# 1 Introduction

My model idea is about how the relationship between business cycles and the labor market is affected by education.

There is a somewhat spread out literature documenting the counter-cyclical nature of enrolment into post-secondary education.[1] The mechanism is intuitive: during an economic downturn wages are suppressed and unemployment is higher, so the opportunity cost of education relative to being in the labor force becomes relatively lower.

Less work, however, has gone into how this stylized fact feeds back into the business cycle, and whether a cheap publicly funded education system amplifies or helps stabilize business cycle shocks. Ex ante, the sign of the effect is not clear. On the supply side, the supply of productive labor would be expected to fall, and thereby also goods supply, ceteris paribus. On the demand side, there are multiple mechanisms. If a household chooses education over working in a period, household income within that period falls, but (given that they chose education) future discounted household income increases to compensate for that. This will decrease demand for savings among these households, putting downward pressure on the interest rate, but whether within-period goods demand falls due to income loss, or raises due to future income expectations, is ambiguous. Further, the lower labor supply when households chooses to study puts upwards pressure on the wage, but (I think) in a general equilibrium effect this won't increase goods demand due to it making everybody poorer due to taxes and transfers.

The model I set up will be based on the HANK models we have seen in lecture 8 and assignment II in the course. I use the intermediary and final goods firms from the HANK model in lecture 8 to get a new Keynesian Phillips curve while having endogenous labor supply at the household level. The bonds market is taken from assignment II, in order to allow transfers to households that choose to study instead of working to be partly debt financed.

The reason for putting my idea into a HANK model as opposed to a HANC, is to better capture business cycle effects on demand and propagation.

I have also ignored unemployment, for now, so the mechanism for choosing education over working only happens through expected wages.

In most models we have seen, households are infinitely lived, and I have opted to stick to this convention, and instead make the productivity effects from education depreciate over time. This avoids the implication of shocks to education choices making permanent changes to the economy if we have no depreciation and infinitely lived households. An alternative approach would be to model finitely lived agents with overlapping generations, as we have seen in De Nardi and Fella (2017).

[1] See e.g. Betts and McFarland (1995), Barr and Turner (2013), Hillman and Orians (2013). There is also some evidence for Denmark, Sønnichsen and Kristensen (2016).

### 1.1 Household problem

The household faces two choices: how much to consume and whether to work or study. If they work, $l_t = 1$, they receive a post-tax wage $(1-\tau_t)w_t$ proportional to their productivity $z_t e_t$, where $z_t$ is they idiosyncratic productivity shock, and $e_t$ is their endogenous and deterministic productivity from education. If they choose not to work, $l_t = 0$, they will instead study, giving a persistent positive shock to $e_t$ of $\varsigma$, which depreciates in each period by the exponent $\gamma \in (0, 1)$. If households never choose to study they will simply have $e_t = 1 \,\forall\, t$. Dividends, $d_t$ are, arbitrarily, given out equally to everyone. I allow for borrowing to give more room for consumption smoothing during periods when households study. This is an easy decision as long as I don't have to implement it: in reality $\underline{\mathrm{a}}$ has to be set close enough to zero to ensure that households do not become insolvent but ideally allow for the fact that they are not expected to always make the minimum possible income.

$$v_t(z_t, a_{t-1}, e_{t-1}) = \max_{c_t, l_t \in \{0,1\}} \frac{c_t^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}\left[v_{t+1}(z_{t+1}, a_t, e_t) \,|\, z_t, a_t, e_t\right] \tag{1}$$

$$\text{s.t. } a_t + c_t = (1 + r_t^a)a_{t-1} + (1 - \tau_t)w_t l_t z_t e_t + (1 - l_t)\,\chi_t + d_t \tag{2}$$

$$\log z_{t+1} = \rho_z \log z_t + \psi_{t+1} \;\; , \psi_t \sim \mathcal{N}(\mu_\psi, \sigma_\psi),\, \mathbb{E}[z_t] = 1 \tag{3}$$

$$e_t = (e_{t-1})^\gamma + \varsigma\,(1 - l_t) \tag{4}$$

$$a_t \geq \underline{\mathrm{a}} \tag{5}$$

This adds an additional state variable, $e_t$, to similar setups we have seen, but the main added difficulty to solving the household problem will be the introduction of the discrete choice, $l_t$ that has to be made, simultaneously with the continuous choice, $c_t$. Luckily, the DC-EGM method proposed by Iskhakov et al. (2017) can help us. Very simplified, this approach performs the EGM-step over the state space *and* the space of discrete choices, then computes value functions for each discrete choice and chosen consumption rules, which allows for finding the optimal switching point between discrete choices. With the known discrete choices, the actual consumption rules can then be constructed. The authors note that each future discrete choice causes the optimal consumption plan to have discontinuities, and they suggest introducing random taste shocks to smooth out the solution, which might be necessary if one were to implement this.

