CENTER FOR
ECONOMIC
BEHAVIOR &
INEQUALITY

2. Consumption-Saving Models

Adv. Macro: Heterogenous Agent Models

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Danmarks
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Danish National
Research Foundation

Introduction

Disclaimer

- Note: The views expressed in this presentation are those of the author and do not represent the views of the Federal Reserve Board or Federal Reserve System.

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- **Plan for today:**
  1. Discuss the MPC, why it matters, and how it looks in the data
  2. Consider a variety of models that attempt to match the data
  3. Study the link between income risk and consumption behavior

MPC

The Marginal Propensity to Consume (MPC)

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- For a comprehensive overview, see Kaplan and Violante (2021)

Why do we care about the MPC?

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- Central concept in modern heterogeneous-agent macroeconomics
- Affects spending response to fiscal stimulus and monetary policy
- Tension between data and models
- Disagreement among economists
- We need macro models that can reproduce the data on MPC

MPC in the Data: Methods

- Three strands of empirical evidence on the size of the MPC:

  1. Quasi-experimental evidence
     Johnson-Parker-Souleles (2006): Economic Impact Payments
     Shapiro et al. (2017): government shutdown
     Fagereng et al. (2020), Golosov et al. (2021): lottery wins

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  3. Structural estimates
     Blundell-Pistaferri-Preston (2008), Commault (2019)

MPC in the Data: Findings

- The quarterly aggregate MPC is between 15% and 25%
    - Size dependence: MPC larger for small income shocks
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  - Size dependence: MPC larger for small income shocks
  - Sign asymmetry: MPC much larger for negative income shocks
- There is large heterogeneity in MPCs across households
  - Liquid wealth: MPC larger for low wealth households
  - Fixed individual characteristics: MPC larger for young, low-income households

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- Question: how can common macro models generate a large MPC?

MPCs in Macro Models

Representative Agent (RA) Model

- No idiosyncratic risk, no borrowing constraint
- Household problem:

$$\max_{\{c_t, b_{t+1}\}} \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma}$$

s.t.

$$c_t + b_{t+1} = Rb_t + y_t$$

- Consumption function:

$$c(b) = \mathfrak{m}^{CE}\left[Rb + \sum_{t=0}^{\infty}\left(\frac{1}{R}\right)^t y_t\right], \text{ where } \mathfrak{m}^{CE} = 1 - R^{-1}(R\beta)^{\frac{1}{\gamma}}$$

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- The consumption function is linear in asset holdings (b) $\rightarrow$ wealth distribution irrelevant for MPC

Representative Agent (RA) Model

- Parameterization:
  1. Log utility ($\gamma = 1$): then we can simplify to: $\mathfrak{m}^{CE} = 1 - \beta$
  2. Plausible (quarterly) calibrations: $\mathfrak{m}^{CE} = 0.5\%$
- Representative Agent model features a tiny MPC

$$c\left(b\right) = 0.005 * \left[Rb + \sum_{t=0}^{\infty} \left(\frac{1}{R}\right)^{t} y_t\right]$$

Main Takeaways for the MPC

Can macro models generate a high MPC, and if so, how?

1. RA model: No

One-Asset Heterogeneous Agent (HA) Model

- Add idiosyncratic income risk, realistic borrowing constraint

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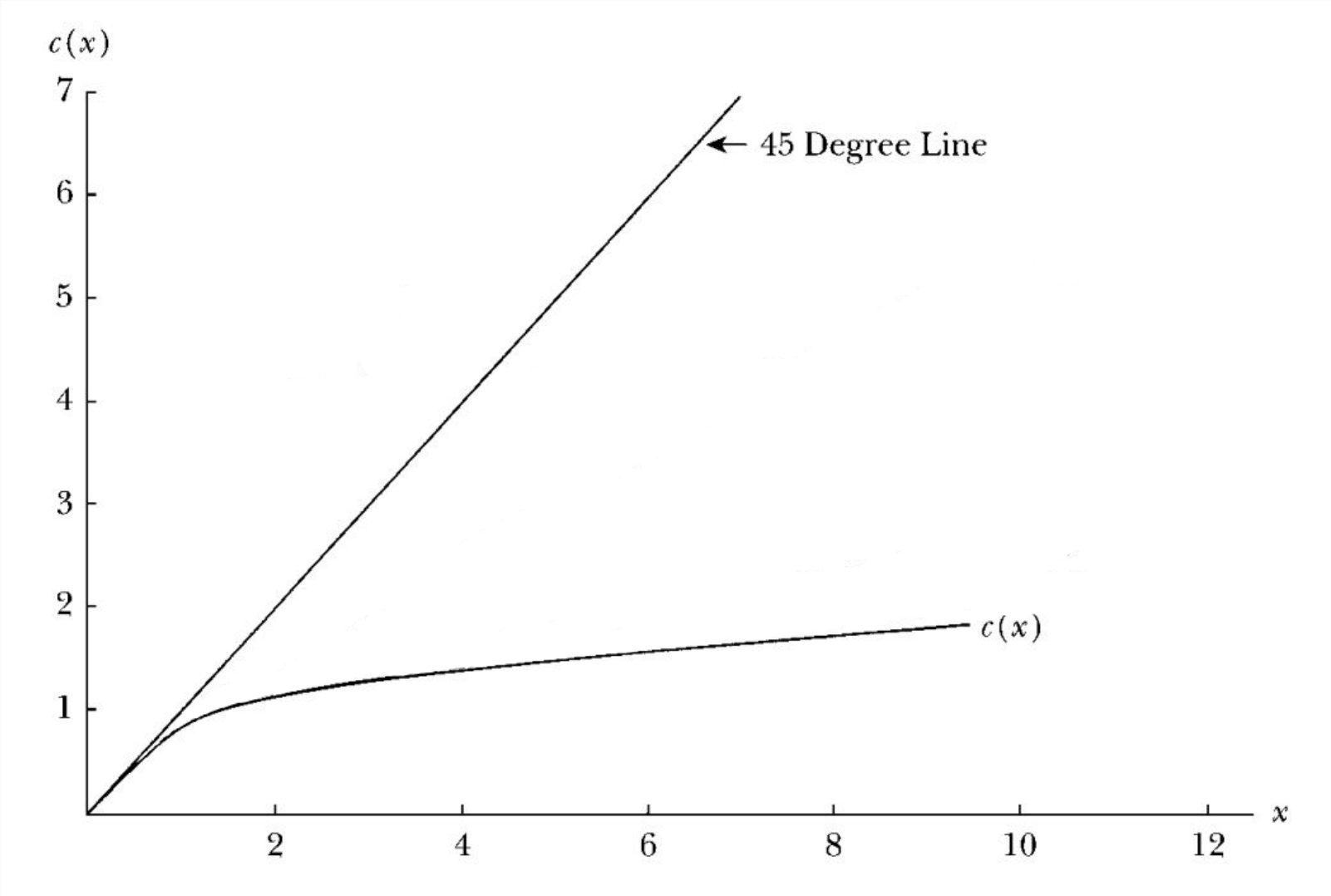
- Main takeaways:
  1. Consumption function $c(b)$ is concave due to precautionary motive
  2. There is an optimal buffer stock of assets that HHs want to achieve

