13a. Global solution methods with aggregate risk

Adv. Macro: Heterogenous Agent Models

Jeppe Druedahl & Patrick Moran

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Introduction

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    1. Non-linear solution
    2. Linear solution $\rightarrow$ simulation with aggregate risk

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     Higher order terms can (in principle) be included
     Reference: Bayer and Luetticke (2020)

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       Problem: *The distribution of households is a state variable*
       References: Krusell and Smith (1998); Algan et al. (2014); Proehl
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- **Disclaimer:** *This is advanced stuff!*

Household problem with aggregate risk

$$v(Z_t, \boldsymbol{D}_t, z_{it}, a_{it-1}) = \max_{c_{it}} \frac{c_{it}^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}_t \left[v(Z_{t+1}, \boldsymbol{D}_{t+1}, z_{it+1}, a_{it}) \right]$$

$$\text{s.t. } a_{it} + c_{it} = (1 + r_t)a_{it-1} + w_t z_{it} \geq 0$$

$$\log z_{it+1} = \rho_z \log z_{it} + \psi_{it+1} \ , \psi_{it+1} \sim \mathcal{N}(\mu_\psi, \sigma_\psi), \ \mathbb{E}[z_{it}] = 1$$

$$K_{t-1} = \int a_{it-1} d\boldsymbol{D}_t$$

$$L_t = \int z_{it} d\boldsymbol{D}_t = 1$$

$$r_t = \alpha Z_t (K_{t-1}/L_t)^{\alpha-1} - \delta$$

$$w_t = (1 - \alpha) Z_t (K_{t-1}/L_t)^{\alpha}$$

$$\boldsymbol{D}_{t+1} = \Gamma(Z_t, \boldsymbol{D}_t)$$

$$Z_{t+1} \sim \Gamma_Z(Z_t)$$

Method

Assuming (strong) approximate aggregation

$$v(Z_t, K_{t-1}, z_{it}, a_{it-1}) = \max_{c_{it}} \frac{c_{it}^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}_t \left[v(Z_{t+1}, K_t, z_{it+1}, a_{it}) \right]$$

$$\text{s.t. } a_{it} + c_{it} = (1 + r_t)a_{it-1} + w_t z_{it} \geq 0$$

$$\log z_{it+1} = \rho_z \log z_{it} + \psi_{it+1} \text{ , } \psi_{it+1} \sim \mathcal{N}(\mu_\psi, \sigma_\psi), \mathbb{E}[z_{it}] = 1$$

$$r_t = \alpha \Gamma_t (K_{t-1})^{\alpha-1} - \delta$$

$$w_t = (1-\alpha)\Gamma_t(K_{t-1})^\alpha$$

$$K_t = \mathsf{PLM}(Z_t, K_{t-1})$$

$$Z_{t+1} \sim \Gamma_Z(Z_t)$$

PLM $\equiv$ »perceived law of motion«

Weak approximation in general

$$v(\boldsymbol{Z}_t, \boldsymbol{S}_{t-1}, \boldsymbol{z}_t, a_{t-1}) = \max_{a_t, c_t} u(c_t) + \beta \mathbb{E}_t \left[v(\boldsymbol{Z}_{t+1}, \boldsymbol{S}_t, \boldsymbol{z}_{t+1}, a_t) \right]$$

$$\text{s.t. } \boldsymbol{S}_t, \boldsymbol{P}_t = \text{PLM}(\boldsymbol{Z}_t, \boldsymbol{S}_{t-1})$$

