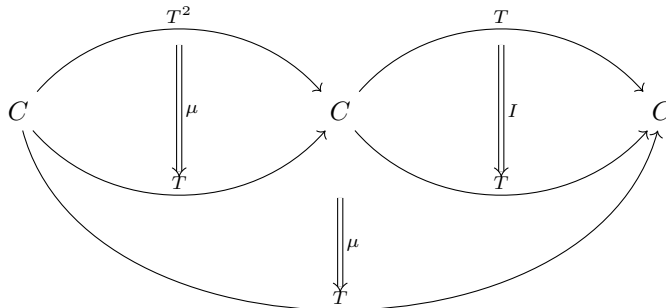
$(m \gg= f) \gg= g == m \gg= (\backslash x \rightarrow f\ x \gg= g)$

In diagram form:

Category level:



is = to:



So,  $\mu \circ (\mu \circ I) = \mu \circ (I \circ \mu)$

Endomorphism level:

$$\begin{array}{c}
 T^3 \\
 \begin{array}{c} \curvearrowright \quad \curvearrowleft \\ \mu \circ I \quad I \circ \mu \end{array} \\
 T^2 \\
 \downarrow \mu \\
 T
 \end{array}$$

### 2.5.6.6.3 Monad type class

```

class Applicative m => Monad m where
  (>>=) :: m a -> (a -> m b) -> m b
  (>>)  :: m a -> m b -> m b
  return :: a -> m a

```

#### 2.5.6.6.3.1 MonadPlus type class

Is a [monoid](#) over [monad](#), with additional rules. The precise [set](#) of rules ([properties](#)) not agreed upon. Class instances obey *monoid* & *left zero* rules, some additionally obey *left catch* and others *left distribution*.

Overall there \* currently reforms ([MonadPlus](#) reform proposal) in several smaller nad strictly defined [type classes](#).

Subclass of an [Alternative](#).

\*

Monadplus

#### 2.5.6.6.4 Functor -> Applicative -> Monad progression

```

<$> :: Functor    f => (a -> b) -> f a -> f b
<*> :: Applicative f => f (a -> b) -> f a -> f b
=<< :: Monad      f => (a -> f b) -> f a -> f b

```

pure & join are [Natural transformations](#) for the fmap.

### 2.5.6.6.5 Monad function

#### 2.5.6.6.5.1 Return function

```
return == pure
```

Nonstrict.

#### 2.5.6.6.5.2 Join function

```
join :: Monad m => m (m a) -> m a
```

Generates knowledge of concat.

[Kleisli composition](#) that flattens two layers of [structure](#) into one.

The way to express ordering in [lambda calculus](#) is to nest.

\*

join

```
join . fmap == (=«)
```

```
-- b = f b
fmap      :: Monad f => (a -> f b) -> f a -> f (f b)
join      :: Monad f => f (f a) -> f a
join . fmap :: Monad f => (a -> f b) -> f a -> f b
flip      >>= :: Monad f => (a -> f b) -> f a -> f b
```

### 2.5.6.6.5.3 Bind function

```
>>=      :: Monad f => f a -> (a -> f b) -> f b
join . fmap :: Monad f => (a -> f b) -> f a -> f b
```

Nonstrict.

The most ubiquitous way to `>>=` something is to use [Lambda function](#):

```
getLine >>= \name -> putStrLn "age pls:"
```

Also a neat way is to bundle and handle [Monad](#) - is to bundle it with [bind](#), and leave [applied](#) partially. And use that partial bundle as a [function](#) - every [evaluation](#) of the [function](#) would trigger [evaluation](#) of internal [Monad structure](#). Thumbs up.

```
printOneOf :: Bool -> IO ()
printOneOf False = putStr "1"
printOneOf True  = putStr "2"

quant :: (Bool -> IO b) -> IO b
quant = (>>=) (randomRIO (False, True))

recursePrintOneOf :: Monad m => (t -> m a) -> t -> m b
recursePrintOneOf f x = (f x) >> (recursePrintOneOf f x)

main :: IO ()
main = recursePrintOneOf (quant) $ printOneOf
*
```

Monadic extend Monadic bind Monad bind Binder

```
(>>=)
>>=
(=<<)
=<<
```

### 2.5.6.6.5.4 Sequencing operator (`>>`) \equiv (`*>`):

Discard any resulting value of the action and [sequence](#) next action. [Applicative](#) has a similar [operator](#).

```
(>>) :: m a -> m b -> m b
(*>) :: f a -> f b -> f b
```

### 2.5.6.6.5.5 Monadic versions of list functions

```
sequence :: (Traversable t, Monad m) => t (m a) -> m (t a)
```

[Sequence](#) gets the traversable of [monadic](#) computations and swaps it into [monad](#) computation of traverse. In the result the collection of [monadic](#) computations turns into one long [monadic](#) computation on traverse of data.

If some step of this long computation fails - [monad](#) fails.

```
mapM :: (Traversable t, Monad m) => (a -> m b) -> t a -> m (t b)
```



mapM gets the AMB [function](#), then takes traversable data. Then applies AMB [function](#) to traversable data, and returns converted [monadic](#) traversable data.

```
foldM :: (Foldable t, Monad m) => (b -> a -> m b) -> b -> t a -> m b
foldl :: Foldable t           => (b -> a -> b) -> b -> t a -> b
```

\* is a [monadic](#) foldl.

b is initial cumulative value, m b is a cumulative bank. Right folding achieved by reversing the input [list](#).

```
filterM :: Applicative m => (a -> m Bool) -> [a] -> m [a]
filter  :: (a -> Bool) -> [a] -> [a]
```

Take Boolean [monadic](#) computation, filter the [list](#) by it.

```
zipWithM :: Applicative m => (a -> b -> m c) -> [a] -> [b] -> m [c]
zipWith  :: (a -> b -> c) -> [a] -> [b] -> [c]
```

Take [monadic](#) combine [function](#) and combine two lists with it.

```
msum :: (Foldable t, MonadPlus m) => t (m a) -> m a
sum  :: (Foldable t, Num a)       => t a -> a
```

#### 2.5.6.6.5.6 liftM\*

liftM Essentially a [fmap](#).

```
liftM :: Monad m => (a -> b) -> m a -> m b
```

Lifts a [function](#) into [monadic](#) equivalent.

liftM2 [Monadic](#) liftA2.

```
liftM2 :: Monad m => (a -> b -> c) -> m a -> m a -> m c
```

Lifts [binary](#) [function](#) into [monadic](#) equivalent.

#### 2.5.6.6.6 Comonad

Category  $\mathcal{C}$  [comonad](#) is a [monad](#) of [opposite](#) category  $\mathcal{C}^{op}$ .

#### 2.5.6.6.7 Kleisli arrow

[Morphism](#) that while doing computation also adds [monadic](#)-able [structure](#).

```
a -> m b
```

##### 2.5.6.6.7.1 \*

Kleisli arrows Kleisli morphism Kleisli morphisms

#### 2.5.6.6.8 Kleisli composition

[Composition](#) of [Kleisli](#) arrows.

```
(<=<) :: Monad m => (b -> m c) -> (a -> m b) -> a -> m c infixr 1
;; compare
(.)   :: (b -> c) -> (a -> b) -> a -> c
```

Often used left-to-right version:

```
(>=>) :: Monad m => (a -> m b) -> (b -> m c) -> a -> m c
;; compare
(>>=) :: Monad m => m a -> (a -> m b) -> m b
```

Which allows to replace [monadic bind](#) chain with [Kleisli composition](#).

```
f1 arg >>= f2 >>= f3
==
f1 >=> f2 >=> f3 $ arg
==
f3 <=< f2 <=< f1 $ arg
```

#### 2.5.6.6.9 Kleisli category

[Category](#)  $\mathcal{C}$ ,  $\langle E, \overrightarrow{\eta}, \overrightarrow{\mu} \rangle$  [monad](#) over  $\mathcal{C}$ .

[Kleisli category](#)  $\mathcal{C}_T$  of  $\mathcal{C}$ :

$$\text{Obj}(\mathcal{C}_T) = \text{Obj}(\mathcal{C}) \quad \text{Hom}_{\mathcal{C}_T}(x, y) = \text{Hom}_{\mathcal{C}}(x, E(y))$$

#### 2.5.6.6.10 Special monad

##### 2.5.6.6.10.1 Identity monad

Wraps data in the [Identity constructor](#).

Useful: Creates [monads](#) from [monad transformers](#).

[Bind](#): Applies internal value to the [bound function](#).

Code: (see: [coerce](#))

```
newtype Identity a = Identity { runIdentity :: a }
```

```
instance Functor Identity where
    fmap      = coerce
```

```
instance Applicative Identity where
    pure      = Identity
    (<*>)     = coerce
```

```
instance Monad Identity where
    m >>= k   = k (runIdentity m)
```

Example:

```
-- derive the State monad using the StateT monad transformer
type State s a = StateT s Identity a
```

##### 2.5.6.6.10.2 Maybe monad

Something that may not be or not return a result. Any lookups into the real world, database queries.

[Bind](#): Nothing input gives Nothing output, Just x input uses x as input to the [bound function](#).

When some computation results in [Nothing](#) - drops the chain of computations and returns [Nothing](#).

[Zero](#): [Nothing](#) Plus: result in first occurrence of Just else [Nothing](#).

Code:

```
data Maybe a = Nothing | Just a
```

```
instance Monad Maybe where
    return     = Just
    fail       = Nothing
```

```

Nothing >>= _ = Nothing
(Just x) >>= f = f x

instance MonadPlus Maybe where
  mzero          = Nothing
  Nothing `mplus` x = x
  x `mplus` _     = x

```

Example: Given 3 dictionaries:

1. Full names to email addresses,
2. Nicknames to email addresses,
3. Email addresses to email preferences.

Create a **function** that finds a person's email preferences based on **either** a full name or a nickname.

```

data MailPref = HTML | Plain
data MailSystem = ...

getMailPrefs :: MailSystem -> String -> Maybe MailPref
getMailPrefs sys name =
  do let nameDB = fullNameDB sys
       nickDB = nickNameDB sys
       prefDB = prefsDB sys
     addr <- (lookup name nameDB) `mplus` (lookup name nickDB)
     lookup addr prefDB

```

#### 2.5.6.6.10.3 Either monad

When computation results in **Left** - drops other computations & returns the recieved **Left**.

#### 2.5.6.6.10.4 Error monad

Something that can fail, throw **exceptions**.

The failure **process** records the description of a failure. **Bind function** uses successful values as input to the **bound function**, and passes failure information on without executing the **bound function**.

Useful: Composing **functions** that can fail. Handle **exceptions**, crate **error** handling **structure**.

**Zero**: empty **error**. Plus: if first **argument** failed then execute second **argument**.

#### 2.5.6.6.10.5 List monad

Computations which may return 0 or more possible results.

**Bind**: The **bound function** is **applied** to all possible values in the input **list** and the resulting lists are concatenated into **list** of all possible results.

Useful: Building computations from **sequences** of non-deterministic operations.

**Zero**: [] Plus: (++)

\*

[] monad

#### 2.5.6.6.10.6 Reader monad

Creates a read-only shared environment for computations.

The pure `function` ignores the environment, while `>=>` passes the inherited environment to both subcomputations.

Today it is defined though `ReaderT` transformer:

```
type Reader r = ReaderT r Identity -- equivalent to ((->) e), (e ->)
```

Old definition was:

```
newtype Reader e a = Reader { runReader :: (e -> a) }
```

For `(e ->)`:

- `Functor` is `(.)`

```
fmap :: (b -> c) -> (a -> b) -> a -> c
fmap = (.)
```

- `Applicative`:

— pure is `const`

```
pure :: a -> b -> a
pure x _ = x
```

- `(<*>)` is:

```
(<*>) :: (a -> b -> c) -> (a -> b) -> a -> c
(<*>) f g = \a -> f a (g a)
```

- `Monad`:

```
(>>=) :: (a -> b) -> (b -> a -> c) -> a -> c
(>>=) m k = Reader $ \r ->
  runReader (k (runReader m r)) r
```

```
join :: (e -> e -> a) -> e -> a
join f x = f x x
```

```
runReader
  :: Reader r a -- the Reader to run
  -> r -- an initial environment
  -> a -- extracted final value
```

Usage:

```
data Env = ...
```

```
createEnv :: IO Env
createEnv = ...
```

```
f :: Reader Env a
f = do
  a <- g
  pure a
```

```
g :: Reader Env a
g = do
  env <- ask -- "Open the environment namespace into env"
  a <- h env -- give env to h
  pure a
```

```
h :: Env -> a
```

```

... -- use env and produce the result

main :: IO ()
main = do
  env <- createEnv
  a = runReader g env
  ...

```

In Haskell under normal circumstances impure [functions](#) should not directly call impure [functions](#). `h` is an impure [function](#), and `createEnv` is impure [function](#), so they should have intermediary.

#### 2.5.6.6.10.7 Writer monad

Computations which accumulate [monoid](#) data to a shared Haskell storage. So `*` is parametrized by [monoidal type](#).

Accumulator is maintained separately from the returned values.

Shared value modified through [Writer monad](#) methods.

`*` frees creator and code from manually keeping the track of accumulation.

**Bind:** The [bound function](#) is [applied](#) to the input value, [bound function](#) allowed to `<>` to the accumulator.

```
type Writer r = WriterT r Identity
```

Example:

```

f :: Monoid b => a -> (a, b)
f a = if _condition_
      then runWriter $ g a
      else runWriter do
        a1 <- h a
        pure a1

g :: Monoid b => Writer b a
g a = do
  tell _value1_ -- accumulator <> _value1_
  pure a -- observe that accumulator stored inside monad
          -- and only a main value needs to be returned.

h :: Monoid b => Writer b a
h a = do
  tell _value2_ -- accumulator <> _value_
  pure a

runWriter :: Writer w a -> (a, w) -- Unwrap a writer computation
                                   -- as a (result, accumulator) pair.
                                   -- The inverse of writer.

```

`WriterT`, `Writer` unnecessarily keeps the entire logs in the memory. Use `fast-logger` for logging.

#### 2.5.6.6.10.8 State monad

Computations that pass-over a state.

The [bound function](#) is [applied](#) to the input value to produce a state transition [function](#) which is [applied](#) to the input state.

[Pure](#) functional language cannot update values in place because it violates [referential transparency](#).

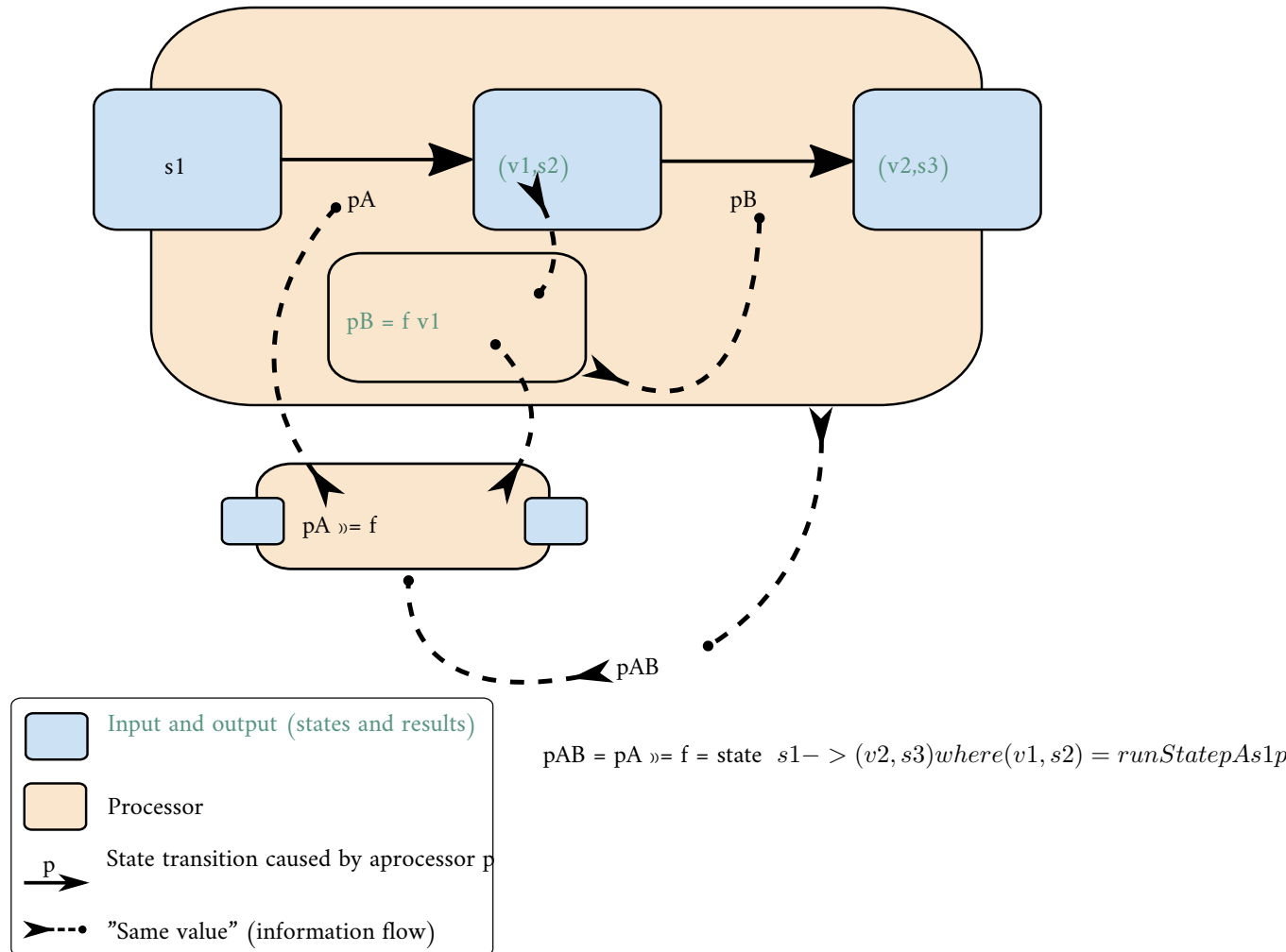
```
type State s = StateT s Identity
```

**Binding** copies and transforms the state **parameter** through the **sequence** of the **bound functions** so that the same state storage is never used twice. Overall this gives the illusion of in-place update to the programmer and in the code, while in fact the autogenerated transition **functions** handle the state changes.

Example **type**: `State st a`

`State` describes **functions** that consume a state and produce a **tuple** of result and an updated state.

**Monad** manages the state with the next **process**:



Where:

- $f$  - processor making **function**
- $pA, pAB, pB$  - state processors
- $sN$  - states
- $vN$  - values

**Bind** with a processor making **function** from state processor ( $pA$ ) creates a new state processor ( $pAB$ ). The wrapping and unwrapping by `State/runState` is implicit.

## 2.5.6.6.11 Monad transformer

\* is a practical solution to the current functional programming situation that generally **monads** do not have **composition** ability. In other words many **monads** can not be **composed**.

\* is a **special monad** that extends other **monad** with extra functionality, it is a convenience mechanism, the functionality itself always can be developed in some other way. Sometimes transformers can make things way harder (especially profound for concurrency ([Michael Snoyman - Monad Transformer State](#))) then other ways of implementation, especially when transformers hold some **structure** information (state-like information, in `ExceptT`, `StateT`)

**Monad** is not **closed** under composition. **Composition** of **monadic types** does not always results in **monadic type**.

Basic **case**: during implementation of **monadic composition**, as a result **type**  $m \rightarrow T \rightarrow m \rightarrow a$  arises, which does not allow join transformation for the  $m$  **monadic** layers or to have a regular **unit** transformation.

**Monads** that are \* are the **monads** that have own **properties** as also ability to **compose** with any other monad and extend it with own **properties**. \* use their implementation to solve the composition **type** layering and allow to attach desirable **property** to result.

\* solve **monad composition** and **type** layering by using own **structure** and information about itself. It is often that **process** involves a **catamorphism** of a \* **type** layer.

Transformers have a light wrapper around the data that tags the modification with this transformer.

In **type** signatures of transformers  $*T \rightarrow m \rightarrow m \rightarrow a$  is already an extended **monad**, so  $*T$  is just a wrapper to point that out.

Main **monadic structure**  $m$  is wrapped around the internal data (core is  $a$ ). The **structure** that corresponds to the transformer creation **properties** (if it emitted by  $\eta$  of a transformer), goes into  $m \rightarrow$ . Open **parameters** go external to the  $m$ .

```
newtype ExceptT e m a =
  ExceptT { runExceptT :: m (Either e a) }
```

```
newtype MaybeT m a =
  MaybeT { runMaybeT :: m (Maybe a) }
```

```
newtype ReaderT r m a =
  ReaderT { runReaderT :: r -> m a }
```

This has an **effect** that on stacking **monad** transformers,  $m$  becomes **monad stack**, and every next transformer injects the transformer creation-specific properties  $\eta$  inside the **stack**, so out-most transformer has inner-most **structure**. Base **monad** is structurally the outermost.

## 2.5.6.6.11.1 MaybeT

\* extends **monads** by injecting **Maybe** layer underneath **monad**, and processing that **structure**:

```
newtype MaybeT m a = MaybeT { runMaybeT :: m (Maybe a) }
```

## 2.5.6.6.11.2 EitherT

\* extends **monads** by injecting **Either** layer underneath **monad**, and processing that **structure**:

```
newtype EitherT e m a = EitherT { runEitherT :: m (Either e a) }
```

`EitherT` of either package gets replaced by `ExceptT` of transformers or `mtl` packages.

\* `ExceptT`

## 2.5.6.6.11.3 ReaderT

Definition:

```
newtype ReaderT r m a = ReaderT { runReaderT :: r -> m a }
```

\* **functions:** input `monad m a`, out: `m a` wrapped it in a free-variable `r` (**partially applied function**). That allows to use transformed `m a`, now it requires and can use the `r` passed environment.

To create a `Reader monad`:

```
type Reader r = ReaderT r Identity
```

## 2.5.6.6.11.4 MonadTrans type class

Allows to **lift monadic** actions into a larger **context** in a neutral way.

`pure` takes a parametric **type** and embodies it into constructed **structure** (talking of **monad** transformers - **structure** of the stacked **monads**).

`lift` takes **monad** and extends it with a transformer.

In fact, for **monad** transformers - `lift` is a last stage of the `pure`, it follows from the **lift property**.

Method:

```
lift :: Monad m => m a -> t m a
```

Lift a computation from the **argument monad** to the constructed **monad**.

Neutral means:

```
lift . return = return
```

```
lift (m >>= f) = lift m >>= (lift . f)
```

The general pattern with **MonadTrans** instances is that it usually lifts the **injection** of the known **structure** of transformer over some **Monad**.

`lift` embeds one **monadic** action into **monad transformer**.

The difference between **pure**, **lift** and **MaybeT** constructor becomes clearer if you look at the **types**:

Example, for `MaybeT IO a`:

```
pure      ::      a -> MaybeT IO a
lift      ::      IO a -> MaybeT IO a
MaybeT   :: IO (Maybe a) -> MaybeT IO a
```

```
x = (undefined :: IO a)
```

```
:t (pure x)
(pure x) :: Applicative t => t (IO a)  -- t recieves one argument of product type
:t (pure x :: MaybeT IO a)
-- Expected type: MaybeT IO a1
-- Actual type: MaybeT IO (IO a0)
```

```
-- While the real type would be
```

```
:t (pure x :: MaybeT IO (IO a))
(pure x :: MaybeT IO (IO a)) :: MaybeT IO (IO a) -- This goes into a conflict of what type&kind (
```

```
:t (lift x)
(lift x) :: MonadTrans t => t IO a  -- result is a proper expected product type
```



-- To belabour

```
:t (lift x :: MaybeT IO a)
(lift x) :: MonadTrans t => t IO a -- result is a proper expected product type
```

lift is a [natural transformation](#)  $\eta$  from an [Identity monad](#) ([functor](#)) with other [monad](#) as content into transformer [monad](#) ([functor](#)), with the preservation of the contained [monad](#):

-- Abstract monads with content as parameters. Define '~>' as a family of morphisms that translate

```
type f ~> g = forall x. f x -> g x
```

-- follows

```
lift :: m ~> t m
```

MonadIO [type class](#) \* - allows to [lift IO](#) action until reaching the [IO monad](#) layer at the top of the [Monad stack](#) (which is allways in the Haskell code that does [IO](#)).

```
class (Monad m) => MonadIO m where
  liftIO :: IO a -> m a
```

liftIO [properties](#):

```
liftIO . pure = pure
```

```
liftIO (m >=> f) = liftIO m >=> (liftIO . f)
```

Which is identical [properties](#) to [MonadTrans lift](#).

Since lift is one step, and liftIO all steps - all steps defined in terms of one step and all other steps, so the most frequent implementation is self-[recursive](#) lift . liftIO:

```
liftIO ioa = lift $ liftIO ioa
```

\*

liftIO

### 2.5.6.7 Alternative type class

[Monoid](#) over [applicative](#). Has left catch [property](#).

Allows to run simultaneously several instances of a computation (or computations) and from them yield one result by [property](#) from ([<|>](#)) :: Type -> Type -> Type.

Minimal complete definition:

```
empty :: f a -- The identity element of <|>
(<|>) :: f a -> f a -> f a -- Associative binary operation
```

#### 2.5.6.7.1 \*

Alternative

## 2.5.7 Monoidal functor

[Functors](#) between [monoidal categories](#) that preserves [monoidal structure](#).

### 2.5.8 \$>

Get & [set](#) a value inside [Functor](#).

#### 2.5.8.1 \*

<\$

### 2.5.9 Multifunctor

Generalizes the concept of [functor](#) between [categories](#), canonical [morphisms](#) between multicategories.

Works over  $N$  [type](#) arguments instead of one.

To put simply - accepts multiple arguments, from that information constructs source [product category](#) ([Cartesian product](#)) of [categories](#), and is a [functor](#) from [product category](#) to target [category](#).

To put even simpler - [functor](#) that takes as an [argument](#) the [product](#) of [types](#).

In Haskell there is only one [category](#), [Hask](#), so in Haskell  $*$  is still  $(Hask \times Hask) \rightarrow Hask \Rightarrow |(Hask \times Hask) \equiv Hask| \Rightarrow Hask \rightarrow Hask$  [endofunctor](#).

Any [product](#) or sum in a Cartesian [category](#) is a  $*$ .

Code definition:

```
class Bifunctor f
  where
    bimap :: (a -> a') -> (b -> b') -> f a a' -> f a a'
    bimap f g = first f . second g
    first :: (a -> a') -> f a b -> f a' b
    first f = bimap f id
    second :: (b -> b') -> f a b -> f a b'
    second = bimap id
```

#### 2.5.9.1 $*$

Bifunctor

## 2.6 Hask category

[Category](#) of Haskell where [objects](#) are [types](#) and [morphisms](#) are [functions](#).

It is a hypothetical [category](#) at the moment, since [undefined](#) and [bottom values](#) break the theory, is not Cartesian [closed](#), it does not have sums, [products](#), or [initial object](#),  $()$  is not a [terminal object](#), [monad](#) identities fail for almost all instances of the [Monad](#) class.

That is why Haskell developers think in subset of Haskell where [types](#) do not have [bottom values](#). This only includes [functions](#) that terminate, and typically only finite values. The corresponding [category](#) has the expected initial and terminal [objects](#), sums and [products](#), and instances of [Functor](#) and [Monad](#) really are [endofunctors](#) and [monads](#).

[Hask](#) contains subcategories, like [Lst](#) containing only [list types](#).

Haskell and [Category](#) concepts:

- Things that take a [type](#) and return another [type](#) are [type constructors](#).
- Things that take a [function](#) and return another [function](#) are higher-order [functions](#).

#### 2.6.1 $*$

[Hask](#)

## 2.7 Magma

[Set](#) with a [binary operation](#) which form a [closure](#).

