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Section 1.1 and 1.2 discusses how the solution is found, section 1.3 shows the results.

1.1 Solving for steady state.

Γ_{ss}, r_{ss}, r^a_{ss}, G_{ss}, χ, L_{ss}, π_{ss}, π^w_{ss}, and A_{ss} are all targets set by the assignment.

Y_{ss} and w_{ss} are given by equations (5) and (7) in the assignment description.

i_{ss} can be derived using the Taylor rule, along with $\pi_{ss} = 0$ and $i_t = i_{t+1} = i_{ss}$:

$$1 + i_t = (1 + i_{t-1})^{\rho_i} \left((1 + r_{ss})(1 + \pi_t)^{\phi_\pi}\right)^{1-\rho_i} \tag{1}$$

$$\Leftrightarrow 1 + i_{ss} = (1 + i_{ss})^{\rho_i} \left((1 + r_{ss})(1)^{\phi_\pi}\right)^{1-\rho_i} \tag{2}$$

$$\Leftrightarrow (1 + i_{ss})^{1-\rho_i} = (1 + r_{ss})^{1-\rho_i} \Leftrightarrow i_{ss} = r_{ss} \tag{3}$$

q_{ss} can be derived from equation (17):

$$\frac{1 + \delta q_{t+1}}{q_t} = 1 + r_t \Leftrightarrow \frac{1 + \delta q_{ss}}{q_{ss}} = 1 + r_{ss} \tag{4}$$

$$\Leftrightarrow 1 + \delta q_{ss} = (1 + r_{ss})\, q_{ss} \Leftrightarrow (1 + r_{ss} - \delta)\, q_{ss} = 1 \tag{5}$$

$$\Leftrightarrow q_{ss} = \frac{1}{1 + r_{ss} - \delta} \tag{6}$$

Since $A_{ss} = q_{ss} B_{ss}$ we now also have B_{ss}.

τ_{ss} can be derived from equation (17) in the assignment:

$$q_t(B_t - \delta B_{t-1}) = B_{t-1} + G_t + \chi_t - \tau_t Y_t \Leftrightarrow q_{ss}(B_{ss} - \delta B_{ss}) = B_{ss} + G_{ss} + \chi_{ss} - \tau_{ss} Y_{ss} \tag{7}$$

$$\Leftrightarrow q_{ss}(1 - \delta) B_{ss} = B_{ss} + G_{ss} + \chi_{ss} - \tau_{ss} Y_{ss} \Leftrightarrow \tau_{ss} Y_{ss} = (1 - q_{ss}(1 - \delta)) B_{ss} + G_{ss} + \chi_{ss} \tag{8}$$

$$\Leftrightarrow \tau_{ss} = \frac{(1 - q_{ss}(1 - \delta)) B_{ss} + G_{ss} + \chi_{ss}}{Y_{ss}} \tag{9}$$

β is then adjusted numerically such that the solution to the household problem clears the assets market condition $q_{ss} B_{ss} - A^{hh}_{ss}$, causing the goods clearing condition to clear by Walras's law. φ is the set to clear the New Keynesian Wage Phillips Curve.

1.2 Solving the non-linear transition path

When solving the non-linear transition path, G, χ, and Γ are exogenous inputs, while π^w and B are adjusted numerically to clear the New Keynesian Wage Phillips Curve and assets market clearing condition.[1] The rest of the variables are found using the equations below. The entire equation

[1] The assignment set it up to use L instead of B, but initially I found it easier to use B, I have also written the code for using L (blocks.py instead of blocks_B.py, giving similar results.