$$a_t + c_t = m(\boldsymbol{z}_t, a_{t-1}, \boldsymbol{P}_t)$$

$$\boldsymbol{z}_{t+1} \sim \Gamma_z(\boldsymbol{z}_t)$$

$$\boldsymbol{Z}_{t+1} \sim \Gamma_Z(\boldsymbol{Z}_t)$$

$$a_t \geq -b$$

1. $\boldsymbol{Z}_t$ are exogenous aggregate shocks.
2. $\boldsymbol{S}_{t-1}$ are pre-determined (finite dimensional) aggregate states.
3. $\boldsymbol{P}_t$ are »prices«.
4. PLM($\bullet$) is the *Perceived-Law-of-Motion*.
5. $\boldsymbol{z}_t$ is stochastic and exogenous idiosyncratic states.
6. c_t is consumption providing utility $u(c_t)$ discounted by β.
7. a_t is end-of-period assets (borrowing constraint given by b).
8. $m(\bullet)$ is cash-on-hand with $\frac{\partial m(\bullet)}{\partial a_{t-1}} > 0$.

EGM can still be used

1. **EGM**

$$q(\boldsymbol{Z}_t, \boldsymbol{S}_{t-1}, \boldsymbol{z}_t, a_t) = \mathbb{E}\left[v_a(\boldsymbol{Z}_{t+1}, \boldsymbol{S}_t, \boldsymbol{z}_{t+1}, a_t)\right]$$

$$\tilde{c}(\boldsymbol{Z}_t, \boldsymbol{S}_{t-1}, \boldsymbol{z}_t, a_t) = (\beta q(\bullet))^{-\frac{1}{\sigma}}$$

$$\tilde{m}(\boldsymbol{Z}_t, \boldsymbol{S}_{t-1}, \boldsymbol{z}_t, a_t) = a_t + c(\bullet)$$

$$c^\star(\boldsymbol{Z}_t, \boldsymbol{S}_{t-1}, \boldsymbol{z}_t, a_{t-1}) = \text{interp } \tilde{m}(\bullet) \to \tilde{c}(\bullet) \text{ at } m(\boldsymbol{z}_t, a_{t-1}, \boldsymbol{P}_t)$$

$$a^\star(\boldsymbol{Z}_t, \boldsymbol{S}_{t-1}, \boldsymbol{z}_t, a_{t-1}) = m(\bullet) - c^\star(\bullet)$$

$$v_a(\boldsymbol{Z}_t, \boldsymbol{S}_{t-1}, \boldsymbol{z}_t, a_{t-1}) = \frac{\partial m(\bullet)}{\partial a_{t-1}} c^\star(\bullet)^{-\sigma}$$

2. **Implied savings:**

$$a^\star(\boldsymbol{Z}_t, \boldsymbol{S}_{t-1}, \boldsymbol{z}_t, m_t) = a^\star(\boldsymbol{Z}_t, \boldsymbol{S}_{t-1}, \boldsymbol{z}_t, a_{t-1})$$

$$a_{t-1} = m^{-1,a}(m_t, \boldsymbol{z}_t, \boldsymbol{P}_t)$$

Simulation

1. Draw $\mathbf{Z}_t$ given $\mathbf{Z}_{t-1}$

2. Find

$$a_t^\star(\mathbf{z}_t, m_t) = a^\star(\mathbf{Z}_t, \mathbf{S}_{t-1}, \mathbf{z}_t, m_t)$$

   by interpolation over $\mathbf{Z}_t$ and $\mathbf{S}_{t-1}$

3. Search for $\mathbf{P}_t$ so

$$\int a_t^\star(\mathbf{z}_t, m(\mathbf{z}_t, a_{t-1}, \mathbf{P}_t)) d\mathbf{D}_t$$

   clears the savings market

Fixed-point iteration for PLM

1. Draw shocks to be used in all simulations

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5. Set the PLM convergence iteration counter $n = 0$

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6. Solve the approximate household problem given $\breve{S}^n$ and $\breve{P}^n$

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7. Simulate the model given household behavior
8. Estimate the PLM on the simulated data
9. Given the PLM compute $\breve{\boldsymbol{S}}_{NEW}$ and $\breve{\boldsymbol{P}}_{NEW}$ on the grid of $\boldsymbol{Z}_t$ and $\boldsymbol{S}_{t-1}$

