Assignment II: The HANK Model

Vision: This project teaches you how to analyze the effects of *deficit financed fiscal stimulus* in a Heterogeneous Agent New Keynesian model.

- **Problem:** The problem consists of
  1. A number of questions (page 2-4)
  2. A model (page 4 onward)
  3. Some code files you can start from
- **Code:** The problem is designed to be solved with the *GEModelTools* package.
- **Structure:** Your project should consist of
  1. A single self-contained pdf-file with all results
  2. A single Jupyter notebook showing how the results are produced
  3. Well-documented *.py* files
- **Hand-in:** Upload a single zip-file on Absalon (and nothing else)
- **Deadline:** 22nd of November 2025
- **Exam:** Your HANK-project will be a part of your exam portfolio.
  You can incorporate feedback before handing in the final version.

Questions

Note: For all the questions where you need to compute a transition, you should compute a linear-transition, except for question 3. (d) where you should also compute a non-linear transition (as specified).

1. Sketch how to derive the Intertemporal Keynesian Cross (IKC) equation for a shock to transfers and discuss the economic intuition it provides.

2. Impact and cumulative multipliers in RANK and TANK:
   Note: You will need to write the code in the functions RA_HHs() and TA_HHs() in blocks.py yourself, based on the equations in section 3 below.
   (a) Solve for the steady-state of the two models.
   (b) Compute the impact and cumulative multipliers in the RANK and the TANK models after a deficit-financed shock to transfers, using the Intertemporal Keynesian Cross formula.
   (c) Discuss their different values, relating them to the IKC equation.

3. Impact and cumulative multipliers in HANK:
   (a) Solve for the steady-state of the HANK model after setting the public debt to $B = 0.92$.
   (b) Compute the impact and cumulative multipliers in the HANK model.
   (c) Discuss and interpret the differences to the RANK and TANK models.
   (d) Compute the fiscal multiplier when the shock to transfers at the initial period is equal to 0.1 (10% of output), using a linear approximation and a non-linear approximation. Comment on the differences between the two methods.

4. Impact and cumulative multipliers with high liquidity
   (a) Computer the impact and cumulative multipliers when $B = 16$ in the RANK, TANK and HANK model (keep $r = 0.005$, adjust only β to obtain the new steady-states).

(b) For the HANK model, plot the iMPCs when $B = 16$ and $B = 0.92$. Give an economic mechanism explaining the change in the iMPCs.

(c) Conclude: What is the impact of an increase in liquidity ($B = 0.92$ vs $B = 16.0$) in the three models?

5. The government decides to implement a policy that targets transfers to households with earnings below the average earnings in the economy.

   (a) Implement this policy in the HANK model (keep $B = 16.0$) and compute the impact / cumulative multipliers with targeting (make sure the goods-market clearing holds in the transition).
   Note: You will need to edit the household_problem.py file.

   (b) Discuss your results.

6. Compute the fiscal multiplier in the HANK model with high liquidity when κ is 0.1 instead of 0.03. Is the multiplier affected by the slope of the NKPC? Which assumption is central for this result?

1. HANK model

Households. The model has a continuum of infinitely lived households indexed by $i \in [0,1]$. Households are *ex post* heterogeneous in terms of their time-varying stochastic productivity, z_t, and their (end-of-period) savings, a_{t-1}. The distribution of households over idiosyncratic states is denoted $\underline{\boldsymbol{D}}_t$ before shocks are realized and $\boldsymbol{D}_t$ afterwards. Households supply labor, ℓ_t, chosen by a union, and choose consumption, c_t, on their own. Households are not allowed to borrow. The return on savings is r_t, the real wage is w_t, labor income and profits are taxed with the rate $\tau \in [0,1]$, and households receive lump-sum transfers, χ_t.

