7. Secular Stagnation

Adv. Macro: Heterogenous Agent Models

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Introduction

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    2. What will happen going forward?
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- **Central economic questions:**
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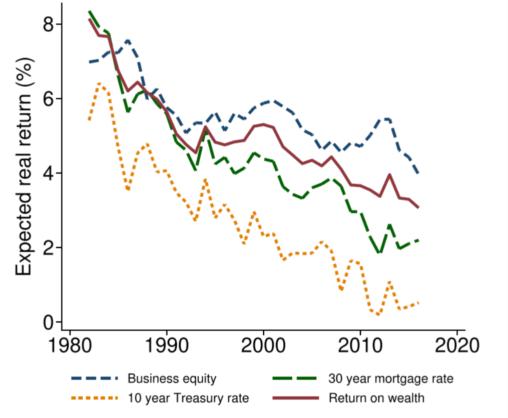
- **Plan for today:** Discuss two possible explanations for observed secular stagnation:
  1. Population aging
  2. Increase in income inequality

Secular Stagnation

- **Secular stagnation**: A state in which private demand is structurally low
  - Low level of growth in the economy
  - Low interest rates
  - Low level of inflation
- Large litterature suggests that advanced economies have been in this state over the past ≈ 20 years

Declining interest rates

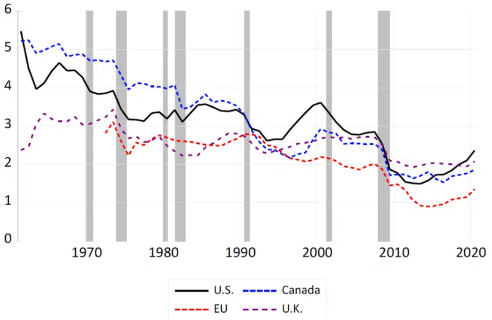
Various interest rates from Mian, Sufi, Straub (2021)



Declining growth

Trend Real Output Growth, 1961 to 2020

Percent



Source: www.cbo.gov/system/files/2020-12/56891-real%20-interest-rates.pdf

Data sources: Holston et al. (2017) and Federal Reserve Bank of New York, "Measuring the Natural Rate of Interest" (accessed December 10, 2020), www.newyorkfed.org/research/policy/rstar.

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Other potential culprit: rising markups.
Implications for fiscal and monetary policy? Low r means more likely to hit ZLB and fiscal stimulus cheap and powerful.

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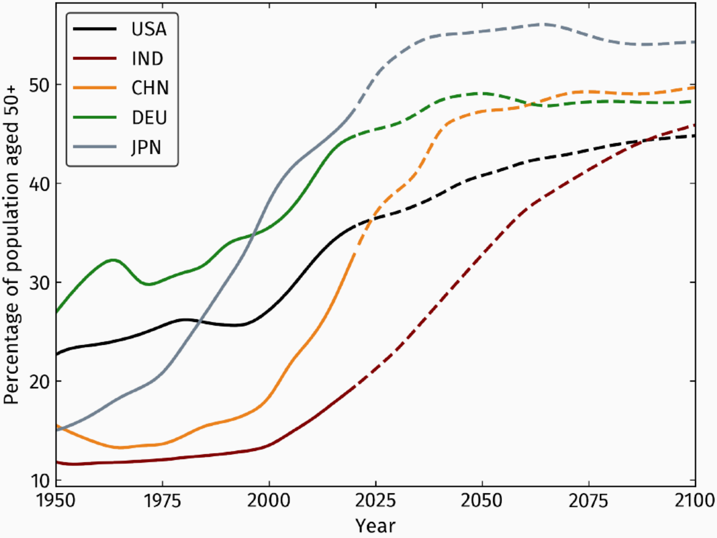
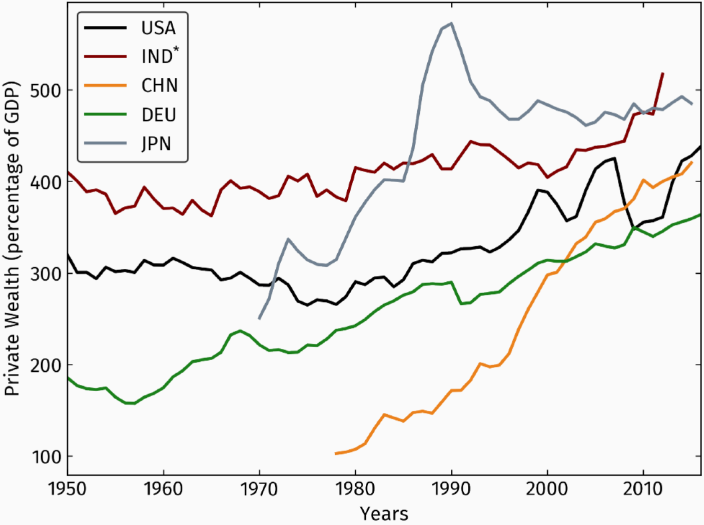
- **Increasing income inequality**
  - Straub (2019), Mian, Sufi & Straub (2021)