### 2.7.1 Mag category

The [category of magmas](#), denoted *Mag*, has as [objects](#) - [sets](#) with a [binary operation](#), and [morphisms](#) given by homomorphisms of operations (in the [universal algebra](#) sense).

#### 2.7.1.1 \*

MAG Magma category Category of magmas

### 2.7.2 Semigroup

[Magma](#) with [associative property](#) of [operation](#).

Defined in Haskell as:

```
class Semigroup a where
  (<>) :: a -> a -> a
```

#### 2.7.2.1 \*

Semigroups

#### 2.7.2.2 Monoid

[Semigroup](#) with [identity](#) element. [Category](#) with a one [object](#).

Ideally fits as an accumulation class.

```
class Monoid m where
  mempty :: m
  mappend :: m -> m -> m
  mappend = (<>)
  mconcat :: [m] -> m
  mconcat = foldr mappend mempty
```

\* can be simplified to [category](#) with a single [object](#), remember that [monoid operation](#) is a [composition](#) of [morphisms operation](#) in [category](#).

For example to represent the whole non-negative integers with the one [object](#) and [morphism](#) "1" is absolutely enough, [composition operation](#) is "+".

```
import Data.Monoid
do
  show (mempty :: Num a => Sum a)
  -- "Sum {getSum = 0}"
  show $ Sum 1
  -- "Sum {getSum = 1}"
  show $ (Sum 1) <> (Sum 1) <> (Sum 1)
  -- "Sum {getSum = 3}"
  -- ...
```

Also backwards - any single-object [category](#) is a [monoid](#). [Category](#) has an [identity](#) requirement and [associativity](#) of [composition](#) requirement, which makes it a free [monoid](#).

#### 2.7.2.2.1 \*

Monoidal Monoids

**2.7.2.2.2 Monoid properties****2.7.2.2.2.1 Monoid left identity property**

`mempty <> x = x`

**2.7.2.2.2.2 Monoid right identity property**

`x <> mempty = x`

**2.7.2.2.2.3 Monoid associativity property**

`x <> mempty = x (y <> z) = (x <> y) <> z`  
`mconcat = foldr (mempty <>)`

Everything [associative](#) can be `mappend`.

**2.7.2.2.3 Commutative monoid**

[Operation](#) that forms [structure](#) has [commutativity property](#):  $x \circ y = y \circ x$

Opens a big abilities in concurrent and distributed processing.

**2.7.2.2.3.1 \***

Abelian monoid

**2.7.2.2.4 Group**

[Monoid](#) that has [inverse](#) for every element.

**2.7.2.2.4.1 \***

Groups

**2.7.2.2.4.2 Commutative group**

[Commutative monoid](#) that is a [group](#).

\*

Abelian group

Ring [Commutative group](#) under  $+$  & [monoid](#) under  $\times$ ,  $+$  &  $\times$  connected by [distributive property](#).

- and  $\times$  are generalized [binary](#) operations of addition and multiplication.  $\times$  has no requirement for [commutativity](#).

Example: [set](#) of same size square matrices of numbers with matrix operations form a [ring](#).

\*

Rings

**2.8 Morphism**

$\mu\omicron\phi\phi\acute{\eta}$  *morphe* form

[Arrow](#) between two [objects](#) inside a [category](#).

[Morphism](#) can be anything.

[Morphism](#) is a generalization ( $f(x*y) \equiv f(x) \diamond f(y)$ ) of [homomorphism](#) ( $f(x*y) \equiv f(x)*f(y)$ ).

Since general [morphisms](#) not so much often ment and discussed - under [morphism](#) people almost always really mean the meaning of [homomorphism](#)-like [properties](#), hence they discuss the [algebraic structures \(types\)](#) and homomorphisms between them.

In most usage, on a level under the [objects](#): \* is most often means a map ([relation](#)) that translates from one mathematical [structure](#) (that source [object](#) represents) to another (that target [object](#) represents) (that is called (somewhat, somehow) "[structure](#)-preserving", but that [phrase](#) still means that translation can be lossy and irrevertable, so it is only bear reassemblance of preservation), and in the end the [morphism](#) can be anything and not hold to this conditions.

[Morphism](#) needs to correspond to [function](#) requirements to be it.

### 2.8.1 \*

Morphisms Arrow Arrows

### 2.8.2 Homomorphism

ὁμός *homos* same (was chosen because of initial English mistranslation to "similar")

μορφή *morphe* form

similar form

\* map between two [algebraic structures](#) of the same [type](#) that preserves the operations.

$f(x * y) \equiv f(x) * f(y)$ , [where](#) for  $f^{A \rightarrow B}$  -  $A, B$  are [sets](#) with additonal [algebraic structures](#) ([algebras](#)) that include [operation](#) \*;  $x, y$  are elements of the [set](#)  $B$ .

\* sends [identity morphisms](#) to [identity morphisms](#) and inverses to inverses.

The concept of \* has been generalized under the name of [morphism](#) to many [structures](#) that [either](#) do not have an underlying [set](#), or are not [algebraic](#), or do not preserve the [operation](#).

#### 2.8.2.1 \*

Homomorphic

### 2.8.3 Identity morphism

[Identity morphism](#) - or simply [identity](#):  $x \in C : id_x = 1_x : x \rightarrow x$  [Composed](#) with other [morphism](#) gives same [morphism](#).

Corresponds to [Reflexivity](#) and [Automorphism](#).

#### 2.8.3.1 Identity

[Identity](#) only possible with [morphism](#). See [Identity morphism](#).

There is also distinct [Zero](#) value.

##### 2.8.3.1.1 Two-sided identity of a predicate

$P(e, a) = P(a, e) = a \mid \exists e \in S, \forall a \in S P()$  is [commutative](#).

[Predicate](#)

##### 2.8.3.1.2 Left identity of a predicate

$\exists e \in S, \forall a \in S : P(e, a) = a$

[Predicate](#)

### 2.8.3.1.3 Right identity of a predicate

$$P(a, e) = a \mid \exists e \in S, \forall a \in S$$

Predicate

### 2.8.3.2 Identity function

Return itself. ( $\backslash x.x$ )

`id :: a -> a`

## 2.8.4 Monomorphism

$\mu\omicron\nu\omicron$  *mono* only

$\mu\omicron\rho\phi\eta$  *morphe* form

Maps one to one (uniquely), so invertable (always has [inverse morphism](#)), so preserves the information/[structure](#). [Domain](#) can be equal or less to the [codomain](#).

$f^{X \rightarrow Y}, \forall x \in X \exists! y = f(x) \models f(x) \equiv f_{mono}(x)$  - from [homomorphism context](#)  $f_{mono} \circ g1 = f_{mono} \circ g2 \models g1 \equiv g2$  - from general [morphism context](#) Thus  $*$  is left cancelable.

If  $*$  is a [function](#) - it is [injective](#). Initial [set](#) of  $f$  is fully uniquely mapped onto the [image](#) of  $f$ .

### 2.8.4.1 \*

Monomorphic Monomorphisms

## 2.8.5 Epimorphism

$\epsilon\pi\iota$  *epi* on, over

$\mu\omicron\rho\phi\eta$  *morphe* form

$*$  is right cancelable [morphism](#).  $f^{X \rightarrow Y}, \forall y \in Y \exists f(x) \models f(x) \equiv f_{epi}(x)$  - from [homomorphism context](#)  $g1 \circ f_{epi} = g2 \circ f_{epi} \Rightarrow g1 \equiv g2$  - from general [morphism context](#)

In [Set category](#) if  $*$  is a [function](#) - it is [surjective](#) ([image](#) of it fully uses [codomain](#)) [Codomain](#) is a called a projection of the [domain](#).

$*$  fully maps into the target.

### 2.8.5.1 \*

Epimorphic Epimorphisms

## 2.8.6 Isomorphism

$\iota\sigma\omicron\varsigma$  *isos* equal

$\mu\omicron\rho\phi\eta$  *morphe* form

Not equal, but equal for current intents and purposes. [Morphism](#) that has [inverse](#). Almost equal, but not quite: ([Integer](#), [Bool](#)) & ([Bool](#), [Integer](#)) - but can be transformed losslessly into one another.

[Bijective homomorphism](#) is also [isomorphism](#).

$$f^{-1, b \rightarrow a} \circ f^{a \rightarrow b} \equiv 1^a, f^{a \rightarrow b} \circ f^{-1, b \rightarrow a} \equiv 1^b$$

2 reasons for non-[isomorphism](#):

- [function](#) at least ones collapses a values of [domain](#) into one value in [codomain](#)

- **image** (of a **function** in **codomain**) does not fill-in **codomain**. Then **isomorphism** can exist for **image** but not whole **codomain**.

**Categories** are **isomorphic** if there  $R \circ L = ID$

### 2.8.6.1 \*

Isomorphic Isomorphisms

### 2.8.6.2 Lax

Holds up to **isomorphism**. (upon the transformation can be used as the same)

## 2.8.7 Endomorphism

ἐνδο *endo* internal

μορφή *morphe* form

**Arrow** from **object** to itself. **Endomorphism** forms a **monoid** (**object** exists and **category** requirements already in place).

### 2.8.7.1 Automorphism

αυτο *auto* self

μορφή *form* form

**Isomorphic endomorphism**.

Corresponds to **identity**, **reflexivity**, **permutation**.

#### 2.8.7.1.1 \*

Automorphic Automorphisms

### 2.8.7.2 \*

Endomorphic Endomorphisms

## 2.8.8 Catamorphism

κατά *kata* downward

μορφή *morphe* form

Unique **arrow** from an initial **algebra structure** into different **algebra structure**.

\* in FP is a generalization folding, deconstruction of a **data structure** into more primitive **data structure** using a **functor** **F-algebra structure**.

\* reduces the **structure** to a lower level **structure**. \* creates a projection of a **structure** to a lower level **structure**.

### 2.8.8.1 \*

Catamorphic Catamorphisms

Table 2.2: [Catamorphism properties](#) in Haskell

Rule name	Haskell
cata-cancel	<code>cata phi . InF = phi . fmap (cata phi)</code>
cata-refl	<code>cata InF = id</code>
cata-fusion	<code>f . phi = phi . fmap f =&gt; f . cata phi = cata phi</code>
cata- <a href="#">compose</a>	<code>eps :: f :~&gt; g =&gt; cata phi . cata (In . eps) = cata (phi . eps)</code>

### 2.8.8.2 Catamorphism property

#### 2.8.8.2.1 Hylomorphism

[catamorphism](#)  $\circ$  [anamorphism](#)

Expanding and collapsing the [structure](#).

##### 2.8.8.2.1.1 \*

Hylomorphic Hylomorphisms

### 2.8.8.3 Anamorphism

Generalizes `unfold`.

[Dual](#) concept to [catamorphism](#).

Increases the [structure](#).

[Morphism](#) from a [coalgebra](#) to the final [coalgebra](#) for that [endofunctor](#).

Is a [function](#) that generates a [sequence](#) by repeated [application](#) of the [function](#) to its previous result.

##### 2.8.8.3.1 \*

Anamorphic Anamorphisms

## 2.8.9 Kernel

[Kernel](#) of a [homomorphism](#) is a number that measures the [degree homomorphism](#) fails to meet [injectivity](#) (AKA be [monomorphic](#)). It is a number of [domain](#) elements that fail [injectivity](#):

- elements not included into [morphism](#)
- elements that collapse into one element in [codomain](#)

thou [Kernel](#)  $[x|x \leftarrow 0||x \geq 2]$ .

Denotation:  $\ker T = \{\mathbf{v} \in V : T(\mathbf{v}) = \mathbf{0}_W\}$ .

### 2.8.9.1 Kernel homomorphism

[Morphism](#) of elements from the [kernel](#). Complementary [morphism](#) of elements that make main [morphism](#) not [monomorphic](#).

## 2.9 Set category

[Category](#) in which [objects](#) are [sets](#), [morphisms](#) are [functions](#).

Denotation: *Set*



## 2.10 Natural transformation

Roughly  $*$  is:

**trans** ::  $F \ a \rightarrow G \ a$

, while  $a$  is **polymorphic variable**.

**Naturality** condition:  $\forall a \exists (F \ a \rightarrow G \ a)$ , or, analogous to **parametric polymorphism** in **functions**. Since  $*$  in a **category**, stating  $\forall (F \ a \rightarrow G \ a)$  **Naturality** condition means that all **morphisms** that take part in **homotopy** of source **functor** to target **functor** must exist, and that is the same, diagrams that take part in transformation, should commute, and different paths brings same result: if  $\alpha$  - **natural transformation**,  $\alpha_a$  **natural transformation component** -  $G \ f \circ \alpha_a = \alpha_b \circ F \ f$ . Since  $*$  are just a **type** of **parametric polymorphic function** - they can **compose**.

$*$  ( $\overrightarrow{\eta}^{\mathcal{D}}$ ) is transforming :  $\overrightarrow{\eta}^{\mathcal{D}} \circ F^{C \rightarrow \mathcal{D}} = G^{C \rightarrow \mathcal{D}}$ .  $*$  **abstraction** creates higher-language of **Category** theory, allowing to talk about the **composition** and transformation of complex entities.

It is a **process** of transforming  $F^{C \rightarrow \mathcal{D}}$  into  $G^{C \rightarrow \mathcal{D}}$  using existing **morphisms** in target **category**  $\mathcal{D}$ .

Since it uses **morphisms** - it is **structure-preserving** transformation of one **functor** into another. It is mostly a lossy transformation. Only existing **morphisms** can make it exist.

Existence of  $*$  between two **functors** can be seen as some **relation**.

Can be observed to be a "**morphism of functors**", especially in **functor category**.  $*$  by  $\overrightarrow{\eta}_{y^{\mathcal{C}}}^{\mathcal{D}}((x, y)^{\mathcal{C}}) \circ F^{C \rightarrow \mathcal{D}}((x, y)^{\mathcal{C}}) = G^{C \rightarrow \mathcal{D}}((x, y)^{\mathcal{C}}) \circ \overrightarrow{\eta}_{x^{\mathcal{C}}}^{\mathcal{D}}((x, y)^{\mathcal{C}})$ , often written short  $\overrightarrow{\eta}_b \circ F(\overrightarrow{f}) = G(\overrightarrow{f}) \circ \overrightarrow{\eta}_a$ . Notice that the  $\overrightarrow{\eta}_{x^{\mathcal{C}}}^{\mathcal{D}}((x, y)^{\mathcal{C}})$  depends on **objects&morphisms** of  $\mathcal{C}$ .

In words:  $*$  depends on  $F$  and  $G$  **functors**, ability of  $\mathcal{D}$  **morphisms** to do a **homotopy** of  $F$  to  $G$ , and  $*$ :

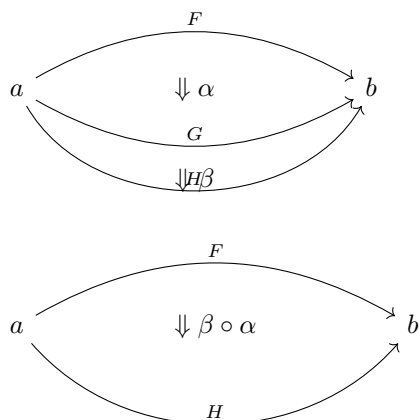
- for every **object** in  $\mathcal{C}$  picks **natural transformation component** in  $\mathcal{D}$ .
- for every **morphism** in  $\mathcal{C}$  picks the **commuting diagram** in  $\mathcal{D}$ , called naturality square.

Also see: **Natural transformation in Haskell**

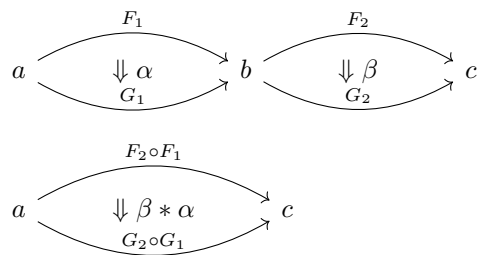
Knowledge of  $*$  forms a **2-category**.

Can be **composed**

- "vertically":



- "horizontally" ("**Godement product**"):



Vertical and horizontal [compositions](#) can be done in any [order](#), they abide the [exchange property](#).

### 2.10.1 \*

Natural transformations Naturality condition Naturality

### 2.10.2 Natural transformation component

$$\vec{\eta}^{\mathcal{D}}(x) = F^{\mathcal{D}}(x) \rightarrow G^{\mathcal{D}}(x) \mid x \in \mathcal{C}$$

#### 2.10.2.1 \*

Component of natural transformation

### 2.10.3 Natural transformation in Haskell

Family of [parametric polymorphism functions](#) between [endofunctors](#).

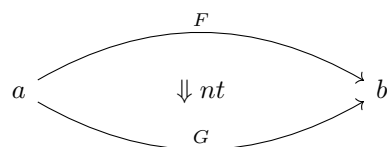
In [Hask](#) is  $F\ a \rightarrow G\ a$ . Can be analogued to repackaging data into another container, never modifies the [object](#) content, it only if - can delete it, because [operation](#) is lossy.

Can be sees as ortogonal to [functors](#).

### 2.10.4 Cat category

[Category](#) where:

	Part	Is	#
*	<a href="#">object</a>	<a href="#">category</a>	0-cell
$\Rightarrow$	<a href="#">morphism</a>	<a href="#">functor</a>	1-cell
$\Rightarrow$	<a href="#">2-morphism</a>	<a href="#">natural transformation</a> , <a href="#">morphisms homotopy</a>	2-cell



Is Cartesian [closed category](#).

#### 2.10.4.1 \*

Cat 2-category

#### 2.10.4.2 Bicategory

[2-category](#) that is [enriched](#) and [lax](#).

For handling relaxed [associativity](#) - introduces associator, and for [identity](#) 1 -eft/right unitor.

Forming from bicategories higher [categories](#) by stacking levels of [abstraction](#) of such [categories](#) - leads to explosion of special cases, differences of every level, and so overall difficulties.

Stacking groupoids (category in which are morphisms are invertable) is much more homogenous up to infinity, and forms base of the homotopy type theory.

## 2.11 Category dual

Category duality behaves like a logical inverse.

Inverse  $\mathcal{C} = \mathcal{C}^{op}$  - inverts the direction of morphisms.

Composition accordingly changes to the morphisms:  $(g \circ f)^{op} = f^{op} \circ g^{op}$

Any statement in the terms of  $\mathcal{C}$  in  $\mathcal{C}^{op}$  has the dual - the logical inverse that is true in  $\mathcal{C}^{op}$  terms.

Opposite preserves properties:

- products:  $(\mathcal{C} \times \mathcal{D})^{op} \cong \mathcal{C}^{op} \times \mathcal{D}^{op}$
- functors:  $(F^{\mathcal{C} \rightarrow \mathcal{D}})^{op} \cong F^{\mathcal{C}^{op} \rightarrow \mathcal{D}^{op}}$
- slices:  $(\mathcal{F} \downarrow \mathcal{G})^{op} \cong (\mathcal{G}^{op} \downarrow \mathcal{F}^{op})$

### 2.11.0.0.1 \*

Opposite category Opposite categories Category duality Duality Dual category Dual

### 2.11.1 Coalgebra

Structures that are dual (in the category-theoretic sense of reversing arrows) to unital associative algebras. Every coalgebra, by vector space duality, reversing arrows - gives rise to an algebra. In finite dimensions, this duality goes in both directions. In infinite - it should be determined.

## 2.12 Thin category

$\forall$  Hom sets contain zero or one morphism.

$$f \equiv g \mid \forall x, y \forall f, g : x \rightarrow y$$

A proset (preordered set).

### 2.12.1 \*

Proset category Prosetal category Poset category Posetal category

## 2.13 Commuting diagram

Establishes equality in morphisms that have same source and target.

Draws the morphisms that are:  $f = g \Rightarrow \{f, g\} : X \rightarrow Y$

### 2.13.1 \*

Diagram commutes Commutes

## 2.14 Universal construction

Algorithm of constructing definitions in Category theory. Specially good to translate properties/definitions from other theories (Set theory) to Categories.

Method:

1. Define a pattern that you defining.
2. Establish ranking for pattern matches.
3. The top of ranking, the best match or [set](#) of matches - is the thing you was looking for. Matches are [isomorphic](#) for defined rules.

\* uses Yoneda lemma, and as such constructions are defined until [isomorphism](#), and so [isomorphic](#) between each-other.

### 2.14.1 \*

Universal constructions

## 2.15 Product

Universal construction:

$$\begin{array}{ccccc}
 & & c' & & \\
 & p \swarrow & \downarrow ! & \searrow q & \\
 a & \xleftarrow{\pi_a} & c & \xrightarrow{\pi_b} & b
 \end{array}$$

Pattern:  $p : c \rightarrow a, q : c \rightarrow b$

Ranking:  $\max \sum^{\forall} (! : c' \rightarrow c \mid p' = p \circ !, q' = q \circ !)$

$c'$  is another candidate.

For [sets](#) - Cartesian product.

\* is a pair. Corresponds to [product data type](#) in [Hask](#) (inhabited with all elements of the [Cartesian product](#)).

Dual is [Coproduct](#).

### 2.15.1 \*

Products

## 2.16 Coproduct

Universal construction:

$$\begin{array}{ccccc}
 & & c' & & \\
 & p \nearrow & \uparrow ! & \nwarrow q & \\
 a & \xrightarrow{\iota_a} & c & \xleftarrow{\iota_b} & b
 \end{array}$$

Pattern:  $i : a \rightarrow c, j : b \rightarrow c$

Ranking:  $\max \sum^{\forall} (! : c \rightarrow c' \mid i' = ! \circ i, j' = ! \circ j)$

$c'$  is another candidate.

For [sets](#) - Disjoint union.

\* is a [set](#) assembled from other two [sets](#), in Haskell it is a tagged [set](#) (analogous to disjoint union).

Dual is [Product](#).

### 2.16.1 \*

Coproducts

## 2.17 Free object

General particular **structure**. In which **structure**, **properties** autofollows from definition, axioms.

Also uses as a term when surcomstances of **structures**, rules, **properties**, axioms used coincide with the definition of a particular **object**  $\therefore$  form **object** of this **type** with the according **properties** and possibilities.

## 2.18 Internal category

**Category** which is included into a bigger **category**.

## 2.19 Hom set

All **morphisms** from source **object** to target **object**.

Denotation:  $hom_C(X, Y) = (\forall f : X \rightarrow Y) = hom(X, Y) = C(X, Y)$  Denotation was not standardized.

**Hom sets** belong to **Set category**.

In **Set category**:  $\exists!(a, b) \iff \exists! Hom, \forall Hom \in Set$ . **Set category** is special, **Hom sets** are also **objects** of it.

**Category** can include **Set**, and **hom sets**, or not.

### 2.19.1 \*

Hom-set Hom sets

### 2.19.2 Hom-functor

$hom : C^{op} \times C \rightarrow Set$  **Functor** from the **product** of  $C$  with its **opposite category** to the **category** of **sets**.

Denotation variants:  $H_A = Hom(-, A)$   $h_A = C(-, A)$   $Hom(A, -) : C \rightarrow Set$

Hom-**bifunctor**:  $Hom(-, -) : C^{op} \times C \rightarrow Set$

### 2.19.3 Exponential object

Generalises the notion of **function set** to internal **object**. As also **hom set** to **internal hom objects**.

Cartesian **closed (monoidal) category** strictly required, as  $*$  multiplication holds **composition** requirement:

$$\circ : hom(y, z) \otimes hom(x, y) \rightarrow hom(x, z)$$

Denotation:  $b^a$

**Universal construction**:

$$\begin{array}{ccccc} c & \xleftarrow{\quad} & c \times a & & \\ \vdots u & & \vdots u \times 1^a & \searrow g & \\ b^a & \xleftarrow{\quad} & b^a \times a & \xrightarrow{\quad eval \quad} & b \end{array}$$

, where in **Category**:  $b^a$  - **exponential object**,  $\times$  - **product bifunctor**,  $a$  - **argument** of  $*$ ,  $b$  - **result**,  $c$  - **candidate**,  $b^a \equiv (a \Rightarrow b) - *$ .

\*  $b^a$  (also as  $(a \Rightarrow b)$ ) represent exponentiation of [cardinality](#) of  $\forall b^a$  possibilities.

### 2.19.3.1 \*

Function object Internal hom Exponential objects Hom object Hom objects

### 2.19.3.2 Enriched category

Uses [Hom objects](#) ([exponential objects](#)), which do not belong into [Set category](#). [Category](#) is no longer small, now may be called large.

$hom(x, y) \in K$ .

Called: \* over  $K$  (which holds [hom objects](#)).

### 2.19.3.2.1 \*

Enriched Large category

# Chapter 3

## Data type

Set of values. For [type](#) to have sense the values must share some sense, [properties](#).

### 3.1 \*

Type Types Data types

### 3.2 Actual type

[Data type](#) recieved by [->inferring->compiling->execution](#).

### 3.3 Algebraic data type

Composite [type](#) formed by combining other [types](#).

#### 3.3.1 \*

AlgDT

### 3.4 Cardinality

Number of possible implementations for a given [type](#) signature.

[Disjunction](#), sum - adds [cardinalities](#). [Conjunction](#), [product](#) - multiplies [cardinalities](#).

#### 3.4.1 \*

Cardinalities

### 3.5 Data constant

\* - [constant](#) value; [nullary data constructor](#).

### 3.6 Data constructor

One instance that [inhabit data type](#).

## 3.7 data declaration

**Data type declaration** is the most general and versatile form to create a new **data type**. Form:

```
data [context =>] type typeVars1..n
  = con1 c1t1..i
  | ...
  | conm cmt1..q
  [deriving]
```

## 3.8 Dependent type

When **type** and values have **relation** between them. **Type** has restrictions for values, value of a **type variable** has a result on the **type**.

### 3.8.1 \*

Dependent types

## 3.9 Gen type

**Generator**. **Gen type** is to generate pseudo-random values for parent **type**. Produces a **list** of values that gets infinitely cycled.

## 3.10 Higher-kinded data type

Any combination of \* and ->

**Type** that take more **types** as arguments.

*Humbly really a **function***

### 3.10.1 \*

Higher-kinded data types

## 3.11 newtype declaration

Create a new **type** from old **type** by attaching a new **constructor**, allowing **type class instance declaration**.

```
newtype FirstName = FirstName String
```

Data will have exactly the same representation at runtime, as the **type** that is wrapped.

```
newtype Book = Book (Int, Int)
```

```
    (,)
    /\
Integer Integer
```

## 3.12 Principal type

The most generic **data type** that still **typechecks**.



### 3.13 Product data type

Is an [algebraic data type](#) representation of a [product](#) construction. Formed by logical [conjunction](#) (AND, '[\\* \\*](#)').

Haskell forms:

```
-- 1. As a tuple (the uncurried & most true-form)
(T1, T2)

-- 2. Curried form, data constructor that takes two types
C T1 T2

-- 3. Using record syntax. =r# <inhabitant>= would return the respective =T#=
C { r1 :: T1
  , r2 :: T2
  }
```

#### 3.13.1 \*

Product type

#### 3.13.2 Sequence

Enumerated (ordered) [set](#).

Denotation:

```
()
( , )
( , , )
( , , ... )
```

More general mathematical denotation was not established, variants:  $(n)_{n \in \mathbb{N}} \omega \rightarrow X \{i : Ord \mid i < \alpha\}$

In Haskell: [Data type](#) that stores multiple ordered values withing a single value.

Table 3.1: [Sequence constructor](#) naming by [arity](#)

Name	Arity	Denotation
<a href="#">Unit</a> , empty	0	()
<a href="#">Singleton</a>	1	( <a href="#">_</a> )
Tuple, pair, two-tuple	2	( , )
Triple, three-tuple	3	( , , )
<a href="#">Sequence</a>	N	( , , ... )

#### 3.13.2.1 \*

Sequences Tuples Ordered pair Ordered triple

#### 3.13.2.2 List

[Sequence](#) of one [type](#) objects.

