



## 2. Consumption-Saving Models

Adv. Macro: Heterogenous Agent Models

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# Introduction

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# Disclaimer

- Note: The views expressed in this presentation are those of the author and do not represent the views of the Federal Reserve Board or Federal Reserve System.

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  3. What is the effect of income risk on consumption dynamics?
- **Plan for today:**
  1. Discuss the MPC, why it matters, and how it looks in the data
  2. Consider a variety of models that attempt to match the data
  3. Study the link between income risk and consumption behavior

# MPC



# The Marginal Propensity to Consume (MPC)

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- For a comprehensive overview, see Kaplan and Violante (2021)

# Why do we care about the MPC?

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- Affects spending response to fiscal stimulus and monetary policy
- Tension between data and models
- Disagreement among economists
- We need macro models that can reproduce the data on MPC

- Three strands of empirical evidence on the size of the MPC:
  1. Quasi-experimental evidence
    - Johnson-Parker-Souleles (2006): Economic Impact Payments
    - Shapiro et al. (2017): government shutdown
    - Fagereng et al. (2020), Golosov et al. (2021): lottery wins



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  3. Structural estimates
    - Blundell-Pistaferri-Preston (2008), Commault (2019)

# MPC in the Data: Findings

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- Size dependence: MPC larger for small income shocks
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  - Size dependence: MPC larger for small income shocks
  - Sign asymmetry: MPC much larger for negative income shocks
- There is large heterogeneity in MPCs across households
  - Liquid wealth: MPC larger for low wealth households
  - Fixed individual characteristics: MPC larger for young, low-income households

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- Question: how can common macro models generate a large MPC?

# MPCs in Macro Models

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# Representative Agent (RA) Model

- No idiosyncratic risk, no borrowing constraint
- Household problem:

$$\max_{\{c_t, b_{t+1}\}} \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma}$$

s.t.

$$c_t + b_{t+1} = Rb_t + y_t$$

- Consumption function:

$$c(b) = m^{CE} \left[ Rb + \sum_{t=0}^{\infty} \left( \frac{1}{R} \right)^t y_t \right], \text{ where } m^{CE} = 1 - R^{-1}(R\beta)^{\frac{1}{\gamma}}$$

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- The consumption function is linear in asset holdings ( $b$ )  $\rightarrow$  wealth distribution irrelevant for MPC

# Representative Agent (RA) Model

- Parameterization:
  1. Log utility ( $\gamma = 1$ ): then we can simplify to:  $m^{CE} = 1 - \beta$
  2. Plausible (quarterly) calibrations:  $m^{CE} = 0.5\%$
- Representative Agent model features a tiny MPC

$$c(b) = 0.005 * \left[ Rb + \sum_{t=0}^{\infty} \left( \frac{1}{R} \right)^t y_t \right]$$

# Main Takeaways for the MPC

Can macro models generate a high MPC, and if so, how?

1. RA model: No

# One-Asset Heterogeneous Agent (HA) Model

- Add idiosyncratic income risk, realistic borrowing constraint
- Household problem:

$$\max_{\{c_t, b_{t+1}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma}$$

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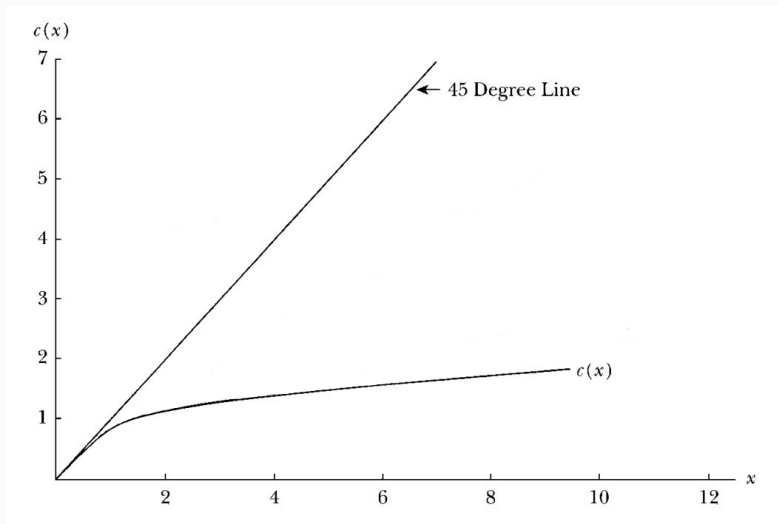
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- Main takeaways:
  1. Consumption function  $c(b)$  is concave due to precautionary motive
  2. There is an optimal buffer stock of assets that HHs want to achieve