Demographics

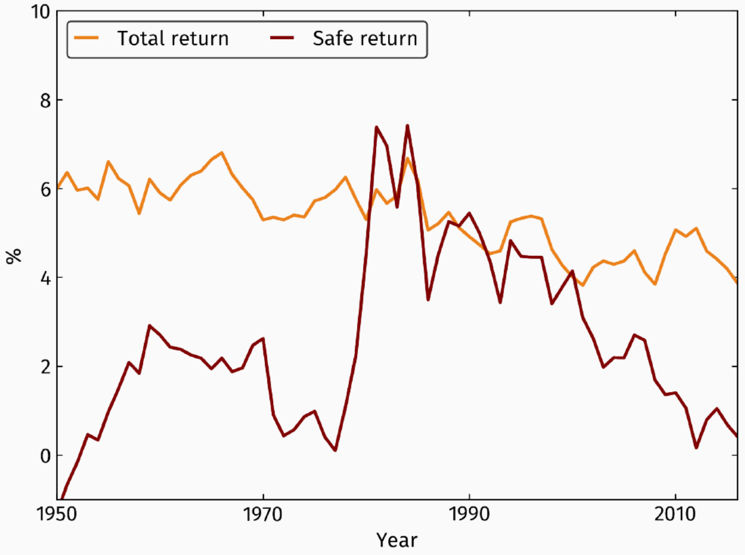
The world population is aging



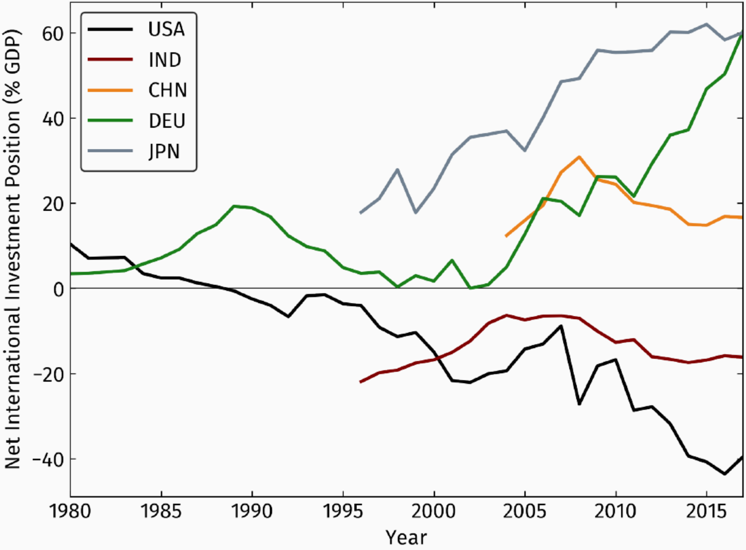
...wealth-to-GDP ratios are increasing...



...rates of return on wealth are falling...



…and “global imbalances” are rising



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- Older population saves more, unevenly across countries
- Much less agreement about how much: Δr for 1970-2015 is

  $> -100\text{bp}$ in Gagnon-Johannsen-Lopez-Salido 2021

  $< -30\text{bp}$ in Eggertsson-Mehrotra-Robbins 2019

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  - "asset market meltdown" hypothesis [Poterba 2001]
    - If large elderly cohort wishes to sell assets to younger, smaller cohort asset prices drop, rates increase
  - "great demographic reversal" hypothesis [Goodhart-Pradhan 2020]
    - Demographic shift may raise rates going forward
    - (**Less**): Young who consume and produce (i.e. supply labor)
    - (**More**): Old who only consume
    - $\Rightarrow$ Increase in demand with lower supply: inflationary

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  - Demographic change alters demand for assets in each country
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- Big challenge: how to take this model to the data to discipline the importance of demographics?

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- Second, they use this framework to measure the importance of demographic change
- Admittedly, this approach requires a lot of simplifying assumptions. The authors solve and simulate the full model and show that it gives similar results

Main results

- The authors reject the “great demographic reversal” hypothesis
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- The authors reject the "great demographic reversal" hypothesis
  - Do not find that aging will decrease savings and increase interest rates
  - Instead, it appears the global savings glut has just begun
- In addition, the authors refute the "asset market meltdown" hypothesis
  - Will dissaving of the old reverse the effects of demographics?
  - Yes, slightly. But it does not cause r to increase
  - As a result, no asset market meltdown

Model

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- Government
  - Flow budget constraint

$$G_t + w_t \sum_{j=0}^{T} N_{jt} \mathbb{E} tr_j + (1 + r_t) B_{t-1} = \tau w_t \sum_{j=0}^{T} N_{jt} \mathbb{E} l_j + B_t$$

  - Balance budget by changing G_t, not τ_t or tr_{jt}, to keep B_t/Y_t constant

Environment: heterogeneous agents

- Problem for heterogeneous agents of cohort k (age $j \equiv t-k$)}

$$\max \mathbb{E}_k \left[\sum_j^J \beta_j \Phi_j \frac{c_{jt}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \right],$$

$$\text{s.t } c_{jt} + a_{j,t} \leq w_t \left((1-\tau)\ell\left(z_j\right) + tr\left(z_j\right) \right) + \left(1+r_t\right) a_{j-1,t-1}$$

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- a_{jt} : savings

Equilibrium

Given demographics and policy, in an integrated world equilibrium:

- Individuals optimize
- Firms optimize
- Global asset markets clear

$$\sum_c W_t^c = \sum_c (K_t^c + B_t^c) \quad \forall t$$

where W_t^c is aggregate household wealth in country c:

$$W_t^c = \sum_{j=0}^{J} N_{jt}^c a_{jt}^c$$

Next: consider small country aging alone, with world at steady state
$\rightarrow r$ constant (will adjust later)

Compositional effects as sufficient statistics

Proposition 1

The wealth-to-GDP ratio of a small country aging alone with constant rate r and growth γ follows

$$\frac{W_t}{Y_t} \propto \frac{\sum_j \pi_{jt} a_{jo}}{\sum_j \pi_{jt} h_{jo}}$$

where $a_{j0} \equiv \mathbb{E} a_{j,0}$ and $h_{j0} = \mathbb{E} w_0 \ell_{j,0}$ are average initial asset holdings and pretax labor income by age, and $\pi_{jt} = N_{jt}/N_t$ is the share of the population of age j.