Denotation:

```
[]
[ , ]
[ , , ]
[ , , ... ]
```

Haskell definition:

```
data [] a = [] | a : [a]
```

Definition is self-referential (self-recursive), can be seen as [anamorphism](#) (unfold) of the [] (empty list, memory cell which is container of particular type) and : (cons operation, pointer). As such - can create non-terminating data type (and computation), in other words - infinite.

## 3.14 Proxy type

[Proxy type](#) holds no data, but has a phantom parameter of arbitrary type (or even kind). Able to provide type information, even though has no value of that type (or it can be may too costly to create one).

```
data Proxy a = ProxyValue
```

```
let proxy1 = (ProxyValue :: Proxy Int) -- a has kind `Type`
let proxy2 = (ProxyValue :: Proxy List) -- a has kind `Type -> Type`
```

## 3.15 Static typing

[Type check](#) takes place at [compile level](#), so compiled program already has expectations of types it should receive.

## 3.16 Structural type

Mathematical type. They form into [structural type system](#).

### 3.16.1 \*

Structural

## 3.17 Structural type system

Strict global hierarchy and relationships of types and their properties. Haskell type system is \*. In most languages typing is name-based, not structural.

### 3.17.1 \*

Structural typing

## 3.18 Sum data type

[Algebraic data type](#) formed by logical [disjunction](#) (OR '|').

## 3.19 Type alias

Create new [type constructor](#), and use all [data structure](#) of the base type.

## 3.20 Type class

Type system [construct](#) that adds a support of [ad hoc polymorphism](#).

Type class makes a nice way for defining behaviour, [properties](#) over many [types/objects](#) at once.

### 3.20.1 \*

Type classes

### 3.20.2 Arbitrary type class

Type class of [QuickCheck.Arbitrary](#) (that is reexported by [QuickCheck](#)) for creating a [generator](#)/distribution of values. Useful [function](#) is [arbitrary](#) - that autogenerates values.

#### 3.20.2.1 Arbitrary function

Depends on [type](#) and generates values of that [type](#).

### 3.20.3 CoArbitrary type class

Pseudogenerates a [function](#) basing on resulting [type](#).

```
coarbitrary :: CoArbitrary a => a -> Gen b -> Gen b
```

#### 3.20.3.1 \*

CoArbitrary

### 3.20.4 Typeable type class

Allows dynamic [type](#) checking in Haskell for a [type](#). Shift a [typechecking](#) of [type](#) from compile time to runtime. \* [type](#) gets wrapped in the universal [type](#), that shifts the [type](#) checks to runtime.

Also allows:

- Get the [type](#) of something at runtime (ex. print the [type](#) of something `typeof`).
- Compare the [types](#).
- Reifying [functions](#) from [polymorphic type](#) to concrete (for [functions](#) like `:: Typeable a => a -> String`).

#### 3.20.4.1 \*

Typeable

### 3.20.5 Type class inheritance

Type class has a [superclass](#).

### 3.20.6 Derived instance

Type class instances sometimes can be automatically [derived](#) from the parent [types](#).

Type classes such as `Eq`, `Enum`, `Ord`, `Show` can have instances generated based on definition of [data type](#).

P.S.

Language options:

- DeriveAnyClass
- DeriveDataTypeable
- DeriveFoldable
- [DeriveFunctor](#)
- DeriveGeneric
- DeriveLift
- DeriveTraversable
- DerivingStrategies
- DerivingVia
- GeneralisedNewtypeDeriving
- StandaloneDeriving

#### 3.20.6.1 \*

Derived Deriving

### 3.21 Type constant

[Nullary type constructor](#).

### 3.22 Type constructor

Name of the [data type](#).

[Constructor](#) that takes [type](#) as an [argument](#) and produces new [type](#).

### 3.23 type declaration

Synonym for existing [type](#). Uses the same [data constructor](#).

[type](#) `FirstName = String`

Used to distinct one entities from other entities, while they have the same [type](#). Also main [type functions](#) can operate on a new [type](#).

### 3.24 Typed hole

\* - is a `_` or `_name` in the [expression](#). On [evaluation](#) GHC would show the [derived type](#) information which should be in place of the \*. That information helps to fill in the gap.

#### 3.24.1 \*

Typed holes

### 3.25 Type inference

Automatic [data type](#) detection for [expression](#).

**3.25.1 \***

Inferring Infer Infers Inferred

**3.26 Type class instance**

Block of implementations of [functions](#), based on unique [type class](#)->[type](#) pairing.

**3.27 Type rank**

Weak ordering of [types](#).

The rank of [polymorphic type](#) shows at what level of nesting forall [quantifier](#) appears. Count-in only [quantifiers](#) that appear to the left of [arrows](#).

```
f1 :: forall a b. a -> b -> a == fi :: a -> b -> c
g1 :: forall a b. (Ord a, Eq b) => a -> b -> a == g1 :: (Ord a, Eq b) => a -> b -> a
```

f1, g1 - [rank-1 types](#). Haskell itself implicitly adds universal [quantification](#).

```
f2 :: (forall a. a->a) -> Int -> Int
g2 :: (forall a. Eq a => [a] -> a -> Bool) -> Int -> Int
```

f2, g2 - [rank-2 types](#). Quantificator is on the left side of a  $\rightarrow$ . Quantificator shows that [type](#) on the left can be overloaded.

[Type inference](#) in Rank-2 is possible, but not higher.

```
f3 :: ((forall a. a->a) -> Int) -> Bool -> Bool
```

f3 - [rank-3-type](#). Has [rank-2 types](#) on the left of a  $\rightarrow$ .

```
f :: Int -> (forall a. a -> a)
g :: Int -> Ord a => a -> a
```

f, g are rank 1. [Quantifier](#) appears to the right of an [arrow](#), not to the left. These [types](#) are not Haskell-98. They are supported in [RankNTypes](#).

**3.27.1 \***

Type ranks Rank type Rank types Rank-1 type Rank-1 types Rank-2 type Rank-2 types Rank-3 type Rank-3 types

**3.28 Type variable**

Refer to an unspecified [type](#) in Haskell [type](#) signature.

**3.29 Unlifted type**

[Type](#) that directly exist on the hardware. The [type abstraction](#) can be completely removed. With [unlifted types](#) Haskell [type](#) system directly manages data in the hardware.

**3.29.1 \***

Unlifted types

### 3.30 Linear type

Type system and algebra that also track the multiplicity of data. There are 3 general linear type groups:

- 0 - exists only at type level and is not allowed to be used at value level. Aka s in ST-Trick.
- 1 - data that is not duplicated
- 1< - all other data, that can be duplicated multiple times.

#### 3.30.1 \*

Linear types

### 3.31 NonEmpty list data type

Data.List.NonEmpty Has a Semigroup instance but can't have a Monoid instance. It never can be an empty list.

```
data NonEmpty a = a :| [a]
    deriving (Eq, Ord, Show)
```

:| - an infix data constructor that takes two (type) arguments. In other words :| returns a product type of left and right

### 3.32 Session type

\* - allows to check that behaviour conforms to the protocol.

So far very complex, not very productive (or well-established) topic.

### 3.33 Binary tree

Tree where every element is a Leaf (structure stub, Nothing) or a Node (split of branches):

```
data BinaryTree a
    = Leaf
    | Node (BinaryTree a) a (BinaryTree a)
```

### 3.34 Bottom value

A \_ non-value in the type or pattern match expression. Placeholder for anything.

```
-- _ fits *.
```

#### 3.34.1 \*

Bottom Bottom values

### 3.35 Bound

Haskell \* type class means to have lowest value & highest value, so a bounded range of values.

**3.35.1 \***

Bounded

**3.36 Constructor**

1. [Type constructor](#)
2. [Data constructor](#)

Also see: [Constant](#)

**3.36.1 \***

Constructors

**3.37 Context**

[Type constraints](#) for [polymorphic variables](#). Written before the main [type](#) signature, denoted:

[TypeClass](#) a => ...

**3.37.1 \***

Contexts

**3.38 Inhabit**

Values that is a component of [data type set](#).

**3.39 Maybe**

```
data Maybe
  = Nothing
  | Just a
```

Does not represent the information why Nothing happened. For [error](#) - use [Either](#). Do not propagate \*.

Handle \* locally to [where](#) it is produced. Nothing does not hold useful info for debugging & short-circuits the processes. Do not expect code [type](#) being bug-free, do not return Maybe to end user since it would be impossible to debug, return something that preserves [error](#) information.

**3.39.0.1 \***

Nodes

**3.40 Expected type**

[Data type inferred](#) from the text of the code.

**3.41 ADT**

1. [Abstract data type](#)
2. [Algebraic data type](#)

## 3.42 Concrete type

Fully resolved, definitive, non-polymorphic type.

## 3.43 Type punning

When [type constructor](#) and [data constructor](#) have the same name.

Theoretically if person knows the rules - \* can be solved, because in Haskell [type](#) and [data declaration](#) have different places of use.

## 3.44 Kind

[Kind](#) -> [Type](#) -> Data

### 3.44.1 \*

Kinds

## 3.45 IO

[Type](#) for values whose evaluations has a possibility to cause side effects or return unpredictable result. Haskell standard uses [monad](#) for constructing and transforming [IO](#) actions. [IO](#) action can be evaluated multiple times.

[IO data type](#) has unpure imperative actions inside. Haskell is [pure Lambda calculus](#), and unpure [IO](#) integrates in the Haskell purely ([type](#) system abstracts [IO](#) unpurity inside [IO data type](#)).

[IO sequences effect](#) computation one after another in [order](#) of needed computation, or occurrence:

```
twoBinds :: IO ()
twoBinds =
  putStrLn "First:" >>
  getLine >>=
  \a ->
  putStrLn "Second:" >>
  getLine >>=
  \b ->
  putStrLn ("\nFirst: "
    ++ a ++ ".\nSecond "
    ++ b ++ ".")
main = twoBinds
```

Sequencing is achieved by compilation of effects performing only while they receive the sugared-in & passed around the RealWorld fake [type](#) value, that value in the every computation gets the new "value" and then passed to the next requested computation. But special thing is about this [parameter](#), this RealWorld [type](#) value passed, but never looked at. GHC realizes, since value is never used, - it means value and [type](#) can be equated to () and moreover reduced from the code, and sequencing stays.



# Chapter 4

## Expression

Finite combination of symbols that is well-formed according to [context-free grammar](#).

Generally meaningless. Meaning gets [derived](#) from an \* & [context](#) (and/or content words) by congruency with knowledge & experience.

### 4.1 \*

Expressions

### 4.2 Closed-form expression

\* - mathematical [expression](#) that can be evaluated in a finite number of operations.

May contain:

- constants
- [variables](#)
- operations (e.g.,  $+$   $-$   $\times$   $\div$ )
- [functions](#) (e.g., nth root, exponent, logarithm, trigonometric [functions](#), and [inverse](#) hyperbolic [functions](#)), but usually no limit.

### 4.3 RHS

Right-hand side of the [expression](#).

### 4.4 LHS

Left-hand side of the [expression](#).

### 4.5 Redex

[Reducible expression](#).

## 4.6 Concatenate

Link together [sequences](#), [expressions](#).

## 4.7 Alpha equivalence

[Equivalence](#) of a processes in [expressions](#). If [expressions](#) have according [parameters](#) different, but the internal processes are literally the same [process](#).

## 4.8 Ground expression

[Expression](#) that does not contain any free [variables](#).

### 4.8.1 \*

Ground formula

## 4.9 Variable

A name for [expression](#).

+===

There fequently can be heard: one of most notable Haskell [properties](#) is Haskell has immutable "variables" (and term here used in the sence that imperative programmers frequently use). Logically we see [statement](#) is contradictory with itself: "[variables](#)" - something that has change as a defining property - are not changing; it is a nonsencical [statement](#). Please, read the saying as: Haskell has immutable values, due to following the value [semantics](#): see "Value". And Haskell [expressions](#) are [functions](#) (that are [referentially transparent](#) - meaning itself immutable) - and they are also values (hense term "functional programming" means - [functions](#) are [first-class](#) values). Since values [bind](#) to [variables](#) - people are wrongly mix-up terms and say their names (according "!=") are immutable.

As you see in the code - Haskell [variables](#) (same names) hold different values at different times. [Variables](#) are reused, meaning "names are reused" - binded to different values on [scope](#) changes. But all values that Haskell holds - are, by the design of the language, are treated immutable, any transformations Haskell resolves by creating new values, and frees the space by freeing-up from no longer needed values.

### 4.9.1 \*

Variables

## 4.10 Phrase

[Composable expression](#).

# Chapter 5

## Function

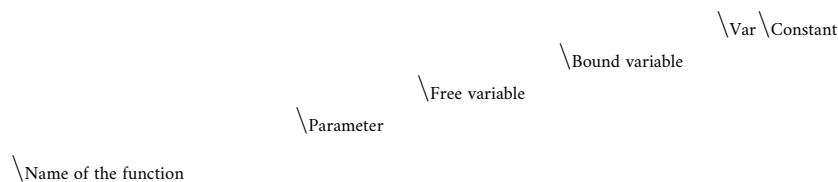
Full dependency of one quantity from another quantity.

Denotation:  $y = f(x)$   $f : X \rightarrow Y$ , where  $X$  is **domain**,  $Y$  is **codomain**.

Directionality and **property** of invariability emerge from one another.

-- **domain func codomain**  
 \*            ->    \*

$$y(x) = (zx^2 + bx + 3 \mid b = 5) \wedge \wedge \wedge \wedge \wedge$$



**Lambda abstraction** is a **function**. **Function** is a mathematical **operation**.

**Function** = Total **function** = **Pure function**. **Function** theoretically can be to memoized.

Also see: **Partial function Inverse function** - often partially exists (**partial function**).

### 5.1 \*

Functions Bound variable

### 5.2 Arity

Number of **parameters** of the **function**.

- **nullary** -  $f()$
- **unary** -  $f(x)$
- **binary** -  $f(x,y)$
- **ternary** -  $f(x,y,z)$
- **n-ary** -  $f(x,y,z..)$

## 5.3 Bijection

**Function** is a complete one-to-one pairing of elements of **domain** and **codomain** (**image**). It means **function** both **surjective** (so **image** == **codomain**) and **injective** (every **domain** element has unique correspondence to the **image** element).

For **bijection inverse** always exists.

**Bijection operation** holds the **equivalence** of **domain** and **codomain**.

Denotation:

$\boxtimes$

$\rightarrow - \rightarrow$

$f : X \boxtimes Y$

LaTeX needed to combine symbols:  $f : X \rightarrowtail Y$

Corresponds to **isomorphism**.

### 5.3.1 \*

Bijection Bijection function

## 5.4 Combinator

**Function** without free **variables**. **Higher-order function** that uses only **function application** and other combinators.

$\backslash a \rightarrow a$

$\backslash a \ b \rightarrow a \ b$

$\backslash f \ g \ x \rightarrow f \ (g \ x)$

$\backslash f \ g \ x \ y \rightarrow f \ (g \ x \ y)$

Not combinators:

$\backslash xs \rightarrow \text{sum } xs$

Informal broad meaning: referring to the style of organizing libraries centered around the idea of combining things.

### 5.4.1 $\Psi$ -combinator

Transforms two of the same **type**, **applying** same mediate transformation, and then transforming those into the result.

`import Data.Function (on)`

`on :: (b -> b -> c) -> (a -> b) -> a -> a -> c`

`a--\b`

`* ---c`

`a--/b`

#### 5.4.1.1 \*

Psi-combinator On-combinator

## 5.5 Function application

\* - **bind** the **argument** to the **parameter** of a **function**, and do a **beta-reduction**.

### 5.5.1 \*

Apply Applied Applying Application

## 5.6 Function body

Expression that haracterizes the process.

## 5.7 Function composition

$(.) :: (b \rightarrow c) \rightarrow (a \rightarrow b) \rightarrow a \rightarrow c$

$a \rightarrow (a \rightarrow b) \rightarrow (b \rightarrow c) \rightarrow c$

In Haskell inline composition requires:

$h.g.f \$ i$

### 5.7.1 \*

Composition Compose Composed

## 5.8 Function head

Is a part with name of the function and it's paramenters. AKA:  $f(x)$

## 5.9 Function range

The range of a function refers to either the codomain or the image of the function, depending upon usage. Modern usage almost always uses range to mean image. So, see Function image.

## 5.10 Higher-order function

Function that has  $\text{arity} > 1$ .

=====

HOF is:

- function that accepts function as a parameter
- function that has more then one parameter.

Application of an argument to \* produces a function that has  $\text{arity} - 1$ .

### 5.10.1 \*

HOF

### 5.10.2 Fold

Catamorphism of a structure to a lower type of structure. Often to a single value.

\* is a higher-order function that takes a function which operates with both main structure and accumulator structure, \* applies units of data structure to a function wich works with accumulator. Upoun traversing the whole structure - the accumulator is returned.

## 5.11 Injection

**Function** one-to-one injects from **domain** into **codomain**.

Keeps distinct pairing of elements of **domain** and **image**. Every element in **image** corresponds to one element in **domain**.

$$\forall a, b \in X, f(a) = f(b) \Rightarrow a = b$$

$$\exists(\text{inverse function}) \mid \forall(\text{injective function})$$

Denotation:

☒

>->

f : X ☒ Y

$f : X \rightarrowtail Y$

Corresponds to **Monomorphism**.

### 5.11.1 \*

Injective Injective function Injectivity

## 5.12 Partial function

One that does not cover all **domain**. **Unsafe** and causes trouble.

## 5.13 Purity

\* means properly abstracted.

If the contrary - **abstraction** is unpure.

Also see: **pure function**.

### 5.13.1 \*

Pure

## 5.14 Pure function

**Function** that is **pure**  $\equiv$  **referentially transparent function**.

## 5.15 Sectioning

Writing **function** in a parentheses. Allows to pass around **partially applied functions**.

## 5.16 Surjection

**Function** uses **codomain** fully.

$$\forall y \in Y, \exists x \in X$$

Denotation:  $f : X \twoheadrightarrow Y$

Corresponds to **Epimorphism**.

**5.16.1 \***

Surjective Surjective function

**5.17 Unsafe function**

Function that does not cover at least one edge case.

**5.17.1 \***

Unsafe

**5.18 Variadic**

\* - having variable arity (often up to indefinite).

**5.19 Domain**

Source set of a function.  $X$  in  $X \rightarrow Y$ .

**5.20 Codomain**

$Y$  in  $X \rightarrow Y$ . Codomain - target set of a function.

**5.21 Open formula**

Logical function that has arity and produces proposition.

**5.22 Recursion**

Repeated function application when sometimes the same function gets called.

Allows computations that may require indefinite amount of work.

**5.22.1 \***

Recursive

**5.22.2 Base case**

A part of a recursive function that trivially produces result.

**5.22.3 Tail recursion**

Tail calls are recursive invocations of itself.

**5.22.4 Polymorphic recursion**

Type of the parameter changes in recursive invocations of function.

Is always a higher-ranked type.

## 5.22.4.1 \*

Milner–Mycroft typability Milner–Mycroft calculus

## 5.23 Free variable

**Variable** in the function that is not **bound** by the head. Until there are \* - **function** stays **partially applied**.

## 5.24 Closure

$f(x) = f^{\mathcal{X} \rightarrow \mathcal{X}} \mid \forall x \in \mathcal{X}, \mathcal{X}$  is **closed** under  $f$ , it is a trivial **case** when **operation** is legitimate for all values of the **domain**.

**Operation** on members of the **domain** always produces a members of the **domain**. The **domain** is **closed** under the **operation**.

In the **case** when there is a **domain** values for which **operation** is not legitimate/not exists:

$f(x) = f^{\mathcal{V} \rightarrow \mathcal{X}} \mid \mathcal{V} \in \mathcal{X}, \forall x \in \mathcal{V}, \mathcal{X}$  is **closed** under  $f$ .

## 5.24.1 \*

Closed

## 5.25 Parameter

para subsidiary metron measure

Named variable of a **function**.

**Argument** is a supplied value to a **function parameter**.

**Parameter** (**formal parameter**) is an **irrefutable** pattern, and implemented that way in Haskell.

## 5.25.1 \*

Parameters Formal parameter Formal parameters

## 5.26 Partial application

Part of **function parameters applied**.

## 5.26.1 \*

Partially applied

## 5.27 Well-formed formula

**Expression**, logical **function** that is/can produce a **proposition**.

## 5.27.1 \*

Well formed formula WFF wff WFFS wffs



## Chapter 6

# Homotopy

Two things are homotopic

One can be "continuously deformed" into the other.

For example - [functions](#), [functors](#). [Natural transformation](#) is a [homotopy](#) of [functors](#).

### 6.1 \*

Homotopies Homotopic

# Chapter 7

## Lambda calculus

Universal model of computation. Which means \* can implement any [Turing machine](#). Based on [function abstraction](#) and [application](#) by substituting [variables](#) and [binding](#) values.

\* has [lambda terms](#):

- [variable](#) ( $x$ )
- [application](#) ( $((ts))$ )
- [abstraction](#) ([lambda function](#)) ( $((\lambda x.t))$ )

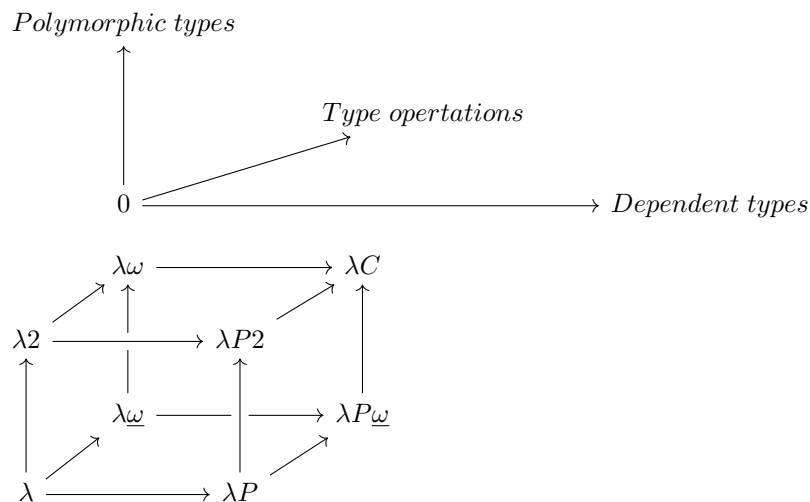
### 7.1 \*

Lambda term Lambda terms

### 7.2 Lambda cube

[λ-cube](#) shows the 3 dimention of generalizations from simply typed [Lambda calculus](#) to [Calculus of constructions](#).

+===



Each dimension of the cube corresponds to extensions (a new [type](#) of [relation](#) of [objects](#) depending on [objects](#)):

Table 7.1: Three degrees of [type](#) systems generalizations

Denotation	Name	Programming	New <a href="#">type</a> of <a href="#">relations</a>
2	<a href="#">Polymorphic types</a>	First-class <a href="#">polymorphism</a> of <a href="#">types</a>	Terms depending on <a href="#">types</a>
$\omega$	<a href="#">Type operation</a>	Type class, <a href="#">type</a> families	<a href="#">Types</a> depending on <a href="#">types</a>
P	<a href="#">Dependent types</a>	Higher-rank <a href="#">polymorphism</a> , <a href="#">dependent types</a>	<a href="#">Types</a> depending on terms

Table 7.2: [λ-cube](#): Names of the [type](#) systems

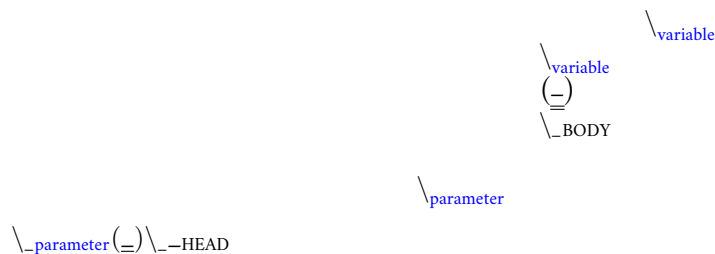
Denotation	Logical system
$\lambda \rightarrow$	(First <a href="#">Order</a> ) <a href="#">Propositional Calculus</a>
$\lambda 2$	Second <a href="#">Order</a> <a href="#">Propositional Caculus</a>
$\lambda \omega$	Weakly Higher <a href="#">Order</a> <a href="#">Propositional Calculus</a>
$\lambda \underline{\omega}$	Higher <a href="#">Order</a> <a href="#">Propositional Calculus</a>
$\lambda P$	(First <a href="#">Order</a> ) <a href="#">Predicate Logic</a>
$\lambda P 2$	Second <a href="#">Order</a> <a href="#">Predicate Calculus</a>
$\lambda P \omega$	Weak Higher <a href="#">Order</a> <a href="#">Predicate Calculus</a>
$\lambda C$	<a href="#">Calculus of Constructions</a>

### 7.2.1 \*

[λ-cube](#) [λ-cube](#)

## 7.3 Lambda function

[Function](#) of [Lambda calculus](#).  $\lambda xy.x^2 + y^3$  ^ ^ ^



### 7.3.1 \*

[Lambda abstraction](#)

### 7.3.2 Anonymous lambda function

[Lambda function](#) that is not binded to any name.

#### 7.3.2.1 \*

[Anonymous lambda function](#)

### 7.3.3 Uncurry

Replace sequenced [lambda functions](#) into single [function](#) taking [sequence/product](#) of values as [argument](#).

## 7.4 $\beta$ -reduction

Equation of a [parameter](#) to a [bound variable](#), then reducing [parameter](#) from the head.

### 7.4.1 \*

$\beta$  reduction Beta-reduction Beta reduction

### 7.4.2 $\beta$ -normal form

No [beta reduction](#) is possible.

#### 7.4.2.1 \*

$\beta$  normal form Beta normal form Beta-normal form

## 7.5 Calculus of constructions

Extends the [Curry–Howard](#) correspondence to the proofs in the full intuitionistic [predicate](#) calculus (includes proofs of [quantified statements](#)). [Type](#) theory, typed programming language, and constructivism (philosophy) foundation for mathematics. Directly relates to Coq programming language.

### 7.5.1 \*

«CoC»

## 7.6 Curry–Howard correspondence

[Equivalence](#) of {[First-order logic](#), computer programming, [Category](#) theory}. They represent each-other, possible in one - possible in the other, so all the definitions and theorems have analogues in other two.

Gives a ground to the [equivalence](#) of computer programs and mathematical proofs.

Lambek added analogue to Cartesian [closed category](#), which can be used to model logic and [type](#) theory.

Table 7.3: Table of basic correspondence

Logic	Type	Category
True	() (any inhabited <a href="#">type</a> )	Terminal
False	<a href="#">Void</a>	Initial
$a \wedge b$	$(a, b)$	$a \times b$
$a \vee b$	$(a \mid b)$	$a \mid b$
$a \Rightarrow b$	$a \rightarrow b$	$b^a$

[Algebra](#) correspondence to [types](#):

$$a^b +^c \sim (b \mid c \rightarrow a) \quad a^b \times a^c \sim (b \rightarrow a, c \rightarrow a)$$

$$a^{b^c} \sim (c \rightarrow b \rightarrow a) \quad a^{b \times c} \sim ((b, c) \rightarrow a)$$

### 7.6.1 \*

Curry–Howard isomorphism Curry–Howard–Lambek

## 7.7 Currying

Translating the [evaluation](#) of a multiple [argument function](#) (or a [tuple](#) of arguments) into evaluating a [sequence](#) of [functions](#), each with a single [argument](#).