Consumption function is concave



$x = {}^{b}/_{y}$ is the share of assets to permanent income (Carroll 2001)

Households try to achieve an optimal buffer stock

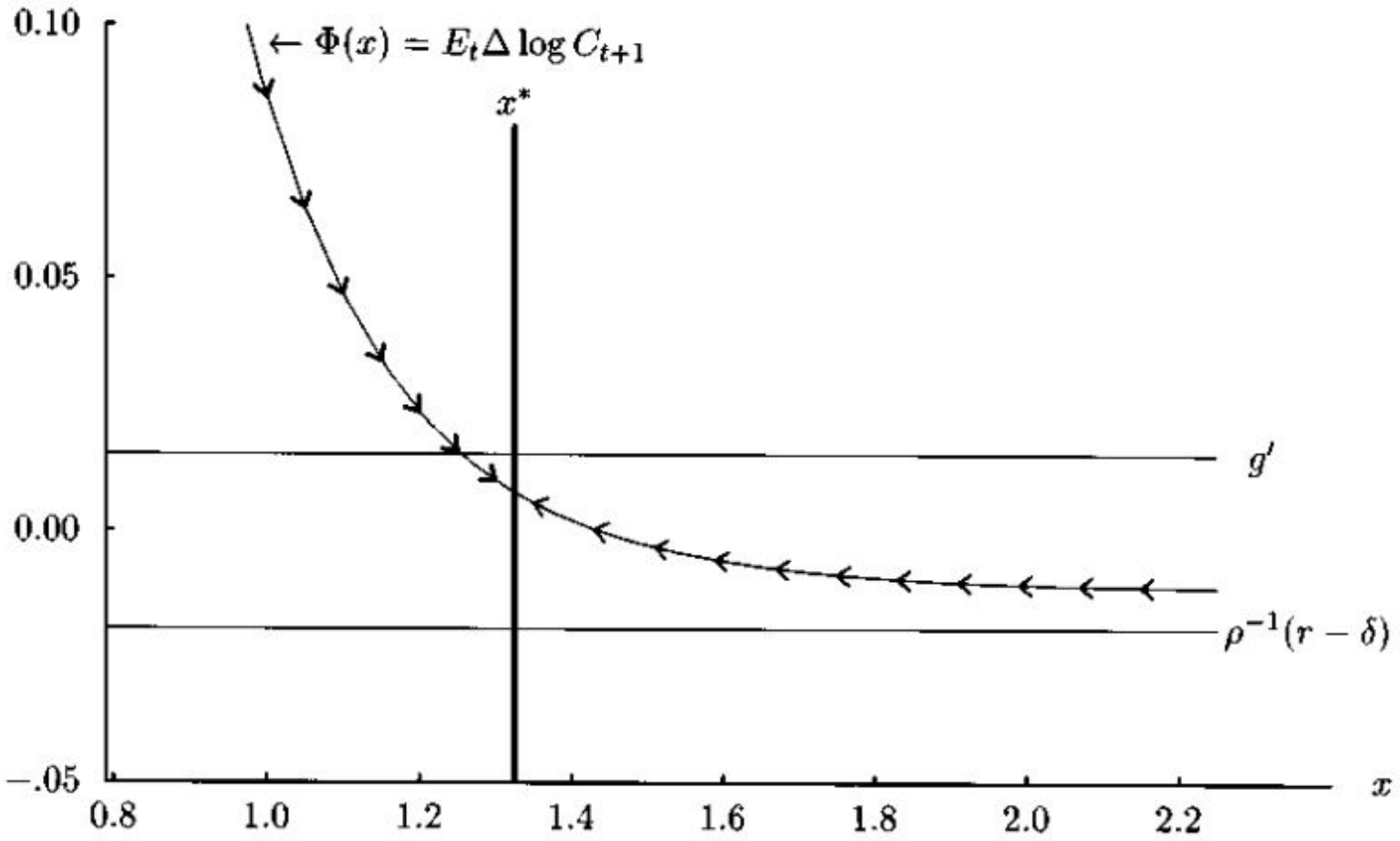


FIGURE Ia
Expected Consumption Growth as a Function of Cash on Hand

Households try to achieve an optimal buffer stock

Takeaways:

1. As $x \to \infty$, the expected growth rate of consumption (and the MPC) converge to their values in the RA model
2. As $x \to 0$, the expected growth rate of consumption approaches infinity, and the MPC approaches one

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2. As $x \to 0$, the expected growth rate of consumption approaches infinity, and the MPC approaches one
3. If the consumer is impatient, there exists a unique target assets-to-permanent-income ratio (x^*)

From the inidividual to the aggregate MPC

- Individual MPC for a household with state (b, y):

$$m(b, y) = \frac{c(b + x, y) - c(b, y)}{x} \simeq \frac{\partial c(b, y)}{\partial b}$$

$$\overline{\mathrm{m}} = \int_{B \times Y} \mathfrak{m}(b, y) d\mu(b, y)$$

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- Two key determinants:
  1. Consumption function $c(b, y) \Longrightarrow$MPC function $m(b, y)$
  2. Wealth distribution $\mu(b, y)$

What determines the size of the aggregate MPC?

- Shape of the consumption function
  - Uninsurable income risk $\rightarrow$ precautionary saving motive
    - Prudence ($u''' > 0$)
    - Occasionally binding borrowing constraint
  - Strength of precautionary saving is decreasing in wealth
  - Consumption function is concave $\rightarrow$ MPC is decreasing in wealth
  - As wealth grows, the MPC $\rightarrow$ MPC in the RA model

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- Shape of the wealth distribution
  - Bigger mass at bottom, where c function is concave $\rightarrow$ large MPC
  - Hand-to-mouth (H2M) households with zero wealth and MPC=1

What is a reasonable calibration of such a model?

- **Calibration Strategy:**
  1. As before, we set $\gamma = 1$, so that we have log utility
  2. Set the interest rate r to be 1% per year
  3. Choose β so that the model matches some target of mean wealth

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  1. Target US data: wealth to income ratio of 4.1
  2. This gives an MPC of 4.6%

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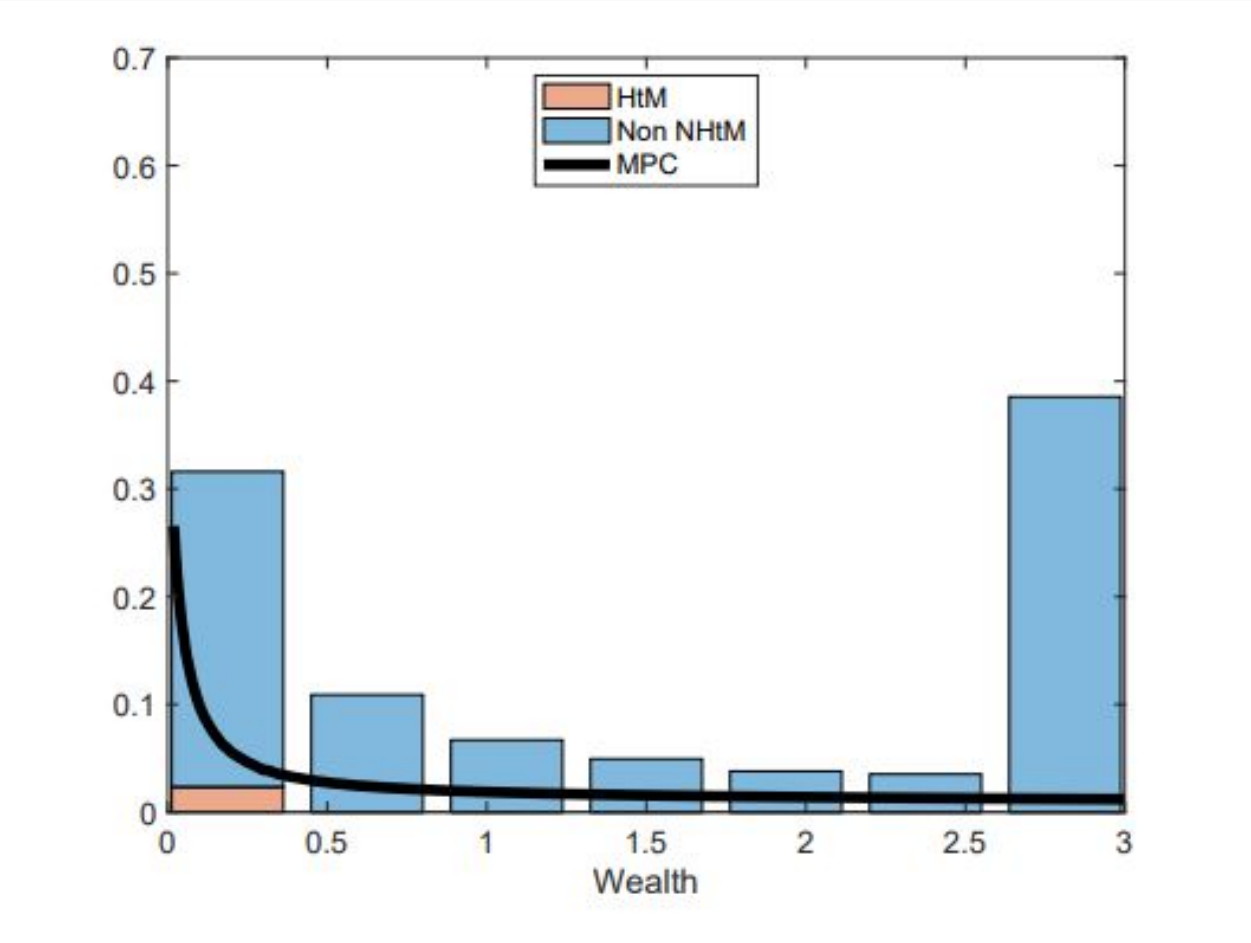

