

9-10. Fiscal Policy in HANK

Adv. Macro: Heterogenous Agent Models

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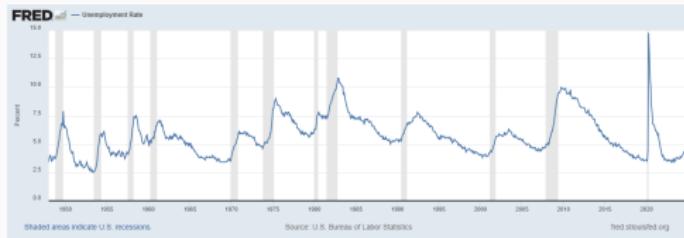
Introduction

Introduction

- **Previously:**
 1. Economic content: Long run trends and outcomes
 2. Methods: Stationary eq., Non-linear transition path and perfect foresight
- **Today:**
 1. Business cycles in the New Keynesian model
 2. Linearized solution in models with aggregate risk
- **Literature:**
 - NK:
 1. Gali textbook ch. 3-4
 2. *Macroeconomics* textbook ch. 16
 - Solution methods:
 1. Auclert et. al. (2021), »Using the Sequence-Space Jacobian to Solve and Estimate Heterogeneous-Agent Models«
 2. Boppert et. al. (2018), »Exploiting MIT shocks in heterogeneous-agent economies: The impulse response as a numerical derivative«
 3. Documentation for GEModelTools

Business cycles

- Macro variables relatively volatile around long-run trends



- Rest of the course:
 - Study how aggregate shocks cause business cycles
 - Does the transmission change with heterogeneous agents?
 - Implications for fiscal and monetary policy
- First point on agenda: Need **role** for monetary policy
 - Models so far in the course have featured **monetary-non neutrality**
 - Monetary policy cannot affect real quantities (unemployment, GDP)

New Keynesian framework

- A proper model of business cycles and stabilization policy requires that output is partially demand determined
- The study of monetary policy requires **monetary-non neutrality**
- The New Keynesian (NK) model addresses these two concerns by adding to the standard model:
 - Monopolistic competition (price-setting)
 - Price rigidities
- The basic NK model is simple (can be reduced to 3 equations) but **extremely influential**

Quick Recap: The IS-LM model

The IS-LM model

- Aggregate demand is $Z = C + G$ (abstracting away from investment)
- Assume $C = c_0 + c_1(Y - T) - c_2r$ with $c_1, in(0, 1)$
- If output is demand determined, then $Y = Z$, and

$$Y = \frac{1}{1 - c_1}(c_0 - c_1T + G - c_2r)$$

- **Core intuition:** all of income is somebody's else spendings!

Policies in IS-LM

- **Fiscal policy:** multiplier is $\frac{dY}{dG} = \frac{1}{1-c_1} > 1$
- **Monetary policy:** multiplier is $\frac{dY}{dr} = -\frac{c_2}{1-c_1} < 0$
 - Direct effect is $c_2 r$
 - Indirect effect through Y is $c_1 \frac{dY}{dr}$
- What's missing?
 - A supply side, micro-foundations, expectations, etc.

The New Keynesian model

- Several versions of the NK model
- I present the simplest version here
- The model consists of the following agents:
 1. A representative household who consumes, saves and supplies labor
 2. Firms with market power who produce output using labor and sets prices subject to nominal rigidities
 3. A central bank which conduct monetary policy

Overview

- **Households:**
 1. Representative agent
 2. Supply labor and choose consumption
- **Intermediary goods firms** (continuum)
 1. Produce differentiated goods with labor
 2. Set price under monopolistic competition
 3. Pay profits to households
- **Final goods firms** (representative)
 1. Produce final good with intermediary goods
 2. Take price as given under perfect competition
- **Central bank:** Sets nominal interest rate

Households

- The **representative household** solves the following problem:

$$\max_{C_t, A_t, L_t} E_0 \sum_{t=0}^{\infty} \beta^t [u(C_t) - \nu(L_t^{hh})]$$

s.t.

$$C_t + A_t = (1 + r_t) A_{t-1} + (w_t L_t^{hh} + \Pi_t)$$

- Note: Expectation taken w.r.t **aggregate shocks** (TFP, monetary policy, markup etc.)
- Standard first-order conditions:

$$u'(C_t) = E_t \beta (1 + r_{t+1}) u'(C_{t+1})$$
$$\nu'(L_t^{hh}) = w_t u'(C_t)$$

Households

A few things to note:

- Households are forward-looking and take into account future income and interest rates when making consumption and labor supply decisions.
- Ricardian equivalence holds: fiscal policy has no impact on aggregate demand

Final goods firms

- Final goods firm buy goods y_{jt} from intermediary goods firms and assemble into aggregate output Y_t using CES technology with sub. elasticity ϵ
- Intermediary firms/intermediary goods indexed by $j \in [0, 1]$
- **Static problem for representative final good firm:**

$$\max_{y_{jt} \forall j} P_t Y_t - \int_0^1 p_{jt} y_{jt} dj \text{ s.t. } Y_t = \left(\int_0^1 y_{jt}^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}$$

for given output price, P_t , and input prices, p_{jt}

- **Demand curve** derived from FOC wrt. y_{jt}

$$\forall j : y_{jt} = \left(\frac{p_{jt}}{P_t} \right)^{-\frac{1}{\epsilon}} Y_t$$

- **Note:** Zero profits (can be used to derive price index)

Intermediary goods firms

- Intermediary goods firms produce using labor, choose price subject to quadratic adjustment cost of changing prices (Rotemberg)
- **Dynamic problem for intermediary goods firms:**

$$J_t(p_{jt-1}) = \max_{y_{jt}, p_{jt}, l_{jt}} \left\{ \frac{p_{jt}}{P_t} y_{jt} - w_t l_{jt} - \Omega(p_{jt}, p_{jt-1}) Y_t + E_t \frac{J_{t+1}(p_{jt})}{1 + r_{t+1}} \right\}$$

s.t. $y_{jt} = \Gamma_t l_{jt}$, $y_{jt} = \left(\frac{p_{jt}}{P_t} \right)^{-\epsilon} Y_t$, $\Omega(p_{jt}, p_{jt-1}) = \frac{\theta}{2} \left[\frac{p_{jt}}{p_{jt-1}} - 1 \right]^2$

- **Symmetry:** In equilibrium all firms set the same price, $p_{jt} = P_t$
- **NKPC** with slope $\kappa = \frac{\epsilon}{\theta}$ and $\mu = \frac{\epsilon}{\epsilon-1}$ derived from FOC wrt. p_{jt} and envelope condition (note $mc_t = \frac{MC_t}{P_t} = \frac{w_t}{\Gamma_t}$):

$$\pi_t(1+\pi_t) = \kappa \left(\frac{w_t}{\Gamma_t} - \frac{1}{\mu} \right) + E_t \frac{Y_{t+1}}{Y_t} \frac{\pi_{t+1}(1+\pi_{t+1})}{1+r_{t+1}}, \quad \pi_t \equiv P_t/P_{t-1}-1$$