### 7.7.1 \*

Curry

## 7.8 Hindley–Milner type system

Classical [type](#) system for the [Lambda calculus](#) with [Parametric polymorphism](#) and [Type inference](#). [Types](#) marked as [polymorphic variables](#), which enables [type inference](#) over the code.

### 7.8.1 \*

Hindley-Milner Damas-Milner Damas-Hindley-Milner

## 7.9 Reduction

Take out something from a [structure](#), make simpler.

See [Beta reduction](#)

### 7.9.1 \*

Reducible

## 7.10 $\beta$ - $\eta$ normal form

All  [\$\beta\$ -reduction](#) and  [\$\eta\$ -abstraction](#) are done in the [expression](#).

### 7.10.1 \*

beta-eta normal form beta eta normal form

## 7.11 $\eta$ -abstraction

$(\lambda x.Mx) \xleftarrow{\eta} M$

$\backslash x \rightarrow g . f \$ x$   
 $\backslash x \rightarrow g . f \quad \text{--eta-abstraction}$

### 7.11.1 \*

$\eta$ -reduction  $\eta$ -conversion  $\eta$  abstraction  $\eta$  reduction  $\eta$  conversion eta-abstraction eta-reduction eta-conversion eta abstraction eta reduction eta conversion

## 7.12 Lambda expression

See [Lambda calculus](#) ([Lambda terms](#)) and [Expression](#). In majority cases meaning some [Lambda function](#).

# Chapter 8

## Operation

Calculation into output value. Can have [zero](#) & more inputs.

### 8.1 Constant

[Nullary operation](#).

Also see: [Type constant](#).

### 8.2 Binary operation

$\forall(a, b) \in S, \exists P(a, b) = f(a, b) : S \times S \rightarrow S$

#### 8.2.1 \*

Binary operations

### 8.3 Operator

Denotation symbol/name for the [operation](#).

#### 8.3.1 Shift operator

[Shift operator](#) defined by Lagrange through [Differential operator](#).  $T^t = e^{t \frac{d}{dx}}$

##### 8.3.1.1 \*

Shift

#### 8.3.2 Differential operator

Denotation.  $\frac{d}{dx}$ ,  $D$ ,  $D_x$ ,  $\partial_x$ . Last one is partial.

$e^{t \frac{d}{dx}}$  - [Shift operator](#).

##### 8.3.2.1 \*

Differential

## 8.4 Infix

Form of writing of [operator](#) or [function](#) in-between [variables](#) for [application](#).

For priorities see [Fixity](#).

## 8.5 Fixity

Declares the presedence of action of a [function/operator](#).

Funciton [application](#) has presedence higher then all [infix](#) operators/[functions](#) (virtually giving it a [priority](#) 10).

Table 8.1: Haskell operators [priority](#) and [fixity](#) association

P	L	Non	R
10			F.A.
9	!!		.
8			^ ^ ^ **
7	* / div		
6	+ -		
5			: , ++
4		<comparison> elem	
3			&&
2			OR
1	» »=		
0			\$ \$! seq

Any [operator](#) lacking a [fixity declaration](#) is assumed to be [infixl](#) 9.

### 8.5.1 \*

Infixl Infixr Priority Precedence

## 8.6 Zero

\* is the value with which [operation](#) always yields [Zero](#) value.  $zero, n \in C : \forall n, zero * n = zero$

\* is distinct from [Identity](#) value.

## 8.7 Bind

Establishing equality between two [objects](#).

Most often:

- equating [variable](#) to a value.
- equating [parameter](#) of a [function](#) to an [argument](#) ([variable](#)/value/[function](#)). This term often is equated to [applying argument](#) to a [function](#), which includes [β-reduction](#).

### 8.7.1 \*

Binds Binding Bindings

## **8.8 Declaration**

[Binding](#) name to [expression](#).

## **8.9 Dispatch**

Sort-out & send.

## **8.10 Evaluation**

For FP see [Bind](#).



## Chapter 9

# Permutation

Bijjective function from domain to itself.

Domain & permutation functions & function composition form a group.

# Chapter 10

## Point-free

Paradigm [where function](#) only describes the [morphism](#) itself.

[Process](#) of converting [function](#) to [point-free](#). If brackets `()` can be changed to `$` then `$` equal to [composition](#):

```
\ x -> g (f x)
\ x -> g $ f x
\ x -> g . f $ x
\ x -> g . f      --eta-abstraction
```

```
\ x1 x2 -> g (f x1 x2)
\ x1 x2 -> g $ f x1 x2
\ x1 x2 -> g . f x1 $ x2
\ x1      -> g . f x1
```

### 10.1 \*

Pointfree Tacit Tacit programming

### 10.2 Blackbird

```
(.).(.) :: (b -> c) -> (a1 -> a2 -> b) -> a1 -> a2 -> c
```

[Composition of compositions](#) `(.).(.)`. Allows to [compose-in](#) a [binary function](#) `f1(c) (.).(.) f2(a,b)`.

```
\ f g x y -> f (g x y)
```

#### 10.2.1 \*

`.) . (.) . (.)` Composition of compositions

### 10.3 Swing

```
swing :: ((a -> b) -> b) -> c -> d -> c -> a -> d
swing = flip . (. flip id)
swing f = flip (f . runCont . return)
swing f c a = f ($ a) c
```

## 10.4 Squish

**f** >>= a . b . c =<< g

# Chapter 11

## Polymorphism

πολύς *polús* many

At once several forms.

In Haskell - [abstract](#) over [data types](#).

\* [types](#):

### 11.1 \*

Polymorphic

### 11.2 Levity polymorphism

Extending [polymorphism](#) to work with unlifted and lifted [types](#).

### 11.3 Parametric polymorphism

[Abstracting](#) over [data types](#) by [parameter](#).

*In most languages named as 'Generics' (generic programming).*

[Types](#):

#### 11.3.1 Rank-1 polymorphism

[Parametric polymorphism](#) in [rank-1 types](#) by [type variables](#).

##### 11.3.1.1 \*

Prenex Prenex polymorphism

#### 11.3.2 Let-bound polymorphism

It is [property](#) chosen for Haskell [type](#) system. Haskell is based on [Hindley-Milner type](#) system, it is [let-bound](#). To have strict [type inference](#) with \* - if [let](#) and [where](#) declarations are [polymorphic](#) -  $\lambda$  declarations - should be not.

See: [Good](#): In Haskell parameters bound by lambda declaration instantiate to only one concrete type.

### 11.3.3 Constrained polymorphism

Constrained [Parametric polymorphism](#).

#### 11.3.3.1 Ad hoc polymorphism

Artificial [constrained polymorphism](#) dependent on incoming [data type](#). It is [interface dispatch](#) mechanism of [data types](#). Achieved by creating a [type class instance functions](#).

*Commonly known as overloading.*

##### 11.3.3.1.0.1 \*

Ad-hoc polymorphism Ad hoc polymorphic Ad-hoc polymorphic Constraint Constraints

### 11.3.4 Impredicative polymorphism

\* allows [type](#)  $\lambda$  entities with [polymorphic types](#) that can contain [type](#)  $\lambda$  itself.  $T = \forall X. X \rightarrow X : T \in X \models T \in T$

The most powerful form of [parametric polymorphism](#). See: [Impredicative](#).

This approach has Girard's paradox ([type systems](#) [Russell's paradox](#)).

#### 11.3.4.1 \*

First-class polymorphism

### 11.3.5 Higher-rank polymorphism

Means that [polymorphic types](#) can appear within other [types](#) ([types of function](#)). There is a cases [where higher-rank polymorphism](#) than the a Ad hoc - is needed. For example [where ad hoc polymorphism](#) is used in [constraints](#) of several different implementations of [functions](#), and you want to build a [function](#) on top - and use the [abstract interface](#) over these [functions](#).

```
-- ad-hoc polymorphism
f1 :: forall a. MyType Class a => a -> String    ==    f1 :: MyType Class a => a -> String
f1 = -- ...

-- higher-rank polymorphism
f2 :: Int -> (forall a. MyType Class a => a -> String) -> Int
f2 = -- ...
```

By moving forall inside the [function](#) - we can achieve [higher-rank polymorphism](#).

From: <https://news.ycombinator.com/item?id=8130861>

Higher-rank polymorphism is formalized using System F, and there are a few implementations of (in

Useful example also a [ST-Trick monad](#).

#### 11.3.5.1 \*

Rank-n polymorphism

## 11.4 Subtype polymorphism

Allows to declare usage of a [Type](#) and all of its Subtypes.  $T$  - [Type](#)  $S$  - Subtype of [Type](#)  $<:$  - subtype of  $S <: T = S \leq T$

Subtyping is: If it can be done to T, and there is subtype S - then it also can be done to S.  $S <: T : f^{T \rightarrow X} \Rightarrow f^{S \rightarrow X}$

## 11.5 Row polymorphism

Is a lot like [Subtype polymorphism](#), but aligns itself on allowance (with | r) of subtypes and [types](#) with requested [properties](#).

```
printX :: { x :: Int | r } -> String
printX rec = show rec.x

printY :: { y :: Int | r } -> String
printY rec = show rec.y

-- type is inferred as `{x :: Int, y :: Int | r } -> String`
printBoth rec = printX rec ++ printY rec
```

## 11.6 Kind polymorphism

Achieved using a phantom [type argument](#) in the [data type declaration](#).

```
;;          * -> *
data Proxy a = ProxyValue
```

Then, by default the [data type](#) can be inhabited and fully work being partially defined. But multiple instances of [kind polymorphic type](#) can be distinguished by their particular [type](#).

Example is the [Proxy type](#):

```
data Proxy a = ProxyValue

let proxy1 = (ProxyValue :: Proxy Int) -- * :: Proxy Int
let proxy2 = (ProxyValue :: Proxy a)   -- * -> * :: Proxy a
```

## 11.7 Linearity polymorphism

Leverages [linear types](#). For example - if [fold](#) over a dynamic array:

1. In basic Haskell - array would be copied at every step.
2. Use low-level [unsafe functions](#).
3. With [Linear type function](#) we guarantee that the array would be used only at one place at a time.

So, if we use a [function](#) `(* -o * -o -o *)` in `foldr` - the [fold](#) will use the initial value only once.

## Chapter 12

# Compositionality

Complex [expression](#) is determined by the constituent [expressions](#) and the rules used to combine them.

If the meaning fully obtainable from the parts and [composition](#) - it is full, [pure compositionality](#).

If there exists [composed idiomatic expression](#) - it is unfull, unpure [compositionality](#), because meaning leaks-in from the sources that are not in the [composition](#).

### 12.1 \*

Principle of compositionality Composition Compositional

## Chapter 13

# Referential transparency

Given the same input return the same output. So: \* [expression](#) can be replaced with its corresponding resulting value without change for program's behavior. \* [functions](#) are [pure](#).

### 13.1 \*

Referentially transparent



# Chapter 14

## Semantics

Philosophical study of meaning. Meaning of symbols, words.

### 14.1 Operational semantics

Constructing proofs from logical [assertions](#) and verifying/checking/asserting things about execution and procedures their [properties](#), such as correctness, safety or security.

Good to solve in-point localized tasks.

Process of [abstraction](#).

#### 14.1.1 Argument

*arguere* make clearmake known, to prove, to shine

\* - evidence, proof, [statement](#) that results systematic changes.

##### 14.1.1.1 Argument of a function

A value binded to the [function parameter](#). Value/topic that the fuction would [process](#)/deal with.

Also see Argument.

##### 14.1.1.1.1 \*

Function argument

##### 14.1.1.2 First-class

Means *it*:

- Can be used as value.
- Passed as an [argument](#).

From 1&2 -> *it* can include itself.

#### 14.1.2 Relation

[Relationship](#) between two [objects](#). By default it is not directed and not limited. In [Set theory](#): some subset of a [Cartesian product](#) between [sets](#) of [objects](#).

## 14.1.2.1 \*

Relations Relationship

## 14.2 Denotational semantics

Construction of [objects](#), that describe/tag the meanings. In Haskell often [abstractions](#) that are ment (denotations), implemented directly in the code, sometimes exist over the code - allowing to reason and implement.

\* are [composable](#).

Good to achive more broad approach/meaning.

Also see [Abstraction](#).

### 14.2.1 Abstraction

abs away from, off (in absentia)

tractus draw, haul, drag

Purified generalization.

Forgetting the details ([axiomatic semantics](#)). Simplified approach. Out of sight - out of mind.

\* creates a new semantic level in which one can be absolutely precise ([operational semantics](#)).

It is a great did to name an [abstraction](#) ([denotational semantics](#)).

The ideal [abstractions](#) are:

- integrative (global):
  - [nothing](#), [void](#), emptiness - "none", [initial object](#)
  - everything - "all", "existance", [terminal object](#)
- [differential](#) (local):
  - point - "this", "is", "one", stasis
  - chaos - "any", "of", "many", [process](#)

They are ideal - because they are the [basis](#), the beginning. Because you can not express any other obstrac-tions without these.

+===

*This is personal idea & the thought of autor of the book regarding basic [abstractions](#) particularly. Other definitions in the book basing on this are the proof that [statement](#) has some ground truth in it. There is ongoing philosophical discussion on the topics like these.*

## 14.2.1.1 \*

Abstractions Abstracting Abstract

#### 14.2.1.2 Leaky abstraction

[Abstraction](#) that leaks details that it is supposed to [abstract](#) away.

## 14.2.1.2.1 \*

Leaky abstractions

**14.2.1.3 Object**

Absolute [abstraction](#).

Point that additionally can have [properties](#).

Often abstracts something, that is why it exposes external [properties](#) on [abstracting](#) something, for example some [structure](#), maybe mathematical. In this book [objects](#) represent [algebraic structures](#), as we are talking about Haskell and [Category](#) theory.

[Objects](#) without [process](#) are in [constant](#) state.

**14.2.1.3.1 \***

Structure Structures Objects

**14.2.1.3.2 Arrow**

Second level of absolute [abstraction](#).

[Arrow](#).

Can have target, can have source. Both often are [objects](#).

Often abstracts [process](#).

Can have [properties](#).

Also alias in [Category](#) Theory for "[morphism](#)", thou theory imposes [properties](#).

**14.2.1.3.2.1 \***

Arrows Process

**14.2.1.3.3 Terminal object**

One that receives unique [arrow](#) from every [object](#).

$\exists! : x \rightarrow 1 \mid \exists 1 \in \mathcal{C}, \forall x \in \mathcal{C}$

\* is an empty [sequence](#) `()` in Haskell.

Called a [unit](#), so receives *terminal* or [unit arrow](#).

[Dual](#) of [initial object](#).

Denotation:

[Category](#) theory 1

Haskell

`()`

**14.2.1.3.4 Initial object**

One that emits unique [arrow](#) into every [object](#).

$\exists! : \emptyset \rightarrow x \mid \exists \emptyset \in \mathcal{C}, \forall x \in \mathcal{C}$

If [initial object](#) is `Void` (most frequently) - emitted [arrows](#) called absurd, because they can not be called.

[Dual](#) of [terminal object](#).

Denotation:

Category theory:  $\emptyset$

Haskell:

Void

#### 14.2.1.3.5 Value

What **object** abstracts. Without any **object** external **structure** (aka **identity** in **Category Theory**). So **\*** is immutable. Such heresy is called "Value **semantics**" and leads such things as **referential transparency**, functional programming and Haskell.

(Except, when you hack Haskell with explicit low-level functions, and start to directly mute values - then you are on your own, Haskell paradigm does not expect that.)

##### 14.2.1.3.5.1 \*

Value **semantics** Values

#### 14.2.1.3.6 Tensor

**Object** existing out of planes, thus it can translate **objects** from one plane into another. **\*** can be tried to be described with knowledge existing inside planes (from projection on the plane), but representation would always be partial.

**Tensor** of rank 1 is a vector.

Translations with **tensor** can be seen as **functors**.

##### 14.2.1.3.6.1 \*

Tensors Tensorial

### 14.2.2 Ambigram

ambi both

γράμμα *grámma* written character

**Object** that from different points of view has the same meaning.

While this word has two contradictory diametrically opposite usages, one was chosen (more frequent).

But it has... Both.

*TODO: For merit of differentiating the meaning about different meaning referring to **Tensor** as **object** with many meanings.*

### 14.2.3 Binary

Two of something.

### 14.2.4 Arbitrary

*arbitrarius* uncertain

Random, any one of.

Used as: Any one with *this* **set** of **properties**. (**constraints**, **type**, etc.).

When there is a talk about any **arbitrary** value - in fact it is a talk about the generalization of computations over the **set** of **properties**.

### 14.2.5 Refutable

One that has an option to fail.

### 14.2.6 Irrefutable

One that can not fail.

### 14.2.7 Superclass

Broader parent class.

### 14.2.8 Unit

Represents existence. Denoted as empty [sequence](#).

()

Type () holds only self-representation [constructor](#) (), & [constructor](#) holds [nothing](#).

Haskell code always should receive something back, hence [nothing](#), emptiness, [void](#) can not be theoretically addressed, practically constructed or received - [unit](#) in Haskell also has a role of a stub in place of emptiness, like in IO ().

### 14.2.9 Nullary

Takes no entries (for example has the [arity](#) of [zero](#)). Has the trivial [domain](#).

### 14.2.10 Syntax tree

Tree of syntactic elements (each [node](#) denotes [construct](#) occurring in the language/source code) that represent the full particular [expression](#)/implementation (or said).

#### 14.2.10.1 Abstract syntax tree

"Abstract" since does not represent every detail of the syntax (ex. parentheses), but rather concentrates on [structure](#) and content.

Widely used in compilers to check the code [structure](#) for accuracy and coherence.

##### 14.2.10.1.1 \*

AST

#### 14.2.10.2 Concrete syntax tree

An ordered, rooted [syntax tree](#) that represents the syntactic [structure](#) of a string according to some [context-free grammar](#).

"Concrete" since (in contrast to "abstract") - concretely reflects the syntax of the input language.

##### 14.2.10.2.1 \*

Parse tree Derivation tree

### 14.2.11 Stream

\* an infinite [sequence](#) that forgets previous [objects](#), and remembers only currently relevant [objects](#).

$E \mid X \rightarrow (X \times A + 1)$ , the [set](#) (or [object](#)) of streams on A (final [coalgebra](#)  $A_*$  of  $E$ ).

[cycle](#) is one of [stream functions](#).

[a](#) = (cycle [[Nothing](#), [Nothing](#), [Just](#) "[Fizz](#)"])

[b](#) = (cycle [[Nothing](#), [Nothing](#), [Nothing](#), [Nothing](#), [Just](#) "[Buzz](#)"])

Can be:

- indexed, timeless, with current [object](#)
- timed:

\* [([timescale](#), [event](#))] \* [([realtime](#), [event](#))]

Has amalgamation with Functional Reactive Programming.

### 14.2.12 Linear

Values consumed once or not used.

$x^2$  consumes/uses x two times ( $x*x$ ).

#### 14.2.12.1 \*

Linearity

### 14.2.13 Predicative

Non-self-referencing definition.

+===

Antonym - [Impredicative](#).

### 14.2.14 Quantifier

Specifies the quantity of specimens.

Two most common [quantifiers](#)  $\forall$  ([Forall](#)) and  $\exists$  (Exists).  $\exists!$  - one and only one (exists only unique).

Turns [predicate](#) into [statement](#).

#### 14.2.14.1 \*

Quantification Quantifiers Quantified

#### 14.2.14.2 Forall quantifier

Permits to not [infer](#) the [type](#), but to use any that fits. The variant depends on the [LANGUAGE option](#) used:

- [ScopedTypeVariables](#)
- [RankNTypes](#)
- [ExistentialQuantification](#)

## 14.2.14.2.1 \*

Forall

## 14.3 Axiomatic semantics

Empirical [process](#) of studying something complex by finding and analyzing true [statements](#) about it.

Good for examining interconnections.

### 14.3.1 Property

Something has a [property](#) in the real world, and [property](#) always yealds an axiom (law) for something.

Meaningful [abstraction](#) denotation always defines through [properties](#) (axioms for that definition).

[Abstraction](#) forms nicely around the boundaries [where](#) the particular [properties](#) spread. [Properties](#) inside [abstraction](#) may have emergence [effect](#) (combination of [properties](#) result into bigger [property](#)), so in that way [abstracting](#) them simplifies outside picture, as [abstraction](#) hides plethora of internal [properties](#) and exposes only emergent [properties](#).

In Haskell under [property/law](#) most often [properties](#) of [algebraic structures](#).

There [property testing](#) wich does what it says.

## 14.3.1.1 \*

Properties

#### 14.3.1.2 Associativity

Joined with common purpose.

$$P(a, P(b, c)) \equiv P(P(a, b), c) \mid \forall (a, b, c) \in S,$$

\* - the operations can be grouped arbitrarily.

[Property](#) that determines how operators of the same [precedence](#) are grouped, (in computer science also in the absence of parentheses).

Etymology: Latin *associatus* past participle of *associare* "[join with](#)", from assimilated form of *ad* "[to](#)" + *sociare* "[unite with](#)", from *socius* "[companion, ally](#)" from PIE *\*sokw-yo-*, suffixed form of root *\*sekw-* "[to follow](#)".

In Haskell \* has influence on parsing when compounds have same [fixity](#).

## 14.3.1.2.1 \*

Associative Associative property Associativity property

#### 14.3.1.3 Left associative

\* - the operations are grouped from the left.

Example: In lambda [expressions](#) same level parts follow grouping from left to right.  $(\lambda x.x)(\lambda y.y)z \equiv ((\lambda x.x)(\lambda y.y))z$

## 14.3.1.3.1 \*

Left associativity Left-associative

**14.3.1.4 Right associative**

\* - the operations are grouped from the right.

**14.3.1.5 Non-associative**

Operations can't be chained.

Often is the [case](#) when the output [type](#) is incompatible with the input [type](#).

**14.3.1.6 Basis**

*βασίς* - stepping

The initial point, unreducible axioms and terms that spawn a theory. AKA see [Category](#) theory, or Euclidian geometry [basis](#).

**14.3.1.6.1 Contravariant**

The [property](#) of [basis](#), in which if new [basis](#) is a [linear](#) combination of the prior [basis](#), and the change of [basis](#) inverse-proportional for the description of a [Tensors](#) in this basis.

Denotation: Components for [contravariant basis](#) denoted in the upper indices:  $V^i = x$

The [inverse](#) of a [covariant](#) transformation is a [contravariant](#) transformation. Whenever a vector should be invariant under a change of [basis](#), that is to say it should represent the same geometrical or physical [object](#) having the same magnitude and direction as before, its components must transform according to the [contravariant](#) rule.

**14.3.1.6.1.1 \***

Contravariant cofunctor Contravariant functor - More inline term is [Contravariant cofunctor](#)

**14.3.1.6.2 Covariant**

The [property](#) of [basis](#), in which if new [basis](#) is a [linear](#) combination of the prior [basis](#), and the change of [basis](#) proportional for a descriptions of [tensors](#) in basis.

Denotation: Components for [covariant basis](#) denoted in the upper indices:  $V_i = x$

**14.3.1.6.2.1 \***

Covariant functor Covariant cofunctor

**14.3.1.7 Commutativity**

$\forall(a, b) \in S : P(a, b) \equiv P(b, a)$

All processes that are independent from one-another, but on manifestation of their results - their combination result into something (create circumstances for something) - are [commutative](#).

**14.3.1.7.1 \***

Commutative Commutative property

**14.3.1.8 Idempotence**

First [application](#) gives a result. Then same [operation](#) can be [applied](#) multiple times without changing the result. Example: Start and Stop buttons on machines.



**14.3.1.8.1 \***

Idempotent Idempotency

**14.3.1.9 Distributive property**

Set  $S$  and two binary operators  $+$   $\times$ :

- $x \times (y + z) = (x \times y) + (x \times z)$  -  $\times$  is left-distributive over  $+$
- $(y + z) \times x = (y \times x) + (z \times x)$  -  $\times$  is right-distributive over  $+$
- left-&right-distributive -  $\times$  is distributive over  $+$

**14.3.1.9.1 \***

Distributive rule Distributive axiom Distributive property Distributive

**14.3.2 Effect**

Observable action.

**14.3.3 Bisimulation**

When systems have exact external behaviour so for observer they are the same.

Binary relation between state transition systems that match each other's moves.

**14.3.3.1 \***

Bisimilar

**14.4 Content word**

Words that name objects of reality and their qualities.

**14.5 Ancient Greek and Latin prefixes****14.5.1 \***

Greek prefix Latin prefix

**14.6 Idiom**

\* - something having a meaning that can not be derived from the conjoined meanings of \* constituents.  
Meaning can be special for language speakers or human with particular knowledge.

\* can also mean applicative functor, people better stop making idiom from the term "idiom".

**14.6.1 \***

Idioms Idiomatic

Table 14.1: Ancient Greek and Latin prefixes

Meaning	Greek prefix	Latin prefix
above, excess	hyper-	super-, ultra-
across, beyond, through	dia-	trans-
after		post-
again, back		re-
against	anti-	contra-, (in-, ob-)
all	pan	omni-
around	peri-	circum-
away or from	apo-, ap-	ab- (or de-)
bad, difficult, wrong	dys-	mal-
before	pro-	ante-, pre-
between, among		inter-
both	amphi-	ambi-
completely or very		de-, ob-
down		de-, ob-
four	tetra-	quad-
good	eu-	ben-, bene-
half, partially	hemi-	semi-
in, into	en-	il-, im-, in-, ir-
in front of	pro-	pro-
inside	endo-	intra-
large	macro-	(macro-, from Greek)
many	poly-	multi-
not*	a-, an-	de-, dis-, in-, ob-
on	epi-	
one	mono-	uni-
out of	ek-	ex-, e-
outside	ecto-, exo-	extra-, extro-
over	epi-	ob- (sometimes)
self	auto-, aut-, auth-	ego-
small	micro-	
three	tri-	tri-
through	dia-	trans-
to or toward	epi-	ad-, a-, ac-, as-
two	di-	bi-
under, insufficient	hypo-	sub-
with	sym-, syn-	co-. com-, con-
within, inside	endo-	intra-
without	a-, an-	dis- (sometimes)

## 14.7 Impredicative

Self-referencing definition.

====

Antonym - *Predicative*.

## 14.8 Context-free grammar

Type of formal grammar that is: a set of production rules that describe all possible string in a given formal language.

Term is invented by Noam Chomsky.

### 14.8.1 \*

CFG

# Chapter 15

## Set

Well-defined collection of distinct [objects](#).

### 15.1 \*

Sets Set theory

### 15.2 Closed set

1. [Set](#) which complements an open [set](#).
2. Is form of [Closed-form expression](#). [Set](#) can be [closed](#) in under a [set](#) of operations.

### 15.3 Power set

For some [set](#)  $S$ , the [power set](#) ( $\mathcal{P}(S)$ ) is a [set](#) of all subsets of  $S$ , including  $\{\}$  &  $S$  itself.

Denotation:  $\mathcal{P}(S)$

### 15.4 Singleton

[Singleton](#) - [unit set](#) - [set](#) with exactly one element. Also 1-[sequence](#).

### 15.5 Russell's paradox

If there exists normal [set](#) of all [sets](#) - it should contain itself, which makes it abnormal.

### 15.6 Cartesian product

$\mathcal{A} \times \mathcal{B} \equiv \sum^{\forall} (a, b) \mid \forall a \in \mathcal{A}, \forall b \in \mathcal{B}$ . [Operation](#), returns a [set](#) of all ordered pairs  $(a, b)$

Any [function](#), [functor](#) is a subset of [Cartesian product](#).

$\sum (elem \in (\mathcal{A} \times \mathcal{B})) = cardinality^{\mathcal{A} \times \mathcal{B}}$

[Properties](#):

- not [associative](#)

- not commutative

### 15.6.1 Pullback

Subset of the cartesian product of two sets.

