CENTER FOR ECONOMIC BEHAVIOR & INEQUALITY

5. Wealth Inequality

Adv. Macro: Heterogenous Agent Models

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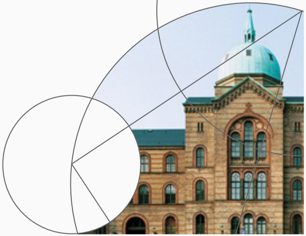

Introduction

Disclaimer

- Note: The views expressed in this presentation are those of the author and do not represent the views of the Federal Reserve Board or Federal Reserve System.

Wealth Inequality

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    3. What explains the rise in wealth inequality in recent decades?

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  2. To what extent can governments affect inequality?
  3. What explains the rise in wealth inequality in recent decades?

- To answer these questions, we need to better understand why people save, and how this translates into wealth inequality

- **Plan for today:**
  1. Study the predictions of a baseline Bewley-Huggett-Aiyagari model
  2. Consider various model extensions that help match the data
  3. Then use such a model to better understand the rise in wealth inequality in the US

Savings and Wealth Inequality

Savings and Wealth Inequality

Basic Facts

Earnings and wealth inequality

- Skewed distributions with thick upper tails
- Wealth more concentrated than earnings

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Top 1%	5%	20%	40%	Perc. at zero or negative
Wealth (SCF)				
29	53	80	93	6
Earnings (LIS)				
6	19	48	72	8

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- Wealth and earnings becoming more concentrated over time
- Rich people (high lifetime income, education, wealth) have a higher saving rate before and after retirement

Savings and Wealth Inequality

Basic models of inequality

Aiyagari Model

- Infinitely lived agents
- Preferences

$$\max_{\{c_t\}_{t=0}^{\infty}} E \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}$$

- Budget constraint

$$a_{t+1} = y_t + (1+r)a_t - c_t, \quad a_{t+1} \geq \underline{a}$$

- Ex-ante identical households hit by earning shocks
- Households are ex-post heterogeneous
- Constant distribution of people over states (assets, age) and individuals face a lot of uncertainty

Aiyagari Model

	Wealth Gini	Wealth in top (%)		
		1 %	5 %	20 %
U.S. data, 1989 SCF	.78	29	53	80
Aiyagari Baseline	.38	3.2	12.2	41.0
Aiyagari higher variability	.41	4.0	15.6	44.6

Finitely lived agents (Huggett model)

- Finitely lived agents with overlapping generations
- Preferences

$$\max_{\{c_t\}_{t=0}^{T}} E \sum_{t=0}^{T} s_t \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}$$

- s_t is the survival probability
- Budget constraint

$$a_{t+1} = y_t + (1+r)a_t - c_t, \quad a_{t+1} \geq \underline{a}$$

- y_t is an exogenous, age-dependent earnings process
  - Stochastic hump shaped income profile over the working life, then flat income stream after exogenous retirement

Closing the model

- Aggregate production function $Y_t = AK_{t-1}^{\alpha} L_t^{1-\alpha}$ (w/ exog. $L = 1$)
- A stationary equilibrium is given by
  1. Quantities K_{ss}
  2. interest rate r_{ss}
  3. the distribution $\boldsymbol{D}_{ss}$ over y_t and a_{t-1}
  4. and the policy functions $a_{ss}^*(z_t, a_{t-1})$ and $c_{ss}^*(z_t, a_{t-1})$

  are such that
  1. Household maximize expected utility (policy functions)
  2. Firms maximize profits (prices)
  3. $\boldsymbol{D}_{ss}$ is the invariant distribution implied by the HH problem
  4. The capital market clears
  5. The goods market clears

Results

- The Huggett model succeeds in matching the US Gini coefficient

	Wealth Gini	Wealth in top (%) 1 %	5 %	20 %	40%	60%	Share with wealth ≤ 0
US data							
	.78	29	53	80	93	98	6
Model							
	.67	7	27	69	90	98	17

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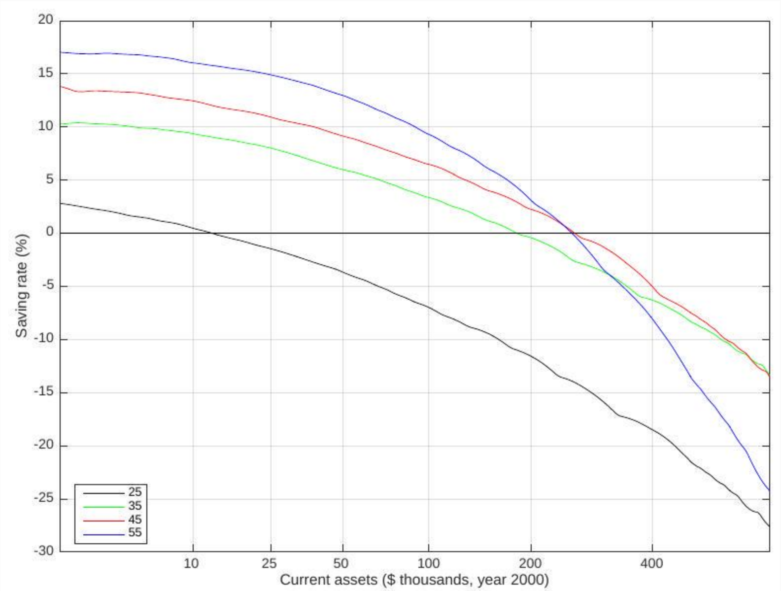
Model	.67	7	27	69	90	98	17

- But this is achieved by having:
  - too many people holding little wealth
  - not by concentrating wealth in the right tail of the wealth distribution