- **Implied production:** $Y_t = y_{jt}$, $L_t = l_{jt}$ (from symmetry)
- **Implied dividends:** $\Pi_t = Y_t - w_t L_t - \frac{\theta}{2} \left[\frac{p_{jt}}{p_{jt-1}} - 1 \right]^2 Y_t$

Derivation of NKPC

- FOC wrt. p_{jt} :

$$0 = (1 - \epsilon) \left(\frac{p_{jt}}{P_t} \right)^{-\epsilon} \frac{Y_t}{P_t} + \epsilon \frac{w_t}{\Gamma_t} \left(\frac{p_{jt}}{P_t} \right)^{-\epsilon-1} \frac{Y_t}{P_t} \\ - \theta \left[\frac{p_{jt}}{p_{jt-1}} - 1 \right] \frac{Y_t}{p_{jt-1}} + E_t \frac{J'_{t+1}(p_{jt})}{1 + r_{t+1}}$$

- Envelope condition: $J'_{t+1}(p_{jt}) = -\theta \left[\frac{p_{jt+1}}{p_{jt}} - 1 \right] \left(\frac{p_{jt+1}}{p_{jt}^2} \right) Y_{t+1}$
- FOC + Envelope + Symmetry + $\pi_t = P_t/P_{t-1} - 1$

$$0 = \left[(1 - \epsilon) + \epsilon \frac{w_t}{\Gamma_t} \right] \frac{Y_t}{P_t} \\ - \theta \left[\frac{P_t}{P_{t-1}} - 1 \right] \frac{Y_t}{P_{t-1}} - E_t \frac{\theta \left[\frac{P_{t+1}}{P_t} - 1 \right] \left(\frac{P_{t+1}}{P_t^2} \right) Y_{t+1}}{1 + r_{t+1}}$$

Central NKPC intuition

$$\pi_t(1 + \pi_t) = \kappa \left(\frac{w_t}{\Gamma_t} - \frac{1}{\mu} \right) + E_t \frac{Y_{t+1}}{Y_t} \frac{1}{1 + r_{t+1}} \pi_{t+1} (1 + \pi_{t+1})$$

1. Zero-inflation steady state:

$\pi_t = 0 \rightarrow w_t = \frac{\Gamma_t}{\mu} \rightarrow$ wage is mark-downed relative to MPL
 $(\mu > 1)$

2. Larger adjustment costs, $\kappa \downarrow$ (more sticky prices):

Less pass-through from marginal costs, $\frac{w_t}{\Gamma_t}$, to inflation, π_t

3. Larger (expected) future inflation, $\pi_{t+1} \uparrow$:

Increase price today, $\pi_t \uparrow$

Especially in a boom, $\frac{Y_{t+1}}{Y_t} > 1$

4. Note:

- Sometimes a β^{firm} is used instead of $\frac{1}{1+r_{t+1}}$
- $\pi_t(1 + \pi_t) \approx \pi_t$ for small π_t

Government and central bank

- **Monetary policy:** Follow Taylor-rule:

$$i_t = i_t^* + \phi\pi_t$$

where i_t^* is a monetary policy shock (target for CB)

- **Fisher relationship:**

$$r_t = (1 + i_{t-1})/(1 + \pi_t) - 1$$

- **Government:** In standard model Government simply supplies bonds that are in net-zero supply, $B = 0$
 - Note: HHs still make consumption-saving decisions (so cannot impose $A = 0$ in budget), but in equilibrium prices will adjust such that $A = B = 0$
 - Simplifying assumption, can easily incorporate more realistic government $\tau_t = r_t B_{ss} + G_t$ with $B_{ss} > 0$ (see HANK later)

Equilibrium

- Three markets that need to clear in the NK model:

1. Goods market:

$$Y_t = C_t + \frac{\theta}{2} \pi_t^2 Y_t$$

2. Labor market:

$$L_t^{hh} = L_t$$

3. Asset market:

$$A = 0$$

- As usual, in practice we will only impose market clearing in two of the markets when solving the model

Aggregate shocks

- In the standard NK model business cycles arise due to fluctuations in aggregate shocks:

1. TFP (supply)

$$\ln \Gamma_t = \bar{\Gamma} + \ln \Gamma_{t-1} + \epsilon_t^{\Gamma}, \quad \epsilon_t^{\Gamma} \sim \mathcal{N}(0, \sigma_{\Gamma}^2)$$

2. Discount factor (demand)

$$\ln \beta_t = \bar{\beta} + \ln \beta_{t-1} + \epsilon_t^{\beta}, \quad \epsilon_t^{\beta} \sim \mathcal{N}(0, \sigma_{\beta}^2)$$

3. Monetary policy

$$i_t^* = \bar{i}^* + \ln i_{t-1}^* + \epsilon_t^{i^*}, \quad \epsilon_t^{i^*} \sim \mathcal{N}(0, \sigma_{i^*}^2)$$

The 3 equation NK model

- Consider the deterministic, zero-inflation steady state of the model (with TFP and prices normalized to 1):

$$\pi_{ss} = 0, \quad Y_{ss} = C_{ss} = 1$$

$$r_{ss} = i_{ss} = \frac{1}{\beta} - 1, \quad w_{ss} = \frac{1}{\mu}$$

- Linearize the model arounds this steady state with notation $\hat{x}_t = x_t - x_{ss}$ for some endo. variable x_t
- The model can be reduced to three equations:

$$\hat{Y}_t = -\sigma(i_t - \pi_{t+1}) + \hat{Y}_{t+1} + \epsilon_t^D \quad (\text{Euler/demand curve})$$

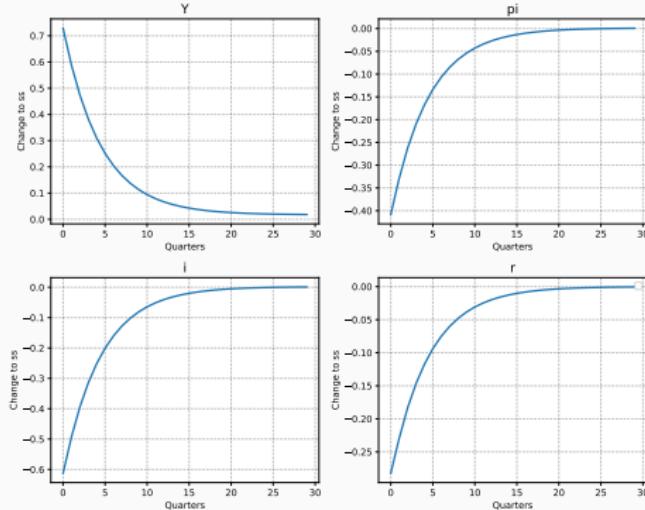
$$\hat{\pi}_t = \tilde{\kappa} \hat{Y}_t + \beta \hat{\pi}_{t+1} + \epsilon_t^S \quad (\text{NKPC/supply curve})$$

$$\hat{i}_t = \phi \hat{\pi}_t + \epsilon_t^I \quad (\text{Monetary policy})$$

- With three unknowns (per period) $\hat{Y}_t, \hat{\pi}_t, \hat{i}_t$

Technology shocks in the NK model

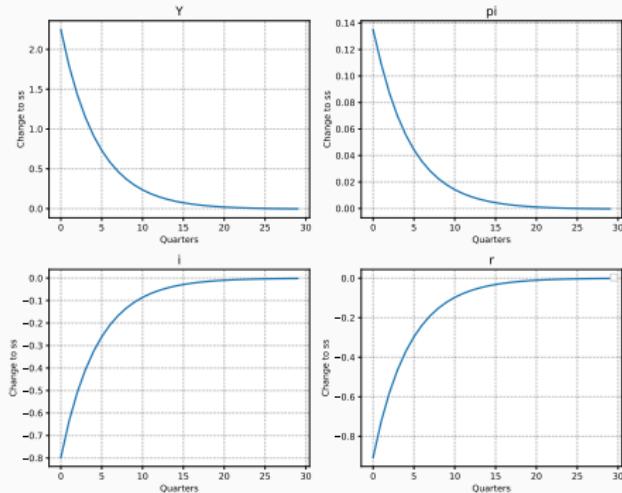
- Effects of a positive TFP shock (increase Γ_t)