#### 15.6.1.1 \*

Pullbacks

# Chapter 16

# Testing

## 16.1 Property testing

Since [property](#) yealds the according [law](#), family of [unit](#) tests for the [property](#) can be abstracted into the [function](#) that test the [law](#).

[Unit](#) test cases come from [generator](#), and test the [law](#) empirically, but repeatedly and automatically.

### 16.1.1 Function property

[Property](#) corresponds to the according [law](#). In [property testing](#) you need to think additionally about [generator](#) and [shrinking](#).

### 16.1.2 Property testing type

Table 16.1: [Property testing types](#)

	Exhaustive	Randomized	<a href="#">Unit</a> test
Whole <a href="#">set</a> of values	Exhaustive <a href="#">property</a> test	Randomised <a href="#">property</a> test	One elem
Special subset of values	Exhaustive specialised <a href="#">property</a> test	Randomised specialised <a href="#">property</a> test	One elem

### 16.1.3 Generator

Seed  
|  
v  
Gen A -> A  
^  
|  
Size

Seed allows reproducibility. There is anyway a need to have some seed. Size allows setting upper [bound](#) on size of generated value. Think about infinity of [list](#).

After failed test - [shrinking](#) tests value parts of contrexample, finds a part that still fails, and recurses [shrinking](#).

#### 16.1.3.1 \*

Generators

### 16.1.3.2 Custom generator

When certain theorem only works for a specific [set](#) of values - the according [generator](#) needs to be produced.

```
arbitrary :: Arbitrary a => Gen a
suchThat :: Gen a -> (a -> Bool) -> Gen a
elements :: [a] -> Gen a
```

### 16.1.4 Reusing test code

Often it is convinient to [abstract testing](#) of same [function properties](#):

It can be done with (aka TestSuite [combinator](#)):

```
-- Definition
{-# LANGUAGE ScopedTypeVariables #-}
{-# LANGUAGE AllowAmbiguousTypes #-}
eqSpec :: forall a. Arbitrary a => Spec

-- Usage
{-# LANGUAGE TypeApplications #-}
spec :: Spec
spec = do
  eqSpec @Int

Eq Int
(==) :: Int -> Int -> Bool
  is reflexive
  is symetric
  is transitive
  is equivalent to (\ a b -> not $ a /= b)
(/=) :: Int -> Int -> Bool
  is antireflexive
  is equivalent to (\ a b -> not $ a == b)
```

#### 16.1.4.1 Test Commutative property

[Commutativity](#)

```
:: Arbitrary a => (a -> a -> a) -> Property
```

#### 16.1.4.2 Test Symmetry property

[Symmetry](#)

```
:: Arbitrary a => (a -> a -> Bool) -> Property
```

#### 16.1.4.3 Test Equivalence property

[Equivalence](#)

```
:: (Arbitrary a, Eq b) => (a -> b) -> (a -> b) -> Property
```

#### 16.1.4.4 Test Inverse property

```
:: (Arbitrary a, Eq b) => (a -> b) -> (b -> a) -> Property
```

### 16.1.5 QuickCheck

Target is a member of the [Arbitrary type class](#). Target -> Bool is something Testable. This [properties](#) can be complex. [Generator](#) arbitrary gets the seed, and produces values of Target. [Function](#) quickCheck runs the loop and tests that generated Target values always comply the [property](#).

#### 16.1.5.1 Manual automation with QuickCheck properties

```
import Test.QuickCheck
import Test.QuickCheck.Function
import Test.QuickCheck.Property.Common
import Test.QuickCheck.Property.Functor
import Test.QuickCheck.Property.Common.Internal

data Four' a b = Four' a a a b
    deriving (Eq, Show)

instance Functor (Four' a) where
    fmap f (Four' a b c d) = Four' a b c (f d)

instance (Arbitrary a, Arbitrary b) => Arbitrary (Four' a b) where
    arbitrary = do
        a1 <- arbitrary
        a2 <- arbitrary
        a3 <- arbitrary
        b <- arbitrary
        return (Four' a1 a2 a3 b)

-- Wrapper around `prop_FunctorId`
prop_AutoFunctorId :: Functor f => f a -> Equal (f a)
prop_AutoFunctorId = prop_FunctorId T

type Prop_AutoFunctorId f a
    = f a
    -> Equal (f a)

-- Wrapper around `prop_AutoFunctorCompose`
prop_AutoFunctorCompose :: Functor f => Fun a1 a2 -> Fun a2 c -> f a1 -> Equal (f c)
prop_AutoFunctorCompose f1 f2 = prop_FunctorCompose (applyFun f1) (applyFun f2) T

type Prop_AutoFunctorCompose structureType origType midType resultType
    = Fun origType midType
    -> Fun midType resultType
    -> structureType origType
    -> Equal (structureType resultType)

main = do
    quickCheck $ eq $ (prop_AutoFunctorId :: Prop_AutoFunctorId (Four' ()))Integer
    quickCheck $ eq $ (prop_AutoFunctorId :: Prop_AutoFunctorId (Four' ())) (Either Bool String)
    quickCheck $ eq $ (prop_AutoFunctorCompose :: Prop_AutoFunctorCompose (Four' ())) String Integer
    quickCheck $ eq $ (prop_AutoFunctorCompose :: Prop_AutoFunctorCompose (Four' ())) Integer String
```

## 16.2 Write tests algorithm

1. Pick the right language/[stack](#) to implement features.



2. How expensive breakage can be.
3. Pick the right tools to test this.

## 16.3 Shrinking

**Process** of reducing complexity in the test **case** - re-run with smaller values and make sure that the test still fails.

# Chapter 17

## Logic

### 17.1 Proposition

Purely abstract & theoretical logical **object** (idea) that has a Boolean value.

\* is expressed by a **statement**.

#### 17.1.1 \*

Propositions

#### 17.1.2 Atomic proposition

Logically undividable **unit**. Does not contain **logical connectives**.

Often abstracts the non-logical ("real") **objects**.

##### 17.1.2.1 \*

Atomic propositions

#### 17.1.3 Compound proposition

Formed by connecting **propositions** by **logical connectives**.

##### 17.1.3.1 \*

Compound propositions

#### 17.1.4 Propositional logic

Studies **propositions** and **argument** flow.

Distinguishes logically indivisible units (**atomic propositions**) and abstracts them as **variable** and **constant** values of Boolean **type properties**. Studies consequences of those value arguments (**atomic proposition**) on **composed propositions**.

Invented by Aristotle.

Classical system that lead humanity to profound advancement. The classical \* language thou is limited and so in modern day extensions (mainly **First-order logic**) are used in the sciences.

Not Turing-complete, for example it is impossible to **construct** an **arbitrary** loop.

**17.1.4.1 \***

Proposition logic Proposition calculus Propositional calculus Statement logic Sentential logic Sentential calculus Zeroth-order logic

**17.1.4.2 First-order logic**

Extension of a [propositional logic](#) that adds [quantifiers](#).

Turing-complete.

**17.1.4.2.1 \***

Predicate logic First-order predicate logic First-order predicate calculus

**17.1.4.2.2 Second-order logic**

Extension over [first-order logic](#) that quantifies over [relations](#).

**17.1.4.2.2.1 Higher-order logic**

Extension over [second-order logic](#) that uses additional [quantifiers](#), stronger [semantics](#).

Is more expressive, but model-theoretic [properties](#) are less well-behaved.

**17.2 Logical connective**

Logical [operation](#).

**17.2.1 \***

Logical connectives

**17.2.2 Conjunction**

Logical AND.

Denotation:  $\wedge$

Multiplies [cardinalities](#).

Haskell [kind](#):

[\\*](#) [\\*](#)

**17.2.3 Disjunction**

Logical OR Denotation:  $\vee$

Summs [cardinalities](#).

**17.3 Predicate**

[Function](#) with Boolean [codomain](#).  $P : X \rightarrow \{true, false\}$  - [\\*](#) on  $X$ .

Notation:  $P(x)$

Im many cases includes [relations](#), [quantifiers](#).

## 17.4 Statement

Declarative **expression** that is a bearer of a **proposition**.

When we talk about **expression** or **statement** being true/false - in fact we refer to the **proposition** that they represent.

Difference between **proposition**, **statement**, **expression**:

1. " $2 + 3 = 5$ "
2. "two plus three equals five"
  - 1 & 2 are **statements**. Each of them is a collection of transmission symbols (linguistic **objects**) from a symbol systems  $\equiv$  **expression**. Each of them is **expression** that bears **proposition** (an idea resulting in a Boolean value)  $\equiv$  **statement**.
  - 1 & 2 represent the same **proposition**. **Proposition** from 1  $\equiv$  **proposition** from 2.
  - **Statement** 1  $\neq$  **statement** 2. They are two different **statements**, written in different systems. And **statement** " $2 + 3 = 5$ "  $\neq$  **statement** " $3 + 2 = 5$ ".

### 17.4.1 \*

Assertion Assertions Statements

## 17.5 Iff

If and only if, exactly when, just. Denotation:  $\iff$

# Chapter 18

## Haskell **structure**

### 18.1 \*

Haskell **structures**

### 18.2 Pattern match

Are not **first-class**. It is a **set** of patter match semantic notations.

Must be **linear**.

\* **precedence** (especially with more then one **parameter**, especially with **\_** used) often changes the **function**.

#### 18.2.1 As-pattern

```
f list@(x, xs) = ...  
f (x:xs)      = x:x:xs -- Can be compiled with reconstruction of x:xs  
f a@(x:_)    = x:a -- Reuses structure without reconstruction
```

##### 18.2.1.1 \*

As-patterns As pattern As patterns

#### 18.2.2 Wild-card

Matches anything and can not be binded. For matching someting that should pass not checked and is not used.

```
head (x:_)      = x  
tail (_:xs)     = xs
```

##### 18.2.2.1 \*

Wild-cards Wildcard Wildcards

#### 18.2.3 Case

```
case x of  
  pattern1 -> ex1  
  pattern2 -> ex2
```

```

pattern3 -> ex3
otherwise -> exDefault

```

Bolting **guards** & **expressions** with syntatic sugar on **case**:

```

case () of _
| expr1      -> ex1
| expr2      -> ex2
| expr3      -> ex3
| otherwise  -> exDefault

```

Pattern matching in **function** definitions is realized with **case expressions**.

### 18.2.4 Guard

Check values against the **predicate** and use the first match definition:

```

f x
| predicate1 = definition1
| predicate2 = definition2
...
| x < 0      = definitionN
...
| otherwise  = definitionZ

```

#### 18.2.4.1 \*

Guards

### 18.2.5 Pattern guard

Allows check a **list** of pattern matches against **functions**, and then proceed.

```
(patternMatch1) <- (funcCheck1)
```

```
, (patternMatch2) <- (funcCheck2) = RHS
```

```
lookup :: Eq a => a -> [(a, b)] -> Maybe b
```

```

addLookup l a1 a2
| Just b1 <- lookup a1 l
, Just b2 <- lookup a2 l
= b1 + b2
{-...other equations...-}

```

Run **functions**, they must succeed. Then **pattern match** results to b1, b2. Only if successful - execute the equation.

Default in Haskell 2010.

#### 18.2.5.1 \*

Pattern guards

### 18.2.6 Lazy pattern

Defers the **pattern match** directly to the last moment of need during execution of the code.

```

f (a, b) = g a b -- It would be checked that the pattern of the pair constructor
-- is present, and that parameters are present in the constructor.
-- Only after that success - work would start on the RHS, aka then construction

```

```
-- g would start only then.
```

```
f ~(a, b) = g a b -- Pattern match of (a, b) deferred to the last moment,
-- RHS starts, construction of g starts.
-- For this lazy pattern the equivalent implementation would be:
-- f p = g (fst p) (snd p) -- RHS starts, during construction of g
-- the arguments would be computed and found, or error would be thrown.
```

Due to full laziness deferring everything to the runtime execution - the [lazy pattern](#) is one-size-fits all ([irrefutable](#)), analogous to `_`, and so it does not produce any checks during compilation, and raises [errors](#) during runtime.

`*` is very useful during [recursive](#) construction of [recursive structure/process](#), especially infinite.

#### 18.2.6.1 \*

Lazy-pattern Lazy patterns

### 18.2.7 Pattern binding

Entire [LHS](#) is a pattern, is a [lazy pattern](#).

```
fib@(1:tfib) = 1 : 1 : [ a+b | (a,b) <- zip fib tfib ]
```

#### 18.2.7.1 \*

Pattern bindings

## 18.3 Smart constructor

[Process](#)/code placing extra rules & [constraints](#) on the construction of values.

## 18.4 Level of code

There are these levels of Haskell code:

### 18.4.1 \*

Code level

### 18.4.2 Type level

[Level of code](#) that works with [data types](#).

#### 18.4.2.1 Type level declaration

```
type ...
newtype ...
data ...
class ...
instance ...
```

##### 18.4.2.1.1 \*

Type level declarations Type-level declaration Type-level declarations

**18.4.2.2 Type check**

if The [type level](#) information is complete ([strongly connected](#) graph)

then

Generalize the [types](#) and check if [type level](#) consistent to [term level](#).

else

[Infer](#) the missing [type level](#) part from the [term level](#). There are certain situations and [structures where](#) ambiguity arises and is unsolvable from the information of the [term level](#) (most basic example is [polymorphic recursion](#)).

**18.4.2.2.1 \***

Typecheck Typechecking Typechecks

**18.4.2.2.2 Complete user-specific kind signature**

[Type level declaration](#) is considered to "have a [CUSK](#)" if it has enough syntactic information to warrant completeness ([strongly connected](#) graph) and start checking [type level](#) correspondence to [term level](#), it is a ad-hock state of [type inferring](#).

In the future GHC would use other algorithm over/instead of [CUSK](#).

**18.4.2.2.2.1 \***

CUSK CUSKs Complete user-specific kind signatures Complete, user-specific kind signature

**18.4.3 Term level**

[Level of code](#) that does logical execution.

**18.4.4 Compile level**

[Level of code](#), about compilation processes/results.

**18.4.4.1 \***

Compilation level

**18.4.5 Runtime level**

[Level of code](#) of main program [operation](#), when machine does computations with compiled [binary](#) code.

**18.4.6 Kind level**

[Level of code where kinds & kind](#) declarations are situated, inferred and checked.

**18.4.6.1 Kind check**

[Applying](#) the [type check](#) to [kind](#) check:

if The [kind](#) level information is complete ([strongly connected](#) graph)

then

Check if [kind](#) level consistent to [term level](#).

else



Infer the missing [kind](#) level parts from the [type level](#). There are certain situations and [structures where](#) ambiguity arises and is unsolvable from the information of the [kind](#) level.

With StandaloneKindSignatures [kind](#) completeness happens against found (standalone) [kind](#) signature.

With CUSKs extension kind completeness happens against "[complete user-specific kind signature](#)"

#### 18.4.6.1.1 \*

Kindcheck Kind checks

## 18.5 Orphan instance

Situation when [module](#) provides [type class](#) but does not provide instance for some publically used [type](#).

That allows/pushes to implement own version of instance. If upstream would add instance - now upstream instance and own instance exist. Locally that would create instance clash. Remotely, through modules usage - that should create inconsistency problems in computations, since instances most probably not [bisimilar](#).

If [module](#) has any orphans - in GHC terms all [module](#) is called "orphan [module](#)". GHC "visits the [interface](#) file of every orphan [module](#) below the [module](#) being compiled. This is usually wasted work, but there is no avoiding it. You should therefore do your best to have as few orphan modules as possible" ("[GHC User's Guide Documentation, Release 8.8.3](#)"). So orphan prolongs the compilation, and moreover - compilation of all dependent code, because requires [recursive](#) lookups into [module](#) dependencies.

See: [Good: Handling orphan instance](#).

## 18.6 undefined

Placeholder value that helps to do [typechecking](#).

## 18.7 Hierarchical module name

Hierarchical naming scheme:

```
Algebra                -- Was this ever used?
  DomainConstructor    -- formerly DoCon
  Geometric             -- formerly BasGeomAlg

Codec                  -- Coders/Decoders for various data formats
  Audio
    Wav
    MP3
    ...
  Compression
    Gzip
    Bzip2
    ...
  Encryption
    DES
    RSA
    BlowFish
    ...
```

```

Image
  GIF
  PNG
  JPEG
  TIFF
  ...
Text
  UTF8
  UTF16
  ISO8859
  ...
Video
  Mpeg
  QuickTime
  Avi
  ...
Binary                                -- these are for encoding binary data into text
  Base64
  Yenc

Control
  Applicative
  Arrow
  Exception                          -- (opt, inc. error & undefined)
  Concurrent                         -- as hslibs/concurrent
    Chan                            -- these could all be moved under Data
    MVar
    Merge
    QSem
    QSemN
    SampleVar
    Semaphore
  Parallel                          -- as hslibs/concurrent/Parallel
    Strategies
  Monad                             -- Haskell 98 Monad library
    ST                              -- ST defaults to Strict variant?
      Strict                        -- renaming for ST
      Lazy                          -- renaming for LazyST
    State                           -- defaults to Lazy
      Strict
      Lazy
    Error
    Identity
    Monoid
    Reader
    Writer
    Cont
    Fix                             -- to be renamed to Rec?
    List
    RWS

Data
  Binary                            -- Binary I/O
  Bits
  Bool                             -- &&, ||, not, otherwise

```

```

Tuple          -- fst, snd
Char           -- H98
Complex        -- H98
Dynamic
Either
Int
Maybe         -- H98
List           -- H98
PackedString
Ratio          -- H98
Word
IORef
STRef          -- Same as Data.STRef.Strict
    Strict
    Lazy       -- The lazy version (for Control.Monad.ST.Lazy)
Binary         -- Haskell binary I/O
Digest
    MD5
    ...        -- others (CRC ?)
Array          -- Haskell 98 Array library
    Unboxed
    IArray
    MArray
    IO         -- mutable arrays in the IO/ST monads
    ST
Trees
    AVL
    RedBlack
    BTree
Queue
    Bankers
    FIFO
Collection
Graph          -- start with GHC's DiGraph?
FiniteMap
Set
Memo           -- (opt)
Unique

Edison         -- (opt, uses multi-param type classes)
    Prelude    -- large self-contained packages should have
    Collection -- their own hierarchy? Like a vendor branch.
    Queue      -- Or should the whole Edison tree be placed

Database
    MySQL
    PostgreSQL
    ODBC

Dotnet
    ...        -- Mirrors the MS .NET class hierarchy

Debug          -- see also: Test
    Trace
    Observe    -- choose a default amongst the variants

```

```

    Textual          -- Andy Gill's release 1
    ToXmlFile        -- Andy Gill's XML browser variant
    GHood            -- Claus Reinke's animated variant

Foreign
  Ptr
  StablePtr
  ForeignPtr  -- rename to FinalisedPtr? to void confusion with Foreign.Ptr
  Storable
  Marshal
    Alloc
    Array
    Errors
    Utils
  C
    Types
    Errors
    Strings

GHC
  Exts          -- hslibs/lang/GlaExts
  ...

Graphics
  HGL
  Rendering
    Direct3D
    FRAN
    Metapost
    Inventor
    Haven
    OpenGL
      GL
      GLU
  Pan
  UI
    FranTk
    Fudgets
    GLUT
    Gtk
    Motif
    ObjectIO
    TkHaskell
  X11
    Xt
    Xlib
    Xmu
    Xaw

Hugs
  ...

Language
  Haskell      -- hslibs/hssource
  Syntax

```

```

    Lexer
    Parser
    Pretty
HaskellCore
Python
C
...

Nhc
...

Numeric                -- exports std. H98 numeric type classes
  Statistics

Network                -- (== hslibs/net/Socket), depends on FFI only
  BER                  -- Basic Encoding Rules
  Socket               -- or rename to Posix?
  URI                  -- general URI parsing
  CGI                  -- one in hslibs is ok?
  Protocol
    HTTP
    FTP
    SMTP

Prelude                -- Haskell98 Prelude (mostly just re-exports
                        -- other parts of the tree).

Sound                  -- Sound, Music, Digital Signal Processing
  ALSA
  JACK
  MIDI
  OpenAL
  SC3                  -- SuperCollider

System                 -- Interaction with the "system"
  Cmd                  -- ( system )
  CPUTime              -- H98
  Directory            -- H98
  Exit                 -- ( ExitCode(..), exitWith, exitFailure )
  Environment          -- ( getArgs, getProgName, getEnv ... )
  Info                 -- info about the host system
  IO                   -- H98 + IOExts - IOArray - IORef
    Select
    Unsafe             -- unsafePerformIO, unsafeInterleaveIO
  Console
    GetOpt
    Readline
  Locale               -- H98
  Posix
    Console
    Directory
    DynamicLinker
      Prim
      Module
    IO

```

```

    Process
    Time
Mem      -- rename from cryptic 'GC'
    Weak      -- (opt)
    StableName -- (opt)
Time      -- H98 + extensions
Win32     -- the full win32 operating system API

Test
    HUnit
    QuickCheck

Text
    Encoding
        QuotedPrintable
        Rot13
    Read
        Lex      -- cut down lexer for "read"
    Show
        Functions -- optional instance of Show for functions.
    Regex      -- previously RegexString
        Posix     -- Posix regular expression interface
    PrettyPrint -- default (HughesPJ?)
        HughesPJ
        Wadler
        Chitil
        ...
    HTML      -- HTML combinator lib
    XML
        Combinators
        Parse
        Pretty
        Types
    ParserCombinators -- no default
        ReadP      -- a more efficient "ReadS"
        Parsec
        Hutton_Meijer
        ...

Training      -- Collect study and learning materials
    <name of the tutor>

```

### 18.7.1 \*

Top-level module name Top-level module names

## 18.8 Reserved word

Haskell has special meaning for:

case, class, data, deriving, do, else, if, import,  
in, infix, infixl, infixr, instance, let,  
of, module, newtype, then, type, where

### 18.8.1 \*

Reserved words

### 18.8.2 import

`import statement` by default imports identifiers from the other [module](#), using [hierarchical module name](#), brings into [scope](#) the identifiers to the global [scope](#) both into unqualified and qualifies by the [hierarchical module name](#) forms.

This possibilities can mix and match:

- `<modName> ()` - [import](#) only instances of [type classes](#).
- `<modName> (x, y)` - [import](#) only declared indentifiers.
- `qualified <modName>` - discards unqualified names, force obligatory namespace for the imports.
- `hiding (x, y)` - skip [import](#) of declared identifies.
- `<modName> as <modName>` - renames [module](#) namespace.
- `<type/class> (..)` - [import](#) class & it's methods, or [type](#), all its data [constructors](#) & field names.

### 18.8.3 let

\* [expression](#) is a [set](#) of cross-recursive lazy pattern bindings.

Declarations permitted:

- [type](#) signatures
- [function bindings](#)
- [pattern bindings](#)

It is an [expression](#) (macro) and that integrates in external [lexical scope expression](#) it [applied](#) in.

Form:

```
let
  b1
  bn
in
  c
```

#### 18.8.3.1 \*

Let expression Let expressions

### 18.8.4 where

Part of the syntax of the whole [function declaration](#), has according [scope](#).

As part of whole [declaration](#) - can extend over definitions of the funtion (pattern matches, [guards](#)).

Form:

```
f match1 = y
f match2 = y
f x =
  | cond1 x = y
  | cond2 x = y
  | otherwise = y
```

```
where
  y = ... x ...
```

#### 18.8.4.1 \*

Where clause

## 18.9 Haskell Language Report

Document that is a standart of language.

#### 18.9.1 \*

Report Haskell Report Haskell 98 Language Report Haskell 98 Report Haskell 1998 Language Report Haskell 2010 Language Report Haskell 2010 Report

## 18.10 Haskell'

Current language development mod.

<https://prime.haskell.org/>

#### 18.10.1 \*

Haskell prime

## 18.11 Lense

Library of combinators to provide Haskell (functional language without mutation) with the emulation of get-ters and set-ters of imperative language.

## 18.12 Pragma

**Pragma** - instruction to the compiler that specifies how a compiler should **process** the code. **Pragma** in Haskell have form:

```
{-# PRAGMA options #-}
```

#### 18.12.1 LANGUAGE pragma

Controls what variations of the language are permitted. It has a **set** of allowed options: [https://downloads.haskell.org/~ghc/latest/docs/html/users\\_guide/glasgow\\_exts.html](https://downloads.haskell.org/~ghc/latest/docs/html/users_guide/glasgow_exts.html), which can be supplied.

##### 18.12.1.1 LANGUAGE option

###### 18.12.1.1.1 \*

Language options

###### 18.12.1.1.2 Useful by default

```
import EmptyCase
import FlexibleContexts
import FlexibleInstances
```



```
import InstanceSigs
import MultiParamTypeClasses
```

#### 18.12.1.1.3 AllowAmbiguousTypes

Allow **type** signatures which appear that they would result in an unusable **binding**. However GHC will still check and complain about a **functions** that can never be called.

#### 18.12.1.1.4 ApplicativeDo

Enables an **alternative** in-depth **reduction** that translates the do-notation to the operators `<$>`, `<*>`, `join` as far as possible.

For GHC to pickup the patterns, the final **statement** must match one of these patterns exactly:

```
pure E
pure $ E
return E
return $ E
```

When the **statements** of do **expression** have dependencies between them, and **ApplicativeDo** cannot **infer** an **Applicative type** - GHC uses a heuristic  $O(n^2)$  algorithm to try to use `<*>` as much as possible. This algorithm usually finds the best solution, but in rare complex cases it might miss an opportunity. There is also  $O(n^3)$  algorithm that finds the optimal solution: `-foptimal-applicative-do`.

Requires `ap = <*>`, `return = pure`, which is true for the most **monadic types**.

- Allows use of do-notation with **types** that are an instance of **Applicative** and **Functor**
- In some **monads**, using the **applicative** operators is more efficient than **monadic bind**. For example, it may enable more parallelism.

The only way it shows up at the source level is that you can have a do **expression** with only **Applicative** or **Functor** constant.

It is possible to see the actual translation by using `-ddump-ds`.

#### 18.12.1.1.5 ConstrainedClassMethods

Enable the definition of further **constraints** on individual class methods.

#### 18.12.1.1.6 CPP

Enable **C preprocessor**.

#### 18.12.1.1.7 DeriveFunctor

Automatic **deriving** of instances for the **Functor type class**. For **type power set functor** is unique, its derivation implementation can be autochecked.

#### 18.12.1.1.8 ExplicitForAll

Allow explicit **forall** quantificator in places **where** it is implicit by Haskell.

#### 18.12.1.1.9 FlexibleContexts

Ability to use complex **constraints** in class **declaration contexts**. The only restriction on the **context** in a class **declaration** is that the class hierarchy must be acyclic.

```
class C a where
  op :: D b => a -> b -> b
```

```
class C a => D a where ...
```

$C \Rightarrow D$ , so in  $C$  we can talk about  $D$ .

Synergizes with [ConstraintKinds](#).

#### 18.12.1.1.10 FlexibleInstances

Allow [type class](#) instances [types](#) contain nested [types](#).

```
instance C (Maybe Int) where ...
```

Implies [TypeSynonymInstances](#).

#### 18.12.1.1.11 GeneralizedNewtypeDeriving

Enable GHC's newtype cunning generalised [deriving](#) mechanism.

```
newtype Dollars = Dollars Int
  deriving (Eq, Ord, Show, Read, Enum, Num, Real, Bounded, Integral)
```

(In old Haskell-98 only Eq, Ord, Enum could be inherited.)

#### 18.12.1.1.12 ImplicitParams

Allow definition of [functions](#) expecting implicit [parameters](#). In the Haskell that has static scoping of [variables](#) allows the dynamic scoping, such as in classic Lisp or ELisp. Sure thing this one can be puzzling as hell inside Haskell.