Saving rate by age and wealth, median earnings level



Key mechanism

- Precautionary savings behavior. People save to self-insure against
  - Earnings risk
  - Longevity risk
  - Retirement
- Once "buffer stock savings" is reached, people start dissaving. Carroll (1997)
- The saving rate of the high wealth households is low or even negative
  - Contrasts with much empirical evidence (Dynan Skinner and Zeldes, 2004 and De Nardi, French and Jones, 2010)

Limitations of the standard model

- Counterfactual saving behavior
- They do not generate the high wealth people that we see in the data
- They allow for very few saving motives. Might miss important saving motives even for households whose saving behavior we think we understand
- Why people save is important

Savings and Wealth Inequality

Richer models of wealth inequality

Explaining wealth inequality

$$\max_{\{c_t\}_{t=0}^{T}} E \sum_{t=0}^{T} \beta^t \left(s_t \frac{c_t^{1-\sigma}}{1-\sigma} + (1 - s_t)\, s_{t-1} \phi\,(a_t) \right)$$

$$a_{t+1} = y_t + (1 + r)a_t - c_t + b_t, \quad a_{t+1} \geq \underline{a}$$

1. Bequests and human capital transmission across generations

Explaining wealth inequality

$$\max_{\{c_t\}_{t=0}^{T}} E \sum_{t=0}^{T} \beta_i{}^t s_t \frac{c_t^{1-\sigma_i}}{1-\sigma_i}$$

$$a_{t+1} = y_t + (1+r)a_t - c_t, \quad a_{t+1} \geq \underline{a}$$

1.
2. Heterogeneous preferences

Explaining wealth inequality

$$\max_{\{c_t\}_{t=0}^{T}} E \sum_{t=0}^{T} \beta^t s_t \frac{c_t^{1-\sigma}}{1-\sigma}$$

$$a_{t+1} = \left[l_e f\left(\theta_t, k_t\right) + \left(1 - l_e\right) y_t \right] + \left(1 + r\right)\left(a_t - k_t\right) - c_t, \quad a_{t+1} \geq \underline{a}$$

1.
2.
3. Entrepreneurship

Explaining wealth inequality

$$\max_{\{c_t\}_{t=0}^T} E \sum_{t=0}^{T} \beta^t s_t \frac{c_t^{1-\sigma}}{1-\sigma}$$

$$a_{t+1} = y_t + \left(1 + r_t^i\right) a_t - c_t, \quad a_{t+1} \geq \underline{a}$$

1.
2.
3.
4. Idiosyncratic rates of return

Savings and Wealth Inequality

Bequests

Bequests and human capital, facts

- A large fraction of wealth is inherited Kotlikoff and Summers (1981), Modigliani (1988), Gale and Scholz (1994)
- Earnings of parents and children are correlated Solon (1992), Zimmermann (1992), Stokey (1996),... Chetty et al. (2014)

Bequests and human capital model (De Nardi, 2004)

- OLG with retirement period.
- Earnings and lifetime uncertainty. Accidental bequests
- Parents value leaving bequests. Voluntary bequests

Bequests and human capital model (De Nardi, 2004)

- OLG with retirement period.
- Earnings and lifetime uncertainty. Accidental bequests
- Parents value leaving bequests. Voluntary bequests
- Children partially inherit parents' earnings ability

  pause

$$\max_{\{c_t\}_{t=0}^{T}} E \sum_{t=0}^{T} \beta^t \left(s_t \frac{c_t^{1-\sigma}}{1-\sigma} + (1-s_t)\, s_{t-1} \phi\left(a_t\right) \right)$$

$$a_{t+1} = y_t + (1+r)a_t - c_t + b_t, \quad a_{t+1} \geq \underline{a}$$

The bequest motive

- "Warm glow altruism."

$$\phi(a_t) = \frac{(a_t + \eta)^{1-\sigma}}{1-\sigma}$$

- The larger is η, the more bequests are luxury goods. Non-homoteticity

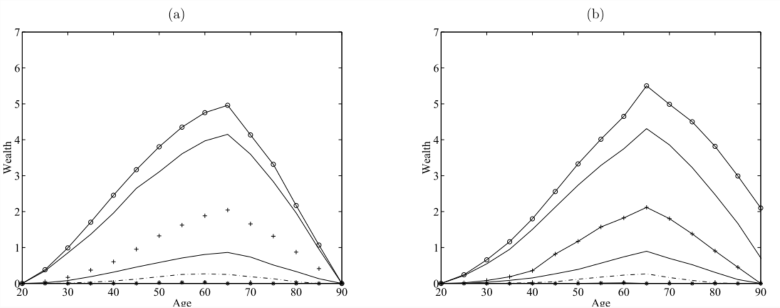
The bequest motive

- "Warm glow altruism."

$$\phi(a_t) = \frac{(a_t + \eta)^{1-\sigma}}{1-\sigma}$$

- The larger is η, the more bequests are luxury goods. Non-homoteticity
  - Many people leave no bequests. Hurd and Smith (2001)
  - The altruistic model has strong implications about risk sharing across generations that have been strongly rejected by data, Altonji, Hayashi, Kotlikoff, 1997
- Do not pick model parameters to match wealth inequality

Age profiles of wealth by quantiles



- A: No bequests - households spend all wealth in retirement
- B: Bequest motive - rich households maintain substantial wealth for children

Data and richer life cycle model

Wealth Gini	Percentage wealth in the top 1%	5%	20%	40%	60%	$\% \leq 0$ Wealth
U.S. data, SCF 1989						
.78	29	53	80	93	98	6
Accidental bequests to one's children						
.68	7	27	69	91	99	17
+ Voluntary bequests						
.74	14	37	76	95	100	19
+ Voluntary bequests + HC inheritance						
.76	18	42	79	95	100	19