- Households want to escape the borrowing limit
- Very few high MPC households

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- **Calibration 1:**
  1. Target US data: wealth-to-income ratio of 4.1
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- **Calibration 2:**
  1. Target a counterfactual wealth-to-income ratio of 0.5
  2. This gives an MPC of 14%

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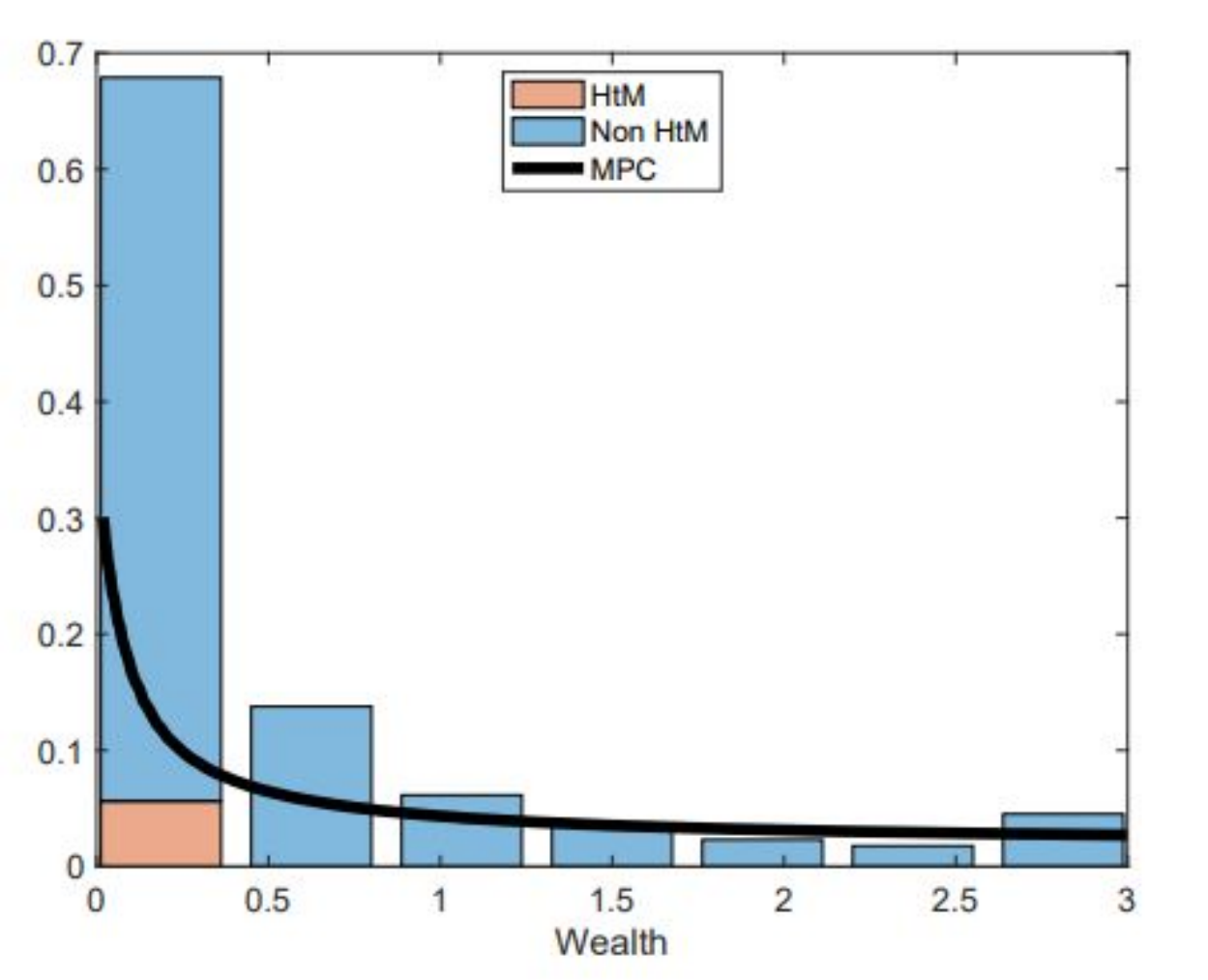

- Now we have a lot more high MPC households
- But we miss the vast majority of wealth in the economy

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- Observation:
  1. Not all household wealth is immediately available for consumption smoothing
  2. Important difference between liquid and illiquid wealth
  3. In line with evidence that MPC declines in liquid wealth

Two-Asset HA Model

- Continuum of households
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- Fixed transaction cost to move funds into / out of illiquid account

