Written Exam Economics winter 2024-25

Advanced Macroeconomics: Heterogeneous Agent Models

January 4 to January 5

This exam question consists of 7 pages in total. Answers only in English.

A take-home exam paper cannot exceed 10 pages – and one page is defined as 2400 keystrokes

You should hand-in a single zip-file. The zip-file should have the following folder and file structure.

Assignment_I
Assignment_I.pdf – with text and all results
files for producing the results

Assignment_II
Assignment_II.pdf – with text and all results
files for producing the results

Assignment_III
Assignment_III.pdf – with text and all results
files for producing the results

Exam
Exam.pdf – with text and all results
files for producing the results

Use of AI tools is permitted. You must explain how you have used the tools. When text is solely or mainly generated by an AI tool, the tool used must be quoted as a source.

A HANK model with Investment

In this exam set, we consider a standard HANK model from the course augmented with capital. The model description is reproduced on page 4 onwards. Code which solves the model using a baseline calibration is provided. You will <u>*not*</u> *need to due any additional coding in the blocks.py, household_problem.py, or steady_state.py files.*

Question 1) Solution method and calibration

a) What is the purpose of discount factor heterogeneity in the model?

b) Say we compute the Jacobians $\boldsymbol{H_U}, \boldsymbol{H_G}$ and solve for the linearized response to a fiscal spending shock $d\boldsymbol{G}$.[1] Do we need to recompute the Jacobians $\boldsymbol{H_U}, \boldsymbol{H_G}$ of the model if we change the parameter ϕ_π in the Taylor rule? Do we need to recompute the Jacobians of the household block?

c) Do we need to recompute the Jacobians $\boldsymbol{H_U}, \boldsymbol{H_G}$ of the model if we change the persistence of the fiscal spending shock? Explain.

Note: You don't have to actually solve for the IRFs in your answers to b) and c) in the code.

Question 2) Monetary policy

In this question we study a monetary policy shock ϵ_t^i of size $d\epsilon_0^i = -0.001$ and a quarterly persistence of 0.80. We focus on *linearized* IRFs.

a) Consider first the model where investment is completely inelastic ($\phi_K \to \infty$).[2] Compute linear IRFs to a monetary policy shock. Compare the effects in HANK and RANK, and explain how monetary policy affects output.

b) Decompose the response of aggregate consumption C in HANK from a) into direct and indirect effects. What would the decomposition look like in RANK?

[1] Here $\boldsymbol{H}$ denotes the residuals (or »targets«) of the model, and $\boldsymbol{U}$ denotes the unknowns.

[2] You may think of this as the standard HANK model without capital. In the code you can set $\phi_K = 100$ as an approximation of $\phi_K \to \infty$.

c) Consider a model with elastic capital and investment ($\phi_K = 3$), and compare the effects of a monetary policy shock in HANK and RANK. Decompose the response of consumption in HANK into direct and indirect effects. Does your results differ compared to the results in a) and b)? Explain.

d) Compute IRFs in the model with investment under *sticky expectations.* Assume that the probability that a household does *not* update their information set in a given period is $\theta = 0.95$. Compare and discuss the effectiveness of monetary policy in HANK and RANK.

Question 3) Fiscal policy

We now consider the effects of a fiscal spending shock of 1% of steady state GDP, $dG_0 = 0.01 \times Y_{ss}$. Assume that the quarterly persistence is 0.8. We still focus only on *linearized* IRFs.

a) Solve for a deficit-financed ($\omega = 0.03$) fiscal spending shock. How does the presence of investment affect the fiscal multiplier in HANK relative to RANK? Explain the role of endogenous monetary policy for your results.

b) Vary the degree of tax financing ω between $[0.03, 1]$ in the HANK model with investment. When is the cumulative government spending multiplier highest?[3]

[3] We define the cumulative multiplier as $\frac{\sum_{t=0}^{\infty}(1+r_{ss})^{-t} dY_t}{\sum_{t=0}^{\infty}(1+r_{ss})^{-t} dG_t}$.

Model

We consider a *closed* economy with heterogeneous agents, flexible prices, sticky wages and investment.

Time is discrete and indexed by t. There is a continuum of households indexed by i.