Bequests and human capital: main results

- Accidental bequests do not help explain the concentration in the upper tail of the wealth distribution
- Voluntary bequests help explain wealth concentration because of non-homoteticity

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Bequests and human capital: main results

- Accidental bequests do not help explain the concentration in the upper tail of the wealth distribution
- Voluntary bequests help explain wealth concentration because of non-homoteticity
- Transmission of earnings ability across generations increases wealth concentration in the upper tail
- But, the wealthy in the model are still not wealthy enough and the poor are too poor

Savings and Wealth Inequality

Heterogeneous preferences

Heterogeneous preferences, facts

Lots of evidence of preference heterogeneity

- Estimate Euler equations. PSID. Lawrence (1991).
- Estimate life cycle model with SMM. PSID. Cagetti (2003)
- Heterogeneity of effects of earnings shocks on consumption. PSID. Alan, Browning, and Ejenaes (2016)
- Estimate life cycle model with ML. Danish registry. Druedhal and Jorgensen (2015)
- Many others...

Heterogeneous preferences

$$\max_{\{c_t\}_{t=0}^{T}} E \sum_{t=0}^{T} s_t \beta_i^t \frac{c_t^{1-\sigma_i}}{1-\sigma_i}$$

- Krusell and Smith (1998)- Infinitely-lived agent model: A little heterogeneity in β generates
    - More wealth concentration
    - But not enough very wealthy people

Heterogeneous preferences

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  - More wealth concentration
  - But not enough very wealthy people
- Hendricks (2007), Paz Pardo (2016) - Life cycle model:
  - Even large heterogeneity in both parameters does not generate very wealthy people

Heterogeneous preferences: main results

- Heterogeneous preferences might drive important difference in savings
- But, little evidence they are the key reason why the wealthiest are so wealthy
- Interesting mechanisms that might interact with other savings motives in richer Bewley models

Savings and Wealth Inequality

Entrepreneurs

Entrepreneurs, facts

Many entrepreneurs are wealthy and many wealthy people are entrepreneurs. Cagetti and De Nardi, 2006

Fraction of entrepreneurs, SCF 1989

Wealth percentile, top	1%	5%	10%	20%
Self-employed business owners	54%	39%	32%	22%

Entrepreneurs, facts

- Entrepreneurs have a high saving rate before and after entry. Quadrini (1999) and (2000) and Buera (2009)

- Entrepreneurs face borrowing constraints Evans and Jovanovic (1989), Gentry and Hubbard (2004), and Cagetti and De Nardi (2006)

- Entrepreneurs hold very undiversified portfolios. (Vissing-Jorgensen and Moskowitz, 2002)

Entrepreneurs models (Cagetti and De Nardi, 2006)

- Every period agents decide whether to be a worker or run a business

- Entrepreneurial technology

$$f(\theta_t, k_t) = \theta_t k_t^{\nu} + (1-\delta)k_t$$
$$k_t \leq k(a_t)$$

- Budget constraint

$$a_{t+1} = [l_e f(\theta_t, k_t) + (1-l_e) y_t] + (1+r)(a_t - k_t) - c_t, \quad a_{t+1} \geq \underline{a}$$

Entrepreneurs, results

- Do not pick model parameters to match wealth inequality

		Percentage wealth in the top			
Wealth Gini	Share entrepreneurs	1%	5%	20%	40%
1989, SCF data					
0.8	7.55%	30	54	81	94
Baseline with entrepreneurs and altruism					
0.8	7.50%	31	60	83	94

Entrepreneurs: main results

- Entrepreneurship can generate a realistic wealth distribution.
- Key mechanism: Some entrepreneurs
  - Have potentially very high rates of returns from investing
  - Are borrowing constrained
  - Have a large optimal firm size
  - Keep saving to grow their business even when they are wealthy
- Model rationalizes entrepreneurial undiversified portfolios, high saving rates, and high wealth

Savings and Wealth Inequality

Heterogeneous returns

Heterogeneous rates of returns, facts

Fagereng, Guiso, Malacrino, Pistaferri (2020) find that rates of returns are

- Heterogeneous across households (over 200 basis points between 10th and 90th percentile of the distribution of returns)
- Also heterogenous within asset classes
- Persistent
- Correlated with household wealth and across generations

Exogenous rates of return (Benhabib, Bisin and Luo, 2015)

$$a_{t+1} = y_t + \left(1 + r_t^i\right) a_t - c_t, \quad a_{t+1} \geq \underline{a}$$

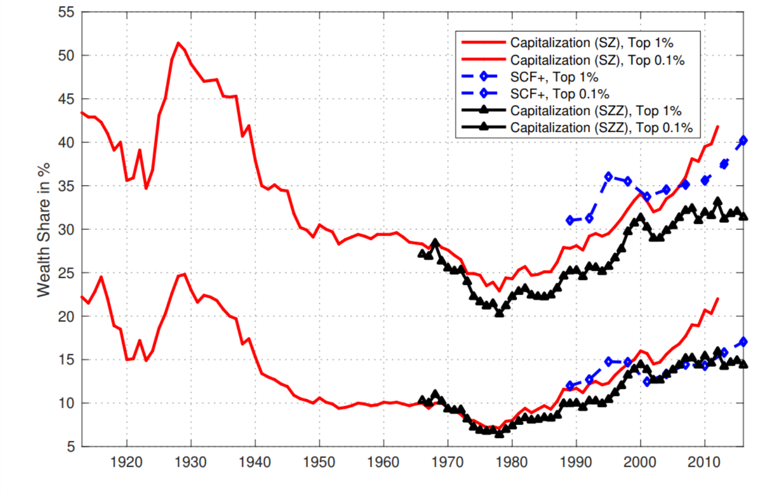
- Choose model parameters to match wealth inequality
- Exogenous and stochastic rates of return can help explain the presence of very rich people
- How do they do it?
  - Idiosyncratic rates of returns r^i are drawn from a distribution at birth, possibly correlated with those of the parent
  - But does it match the data?