system is given by the assignment description:

$$\boldsymbol{H}(\boldsymbol{\pi^w}, \boldsymbol{L}, \boldsymbol{G}, \boldsymbol{\chi}, \boldsymbol{\Gamma}) = \begin{bmatrix} w_t - \Gamma_t \\ 1 + \pi_t - \frac{1+\pi_t^w}{\Gamma_t/\Gamma_{t-1}} \\ Y_t - \Gamma_t L_t \\ 1 + i_t - (1+i_{t-1})^{\rho_i} \left((1+r_{ss}) (1+\pi_t)^{\phi_\pi} \right)^{1-\rho_i} \\ 1 + r_t - \frac{1+i_t}{1+\pi_{t+1}} \\ \frac{1+\delta q_{t+1}}{q_t} - (1+r_t) \\ 1 + r_t^a - \frac{1+\delta q_t}{q_{t-1}} \\ \tau_t - \left[\tau_{ss} + \omega q_{ss} \frac{B_{t-1} - B_{ss}}{Y_{ss}} \right] \\ q_t(B_t - \delta B_{t-1}) - [B_{t-1} + G_t + \chi_t - \tau_t Y_t] \\ q_t B_t - A_t^{hh} \\ \pi_t^w - \left[\kappa \left(\varphi \left(L_t^{hh} \right)^\nu - \frac{1}{\mu} (1-\tau_t) w_t \left(C_t^{hh} \right)^{-\sigma} \right) + \beta \pi_{t+1}^W \right] \end{bmatrix} = \mathbf{0} \tag{10}$$

w_t and π_t are given by first two equations.

i_t is found by using the fisher equation:

$$1 + i_t = (1+i_{t-1})^{\rho_i} \left((1+r_{ss}) (1+\pi_t)^{\phi_\pi} \right)^{1-\rho_i} \tag{11}$$

$$i_t = (1+i_{t-1})^{\rho_i} \left((1+r_{ss}) (1+\pi_t)^{\phi_\pi} \right)^{1-\rho_i} - 1 \tag{12}$$

Looping from $t = 0$ and forward to $t = T$, using the fact that the economy was in steady state prior to the shock, and thus $i_{-1} = i_{ss}$.

r_t is found using the 5th equation.

To get q_t in the transition path we rewrite (17)

$$1 + r_t = \frac{1 + \delta q_{t+1}}{q_t} \Leftrightarrow q_t = \frac{1 + \delta q_{t+1}}{1 + r_t} \tag{13}$$

Which can be found for all time periods by looping from the last value and backwards, now using the fact the model converges to steady state, and thus $q_T = q_{ss}$

r_t^a, and τ_t is given by the 7th and 8th equations in the system. To find Y along the transition path use:

$$q_t(B_t - \delta B_{t-1}) = [B_{t-1} + G_t + \chi_t - \tau_t Y_t] \tag{14}$$

$$\Leftrightarrow \tau_t Y_t = -q_t B_t + q_t \delta B_{t-1} + B_{t-1} + G_t + \chi_t \tag{15}$$

$$\Leftrightarrow \tau_t Y_t = (1 + q_t \delta) B_{t-1} - q_t B_t + G_t + \chi_t \tag{16}$$

$$\Leftrightarrow Y_t = \frac{1}{\tau_t} \left((1 + q_t \delta) B_{t-1} - q_t B_t + G_t + \chi_t \right) \tag{17}$$