Fixed-point iteration for PLM

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9. Given the PLM compute $\breve{\boldsymbol{S}}_{NEW}$ and $\breve{\boldsymbol{P}}_{NEW}$ on the grid of $\boldsymbol{Z}_t$ and $\boldsymbol{S}_{t-1}$

10. Stop if $|\breve{\boldsymbol{S}}_{NEW} - \breve{\boldsymbol{S}}^n|_\infty <$ tol. and $|\breve{\boldsymbol{P}}_{NEW} - \breve{\boldsymbol{P}}^n|_\infty <$ tol.

Fixed-point iteration for PLM

1. Draw shocks to be used in all simulations

2. Solve and simulate a linearized version of the model

3. Estimate the PLM on the simulated data

4. Given the PLM compute $\breve{\boldsymbol{S}}^0$ and $\breve{\boldsymbol{P}}^0$ on the grid of $\boldsymbol{Z}_t$ and $\boldsymbol{S}_{t-1}$

5. Set the PLM convergence iteration counter $n = 0$

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10. Stop if $|\breve{\boldsymbol{S}}_{NEW} - \breve{\boldsymbol{S}}^n|_\infty <$ tol. and $|\breve{\boldsymbol{P}}_{NEW} - \breve{\boldsymbol{P}}^n|_\infty <$ tol.

11. Update $\breve{\boldsymbol{S}}$ and $\breve{\boldsymbol{P}}$ by relaxation with $\omega \in (0, 1)$

$$\breve{\boldsymbol{S}}^{s+1} = \omega \breve{\boldsymbol{S}}_{NEW} + (1 - \omega)\breve{\boldsymbol{S}}^s$$
$$\breve{\boldsymbol{P}}^{s+1} = \omega \breve{\boldsymbol{P}}_{NEW} + (1 - \omega)\breve{\boldsymbol{P}}^s.$$

Fixed-point iteration for PLM

1. Draw shocks to be used in all simulations

2. Solve and simulate a linearized version of the model

3. Estimate the PLM on the simulated data

4. Given the PLM compute $\breve{\boldsymbol{S}}^0$ and $\breve{\boldsymbol{P}}^0$ on the grid of $\boldsymbol{Z}_t$ and $\boldsymbol{S}_{t-1}$

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$$\breve{\boldsymbol{P}}^{s+1} = \omega \breve{\boldsymbol{P}}_{NEW} + (1-\omega)\breve{\boldsymbol{P}}^s.$$

12. Increment n and return to step 6

PLMs

- **Input:** $X_{it} \in \mathbf{Z}_t, \mathbf{S_{t-1}}$, i'th input to the PLM for $i \in \{1, \ldots, \#_{ZS}\}$

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- **Linear:** Estimated by OLS

$$Y_{jt} = \Psi_{j0} + \sum_{i=1}^{\#_{zs}} \Psi_{ji} X_{it}$$

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$$Y_{jt} = \Psi_{j0} + \sum_{i=1}^{\#_{ZS}} \Psi_{ji} X_{it}$$

- **Non-linear:** Estimated with *Radial Basis Functions* (RBF)

$$Y_{jt} = \Psi_{j00} + \sum_{i=1}^{\#_{ZS}} \Psi_{j0i} X_{it} + \sum_{\tau=1}^{\mathcal{T}} \Psi_{jk} \phi \left(\sum_{i=1}^{\#_{ZS}} \sqrt{(X_{it} - X_{i\tau}^{\text{sim}})^2} \right)$$

$X_{i\tau}^{\text{sim}}$ is simulation outcome

$\phi(x) = x^2 \log x$

PLMs

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- **Paper:** Also neural net

Model

Firms

- **Technology shocks:**

$$Z_{t+1} - Z_{ss} = \rho_Z(Z_t - Z_{ss}) + \epsilon_{t+1}^Z, \ \ \epsilon_{t+1}^Z \sim \mathcal{N}(0, \sigma_Z^2)$$

