1. Introduction

Adv. Macro: Heterogenous Agent Models

Jeppe Druedahl & Raphaël Huleux

2025

Plan

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- **Central technical method:** Programming in Python
  Prerequisite: *Intro. to Programming and Numerical Analysis*
  Complicated: *Close to the research frontier*

Macroeconomic Models with Heterogeneous Agents

- **Model components:**
  1. Optimizing individual agents (households + firms)
  2. Idiosyncratic and aggregate risk
  3. Information flows (who knows what when $\Rightarrow$ often everything)
  4. Market clearing (Walras vs. search-and-match)

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- **HANK:** Heterogeneous Agent *New Keynesian* model
  (i.e. include price and wage setting frictions)

History of heterogeneous agent macro

1. Heathcote et al. (2009), »Quantitative Macroeconomics with Heterogeneous Households«
2. Kaplan and Violante (2018), »Microeconomic Heterogeneity and Macroeconomic Shocks«
3. Cherrier et al. (2023), »Household Heterogeneity in Macroeconomic Models: A Historical Perspective«
4. Auclert et. al. (2025), »Fiscal and Monetary Policy with Heterogeneous Agents«

Structure

Teaching method

- **Lectures:** Tuesday 12-15
  $\sim$2 hours of »normal« lecture
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- **Material:**

  Web: sites.google.com/view/numeconcph-advmacrohet/

  Git: github.com/numeconcopenhagen/adv-macro-het

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- **Code:**

  1. We provide code you will build upon
  2. Based on the GEModelTools package

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- **Exam:**
  1. Hand-in 3×**assignments**
  2. **36 hour take-home:** Programming of new extension
     + analysis of model + interpretation of results

Python

1. **Assumed knowledge:** From Introduction to Programming and Numerical Analysis you are assumed to know the basics of
   1.1 Python
   1.2 VSCode
   1.3 git
2. **Updated Python:** Install (or re-install) newest Anaconda
3. **Packages:** `pip install quantecon, EconModel, consav`
4. **GEMoodel tools:**
   4.1 Clone the `GEModelTools` repository
   4.2 Locate repository in command prompt
   4.3 Run `pip install -e .`

Course plan

- **Lecture 1-2:** Consumption-saving models
- **Lecture 3-6:** General equilibrium model (stationary equilibrium + transition path)
- **Lecture 7:** Wealth inequality
- **Lecture 8-13:** HANK
- **Lecture 14:** Exam and perspectives

Learning goals

Knowledge

1. Account for, formulate and interpret precautionary saving models
2. Account for stochastic and non-stochastic simulation methods
3. Account for, formulate and interpret general equilibrium models with ex ante and ex post heterogeneity, idiosyncratic and aggregate risk, and with and without pricing frictions
4. Discuss the difference between the stationary equilibrium, the transition path and the dynamic equilibrium
5. Discuss the relationship between various equilibrium concepts and their solution methods
6. Identify and account for methods for analyzing the dynamic distributional effects of long-run policy (e.g. taxation and social security) and short-run policy (e.g. monetary and fiscal policy)

Skills

1. Solve precautionary saving problems with dynamic programming and simulate behavior with stochastic and non-stochastic techniques
2. Solve general equilibrium models with ex ante and ex post heterogeneity, idiosyncratic and aggregate risk, and with and without pricing frictions (stationary equilibrium, transition path, dynamic equilibrium)
3. Analyze dynamics of income and wealth inequality
4. Analyze transitional and permanent structural changes (e.g. inequality trends and the long-run decline in the interest rate)
5. Analyze the dynamic distributional effects of long-run policy (e.g. taxation and social security) and short-run policy (e.g. monetary and fiscal policy)

Competencies

1. Independently formulate, discuss and assess research on both the causes and effects of heterogeneity and risk for both long-run and short-run outcomes
2. Discuss and assess the importance of how heterogeneity and risk is modeled for questions about both long-run and short-run dynamics

Programming in Python

Classes

- A **class** defines the **type** of an object
  - `.attribute`, state
  - `.method()`, action (incl. changing self)
- **Inheritance** (of methods) (`class Child(Parent)`)

```python
class Parent:
    def __init__(self,value): self.value = value
    def double(self): return self.value * 2

class Child(Parent):
    def half(self): return self.value / 2

child = Child(10)
print(child.value) # 10
print(child.double()) # 20
print(child.half()) # 5.0
```

