6. Wealth Inequality

Adv. Macro: Heterogenous Agent Models

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2024

Introduction

Wealth inequality

- **Goal for today:** Better understand wealth inequality through the lens of heterogeneous agent general equilibrium models
- **Central economic questions:**
  1. Why are some people rich while others are poor?
  2. To what extent can governments affect inequality?
     - What does the baseline Aiyigari model predict in terms of wealth inequality?
     - How can we augment the baseline model to obtain a closer match of reality?
  3. What explains the rise in wealth inequality in recent decades?
- To answer these questions, we need to better understand why people save, and how this translates into wealth inequality
- **Plan for today:**
  1. Study the predictions of a baseline Bewley-Huggett-Aiyagari model
  2. Consider various model extensions that help match the data
  3. Given such a model, what can we say about optimal wealth taxation?

Wealth inequality in the data

Earnings and wealth inequality

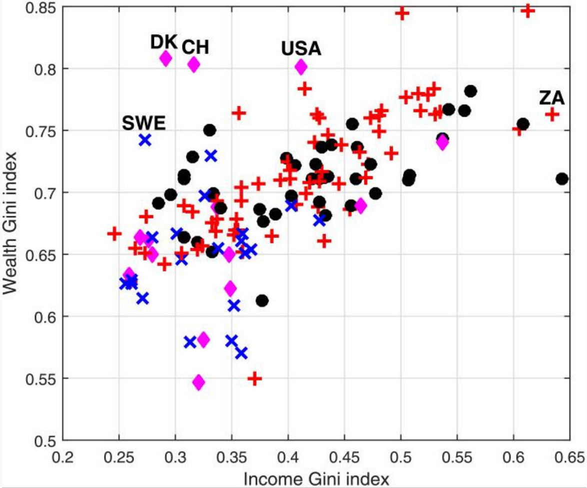
- US data on distribution of income and wealth (SCF, 1989)

	Top 1%	Top 5%	Top 20%	Top 40%	Percent at zero or negative
Wealth	29	53	80	93	6
Earnings	6	19	48	72	8

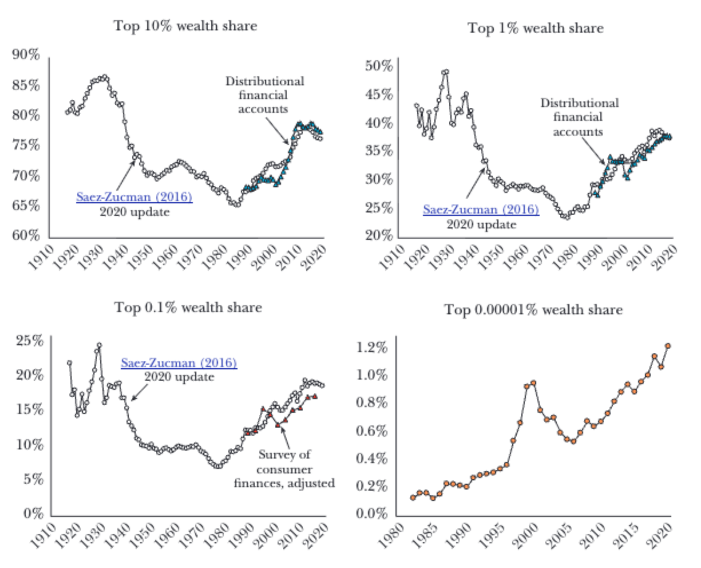
- Wealth more concentrated than earnings
- Skewed distributions with thick upper tails

Wealth more concentrated than earnings

Not only in the US, but also Denmark and almost all other countries



Top wealth shares in the US over time



Income inequality has increased since the 70s (US)

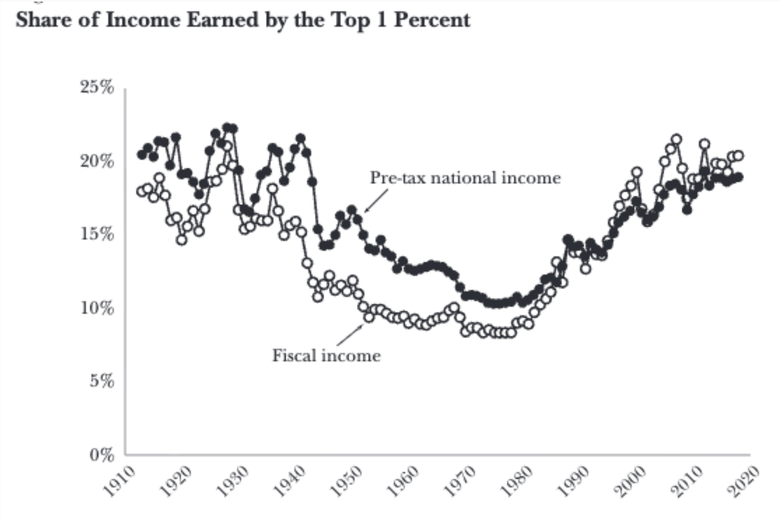


Figure 2: Figure 3 from Saez, Zucman (2020)

Income growth by decile in the U.S.

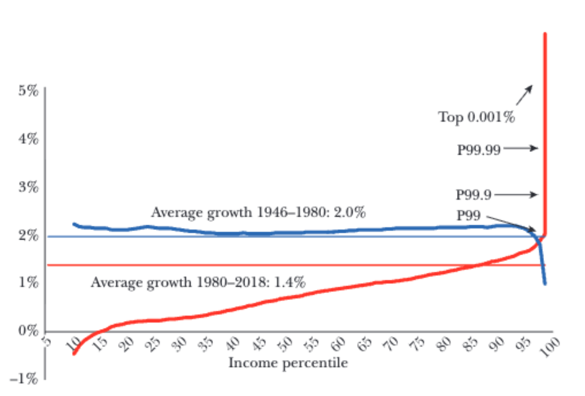


Figure 3: Figure 4 from Saez, Zucman (2020)