- Increase in productivity decreases marginal costs w_t/Γ_t
- Firms reduce prices \Rightarrow CB reduce nominal interest rate
- Intertemporal sub. $\Rightarrow C, Y \uparrow$

Monetary policy shocks in the NK model

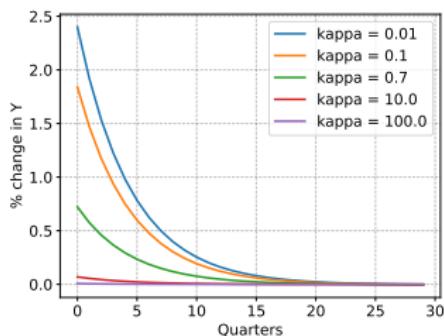
- Effects of accomodating monetary policy (easing) with persistent decline in i_t^*



- Decrease real rate r which induce intertemporal substitution, so $C, Y \uparrow$
- Increase in employment pushes up wages (marginal costs), so inflation increases

Monetary neutrality

- Monetary policy can affect consumption, employment and output in the short run because the model features **monetary non-neutrality**
 - Comes from sticky prices of firms
- Consider monetary policy shock with increasing slope of NKPC κ
 - Recall $\kappa \rightarrow \infty$ is flexible prices (think Ramsey) and constant markup



- Why? With completely flexible prices monetary policy just increases inflation 1-1 without affecting r

Some limits of the NK model

- **No role for fiscal policy:** Ricardian equivalence holds (impact and cumulative multipliers of 1)
- **Not Keynesian?:** aggregate demand moves essentially because of intertemporal substitution rather than income effects (aggregate demand plays a very small role)
- **Can't speak to redistribution effects:** representative agent framework, no inequality
- **Calibration:** cannot take the model to micro-data

Review questions

- Consider the standard New Keynesian model
- Review questions
 1. How does a positive demand shock ϵ_t^β (which decrease β) affect output Y , inflation π , and interest rates i, r ?
 2. Are firm markups pro-cyclical or counter-cyclical (w.r.t Y) in response to the demand shock?
 3. Consider an extension with a government that spends G and raises lumpsum taxes τ
 - What is the effect of a shock to G ? Is the fiscal multiplier $\frac{dY}{dG}$ above or below one?
 - Does the effects of the shock dependent on the method of financing (debt vs taxes)?

Exercise

Exercise - NK model with government

1. Familiarize yourself with the model equations in *blocks.py*. Do you understand all the equations?
2. Compute the non-linear response to a temporary increase in government spending
 - 2.1 Use *model.find_transition_path()* for the non-linear response (results are in *model.path*)
 - 2.2 Use *model.find_IRFs()* for the linear response (results are in *model.IRF*)
3. Add a zero lower bound to the model:

$$i_t = \max \{ i_{ss} + \phi \pi_t, 0 \}$$

Compute linear and non-linear responses to a β -shock of size 0.05 and compare.

4. Assume that the government tries to stabilize the economy after the demand shock. Compute linear and non-linear responses to a simultaneous shock to β ($d\beta_0 = 0.05$) and G ($dG_0 = 0.03$).
5. Is stabilization policy more or less efficient once we take the ZLB into account?

Hint: Compare the multipliers $\frac{dY^{\beta,G} - dY^\beta}{dG}$ for the linear and non-linear responses and compare.

HANK: Introduction

Introduction

- **Today:**
 - The canonical HANK model
 - Model with sticky wages
 - **Application:** Fiscal policy in HANK
- **Literature:** Auclert et. al. (2023),
"The Intertemporal Keynesian Cross"
 - Long paper with many (technical) details
 - We will focus on the main results

Sticky Wages

Detour

- Early HANK papers formulated *Heterogeneous Agent New Keynesian Models* by:
 - Take standard NK model from last lecture
 - Replace Representative agent HH block with Heterogeneous Agents HH block
 - See e.g. McKay, Nakamura, and Steinsson (2016), Hagedorn, Manovskii, and Mitman (2019)
- Turns out that just doing this has undesirable properties when:
 - Prices are sticky
 - Wages are fully flexible
- Need to make one adjustment to standard NK model before adding HA

Sticky wages

- Canonical NK model features **sticky** prices, **flexible** wages
 - Fine for some questions, but can cause issues in HA models (see Auclert, Bardóczy, and Rognlie (2023))
- **EX:** Standard HA model where households have flexible labor supply with FOC ($EIS=1$, $Frisch=1$):

$$\begin{aligned} u'(c_i)w &= \nu'(l_i) \\ \Leftrightarrow c_i^{-1}w &= \varphi l_i \end{aligned}$$

- What happens if we give household i a lumpsum transfer T ?

$$-c_i^{-2} \frac{\partial c_i}{\partial T} w = \varphi \frac{\partial l_i}{\partial T}$$

- Define $MPC_i = \frac{\partial c_i}{\partial T}$, $MPE_i = -\frac{\partial wl_i}{\partial T}$ as the change in consumption and change in earnings following transfer:

$$MPC_i = \frac{c_i}{wl_i} MPE_i$$

- If HHs have little saving, $c_i \approx wl_i \Rightarrow MPC_i = MPE_i$!