Emergence of Wealthy HtM Households

- Three types of households in the model:
  - Unconstrained (60%)
  - Poor HtM: zero net worth (14%)
  - Wealthy HtM: zero liquid wealth, but positive illiquid wealth (26%)

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- If gains exceeds costs $\Longrightarrow$ Wealthy HtM

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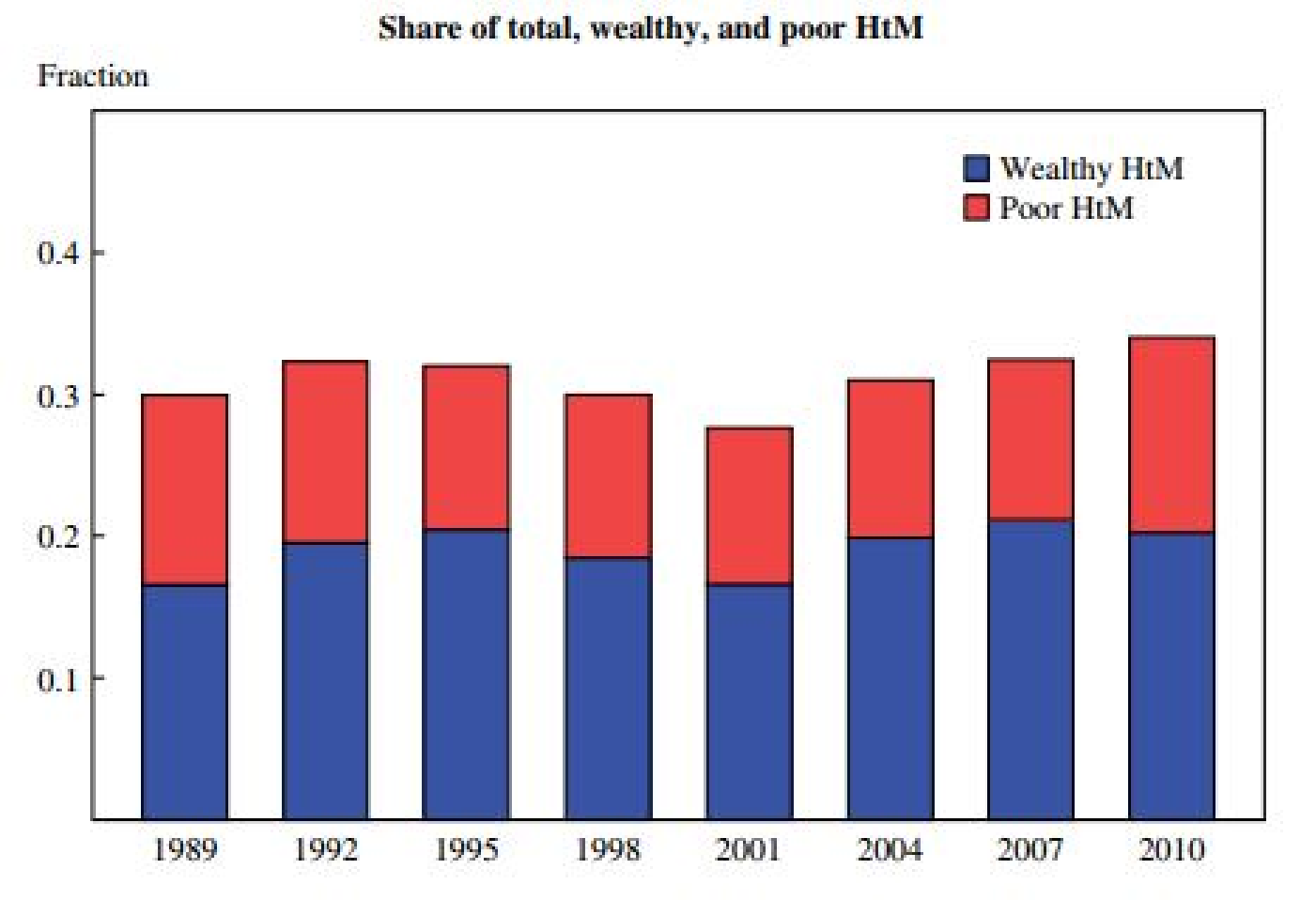