Average tax rates by income groups

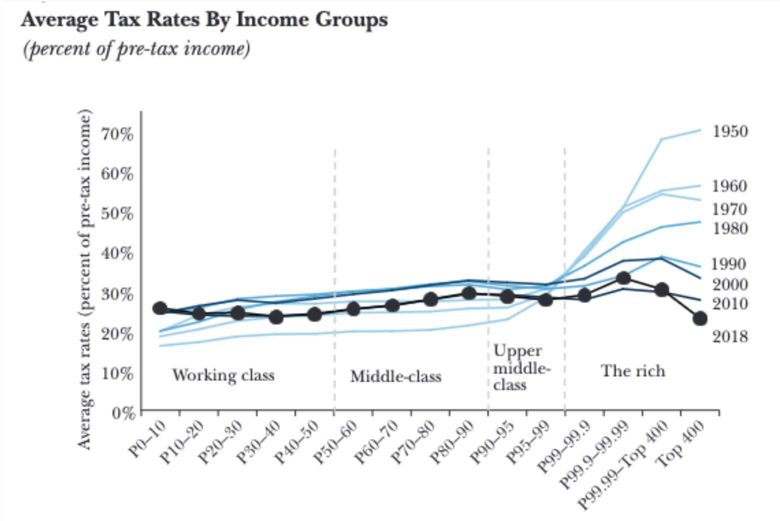


Figure 4: Figure 5 from Saez, Zucman (2020)

Richer households hold more risky assets

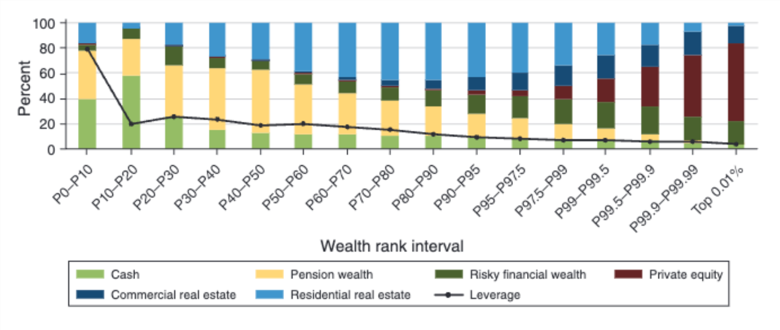


Figure 5: Figure 2 from Bach et al (2020)

Richer households have higher returns

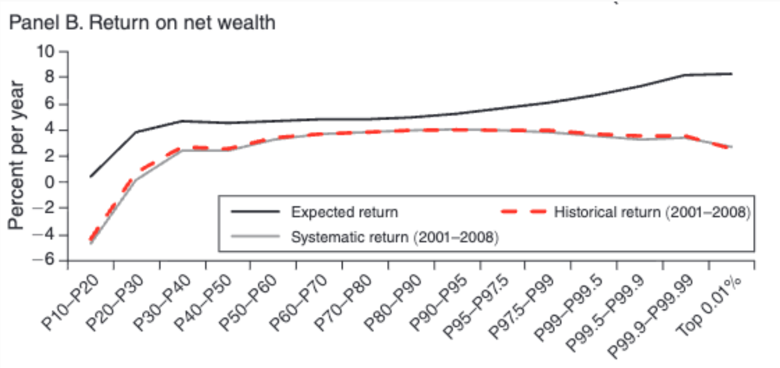


Figure 6: Figure 3 from Bach et al (2020)

→ But still a debate in the literature: is it because of higher risk or higher skill (Fagereng et al, 2020)?

The rich save more, because of capital gains

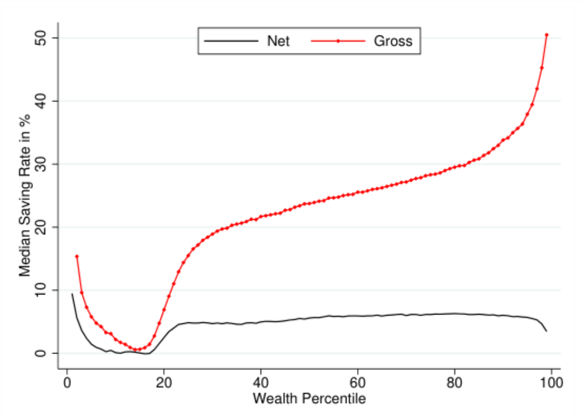


Figure 7: Figure 1 from Fagereng et al (2019)

Taking stock

- Wealth inequality is higher than income inequality
- Both wealth and income inequality have increased over time
- Taxes became less progressive over time (at least in the US)
- Returns are heterogeneous, and higher for richer households
- The rich save more, mostly because of capital gains

Explaining wealth inequality

Aiyagari Model

- Infinitely lived agents with preferences

$$\max_{\{c_t\}_{t=0}^{\infty}} E \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}$$

- Budget constraint and borrowing constraint

$$a_t = y_t + (1+r)a_{t-1} - c_t, \quad a_t \geq \underline{a}$$

- Idiosyncratic earnings risk:

$$\ln y_t = \rho \ln y_{t-1} + \epsilon_t, \quad \epsilon_t \sim \mathcal{N}\left(0, \sigma_\epsilon^2\right)$$

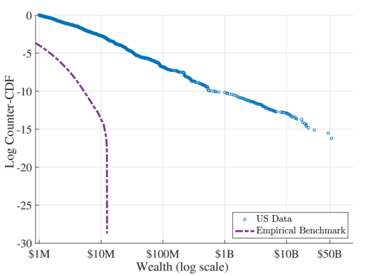
- As usual, calibrate parameters in earnings process $\left(\rho, \sigma_\epsilon^2\right)$ based on estimates from panel data on earnings, i.e. Floden and Linde (2001)

Aiyagari Model - wealth inequality fit

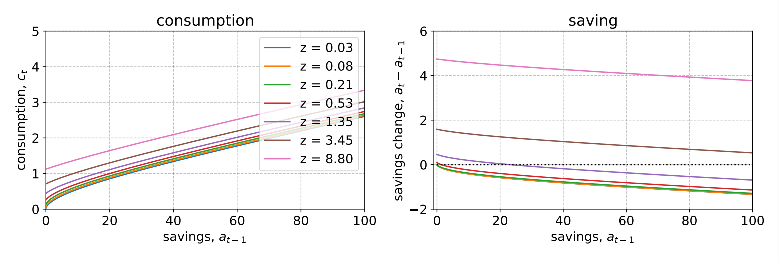
	Wealth Gini	Wealth in top (%)		
		1%	5 %	20 %
U.S. data, 1989 SCF				
	.78	29	53	80
Aiyagari Baseline				
	.38	3.2	12.2	41.0