# Consumption function is concave



$x = b/y$  is the share of assets to permanent income (Carroll 2001)

# Households try to achieve an optimal buffer stock

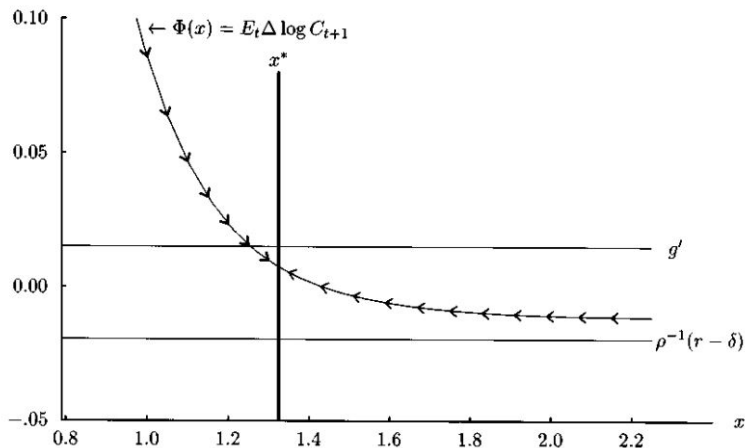


FIGURE 1a  
Expected Consumption Growth as a Function of Cash on Hand



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Takeaways:

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Takeaways:

1. As  $x \rightarrow \infty$ , the expected growth rate of consumption (and the MPC) converge to their values in the RA model
2. As  $x \rightarrow 0$ , the expected growth rate of consumption approaches infinity, and the MPC approaches one
3. If the consumer is impatient, there exists a unique target assets-to-permanent-income ratio ( $x^*$ )

# From the individual to the aggregate MPC

- Individual MPC for a household with state  $(b, y)$ :

$$m(b, y) = \frac{c(b + x, y) - c(b, y)}{x} \simeq \frac{\partial c(b, y)}{\partial b}$$

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- Two key determinants:
  1. Consumption function  $c(b, y) \implies$  MPC function  $m(b, y)$
  2. Wealth distribution  $\mu(b, y)$

# What determines the size of the aggregate MPC?

- Shape of the consumption function
  - Uninsurable income risk → precautionary saving motive
    - Prudence ( $u'' > 0$ )
    - Occasionally binding borrowing constraint
  - Strength of precautionary saving is decreasing in wealth
  - Consumption function is concave → MPC is decreasing in wealth
  - As wealth grows, the MPC → MPC in the RA model

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  - Consumption function is concave  $\rightarrow$  MPC is decreasing in wealth
  - As wealth grows, the MPC  $\rightarrow$  MPC in the RA model
- Shape of the wealth distribution
  - Bigger mass at bottom, where  $c$  function is concave  $\rightarrow$  large MPC
  - Hand-to-mouth (H2M) households with zero wealth and  $MPC=1$



# What is a reasonable calibration of such a model?

- **Calibration Strategy:**

1. As before, we set  $\gamma = 1$ , so that we have log utility
2. Set the interest rate  $r$  to be 1% per year
3. Choose  $\beta$  so that the model matches some target of mean wealth

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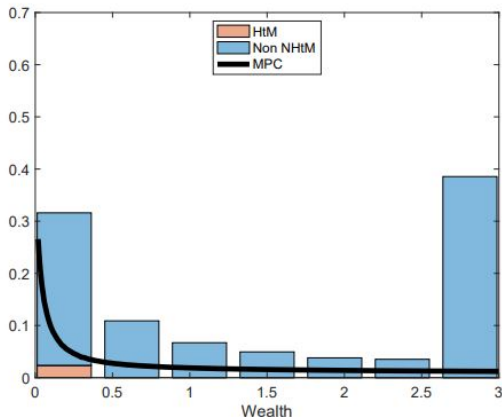
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- **Calibration 1:**