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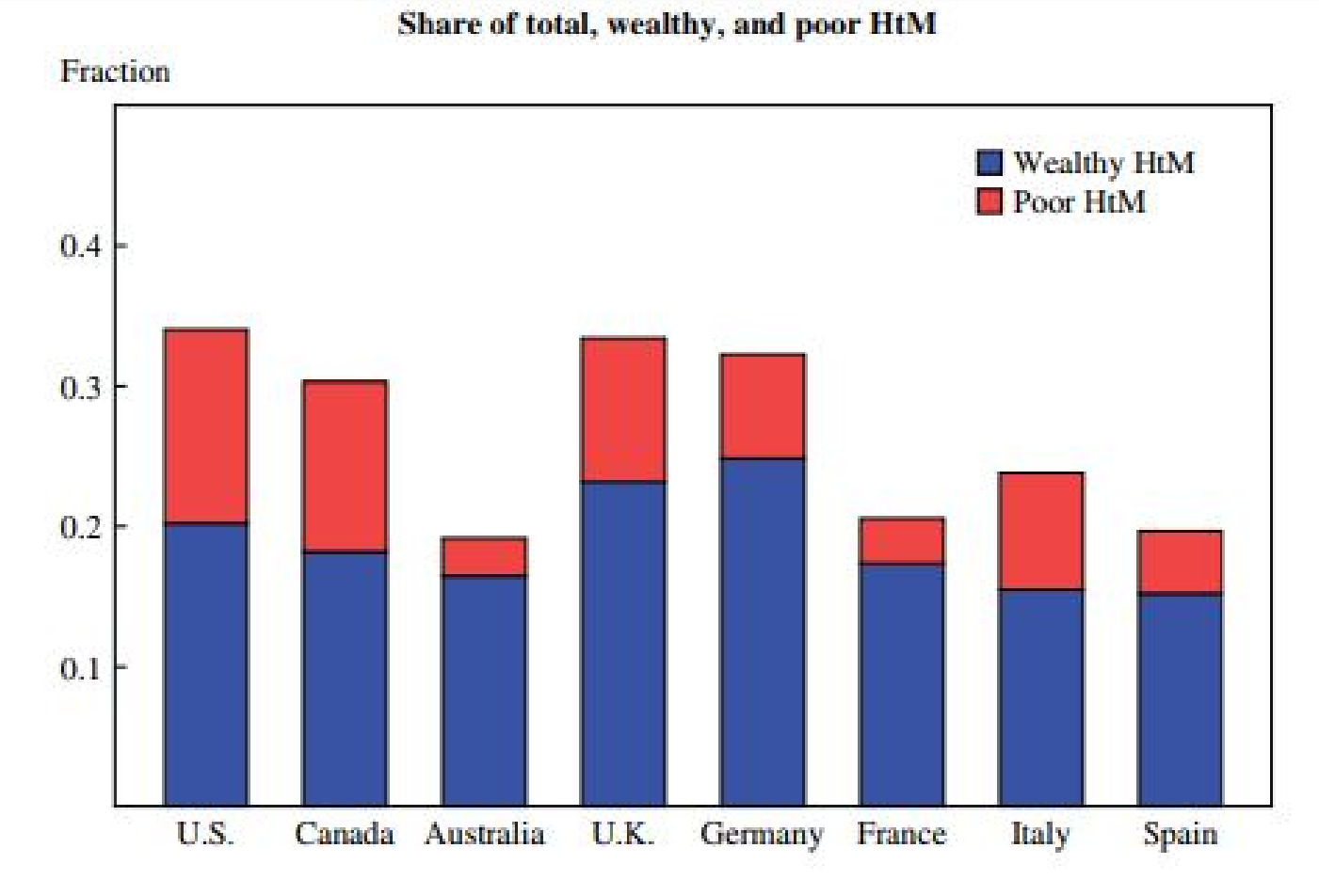

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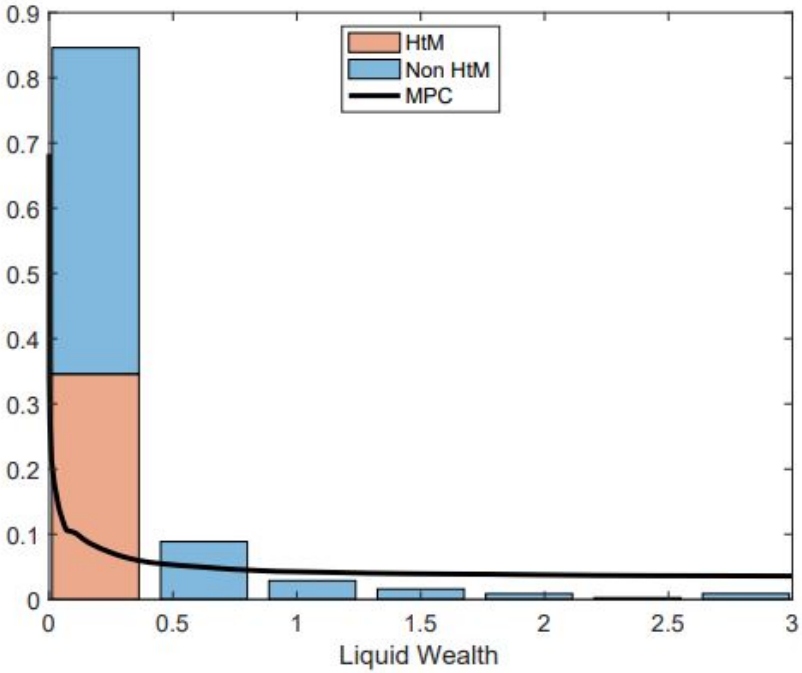
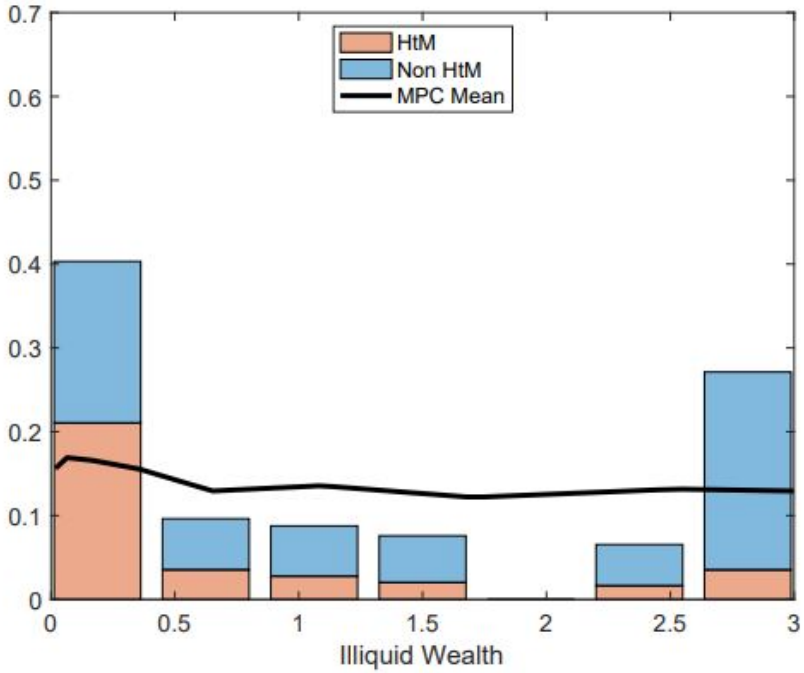
    - Transaction cost κ
  - Choose these three parameters so the model matches three targets:
    - Mean wealth-to-income ratio (4.1)
    - Share of HtM households (34%)
    - Share of wealthy HtM households (25%)