Top wealth inequality

- What about top wealth inequality?
  - Think top 0.01% or 0.001% (Bezos,Musk, Gates etc.)
- The probability of having wealth a above threshold X described as Pareto dist, $P\left(a > X\right) \sim x^{-\alpha}$
  - In logs, $\ln P\left(a > X\right) \sim -\alpha \ln x$, so linear in log wealth with α describing the "thickness of the tail"



Policies in the Buffer-Stock model



Key mechanism

- Precautionary savings behavior. People save to self-insure against earnings risk
- Once buffer stock savings is reached, people start dissaving (Carroll 1997)
- In the model: The saving rate of the high wealth households is low or even negative
  - Contrasts with much empirical evidence (Dynan Skinner and Zeldes, 2004 and De Nardi, French and Jones, 2010)
  - Will discuss this in next lecture
- Note also: Only driver of wealth inequality is earnings risk
  - Income inequality in data typically lower than *wealth* inequality
  - In reality multiple drivers such as entrepreneurship, preferences, bequests, return heterogeneity

Explanations

- Standard Aiyagari model: Income inequality
- Preference heterogeneity
  - Krussel and Smith (1998), Alan, Browning, and Ejenaes (2016), Druedahl and Jorgensen (2015)
- Bequests
  - Kotlikoff and Summers (1981), Modigliani (1988), Gale and Scholz (1994), Di Nardi (2004)
- Entrepreneurship
  - Cagetti and De Nardi (2006), Di Nardi et al. (2007), Guvenen et al. (2023)
- Return heterogeneity
  - Hubmer, Krusell, Smith (2021), Ozkan et al. (2023), Guvenen et al. (2023)

Bequests

$$\max_{\{c_t\}_{t=0}^{T}} E \sum_{t=0}^{T} \beta^t \left(s_t \frac{c_t^{1-\sigma}}{1-\sigma} + (1-s_t)\,\phi\,(a_{t-1}) \right)$$

$$c_t + a_t = y_t + (1+r)a_{t-1} + b_t, \quad a_t \geq \underline{a}$$

1. Bequests and human capital transmission across generations (*warm glow*)

Explaining wealth inequality

$$\max_{\{c_t\}_{t=0}^{T}} E \sum_{t=0}^{T} \beta_i{}^t s_t \frac{c_t^{1-\sigma_i}}{1-\sigma_i}$$

$$c_t + a_t = y_t + (1+r)a_{t-1}, \quad a_t \geq \underline{a}$$

1.
2. Heterogeneous preferences

Explaining wealth inequality

$$\max_{\{c_t\}_{t=0}^{T}} E \sum_{t=0}^{T} \beta^t s_t \frac{c_t^{1-\sigma}}{1-\sigma}$$

$$c_t + a_t = [I_e f(\theta_t, k_{t-1}) + (1 - I_e) y_t] + (1 + r)(a_{t-1} - k_{t-1}), \quad a_t \geq \underline{a}$$

1.
2.
3. Entrepreneurship.

Explaining wealth inequality

$$\max_{\{c_t\}_{t=0}^{T}} E \sum_{t=0}^{T} \beta^t s_t \frac{c_t^{1-\sigma}}{1-\sigma}$$

$$c_t + a_t = y_t + \left(1 + r_t^i\right) a_{t-1}, \quad a_t \geq \underline{a}$$

1.
2.
3.
4. Idiosyncratic rates of return

Gaillard, Hellwig, Wanger and Werguin (2024)

Ranking of Pareto tails in the Data

Empirical ranking of Pareto tails (US):

capital income $<$ wealth $<$ labor income $<$ consumption

Table 1. Top consumption, income, and wealth ineq

Data	Variable	Best fit Pareto[b] $\hat{\underline{x}}^{OLS}$	$\hat{\zeta}^{OLS}$	$\hat{\underline{x}}^{MLE}$	$\hat{\zeta}^{MLE}$
PSID	Capital income	0.96	**1.22**	0.96	**1.21**
		(0.02)	(0.15)	(0.02)	(0.14)
	Wealth	0.93	**1.48**	0.92	**1.47**
		(0.03)	(0.09)	(0.03)	(0.09)
	Labor income	0.88	**2.42**	0.89	**2.50**
		(0.04)	(0.15)	(0.04)	(0.13)
	Consumption	0.89	**3.11**	0.90	**3.13**
		(0.04)	(0.28)	(0.04)	(0.20)
	Food consumption	0.93	**4.40**	0.93	**4.26**
		(0.05)	(0.33)	(0.05)	(0.43)

Theoretical results

Using a continuous time HA model, they show that we need two key features to match this ranking:

1. Non-homothetic preference for wealth
2. Scale-dependent returns

(they also allow for random returns, death probability, progressive taxes)

Quantitative model overview

HA households with:

- Idiosyncratic income shocks
- Scale dependent returns
- Type dependent returns
- Non-homothetic taste for wealth

On the supply side, classic Cobb-Douglas production function with perfect competition.

Quantitative results - Bellman equation

$$
\begin{aligned}
V(y,z,a) = \max_{c,a' \geq a} \frac{c^{1-\gamma}}{1-\gamma} + \kappa \frac{(a/A)^{1-\nu}}{1-\nu} \\
+ \beta(1-\xi) \sum_{y' \in \mathcal{Y}} \sum_{z' \in \mathcal{Z}} P\left(y' \mid y\right) P\left(z' \mid z\right) V\left(y', z', a'\right) \\
\text{s.t. } c + a' = wy - T(wy) + \left(1-\tau_K\right) rzS(a)a + a
\end{aligned}
$$

with

- y is the idiosyncratic productivity type
- ξ is the death probability
- z is the idiosyncratic return type
- $T(wy) = wy - \frac{1-\tau_0}{1-\tau_L}(wy)^{1-\tau_L}$ (progressive taxations, HSV)
- $S(a) = 1 + \psi a^\eta$ (scale dependence)

A note on death probability

Many HA models with a Pareto tail have a death probability:

- Main assumption: every period, households face a constant probability to die ξ
- They are replaced by new houeholds who start with zero wealth
- Because death is iid, does not add an extra state. But needs to change the forward step
- Key paper: perpetual youth model of Blanchard (1985) Yaari
- Need to make assumptions on what happens to accidental bequests (paid to surviving households through annuity markets, taxes by governments, destroyed, etc)

$\rightarrow$ Especially important in non-homothetic model to have a non-degenerate distribution of wealth: we need a force to stop them from accumulating infinite amounts of wealth.