# 2 Model

The full equation system proposed is in equation 6. But as mentioned in the introduction, the rest of the model is just mixing together the HANK models we have already seen, only adding transfers to students, paid by the government, and productivity from education in the market clearing for effective labor.

$$\boldsymbol{H}(\circ) = \begin{bmatrix} Y_t - Z_t N_t \\ \Omega_t - \frac{\mu}{\mu-1}\frac{1}{2\kappa}\left(\log\left(1+\pi_t\right)\right)^2 Y_t \\ d_t - \left[Y_t - w_t N_t - \Omega_t\right] \\ \log(1+\pi_t) - \left[\kappa\left(\frac{w_t}{Z_t} - \frac{1}{\mu}\right) + \frac{Y_{t+1}}{Y_t}\frac{\log(1+\pi_{t+1}}{1+r_{t+1}})\right] \\ 1 + i_t - (1+i_{t-1})^{\rho_i}\left(\left(1+r_{ss}\right)\left(1+\pi_t\right)^{\phi_\pi}\right)^{1-\rho_i} \\ 1 + r_t - \frac{1+i_{t-1}}{1+\pi_t} \\ \frac{1+\delta q_{t+1}}{q_t} - (1+r_t) \\ 1 + r_t^a - \frac{1+\delta q_t}{q_{t-1}} \\ \tau_t - \left[\tau_{ss} + \omega q_{ss}\frac{B_{t-1}-B_{ss}}{Y_{ss}}\right] \\ q_t(B_t - \delta B_{t-1}) - \left[B_{t-1} + G_t + \chi_t \int\left(1 - l_t^*\right) d\mathbf{D}_t - \tau_t Y_t\right] \\ q_t B_t - \boldsymbol{a}_t^{*\prime}\boldsymbol{D}_t \\ N_t - \int l_t^* z_t e_t d\mathbf{D}_t \\ \boldsymbol{D}_t - \Pi_z^\prime \underline{\boldsymbol{D}}_t \\ \underline{\boldsymbol{D}}_t - \Lambda_t^\prime \boldsymbol{D}_t \end{bmatrix} = \mathbf{0} \tag{6}$$

The first 4 equations are all derived from the final and intermediary goods firms set up in lecture 8. The first equation is the production functions of the intermediary goods firms, which, because of symmetry, can simply be written with aggregate inputs and outputs. Because of symmetry and the production of the final goods firms aggregate intermediary output is also aggregate final output.

In equation 2 I have written up the adjustment cost as they are the same for all intermediary firms because of symmetry. Dividends paid out to households are specified in the third equation as earnings minus labor and adjustment costs.

The third equation is New Keynesian Phillips curve implied by the intermediary firms facing costs to adjusting their prices.

In the fourth equation I use the Taylor rule with persistence from assignment II.

The fifth equation denotes the ex ante real interest rate via the fisher equation. The sixth equation denotes the ex post real return on government bonds bought in period $t-1$. The seventh, eighth, and ninth equations are the same bonds market equations as in assignment II, with the exception that $\chi_t$ is only paid out to the part of households that are not studying. The labor market, noted in effective labor, also needs to clear, but only the non-studying part of the households supply labor.

# 3 Applications

The main interest of this model would be to look at how the education system feeds back into the economy when the economy is hit by a shock. One way of doing this would be to see the IRF to a supply shock, e.g. $Z$, differs by changing $\chi$, or to compare the IRFs of this model with the IRFs of a simpler model without an education system.

My expectation would be that a negative shock to $Z$, would lower the wages, which would push more households to study, exacerbating the shock initially as households reduce their labor supply, but as the shock disappears the economy would rebound faster as the better-educated households come back to the labor market. This argument suggests that the education system exacerbates macro volatility from supply shocks.

Analogously, one could look a demand shock, the easiest being wasteful government spending, $G_t$. And look at whether the education system interacts with the different shocks in different ways. I could also be interesting to investigate which households that choose education during an economic downturn. Are the the high productivity households who gain the most form it (because the gain of education is proportional to productivity. Or do the low productivity household choose education as the difference between work income and study subsidy becomes smaller?

One could also do a steady state analysis and look at the optimal level of public subsidy of education, $\chi$. The common market failure associated with education, that is has a positive externality on production is not included. But if there are many households that are credit constrained and would benefit from education, so there could be positive gains from a public subsidy through this mechanism.

# References

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