MPCs and MPEs

- HA model with flexible labor supply implies tight link btw. MPC_i and MPE_i
 - Plausible?
- **MPCs in the data:**
 - Annual estimates between 0.3 and 0.7
 - Typical value in HA litterature: 0.5
- **MPEs in the data:**
 - Range from 0 to 0.04 annually \Rightarrow wealth effects are **small**
 - Increase in (non-wage) income of 1\$ decreases labor earnings by 0-0.04\$
- Tension between model and data \Rightarrow Standard model with high MPCs imply too large wealth effects on labor supply

Union setup

- **Solution:** Take HHs off their individual labor supply curves through **union membership**
 - Literature: Erceg, Henderson and Levin (2000)
- Households solve the same consumption/saving problem as usual but do not choose labor supply l_{it} themselves
- Instead, they belong to a labor union that determines the level of labor supply
- Will also use this a method of introducing **sticky wages**
 - Empirical evidence typically show that wages adjust more sluggishly than prices w.r.t aggregate shocks

Union problem

- Each household i belong to a union j and face labor demand from firms

$$L_t^j = \left(\frac{W_t^j}{W_t} \right)^{-\epsilon^W} L_t,$$

- Unions maximize aggregate utility of their members, choosing both wages and labor subject to demand:

$$\max_{W_t^j, L_t^j} \sum_{t=0}^{\infty} \beta^t \left(\int \left\{ u(c_{i,t}) - \nu(L_t^j) \right\} dD_{i,t} - \frac{\theta^W}{2} \left(\frac{W_t^j}{W_{t-1}^j} - 1 \right)^2 \right)$$

- where $\frac{\theta^W}{2} \left(W_t^j / W_{t-1}^j - 1 \right)^2$ is a quadratic utility cost from changing wages \Rightarrow sticky wages

New Keynesian Wage Phillips curve

- Solving the union's maximization problem in a symmetric equilibrium $L_t^j = L_t$, $W_t^j = W_t$:

$$\pi_t^w (1 + \pi_t^w) = \kappa^w \left\{ \nu' (L_t) - \frac{w_t}{\mu^w} \int z_t u' (c_{i,t}) d\mathcal{D}_{i,t} \right\} L_t + \beta \pi_{t+1}^w (1 + \pi_{t+1}^w)$$

- with $\kappa^w = \frac{\epsilon^w}{\theta^w}$, $\mu^w = \frac{\epsilon^w}{\epsilon^w - 1}$ and z_{it} is individual productivity ($z_{it} = 1$ in RA)
- Philips curve** for wages. For given future inflation π_{t+1}^w
 - Increase in labor demand L_t will push up nominal wage inflation
 - Increase in real wage w_t will decrease nominal wage inflation
 - Higher consumption will increase wage inflation (wealth effect)
 - Pass-through depends on slope κ^w
- Breaks wealth effect on labor supply through two mechanisms:
 - Sticky wages (low κ^w)
 - Wealth effect on agg. L depends on agg. marginal utility
$$\int z_t u' (c_{i,t}) d\mathcal{D}_{i,t}$$

More tractable version

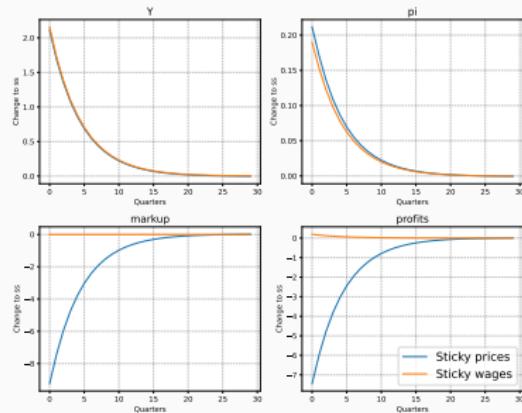
- Sometimes assume that union maximize the utility of an agent with average consumption $u(C_t) = u\left(\int c_{it} d\mathcal{D}_{it}\right)$ instead of $\int u(c_{i,t}) d\mathcal{D}_{it}$
- NKWPC:

$$\pi_t^w (1 + \pi_t^w) = \kappa^w \left\{ \nu' (L_t) - \frac{w_t}{\mu^w} u' (C_t) \right\} L_t + \beta \pi_{t+1}^w (1 + \pi_{t+1}^w)$$

- $u'(C_t)$ instead of $\int z_t u'(c_{i,t}) d\mathcal{D}_{i,t}$
- Slightly easier to implement in code and when working analytically

Profits

- Additional benefit of introducing sticky wages is the **cyclicality of profits**
- In the data profits are *pro-cyclical* (i.e. go up in boom)
- Not consistent with standard NK model with sticky prices
 - Wages rise more than prices in response to demand shock
- Sticky wages solve this



HANK

Model elements

- The Heterogeneous Agent New Keynesian model
 - Similar to standard NK model but substitute sticky prices with wages + HA instead of RA
- **Households**
 - Standard HA: Face idiosyncratic income risk + borrowing constraint
- **Firms**
 - Produce using labor
- **Unions**
 - Decide on labor supply for HHs subject to wage adjustment cost
- **Mutual fund**
 - Collect household savings and invest in available assets (today: gov' bonds)
- **Central bank**
- **Government**

Households

- **Household problem:**

$$v_t(z_t, a_{t-1}) = \max_{c_t} \frac{c_t^{1-\sigma}}{1-\sigma} - \varphi \frac{\ell_t^{1+\nu}}{1+\nu} + \beta \mathbb{E}_t [v_{t+1}(z_{t+1}, a_t)]$$

$$\text{s.t. } a_t + c_t = (1 + r_t^a) a_{t-1} + (1 - \tau_t) w_t \ell_t z_t + \chi_t$$

$$\log z_{t+1} = \rho_z \log z_t + \psi_{t+1}, \psi_t \sim \mathcal{N}(\mu_\psi, \sigma_\psi), \mathbb{E}[z_t] = 1$$

$$a_t \geq 0$$

- **Active decisions:** Consumption-saving, c_t (and a_t)
- **Union decision:** Labor supply, ℓ_t
- **Aggregate Consumption:** $C_t^{hh} = \int c_t d\mathcal{D}_t$
- **Consumption function:** $C_t^{hh} = C^{hh}(\{r_s^a, (1 - \tau_s) w_s \ell_s, \chi_s\}_{s=0}^\infty)$

- Production and profits:

$$Y_t = \Gamma_t L_t$$

$$\Pi_t = Y_t - \frac{W_t}{p} L_t$$

- First order condition:

$$w_t = \Gamma_t$$

Mutual fund and assets

- Mutual fund collect households savings A_t
- Invest in available assets
 - Real government bonds which pay real interest rate r
- **Problem:**

$$\max_{B_t} (1 + r_{t+1}) B_t - (1 + r_{t+1}^a) A_t \quad \text{s.t.} \quad B_t = A_t$$

- **FOCs** (no arbitrage conditions):

$$1 + r_t = 1 + r_t^a$$

- Everybody works the same:

$$\ell_t = L_t^{hh}$$

- Maximization subject to wage adjustment cost imply a **New Keynesian Wage (Phillips) Curve** (NKWPC or NKWC)

$$\pi_t^w = \kappa \left(\varphi (L_t^{hh})^\nu - \frac{1}{\mu} (1 - \tau_t) w_t (C_t^{hh})^{-\sigma} \right) + \beta \pi_{t+1}^w$$

Government

- **Spending:** G_t

$$B_t + T_t = (1 + r_t)B_{t-1} + G_t + \chi_t$$

- where T_t captures labor income tax revenue:

$$T_t = \tau_t w_t L_t$$

- In response to shocks r_t moves which affect budget
- Need rule to ensure that government debt returns to ss in the long run
- Potential **tax-rule:**

$$\tau_t = \tau_{ss} + \omega \frac{B_{t-1} - B_{ss}}{Y_{ss}}$$

- Or simply:

$$B_t = B_{ss}$$

- In which case taxes T_t adjust fully every period to ensure that the budget holds