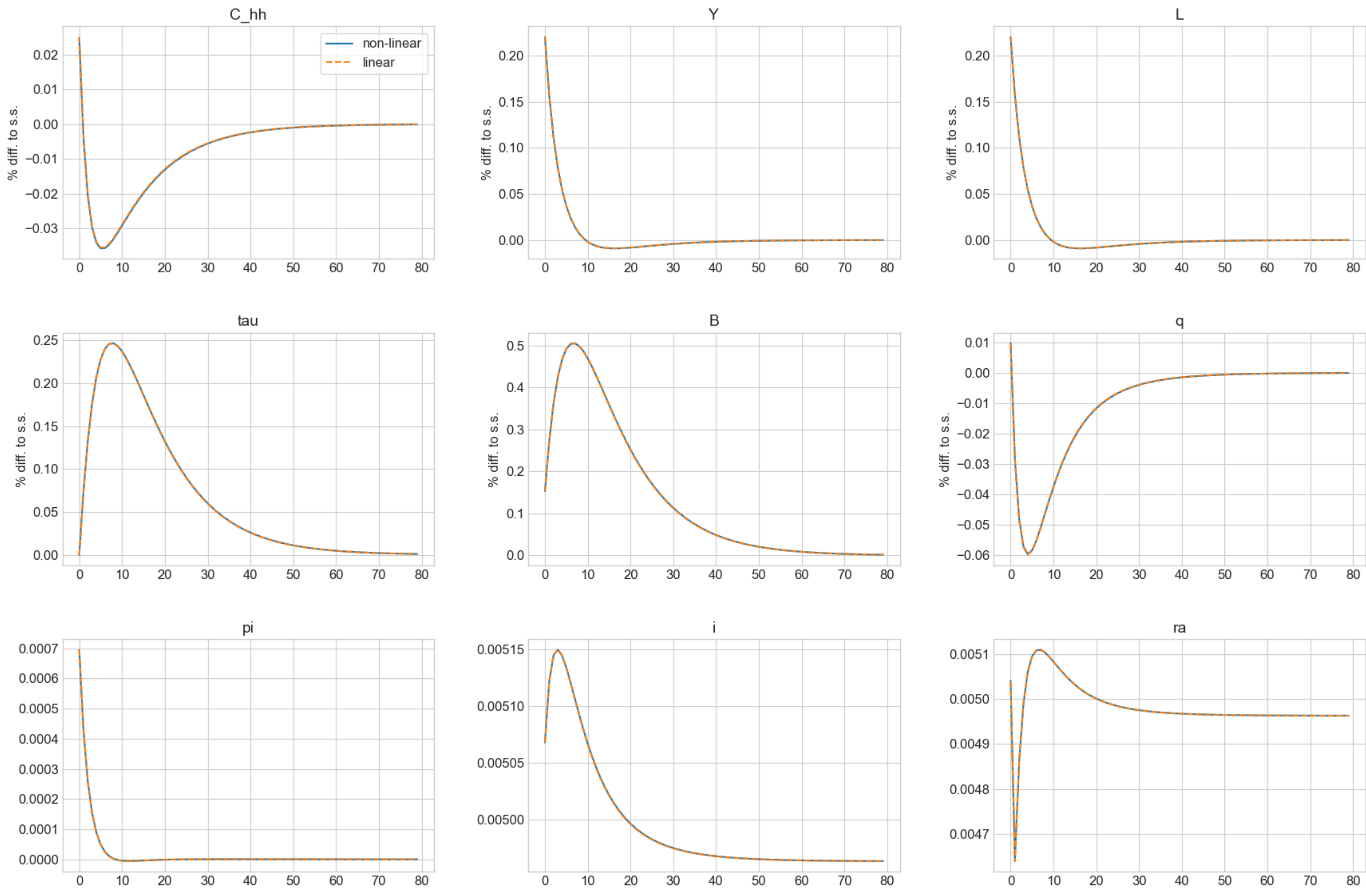
L_t is given by the second equation in the system.[2]

1.3 Results

	$\int c^*_{ss} d\mathbf{D}_{ss}$	Y	L	τ	B	q	π	i	r^a	G	χ
ss	0.800	1.000	1.000	0.205	0.205	4.879	0.00%	0.50%	0.50%	0.200	0.000

Table 1: Steady state values

Figure 1: IRFs from a shock in G of 1%



[2]Alternatively, if guessing on L: To find B and τ along the transition path use:

$$q_t(B_t - \delta B_{t-1}) = [B_{t-1} + G_t + \chi_t - \tau_t Y_t]$$

$$B_t = \frac{1}{q_t}\left[(1 + q_t\delta) B_{t-1} + G_t + \chi_t - \tau_t Y_t\right]$$