In a partial equilibrium world (where r does not adjust to changing demographics) then all changes in W/Y reflect the changing age composition π_{jt} of the population, given fixed profiles of asset holdings by age (a_{j0}) and income by age (h_{j0}).

Compositional effects as sufficient statistics

Based on Proposition 1, we can compute the change in log wealth to GDP ratio as follows:

$$\log\left(\frac{W_t}{Y_t}\right) - \log\left(\frac{W_o}{Y_o}\right) = \log\left(\frac{\sum_j \pi_{jt} a_{jo}}{\sum_j \pi_{jt} h_{j0}}\right) - \log\left(\frac{\sum_j \pi_{j0} a_{jo}}{\sum_j \pi_{jo} h_{j0}}\right) \equiv \Delta_t^{comp}$$

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  - **Note**: Comes from SOE **assumption** (constant r)
- Later: we'll think about how Δ_t^{comp} affects general equilibrium outcomes

Measuring compositional effects

Measuring Δ^{comp}

- Calculate Δ_t^{comp} for 25 countries:

$$\Delta_t^{comp} \equiv \log\left(\frac{\sum \pi_{jt} a_{j0}}{\sum \pi_{jt} h_{j0}}\right) - \log\left(\frac{\sum \pi_{j0} a_{jo}}{\sum \pi_{j0} h_{j0}}\right)$$

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- Data:
  - π_{jt} : projections of age distributions over individuals 2019 UN World Population Prospects
  - a_{jo}, h_{jo} age-wealth and labor income profiles in base year
    - For US: SCF, LIS/CPS, and Sabelhaus-Henriques Volz (2019)
    - a_{j0} includes funded part of DB pensions
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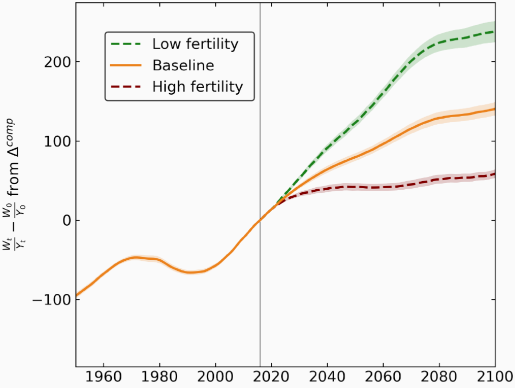
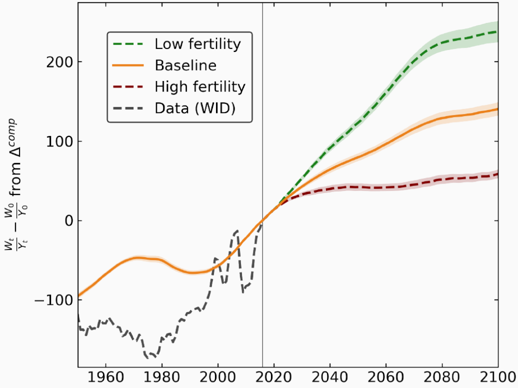
- Report implied level change $\frac{W_t}{Y_t} - \frac{W_0}{Y_0} = \frac{W_0}{Y_0}\left(\exp\left\{\Delta_t^{comp}\right\} - 1\right)$

$\triangle^{\text{comp}}$ in the United States: 1950-2100

- Composition effect implies that increase in average population age (low fertility scenario) increases W/Y



Δ^{comp} in the United States: 1950-2100



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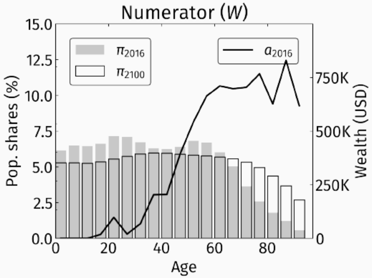

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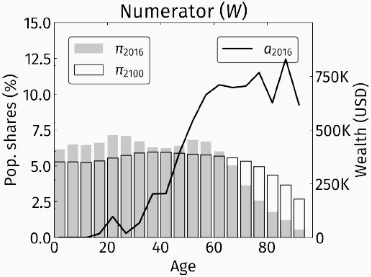