Central bank

- Two options for monetary policy
- 1. Government bonds are nominal, CB chooses nominal interest rate:

$$i_t = i_{ss} + \phi \pi_t$$

- And fisher equation links nominal rate i to real rate r :

$$1 + r_t = \frac{1 + i_t}{1 + \pi_{t+1}}$$

- 2. Alternative: Real rate rule. CB chooses real rate r_t directly

$$r_t = r_{ss} + (\phi - 1) \pi_t$$

- Implicit assumption: CB can control inflation.
 - Today will study special case with »passive« monetary policy, $\phi = 1$
 $\Rightarrow r_t = r_{ss}$
- Indeterminacy: Consider limit for nominal rule $i_t = i_{ss} + \phi \pi_{t+1}$ or assume future tightening

Market clearing

1. Asset market: $B_t = A_t^{hh}$
2. Labor market: $L_t = L_t^{hh}$
3. Goods market: $Y_t = C_t^{hh} + G_t$

Fiscal Policy

Simpler consumption function

- **Assumptions:**

1. One-period real bond
2. No lump-sum transfers, $\chi_t = 0$
3. Fiscal policy in terms of dG_t and dT_t satisfying IBC (e.g. government needs to repay excess debt dB eventually)

$$\sum_{t=0}^{\infty} (1 + r_{ss})^{-t} (dG_t - dT_t) = 0$$

- **Tax-bill:** $T_t = \tau_t w_t \int \ell_t z_t d\mathbf{D}_t = \tau_t \Gamma_t L_t = \tau_t Y_t$
- **Household income:** $(1 - \tau_t) w_t \ell_t z_t = \underbrace{(Y_t - T_t)}_{\equiv Z_t} z_t = Z_t z_t$
- **Consumption function in sequence-space:** Simplifies to

$$C_t^{hh} = C^{hh}(\{Y_s - T_s, r_s\}_{s \geq 0}) \Rightarrow \mathbf{C}^{hh} = C^{hh}(\mathbf{Y} - \mathbf{T}, \mathbf{r}) = C^{hh}(\mathbf{Z}, \mathbf{r})$$

Side-note: Two-equation version in Y and r

$$Y = G + C^{hh}(r, Y - T)$$
$$r = \mathcal{R}(Y)$$

- **First equation:** Goods market clearing
- **Second equation (Firms + NKWPC):**
 1. Given output Y , can compute L
 2. Firm behavior I: $\Gamma, Y \rightarrow L, w$
 3. NKWC: $L, C, w, \tau \rightarrow \pi^w$
 4. Firm behavior II: $\pi^w, \Gamma \rightarrow \pi$
 5. Central bank: $\pi \rightarrow i$
 6. Fisher: $i, \pi \rightarrow r$
- Final assumption for today: **Constant r**
 - Can replace $r = \mathcal{R}(Y)$ with $r = r_{ss}$
 - Entire model boils down to 1 equation

Intertemporal Keynesian Cross

$$Y_t = G_t + C_t^{hh} (\{Y_s - T_s\}_{s=0}^{\infty}) \quad \text{Static}$$

$$\mathbf{Y} = \mathbf{G} + C^{hh}(\mathbf{Y} - \mathbf{T}) \quad \text{Sequence-space/vector}$$

- Total differentiation/linearize around ss:

$$dY_t = dG_t + \sum_{s=0}^{\infty} \frac{\partial C_t^{hh}}{\partial Z_s} dZ_s = dG_t + \sum_{s=0}^{\infty} \frac{\partial C_t^{hh}}{\partial Z_s} (dY_s - dT_s)$$

- Intertemporal Keynesian Cross in vector form

$$\begin{aligned} d\mathbf{Y} &= d\mathbf{G} + \mathbf{M}(d\mathbf{Y} - d\mathbf{T}) \Leftrightarrow \\ (\mathbf{I} - \mathbf{M})d\mathbf{Y} &= d\mathbf{G} - \mathbf{M}d\mathbf{T} \end{aligned}$$

where $M_{t,s} = \frac{\partial C_t^{hh}}{\partial Z_s}$ encodes the entire *complexity of HH behavior*

Illustration

- Writing out the IKC:

$$\begin{bmatrix} dY_0 \\ dY_1 \\ dY_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} dG_0 \\ dG_1 \\ dG_2 \\ \vdots \end{bmatrix} + \begin{bmatrix} \frac{\partial C_0^{hh}}{\partial Z_0} & \frac{\partial C_0^{hh}}{\partial Z_1} & \frac{\partial C_0^{hh}}{\partial Z_2} & \dots \\ \frac{\partial C_1^{hh}}{\partial Z_0} & \frac{\partial C_1^{hh}}{\partial Z_1} & \frac{\partial C_1^{hh}}{\partial Z_2} & \dots \\ \frac{\partial C_2^{hh}}{\partial Z_0} & \frac{\partial C_2^{hh}}{\partial Z_1} & \frac{\partial C_2^{hh}}{\partial Z_2} & \dots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \left(\begin{bmatrix} dY_0 \\ dY_1 \\ dY_2 \\ \vdots \end{bmatrix} - \begin{bmatrix} dT_0 \\ dT_1 \\ dT_2 \\ \vdots \end{bmatrix} \right)$$

iMPC matrix

- \mathbf{M} is the Jacobian of aggregate C w.r.t (post-tax) labor income
 - Column s : Response of C at different dates to unit change in Z at date s (IRF)
 - Row s : Change in C at date s to change in income Z at different dates

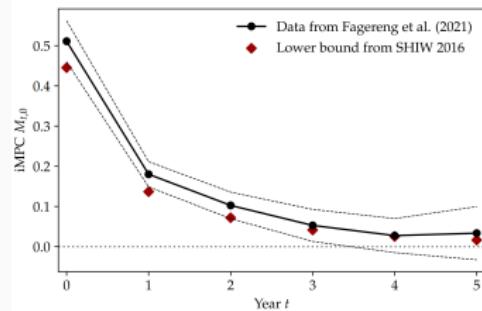
$$\mathbf{M} = \begin{bmatrix} \frac{\partial C_0^{hh}}{\partial Z_0} & \frac{\partial C_0^{hh}}{\partial Z_1} & \dots \\ \frac{\partial C_1^{hh}}{\partial Z_0} & \frac{\partial C_1^{hh}}{\partial Z_1} & \dots \\ \vdots & \vdots & \ddots \end{bmatrix}$$

- In a quarterly model $\frac{\partial C_0^{hh}}{\partial Z_0}$ is essentially the quarterly MPC
 - Note: Typically define MPCs following change in lump-sum transfer and not labor income, so not quite MPC

iMPCs in the data

- What can we say about the iMPC matrix \mathbf{M} ?
 - Can say a lot about first element $\frac{\partial C_0^{hh}}{\partial Z_0}$ (see lecture 2)
- Fagareng et al (2016) estimate dynamic response of C to income shocks (lottery winnings)
 - Info on first column $\left[\frac{\partial C_0^{hh}}{\partial Z_0}, \frac{\partial C_1^{hh}}{\partial Z_0}, \frac{\partial C_2^{hh}}{\partial Z_0}, \frac{\partial C_3^{hh}}{\partial Z_0}, \dots \right]'$