1. Target US data: wealth to income ratio of 4.1
2. This gives an MPC of 4.6%

# What is a reasonable calibration of such a model?



- Households want to escape the borrowing limit
- Very few high MPC households

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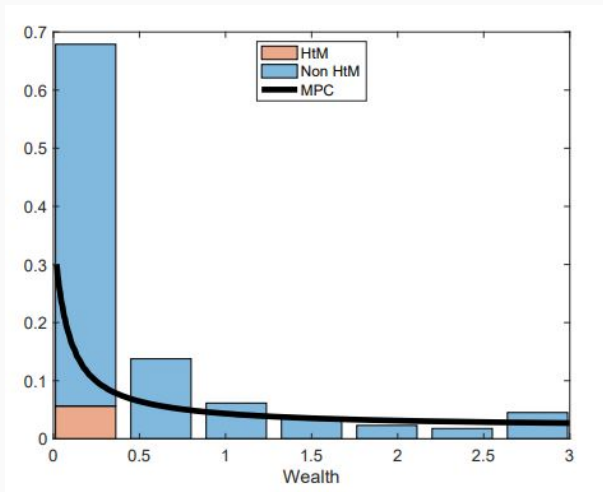
- **Calibration 1:**

1. Target US data: wealth-to-income ratio of 4.1
2. This gives an MPC of 4.6%

- **Calibration 2:**

1. Target a counterfactual wealth-to-income ratio of 0.5
2. This gives an MPC of 14%

# What is a reasonable calibration of such a model?



- Now we have a lot more high MPC households
- But we miss the vast majority of wealth in the economy

# Main Takeaways for the MPC

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- Wanted: a version of the HA model that:
  1. Generates a large aggregate MPC
  2. Matches wealth holdings as in the data
- Observation:
  1. Not all household wealth is immediately available for consumption smoothing
  2. Important difference between liquid and illiquid wealth
  3. In line with evidence that MPC declines in liquid wealth

# Two-Asset HA Model

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- Two assets: liquid ( $m$ ) and illiquid ( $a$ ) with  $r^a > r^m$ 
  - Liquid: cash + deposits + directly held stock - unsecured debt
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  - Liquid: cash + deposits + directly held stock - unsecured debt
  - Illiquid: housing equity + retirement account (85% of net worth)
- Fixed transaction cost  $\kappa$  to move funds into / out of illiquid account

# Two-Asset HA Model

- Value function is the max of the value if you do not ( $N$ ) or do adjust ( $A$ ) illiquid assets

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subject to

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- Choices:  $(c_j, m_{j+1})$  = consumption, liquid asset tmrw

- Value function if you adjust:

$$V_j^A(a_j, m_j, z_j) = \max_{c_j, a_{j+1}, m_{j+1}} u(c_j) + \beta \mathbb{E}_j[v_{j+1}(a_{j+1}, m_{j+1}, z_{j+1})]$$

subject to

$$c_j + a_{j+1} + m_{j+1} \leq a_j(1 + r^a) + m_j(1 + r^m) - \kappa + y_j(z_j)$$

$$a_{j+1} \geq 0, m_{j+1} \geq \underline{m}$$

- Choices:  $(c_j, a_{j+1}, m_{j+1})$  = consumption, illiquid asset tmrw, liquid asset tmrw

## Result: Two different Euler equations

- Short-Run Euler Equation - governed by saving vs dissaving in the liquid asset

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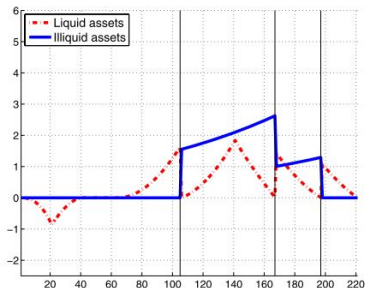
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- Long-Run Euler Equation - governed by saving vs dissaving in the illiquid assets

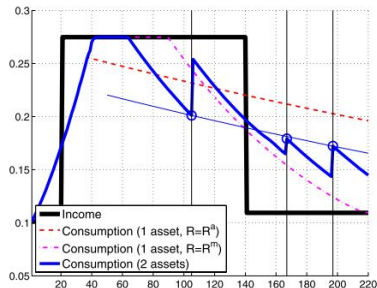
$$u'(c_j) = \beta(1 + r^m)^N u'(c_{j+N})$$

- where  $N$  is the number of periods between adjustment

# Example 1



(a) Life-cycle asset accumulation

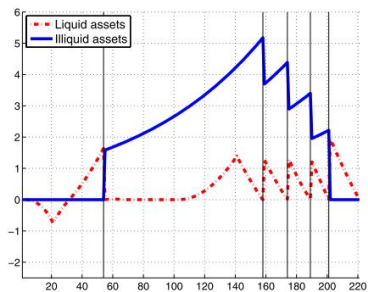


(b) Life-cycle income and consumption path

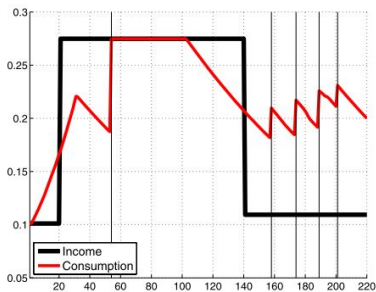
FIGURE 1.—Example of life-cycle of a poor hand-to-mouth agent in the model.