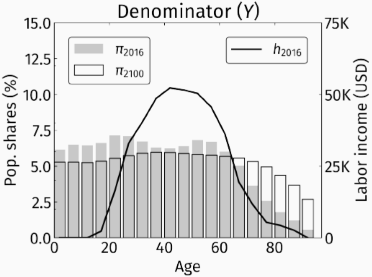

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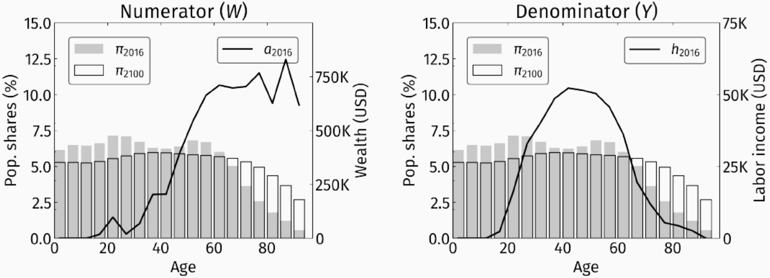

- In paper: separate contribution of numerator (wealth) and denominator (income)
  - Going forward: W contributes $\sim 2/3$, Y contributes $\sim 1/3$
  - Historically demographic dividend pushed Y up, reversed in 2010

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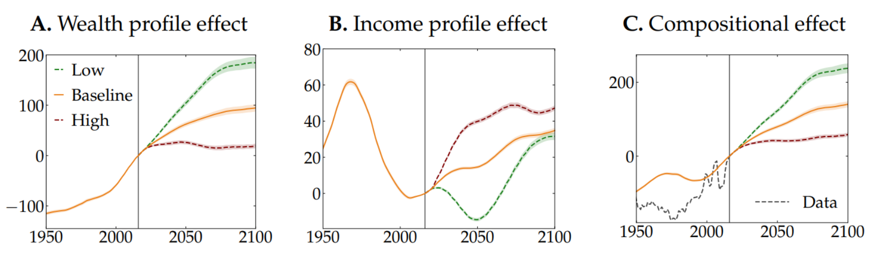

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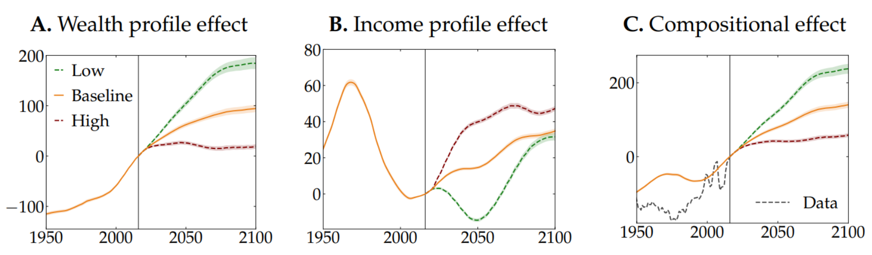
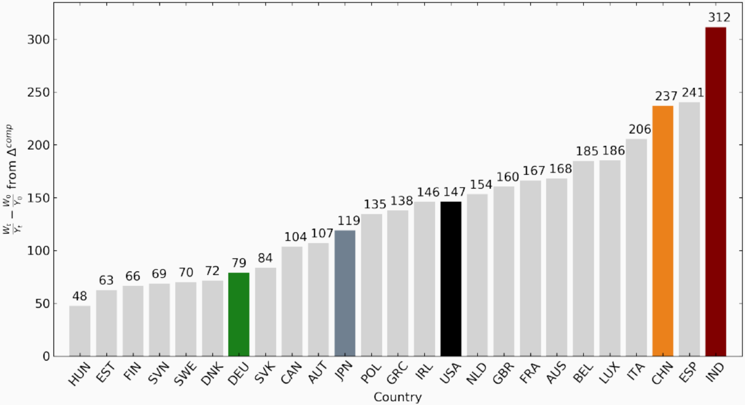


- Historically (btw. 1970 and 2010) "demographic dividend" pushed up Y, decreased W/Y as a larger share of households were at peak working age where labor income is highest
- But this effect has been less pronounced recently, as elderly households earn less

Across countries, Δ^{comp} large and heterogeneous by 2100

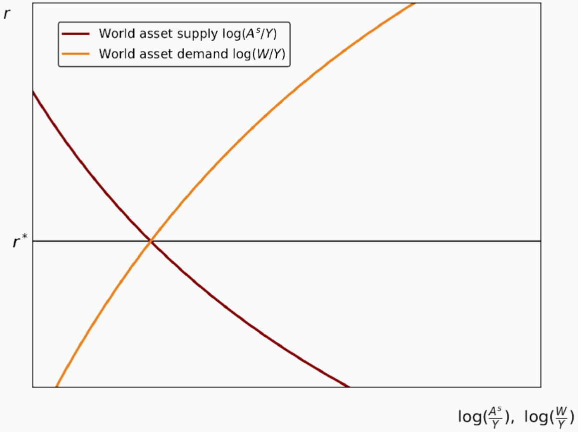


- **Note**: Uses US saving/income age profiles

General equilibrium

- So far: Change age distribution keeping
  - World interest rate r fixed
  - Household consumption/saving behavior fixed
- Now: **General equilibrium**
- Changing the age distribution will affect *supply* of wealth (demand for assets)
- Equilibrium r will depend on supply of assets (gov. bonds + firm capital) as well

General equilibrium implications



Semielasticity of asset demand $\bar{\epsilon}_d = \frac{\partial \ln(W/Y)}{\partial r}$: depends on elasticity of intertemporal substitution σ and observables (HHs)

Semielasticity of asset supply $\bar{\epsilon}_s = -\frac{\partial \ln((K+B)/Y)}{\partial r}$: depends on elasticity of substitution between labor and capital