Figure 1: iMPCs in the Norwegian and Italian data



- Very hard to say something about remaining columns (announcement effects)
 - Best we can do: Calibrate model to first column, get rest of \mathbf{M} from model

Perspective: Static Keynesian Cross

- **Old Keynesians:** Consumption only depends on current income

$$Y_t = G_t + C^{hh}(Y_t - T_t)$$

- **Total differentiate:**

$$\begin{aligned} dY_t &= dG_t + \frac{\partial C_t^{hh}}{\partial Z_t} (dY_t - dT_t) \\ &= dG_t + mpc \cdot (dY_t - dT_t) \end{aligned}$$

- **Solution**

$$dY_t = dG_t + \frac{mpc}{1 - mpc} (dG_t - dT_t)$$

from multiplier-process $mpc \times (1 + mpc + mpc^2 \dots) = \frac{mpc}{1 - mpc}$

NPV-vector

- **NPV-vector:** $\mathbf{q} \equiv [1, (1 + r_{ss})^{-1}, (1 + r_{ss})^{-2}, \dots]'$ - implies
 $\sum_{t=0}^{\infty} (1 + r_{ss})^{-t} x_t = \mathbf{q}' \mathbf{x}$
- **Government:** IBC holds

$$\sum_{t=0}^{\infty} (1 + r_{ss})^{-t} (dG_t - dT_t) = 0 \Leftrightarrow \\ \mathbf{q}' (\mathbf{dG} - \mathbf{dT}) = 0$$

- **Households:** IBC holds

$$\sum_{t=0}^{\infty} (1 + r_{ss})^{-t} C_t^{hh} = (1 + r_{ss}) A_{-1} + \sum_{t=0}^{\infty} (1 + r_{ss})^{-t} Z_t \Rightarrow \\ \sum_{t=0}^{\infty} (1 + r_{ss})^{-t} M_{t,s} = \frac{1}{(1 + r)^s} \Rightarrow \mathbf{q}' \mathbf{M} = \mathbf{q}' \Leftrightarrow \mathbf{q}' (\mathbf{I} - \mathbf{M}) = \mathbf{0}$$

- Present value of \mathbf{M} columns is 1 - HHs must *eventually* spend income they receive

- $\sum_{t=0}^{\infty} \frac{MPC_{t,s}}{(1 + r_{ss})^{t-s}} = 1$

Form of unique solution

- Back to IKC:

$$(\mathbf{I} - \mathbf{M})d\mathbf{Y} = d\mathbf{G} - \mathbf{M}d\mathbf{T}$$

- **Problem:** $(\mathbf{I} - \mathbf{M})^{-1}$ cannot exist because this leads to a contradiction

$$\mathbf{q}'(\mathbf{I} - \mathbf{M})(\mathbf{I} - \mathbf{M})^{-1} = \mathbf{0} (\mathbf{I} - \mathbf{M})^{-1} \Leftrightarrow \\ \mathbf{q}' = \mathbf{0}$$

- **Result:** If unique solution then on the form

$$d\mathbf{Y} = \mathcal{M}(d\mathbf{G} - \mathbf{M}d\mathbf{T})$$

$$\mathcal{M} = (\mathbf{K}(\mathbf{I} - \mathbf{M}))^{-1} \mathbf{K}$$

- **Note:** This is only an issue in infinite horizon

- When solving numerically we truncate at horizon T , implying that columns in \mathbf{M} do not exactly add to 1
- Can then invert $(\mathbf{I} - \mathbf{M})$ (but precision becomes worse as horizon T increases)

Response of consumption

$$\begin{aligned} d\mathbf{Y} = d\mathbf{G} + \mathbf{M}(d\mathbf{Y} - d\mathbf{T}) &\Leftrightarrow \\ d\mathbf{Y} - d\mathbf{G} = \mathbf{M}(d\mathbf{G} - d\mathbf{T}) + \mathbf{M}(d\mathbf{Y} - d\mathbf{G}) &\Leftrightarrow \\ (\mathbf{I} - \mathbf{M})(d\mathbf{Y} - d\mathbf{G}) = \mathbf{M}(d\mathbf{G} - d\mathbf{T}) &\Leftrightarrow \\ d\mathbf{Y} - d\mathbf{G} = \mathcal{M}\mathbf{M}(d\mathbf{G} - d\mathbf{T}) &\Leftrightarrow \\ d\mathbf{C} = \mathcal{M}\mathbf{M}(d\mathbf{G} - d\mathbf{T}) \end{aligned}$$

Fiscal multipliers

$$dY = dG + \underbrace{MM(dG - dT)}_{dC}$$

- **Balanced budget multiplier:**

$$dG = dT \Rightarrow dY = dG, dC = 0$$

Note: Central that income and taxes affect household income proportionally in exactly the same way = no redistribution

- **Deficit multiplier:** $dG \neq dT$

1. Potentially fiscal multiplier **above 1**
2. Larger effect of dG than dT
3. *Numerical results needed*

Fiscal multiplier

Impact-multiplier:

$$\frac{\partial Y_0}{\partial G_0}$$

Cumulative-multiplier:

$$\frac{\sum_{t=0}^{\infty} (1 + r_{ss})^{-t} dY_t}{\sum_{t=0}^{\infty} (1 + r_{ss})^{-t} dG_t}$$

Comparison with RA model

- From lecture 1: $\beta(1 + r_{ss}) = 1$ implies

$$C_t = (1 - \beta) \sum_{s=0}^{\infty} \beta^s Y_{t+s}^{hh} + r_{ss} a_{-1}$$

- The iMPC-matrix becomes (**1** is a square matrix of 1's)

$$\mathcal{M}^{RA} = \begin{bmatrix} (1 - \beta) & (1 - \beta)\beta & (1 - \beta)\beta^2 & \dots \\ (1 - \beta) & (1 - \beta)\beta & (1 - \beta)\beta^2 & \dots \\ (1 - \beta) & (1 - \beta)\beta & (1 - \beta)\beta^2 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} = (1 - \beta)\mathbf{1}\mathbf{q}'$$

- Consumption response is zero

$$\begin{aligned} d\mathbf{C}^{RA} &= \mathcal{M}\mathcal{M}^{RA}(d\mathbf{G} - d\mathbf{T}) \\ &= \mathcal{M}(1 - \beta)\mathbf{1}\mathbf{q}'(d\mathbf{G} - d\mathbf{T}) \\ &= \mathbf{0} \Leftrightarrow d\mathbf{Y} = d\mathbf{G} \end{aligned}$$

- Fiscal multiplier is 1

- Note: Lower when real rate responds

Details on matrix formulation

$$(1-\beta)\mathbf{1}\mathbf{q}' = \begin{bmatrix} (1-\beta) & (1-\beta) & (1-\beta) & \cdots \\ (1-\beta) & (1-\beta) & (1-\beta) & \cdots \\ (1-\beta) & (1-\beta) & (1-\beta) & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} 1 & (1+r_{ss})^{-1} & (1+r_{ss})^{-2} & \cdots \end{bmatrix}$$
$$= \begin{bmatrix} (1-\beta) & (1-\beta) & (1-\beta) & \cdots \\ (1-\beta) & (1-\beta) & (1-\beta) & \cdots \\ (1-\beta) & (1-\beta) & (1-\beta) & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} 1 & \beta & \beta^2 & \cdots \end{bmatrix}$$
$$= \begin{bmatrix} (1-\beta) & (1-\beta)\beta & (1-\beta)\beta^2 & \cdots \\ (1-\beta) & (1-\beta)\beta & (1-\beta)\beta^2 & \cdots \\ (1-\beta) & (1-\beta)\beta & (1-\beta)\beta^2 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