Endogenous rates of returns

- Rates of return depend on investment choices
- Important to study their determinants. What might affect them?
  - Entrepreneurial choices: Quadrini (1999), Cagetti and De Nardi (2006 and 2009), Bassetto, Cagetti, and De Nardi (2015)
  - Portfolio choice: Khan and Kim (2015)
  - Heterogeneous investor sophistication: Kacperczyk, Nosal, and Stevens (2015)
- Would be very difficult to model all these different decisions
- Recent approach that has gained increasing popularity: assume that returns scale in wealth: $r(a_t)$
- Next: we'll study a model that includes such a specification

Sources of US Wealth Inequality

Evolution of top wealth inequality in the U.S.



Data sources: Saez & Zucman (2016), Smith, Zidar & Zwick (2019).

Overview

- examine a quantitative macro model with sharp implications for the distribution of wealth: can it match the data?
    - its average shape
    - its evolution over time
- in particular, study the role of a number of wealth inequality determinants: tax rates, labor income, and portfolio returns—all varying across households and over time
- we discipline the model by tying all parameters to micro data
    - does the benchmark framework do an adequate job?

Quantitative model

- Extended Aiyagari 1994 framework:
  - exogenous labor supply with idiosyncratic risk: persistent and transitory component, plus Pareto tail
  - heterogeneous returns: increasing in wealth, i.i.d. idiosyncratic component
  - progressive taxation
  - lumpsum transfer
  - stochastic discount factor
- time-varying: tax system, labor income process, and aggregate asset return premia
- finding: saving rates (key consumer choice) very robust and unresponsive to all drivers

Consumer problem

$$V_t(x_t, p_t, \beta_t) = \max_{a_{t+1} \geq \underline{a}} \{u(x_t - a_{t+1}) + \beta_t \mathbb{E}[V_{t+1}(x_{t+1}, p_{t+1}, \beta_{t+1}) | p_t, \beta_t]\}$$

$$\text{subject to } x_{t+1} = a_{t+1} + y_{t+1} - \tau_{t+1}(y_{t+1}) + (1 - \tilde{\tau}_{t+1})\tilde{y}_{t+1} + T_{t+1}$$

$$y_{t+1} = \left(\underline{r}_{t+1} + r^X_{t+1}(a_{t+1})\right) a_{t+1} + w_{t+1} l_{t+1}(p_{t+1}, \nu_{t+1})$$

$$\tilde{y}_{t+1} = \sigma^X(a_{t+1}) \eta_{t+1} a_{t+1}$$

- cash-on-hand x_t
- persistent component of labor income process p_t and discount factor β_t follow Markov processes
- transitory shocks to labor income ν_t and capital income η_t
- progressive tax on ordinary income $\tau_t(\cdot)$; flat on cap. gains $\tilde{\tau}_t$
- Lumpsum transfer T_t

Equilibrium: capital market clearing

- need to find two equil. objects $(K_t, \underline{r}_t)$ for capital market clearing:
  1. aggregate capital (as usual)
  $$K_t = \int a_t d\Gamma(a_t)$$
  2. aggregate capital income (redundant if $r_t^X(\cdot) = 0$)
  $$(MPK(K_t) - \delta)K_t = \int \left(\underline{r}_t + r_t^X(a_t)\right) a_t d\Gamma(a_t)$$
- for initial $(K_t^*, \underline{r}_t^*)$ and new steady state $(K_t^{**}, \underline{r}_t^{**})$, as well as over transition $(K_t, \underline{r}_t)_{t=t_0}^{t_1}$

Multiplicative shocks and Pareto tails

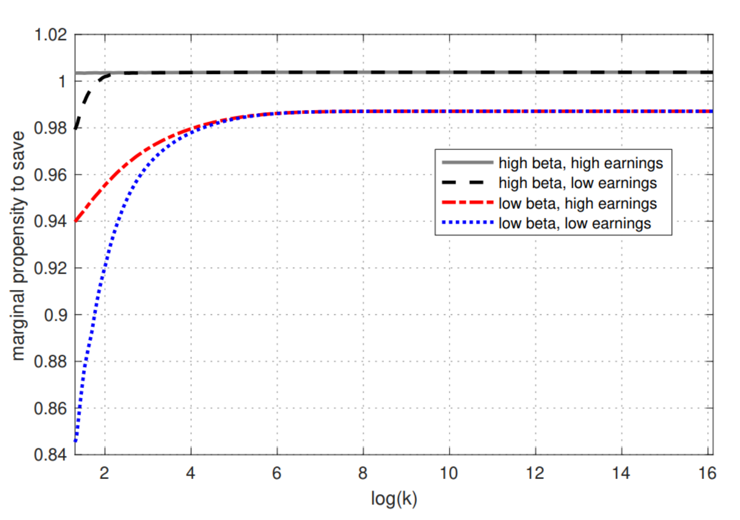
- linear savings rules as wealth grows large (Bewley 1977; Carroll 2012; Benhabib et al. 2015): $\lim_{x\to\infty} s(x,\beta) = \bar{s}_\beta x$.
- asset accumulation for large x:

$$
\begin{aligned}
a_{t+1} &= s(x_t, \beta) \\
&= s(a_t + y_t - T(y_t), \beta) \\
&\approx \bar{s}_\beta a_t(1 + (1-\tau_{\max})r) + \bar{s}_\beta(1-\tau_{\max})e_t \\
&\equiv \hat{s} a_t + z_t,
\end{aligned}
$$