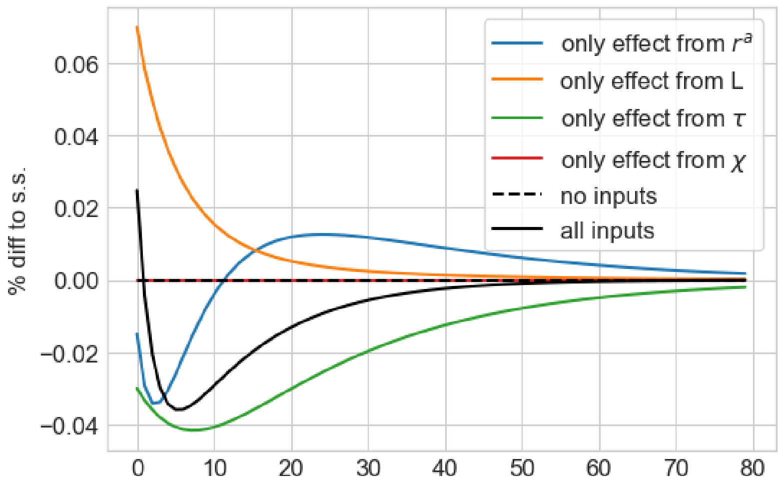
Combine with:

$$\tau_t = \left[\tau_{ss} + \omega q_{ss} \frac{B_{t-1} - B_{ss}}{Y_{ss}}\right]$$

And loop from period 0 and forwards.

The results are shown in figure 1. A 1% increase in G is in absolute terms 0.002. As Y increases by more than 0.2% in period 0 the famous Keynesian multiplier is present in this model. The sudden increase in wasteful government spending works as a positive demand shock which increases L and thereby household income, as can be seen in figure 2 this is what drives up household consumption in the first few periods. The government bill must however be paid and increased taxes can be seen to be suppressing consumption in the subsequent periods. r^a is also seen to have a negative effect on consumption in the early periods. Through the NKWPC the demand shock increases inflation which causes the central bank to adjust i, however monetary policy moves sluggishly because of ρ_i, and persists after inflation has disappeared.

Figure 2: Decomposition of household consumption behavior from a shock in G of 1%



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The linear impulse-response functions are also shown in figure 1 overlaying non-linear IRFs, and can be seen to be virtually identical.

Figure 3 plots the residuals of the Phillips curve and the assets clearing and shows that while they are not exactly the same, it at an insignificant magnitude.

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κ adjusts the sensitivity of inflation in the NKWPC to imbalances between the marginal dis-utility of working more and the marginal utility from consumption from working more.

As can be seen in figure 4, inflation reacts much more vehemently to the demand shock. Correspondingly, i is raised more, and with its persistence, keeps r^a higher than for the lower κ.

As consumption initially drops, the multiplier is effectively killed for this dramatization.