- **RANK** cannot produce fiscal multipliers above 1 because it features ricardian equivalence
- Large litterature on Two Agent New Keyniesan models (TANK) starting with Campbell and Mankiw (1989)
 - Share $1 - \lambda$ is unconstrained; Always on Euler (Ricardian, permanent income HHs, optimizing HHs)
 - Share λ is constrained (no savings) and consume entire income each period, $MPC = 1$ (hand to mouth)
- M matrix is:

$$M^{TA} = (1 - \lambda) M^{RA} + \lambda I$$

- Simple to implement+tractable, but some drawbacks
 - No intertemporal MPCs
 - Extremely stylized level of inequality
 - Hard to connect to micro data
 - No precautionary saving

Comparison with TANK model

- **Hand-to-Mouth (HtM) households:** λ share have $C_t = Y_t^{hh}$

$$\mathbf{M}^{TA} = (1 - \lambda)\mathbf{M}^{RA} + \lambda\mathbf{I}$$

- **Intertemporal Keynesian Cross** becomes

$$d\mathbf{Y} = d\mathbf{G} + \mathbf{M}^{TA}(d\mathbf{Y} - d\mathbf{T})$$

$$(\mathbf{I} - \mathbf{M}^{TA})d\mathbf{Y} = d\mathbf{G} - \mathbf{M}^{TA}d\mathbf{T}$$

$$(\mathbf{I} - \mathbf{M}^{RA})d\mathbf{Y} = \frac{1}{1 - \lambda} [d\mathbf{G} - \lambda d\mathbf{T}] - \mathbf{M}^{RA}d\mathbf{T}$$

- **Solution:**

$$d\mathbf{Y} = d\mathbf{G} + \frac{\lambda}{1 - \lambda} [d\mathbf{G} - d\mathbf{T}]$$

- **Amplification** of fiscal policy with deficit financing $d\mathbf{G} > d\mathbf{T}$

- Size of amplification increasing in share of constrained agents λ
- Solution **very** similar to static, old Keynesian cross (multiplier:

$$\frac{\text{mpc}}{1 - \text{mpc}}$$

TANK Proof

- In TANK we have:

$$\begin{aligned} d\mathbf{Y} &= d\mathbf{G} + \mathbf{M}^{TA}(d\mathbf{Y} - d\mathbf{T}) \\ (\mathbf{I} - \mathbf{M}^{TA})d\mathbf{Y} &= d\mathbf{G} - \mathbf{M}^{TA}d\mathbf{T} \\ (\mathbf{I} - \mathbf{M}^{RA})d\mathbf{Y} &= \frac{1}{1-\lambda} [d\mathbf{G} - \lambda d\mathbf{T}] - \mathbf{M}^{RA}d\mathbf{T} \\ d\mathbf{Y} &= \mathbf{M}^{RA}d\mathbf{Y} - \mathbf{M}^{RA}d\mathbf{T} + \frac{1}{1-\lambda} [d\mathbf{G} - \lambda d\mathbf{T}] \\ d\mathbf{Y} &= d\tilde{\mathbf{G}} + \mathbf{M}^{RA}(d\mathbf{Y} - d\mathbf{T}) \end{aligned}$$

- where $d\tilde{\mathbf{G}} = \frac{1}{1-\lambda} [d\mathbf{G} - \lambda d\mathbf{T}]$
- Recall that in RANK $d\mathbf{Y} = d\mathbf{G} + \mathbf{M}^{RA}(d\mathbf{Y} - d\mathbf{T})$ with solution $d\mathbf{Y} = d\mathbf{G}$ thus implying $d\mathbf{Y} = d\tilde{\mathbf{G}}$ in TANK:

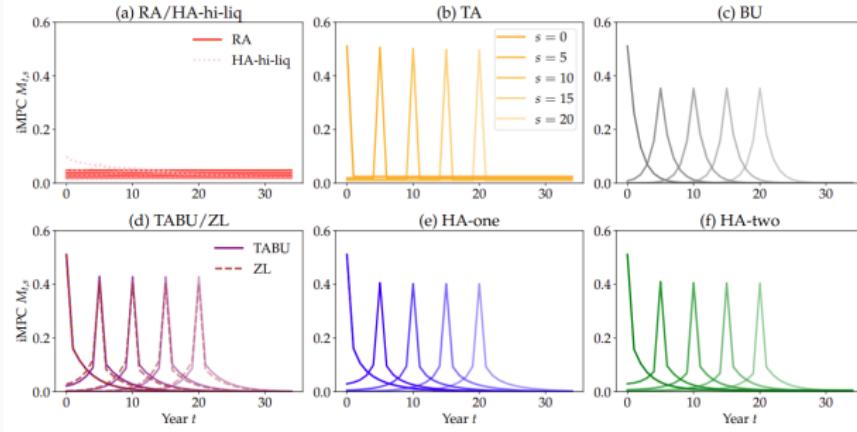
$$\begin{aligned} d\mathbf{Y} &= \frac{1}{1-\lambda} [d\mathbf{G} - \lambda d\mathbf{T}] \\ d\mathbf{Y} &= d\mathbf{G} - d\mathbf{G} + \frac{1}{1-\lambda} [d\mathbf{G} - \lambda d\mathbf{T}] \\ d\mathbf{Y} &= d\mathbf{G} + \frac{\lambda}{1-\lambda} [d\mathbf{G} - \lambda d\mathbf{T}] \end{aligned}$$

Cumulative multiplier still one

$$\frac{q' d Y}{q' d G} = \frac{q' d G_t + \frac{\lambda}{1-\lambda} q' [d G - d T]}{q' d G}$$
$$= 1$$

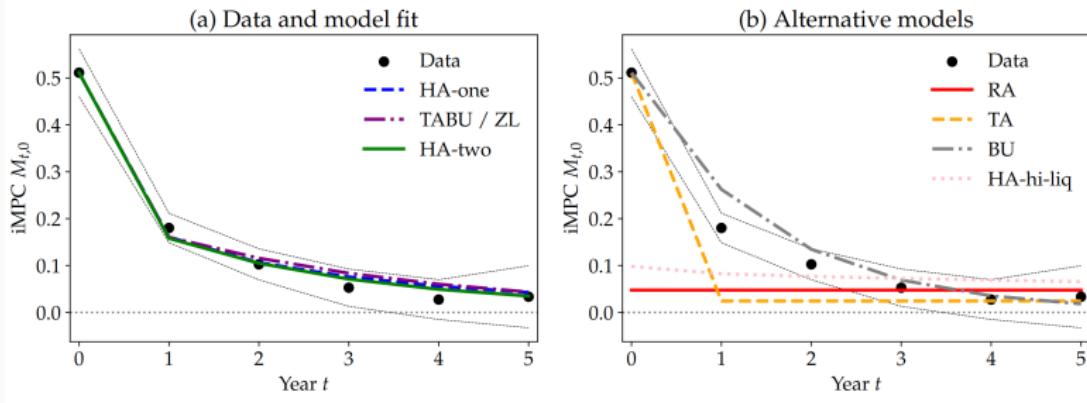
Jacobian columns

- TANK produces positive C response, fiscal multiplier above 1 - do we need HANK?
- Plot columns of \mathbf{M} in TANK, HANK + other models
 - Recall columns: dynamic C response to change in Z at various dates