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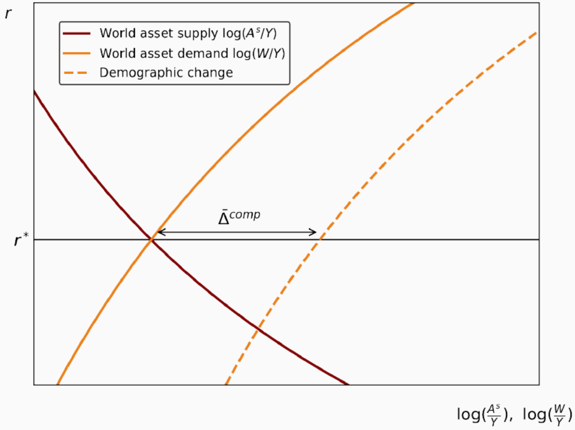
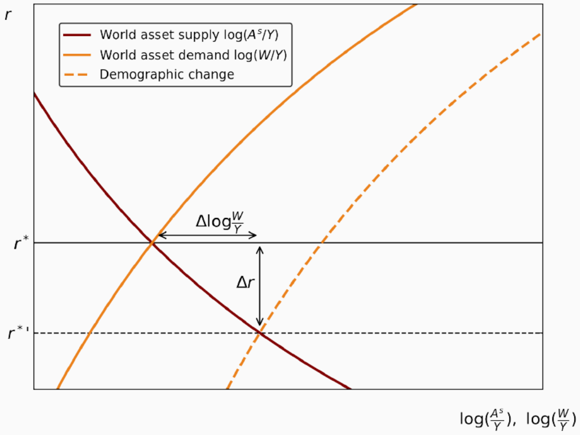


Asset demand shift of $\overline{\Delta}^{\text{comp}}$: wealth-weighted average of $\Delta^{\text{comp, c}}$

Large and positive in the data.

General equilibrium implications



General equilibrium implications

Proposition 2

If the age profiles of assets and consumption are constant, net foreign assets are zero, and governments maintain constaint debt-to-GDP ratios, then the long run change in the rate of return is:

$$\Delta r \approx -\frac{\bar{\Delta}^{\text{comp}}}{\bar{\epsilon}_S + \bar{\epsilon}_d}$$

where $\bar{\epsilon}_S$ is the average semielasticity of asset supply to r,and $\bar{\epsilon}_d$ is the average semielasticity of asset holdings to r, and $\bar{\Delta}^{\text{comp}}$ is the average compositional change.

- If asset demand/capital supply is very elastic: Small decline in r
  - HHs respond a lot to initial decline in r by saving less (crowding out direct effect $\bar{\Delta}^{\text{comp}}$), which stabilizes r in eq.
  - Firms respond by investing a lot in capital thereby driving up r in eq.

What determines the asset demand semielasticity?

$$\epsilon^d = \underbrace{\sigma \frac{C}{(1+g)W} \frac{\text{Var}\, Age_c}{1+r}}_{\equiv \epsilon^d_{\text{substitution}}} + \underbrace{\frac{\mathbb{E}Age_c - \mathbb{E}Age_a}{1+r}}_{\equiv \epsilon^d_{\text{income}}}$$

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- The above can be measured assuming fixed Age_a and Age_c
  - The authors find $\epsilon^d_{\text{substitution}} = 39.5$, $\epsilon^d_{\text{income}} = -2$, thus $\epsilon^d > 0$

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- Note: Holds for fixed gov. bond supply

Change in world interest rate

Since $\bar{\epsilon}_S + \bar{\epsilon}_d > 0$, then the change in the world interest rate must be negative:

$$\Delta r \approx -\frac{\bar{\Delta}^{\text{comp}}}{\bar{\epsilon}_S + \bar{\epsilon}_d} < 0$$

With different assumptions on the elasticity of intertemporal substitution (σ) and the elasticity of substitution between capital and labor (η), this gives:

	σ		
η	0.25	0.50	1.00
0.60	−3.24	−1.59	−0.79
1.00	−2.09	−1.25	−0.70
1.25	−1.71	−1.10	−0.65

Change in capital to income ratio

Proposition 2 gives a similar formula for the change in capital to income:

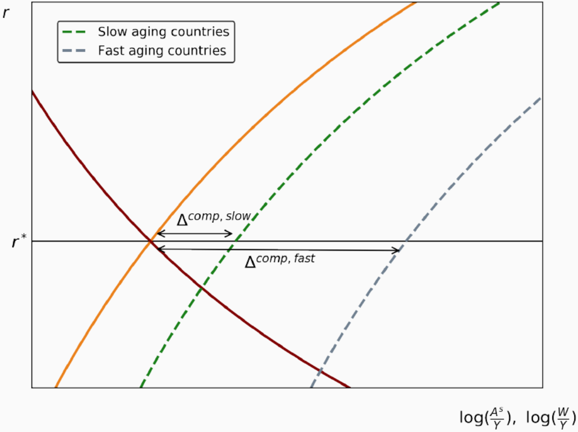
$$\overline{\Delta \log \left(\frac{W}{Y}\right)} \approx \frac{\bar{\epsilon}_{\mathrm{S}}}{\bar{\epsilon}_{\mathrm{S}} + \bar{\epsilon}_{d}} \bar{\Delta}^{\text{comp}} > 0$$