- **Production firm problem:**

$$\max_{L_t, K_{t-1}, u_t} u_t Z_t K_{t-1}^{\alpha} L_t^{1-\alpha} - w_t L_t - r_t^k K_{t-1} - \chi_1 \left(u_t - \tilde{u}\right) - \frac{\chi_2}{2} \left(u_t - \tilde{u}\right)^2$$
$$\text{s.t. } u_t \leq \bar{u}.$$

implies

$$r_t^k = \alpha u_t Z_t (K_{t-1}/L_t)^{\alpha-1} \equiv r^k(u_t, Z_t, K_{t-1}, L_t)$$
$$w_t = (1-\alpha) u_t Z_t (K_{t-1}/L_t)^{\alpha} \equiv w(u_t, Z_t, K_{t-1}, L_t)$$
$$u_t = \max \left[\frac{Z_t K_{t-1}^{\alpha} L_t^{1-\alpha} - \chi_1 + \chi_2 \tilde{u}}{\chi_2}, \bar{u} \right] \equiv u(Z_t, K_{t-1}, L_t)$$

Capital producers

- **Capital producer problem**

$$\max_{\{I_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t I_t \left\{ q_t \left[1 - \frac{\phi}{2} \left(\log \frac{I_t}{I_{t-1}} \right)^2 \right] - 1 \right\}$$

implies

$$q_t \left[1 - \phi \log \frac{I_t}{I_{t-1}} \right] = 1 - \beta \mathbb{E}_t \left[q_{t+1} \phi \log \left(\frac{I_{t+1}}{I_t} \right) \right]$$

- **Accumulation:** $K_t = I_t + (1 - \delta) K_{t-1}$
- **Real interest rate:**

$$r_t = r_t^k - q_t \delta = r^k(u_t, Z_t, K_{t-1}, L_t) - q_t \delta \equiv r(u_t, Z_t, K_{t-1}, L_t, q_t).$$

Households

$$v(Z_t, K_{t-1}, l_{t-1}, z_t, a_{t-1}) = \max_{a_t, c_t} \frac{c_t^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}_t \left[v(Z_{t+1}, K_t, l_t, z_{t+1}, a_t) \right]$$

$$\text{s.t.}$$

$$r_t, w_t, K_t, l_t = \text{PLM}(Z_t, K_{t-1}, l_{t-1})$$

$$a_t + c_t = (1 + r_t)a_{t-1} + w_t z_t$$

$$\log z_{t+1} = \rho_z \log z_t + \psi_{t+1} \ , \psi_{t+1} \sim \mathcal{N}(\mu_\psi, \sigma_\psi), \ \mathbb{E}[z_{t+1}] = 1$$

$$Z_{t+1} = Z_{ss} + \rho_Z(Z_t - Z_{ss}) + \epsilon_{t+1}^Z, \ \epsilon_{t+1}^Z \sim \mathcal{N}(0, \sigma_Z^2)$$

$$a_t \geq 0$$

Solution method (1/2)

1. The shocks are $\boldsymbol{Z}_t = \{Z_t\}$.

2. The aggregate states are $\boldsymbol{S}_t = \{K_t, I_t\}$.

3. The »prices« are $\boldsymbol{P}_t = \{r_t, w_t\}$,

4. The PLM is

$$K_t = \text{PLM}_K(Z_t, I_{t-1}, K_{t-1}; \Psi)$$
$$q_t = \text{PLM}_q(Z_t, I_{t-1}, K_{t-1}; \Psi)$$
$$u_t, w_t, r_t^k, r_t = u(\bullet), w(\bullet), r_t^k(\bullet), r(\bullet)$$