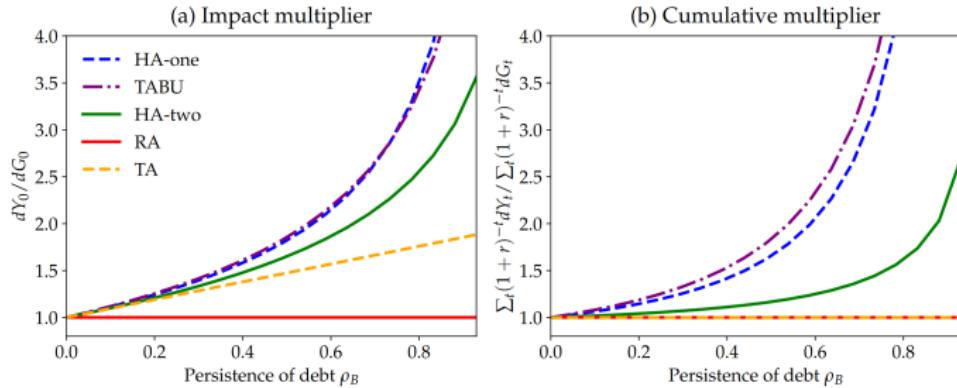
iMPCs in models

Figure 2: iMPCs in the Norwegian data and several models



Multipliers and debt-financing

Figure 5: Multipliers according to the IKC



Note. These figures assume a persistence of government spending equal to $\rho_G = 0.76$, and vary ρ_B in $dB_t = \rho_B(dB_{t-1} + dG_t)$. See section 7.1 for details on calibration choices.

Summary in table

- Summary:

Table 1: Government spending multipliers in the intertemporal Keynesian cross

Fiscal rule	Multiplier	Rep. agent (RA) doesn't match MPC	Two agents (TA) matches MPC	Het. agents (HA) matches iMPCs
balanced budget	impact	1	1	1
	cumulative	1	1	1
deficit financing	impact	1	> 1	> 1
	cumulative	1	1	> 1

Interest rate effects

- We assumed real bonds for tractability - In reality, bonds are typically **nominal**:

-

$$(1 + r_t) B_{t-1} \quad \text{versus} \quad \frac{1 + i_{t-1}}{1 + \pi_t} B_{t-1}$$

- **Fisher:** $(1 + r_t) = \frac{1 + i_{t-1}}{1 + \pi_t} \approx r_t = i_{t-1} - \pi_t$
- Even if CB keeps ex-ante real rate constant $i_t = i_{ss} + \pi_{t+1}$ real returns on bonds will differ in period 0:
- Period 0:

$$\frac{1 + i_{ss}}{1 + \pi_0} B_{ss}$$

- Period 1:

$$\frac{1 + i_0}{1 + \pi_1} B_0 = (1 + r) B_0$$

- With nominal bonds **surprise inflation** affects returns - implies capital for households in period 0
 - Positive fiscal shock generates inflation, so **negative effect on return**

Generalized IKC

- **Budget constraint** can be written with initial capital gain

$$a_t + c_t = (Y_t - T_t)z_t + \chi_t + \begin{cases} (1 + r_{t-1}^a)a_{t-1} & \text{if } t > 0 \\ (1 + r_{ss} + \text{cap}_0)a_{t-1} & \text{if } t = 0 \end{cases}$$

1. Real bond: $\text{cap}_0 = 0$
2. Nominal bond:

$$\text{cap}_0 = \frac{(1 + r_{ss})(1 + \pi_{ss})}{1 + \pi_0} - (1 + r_{ss})$$

Generalized IKC

- Consumption-function $\mathbf{C}^{hh} = C^{hh}(r, \mathbf{Y} - \mathbf{T}, \text{cap}_0)$ implies

$$d\mathbf{C}^{hh} = \mathbf{M}^r dr + \mathbf{M}(d\mathbf{Y} - d\mathbf{T}) + \mathbf{m}^{\text{cap}} \text{cap}_0$$

where

$$\mathbf{M}_{t,s}^r = \left[\frac{\partial C_t^{hh}}{\partial r_s} \right], \quad \mathbf{m}_t^{\text{cap}} = \left[\frac{\partial C_t^{hh}}{\partial \text{cap}_0} \right]$$

- Capital return effect is negative - can this overturn fiscal multiplier > 1 ?
 - No - entries in \mathbf{M} are large
 - Entries in \mathbf{m}^{cap} are **small**
- MPC out of capital gains approximatly 1-4% per year

Fiscal policy in HANK - litterature

- Seminal paper: **Intertemporal Keynesian Cross**
- **Many** other interesting papers:
- McKay and Reis - *The role of automatic stabilizers in the US business cycle* (2016)
 - Analyze the role of automatic stabilizers in a HANK model
- Bayer, Born, Luetticke - *The liquidity channel of fiscal policy* (2023)
 - Role of liquid and illiquid effects of fiscal policy
- Hagedorn, Manovskii, and Mitman - *The fiscal multiplier* (2019)
 - Systematic evaluation of multiplier in HANK + ZLB
- Druedahl, Ravn, Sunder-Plassmann, Sundram, & Waldstrøm - *Fiscal Multipliers in Small Open Economies With Heterogeneous Households* (2024)
 - Generalize to small open economies

Exercise

Exercise

Consider the standard HANK model outlined in section 2

1. Compute and plot selected columns of the household Jacobian of C w.r.t Z, r, χ

Hint: use `model._compute_jac_hh()`to compute the jacobians. You can find the results in `model.jac_hh`

- 1.1 Are the MPCs out of labor income Z and a transfer χ different?
why?
2. Compute IRFs to a deficit financed ($\omega = 0.1$) and tax financed (ω large) fiscal spending shock. Compare the responses.
3. Compute the output IRF $d\mathbf{Y}$ using the formula
$$d\mathbf{Y} = d\mathbf{G} + \mathcal{M}\mathcal{M}[d\mathbf{G} - d\mathbf{T}] \text{ where } \mathcal{M} = (\mathbf{I} - \mathbf{M})^{-1}$$
. Check that you get the same as when using `model.find_IRFs()`
4. Redo Q2 with active monetary policy, $\phi_\pi = 1.5$. How does the fiscal multiplier change?
5. Redo Q2 with active monetary policy *and* a flatter NKWPC, $\kappa = 0.01$. How does the fiscal multiplier change?

Summary

Summary and next week

- **Today:** Fiscal policy in a HANK model with sticky wages
- **Next week:** Assignment workshop
- **Homework:**
 1. Work on exercise
 2. Work on assignment