Again with different assumptions on the IES (σ) and the elasticity of substitution between capital and labor (η)

	σ		
η	0.25	0.50	1.00
0.60	15.6	7.7	3.8
1.00	16.7	10.0	5.6
1.25	17.1	11.1	6.5

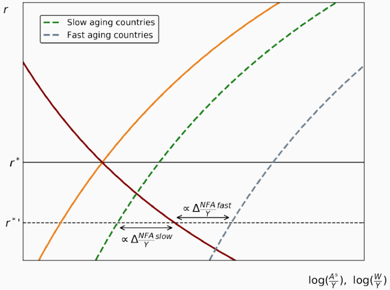
The authors argue that simulations from the general model deliver similar outcomes

Change in net foreign assets



Country specific Δ^{comp} large and heterogeneous in the data

Change in net foreign assets



- Countries that age faster will accumulate more wealth, which it supplies to the rest of the world (NFA>0)
  - Particularly to countries that age slowly (less wealth accumulation) where domestic firms need to go abroad for investments

$$\Delta\left(\frac{NFA}{Y}\right) \approx \frac{W_0}{Y_0}\left(\Delta^{\text{comp,c}} - \bar{\Delta}^{\text{comp}}\right)$$

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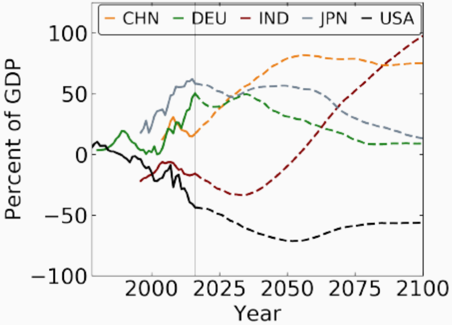

→ Data suggest large global imbalances going forward

Limitation to baseline model

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  - No changes in mortality (only birth rates)
  - Demographics have no effect on TFP growth
- To study some of these changes, the authors extend their baseline model $\rightarrow$ then simulate the transition path

Results from richer model

- Compositional effect by country from analytical model product of:
  - Combination of demographic changes (exogenous in model) and labor supply/wealth profiles (endogenous)

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	$\Delta^{comp,c}$	
Country	Model	Data
AUS	30	29
CAN	21	20
CHN	47	45
DEU	21	20
ESP	42	37
FRA	31	30
GBR	27	26
IND	65	56
ITA	34	30
JPN	24	22
NLD	34	33
USA	32	29

Results from richer model

- GE Effects from the model are also roughly similar

	Δr	$\overline{\Delta \log \frac{W}{Y}}$	$\bar{\Delta}^{comp}$	$\bar{\Delta}^{soe}$	$\bar{\epsilon}^d$	$\bar{\epsilon}^s$
Sufficient statistic analysis	-1.23	9.9	31.8		17.8	8.0
Preferred model specification	-1.23	10.3	34.1	30.3	17.1	8.0
Alternative model specifications						
+ Constant bequests	-1.18	10.0	34.1	27.0	14.9	8.0
+ Constant mortality	-1.23	10.9	34.1	27.1	13.8	8.0
+ Constant taxes and transfers	-1.33	11.9	34.1	30.1	14.5	8.0
+ Constant retirement age	-1.49	13.4	34.1	34.1	14.6	8.0
+ No income risk	-1.47	13.2	33.9	33.9	13.8	8.0
+ Annuities	-1.33	11.5	34.2	34.2	17.2	8.0
Alternative fiscal rules						
Only lower expenditures	-1.29	11.0	34.1	32.6	17.9	8.0
Only higher taxes	-0.88	6.7	34.1	19.4	14.6	8.0
Only lower benefits	-1.50	12.9	34.1	39.1	18.4	8.0

Notes: Δr, $\overline{\Delta \log \frac{W}{Y}}$, $\bar{\Delta}^{comp}$, and $\bar{\Delta}^{soe}$ denote the changes in the model simulation between 2016 and 2100, with Δr reported in percentage points and the other three reported in percent ($100 \cdot \log$).

Conclusion

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  - what matters is the compositional effect Δ^{comp}
  - large and heterogeneous in the data
- The approach developed by the authors:
  - Refutes the asset market meltdown hypothesis: r falls
  - Suggests wealth-to-income ratio will keep rising
  - Larger global imbalances (dispersion of NFAs)

Income Inequality

Straub (2019)

Secular stagnation and income inequality

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  - See i.e. Mian, Sufi, Straub (2021): *What explains the decline in r*? Rising income inequality versus demographic shifts*
- **Example**: If saving rates increase with income (richer save more) then:
  - Redistribution from poor to rich households *increase* the aggregate supply of savings $\Rightarrow$ lower rates in GE
  - Suggests link between increasing income inequality and secular stagnation
  - Straub (2019, WP) tests this hypothesis

Stylized model

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$$\max_{c_t, a_{t+1}} u(c_t) + \beta U(a_{t+1})$$

$$c_t + R^{-1} a_{t+1} \leq a_t + w_t$$

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with $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ and $U(a) = \frac{a^{1-\Sigma}}{1-\Sigma}$
We can show that $c_t \approx k(a_t + w)^{\phi}$, with $\phi = \frac{\Sigma}{\sigma}$

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- Two questions
  - What does this elasticity look like emprically?
  - If $\phi \neq 1$ how can we accomodate this in a HA model?