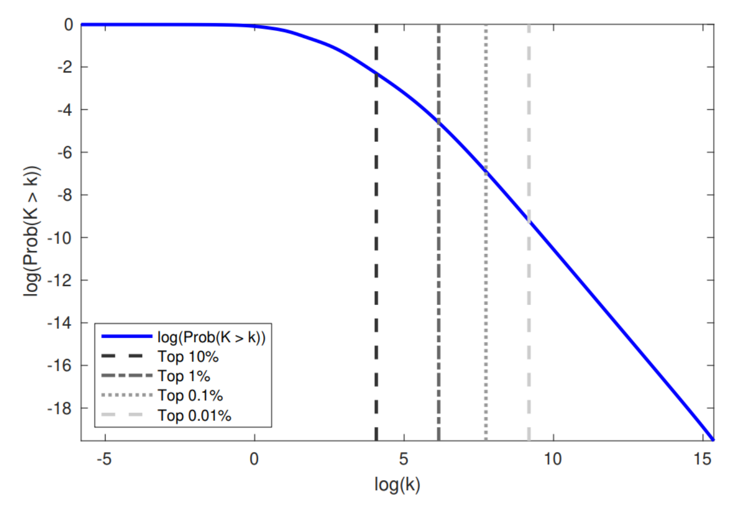
where e_t is earnings.

- β and/or r random $\to$ $\hat{s}$ is random.
- with reflecting barrier (borrowing constraint) and/or random earnings, the invariant distribution for wealth has a Pareto tail with coefficient ζ solving: $\mathbb{E}[\hat{s}^\zeta] = 1$.

Stochastic-β yields stochastic, linear savings decisions



Gives rise to a Pareto tail in the wealth distribution



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1. calibrate earnings process, tax rates, return process, social safety net to observables
2. choose randomness in discount factor β residually so as to replicate the wealth distribution in the initial steady state (1967)
3. then feed in exogenous changes in tax rates, earnings inequality, etc. between 1967 and 2015 to understand the role of these different factors

Return heterogeneity

- overall return given asset holdings a_t equals

$$\underline{r}_t + r_t^X(a_t) + \sigma^X(a_t)\eta_t$$

- $\underline{r}_t$ is endogenous
- $r_t^X(\cdot)$ and $\sigma^X(\cdot)$ are exogenous excess return schedules (mean and st.dev.), taken from the data
- η_t is an i.i.d. standard normal shock
- reduced form portfolio choice

Calibration: return process

$$r_t^X(a_t) = \sum_{c\in C} w_c(a_t)\left(\bar{r}_{c,t} + \tilde{r}_c^X(a_t)\right)$$

$$\left(\sigma^X(a_t)\right)^2 = \sum_{c\in C}\left(w_c(a_t)\tilde{\sigma}_c^X(a_t)\right)^2$$

- asset classes C: risk-free, public equity, private equity, housing
- $\bar{r}_{c,t}$: aggregate return on asset class c (U.S. data), time-varying
- fixed over time, based on Swedish administrative data from Bach, Calvet, Sodini (2016):
  - $w_c(\cdot)$: portfolio weights
  - $\tilde{r}_c^X(\cdot)$: within asset class return heterogeneity
  - $\tilde{\sigma}_c^X(\cdot)$: asset c idiosyncratic return standard deviation

Public and private equity

Public Equity

- U.S. stock market return

Private Equity

- Kartashova (AER, 2014) documents private equity premium over stock market
- aggregate time series for U.S. starting in 1960

Housing details

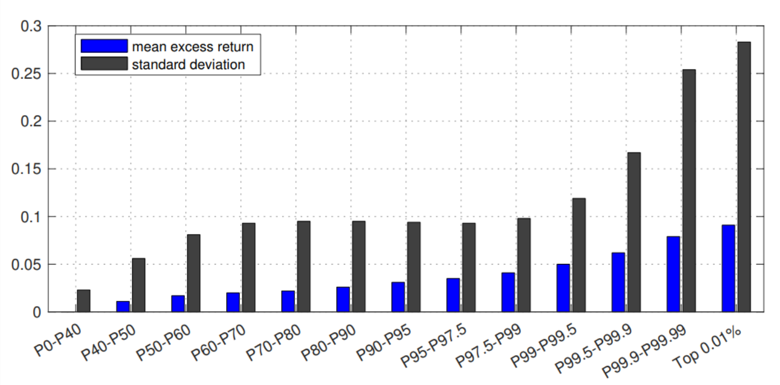
- financial return on housing as sum of capital gains term and rental income
- we set capital gains term to zero in steady states (in long run 0-0.5% real price growth)
- over transition, use growth in aggregate house price index (Case-Shiller)
- rental income set to 5.33% (average for U.S. from Jorda, Knoll, Kuvshinov, Schularick, Tayler "Rate of Return on Everything")

Excess return schedule details

- Aggregate Excess Returns in 1967 steady state:
  - public equity 0.067 (U.S., Kartashova 2014)
  - private equity 0.129 (U.S., Kartashova 2014)
  - housing 0.037 (incl. imputed rent; Jorda, Knoll, Kuvshinov, Schularick, Tayler 2017)
- and cross-sectional data from Bach, Calvet, Sodini (2019) implies