#### 18.12.1.1.13 LambdaCase

Enables [expressions](#) of the form:

```
\case { p1 -> e1; ...; pN -> eN }
```

-- OR

```
\case
  p1 -> e1
  ...
  pN -> eN
```

#### 18.12.1.1.14 MultiParamTypeClasses

Implies: [ConstrainedClassMethods](#) Enable the definitions of typeclasses with more than one [parameter](#).

```
class Collection c a where
```

#### 18.12.1.1.15 MultiWayIf

Enable multi-way-if syntax.

```
if | guard1 -> code1
   | ...
   | guardN -> codeN
```

**18.12.1.1.16 OverloadedStrings**

Enable overloaded string literals (string literals become desugared via the `IsString` class).

With overload, string literals has `type`:

```
(IsString a) => a
```

The usual string syntax can be used, e.g. `ByteString`, `Text`, and other variations of string-like `types`. Now they can be used in pattern matches as `char->integer` translations. To `pattern match` `Eq` must be `derived`.

To use class `IsString` - `import` it from `GHC.Ext`.

**18.12.1.1.17 PartialTypeSignatures**

Partial `type` signature contains `wildcards`, placeholders (`_`, `_name`). Allows programmer to which parts of a `type` to annotate and which to `infer`. Also applies to `constraint` part.

As untaped `expression`, partly typed can not polymorphically recurse.

`-Wno-partial-type-signatures` suppresses `infer` warnings.

**18.12.1.1.18 RankNTypes**

Enable `types` of arbitrary rank. See `Type rank`.

Implies `ExplicitForAll`.

Allows `forall` `quantifier`:

- Left side of  $\rightarrow$
- Right side of  $\rightarrow$
- as `argument` of a `constructor`
- as `type` of a field
- as `type` of an implicit `parameter`
- used in pattern `type` signature of `lexically scoped type variables`

**18.12.1.1.19 ScopedTypeVariables**

By default `type variables` do not have a `scope` except inside `type` signatures `where` they are used.

When there are internal `type` signatures provided in the code block (`where`, `let`, etc.) they (main `type` description of a `function` and internal `type` descriptions) restrain one-another and become not truly `polymorphic`, which creates a bounding interdependency of `types` that GHC would complain about.

\* option provides the `lexical scope` inside the code block for `type variables` that have `forall` `quantifier`. Because they are now lexically scoped - those `type variables` are used across internal `type` signatures.

For details see: <https://ocharles.org.uk/guest-posts/2014-12-20-scoped-type-variables.html>

Implies `ExplicitForAll`.

**18.12.1.1.20 TupleSections**

Allow `tuple` section syntax:

```
(, True)
(, "I", , , "Love", , 1337)
```

**18.12.1.1.21 TypeApplications**

Allow [type application](#) syntax:

```
read @Int 5

:type pure @[]
pure @[] :: a -> [a]

:type (<*>) @[]
(<*>) @[] :: [a -> b] -> [a] -> [b]

--

instance (CoArbitrary a, Arbitrary b) => Arbitrary (a -> b)

λ> ($ 0) <$> generate (arbitrary @(Int -> Int))
```

**18.12.1.1.22 TypeSynonymInstances**

Now [type](#) synonym can have it's own [type class](#) instances.

**18.12.1.1.23 UndecidableInstances**

Permit instances which may lead to [type](#)-checker non-termination.

GHC has Instance termination rules regardless of [FlexibleInstances](#) [FlexibleContexts](#).

**18.12.1.1.24 ViewPatterns**

```
foo (f1 -> Pattern1) = c1
foo (fn -> Pattern2 a b) = g1 a b
```

([expression](#) → [pattern](#)): take what is came to match - [apply](#) the [expression](#), then do [pattern](#)-match, and return what originally came to match.

Semantics:

- [expression](#) & [pattern](#) share the [scope](#), so also [variables](#).

[expression](#) :: t1 -> t2) && (pattern t2)=) then (ViewPattern (/expression/ -> /pattern/) :: t1) (return what originally was recieved into [pattern match](#)) else skip

\* are like [pattern guards](#) that can be nested inside of other patterns. \* are a convenient way to pattern-match [algebraic data type](#).

Additional possible usage:

```
foo a (f2 a -> Pattern3 b c) = g2 b c -- only for function definitions
foo ((f,_), f -> Pattern4) = c2 -- variables can be bount to the left in data constructors and t
```

**18.12.1.1.25 DatatypeContexts**

Allow [contexts](#) in [data types](#).

```
data Eq a => Set a = NilSet | ConsSet a (Set a)

-- NilSet :: Set a
-- ConsSet :: Eq a => a -> Set a -> Set a
```

Considered misfeature. Deprecated. Going to be removed.

**18.12.1.1.26 StandaloneKindSignatures**

Type signatures for [type-level declarations](#).

```
type <name_1> , ... , <name_n> :: <kind>
```

```
type MonoTagged :: Type -> Type -> Type
data MonoTagged t x = MonoTagged x
```

```
type Id :: forall k. k -> k
type family Id x where
  Id x = x
```

```
type C :: (k -> Type) -> k -> Constraint
class C a b where
  f :: a b
```

```
type TypeRep :: forall k. k -> Type
data TypeRep a where
  TyInt    :: TypeRep Int
  TyMaybe :: TypeRep Maybe
  TyApp    :: TypeRep a -> TypeRep b -> TypeRep (a b)
```

< GHC 8.10.1 - [type](#) signatures were only for [term level](#) declarations.

Extension makes signatures feature more uniformal.

Allows to [set](#) the [order](#) of [quantification](#), [order](#) of [variables](#) in a [kind](#). For example when using [TypeApplications](#).

Allows to [set](#) full [kind](#) of derivable class, solving situations with [GADT](#) return [kind](#).

**18.12.1.1.26.1 \***

SAKS Standalone kind signatures

**18.12.1.1.27 PartialTypeSignatures**

Very helpful. Helps to solve [type level](#), helps to establish [type](#) signatures and [constraints](#). Allow to provide [\\_](#) in the [type](#) signatures to automatically infer-in the [type](#) information there.

Wild cards:

- [Type](#)

```
f :: _ -> _ -> a
```

- [Constraint](#)

```
f :: _ => a -> b -> c
```

- [Named](#)

```
f :: _x -> _x -> a
```

allows to identify the same [wildcard](#).

**18.12.1.2 How to make a GHC LANGUAGE extension**

In `libraries/ghc-boot-th/GHC/LanguageExtensions/Type.hs` add new [constructor](#) to the `Extension` [type](#)

```
data Extension
  = Cpp
  | OverlappingInstances
  ...
  | Foo
```

/main/DynFlags.hs extend xFlagsDeps:

```
xFlagsDeps = [
  flagSpec "AllowAmbiguousTypes" LangExt.AllowAmbiguousTypes,
  ...
  flagSpec "Foo"                  LangExt.Foo
]
```

It is for basic [case](#). For [testing](#), parser see further: <https://blog.shaynefletcher.org/2019/02/adding-ghc-language-extension.html>

# Chapter 19

## Computer science

### 19.1 Guerrilla patch

\* changing code/[applying](#) patch sneakily - and possibility incompatibility with other at runtime. [Monkey patch](#) is derivative term.

#### 19.1.1 Monkey patch

From [Guerrilla patch](#).

\* is a way for program to modify supporting system software affecting only the running instance of the program.

### 19.2 Interface

Point of mutual meeting. Code behind [interface](#) determines how data is consumed.

### 19.3 Module

Importable organizational [unit](#).

### 19.4 Scope

Area [where binds](#) are accessible.

#### 19.4.1 Dynamic scope

The name resolution depends upon the program state when the name is encountered, which is determined by the execution [context](#) or calling [context](#).

#### 19.4.2 Lexical scope

[Scope bound](#) by the [structure](#) of source code [where](#) the named entity is defined.

##### 19.4.2.1 \*

Static scope

### 19.4.3 Local scope

Scope applies only in (current) area.

#### 19.4.3.1 \*

Local

## 19.5 Shadowing

When in the local scope bigger scope variable overridden by same name variable from the local scope.

## 19.6 Syntactic sugar

Artificial way to make language easier to read and write.

## 19.7 System F

Is parametric polymorphism in programming.

Extends the Lambda calculus by introducing  $\forall$  (universal quantifier) over types.

#### 19.7.1 \*

Girard–Reynolds polymorphic lambda calculus Girard–Raynolds

## 19.8 Tail call

Final evaluation inside the function. Produces the function result.

## 19.9 Thunk

Not evaluated calculation. Can be dragged around, until be lazily evaluated.

## 19.10 Application memory

Table 19.1: Application memory structural parts

Storage of	Block name
All not currently processing data	Heap
Function call, local variables	Stack
Static and global variables	Static/Global
Instructions	Binary code

When even Main invoked - it work in Stack, and called Stack frame. Stack frame size for function calculated when it is compiled. When stacked Stack frames exceed the Stack size - stack overflow happens.



## 19.11 Turing machine

Mathematical model of computation that defines [abstract Turing machine](#). [Abstract](#) machine which manipulates symbols on a strip of tape, according to a table of rules.

### 19.11.1 Turing complete

[Set](#) of action rules that can simulate any [Turing machine](#).

#### 19.11.1.1 \*

Turing incomplete Turing incompleteness Turing completeness Computationally universal

## 19.12 REPL

Read-eval-print loop, aka interactive shell.

## 19.13 Domain specific language

Language design/fitted for particular [domain](#) of [application](#). Mainly should be [Turing incomplete](#), since general-purpose language implies [Turing completeness](#).

### 19.13.1 \*

Domain-specific language DSL

### 19.13.2 Embedded domain specific language

[DSL](#) used inside outer language.

Two levels of embedding:

- Shallow: [DSL](#) translates into Haskell directly
- Deep: Between [DSL](#) and Haskell there is a [data structure](#) that reflects the [expression](#) tree, AKA stores the [syntax tree](#).

#### 19.13.2.1 \*

eDSL

## 19.14 Data structure

### 19.14.1 Cons cell

Cell that values may [inhabit](#).

### 19.14.2 Construct

`(:) :: a -> [a] -> [a]`

#### 19.14.2.1 \*

Cons

**19.14.3 Leaf**

-

**19.14.4 Node**\*  
/ \**19.14.5 Spine**

Is a chain of memory cells, each points to the both value of element and to the next memory cell.

Array:

```

      :
     / \
1    :
    / \
  2    :
    / \
  3    []

```

1:2:3: []

Spine:

```

      :
     / \
-    :
    / \
  -    :
    / \
  -    []

```

# Chapter 20

## Graph theory

### 20.1 Successor

[Object](#) that receives the [arrow](#).

#### 20.1.1 Direct successor

Immediate [successor](#).

### 20.2 Predecessor

[Object](#) that sends [arrow](#).

#### 20.2.1 Direct predecessor

Immediate [predecessor](#).

### 20.3 Degree

Number of [arrows](#) of [object](#).

#### 20.3.1 Indegree

Number of ingoing [arrows](#).

#### 20.3.2 Outdegree

Number of outgoing [arrows](#).

### 20.4 Adjacency matrix

Matrix of connection of objects  $\{-1, 0, 1\}$ .

#### 20.4.0.1 InstanceSigs

Allow adding [type](#) signatures to [type class function](#) instance [declaration](#).

## 20.5 Strongly connected

If every vertex in a graph is reachable from every other vertex.

It is possible to find all **strongly connected components** (and that way also test graph for strong connectivity), in **linear** time ( $\Theta(V+E)$ ).

**Binary relation** of being **strongly connected** is an **equivalence relation**.

### 20.5.1 \*

Strongly-connected

### 20.5.2 Strongly connected component

Full **strongly connected** subgraph of some graph.

\* of a directed graph  $G$  is a subgraph that is **strongly connected**, and has **property**: no additional edges or vertices from  $G$  can be included in the subgraph without breaking its **property** of being **strongly connected**.

#### 20.5.2.1 \*

SCC Strongly connected components Strongly-connected component Strongly-connected components

# Chapter 21

## Tagless-final

Method of embedding [eDSL](#) in a typed functional host language (Haskell). [Alternative](#) to the embedding as a (generalized) [algebraic data type](#). For parsers of DLS [expressions](#): (1/partial) evaluator, compiler, pretty printer, multi-pass optimizer.

\* embedding is writing [denotational semantics](#) for the [DSL](#) in the host language.

Approach can be used [iff eDSL](#) is typed. Only well-typed terms become embeddable, and host language can implemen also a [eDSL type](#) system. Approach that [eDSL](#) code interpretations are [type](#)-preserving.

One of main pros of \* - extensibility: implementation of [DSL](#) can be used to analyze/evaluate/transform/pretty-print/compile and interpreters can be extended to more passes, optimizations, and new versions of [DSL](#) while keeping/using/reusing the old versions.

Example fields of [application](#): language-integrated queries, non-deterministic & probabilistic programming, delimiter continuation, computability theory, [stream](#) processing, hardware description languages, generation of specialized numerical kernels, [semantics](#) of natural language.

## Chapter 22

# Prefix notation

Operators then their operands.

### **22.1   \***

Polish notation PN

### **22.2   Postfix notation**

Operands then their operation.

### **22.3   \***

Reverse Polish notation PRN

## **Part III**

# **Give definitions**

## Chapter 23

# Identity type



## Chapter 24

# Constant type

## Chapter 25

### Gen

## Chapter 26

# Tensorial strength

## Chapter 27

# Strong monad

## Chapter 28

# Weak head normal form

### 28.1 \*

WHNF

## Chapter 29

# Function image

### 29.1 \*

Image

## Chapter 30

# Invertible

## Chapter 31

# Invertibility



## Chapter 32

# Define LANGUAGE pragma options

### 32.1 ExistentialQuantification

### 32.2 GADTs

**GADT** is a generalization over parametric [algebraic data types](#) which allow explicitly denote the [types](#) ([type](#) matching) of the [constructors](#) and define [data types](#) using pattern matching on the left side of "data" [statements](#).

### 32.3 \*

GADT Generalized algebraic data type First-class phantom data type Guarded recursive data type Equality-qualified data type

### 32.4 GeneralizedNewTypeClasses

### 32.5 FuncitonalDependencies

## Chapter 33

# GHC check keys

### 33.1 -Wno-partial-type-signatures

Supresses [PartialTypeSignatures wildcard infer](#) warning.

## Chapter 34

# Generalised algebraic data types

LANGUAGE [GADTs](#)

### 34.1 \*

GADT

# Chapter 35

## Order theory

Investigates in thepht the intuitive notion of [order](#) using [binary relations](#).

### 35.1 Domain theory

Formalizes approximation and convergense. Has close [relation](#) to Topology.

### 35.2 Lattice

[Abstract structure](#) that consists of [partially ordered set](#), [where](#) every two elements have unique supremum and infimum. == \* [algebraic structure](#) satisfying certain axiomatic identities. \* [order-theory](#) & [algebraic](#).

### 35.3 Order

#### 35.3.1 Preorder

$R^X \rightarrow^X$  : [Reflexive](#) & [Transitive](#):  $aRa \ aRb, bRc \Rightarrow aRc$

Generalization of [equivalence relations](#) [partial orders](#).

\* [Antisymmetric](#)  $\Rightarrow$  Partial ordering, \* [Symmetric](#)  $\Rightarrow$  [Equivalence](#).

##### 35.3.1.1 \*

Preordered

##### 35.3.1.2 Total preorder

$\forall a, b : a \leq b \vee b \leq a \Rightarrow$  [Total Preorder](#).

#### 35.3.2 Partial order

A [binary relation](#) must be [reflexive](#), [antisymmetric](#) and [transitive](#).

Partial - not every elemponents between them need to be comparable.

Good example of \* is a genealogical descendancy. Only related people produce [relation](#), not related do not.

**35.3.2.1   \***

Partial orders   Partially ordered set   Partially ordered sets   Poset   Posets

**35.4   Partial order****35.5   Total order**

## Chapter 36

# Universal algebra

Studies [algebraic structures](#).

# Chapter 37

## Relation

### 37.1 Reflexivity

$R^{X \rightarrow X}, \forall x \in X : xRx$  **Order** theory:  $a \leq a$

\* - each element is comparable to itself.

Corresponds to **Identity** and **Automorphism**.

#### 37.1.1 \*

Reflexive Reflexive relation

### 37.2 Irreflexivity

$R^{X \rightarrow X}, \forall x \in X : \neg R(x, x)$

#### 37.2.1 \*

Anti-reflexive Anti-reflexive relation Irreflexive Irreflexive relation

### 37.3 Transitivity

$\forall a, b, c \in X, \forall R^{X \rightarrow X} : (aRb \wedge bRc) \Rightarrow aRc$

\* - the start of a chain of **precedence relations** must precede the end of the chain.

#### 37.3.1 \*

Transitive Transitive relation

### 37.4 Symmetry

$\forall a, b \in X : (aRb \iff bRa)$

#### 37.4.1 \*

Symmetric Symmetric relation

## 37.5 Equivalence

Reflexive	Symmetric	Transitive
$\forall x \in X, \exists R : xRx$ $a = a$	$\forall a, b \in X : (aRb \iff bRa)$ $a = b \iff b = a$	$\forall a, b, c \in X, \forall R^{X \rightarrow X} : (aRb \wedge bRc) \Rightarrow aRc$ $a = b, b = c \Rightarrow a = c$

### 37.5.1 \*

Equivalent Equivalent relation

## 37.6 Antisymmetry

$\forall a, b \in X : aRb, bRa \Rightarrow a = b \sim aRb, a \neq b \Rightarrow \nexists bRa$ . **Antisymmetry** does not say anything about  $R(a, a)$ .

\* - no two different elements precede each other.

### 37.6.1 \*

Antisymmetric Antisymmetric relation

## 37.7 Asymmetry

$\forall a, b \in X (aRb \Rightarrow \neg(bRa)) \iff \text{Antisymmetric} \wedge \text{Irreflexive}$ . **Asymmetry**  $\neq$  "not **symmetric**"  
**Symmetric**  $\wedge$  **Asymmetric** is only empty **relation**.

### 37.7.1 \*

Asymmetric Asymmetric relation



## Chapter 38

# Cryptomorphism

[Equivalent](#), interconvertable with no loss of information.

### 38.1 \*

Crypromorphic

## Chapter 39

# Lexically scoped type variables

Enable [lexical scope](#) for [forall quantifier](#) defined [type variables](#)

Implemented in [ScopedTypeVariables](#)

## Chapter 40

# Abstract data type

Several definitions here, reduce them.

**Data type** mathematical model, defined by its **semantics** from the user point of view, listing possible values, operations on the data of the **type**, and behaviour of these operations.

\* class of **objects** whose logical behaviour is defined by a **set** of values and **set** of operations (analogue to **algebraic structure** in mathematics).

A specification of a **data type** like a **stack** or queue **where** the specification does not contain any implementation details at all, only the operations for that **data type**. This can be thought of as the contract of the **data type**.

### 40.1 \*

AbsDT

## Chapter 41

# Functional dependencies

## Chapter 42

# MonoLocalBinds

## Chapter 43

# KindSignatures

## Chapter 44

# ExplicitNamespaces

## Chapter 45

# Combinator pattern



## Chapter 46

# Symbolic expression

Nested tree [data structure](#).

Introduced & used in Lisp. Lisp code and data are \*.

\* in Lisp: Atom or [expression](#) of the form (x . y), x and y are \*.

Modern abbreviated notation of \*: (x y).

### 46.1 \*

S-expression S-expressions Sexpression Sexpressions Sexp Sexps Sexpr Sexprs

## Chapter 47

# Polynomial

**Expression** consisting of:

- **variables**
- coefficients
- addition
- subtraction
- multiplication (including positive integer **variable** exponentiation)

**Polynomials** form a **ring**. **Polynomial ring**.

### 47.1 \*

Polynomials

## Chapter 48

# Data family

Indexed form of data and newtype definitions.

## Chapter 49

# Type synonym family

Indexed form of [type](#) synonyms.

## Chapter 50

# Indexed type family

\* additional structure in language that allows ad-hoc overloading of [data types](#). AKA are to [types](#) as [type class](#) to methods.

Varieties:

- [data family](#)
- [type](#) synonym families

Defined by pattern matching the partial [functions](#) between [types](#). Associates [data types](#) by [type-level function](#) defined by open-ended collection of valid instances of input [types](#) and corresponding output [types](#).

Normal [type classes](#) define partial [functions](#) from [types](#) to a collection of named values by pattern matching on the input [types](#), while [type](#) families define partial [functions](#) from [types](#) to [types](#) by pattern matching on the input [types](#). In fact, in many uses of [type](#) families there is a single [type class](#) which logically contains both values and [types](#) associated with each instance. A [type family](#) declared inside a [type class](#) is called an associated [type](#).

### 50.1 \*

Type family

## Chapter 51

# TypeFamilies

Allow use and definition of indexed [type](#) families and data families.

\* are [type](#)-level programming. \* are overload [data types](#) in the same way that [type classes](#) overload [functions](#). \* allow handling of [dependent types](#). Before it [Functional dependencies](#) and [GADTs](#) were used to solve that. \* useful for generic programming, creating highly parametrised interfaces for libraries, and creating interfaces with enhanced static information (much like [dependent types](#)).

Implies: [MonoLocalBinds](#), [KindSignatures](#), [ExplicitNamespaces](#)

Two [types](#) of \* are:

## Chapter 52

# Error

Mistake in the program that can be resolved only by fixing the program.

`error` is a sugar for `undefined`.

Distinct from [Exception](#).

### 52.1 \*

Errors

## Chapter 53

# Exception

Expected but irregular situation.

Distinct from [Error](#). Also see [107](#)

### 53.1 \*

Exceptions



## Chapter 54

# ConstraintKinds

`Constraints` are just handled as `types` of a particular `kind` (`Constraint`). Any `type` of the `kind` `Constraints` can be used as a `constraint`.

- Anything which is already allowed in code as a `constraint` without `*`. Saturated applications to `type` `classes`, implicit `parameter` and equality `constraints`.
- `Tuples`, all of whose component `types` have `kind` `Constraint`.

```
type Some a = (Show a, Ord a, Arbitrary a) -- is of kind Constraint.
```

- Anything form of which is not yet known, but the user has declared for it to have `kind` `Constraint` (for which they need to `import` it from `GHC.Exts`):

```
Foo (f :: Type -> Constraint) = forall b. f b => b -> b -- is allowed
-- as well as examples involving type families:
type family Typ a b :: Constraint
type instance Typ Int b = Show b
type instance Typ Bool b = Num b

func :: Typ a b => a -> b -> b
func = ...
```

## Chapter 55

# Specialisation

Turns [ad hoc polymorphic function](#) into compiled [type](#)-specific inmpementations.

### 55.1 \*

Specialise Specialize Specialization

## Chapter 56

# Diagram

For categories  $\mathcal{C}$  and  $\mathcal{J}$ , a diagram of type  $\mathcal{J}$  in  $\mathcal{C}$  is a covariant functor  $D : \mathcal{J} \rightarrow \mathcal{C}$ .

## Chapter 57

# Category theoretical presheaf

For categories  $C$  and  $J$ , a  $J$ -presheaf on  $C$  is a contravariant functor  $D : C \rightarrow J$ .

## Chapter 58

# Topological presheaf

If  $X$  is a topological space, then the open sets in  $X$  form a partially ordered set  $\text{Open}(X)$  under inclusion. Like every partially ordered set,  $\text{Open}(X)$  forms a small category by adding a single arrow  $U \rightarrow V$  if and only if  $U \subseteq V$ . Contravariant functors on  $\text{Open}(X)$  are called presheaves on  $X$ . For instance, by assigning to every open set  $U$  the associative algebra of real-valued continuous functions on  $U$ , one obtains a presheaf of algebras on  $X$ .

## Chapter 59

# Diagonal functor

The [diagonal functor](#) is defined as the [functor](#) from  $D$  to the [functor category](#)  $D^C$  which sends each [object](#) in  $D$  to the [constant functor](#) at that [object](#).

## Chapter 60

# Limit functor

For a fixed index [category](#)  $J$ , if every [functor](#)  $J \rightarrow C$  has a limit (for instance if  $C$  is complete), then the [limit functor](#)  $C^J \rightarrow C$  assigns to each [functor](#) its limit. The existence of this [functor](#) can be proved by realizing that it is the right-adjoint to the [diagonal functor](#) and invoking the Freyd adjoint [functor](#) theorem. This requires a suitable version of the axiom of choice. Similar remarks [apply](#) to the colimit [functor](#) (which is [covariant](#)).

## Chapter 61

# Dual vector space

The map which assigns to every vector space its [dual](#) space and to every [linear](#) map its [dual](#) or transpose is a [contravariant functor](#) from the [category](#) of all vector spaces over a fixed field to itself.



## Chapter 62

# Fundamental group

Consider the [category](#) of pointed topological spaces, i.e. topological spaces with distinguished points. The [objects](#) are pairs  $(X, x_0)$ , [where](#)  $X$  is a topological space and  $x_0$  is a point in  $X$ . A [morphism](#) from  $(X, x_0)$  to  $(Y, y_0)$  is given by a continuous map  $f : X \rightarrow Y$  with  $f(x_0) = y_0$ .

To every topological space  $X$  with distinguished point  $x_0$ , one can define the [fundamental group](#) based at  $x_0$ , denoted  $\pi_1(X, x_0)$ . This is the [group](#) of [homotopy](#) classes of loops based at  $x_0$ . If  $f : X \rightarrow Y$  is a [morphism](#) of pointed spaces, then every loop in  $X$  with base point  $x_0$  can be [composed](#) with  $f$  to yield a loop in  $Y$  with base point  $y_0$ . This [operation](#) is compatible with the [homotopy equivalence relation](#) and the [composition](#) of loops, and we get a [group homomorphism](#) from  $\pi_1(X, x_0)$  to  $\pi_1(Y, y_0)$ . We thus obtain a [functor](#) from the [category](#) of pointed topological spaces to the [category](#) of [groups](#).

In the [category](#) of topological spaces (without distinguished point), one considers [homotopy](#) classes of generic curves, but they cannot be [composed](#) unless they share an endpoint. Thus one has the fundamental groupoid instead of the [fundamental group](#), and this construction is [functorial](#).

## Chapter 63

# Algebra of continuous function

A [contravariant functor](#) from the [category](#) of topological spaces (with continuous maps as [morphisms](#)) to the [category](#) of real [associative algebras](#) is given by assigning to every topological space  $X$  the [algebra](#)  $C(X)$  of all real-valued continuous [functions](#) on that space. Every continuous map  $f : X \rightarrow Y$  induces an [algebra homomorphism](#)  $C(f) : C(Y) \rightarrow C(X)$  by the rule  $C(f)(\varphi) = \varphi \circ f$  for every  $\varphi$  in  $C(Y)$ .

## Chapter 64

# Tangent and cotangent bundle

The map which sends every differentiable manifold to its tangent bundle and every smooth map to its derivative is a [covariant functor](#) from the [category](#) of differentiable manifolds to the [category](#) of vector bundles.

Doing this constructions pointwise gives the tangent space, a [covariant functor](#) from the [category](#) of pointed differentiable manifolds to the [category](#) of real vector spaces. Likewise, cotangent space is a [contravariant functor](#), essentially the [composition](#) of the tangent space with the [dual](#) space above.

## Chapter 65

# Group action / representation

Every **group**  $G$  can be considered as a **category** with a single **object** whose **morphisms** are the elements of  $G$ . A **functor** from  $G$  to **Set** is then **nothing** but a **group** action of  $G$  on a particular **set**, i.e. a  **$G$ -set**. Likewise, a **functor** from  $G$  to the **category** of vector spaces,  $\text{Vect}_K$ , is a **linear** representation of  $G$ . In general, a **functor**  $G \rightarrow C$  can be considered as an "action" of  $G$  on an **object** in the **category**  $C$ . If  $C$  is a **group**, then this action is a **group homomorphism**.