Results from the two-asset model





- What matters most for the MPC is liquid wealth, not total wealth
- Wealthy HtM have a very high MPC
- MPC remains high even for households with sizeable illiquid wealth
- Average MPC $= 15\%$

Main Takeaways for the MPC

- Can macro models generate a high MPC, and if so, how?
  1. RA model: No
  2. One-asset HA model: only by neglecting the majority of wealth
  3. Two-asset HA model: successful, while also matching wealth holdings

Unemployment Risk

Unemployment Risk and Consumption Dynamics

- **Question:** How does unemployment risk affect household spending?
  - During recessions, unemployment risk increases
  - This may induce HHs to increase their buffer stock of assets
  - The resulting fall in consumption may increase output volatility
  - This channel has been difficult (if not impossible) to capture with RA models

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  - This channel has been difficult (if not impossible) to capture with RA models
- **Our goal:** Study a HA model that can capture this channel
  - We will closely follow Harmenberg and Öberg (2021)
  - Consumption falls in response to increased risk during recessions
  - Households increase their precautionary savings and postpone irreversible durable investments.

Model

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- Subject to

$$C_{it} + D_{it} + qB_{it} \leq \Upsilon\left(Y_{it}, n_{it}\right) + (1-\delta)D_{it-1} + B_{it-1} - A\left(D_{it}, D_{it-1}\right),$$
$$C_{it}, D_{it}, B_{it} \geq 0.$$

Model

- Adjustment costs to durable consumption

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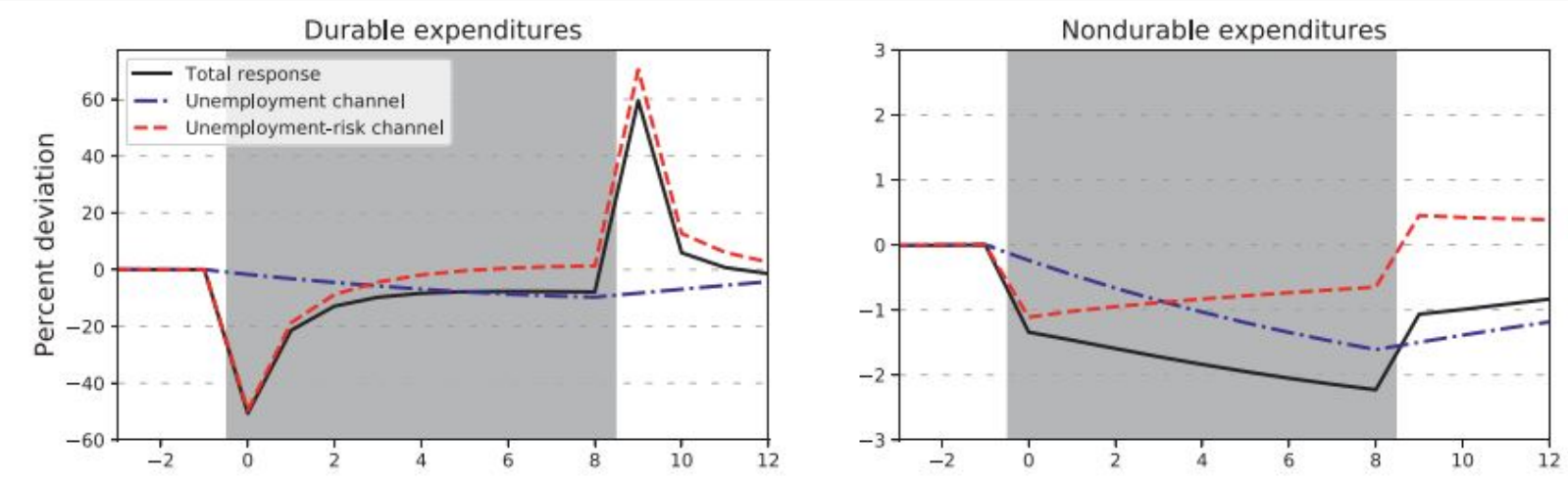
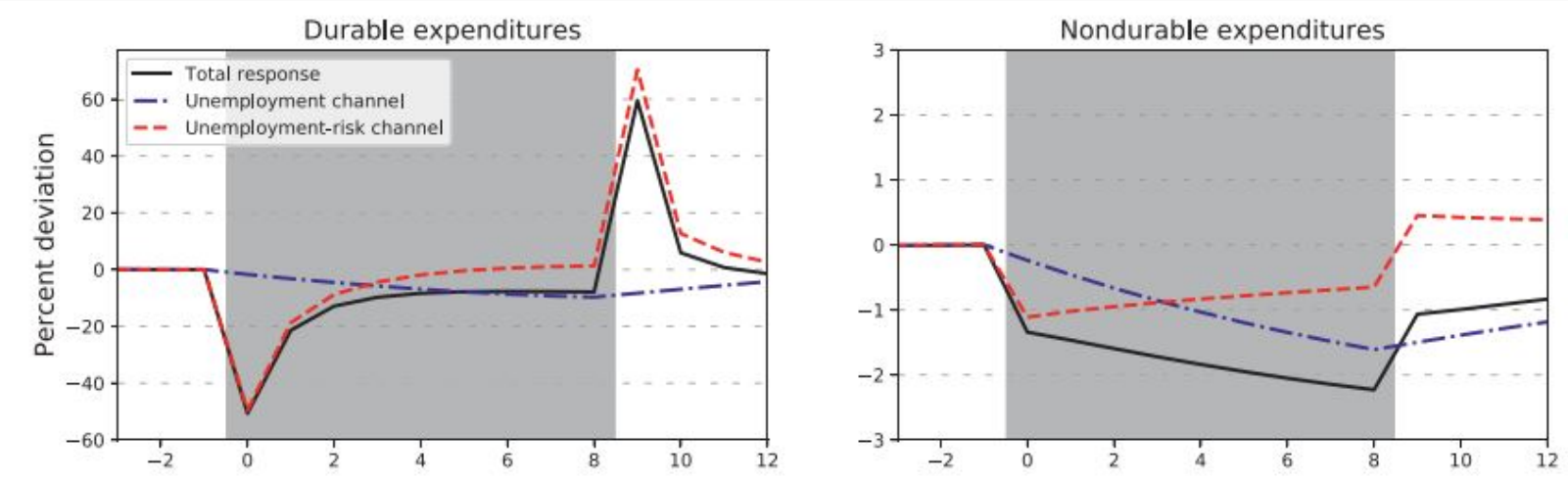
- Which of these channels is more important?