	P0-P40	P40-P50	P50-P60	P60-P70	P70-P80	P80-P90	P90-P95	P95-P97.5	P97.5-P99	P99-P99.5	P99.5-P99.9	P99.9-P99.99	Top 0.01%
fixed portfolio weights													
risk-free	0.722	0.412	0.248	0.182	0.156	0.134	0.115	0.102	0.090	0.079	0.071	0.051	0.029
housing	0.162	0.394	0.580	0.662	0.678	0.674	0.658	0.626	0.572	0.482	0.363	0.253	0.155
public equity	0.113	0.189	0.165	0.147	0.153	0.170	0.189	0.207	0.219	0.232	0.230	0.185	0.179
private equity	0.002	0.005	0.007	0.009	0.013	0.021	0.038	0.065	0.118	0.207	0.336	0.511	0.637
difference from aggregate return on asset class													
risk-free	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
housing	0.000	0.000	0.002	0.004	0.005	0.007	0.009	0.010	0.010	0.011	0.010	0.010	0.011
public equity	0.000	0.000	0.001	0.002	0.003	0.005	0.008	0.012	0.014	0.015	0.016	0.016	0.016
private equity	0.000	0.000	-0.019	-0.030	-0.054	-0.055	-0.049	-0.066	-0.064	-0.063	-0.063	-0.059	-0.060
standard deviation of return on asset class													
risk-free	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
housing	0.140	0.140	0.140	0.140	0.140	0.140	0.140	0.140	0.140	0.140	0.140	0.140	0.140
public equity	0.035	0.035	0.031	0.031	0.031	0.031	0.032	0.033	0.035	0.038	0.042	0.046	0.053
private equity	0.664	0.664	0.621	0.595	0.544	0.525	0.518	0.480	0.474	0.470	0.474	0.492	0.443
private equity (re-scaled)	0.345	0.345	0.323	0.309	0.283	0.273	0.269	0.249	0.246	0.245	0.246	0.256	0.230
excess return schedule in 1967													
mean excess return	0.000	0.011	0.017	0.020	0.022	0.026	0.031	0.035	0.041	0.050	0.062	0.079	0.091
standard deviation	0.023	0.056	0.081	0.093	0.095	0.095	0.094	0.093	0.098	0.119	0.167	0.254	0.283
st. dev. (priv.equ. re-scaled)	0.023	0.056	0.081	0.093	0.095	0.095	0.093	0.089	0.086	0.085	0.098	0.136	0.149

Schedule of excess returns



Data sources: Bach, Calvet, Sodini (2019); Kartashova (2014); Jorda, Knoll, Kuvshinov, Schularick, Taylor (2019); Case-Shiller.

Results, I: steady state (1967)

	Top 10%	Top 1%	Top 0.1%	Top 0.01%
Data	70.8%	27.8%	9.4%	3.1%
Single-β Model	66.6%	23.7%	11.2%	7.2%
Benchmark Model	73.8%	27.4%	8.4%	3.2%
	Bottom 50%	Fraction $a < 0$		
Data	4.0%	8.0%		
Single-β Model	3.5%	7.3%		
Benchmark Model	3.0%	6.6%		

- model matches wealth distribution well on its entire domain
  - return heterogeneity is key ingredient
  - wealth concentration is mitigated by progressive taxation and labor income risk

Next step: transition

Now the authors feed in four different factors that have changed during the past 50 years

- Decrease in tax progressivity
- Increase in labor income risk
- Increase in income going to the top
- Changing return premia to different asset classes

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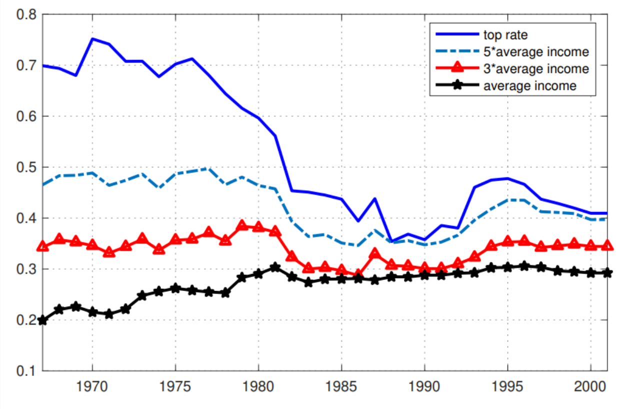
Note: in two weeks, we will learn more about the solution method for solving for the transition from one steady state to another

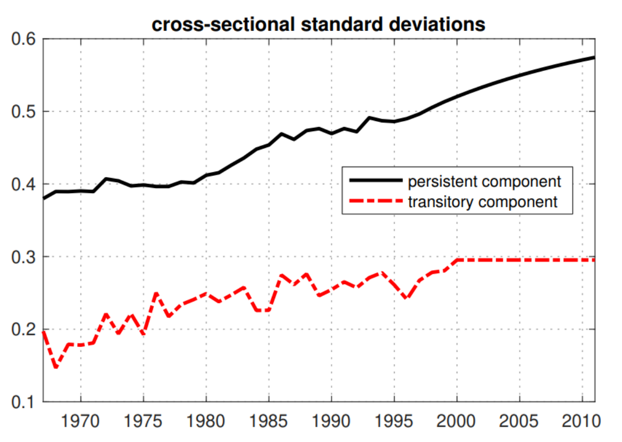
Observed change 1: decrease in tax progressivity

- federal effective tax rates (Piketty & Saez 2007): income, payroll, corporate and estate taxes