5. The cash-on-hand function is

$$m(z_t, a_{t-1}, \boldsymbol{P}_t) = (1 + r_t)a_{t-1} + w_t z_t.$$

Solution method (2/2)

6. The market clearing condition is

$$\int a_t^\star(\mathbf{z}_t, m(\mathbf{z}_t, a_{t-1}, w_t, r_t)) dD_t = K_t,$$

where we guess on I_t and get r_t from

$$q_t = \frac{1 - \beta \mathbb{E}_t \left[q_{t+1} \phi \log \left(\frac{I_{t+1}}{I_t} \right) \right]}{1 - \phi \log \left(\frac{I_t}{I_{t-1}} \right)}$$

$$K_{t+1} = \mathrm{PLM}_K(Z_{t+1}, I_t, K_t; \Psi)$$

$$I_{t+1} = K_{t+1} - (1 - \delta) K_t$$

$$q_{t+1} = \mathrm{PLM}_q(Z_{t+1}, I_t, K_t; \Psi)$$

$$u_t, w_t, r_t^k, r_t = u(\bullet), w(\bullet), r_t^k(\bullet), r(\bullet)$$

where expectations are evaluated using Gauss-Hermite quadrature

Results

Figure 1: Perfect foresight and linearized impulse responses
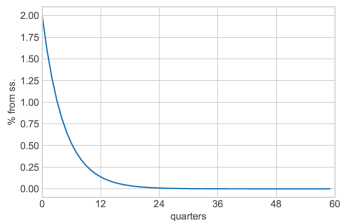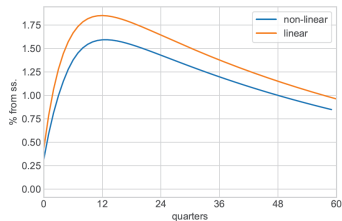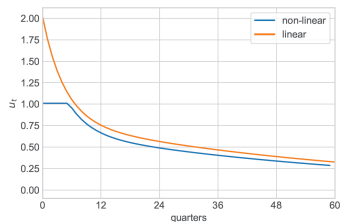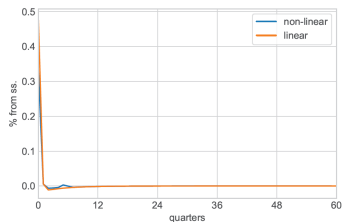
Figure 2: Simulation (in-sample): RBF vs. linear
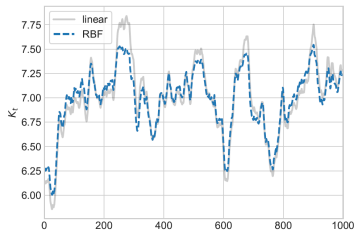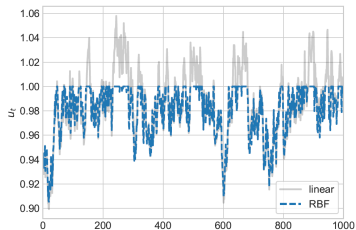
OLS vs. RBF: Some differences