Figure 3: Residuals from targets from a shock in G of 1%

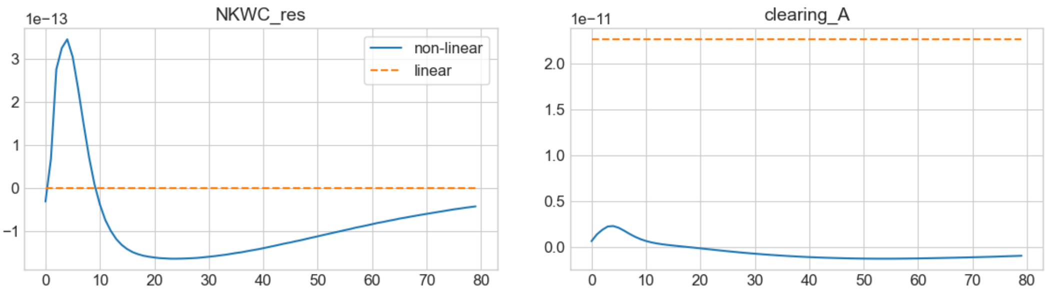
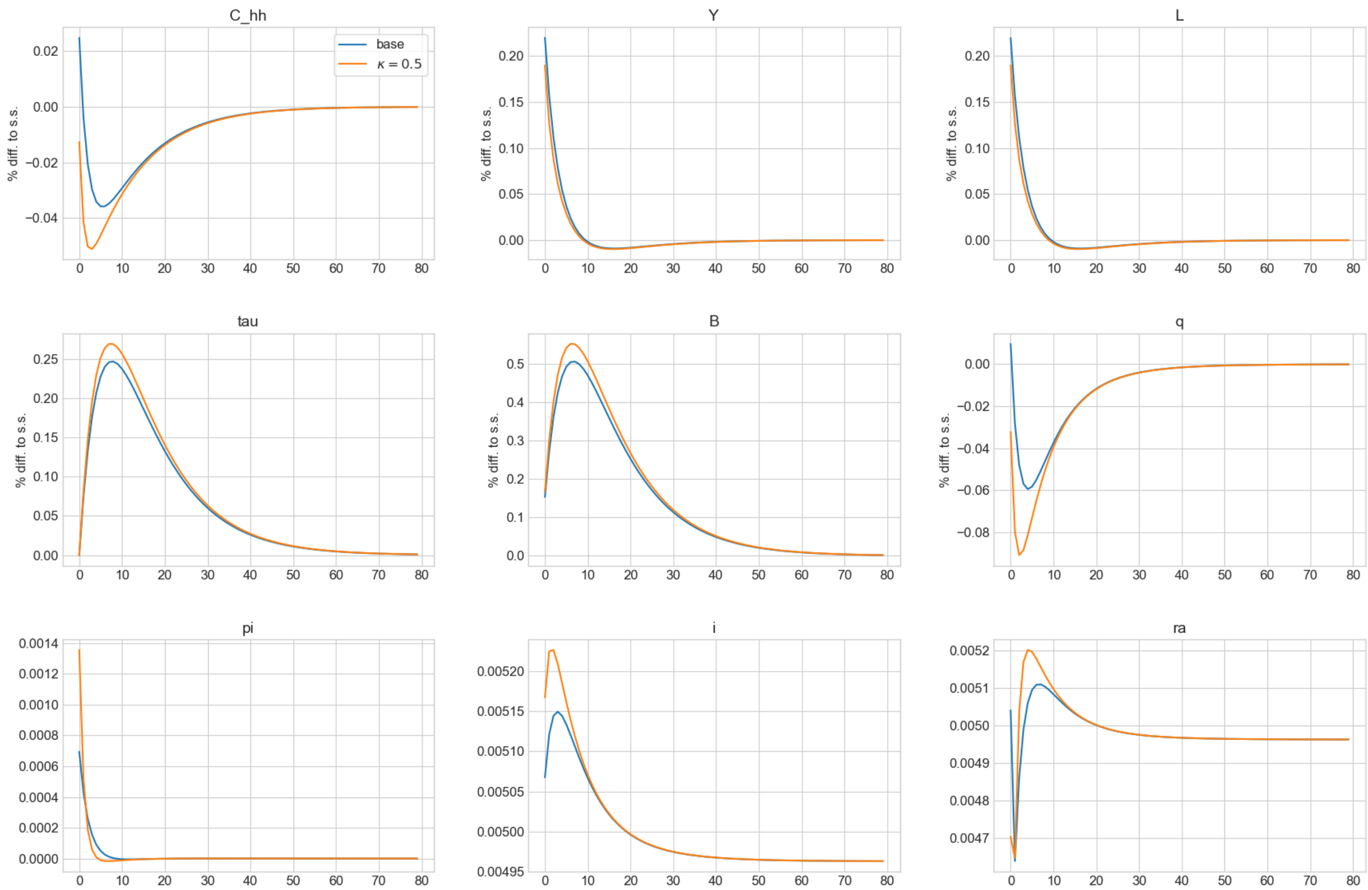


Figure 4: IRFs from a shock in G of 1%, baseline model and $\kappa = 0.5$



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The results of a shock in χ are seen in Figure 5. Instead of going to wasteful government spending χ goes back to the households through the budget constraint. This is seen in the response of C_t^{hh} which increases much more than for the shock in G. However, the chock in χ is a much worse demand shock, as it doesn't enter directly into demand (only C_hh and G does), and thus only works through increasing consumption of the households. In steady state the average MPC is 29.3%, so the increase χ will not pass fully into demand and some will we saved instead polstring consumption somewhat for the later periods where Y is below steady state. The initial upward thrust in Y and L is much smaller and subsequently falls further below the steady state level.