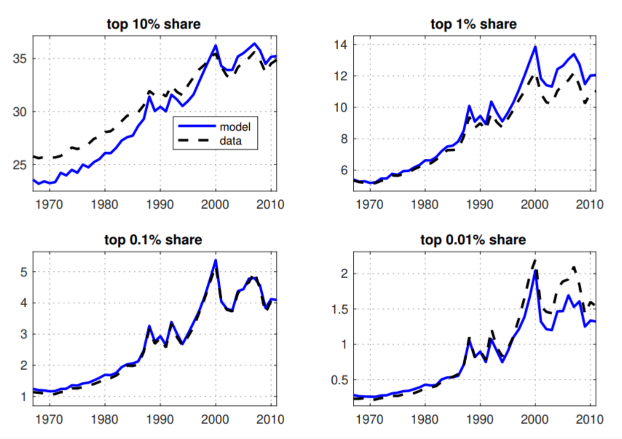
Observed change 2: increase in labor income risk

- estimates for variance of persistent and temporary components 1967-2000 (Heathcote, Storesletten & Violante 2010)



Observed change 3: increase in top labor income shares

- adjust standard AR(1) in idiosyncratic productivity by imposing a Pareto tail for the top 10% earners: calibrated tail coefficient decreases from 2.8 to 1.9 (updated Piketty & Saez 2003 series)



Observed change 4: return premia

- feed in (smoothed) time series of aggregate U.S. asset premia (Kartashova 2014, Case-Shiller index)

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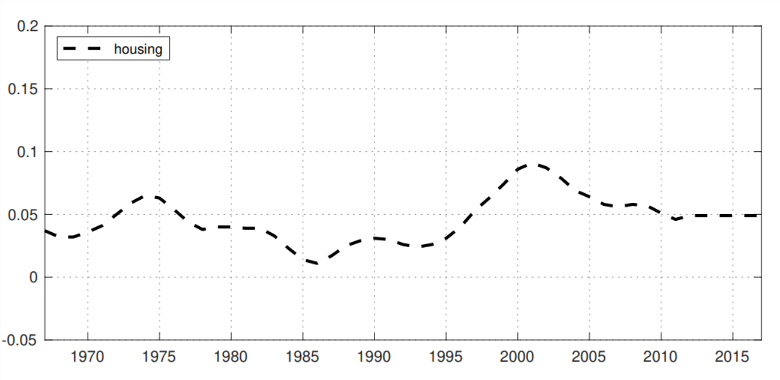

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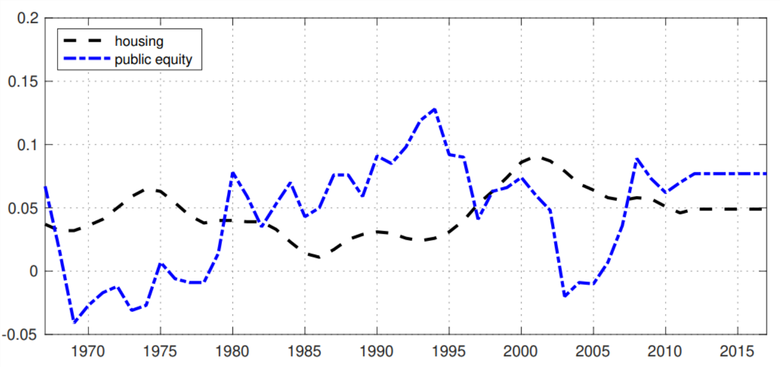

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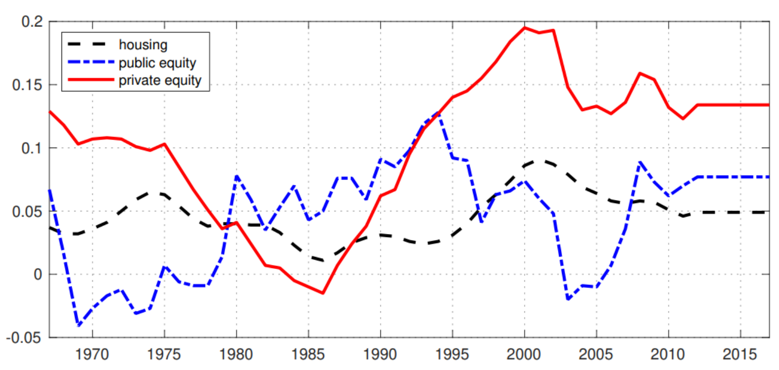
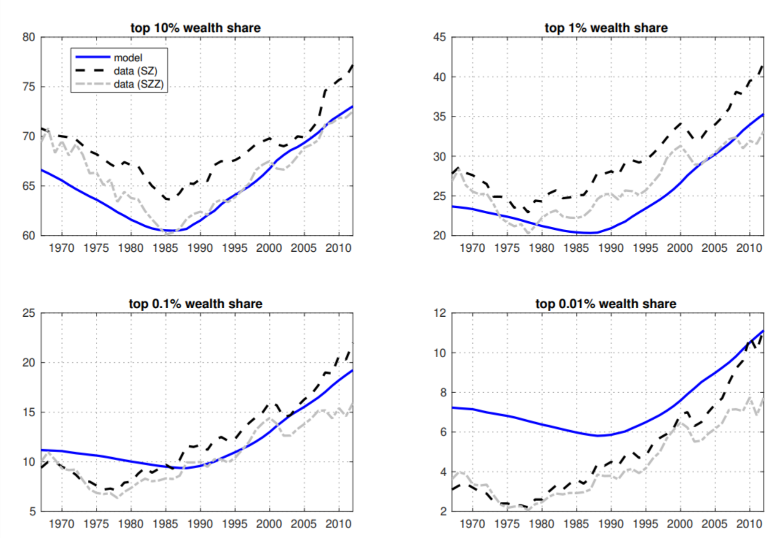
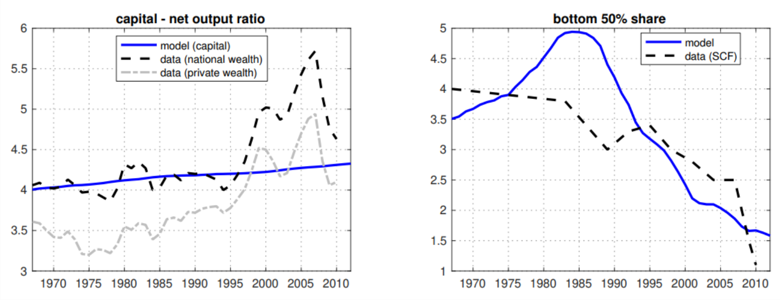


