Assignment I: Labor Income Inequality in the HANC Model

Objective: This assignment focuses on the impact of rising labor income inequality (and risk) on macroeconomics variables, using the Heterogeneous-Agent Neo-Classical model.

- **Problem:** The problem consists of
  1. A number of questions (page 2)
  2. A model (page 3 onward, incl. solution tricks)
- **Code:** The problem is designed to be solved with the *GEModelTools* package.
- **Structure:** Your project should consist of
  1. A single self-contained pdf-file with all results
  2. A single Jupyter notebook showing how the results are produced
  3. Well-documented *.py* files
- **Hand-in:** Upload a single zip-file on Absalon (and nothing else)
- **Deadline:** 23rd of October 2025
- **Exam:** This project will be a part of your exam portfolio.
  You can incorporate feedback before handing in the final version.

Questions

1. **Stationary equilibrium:** Define the stationary equilibrium in this economy, and prove that clearing the asset market and labor market implies that the goods market also clears (Walras' law).

2. Compute the stationary equilibrium with low income risk ($v = 1$) and with high inequality ($v = 1.5$). v is a parameter that scales the variance of idiosyncratic shocks in the model. See Sections 1 and 3 for more details.

   (a) For the steady state with low inequality, use the β calibration method, and target a $K/Y = 4$, normalize $Y = 1$, and $r = 5\%$.

   (b) For the high-inequality steady-state, use the direct-method and keep the same calibration of β as in the low-inequality steady state. Check that the goods-market clearing holds.

3. **Comparative statics:**

   (a) Compare the values of output, consumption, capital, wages, and interest rate in the two steady-states. What is the main mechanism?

   (b) Compare the change in consumption to the change in welfare.

   (c) Compute the wealth shares of the bottom 50%, next 40%, top 10%, top 1%, and top 0.1% in the two-steady states, and compare it to the data included in the *wealth_psz.csv* file in 2019. Has wealth inequality increased or decreased when v increases? Why?

4. **Transition:**

   (a) Compute a permanent transition from the low-inequality steady state to the high inequality steady state where v directly jump to its new value.

   (b) Compute a permanent transition from the low-inequality steady-state to he high inequality steady state where v_t gradually increases to its new value. For this question, assume that v_t increases linearly from 1.0 to 1.5 over 30 periods.

   (c) Plot the change in the share of each productivity type over time in the jump-transition and the gradual one. How do they evolve over time? Why?

(d) Plot the results of the two transitions in one plot, and comment on the differences.

5. **Myopic transition (hard):**

   (a) Compute a transition where v_t increases from 1.0 to 1.5 over 30 periods in 10 equally sized small jumps at time $0, 3, 6, \ldots, 27$.

   (b) Compute a myopic transition where the true v_t are as in the previous questions, but households are myopic and do always think the future v will permanently be the current one.

   (c) Plot your results and compare them to the previous transitions. Comment on the type of expectation implicitly used when computing this type of transition: Is it a departure from rational expectation? With respect to which variables?

1. Model

Households. The model has a continuum of infinitely lived households indexed by $i \in [0,1]$. Households are *ex post* heterogeneous in terms of their time-varying stochastic productivity, z_t, and their (end-of-period) savings, a_t. The distribution of households over idiosyncratic states is denoted $\underline{\boldsymbol{D}}_t$ before shocks are realized and $\boldsymbol{D}_t$ afterwards. Households choose consumption, c_t and savings a_t. Households are not allowed to borrow. The real interest rate is r_t, the real wage is w_t, and real profits are Π_t.

The household problem is

$$
\begin{aligned}
v_t(z_t, a_{t-1}) &= \max_{c_t, \ell_t} \frac{c_t^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}\left[v_{t+1}(z_{t+1}, a_t) \,|\, z_t, a_t\right] \qquad (1)\\
\text{s.t. } a_t + c_t &= (1+r_t)a_{t-1} + w_t z_t + \Pi_t \\
\log z_{t+1} &= \rho_z \log z_t + \psi_{t+1} \ \ , \psi_t \sim \mathcal{N}(v_t \mu_\psi, \sigma_\psi),\ \mathbb{E}[z_t] = 1 \\
a_t &\geq 0
\end{aligned}
$$

Note that the variance of idiosyncratic shocks is scaled by the parameter v_t. Aggregate quantities are

$$L_t^{hh} = \int z_t d\boldsymbol{D}_t = 1 \qquad (2)$$

$$C_t^{hh} = \int c_t d\boldsymbol{D}_t \qquad (3)$$

$$A_t^{hh} = \int a_t d\boldsymbol{D}_t \qquad (4)$$

Firms. A representative firm rents capital, K_{t-1}, and hire labor, L_t, to produce goods, with the production function

$$Y_t = K_{t-1}^{\alpha} L_t^{1-\alpha} \qquad (5)$$

Capital depreciates with the rate $\delta \in (0,1)$. The real rental price of capital is r_t^K and the real wage is w_t. Profits are

$$\Pi_t = Y_t - w_t L_t - r_t^K K_{t-1}. \qquad (6)$$

Because of constant returns to scale, firms make zero profits in equilibrium. The law-of-motion for capital is

$$K_t = (1 - \delta)K_{t-1} + I_t \tag{7}$$

Mutual fund. Households own share in a mutual fund, who owns the stock of physical capital and rent it to the firms. The balance sheet of the mutual fund is $A_t = K_t$, and it pays a return $r_t = r_t^k - \delta$ to households.

Market clearing. Market clearing implies

1. Labor market: $L_t = L_t^{hh}$
2. Goods market: $Y_t = C_t^{hh} + I_t$
3. Asset market: $A_t = K_t = A_t^{hh}$

2. Calibration

The parameters and steady state government behavior are as follows:

1. **Preferences and abilities:** $\sigma = 2$,
2. **Income:** $\rho_z = 0.96$, $\sigma_\psi = 0.257$
3. **Production:** $\alpha = 0.33$. Γ_{ss} and δ are chosen to match calibration targets.

3. Numerical implementation

This assignment focuses on the impact of rising labor income risk on macroeconomic variables, welfare, and the distribution of wealth. In most models we will study in this course, we discretize the AR-1 income process using either the Tauchen-method or the Rouwenhorst-method. This gives us a grid of productivity levels and a corresponding Markov transition matrix.

In this problem set, we use the Tauchen method. In order to keep the grids of productivity constant in different economies with different levels of idiosyncratic productivity risk, we modify only the probabilities in the Markov transition matrix when risk changes.

In the code, this is captured by two variables:

1. v shifts the variance of idiosyncratic risk upward or downward
2. z_{scale} adjusts to keep the aggregate productivity endowment equal to one.

When we compute the steady-state and the transitions, we set v to a new value, and need to find the z_{scale} such that $\mathbb{E}[z_i] = 1$. This is done by imposing that the condition $\mathbb{E}[z_i] - 1 = 0$ at all times. Check the code for more details.