Quantitative results - details on heterogeneous returns

z is a random variable that captures the return type:

- $z \in (z_l, z_h)$, $z_l = 1$, 'worker type', $z_h > 1$, 'entrepreneur type'
- Follows a Markov chain: probability to become an entrepreneur is calibrated on data $q_{LH} = 0.02$
- Probability to switch to worker type: $q_{HL} = 0.2$

$\rightarrow$ as an entrepreneur, you want to save a lot because you get temporarily very high returns on your wealth.

Quantitative results - supply side and market clearing

Rest of the model is standard:

- $Y = K^\alpha L^{1-\alpha}$, factors paid their marginal productivity
- Asset market clearing: $K = A = \int zS(a)adF(y,z,a)$
- Government budget balances (government fully taxes accidental bequests).

Quantitative results - main exercise

They calibrate most of the model parameters, and estimate:

1. κ: strength of taste for wealth
2. ν: exponent of the taste for wealth
3. z_h: excess returns of high-return type
4. ψ: strength of scale dependence
5. η: exponent of scale dependence

And they target the following moments (at the steady-state)

1. Ratio of capital income / wealth Pareto coefficients
2. Ratio of consumption to wealth Pareto coefficients
3. Ratio of wealth to labor income Pareto coefficients

+ Capital income to wealth tail for the top 1% + W/Y=3.8 + Top 1% wealth share

Results

Table 4. Counterfactual models and selected moments.

	Data/Model	Pareto tails: mean MLE estimate[a] ζ_c	ζ_y	ζ_y^{net}	ζ_a	ζ_{ra}	Top 1% wealth	$\frac{\text{Wealth}}{\text{income}}$ ratio
	Adjusted PSID (2005–2021)	3.06	2.25	2.57	1.38	1.20	0.35	3.8
Homothetic preferences								
(1)	Homogeneous returns	3.04	2.25	2.57	2.42	2.42	0.09	3.8
(2)	Type-dependence	2.65	2.25	2.57	1.32	1.02	0.29	3.7
(3)	Scale-dependence	2.56	2.25	2.57	1.30	1.16	0.32	3.7
(4)	Type- and scale-dependence	2.65	2.25	2.57	1.34	1.08	0.35	3.6
Non-homothetic preferences								
(5)	Type-dependence	3.08	2.25	2.57	1.37	1.19	0.34	3.7
(6)	Type- and scale-dependence	3.06	2.25	2.57	1.36	1.18	0.35	3.8

$\rightarrow$ many models can generate high degree of wealth inequality.
But both non-homothetic and type-dependence are key to match relative ranking of Pareto tails!

Hubmer, Krussel and Smith (2021)

Explaining wealth inequality

- Hubmer, Krussel and Smith (2021): *Sources of US wealth inequality: Past, present, and future*
  - Model which matches key features of US wealth inequality in 1967
  - Can we account for changes in wealth inequality going forward from 1967 based on observables?
    - I.e. changes in income inequality, taxes, asset returns

Model

- Household problem features non-linear tax schedules, heterogeneous returns and β-het.

$$V_t(a_{t-1}, p_t, \beta_t) = \max_{a_{t+1} \geq 0} \{u(c_t) + \beta_t \mathbb{E}[V_{t+1}(a_t, p_{t+1}, \beta_{t+1}) | p_t, \beta_t]\}$$

$$\begin{aligned} c_t + a_t =& y_t - \tau_t(y_t) + (1 - \tilde{\tau}_t)\tilde{y}_t + T_t \\ y_t =& (\underline{r_t} + r_t^X(a_{t-1}))a_{t-1} + w_t l_t(p_t) \\ \tilde{y}_t =& \sigma_t^X(a_{t-1})\eta_t a_{t-1} \end{aligned}$$

- Mean excess return $r_t^X(a_{t-1})$
  - How does mean returns vary with wealth?
- Standard deviation of excess returns: $\sigma_t^X(a_{t-1})$
  - How does return uncertainty vary with wealth?
- Example: If rich HHs primarily invest in stocks, poorer HHs in bonds. Would expect both $r_t^X(a_{t-1}), \sigma_t^X(a_{t-1})$ to be increasing in a_{t-1}

Facts

- Fagereng, Guiso, Malacrino, Pistaferri (2020) find that rates of returns are:
  - Heterogeneous across households (over 200 basis points between 10th and 90th percentile of the distribution of returns)
  - Also heterogenous within asset classes
    - So return differences cannot be explained only by poorer HHs holding banket deposits and rich HHs investing in stocks
  - Persistent
  - Correlated with household wealth and across generations

Equilibrium: capital market clearing

- Need to find two equil. objects $(K_t, \underline{r}_t)$ for capital market clearing:
  1. aggregate capital (as usual)

$$K_t = \int a_t d\Gamma(a_t)$$

  2. aggregate capital income (redundant if $r_t^X(\cdot) = 0$)

$$(MPK(K_t) - \delta)K_t = \int \left(\underline{r}_t + r_t^X(a_t)\right) a_t d\Gamma(a_t)$$

- Plus goods market clearing, but redundant given other 2

Calibration strategy summary

1. Calibrate earnings process, tax rates, return process, social safety net to observables
2. Choose randomness in discount factor β residually so as to replicate the wealth distribution in the initial steady state (1967)
3. Then feed in exogenous changes in tax rates, earnings inequality, etc. between 1967 and 2015 to understand the role of these different factors

Return heterogeneity

- Overall return given asset holdings a_{t-1} equals

$$\underline{r}_t + r_t^X(a_{t-1}) + \sigma^X(a_{t-1})\eta_t$$

- $\underline{r}_t$ is endogenous
- $r_t^X(\cdot)$ and $\sigma^X(\cdot)$ are exogenous excess return schedules (mean and st.dev.), taken from the data
- η_t is an i.i.d. standard normal shock
- Reduced form portfolio choice

Calibration: return process

$$r_t^X(a_t) = \sum_{c \in C} w_c(a_t) \left(\bar{r}_{c,t} + \tilde{r}_c^X(a_t) \right)$$

$$\sigma^X(a_t)^2 = \sum_{c \in C} \left(w_c(a_t) \tilde{\sigma}_c^X(a_t) \right)^2$$