References

- **Variables are references to an instance of an object**
- = **assigns a reference** (*not a copy!*)

Question: What does a end up as? What if a = [1,2,3]?

```
a = np.array([1,2,3])
b = a
c = a[1:] # slicing
b[0] = 3 # indexing
c[0] = 3
```

Types and in-place operations

- **Atomic types:** `int`, `float`, `str`, `bool`, etc.
- **Containers** `list`, `tuple`, `dict`, `set`, `np.array`, etc.
- **Mutables** (e.g. `list`, `np.array`) can change in-place
  1. **In-place operators** (+=, -= etc.)
  2. **Slicing**: `x[:] = x + y`
- **Immutables** (e.g. atomic types and tuples) can never change

Questions: What does y end up as?

```
x = np.array([1,2,3])
y = x
x += 1
x[:] = x + 1
x = x + 1
```

Functions and scope

- **Functions are objects** (can e.g. be arguments in functions)
  Unlike in math:
  1. Can change its arguments (side-effects)
  2. Can call itself (recursion)
- Decorators change function behavior (e.g. @numba.njit)
- Variables can both be **local scope** (good) or **global scope** (bad)

Questions: What is the output?

```
a = 1
def f(x):
    return x+a
print(f(1))
a = 2
print(f(1))
```

Conditionals and loops

- **Comparison** (`==`, `!=`, `<`, `<=`, `not`, `and`, `or` etc.)
- **Conditionals** (`if`, `elif`, `else`)
- **Loops** (`for`, `while`, `continue`, `break`)
- **Convergence** (tolerance in optimizer or root-finder/equation-solver)

Questions: How could this be implemented with a `while` loop?

```
x = x0
for i in range(n):
    y = evaluate(x)
    if check(y): break
    x = update(x,y)
else:
    raise ValueError('did not converge')
```

Decimal numbers are not exact

- **Never use exactness for decimal numbers**
  - Order of computation matter
  - Best with numbers are around 1 (underflow and overflow)
- Division, exp, log etc. are (costly) approximations
- **Function approximation and interpolation often needed**

Questions: Which are `True` and which are `False`?

```
print(0.1 + 0.2 == 0.3)
print(0.5 + 0.5 == 1.0)
print(np.isclose(0.1+0.2,0.3))
print(np.isclose(1e-200*1e200*1e200*1e-200,1.0))
print(np.isinf(1e-200*(1e200*1e200)*1e-200))
print(np.isclose(1e200*(1e-200*1e-200)*1e200,0.0))
```

Pseudo random numbers

- **Only one seed** (randomness not assured across seeds)
- State of random number generator can be reset
- **Monte Carlo** simulation and integration
    1. Static alternative: Use **quadrature rules**
    2. Dynamic alternative: Discretize and derive **transition matrix**

Questions: What is z equal to?

```
rng = np.random.default_rng(123)
s = rng.bit_generator.state
x = rng.normal(size=5)
y = rng.normal(size=5)
rng.bit_generator.state = s
z = rng.normal(size=5)
```

Documentation and debugging

- **No code is self-explanatory** (for others, incl. future you)
- **Write documentation** (use *github-copilot*)
  1. The comments explain humans what the code does.
  2. The code makes the computer do what the comments say
- Important **design patterns**:
  1. Use namespaces (be aware of scope) and meaningful names
  2. No repetition of code-lines $\Rightarrow$ single-purpose functions/methods
  3. Use `assert` (also print and plot intermediate results)
  4. Use `try-except`
- **Run from top to bottom** (make shortcut)
  Replication: datacodestandard.org
- **Debugging** (see 02. Debugging.ipynb)
  1. Errors are (almost) always simple
  2. Go through code step-by-step (*manually* or *debugger*)

Numba and EconModelClass

- **Numba:** Faster code (03. Numba.ipynb)
- **EconModel** (04. EconModelClass.ipynb)
  1. Make it easy to write well-structured code
  2. Provide standard functionality for copying, saving and loading
  3. Provide an easy interface to call numba JIT compilled functions
  4. Provide an easy interface to call C++ functions
     (not relevant in this course)

Consumption-Saving

Consumption-saving

We start on the slides for lecture 2