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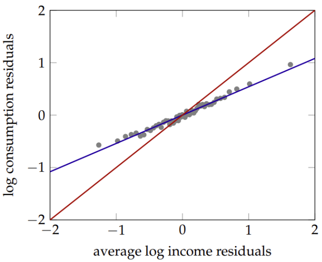
Figure 3: Consumption and average income.



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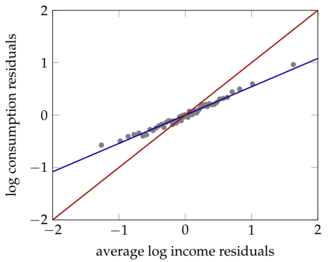

- More detalied estimates in paper, suggest $\phi \approx 0.7$

Permanent redistribution in the canonical HA model

- Consider standard HA model with permanent income state p :

$$V\left(a_{t-1}, z_t, p\right) = \max_{c_t} \frac{c_t^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}\left[V\left(a_t, z_{t+1}, p\right)\right]$$

subject to

$$c_t + a_t = Ra_{t-1} + z_t p$$

$$a_t \geq 0$$

$$\ln z_t = \rho \ln z_{t-1} + \epsilon_t$$

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- Note: preferences are *homothetic* (homogeneous of degree 1)
  - »Scale independent«

Homothetic household problem I

- Normalize constraints by p and Bellman by $p^{1-\sigma}$

$$\frac{V\left(a_{t-1}, z_t, p\right)}{p^{1-\sigma}} = \max_{c_t} \frac{\frac{c_t^{1-\sigma}}{1-\sigma}}{p^{1-\sigma}} + \beta \frac{\mathbb{E}\left[V\left(a_t, z_{t+1}, p\right)\right]}{p^{1-\sigma}}$$

$$\text{subject to}$$

$$\frac{c_t}{p} + \frac{a_t}{p} = R\frac{a_{t-1}}{p} + z_t\frac{p}{p}$$

$$\frac{a_t}{p} \geq \frac{0}{p}$$

Homothetic household problem II

Define $\tilde{c}_t = \frac{c_t}{p}$,$\tilde{a}_t = \frac{a_t}{p}$, $\tilde{V}_t = \frac{V_t}{p^{1-\sigma}}$ to get:

$$\tilde{V}\left(a_{t-1}, z_t\right) = \max_{c_t} \frac{\tilde{c}_t^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}\left[\tilde{V}\left(a_t, z_{t+1}\right)\right]$$

subject to

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Homothetic household problem II

Define $\tilde{c}_t = \frac{c_t}{p}, \tilde{a}_t = \frac{a_t}{p}, \tilde{V}_t = \frac{V_t}{p^{1-\sigma}}$ to get:

$$\tilde{V}\left(a_{t-1}, z_t\right) = \max_{c_t} \frac{\tilde{c}_t^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}\left[\tilde{V}\left(a_t, z_{t+1}\right)\right]$$

subject to

$$\tilde{c}_t + \tilde{a}_t = R\tilde{a}_{t-1} + z_t$$

$$\tilde{a}_t \geq 0$$

$$\ln z_t = \rho \ln z_{t-1} + \epsilon_t$$

Then HH problem is independent of p up to scale!

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- Increase in permanent income by 1% increases c_t, a_t by 1%
- Homothetic preferences (scale independence) imply $\phi = 1$ since $\tilde{c}_t = \frac{c_t}{p}$
- Permanent redistribution has *no effect* on aggregates because the dissavings by one group is exactly offset by increase in savings from other groups

Non-homothetic HA model

- Extend standard HA model with *taste for wealth* (»status«)

$$V\left(a_{t-1}, z_t, p\right) = \max_{c_t} \frac{c_t^{1-\sigma}}{1-\sigma} + \phi_a \frac{a_t^{1-\sigma_a}}{1-\sigma_a} + \beta \mathbb{E}\left[V\left(a_t, z_{t+1}, p\right)\right]$$

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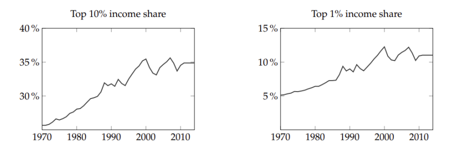

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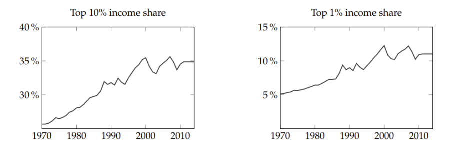
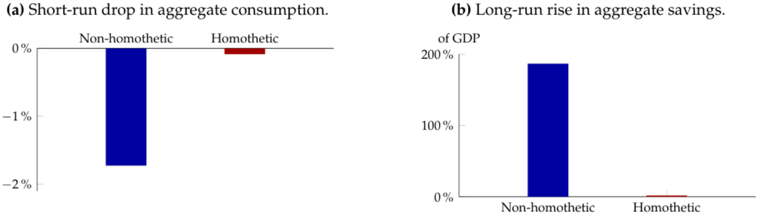


- Solve for GE (Supply side as in HANC)

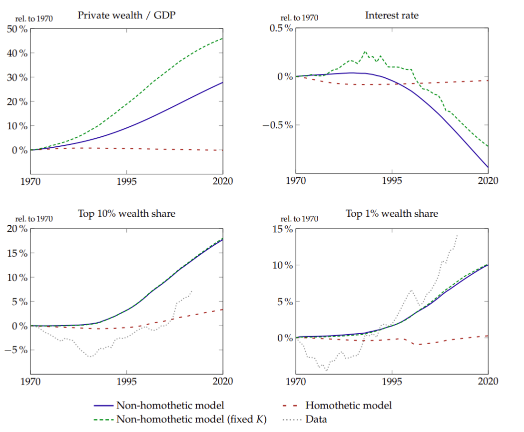
Results

Figure 6: The partial equilibrium effects of a rising income inequality.