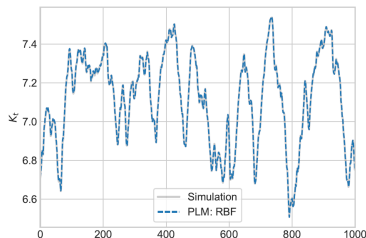Figure 3: Simulation (out-of-sample): PLM methods
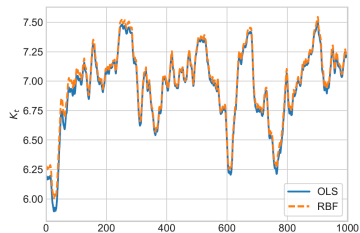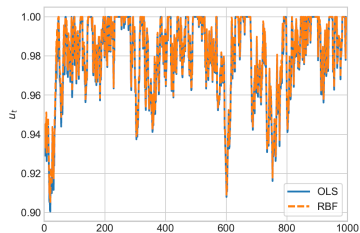
Figure 4: One-step ahead PLM errors
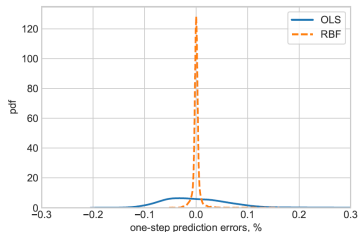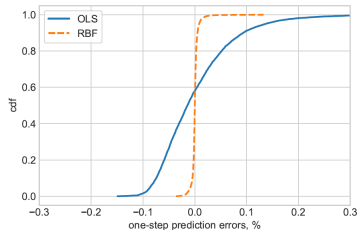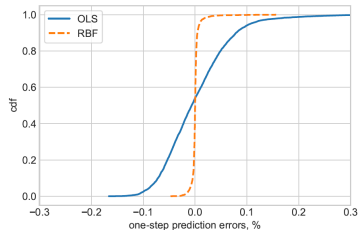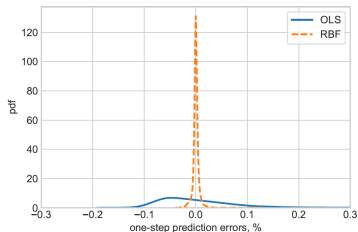
Figure 5: Dynamic PLM errors

(a) OLS
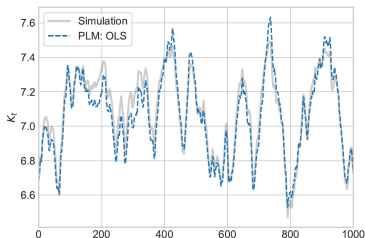
(b) RBF

Pure PLM simulation:

$$K_t^{\text{PLM}} = \text{PLM}_K(Z_t, K_{t-1}^{PLM}, I_{t-1}^{PLM})$$
$$q_t^{\text{PLM}} = \text{PLM}_K(Z_t, K_{t-1}^{PLM}, I_{t-1}^{PLM})$$
$$I_t^{\text{PLM}} = K_t^{\text{PLM}} - (1 - \delta)K_{t-1}^{PLM}.$$

Table 1: HANC: Prediction Errors

	OLS	RBF	NN
dynamic log prediction errors × 100			
max	3.75	0.26	
mean	0.88	0.04	
median	0.79	0.03	
99th perc.	3.16	0.18	
90th perc.	1.62	0.07	
timings (secs.)			
total	666.3	722.2	
- solve household problem	356.2	315.4	
- simulate with market clearing	310.0	356.3	
- estimate PLMs	0.0	50.5	
iterations	13	14	

Figure 8: Impulse-responses: Global vs. perfect foresight
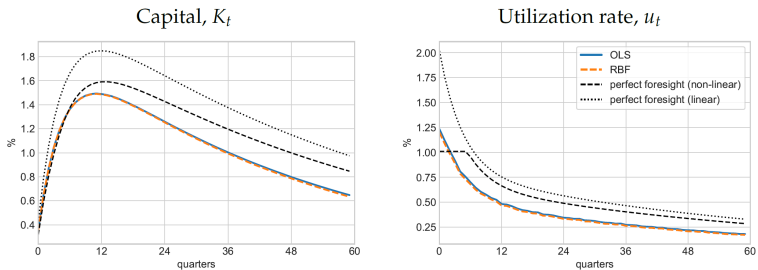
Figure 9: Impulse-responses: State-dependence

Conclusion

Conclusion (1/2)

- **Expanding literature:**
  1. **Hardware:** Graphic cards
  2. **Software:** Automatic differentiation
  3. **Discretization:** Polynomial chaos
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- **Agent-based models:** $\div$ intention, $\div$ forward-looking

My sapere aude project: **»Modeling economic agents as deep reinforcement learners«** (optimizing → »satisfying«)