Households. Households are *ex ante* heterogeneous in terms of their discount factor β_i,and *ex post* heterogeneous in terms of their time-varying stochastic productivity, z_{it}, and their (end-of-period) savings, a_{it-1}. The distribution of households over idiosyncratic states is denoted $\underline{\boldsymbol{D}}_t$ before shocks are realized and $\boldsymbol{D}_t$ afterwards. Households supply labor, ℓ_{it} chosen by a union (implying $\ell_{it} = L_t$) and choose consumption, c_{it}, on their own. Households are not allowed to borrow. The return on savings is r_t^a, the real wage is w_t, and households recieve transfers, χ_t. We denote by $Z_t = (1-\tau_t)\, w_t L_t$ average real, post-tax income.

The household problem is

$$
\begin{aligned}
v_t(z_{it}, a_{it-1}, \beta_i) &= \max_{c_t} \frac{c_{it}^{1-\sigma}}{1-\sigma} - \varphi \frac{\ell_{it}^{1+\nu}}{1+\nu} + \beta_i \mathbb{E}_t\left[v_{t+1}(z_{it+1}, a_{it}, \beta_i)\right] \qquad (1)\\
\text{s.t. } a_{it} + c_{it} &= (1+r_t^a)a_{it-1} + Z_t z_{it} + \chi_t \\
\log z_{it+1} &= \rho_z \log z_{it} + \psi_{it+1} \ \ , \psi_{it} \sim \mathcal{N}(\mu_\psi, \sigma_\psi),\ \mathbb{E}[z_{it}] = 1 \\
a_{it} &\geq 0.
\end{aligned}
$$

where σ is the inverse elasticity of substitution, φ controls the disutility of supplying labor and ν is the inverse of the Frish elasticity. We define the average discount factor in the population as $\overline{\beta} = \int \beta_i d\boldsymbol{D}_{ss}$.

Aggregate quantities are

$$A_t^{hh} = \int a_t^*\left(z_{it}, a_{it-1}, \beta_i\right) d\boldsymbol{D}_t \qquad (2)$$

$$L_t^{hh} = \int \ell_t^*\left(z_{it}, a_{it-1}, \beta_i\right) z_{it} d\boldsymbol{D}_t \qquad (3)$$

$$C_t^{hh} = \int c_t^*\left(z_{it}, a_{it-1}, \beta_i\right) d\boldsymbol{D}_t. \qquad (4)$$

Firms. A representative firm hires labor, L_t and rent capital K_{t-1} to produce goods Y_t. The production function is

$$Y_t = \Gamma_t L_t^{1-\alpha} K_{t-1}^{\alpha}. \qquad (5)$$

where Γ_t is the exogenous technology level. Dividends are

$$D_t = Y_t - w_t L_t - I_t - f\left(\frac{K_t}{K_{t-1}}\right) K_{t-1}. \tag{6}$$

where w_t is the real wage level, I_t is investment, and $f\left(\frac{K_t}{K_{t-1}}\right) = \frac{\phi_K}{2}\left(\frac{K_t}{K_{t-1}} - 1\right)^2$ is a capital adjustment cost. Capital accumulates according to the law of motion

$$K_t = (1-\delta) K_{t-1} + I_t. \tag{7}$$

Firms have market power, and optimize subject to a standard CES demand function. The first-order conditions for labor demand and investment are

$$\frac{1}{\mu}\frac{\partial Y_t}{\partial L_t} = w_t \tag{8}$$

$$1 + f'\left(\frac{K_t}{K_{t-1}}\right) = \frac{1}{1+r_t}\left[\frac{1}{\mu}\frac{\partial Y_{t+1}}{\partial K_t} + (1-\delta) - f\left(\frac{K_{t+1}}{K_t}\right) + f'\left(\frac{K_{t+1}}{K_t}\right)\frac{K_{t+1}}{K_t}\right], \tag{9}$$

where μ is the markup and r_t is the ex-ante real interest rate. Given the real wage w_t the relationship between nominal price inflation π_t and nominal wage inflation π_t^W is

$$\pi_t = \left(\frac{w_t}{w_{t-1}}\right)\left(1 + \pi_t^W\right) - 1. \tag{10}$$

Union. A union chooses the labor supply of each household and sets wages. Each household is chosen to supply the same amount of labor,

$$\ell_{it} = L_t^{hh}. \tag{11}$$

Nominal wage adjustment costs imply a *New Keynesian Wage Philips Curve*,

$$\pi_t^w = \kappa \left(\underbrace{\varphi\left(L_t^{hh}\right)^{\nu}}_{\text{mar. disutility of labor}} - \frac{1-\tau_t}{\mu^W} \underbrace{w_t\left(C_t^{hh}\right)^{-\sigma}}_{\text{marg. utility of consumption}} \right) + \overline{\beta}\pi_{t+1}^w. \tag{12}$$

where κ is the slope parameter and μ^W is a wage mark-up.