## Chapter 66

# Lie algebra

Assigning to every real (complex) Lie [group](#) its real (complex) [Lie algebra](#) defines a [functor](#).

## Chapter 67

# Tensor product

If  $\mathcal{C}$  denotes the [category](#) of vector spaces over a fixed field, with [linear](#) maps as [morphisms](#), then the [tensor product](#)  $V \otimes W$  defines a [functor](#)  $\mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}$  which is [covariant](#) in both arguments.

## Chapter 68

# Forgetful functor

The functor  $U : \mathbf{Grp} \rightarrow \mathbf{Set}$  which maps a group to its underlying set and a group homomorphism to its underlying function of sets is a functor.<sup>[8]</sup> Functors like these, which "forget" some structure, are termed forgetful functors. Another example is the functor  $\mathbf{Rng} \rightarrow \mathbf{Ab}$  which maps a ring to its underlying additive abelian group. Morphisms in  $\mathbf{Rng}$  (ring homomorphisms) become morphisms in  $\mathbf{Ab}$  (abelian group homomorphisms).

## Chapter 69

# Free functor

Going in the opposite direction of [forgetful functors](#) are free [functors](#). The [free functor](#)  $F : \mathbf{Set} \rightarrow \mathbf{Grp}$  sends every [set](#)  $X$  to the free [group](#) generated by  $X$ . [Functions](#) get mapped to [group](#) homomorphisms between free [groups](#). Free constructions exist for many [categories](#) based on structured [sets](#). See [free object](#).



## Chapter 70

# Homomorphism group

To every pair  $A, B$  of abelian groups one can assign the abelian group  $\text{Hom}(A, B)$  consisting of all group homomorphisms from  $A$  to  $B$ . This is a functor which is contravariant in the first and covariant in the second argument, i.e. it is a functor  $\text{Ab}^{\text{op}} \times \text{Ab} \rightarrow \text{Ab}$  (where  $\text{Ab}$  denotes the category of abelian groups with group homomorphisms). If  $f : A_1 \rightarrow A_2$  and  $g : B_1 \rightarrow B_2$  are morphisms in  $\text{Ab}$ , then the group homomorphism  $\text{Hom}(f, g) : \text{Hom}(A_2, B_1) \rightarrow \text{Hom}(A_1, B_2)$  is given by  $\varphi \mapsto g \circ \varphi \circ f$ . See Hom functor.

## Chapter 71

# Representable functor

We can generalize the previous example to any category  $C$ . To every pair  $X, Y$  of objects in  $C$  one can assign the set  $\text{Hom}(X, Y)$  of morphisms from  $X$  to  $Y$ . This defines a functor to  $\mathbf{Set}$  which is contravariant in the first argument and covariant in the second, i.e. it is a functor  $C^{\text{op}} \times C \rightarrow \mathbf{Set}$ . If  $f : X_1 \rightarrow X_2$  and  $g : Y_1 \rightarrow Y_2$  are morphisms in  $C$ , then the group homomorphism  $\text{Hom}(f, g) : \text{Hom}(X_2, Y_1) \rightarrow \text{Hom}(X_1, Y_2)$  is given by  $\phi \mapsto g \circ \phi \circ f$ .

Functors like these are called representable functors. An important goal in many settings is to determine whether a given functor is representable.

## Chapter 72

# Corecursion

## Chapter 73

# Coinduction

*proper definition*

\* [dual](#) to induction. Generalises to [corecursion](#).

## Chapter 74

# Initial algebra of an endofunctor

## Chapter 75

# Terminal coalgebra for an endofunctor

## Chapter 76

# Continuation

### 76.1 Continuation passing style

#### 76.1.1 \*

CPS

## Chapter 77

# Control.Concurrent.Async

Good library for concurrency programming.



## Chapter 78

# Semilattice

# **Part IV**

## **Citation**

---

"One of the finer points of the Haskell community has been its propensity for recognizing [abstract](#) patterns in code which have well-defined, lawful representations in mathematics." (Chris Allen, Julie Moronuki - "Haskell Programming from First Principles" (2017))

## **Part V**

# **Good code**

## Chapter 79

# Good: Type aliasing

Use [data type](#) aliases to deferentiate logic of values.

## Chapter 80

# Good: Type wideness

Wider the [type](#) the more it is [polymorphic](#), means it has broader [application](#) and fits more [types](#).

The more constrained system has more usefulness.

Unconstrained means most flexible, but also most useless.

## Chapter 81

# Good: Print

```
print :: Show a => a -> IO ()  
print a = putStrLn (show a)
```

## Chapter 82

# Good: Fold

`foldr` [spine recursion](#) intermediated by the folding. Can terminate at any point. `foldl` [spine recursion](#) is unconditional, then folding starts. Unconditionally recurses across the whole [spine](#), if it infinite - infinitely.



## Chapter 83

# Good: Computation model

Model the [domain](#) and [types](#) before thinking about how to write computations.

## Chapter 84

**Good: Make bottoms only  
local**

## Chapter 85

**Good: Newtype wrap is ideally transparent for compiler and does not change performance**

## Chapter 86

**Good: Instances of types/type classes must go with code you write**

## Chapter 87

**Good: Functions can be  
abstracted as arguments**

## Chapter 88

**Good: Infix operators can be  
bind to arguments**

## Chapter 89

# Good: Arbitrary

Product types can be tested as a product of random generators. Sum types require to implement generators with separate constructors, and picking one of them, use oneof or frequency to pick generators.

## **Chapter 90**

# **Good: Principle of Separation of concerns**



## Chapter 91

# Good: Function composition

In Haskell inline [composition](#) requires:

```
h.g.f $ i
```

[Function application](#) has a higher [priority](#) than [composition](#). That is why parentheses over [argument](#) are needed. This [precedence](#) allows idiomatically [compose partially applied functions](#).

But it is a way better then:

```
h (g (f i))
```

## Chapter 92

# Good: Point-free

Use `Tacit` very carefully - it hides `types` and harder to change code `where` it is used. Use just enough `Tacit` to communicate a bit better. Mostly only partial `point-free` communicates better.

### 92.1 Good: Point-free is great in multi-dimensions

BigData and OLAP analysis.

## Chapter 93

# Good: Functor application

Function application on n levels beneath:

```
(fmap . fmap) function twoLevelStructure
```

How `fmap . fmap` typechecks:

```
(.)      :: (b -> c) -> (a -> b) -> a -> c
fmap     :: Functor f => (m -> n) -> f m -> f n
fmap     :: Functor g => (x -> y) -> g x -> g y

fmap . fmap :: (Functor f, Functor g)
            => ((g x -> g y) -> f . g x -> f . g y)
            -> (( x -> y) -> g x -> g y)
            -> ( x -> y) -> f . g x -> f . g y
fmap . fmap :: (x -> y) -> f . g x -> f . g y
```

## Chapter 94

# Good: Parameter order

In [functions parameter order](#) is important. It is best to use first the most reusable [parameters](#). And as last one the one that can be the most [variable](#), that is important to chain.

## Chapter 95

# Good: Applicative monoid

There can be more than one valid [Monoid](#) for a [data type](#). && There can be more than one valid [Applicative](#) instance for a [data type](#). -> There can be different [Applicatives](#) with different [Monoid](#) implementations.

## Chapter 96

### Good: Creative process

- 96.1 Pick philosophy principles one to three the more -  
the harder the implementation
- 96.2 Draw the most blurred representation
- 96.3 Deduce **abstractions** and write remotely what they  
are
- 96.4 Model of computation
  - 96.4.1 Model the **domain**
  - 96.4.2 Model the **types**
  - 96.4.3 Think how to write computations
- 96.5 Create

## Chapter 97

# Good: About operators ( $\langle \$ \rangle$ ) ( $\langle * \rangle$ ) ( $\langle * \rangle$ ) ( $\langle * \rangle$ )

Where character is not present - discard the according processing of a [parameter](#). ( $\langle * \rangle$ ) is an [exception](#), it does the reverse. ignores the first [parameter](#), in fact  $\langle * \rangle \equiv \langle * \rangle$ .

$\langle * \rangle$  does the proper action: does calculation, but ignores the value from the first [argument](#).

## Chapter 98

# Good: About functions like `{mapM, sequence}_`

Trailing `_` means ignoring the result.



# Chapter 99

## Good: Guideliles

### 99.1 Wiki.haskell

#### 99.1.1 Documentation

99.1.1.1 Comments write in **application** terms, not technical.

99.1.1.2 Tell what code needs to do not how it does.

#### 99.1.2 Haddock

99.1.2.1 Put haddock comments to ever exposed **data type** and **function**.

99.1.2.2 Haddock header

```
{- |  
Module      : <File name or $Header$ to be replaced automatically>  
Description : <optional short text displayed on contents page>  
Copyright   : (c) <Authors or Affiliations>  
License     : <license>  
  
Maintainer  : <email>  
Stability   : unstable | experimental | provisional | stable | frozen  
Portability : portable | non-portable (<reason>  
  
<module description starting at first column>  
-}
```

#### 99.1.3 Code

99.1.3.1 Try to stay closer to portable (Haskell98) code

99.1.3.2 Try make lines no longer 80 chars

99.1.3.3 Last char in file should be newline

99.1.3.4 Symbolic **infix** identifiers is only library writer right

99.1.3.5 Every **function** does one thing.

## Chapter 100

# Good: Use Typed holes to progress the code

[Typed holes](#) help build code in complex situations.

## Chapter 101

# Good: Haskell allows infinite terms but not infinite types

That is why infinite `types` throw infinite `type error`.

## Chapter 102

# Good: Use type synonyms to differ the information

Even if there is `types` - define `type` synonyms. They are free. That distinction with synonyms, would allow `TypeSynonymInstances`, which would allow to create a different `type class` instances and behaviour for different information.

## Chapter 103

**Good: Use `Control.Monad.Except`  
instead of `Control.Monad.Error`**

## Chapter 104

# Good: Monad OR Applicative

### 104.0.1 Start writing `monad` using 'return', 'ap', 'liftM', 'liftM2', '»' instead of 'do', '»='

If you wrote code and really needed only those - move that code to [Applicative](#).

```
return -> pure
ap -> <*>
liftM -> liftA -> <$>
>> -> *>
```

### 104.0.2 Basic [case](#) when [Applicative](#) can be used

Can be rewritten in [Applicative](#):

```
func = do
  a <- f
  b <- g
pure (a, b)
```

Can't be rewritten in [Applicative](#):

```
somethingdoSomething' n = do
  a <- f n
  b <- g a
pure (a, b)
```

(f n) creates [monadic structure](#), [binds](#) ot to *a* wich is consumed then by *g*.

### 104.0.3 [Applicative](#) block vs [Monad](#) block

With [Type Applicative](#) every condition fails/succseeds independently. It needs a boilerplate [data constructor](#)/value pattern matching code to work. And code you can write only for so many cases and [types](#), so boilerplate can not be so flexible as [Monad](#) that allows [polymorphism](#). With [Type Monad](#) computation can return value that dependent from the previous computation result. So abort or dependent processing can happen.

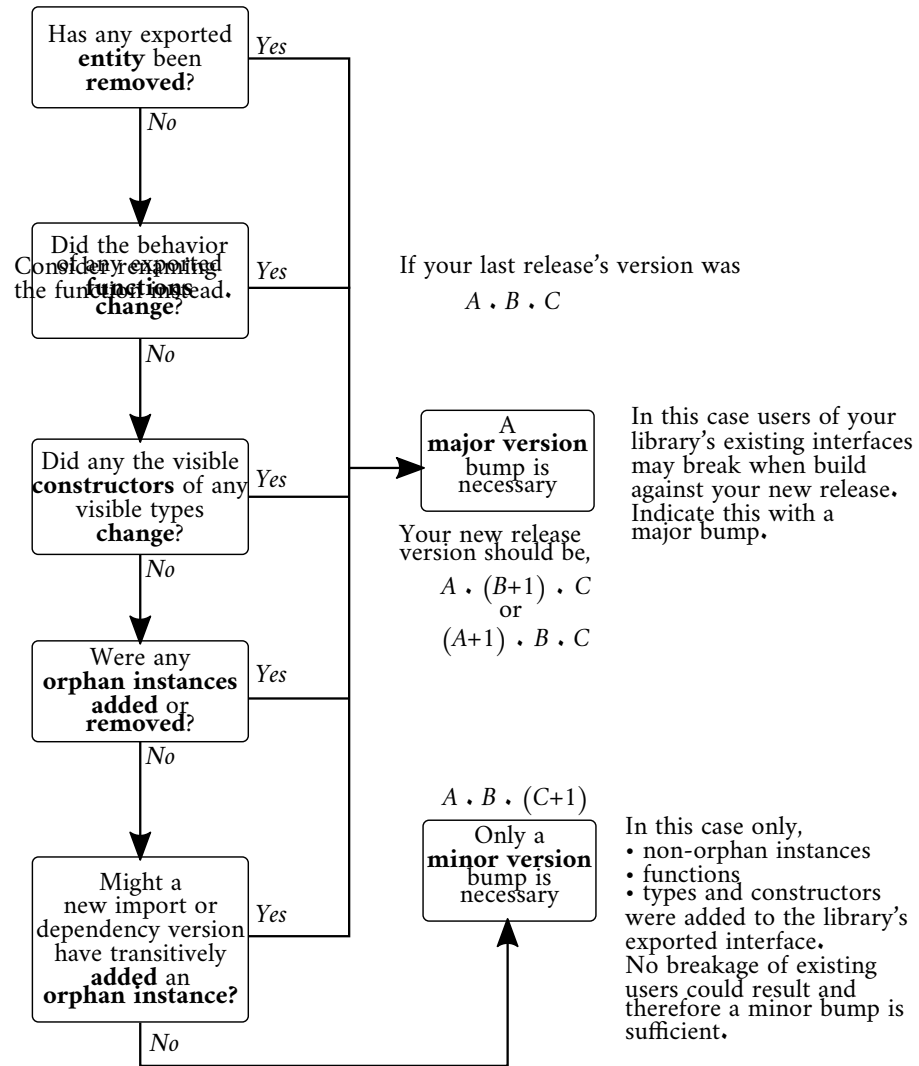
## Chapter 105

# Good: Haskell Package Versioning Policy

Version policy and dependency management.

So you are releasing a new package version?

Use this decision graph to determine how you should version your new release under Haskell Package Versioning Policy.



## 105.1 \*

PVP Good: PVP



## Chapter 106

# Good: Linear type

[Linear types](#) are great to control/minimize resource usage.

## Chapter 107

# Good: **Exception** vs **Error**

Many languages and Haskell have it all mixup. Here is table showing what belongs to one or other in standard libraries:

---

<b>Exception</b>	Prelude.catch, Control. <b>Exception</b> .catch, Control. <b>Exception</b> .try, IOError, Control. <b>Monad.Error</b>
<b>Error</b>	<b>error</b> , assert, Control. <b>Exception</b> .catch, Debug.Trace.trace

## Chapter 108

# Good: Let vs. Where

`let ... in ...` is a separate [expression](#). In contrast, `where` is [bound](#) to a surrounding syntactic [construct](#) (namespace).

## Chapter 109

# Good: RankNTypes

Can powerfully synergyze with [ScopedTypeVariables](#).

## Chapter 110

# Good: Handling orphan instance

Practice to address orphan instances:

Does `type class` or `type` defined by you:

Type class	Type	Recommendation
	✓	{ <code>Type</code> , instance} in the same <code>module</code>
✓		{ <code>Type class</code> & instance} in the same <code>module</code> {Define newtype wrap, its instances} in the same <code>module</code>

## Chapter 111

# Good: Smart constructor

Only proper smart `constructors` should be exported. Do not export `data type constructor`, only a `type`.

## Chapter 112

# Good: Thin category

In \* all [morphisms](#) are [epimorphisms](#) and [monomorphisms](#).

## Chapter 113

# Good: Recursion

Writing/thinking about [recursion](#):

1. Find the base cases, om input of which the answer can be provided right away. There is mosly one [base case](#), but sometimes there can be several of them. Typical base cases are: [zero](#), the empty [list](#), the empty tree, null, etc.
2. Do inductive [case](#). The [recursive](#) invocation. The [argument](#) of a [recursive](#) call needs to be smaller then the current [argument](#). So it would be gradually closer to the [base case](#). The idea is that processes eventually hits the [base case](#).

Simple functional [application](#) is used in the [recursion](#). Assume that the [functions](#) would return the right result.



## Chapter 114

# Good: Monoid

<>: [Sets](#) - union. Maps - left-biased union. Number - Sum, Product form separate [monoid categories](#).

## Chapter 115

# Good: Free monad

The main [case](#) of usage of Free [monads](#) in Haskell:

Start implementation of the [monad](#) from a Free [monad](#), drafting the base [monadic](#) operations, then add custom operations.

Gradually build on top of Free [monad](#) and try to find homomorphisms from [monad](#) to [objects](#), and if only [objects](#) are needed - get rid of the free [monad](#).

## Chapter 116

**Good: Use mostly where clauses**

## Chapter 117

**Good: Where clause is in a scope  
with function parameters**

## Chapter 118

**Good: Strong preference towards pattern matching over {head, tail, etc.} functions**

head and tail and alike [functions](#) are often partial ([unsafe](#)) functions.

## Chapter 119

# Good: Patternmatching is possible on monadic bind in do

Example:

```
instance (Monad m) => Functor (StateT s m) where
  fmap f m = StateT $ \s -> do
    (x, s') <- runStateT m s  -- Here is a pattern matching bind
    return (f x, s')
```

## Chapter 120

# Good: Applicative vs Monad

Giving not Monad but Applicative requirement allows parallel computation, but if there should be a chaining of the intermediate state - it must be [monadic](#).

## Chapter 121

# Good: StateT, ReaderT, WriterT

Reader trait: (r ->).

Writer trait: (a, w).

State trait is combination of both:

```
newtype StateT s m a =  
  StateT { runStateT :: s -> m (a, s) }
```

```
newtype ReaderT r m a =  
  ReaderT { runReaderT :: r -> m a }
```

```
newtype WriterT w m a =  
  WriterT { runWriterT :: m (a, w) }
```

State trait fully replaces writer.



## Chapter 122

# Good: Working with MonadTrans and lift

From the `lift . pure = pure` follows that `MonadTrans` [type](#) can have a `pure` defined with `lift`.

Stacking of `MonadTrans` [monads](#) can result in a lot of chained `lift` and `unwraps`. There is many ways to cope with that but the most robust and common is to [abstract](#) representation with `newtype` on the `Monad` [stack](#). This can reduce caining or remove the manual [lifting](#) withing the `Monad`. For perfect combination for contributors to be able to extend the code - keep the `Internal` [module](#) that has a raw representation.

## Chapter 123

# Good: Don't mix Where and Let

let and where create a [recursive set](#) of definitions with can explode, don't mix them together in code.

## Chapter 124

# Good: Where vs. Let

**Let** is self-**recursive lazy pattern**. It is checked and **errors** only at execution time. **Binds** only inside **expression** it is binded to.

**Where** is a part of definition, scoped over definition implemetations and **guards**, not self-**recursive**.

## Chapter 125

# Good: The proper nature algorithm that models behaviour of many objects is computation heavy

God does not care about our mathematical difficulties. He integrates empirically.

One who is found of mathematical meaning loves to [apply](#) it. But if we implement the "real" algorithms behind nature processes, we face the need to go through the computations of [properties](#) of all particles.

Computation of nature is always a middle way between ideal theory behaviour and computation simplification.

## Chapter 126

**Good: In Haskell parameters bound by lambda declaration instantiate to only one concrete type**

Because of [let-bound polymorphism](#):

This is illegal in Haskell:

```
foo :: (Int, Char)
foo = (\f -> (f 1, f 'a')) id
```

Lambda-bound function (i.e., one passed as [argument](#) to another [function](#)) cannot be instantiated in two different ways, if there is a [let-bound polymorphism](#).

## Chapter 127

# Good: Instance is a good structure to draw a type line

Instances for data type can differentiate by constraints & types of arguments. So instance can preserve type boundary, and data type declaration can stay very polymorphic. If the need to extend the type boundaries arrives - the instances may extend, or new instances are created, while used data type still the same and unchanged.

## Chapter 128

# Good: MTL vs. Transformers

Default of mtl.

Transformers is Haskell-98, doesn't have functional dependencies, lacks the [monad](#) classes, has manual [lift](#) of operations to the composite [monad](#).

MTL extends transformers, providing more instances, features and possibilities, may include [alternative](#) packages features as mtl-tf.

## Part VI

# Bad code



## Chapter 129

# Bad pragma

### 129.1 Bad: Dangerous **LANGUAGE pragma** option

- [DatatypeContexts](#)
- `OverlappingInstances`
- `IncoherentInstances`
- `ImpredicativeTypes`
- `AllowAmbiguousTypes`
- [UndecidableInstances](#) - often

## Part VII

Useful **functions** to remember

# Chapter 130

## Prelude

```
enumFromTo
enumFromThenTo
reverse
show :: Show a => a -> String
flip
sequence - Evaluate each monadic action in the structure from left to right, and collect the results
:sprint - show variables to see what has been evaluated already.
minBound - smaller bound
maxBound - larger bound
cycle :: [a] -> [a] - indefinitely cycle s list
repeat - indefinitely list from value
elemIndex e l - return first index, returns Maybe
fromMaybe (default if Nothing) e :: Maybe a -> a
lookup :: Eq a => a -> [(a, b)] -> Maybe b
```

### 130.1 Ord

compare

### 130.2 Calc

div - always makes rounding down, to infinity divMod - returns a tuple containing the result of integral division and modulo

### 130.3 List operations

```
concat - [ [a] ] -> [a]
elem x xs - is element a part of a list
zip :: [a] -> [b] -> [(a, b)] - zips two lists together. Zip stops when one list runs out.
zipWith :: (a -> b -> c) -> [a] -> [b] -> [c] - do the action on corresponding elements of list a
```

## Chapter 131

### Data.List

`intersperse :: a -> [a] -> [a]` - gets the value and incerts it between values `in` array  
`nub` - remove duplicates from the list

## Chapter 132

# Data.Char

```
ord (Char -> Int)
chr (Int -> Char)
isUpper (Char -> Bool)
toUpper (Char -> Char)
```

## Chapter 133

# QuickCheck

```
quickCheck :: Testable prop => prop -> IO ()
```

```
quickCheck . verbose - run verbose mode
```

## Part VIII

# Tool

## Chapter 134

# ghc-pkg

[List](#) installed packages:

```
ghc-pkg list
```



## Chapter 135

# Integration of NixOS/Nix with Haskell IDE Engine (HIE) and Emacs (Spacemacs)

### 135.1 1. Install the Cachix

Upstream doc: <https://github.com/cachix/cachix>

### 135.2 2. Installation of HIE

Upstream doc: <https://github.com/infinisil/all-hies/#cached-builds>

#### 135.2.1 2.1. Provide cached builds

```
cachix use all-hies
```

#### 135.2.2 2.2.a. Installation on NixOS distribution:

```
{ config, pkgs, ... }:
```

```
let
```

```
    all-hies = import (fetchTarball "https://github.com/infinisil/all-hies/tarball/master") {};
```

```
in {
```

```
    environment.systemPackages = with pkgs; [
```

```
        (all-hies.selection { selector = p: { inherit (p) ghc865 ghc864; }; })
```

```
    ];
```

```
}
```

Insert your GHC versions.

Switch to new configuration:

```
sudo -i nixos-rebuild switch
```

### 135.2.3 2.2.b. Installation with Nix package manager:

```
nix-env -iA selection --arg selector 'p: { inherit (p) ghc865 ghc864; }' -f 'https://github.com/i
```

Insert your GHC versions.

## 135.3 3. Emacs (Spacemacs) configuration:

```
dotspacemacs-configuration-layers
'(
  auto-completion

  (lsp :variables
    default-nix-wrapper (lambda (args)
      (append
        (append (list "nix-shell" "-I" "." "--command" )
          (list (mapconcat 'identity args " ")))
        )
      (list (nix-current-sandbox))
      )
    )

    lsp-haskell-process-wrapper-function default-nix-wrapper
  )

  (haskell :variables
    haskell-enable-hindent t
    haskell-completion-backend 'lsp
    haskell-process-type 'cabal-new-repl
  )
)

dotspacemacs-additional-packages '(
  direnv
  nix-sandbox
)

(defun dotspacemacs/user-config ()

  (add-hook 'haskell-mode-hook 'direnv-update-environment) ;; If direnv configured

)
```

Where:

auto-completion configures YASnippet.

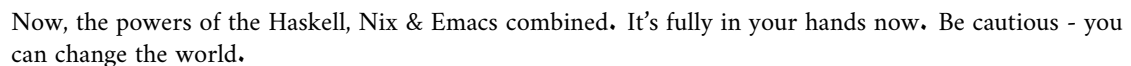
nix-sandbox (<https://github.com/travisbhartwell/nix-emacs>) has a great helper functions. Using nix-current-sandbox function in default-nix-wrapper that used to properly configure lsp-haskell-process-wrapper-function.

Configuration of the lsp-haskell-process-wrapper-function default-nix-wrapper is a key for HIE to work in nix-shell

Inside nix-shell the haskell-process-type 'cabal-new-repl is required.

### 135.4 4. Open the Haskell file from a project

135.5 5. Be pleased writing code



## 135.6 6. (optional) Debugging

1. If HIE keeps getting ready, failing & restarting - check that the projects `ghc --version` is declared in your `all-hie` NixOS configuration.

# Chapter 136

## GHC

### 136.1 GHC code check flags

Additional to default settings it is useful to use `-W`, `-Wcompat`. `-Wall` is for purists and would raise noise. They can be supplied in CLI as also in `.cabal ghc-option`. fr

`-W` turns on additional useful warnings:

- `-Wunused-binds`
- `-Wunused-matches`
- `-Wunused-foralls`
- `-Wunused-imports`
- `-Wincomplete-patterns`
- `-Wdodgy-exports`
- `-Wdodgy-imports`
- `-Wunbanged-strict-patterns`

`-Wall` turns on all warnings that indicate potentially suspicious code.

`-Weverything` turns on all warnings supported by compiler.

`-Wcompat` turns on warnings that will be enabled by default in the future GHC releases, allows library authors make the code compatible in advance for future GHC releases.

`-Werror` promotes warnings into fatal [errors](#), may be useful for CI runs.

`-w` turns off all warnings.

# Chapter 137

## GHCI

### 137.1 Debugging in GHCI

Provides:

- [set](#) a breakpoints
- observe step-by-step [evaluation](#)
- tracing mode

Breakpoints

```
:break 2
:show breaks
:delete 0
:continue
```

Step-by-step

```
:step main
```

[List](#) information at the breakpoint

```
:list
```

What been evaluated already

```
:sprint name
```

## Chapter 138

# GHCID

Commands to run the compile/check loop:

cabal > 3.0 command:

```
ghcid --command='cabal v2-repl --repl-options=-fno-code --repl-options=-fno-break-on-exception --
```

cabal < 3.0 command:

```
ghcid --command='cabal new-repl --ghc-options=-fno-code --ghc-options=-fno-break-on-exception --g
```

nix-shell cabal > 3.0 command:

```
nix-shell --command 'ghcid --command="cabal v2-repl --repl-options=-fno-code --repl-options=-fno-
```

nix-shell cabal < 3.0 command:

```
nix-shell --command 'ghcid --command="cabal new-repl --ghc-options=-fno-code --ghc-options=-fno-b
```

## Chapter 139

# Continuous integration platrorms (CIs) for Open Source Haskell projects

Since Open Source projects mostly use free tiers of CIs, and different CIs have different features - there is a [constant](#) flux of how to [construct](#) the best possible integration pipeline for Haskell projects.