- Asset classes C: risk-free, public equity, private equity, housing
- $\bar{r}_{c,t}$: aggregate return on asset class c (U.S. data), time-varying
- Fixed over time, based on Swedish administrative data from Bach, Calvet, Sodini (2016):
  - $w_c(\cdot)$: portfolio weights
  - $\tilde{r}_c^X(\cdot)$: within asset class return heterogeneity
  - $\tilde{\sigma}_c^X(\cdot)$: asset c idiosyncratic return standard deviation

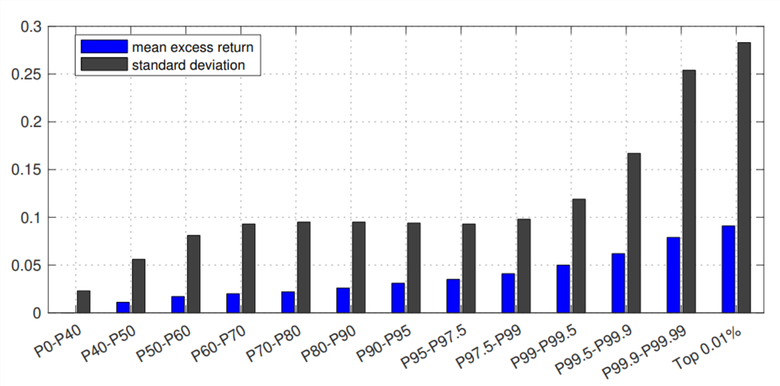
Excess return schedule details

- Aggregate Excess Returns in 1967 steady state:
  - public equity 0.067 (U.S., Kartashova 2014)
  - private equity 0.129 (U.S., Kartashova 2014)
  - housing 0.037 (incl. imputed rent; Jorda, et al, 2017)

- and cross-sectional data from Bach, Calvet, Sodini (2019) implies

	P0-P40	P40-P50	P50-P60	P60-P70	P70-P80	P80-P90	P90-P95	P95-P97.5	P97.5-P99	P99-P99.5	P99.5-P99.9	P99.9-P99.99	Top 0.01%
fixed portfolio weights													
risk-free	0.722	0.412	0.248	0.182	0.156	0.134	0.115	0.102	0.090	0.079	0.071	0.051	0.029
housing	0.162	0.394	0.580	0.662	0.678	0.674	0.658	0.626	0.572	0.482	0.363	0.253	0.155
public equity	0.113	0.189	0.165	0.147	0.153	0.170	0.189	0.207	0.219	0.232	0.230	0.185	0.179
private equity	0.002	0.005	0.007	0.009	0.013	0.021	0.038	0.065	0.118	0.207	0.336	0.511	0.637

Schedule of excess returns



Data sources: Bach, Calvet, Sodini (2019); Kartashova (2014); Jorda, Knoll, Kuvshinov, Schularick, Taylor (2019); Case-Shiller.

Hubmer, Krussel and Smith (2021)

Results

Results, I: Steady state (1967)

- Steady state fit (with and without β-het)

	Top 10%	Top 1%	Top 0.1%	Top 0.01%
Data	70.8%	27.8%	9.4%	3.1%
Single-β Model	66.6%	23.7%	11.2%	7.2%
Benchmark Model	73.8%	27.4%	8.4%	3.2%
	Bottom 50%	Fraction $a < 0$		
Data	4.0%	8.0%		
Single-β Model	3.5%	7.3%		
Benchmark Model	3.0%	6.6%		

Results, I: steady state (1967)

#		top 10%	top 1%	top 0.1%	top 0.01%	Gini
1	β-heterogeneity	8.8%	7.7%	3.8%	2.0%	0.050
2	earnings heterogeneity	−27.5%	−17.8%	−9.5%	−6.4%	−0.173
3	persistent	−5.0%	−7.5%	−4.2%	−2.9%	0.009
4	transitory	−11.6%	−4.3%	−1.7%	−0.9%	−0.109
5	tax progressivity	−21.3%	−61.8%	−71.2%	−67.1%	−0.148
6	return heterogeneity	29.5%	18.4%	6.6%	2.8%	0.192
7	mean differences	25.8%	16.7%	6.0%	2.6%	0.174
8	return risk	0.7%	2.2%	3.3%	2.5%	0.004

- How to read: Shutting of β-het reduces top 10% wealth share by 8.8%
- Model matches wealth distribution well on its entire domain
  - return heterogeneity is key ingredient
  - wealth concentration is mitigated by progressive taxation and labor income risk

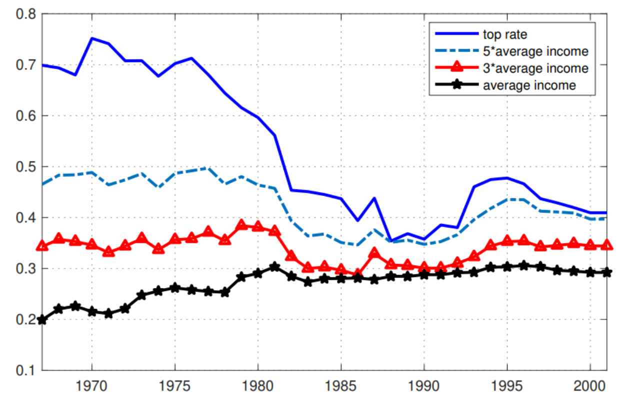
Next step: transition

The authors feed in four different factors that have changed during the past 50 years

- Decrease in tax progressivity
- Increase in labor income risk
- Increase in income going to the top
- Changing return premia to different asset classes

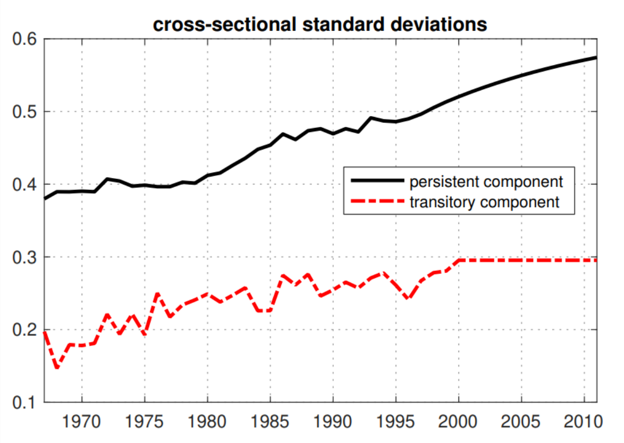
Observed change 1: Decrease in tax progressivity