Results



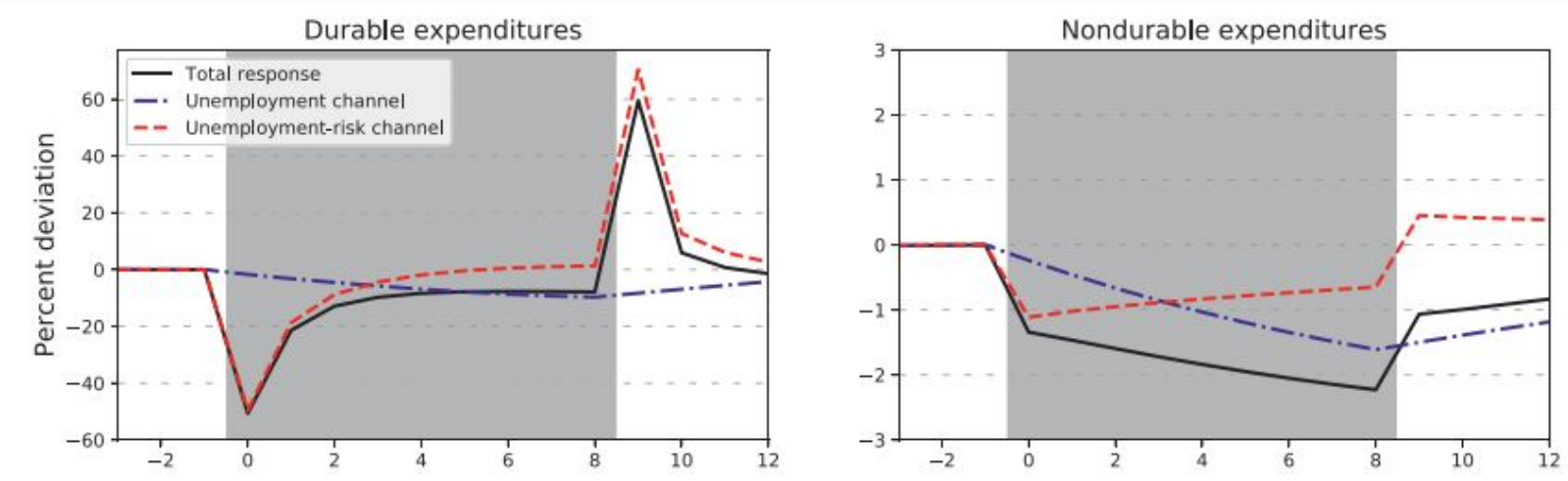
- Response of durables is much larger than nondurables

Results



- Response of durables is much larger than nondurables
- For durables: unemployment-risk channel is most important

Results



- Response of durables is much larger than nondurables
- For durables: unemployment-risk channel is most important
- For nondurables: unemployment-risk matters initially, but unemployment accounts for the majority in the long-term

Summary

Summary and next week

- **Today:** Three applications of dynamic programming to understand household spending dynamics
  1. The role of credit constraints
  2. Modeling the large average MPC to income shocks
  3. Consumption dynamics with time-varying unemployment risk

- **Next week:** Life-cycle consumption-saving models with deviations from full rationality

- **Homework exercises:**
  1. Plot the MPC function in the one-asset HA model
  2. As you adjust average wealth holdings in the HA model, how does the average MPC change?