- Agent exhibits poor hand-to-mouth behavior between periods 40-60, when she consumes all of her income and holds zero liquid assets

## Example 2



(a) Life-cycle asset accumulation



(b) Life-cycle income and consumption path

FIGURE 2.—Example of life-cycle of a wealthy hand-to-mouth agent in the model.

- Agent exhibits wealthy hand-to-mouth behavior between periods 55 to 100, when she owns illiquid wealth, but zero liquid wealth

# Result: Emergence of Wealthy HtM Households

- Three types of households in the model:
  - Unconstrained (60%)
  - Poor HtM: zero net worth (14%)
  - Wealthy HtM: zero liquid wealth, but positive illiquid wealth (26%)



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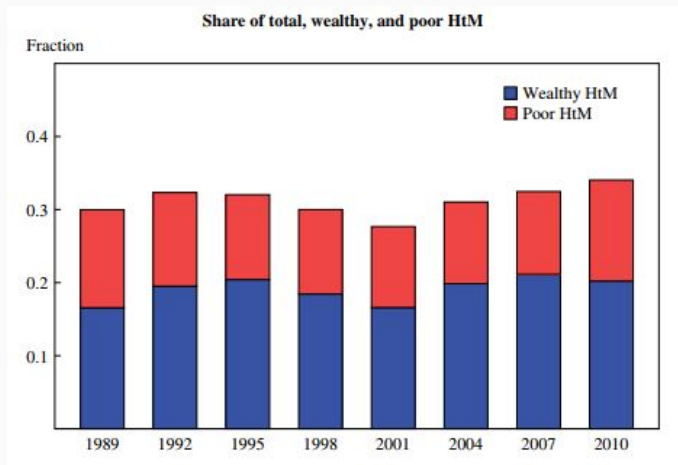
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  - Short-run cost: worse consumption smoothing

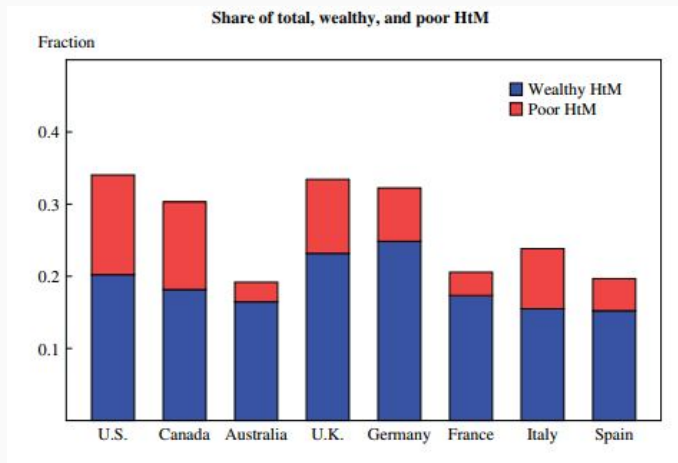
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- If gains exceeds costs  $\implies$  Wealthy HtM

# Wealthy HtM households in the data



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# What is a reasonable calibration of such a model?

- **Calibration Strategy:**

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  - Transaction cost  $\kappa$

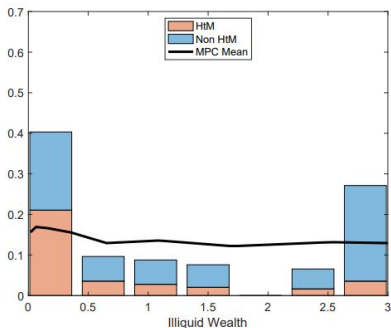
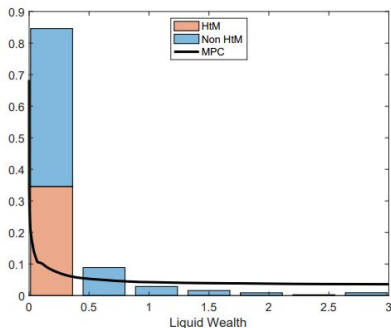
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- Choose these three parameters so the model matches three targets:
  - Mean wealth-to-income ratio (4.1)
  - Share of HtM households (34%)
  - Share of wealthy HtM households (25%)