- Federal effective tax rates (Piketty & Saez 2007): income, payroll, corporate and estate taxes



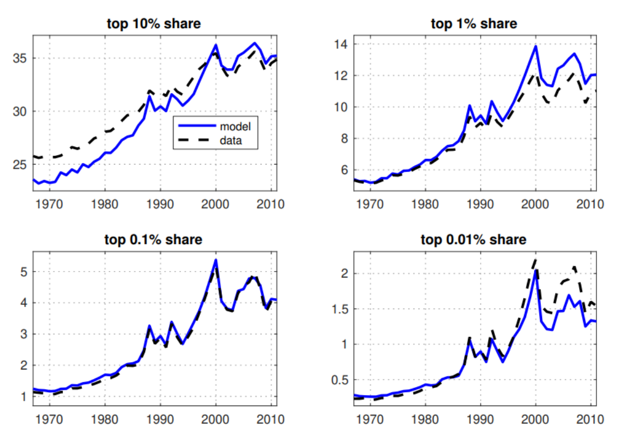
Observed change 2: Increase in labor income risk

- Estimates for variance of persistent and temporary components 1967-2000 (Heathcote, Storesletten & Violante 2010)



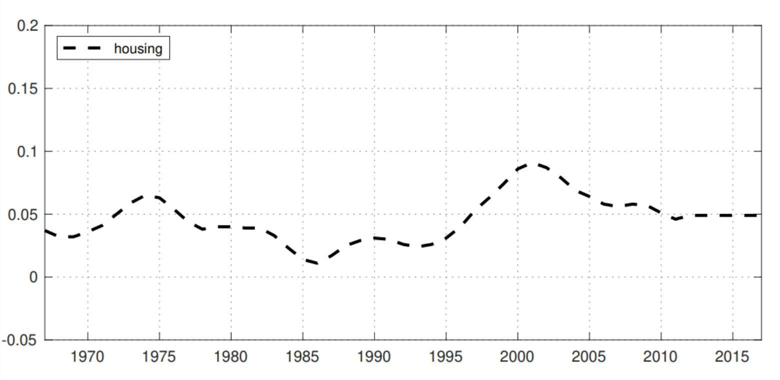
Observed change 3: Increase in top labor income shares

- Adjust standard AR(1) in idiosyncratic productivity by imposing a Pareto tail for the top 10% earners: calibrated tail coefficient decreases from 2.8 to 1.9 (updated Piketty & Saez 2003 series)



Observed change 4: return premia

- Feed in (smoothed) time series of aggregate U.S. asset premia (Kartashova 2014, Case-Shiller index)



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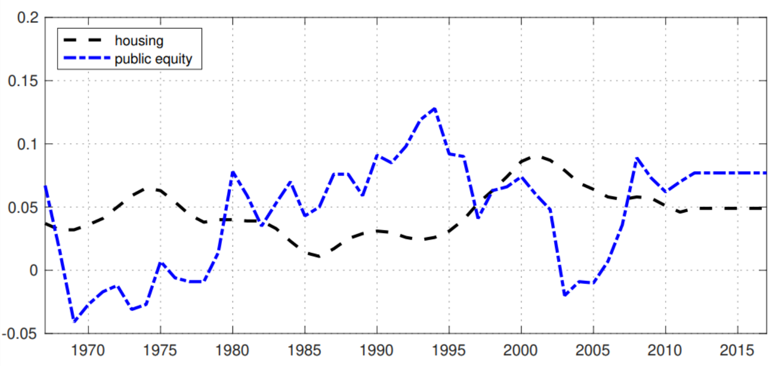

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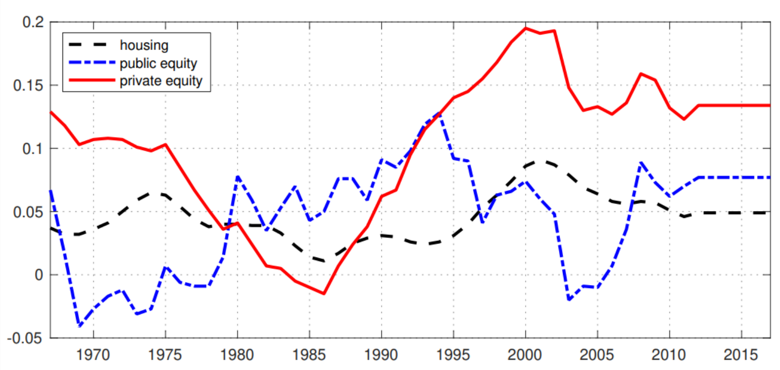
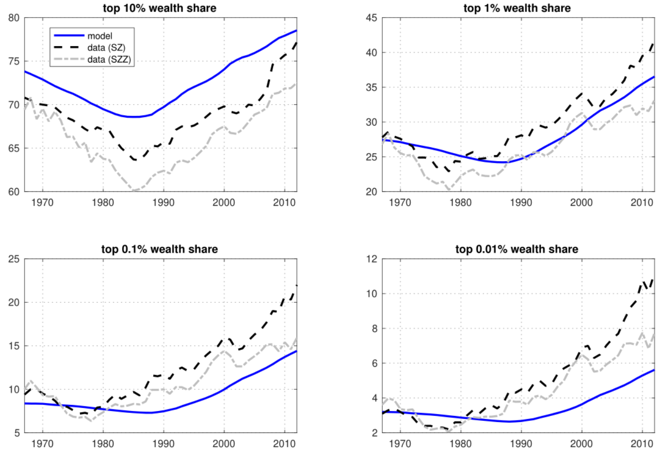
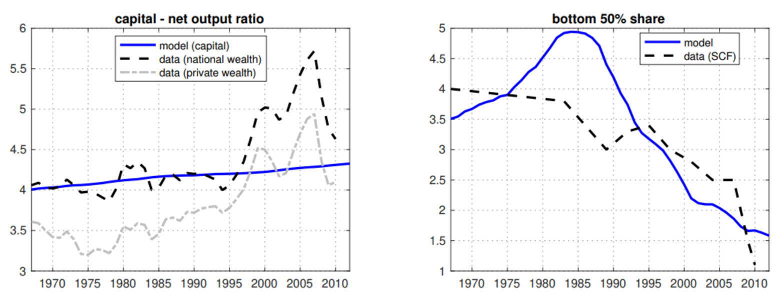