GE implications of rising income inequality

- Compare effects in homothetic and non-homothetic models:



- Note: »Fixed K« is Lucas-tree economy, where wealth grows dueto rising asset prices

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- Increase in savings might not finance more physical capital, but just higher prices of financial assets!

Mian, Sufi, Straub (2021)
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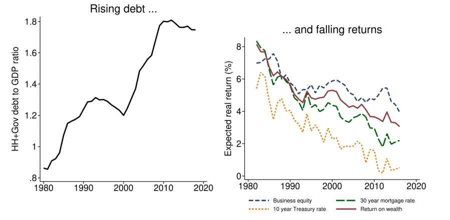

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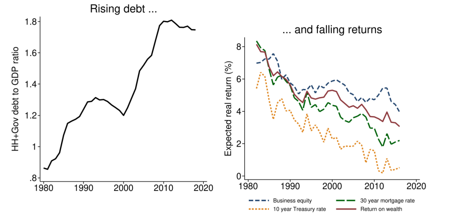

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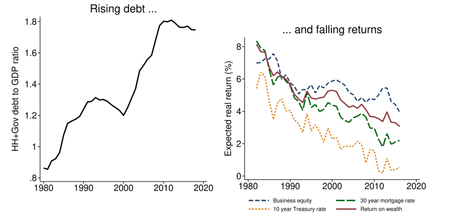

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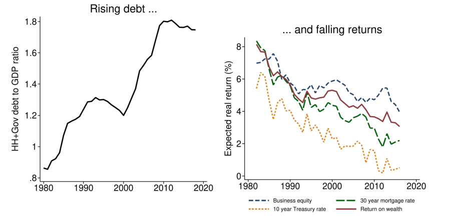

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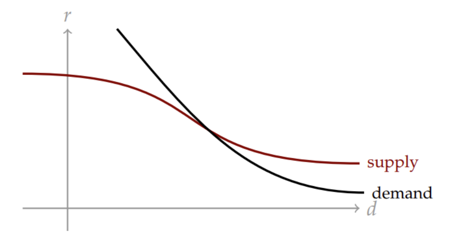

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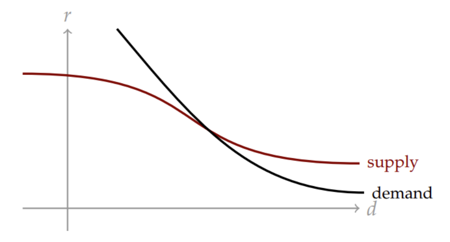

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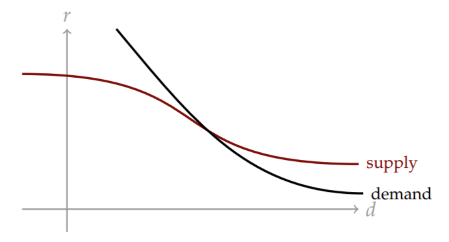

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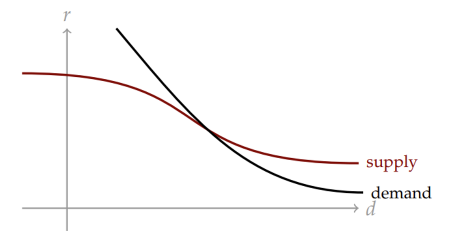

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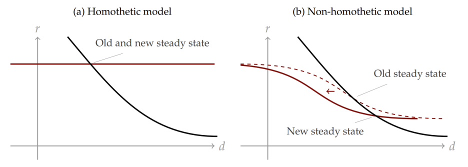

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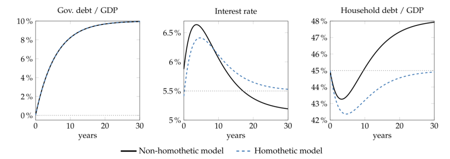

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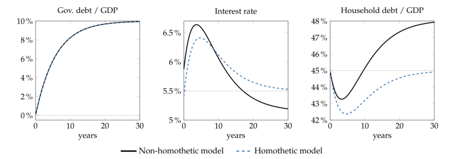

- For monetary policy: lower r means ZLB more likely, "less ammunitions"

Exercise

- Consider the following PE HA model:

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- **Q1**: Derive the Euler equation of the model
- **Q2:** Update the EGM algorithm in *household_problem.py* with the new Euler
- **Q3:** Solve the model with 1) $\phi_a = 0$, 2) $\phi_a = 0.1, \sigma = \sigma_a = 1$, 3) $\phi_a = 0.1, \sigma = 1, \sigma_a = 0.7$
  - Compare the normalized policy function c/p
- **Q4:** Conduct an experiment where you permanently redistribute resources across households (change p). What are the aggregate effects across the models?

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