Mutual fund. There is a mutual fund which collect total household savings A_t^{hh} and invest in either firm equity or real government bonds.

There exists a continuum of shares v_{jt} in firms with price p_{jt}^D that each pays a dividend D_{jt}. Shares bought must then sum to one. Government bonds pay the real interest rate r_t each period. The flow-of-funds constraint at the beginning of the period states that the value of liabilities must be equal to the liquidation value of the intermediaries portfolio

$$(1+r_t^a)A_{t-1}^{hh} = \frac{1+i_{t-1}}{1+\pi_t}B_{t-1} + \int (p_{jt}^D + D_{jt})v_{jt-1}dj.$$

At the end of the period, the new investment in bonds and shares must equal the intermediaries liabilities that equal aggregate savings (end-of-period flow of funds constraint)

$$\int p_{jt}^D v_{jt} dj + B_t = A_t^{hh}.$$

The financial intermediary maximizes the return $\mathbb{E}\left[1+r_{t+1}^a\right]$ for the households by choosing v_{jt}, B_t through adjusting the portfolio composition. The asset pricing equation states that the expected return on the different assets has to be equalized such that all arbitrage opportunities are exhausted

$$\mathbb{E}_t[1+r_{t+1}^a] = 1+r_t = \frac{\mathbb{E}_t[p_{jt+1}^D + D_{jt+1}]}{p_{jt}^D} = \frac{1+i_t}{1+\pi_{t+1}}. \tag{13}$$

where r_t is the ex-ante real interest rate.

Central bank. The central bank follows a standard Taylor rule with persistence,

$$i_t = i_{ss} + \phi_\pi \pi_t + \epsilon_t^i, \tag{14}$$

where i_t is the nominal return from period t to period $t+1$, ϕ_π is the Taylor coefficient, and ϵ_t^i is a monetary policy shock.

Government. The government chooses spending, G_t, transfers, χ_t, and the labor income tax rate, τ_t. The total tax bill is

$$\mathcal{T}_t \equiv \tau_t w_t L_t^{hh}. \tag{15}$$

The government can finance its expenses with bonds, B_t. The budget constraint for the government then is

$$B_t = \frac{1+i_{t-1}}{1+\pi_t} B_{t-1} + G_t + \chi_t - \mathcal{T}_t. \tag{16}$$

Spending, G_t is chosen exogenously. Transfers are assumed to be zero, $\chi_t = 0$. Total taxes follows the rule

$$\mathcal{T}_t = \mathcal{T}_{ss} + \omega \frac{(B_{t-1} - B_{ss}) + (G_t - G_{ss})}{Y_{ss}}. \tag{17}$$

where ω controls the sensitivity of the taxes to public debt and expenses.

Market clearing. Market clearing implies

1. Asset market: $B_t + p_t^D = A_t^{hh}$
2. Labor market: $L_t = L_t^{hh}$
3. Goods market: $Y_t = C_t^{hh} + I_t + G_t + f\left(\frac{K_t}{K_{t-1}}\right) K_{t-1}$

Calibration

The parameters and steady state government behavior are as follows

1. **Preferences and abilities:** $\sigma = 1.0$, $\nu = 1.0$
2. **Income:** $\rho_z = 0.95$, $\sigma_\psi = 0.10$
3. **Production:** $\delta = 0.02, \mu = 1.01$
4. **Union:** $\kappa = 0.01$, $\mu^W = 1.01$
5. **Central bank:** $r_{ss} = 1.02^{\frac{1}{4}} - 1$, $\phi^\pi = 1.25$
6. **Government:** $G_{ss} = 0.1$, $\chi_{ss} = 0$, $B_{ss} = 0.5$, $\omega = 0.03$

We assume that the discount factors β_i takes two values in the population, $\beta_i \in \{\beta_{low}, \beta_{high}\}$. We let $\beta_{low}, \beta_{high}, \Gamma_{ss}$, α and τ_{ss} be unspecified and adjust those to obtain the steady state we want.