The current state of affairs is best put in this quote:

Probably the biggest [constraint](#) is whether or not CI needs to test Windows or OS X, since build machines for those are harder to come by. We currently use AppVeyor for Windows builds and Travis for OS X builds since they are free. For Linux you can basically use any CI provider, but in this [case](#) I pay for a Linode VM which I use to host all Dhall-related infrastructure (i.e. all of the \*.dhall-lang.org domains), so I reuse that to host Hydra for Nix-related CI so that I can use more parallelism and more efficient caching to test a wider range of GHC versions on a budget.

For [testing](#) OS X and Windows platforms we use [stack](#). The main reason we don't use Nix for [either](#) platform is that Nix only supports building release binaries on Linux (and even then it's still experimental).

So the basic summary I can give is:

For [testing](#) everything other than cross-platform support: Nix + Linux is best in my opinion ... because you get much more control and intelligent build caching, which is usually [where](#) most CI solutions fall short

For cross-platform support: [stack](#) + whatever CI provider provides free builds for that platform

Also, if you ever can pay for your own NixOS VM and you want to reuse the setup I built, you can find the NixOS configuration for dhall-lang.org here:

<https://github.com/dhall-lang/dhall-lang/tree/master/nixops>

**Part IX**

**Library**



## Chapter 140

# Exceptions

- 140.1 **Exceptions** - optionally **pure** extensible **exceptions** that are compatible with the mtl
- 140.2 **Safe-exceptions** - safe, simple API **equivalent** to the underlying implementation in terms of power, encourages best practices minimizing the chances of getting the **exception** handling wrong.
- 140.3 **Enclosed-exceptions** - capture **exceptions** from the enclosed computation, while reacting to asynchronous **exceptions** aimed at the calling thread.

## Chapter 141

# Memory management

### 141.1 membrain - [type](#)-safe memory units

## **Chapter 142**

# **Parsers - megaparsec**

## Chapter 143

### CLIs - `optparse-`[applicative](#)

## **Chapter 144**

# **HTML - Lucid**

## Chapter 145

# Web applications - Servant

## Chapter 146

### IO libraries

**146.1** Conduit - practical, monolythic, guarantees termination return

**146.2** Pipes + Pipes Parse - modular, more primitive, theoretically driven

## **Chapter 147**

### **JSON - aeson**



## Chapter 148

# Backpack

On 1-st compilation - \* analyzes the [abstract](#) signatures without loading side modules, doing the [type check](#) with assumption that modules provide right [type](#) signatures, the [process](#) does not emit any [binary](#) code and stores the intermediate code in a special form that allows flexibly connect modules provided. Which allows later to compile project with particular instantiations of the modules. Major work of this [process](#) being done by internal Cabal \* support and \* system that modifies the intermediate code to fit the [module](#).

**Part X**

**Draft**

## Chapter 149

# Exception handling

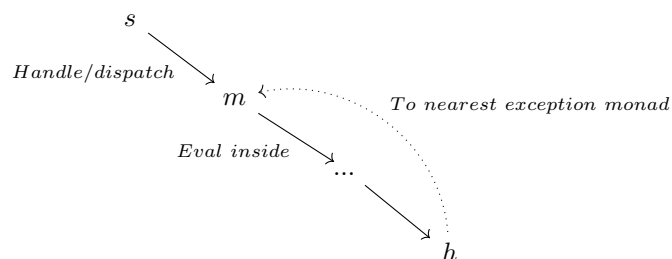
Process of **exception** handling has:

- raising an **exception**
- gathering information and handling an **exception**
- ability to finish important sessions/actions independently of whether **exception** happened or not. That is why it is called guaranteed finalization of important processes.

**Exceptions** and their handling are for the boundaries that receive external things that are not under Haskell control. It is mainly an **IO** handling. The **exception** mechanism may be used for internal **pure** Haskell part - if it grew too complex to sort-out some situation try/catch mechanism can be used, but then avoid use of runtime system **exceptions** catch and sort programmically and generally avoid it.

It's better to promote **exceptions** to just checking preconditions.

Wrapper with **exception** handler around **function** (**f**) call means that all untreated **exception** of **function** or its subfunctions would be caught by this wrapper into the **scope where** wrapper was used (one syntactic level above **function f**).



Any **monad** that short-circuits after some condition check of first **argument** - has **exception** handling potential.

Laziness as **exceptions** are computations - means that some issues would be skipped altogether, in parts that are/were not used would never throw **exceptions**, but also just as computations - **exceptions** would be raised at different times and states during computations.

**Exception** throw breaks **purity**, **function** was called but returned a result.

Try to raise and resolve all **exceptions** before acquiring external **IO** resources. And release all resources when or before the exception can happen.

With concurrency thread could be killed by other threads (that is called to raise an asynchronous **exception** in the thread).

## 149.1 Ideal catching

- Choose what [exceptions](#) to catch. Selection depends on the [type](#).
- No execution of continuation after throw, only handling.
- Handle  $\{,a\}$  synchronous [exceptions](#).

## 149.2 Control.Exception.Safe main sets of functions

- `try*` - allows handle [Either](#) returning [types](#), bridges the [exception](#) handling and basic Haskell computation.
- `handle*` - describes how to handle [exception](#) before the [monadic](#) action itself.
- `catch*` - describes how to handle the [exception](#) after the [monadic](#) action itself.

For asynchronous [exceptions](#) there are special [function](#) to catch them: `catchAsync` and `handleAsync`.

`catch` and `handle` are to catch specific [exception type](#).

`catches`, `catchesDeep` and `catchesAsync` allows to catch matching an elements in the [list](#), and then handle them.

## 149.3 Clean-up of actions/resources

- `bracket*` - computations to acquire and release resource and computation to run in between.
- `finally` - allows to run declared computations afterward (even if an [exception](#) was raised).
- `onException` - run computations only if [exception](#) happened.

## 149.4 Ideal model

- ☒ [Exception](#) must include all [context](#) information that may be useful.
- ☒ Store information in a form for further probable deeper automatic diagnostic.
- ☒ Sensitive data/dummies for it - can be useful during development.
- ☒ Sensitive data should be stripped from a program logging & [exceptions](#).
- ☒ [Exception](#) system should be extendable, data storage & representation should be easily extendable.
- ☒ [Exception](#) system should allow easy exhaustive checking of [errors](#), since the different [errors](#) can happen.
- ☒ [Exception](#) system should be automatically well-documented and transparent.
- ☒ [Exception](#) system should have controllable breaking changes downstream.
- ☒ [Exception](#) system should allow complex composite ([sets](#)) [exceptions](#).
- ☒ [Exception](#) system should be lightweight on the [type](#) signatures of other [functions](#).
- ☒ [Exception](#) system should automate the collection of [context](#) for a [exception](#).
- ☒ [Exception](#) system should have [properties](#) and according [functions](#) for particular [types](#) of [errors](#).

`String` is simple and convinient to throw [exception](#), but really a mistake because it the most cumbersome choise:

- ☒ Any [Exception](#) instance can be converted to a `String` with [either](#) `show` or `displayException`.

- ☐ Does not include key debugging information in the **error** message.
- ☐ Does not allow developer to access/manage the **Exception** information.
- ☐ **Exception** messages need to be constructed ahead of time, it can not be internationalized, converted to some data/file format.
- ☐ **Exception** can have a sensitive information that can be useful for developer during work, but should not be logged/shown to end-user. Stripping it from **Strings** in the changing project is a hard task.
- ☐ Impossible to rely on this representation for further/deeper inspection.
- ☐ Impossible to have exhaustive checking - no knowledge no check, no warning if some cases are not handled.

## 149.5 Universal **exception type**

- ☒ Able to inspect every possible **error case** with **pattern match**.
- ☒ Self-documenting. Shows the hierarchical system of all **exceptions**.
- ☒ Transparent. Ability to discern in current situation what **exceptions** can happen
- ☐ New **exception constructor** causes breaking change to downstream.
- ☐ Wrongly implies completeness. Untreated **Errors** can happen, different **exception** can arrive from the outside code.

Sum **type** must be separate, and **product type structure** over it. Separate **exception type** of

## 149.6 Individual **exception types**

- ☒ Writing & seing & working with exactly what will go wrong because there is only one possible **error** for this **type of exception**. **Pattern match** happens only onconditions, **constructors** that should happen.
- ☒ Knowledge what exactly goes wrong allows wide usage of **Either**.
- ☐ It is hard to handle complex **exceptions** in the unitary system. Real wrorld can return not a particular **case**, but a **set** of cases {**object** not found, path is unreachable, access is denied}.
- ☐ **Type** signatures grow, and even can become complex, since every **case of exception** has its own **type**.
- ☐ Impure **throw** that users can/should use for your code must account for all your **exception types**.

## 149.7 **Abstract exception type**

**Exception type** entirely opaque and inspectable only by accessor **functions**.

- ☒ Updating the internals without breaking the API
- ☒ Semi-automates the **context of exception** with passing it to accessors.
- ☒ Predicates can be **applied** to more than one **constructor**. Which are **properties** that allows to make complex **exceptions** much easier to handle.
- ☐ Not self-documenting.
- ☐ Possible options by design are hidden from the downstream, documentation must be kept.
- ☐ When you change the **exception** handling/throwing **errors** it does not shows to the downstream.

## 149.8 Composit approach

Provide the [set](#) of [constructors](#) and also a [set](#) of predicates and [set](#) of accessors. Use [pattern synonyms](#) to provide a documented accessor [set](#) without exposing internal [data type](#).

## 149.9 The changes in GHC 8.8

The `fail` method of [Monad](#) has been removed in favor of the method of the same name in the `MonadFail` class.

`MonadFail(..)` is now exported from the `Prelude` and `Control.Monad` modules. The `MonadFailDesugaring` language extension is now deprecated, as its effects are always enabled.

So:

```
import           Control.Monad.Fail
...
class MonadFail m => MonadFile m
...
-- use error instead of fail
Nothing      -> error ("Message " <> show x)
```

## 149.10 Diversity in [exceptions](#)

[Exception](#) cause: external or internal.

[Exceptions](#) used by runtime system

```
div 1 0
-- *** Exception: divide by zero
```

[Exceptions](#) used by programmers:

- Language feature
- Programmable (implemented at library level)

## 149.11 [Exception](#) handling strategies

- Ignore
- Print
- Repeat
- Wait, stop, exit
- Substitute with default
- Throw
- Handle
- Rethrow
- Emergency exit

## 149.12 Asynchronous [exception](#)

[Exceptions](#) raised as a result of an "external event", such as signal from another thread.

Are raised by `throwTo`. Are by termin and design should not be caught/handled, by default catching/handling [functions](#) are not catching them, if someone still wants to catch them - there are special [function](#): `catchAsync`.

Further reading: termin and apparatus were introduced by "[Asynchronous Exceptions in Haskell](#)" (Simon Marlow, Simon Peyton Jones, Andrew Moran, John Reppy).

# Chapter 150

## Constraints

Very strong Haskell [type](#) system makes possible to work with code from the top down, an [axiomatic semantics](#) approach, from [constraints](#) into [types](#).

- Helps to form the [type level](#) code (aka [join](#) points of the code).
- Uses the piling up of [constraints/types](#) information. At some point pick and satisfy [constraints](#), can be done one at a time.
- Provides hints through [type level](#) formulation for [term level](#) calculations, does not formulate the [term level](#).
- Tedious method (a lot of boilerplate and rewriting it) but pretty simple and relaxing.
- [Set](#) of [constraints](#).
- When it is needed or convenient, single [constraint](#) gets a little more realistically concrete/abstracted.

Main [type](#) detail annotation thread can happen in [main](#) or special wrapper [function](#), localization is inside [functions](#).

1. Rest of [constraints set](#) shifts to source [type](#).

3.a. For the class handled or known how to handle - write a [base case](#) instance description.

```
instance (Monad m) => MonadReader r (ReaderT r m)
```

3.b. For others write [recursive](#) instance descriptions:

All other unsolved [constraints](#) move into the source [polymorphic variable](#).

```
instance (MonadError e m) => MonadError e (ReaderT r m)
instance (MonadState s m) => MonadState s (ReaderT r m)
```

1. Repeat from 1 until considered done.
2. Code condensed into terse form.

MonadError [constraints](#) is [IOException](#), not for the [String](#). [IOException](#) vs [String](#).

Reverse pluck MonadReader [constraint](#) with [runReader](#) on the [object](#).

MonadState - StateT



## Chapter 151

# Monad transformers and their type classes

## Chapter 152

# Layering **monad** transformers

Different layering of the same **monad** transformers is functionality is the same, but the form is different. Surrounding handling **functions** would need to be different.

# Chapter 153

## Hoogle

### 153.1 Search

Text search ([case](#) insensitive):

- `a`
- `map`
- `con map`

[Type](#) search:

- `:: a`
- `:: a -> a`

Text & [type](#):

`=id a -> a =`

### 153.2 Scope

#### 153.2.1 Default

[Scope](#) is [Haskell Platform](#) (and [Haskell keywords](#)).

All [Package](#) packages are available to search with:

#### 153.2.2 [Hierarchical module name](#) system (from big letter):

- `fold +Data.Map` finds results in the `Data.Map` [module](#)
- `file -System` excludes results from modules such as `System.IO`, `System.FilePath.Windows` and `Distribution.System`

#### 153.2.3 Packages (lower [case](#)):

- `mode +platform`
- `mode +cmdargs` (only)
- `mode +platform +cmdargs`
- `file -base` (Haskell Platform, excluding the "base" package)

## Chapter 154

# ST-Trick monad

ST is like a [lexical scope](#), where all the [variables](#)/state disappear when the [function](#) returns <https://wiki.haskell.org/ST> <https://www.schoolofhaskell.com/school/to-infinity-and-beyond/older-but-still-interesting-monads/deamortized-strg/Monad/ST> <https://dev.to/jvanbruegge/what-the-heck-is-polymorphism-nmh>

### 154.1 \*

ST-Trick

## Chapter 155

# Either

Allows to separate and preserve information about happened, ex. [error](#) handling.

### 155.1 \*

Either data type

## Chapter 156

# Inverse

1. [Inverse function](#)
2. In logic:  $P \rightarrow Q \Rightarrow \neg P \rightarrow \neg Q$ , & same for [category duality](#).
3. For [operation](#): element that allows reversing [operation](#), having an element that with the [dual](#) produces the [identity](#) element.
4. See [Inversion](#).

## Chapter 157

# Inversion

1. Is a [permutation where](#) two elements are out of [order](#).
2. See [Inverse](#)

## Chapter 158

# Inverse function

$$f_{x \rightarrow y} \circ (f_{x \rightarrow y})^{-1} = 1_x$$

\*  $\iff$  function is bijective. Otherwise - partial inverse



## Chapter 159

# Inverse morphism

For  $f : x \rightarrow y$ :  $\exists g : g \circ f = 1^x$  -  $g$  is left [inverse](#) of  $f$ ,  $\exists g : f \circ g = 1^y$  -  $g$  is right [inverse](#) of  $f$ .

## Chapter 160

# Partial inverse

\* when [function](#) is now [bijective](#). When [bijective](#) see [inverse function](#).

## Chapter 161

# PatternSynonyms

Enables [pattern synonym declaration](#), which always begins with the `pattern` word. Allows to [abstract](#) away the [structures](#) of pattern matching.

### 161.1 \*

Pattern synonym Pattern synonyms

## Chapter 162

# GHC debug keys

### 162.1 -ddump-ds

Dump desugarer output.

#### 162.1.1 \*

Desugar GHC desugar

## Chapter 163

# GHC optimize keys

### 163.1 -foptimal-applicative-do

$O(n^3)$  Always finds optimal [reduction](#) into `<*>` for [ApplicativeDo](#) do notation.

## Chapter 164

# Computational trinitarianism

Taken from: <https://ncatlab.org/nlab/show/computational+trinitarianism>

Under the [statements](#):

- [propositions](#) as [types](#)
- programs as proofs
- [relation](#) between [type](#) theory and [category](#) theory

the following notions are [equivalent](#):

== [proposition](#) proof (Logic)

== generalized element of an [object](#) ([Category](#) theory)

== typed program with output ([Type](#) theory & Computer science)

### 164.1 \*

Trinitarism

Table 164.1: Computational trinitarianism

Logic	Category theory	Type theory
true	terminal object / (-2)-truncated object	h-level 0-type/unit type
false	initial object	empty type
proposition	(-1)-truncated object	h-proposition, mere proposition
proof	generalized element	program
cut rule	composition of classifying morphisms / pullback of display maps	substitution
cut elimination for implication	countit for hom-tensor adjunction	beta reduction
introduction rule for implication	unit for hom-tensor adjunction	eta conversion
logical conjunction	product	product type
disjunction	coproduct ((-1)-truncation of)	sum type (bracket type of)
implication	internal hom	function type
negation	internal hom into initial object	function type into empty type
universal quantification	dependent product	dependent product type
existential quantification	dependent sum ((-1)-truncation of)	dependent sum type (bracket type of)
equivalence	path space object	identity type
equivalence class	quotient	quotient type
induction	colimit	inductive type, W-type, M-type
higher induction	higher colimit	higher inductive type
completely presented set	discrete object/0-truncated object	h-level 2-type/preset/h-set
set	internal 0-groupoid	Bishop set/setoid
universe	object classifier	type of types
modality	closure operator, (idempotent) monad	modal type theory, monad (in computer science)
linear logic	(symmetric, closed) monoidal category	linear type theory/quantum computation
proof net	string diagram	quantum circuit
(absence of) contraction rule	(absence of) diagonal	no-cloning theorem
	synthetic mathematics	domain specific embedded programming language

## Chapter 165

# Techniques functional programming deals with the state

### 165.1 Minimizing

Do not rely on state, try not to change the state. Use it only when it is very necessary.

### 165.2 Concentrating

Concentrate the state in one place.

### 165.3 Deferring

Defer state to the last step of the program, or to external system.



## Chapter 166

# Monadic Error handling

```
(>>=) :: m a -> (a -> m b) -> m b --  $\lambda A.E \boxtimes A$  - computes and drops if error value happens.  
catch :: c a -> (e -> c a) -> c a --  $\lambda E.E \boxtimes A$  - handles "errors" as "normal" values and stops when
```

## Chapter 167

# Functions

Total **function** uses **domain** fully, but takes only part of the **codomain**. **Function** allows to collapse **domain** values into **codomain** value. Meaning the **function** allows to loose the information. So total **function** is a computation that looses the information or into bigger codomains. That is why the **function** has a directionality, and **inverse** total **process** is partially possible.

Directionality and invertability are terms.

# Chapter 168

## Void

Emptiness.

Can not be grasped, touched.

A logically uninhabited [data type](#).

(Since [basis](#) of logic is tautologically True and [Void](#) value can not be addressed - there is a logical paradox with the [Void](#)).

Is an [object](#) included into the [Hask category](#), since:

```
:t (id :: Void -> Void)
(id :: Void -> Void) :: Void -> Void
```

id for it exists.

[Type](#) system corresponds to [constructive logic](#) and not to the classical logic. Classical logic answers the question "Is this actually true". Constructive (Intuitionistic) logic answers the question "Is this provable".

Also has [functions](#):

```
-- Represents logical principle of explosion: from falsehood, anything follows.
absurd :: Void -> a
```

```
-- If Functor holds only Void - it holds no values.
vacuous :: Functor f => f Void -> f a
```

```
-- If Monad holds only Void - it holds no values.
vacuousM :: Monad m => m Void -> m a
```

Design pattern: use [polymorphic data types](#) and [Void](#) to get rid of possibilities when you need to.

### 168.1 \*

Nothing, Haskell [expressions](#) can't return [Void](#).

Also see: [Maybe](#).

## Chapter 169

# Constructive proof

Method of proof that demonstrates the existence of a mathematical [object](#) by creating or providing a method for creating the [object](#).

## Chapter 170

# Intuitionistic logic

[Proposition](#) considered True due to direct evidence of existence through constructive proof using [Curry-Howard isomorphism](#).

\* does not include classic logic fundamental axioms of the excluded middle and double negation elimination. Hence \* is weaker than classical logic. Classical logic includes \*, all theorems of \* are also in classical logic.

### 170.1 \*

Constructive logic

## Chapter 171

# Principle of explosion

If asserted **statement** contains some **error** or contradiction - anything can be proven through it. The more there is an **error** - the easier logic chain arrives at any target.

Ancient principle of logic. Both in classical & intuitionistic logic.

### 171.1 \*

Ex falso quodlibet Ex falso sequitur quodlibet EFG Ex contradictione quodlibet Ex contradictione sequitur quodlibet ECQ Deductive explosion Pseudo-Scotus

## Chapter 172

# Universal **property**

A **property** of some construction which boils down to (is manifestly **equivalent** to) the **property** that an associated **object** is a universal **initial object** of some (auxiliary) **category**.

## Chapter 173

# Yoneda lemma

Allows the embedding of any [category](#) into a [category](#) of [functors](#) ([contravariant set-valued functors](#)) defined on that [category](#). It also clarifies how the embedded [category](#), of representable [functors](#) and their [natural transformations](#), relates to the other [objects](#) in the larger [functor category](#).

The Yoneda lemma suggests that instead of studying the (locally small) [category](#)  $\mathcal{C}^{\{\{C\}\}}$ , *one should study the [category](#) of al*



## Chapter 174

# Monoidal category, functoriality of ADTs, Profunctors

Category equipped with tensor product.

<>

wich is a functor for \*.

Set category can be monoidal under both product (having terminal object) or coproduct (having initial object) operations, if according operation exist for all objects.

Any one-object category is \*.

$(a, ()) \sim a$  up to unique isomorphism, which is called Lax monoidal functor.

Product and coproduct are functorial, so, since: Algebraic data type construction can use:

- Type constructor
- Data constructor
- Const functor
- Identity functor
- Product
- Coproduct

Any algebraic data type is functorial.

## Chapter 175

# Const functor

Maps all **objects** of source **category** into one (fixed) **object** of target **category**, and all **morphisms** to **identity morphism** of that fixed **object**.

```
instance Functor (Const c)
  where
    fmap :: (a -> b) -> Const c a -> Const c b
    fmap _ (Const c) = Const c
```

In **Category** theory denoted:

$\Delta$

Last **type parameter** that bears the target **type** of lifted **function** (b) and is a **proxy type**.

Analogy: the container that allways has an **object** attached to it, and everything that is put inside - changes the container **type** accordingly, and dissapears.

## Chapter 176

# Arrow in Haskell

```
(->) a b = a -> b
```

`Functorial` in the last `argument` & called Reader `functor`.

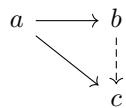
```
newtype Reader c a = Reader (c -> a)
```

```
fmap = ( . )
```

## Chapter 177

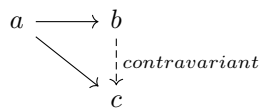
# Contravariant functor

```
fmap :: (a -> b) -> Op c a -> Op c b
      (a -> c) -> (b -> c)
```



$$(a \rightarrow b)^C = (a \leftarrow b)^{C^{op}}$$

```
class Contravariant f
where
  contramap :: (b -> a) -> (f a -> f b)
```



If [arrows](#) does not commute Contravariant functor anyway allows to [construct](#) transformation between these such [arrows](#) to other [arrow](#).

## Chapter 178

# Profunctor

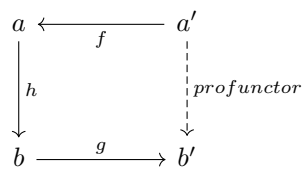
$(\multimap) \ a \ b$

$C^{op} \times C \rightarrow C$

It is called profunctor.

`dimap :: (a' -> a) -> (b -> b') -> p a b -> p a' b'`

So, profunctor in [case](#) of [arrow](#):



```
dimap :: (a' -> a) -> (b -> b') -> p a b -> p a' b'
dimap ::      f      g      -> (a -> b) -> (a' -> b')
dimap ::      f      g      ->      h      -> (a' -> b')
dimap = g . h . f
```

It is [contravariant functor](#) in the first [argument](#), and [covariant functor](#) in the second [argument](#).

```
dimap id <==> fmap
(flip dimap) id <==> contramap
```

## Chapter 179

# Coerce

Operates under condition that source and target [types](#) have same representation. Same representation means they are [type](#) aliases, or it the compiler can [infer](#) that they have the same representation. Directly shares the values from the source [type](#) to the target [type](#). Conversion is free, there is no run-time computations.

The [function](#) implementing the transition:

```
coerce :: Coercible a b => a -> b
```

[Type class](#) implementing the instances for transitions:

```
class a ~R# b => Coercible (a :: k0) (b :: k0)
```

When compiler detects [types](#) have same [structure](#), [type class](#) instances coerse implementation for this pairs of [types](#). This [type class](#) does not have regular instances; instead they are created on-the-fly during [type](#)-checking. Trying to manually declare an instance of Coercible is an [error](#).

### 179.1 \*

Coercible

## Chapter 180

# Universal/Existential quantification

$\forall$  Universal [quantifier](#) - a general solution.  $\exists$  Existential [quantifier](#) - a [local](#) solution.

Haskell existentials always result in throwing away [type](#) information.

Useful to be able to work with data from an external world that we do not know definite [type](#) at compile time.

Some information about existentially [quantified type](#) should be preserved to be able to transform it.

Additional reading: <https://markkarpov.com/post/existential-quantification.html>

**Part XI**

**Reference**



## Chapter 181

# Functor-Applicative-Monad Proposal

Well known event in Haskell history: [https://github.com/quchen/articles/blob/master/applicative\\_monad.md](https://github.com/quchen/articles/blob/master/applicative_monad.md).

Math justice was restored with a RETroactive CONtinuity. Invented in computer science term [Applicative](#) ([lax monoidal functor](#)) become a [superclass](#) of [Monad](#).

& that is why:

- `return = pure`
- `ap = <*>`
- `>> = *>`
- `liftM = liftA = fmap`
- `liftM* = liftA*`

Also, a side-kick - [Alternative](#) became a [superclass](#) of [MonadPlus](#). Hence:

- `mzero = empty`
- `mplus = (<|>)`

Work of unification continues under: <https://gitlab.haskell.org/ghc/ghc/wikis/proposal/monad-of-no-return>

### 181.1 \*

Applicative-Monad proposal AMP

# Chapter 182

## Haskell-98

### 182.1 Old instance termination rules

1.  $\forall$  class **constraint**  $(C\ t_1 \dots t_n)$ : 1.1. **type variables** have occurrences  $\leq$  head 1.2. **constructors+variables+repetitions**  $<$  head 1.3.  $\neg$  **type functions** (**type** func **application** can expand to **arbitrary** size)
2.  $\forall$  **functional dependencies**,  $\text{[]tv}\text{[]}_{\text{left}} \rightarrow \text{[]tv}\text{[]}_{\text{right}}$ , of the class, every **type variable** in  $S(\text{[]tv}\text{[]}_{\text{right}})$  must appear in  $S(\text{[]tv}\text{[]}_{\text{left}})$ , **where**  $S$  is the substitution mapping each **type variable** in the class **declaration** to the corresponding **type** in the instance head.

## Chapter 183

# Performance results and comparisons of **types** & solutions

Haskell performance

## Chapter 184

# Literature

- "GHC User's Guide Documentation" (GHC Team): [PDF](#)
- "What I Wish I Knew When Learning Haskell" (Stephen Diehl & contributors): [PDF](#)
- "[Category](#) Theory for Programmers" (Bartosz Milewski & contributors): [PDF](#)

**Part XII**

**Giving back**

---

λειτ <- λαός *Laos* the people ουργός <- ἔργο *ergon* work λειτουργία *leitourgia* public work

Moral value of people developed from the community to give back, improving the community.

The life is beautiful. For all humans that make the life have more magic.

This study and work would not be possible without the community: teachers, mathematicians, Haskellers, scientists, creators, contributors. These sides of people are fascinating.

Special accolades for the guys at Serokell. They were the force that got me inspired & gave resources to seriously learn Haskell and create this pocket guide.