Results, II: historical evolution

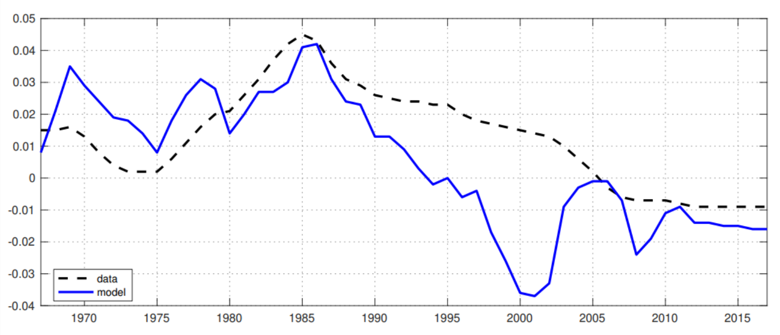


Results: Capital-output ratio and bottom 50 %

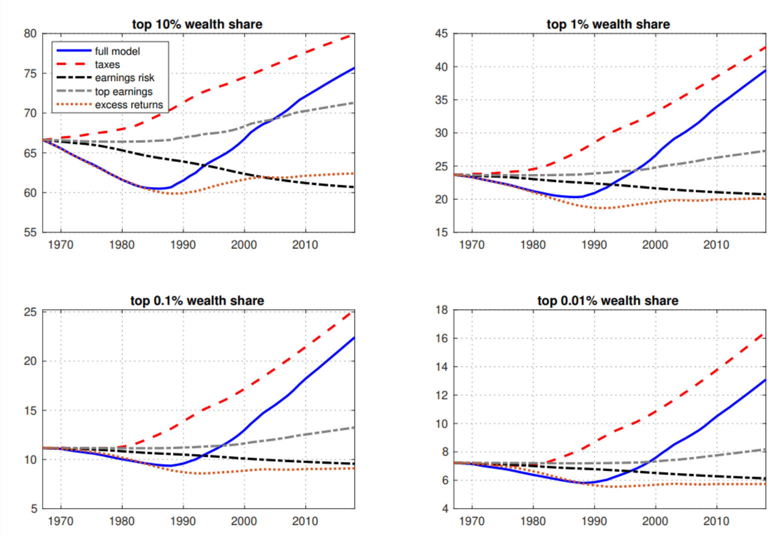


Results: Risk-free rate

- return premia are matched in model by construction
- risk-free rate is endogenous: comparable level and decline



Decomposition of transitional dynamics



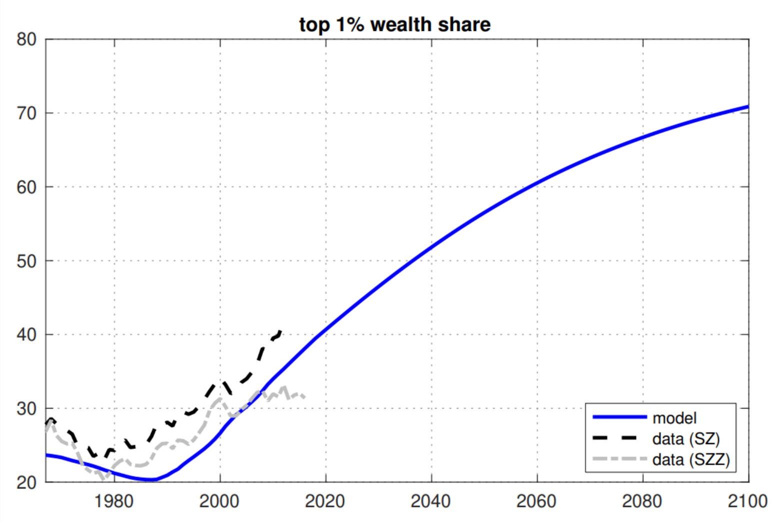
Decomposition of transitional dynamics

- overall increase in wealth inequality (more than) fully explained by declining tax progressivity
  - primarily due to direct effect on resource distribution and not due to changing savings behavior

- time-varying return premia account for U-shape in wealth inequality

- subtle role of increasing earnings dispersion
  - thickening Pareto tail in labor income contributes slightly positively to wealth inequality
  - increase in overall earnings risk decreases wealth inequality

Capital in the 21st century?



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  - asset-price movements account well for medium-run dynamics
- cautious prediction: unless stronger tax progressivity restored, wealth concentration will continue to rise
- remaining questions from perspective of this paper:
  - why are portfolios heterogeneous (both across and within wealth levels), what drives returns?

Summary

Summary and next week

- **Today:** Discuss the predictions for wealth inequality arising from Bewley-Huggett-Aiyagari models and their various extensions
    1. Difficult to match the fact that wealth more unequal than earnings
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- **Homework:**
  1. Continue to work on previous exercises
  2. Begin work on the first assignment