Results, II: historical evolution



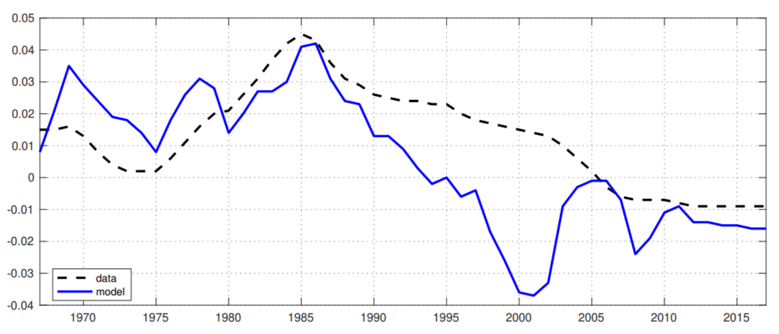
Data sources: dashed black lines refer to Saez & Zucman (2016); dash-dotted gray lines refer to Smith et al. (2020).

Results: Capital-output ratio and bottom 50 %

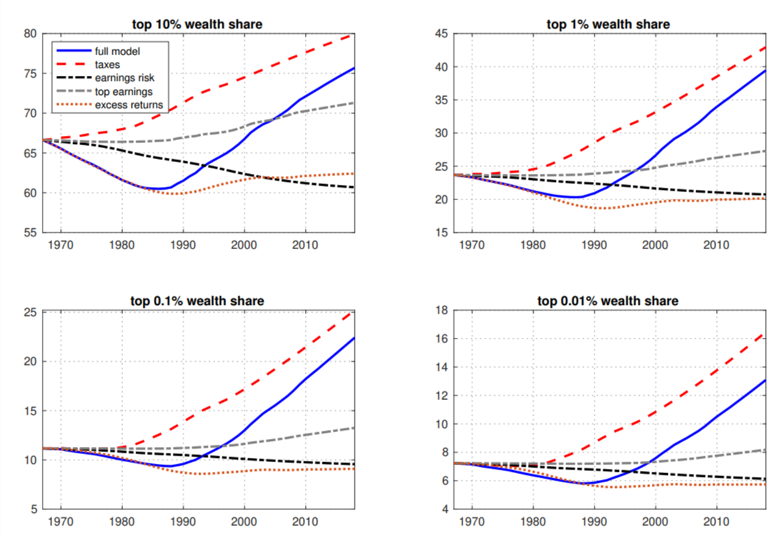


Results: Risk-free rate

- Return premia are matched in model by construction
- Risk-free rate r is endogenous: comparable level and decline



Decomposition of transitional dynamics



Decomposition of transitional dynamics

- Overall increase in wealth inequality (more than) fully explained by declining tax progressivity
  - Primarily due to direct effect on resource distribution and not due to changing savings behavior

- Time-varying return premia account for U-shape in wealth inequality

- Subtle role of increasing earnings dispersion
  - Thickening Pareto tail in labor income contributes slightly positively to wealth inequality
  - Increase in overall earnings risk *decreases* wealth inequality because precautionary savings motive is stronger for poorer HHs

Summary

- **Hubmer, Krussel and Smith (2021)**
- HANC with:
    - Income risk
    - Return heterogeneity
    - β-heterogeneity
    - Tax system
- Main finding:
    - Return heterogeneity key in matching initial (1967) wealth inequality
    - Can roughly explain evolution in US wealth inequality with observable changes in tax systems

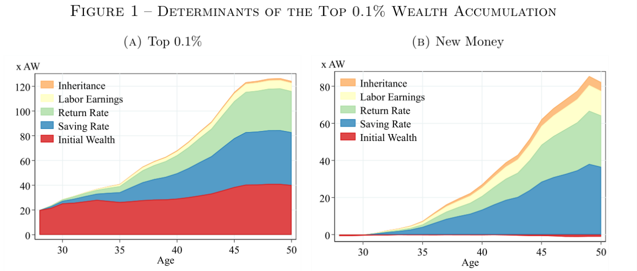
Ozkan et al. (2024)

- Ozkan et al. (2024) takes a *lifecycle perspective*:
  - Why do some people become wealther than others?
  - Use detailed Norwegian admin data
- Evaluate contribution from
  1. Inheritances (bequests) H_{it}
  2. Return heterogeneity r_{it}
  3. Saving rate heterogeneity s_{it}
  4. Labor earnings L_{it}
  5. Initial wealth a_{i0}
- Using budget constraint:

$$a_{it} = a_{it-1} + \left(L_{it} + H_{it} + r_{it} a_{it-1}\right) \times s_{it}$$

Results from Ozkan et al. (2024)

- Left panel: Decomposition of wealth for top 0.1%
- Right panel: »Poorest« HHs *within* top 0.1% (*New Money*)



Application to Wealth Taxation

Wealth taxation I

- Spend a lot of time understanding *what* drives wealth inequality
- We will now see an application where the specific source of inequality matters
- **Wealth taxation**
  - Why would we want to tax wealth?
  - Why not?

Wealth taxation II

- Guvenen et al. (2023): *Use It or Lose It: Efficiency and Redistributional Effects of Wealth Taxation*
- Study optimal taxation in two tax systems:
  - Wealth tax: a_i
  - Capital income tax : $r \times a_i$
- Note: Without return heterogeneity two tax system are equivalent
  - After tax wealth /w CI tax : $a_i + (1 - \tau_k)\, ra_i$
  - After tax wealth /w wealth tax : $(1 - \tau_a)\, a_i + ra_i$
- Social planner can implement same allocation using these two different instruments by setting $\tau_a = r\tau_k$

Taxation with return heterogeneity

- What if returns differ across agents, r_i?
  - No equivalence between tax systems
- With capital income taxation:
  - A highly productive agent (high r_i) will be taxed more than less productive agents (low r_i)
    - Tax burden falls proportinally more on productive agents $\Rightarrow$distortionary
- With wealth taxation:
  - All agents with same wealth pay same tax regardless of return r_i
  - Shifts tax base towards unproductive agents
- Note: We say HHs with high r_i are more **productive**
  - Think in terms of *entrepreneurial* models
  - High productivity HHs have better technology (i.e. are better entrepreneurs) and can make their wealth growth faster (high r_i)