Figure 5: IRFs from a shock in G of 1% compared with a shock of the same size in χ

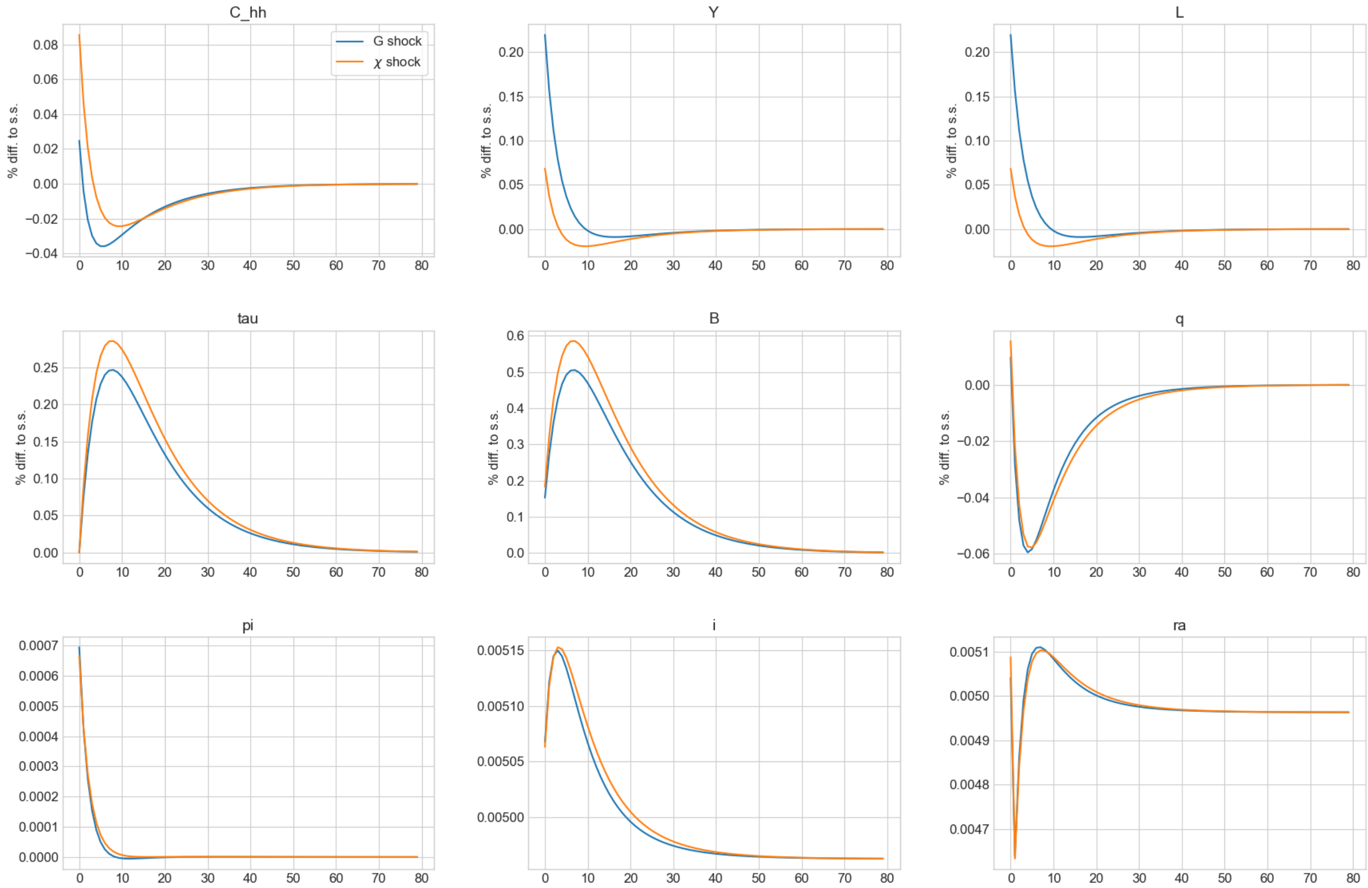

Figure 6: Decomposition of household consumption behavior from a shock in χ of 0.002