Written Exam Economics winter 2022-23

Advanced Macroeconomics: Heterogeneous Agent Models

January 7 to January 9

This exam question consists of 4 pages in total

Answers only in English.

You should hand-in a single zip-file. The zip-file should have the following folder and file structure:

Assignment_I
Assignment_I.pdf – with text and all results
files for producing the results

Assignment_II
Assignment_II.pdf – with text and all results
files for producing the results

Assignment_III
Assignment_III.pdf

Exam
Exam.pdf
files for producing the results

Be careful not to cheat at exams!

Exam cheating is for example if you:

- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text
- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts
- Reuse parts of a written paper that you have previously submitted and for which you have received a pass grade without making use of quotation marks or source references (self-plagiarism)
- Receive help from others in contrary to the rules in the Faculty of Social Science's common part of the curriculum

You can read more about the rules on exam cheating on your Study Site and in the Faculty of Social Science's common part of the curriculum.

Exam cheating is always sanctioned by a written warning and expulsion from the exam in question. In most cases, the student will also be expelled from the University for one semester.

Income and Wealth Inequality

Code to start from is provided as supplementary material. This code solves for the stationary equilibrium of the model in the special case of $\sigma_\beta = \sigma_\chi = \kappa = 0$. Your first task thus is to extend the code to solve the full model.

The first set of questions is related to the **stationary equilibrium**.

a) **Discuss how $\sigma_\psi \in \{0.10, 0.12, 0.14\}$ affects inequality**

   Report at least the stock of capital, the standard deviation of income and the standard deviation and skewness of savings.

b) **Discuss how $\sigma_\beta \in \{0.01, 0.02, 0.03\}$ affects inequality**

   Report the same outcomes as in question a).

c) **Discuss how $\sigma_\chi \in \{0.10, 0.20, 0.30\}$ affects inequality**

   Report the same outcomes as in question a).

Data shows that wealth inequality is much larger than income inequality, and that the distribution of wealth is highly right skewed.

d) **Interpret your results in light of this information.**

The second set of questions is related to the transition path when $\alpha_t = \alpha_{ss} + 0.01 \cdot 0.9^t$.

e) **Find the baseline transition path**

   Discuss the implications for the stock of capital, capital income and inequality.

Consider a government which dislikes increases in capital income. They therefore impose a tax on capital income so that post-tax capital income is constant at the steady state level along the transition path. The tax proceeds are rebated to the households as a lump-sum transfer.

f) **Implement the proposed tax-and-transfer scheme and discuss whether the policy is a good idea.**

1. Model

Households. The model has a continuum of infinitely lived households indexed by $i \in [0,1]$. Households are *ex ante* heterogeneous in terms of their discount factors. Households choose consumption and exogenously supply labor. Savings is in terms of capital, which is rented out to firms at the idiosyncratic rental rate, r_{it}^K. There are no possibilities to borrow. Households are *ex post* heterogeneous in terms of their stochastic labor productivity, s_{it}, their stochastic capital productivity indicator, $\chi_{it} \in \{0,1\}$, and their (end-of-period) savings, a_{it-1}. The distribution of households over idiosyncratic states is denoted $\underline{\boldsymbol{D}}_t$ before shocks are realized and $\boldsymbol{D}_t$ afterwards. The real wage is w_t, and real-profits are Π_t.

The household problem is

$$v_t(s_{it}, \chi_{it}, a_{it-1}) = \max_{c_t} \frac{c_{it}^{1-\sigma}}{1-\sigma} - \kappa \frac{(a_{it} + \underline{a})^{1-\sigma}}{1-\sigma} + \beta_i \mathbb{E}_t \left[v_{t+1}(s_{it+1}, \chi_{it+1}, a_{it}) \right] \tag{1}$$

$$\begin{aligned}
\text{s.t. } a_{it} + c_{it} &= (1 + r_{it}^K - \delta) a_{it-1} + w_t s_{it} + \Pi_t \\
\log s_{it+1} &= \rho_s \log s_{it} + \psi_{it+1}, \;\; \psi_{it+1} \sim \mathcal{N}(\mu_\psi, \sigma_\psi),\; \mathbb{E}[s_{it}] = 1 \\
r_{it}^K &= r_t^K \cdot \begin{cases} 1 + \sigma_\chi & \text{if } \chi_{it} = 1 \\ 1 - \sigma_\chi \frac{\overline{\pi}_\chi}{\underline{\pi}_\chi} & \text{if } \chi_{it} = 0 \end{cases}, \; \mathbb{E}\left[r_{it}^K \right] = r_t^K \\
\Pr[\chi_{it+1} = 1 \,|\, \chi_{it} = 0] &= \overline{\pi}_\chi \\
\Pr[\chi_{it+1} = 0 \,|\, \chi_{it} = 1] &= \underline{\pi}_\chi \\
a_{it} &\geq 0.
\end{aligned}$$

The discount factors are drawn with equal probabilities from a three element set,

$$\beta_i \in \{\breve{\beta} - \sigma_\beta, \breve{\beta}, \breve{\beta} + \sigma_\beta\}.$$

The Euler-equation is

$$c_{it}^{-\rho} = \kappa (a_{it} + \underline{a})^{-\sigma} + \beta_i \mathbb{E} \left[v_{a,it+1}(s_{it+1}, \chi_{it+1}, a_{it}) \right] \tag{2}$$

$$v_{a,it} = (1 + r_{it}^K - \delta) c_{it}^{-\sigma}. \tag{3}$$

The aggregate quantities of central interest are

$$L_t^{hh} = \int s_{it} d\boldsymbol{D}_t = 1 \tag{4}$$

$$C_t^{hh} = \int c_{it} d\boldsymbol{D}_t \tag{5}$$

$$A_t^{hh} = \int a_{it} d\boldsymbol{D}_t \tag{6}$$

Firms. A representative firm rents capital, K_{t-1}, and hires labor, L_t, to produce goods, with the production function

$$Y_t = \Gamma_t K_{t-1}^{\alpha_t} L_t^{1-\alpha_t} \tag{7}$$

where Γ_t is technology and α_t is the Cobb-Douglas weight parameter. Capital depreciates with the rate $\delta \in (0,1)$. The real rental price of capital is r_t^K and the real wage is w_t. Profits are $\Pi_t = Y_t - w_t L_t - r_t^K K_{t-1}$. The households own the representative firm in equal shares.

The law-of-motion for capital is $K_t = (1-\delta)K_{t-1} + I_t$.

Market clearing. Market clearing implies

1. Labor market: $L_t = \int s_{it} d\boldsymbol{D}_t = 1$
2. Asset market: $K_t = A_t^{hh}$
3. Goods market: $Y_t = C_t^{hh} + I_t$

2. Calibration

1. **Preferences:** $\sigma = 2$, $\breve{\beta} = 0.96$, $\sigma_\beta = 0.01$, $\kappa = 0.5$, $\underline{a} = 5$
2. **Income process:** $\rho_s = 0.95$, $\sigma_\psi = 0.10$
3. **Return process:** $\sigma_\chi = 0.10$, $\overline{\pi}_\chi = 0.01$, $\underline{\pi}_\chi = 0.10$
4. **Production:** $\Gamma_{ss} = 1$, $\alpha_{ss} = 0.30$, $\delta = 0.10$