Model

- HH problem:

$$\max_{\{c_t\}_{t=0}^{T}} E \sum_{t=0}^{T} \beta^t \left(s_t \frac{c_t^{1-\sigma}}{1-\sigma} + (1-s_t)\,\phi(a_t) \right)$$

$$a_t + c_t = \mathcal{W}(a_{t-1}, z_{t-1}) + w_t(e_t)\,\ell_t, \quad a_t \geq \underline{a}$$

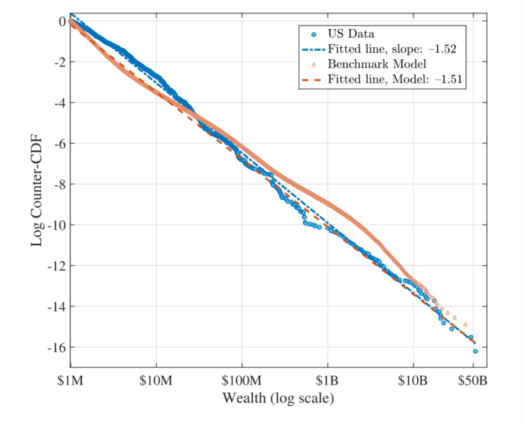
$$\mathcal{W}(a_{t-1}, z_t) = \begin{cases} a_{t-1} + (\pi(a_{t-1}, z_t) + r a_{t-1})(1-\tau_k) & \text{if CI tax} \\ a_{t-1}(1-\tau_a) + (\pi(a_{t-1}, z_t) + r a_{t-1}) & \text{if wealth tax} \end{cases}$$

- Entrepreneurial abilitiy z follow markov chain with values $z = [0, z_L, z_H]'$ and transition matrix Π_z
  - HHs with $z = 0$ are normal workers
  - HHs with $z = z_L$ are »unproductive« entrepreneurs
  - HHs with $z = z_H$ are »productive« entrepreneurs
- Entrepreneurial profit $\pi(a_{t-1}, z_{t-1})$ given by:

$$\pi(a_{t-1}, z_t) = \max_{k_t < \kappa a_{t-1}} \{p_t z_t k_t - (r+\delta)\,k_t\}$$

Empirical fit

- Calibrate model to US. Model reproduces wealth inequality in the data, also for the extremely rich



Results

- Exercise: Replace capital income tax $\tau_k = 25\%$ with wealth tax $\tau_a > 0$ in a government revenue-neutral way (requires $\tau_a = 1.2\%$)

TABLE V

TAX REFORM: CHANGE IN MACRO VARIABLES FROM CURRENT U.S. BENCHMARK

	Quantities (% change)						Prices (change)			
	K	Q	TFP_Q	L	Y	C	$\overline{w}$	$\overline{w}$ (net)	$\Delta r^{\dagger}$	$\Delta r^{\dagger}$ (net)
RN reform	16.4	22.6	5.3	1.2	9.2	9.5	8.0	8.0	0.21	–0.36
BB reform	9.2	16.0	6.2	1.2	6.9	7.7	5.6	5.6	0.67	–0.38

- Capital, productivity output, consumption, wages increases
  - Efficency gain from shifting tax base away from productive agents
- Also generates large welfare gain (around 7% consumption equivalent gains)

Results - optimal taxation

- Now find tax rates that maximize aggregate welfare
  - Wealth taxation (OWT) vs. capital income taxation (OKIT)
- Results:

OPTIMAL TAXATION: TAX RATES AND AVERAGE WELFARE EFFECTS

	Benchmark U.S. economy	RN reform (1)	OWT (2)	OWT L-INEQ (3)	OWT-X (4)	WTE-X (5)	OKIT (6)
Tax rates							
τ_k	25.0	—	—	—	—	—	–13.6
τ_a	—	1.19	3.03	2.54	3.80†	3.30	—
τ_ℓ	22.4	22.4	15.4	18.1	14.4	17.7	31.2
ΔWelfare							
$\overline{CE}_1$	—	6.8	9.0	6.0	9.1	8.4	4.2
$\overline{CE}_2$	—	7.2	8.7	5.2	8.8	8.6	5.1

- Wealth taxation: Positive taxation $\tau_a = 3.03\%$, large welfare gain of 9%
- Capital income taxation: *Subsidy* $\tau_K = -13.6\%$ and smaller welfare gain of 4.2%

Summary

- Guvenen et al. (2023) study optimal wealth taxation
- Source of wealth inequality matters for optimal taxation
- If driven by return heterogeneity **wealth tax** strongly preffered to **capital income tax**
  - Why? It distorts investment decisions of high productivity HHs less than a capital income tax

Exercise

Standard HANC model with return heterogeneity

- HH problem:

$$v_t(, e_{it} r_{it}^x, , a_{it-1}) = \max_{c_t} u(c_t) + \beta \underline{v}_{t+1}(e_{it+1}, r_{it+1}^x, a_{it})$$

$$\text{s.t.}$$

$$\begin{aligned} a_{it} &= (1 + r_t + r_{it}^x) a_{it-1} + w_t e_{it} - c_{it} \\ \log e_{it+1} &= \rho_e \log e_{it} + \psi_{it+1}^e, \quad \psi_{it+1}^e \sim \mathcal{N}\left(0, \sigma_e^2\right) \\ r_{it+1}^x &= \overline{r}^x + \rho_z r_{it}^x + \psi_{it+1}^{r^x}, \quad \psi_{it+1}^{r^x} \sim \mathcal{N}\left(0, \sigma_{r^x}^2\right) \\ a_{it} &\geq 0 \end{aligned}$$

- **Q1**: Solve the PE HA model with return heterogeneity
- **Q2**: Calibrate the HANC model such that average returns are 4%
- **Q3**: Calibrate a standard HA model without return heterogeneity. Compare the wealth distributions obtained in the two models.

Summary

Summary and next week

- **Today:** Various explanations of wealth inequality
  1. Preferences
  2. Bequests
  3. Returns
- **Next week:** Secular stagnation
- **Midterm evaluation:** Don't forget to fill out questionnaire
- **Homework exercise**: Solve model with return heterogeneity
  - See Github repo