# Results from the two-asset model



- What matters most for the MPC is liquid wealth, not total wealth
- Wealthy HtM have a very high MPC
- MPC remains high even for households with sizeable illiquid wealth
- Average MPC = 15%

# Main Takeaways for the MPC

- Can macro models generate a high MPC, and if so, how?
  - RA model: No.
    - $MPC = 0.05\%$
  - One-asset HA model:
    - Realistic wealth calibration:  $MPC = 0.5\%$
    - Low wealth calibration:  $MPC = 15\%$
  - Two-asset HA model:
    - Realistic wealth calibration:  $MPC = 15\%$

# Unemployment Risk

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- **Question:** How does unemployment risk affect household spending?
  - During recessions, unemployment risk increases
  - This may induce HHs to increase their buffer stock of assets
  - The resulting fall in consumption may increase output volatility
  - This channel has been difficult (if not impossible) to capture with RA models

# Unemployment Risk and Consumption Dynamics

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  - This may induce HHs to increase their buffer stock of assets
  - The resulting fall in consumption may increase output volatility
  - This channel has been difficult (if not impossible) to capture with RA models
- **Our goal:** Study a HA model that can capture this channel
  - We will closely follow Harmenberg and Öberg (2021)
  - Consumption falls in response to increased risk during recessions
  - Households increase their precautionary savings and postpone irreversible durable investments.

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$$C_{it}, D_{it}, B_{it} \geq 0.$$



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  - Unemployment-risk channel (ex-ante)
  - Unemployment channel (ex-post)

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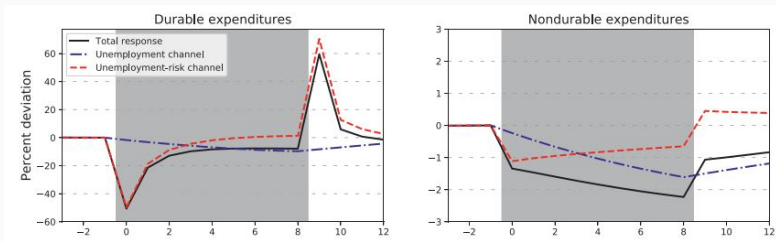
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- Which of these channels is more important?

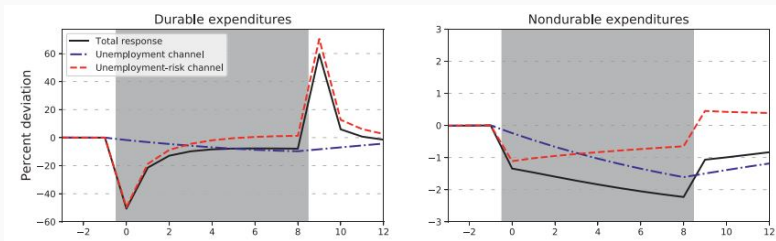


# Results



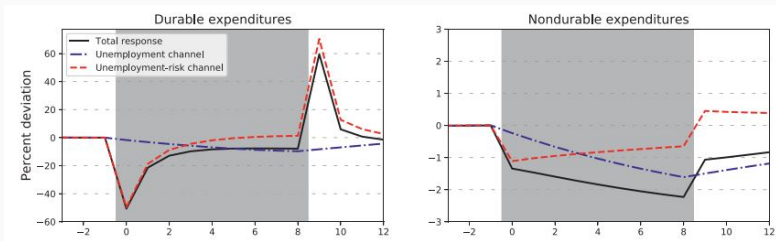
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- Response of durables is much larger than nondurables
- For durables: unemployment-risk channel is most important
- For nondurables: unemployment-risk matters initially, but unemployment accounts for the majority in the long-term

# Summary

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  2. Modeling the large average MPC to income shocks
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- **Homework exercises:**
  1. Plot the MPC function in the model from the previous lecture
  2. Adjust beta to target different levels of average wealth. How does it change the average MPC?
  3. Add unemployment risk to the income process you saw last week. How does it change average savings?