The household problem is

$$
\begin{aligned}
v_t(z_t, a_{t-1}) &= \max_{c_t} \frac{c_t^{1-\sigma}}{1-\sigma} - \varphi \frac{\ell_t^{1+\nu}}{1+\nu} + \beta \mathbb{E}\left[v_{t+1}(z_{t+1}, a_t) \mid z_t, a_t\right] \qquad (1)\\
\text{s.t. } a_t + c_t &= (1+r_t)a_{t-1} + (1-\tau)\left(\Pi_t + w_t \ell_t z_t\right) + \chi_t \\
\log z_{t+1} &= \rho_z \log z_t + \psi_{t+1} \ \ , \psi_t \sim \mathcal{N}(\mu_\psi, \sigma_\psi),\ \mathbb{E}[z_t] = 1 \\
a_t &\geq 0
\end{aligned}
$$

where β is the discount factor, σ is the inverse elasticity of substitution, φ controls the disutility of supplying labor and ν is the inverse of the Frish elasticity.

Aggregate quantities are

$$L_t^{hh} = \int \ell_t z_t d\boldsymbol{D}_t \qquad (2)$$

$$C_t^{hh} = \int c_t d\boldsymbol{D}_t \qquad (3)$$

$$A_t^{hh} = \int a_t d\boldsymbol{D}_t \qquad (4)$$

$$MUC_t^{hh} = \int z_t c_t^{-\sigma} d\boldsymbol{D}_t \qquad (5)$$

Firms. A representative firm hires labor, L_t, to produce goods, with the production function

$$Y_t = L_t \qquad (6)$$

Profits are

$$\Pi_t = Y_t - \frac{W_t}{P_t} L_t \qquad (7)$$

where P_t is the price level and W_t is the wage level. Prices are flexible and the firm is in perfect competition so that

$$w_t = 1$$

and $w_t = \frac{W_t}{P_t}$ is the real wage and $\Pi_t = 0$ in equilibrium.

Union. A union chooses the labor supply of each household. Each household is chosen to supply the same amount of labor,

$$\ell_t = L_t^{hh} \tag{8}$$

The union maximizes aggregate utility of all households subject to a nominal rigidity constraint and a demand constraint. Optimal labor supply is then given by the New-Keynesian Wage Phillips curve:

$$\pi_t = \kappa \left[\varphi \left(L_t^{hh} \right)^{\nu} - \frac{1-\tau_t}{\mu} MUC_t^{hh} \right] + \beta \pi_{t+1} \tag{9}$$

Central bank. The central bank follows a real-rate rule, adjusting nominal inflation such that the real interest rate remains constant

$$r_t = r_{ss} \tag{10}$$

Government. The government lump-sum transfers, χ_t, and the income tax rate, τ. Total tax revenue from income taxes is given by:

$$\mathcal{T}_t \equiv \tau \left(\Pi_t + w_t L_t^{hh} \right) = \tau Y_t \tag{11}$$

The government can finance its expenses with real bonds, B_t which pay out the real rate r_t after one period. The budget constraint for the government is given by:

$$B_t + \mathcal{T}_t = (1 + r_t) B_{t-1} + G + \chi_t. \tag{12}$$

Along with a tax rule

$$\tau_t = \tau_{ss} + \phi_B (B_{t-1} - B_{ss}).$$

Market clearing. Market clearing implies

1. Labor market: $L_t = L_t^{hh}$

2. Goods market: $Y_t = C_t^{hh}$

3. Asset market: $B_t = A_t^{hh}$

2. Calibration

The model is calibrated at the quarterly frequency. The parameters and steady state government behavior are as follows:

1. **Preferences and abilities:** $\sigma = 1.0, \nu = 1.0$

2. **Income:** $\rho_z = 0.96, \sigma_\psi = 0.16$

3. **Production:** $\mu = 1.0, \kappa = 0.03$

4. **Central bank:** $r_{ss} = 0.005$

5. **Government:** $\tau = 0.2$

We calibrate β and φ to obtain the steady-state with $Y = 1$ and market clearing.

3. RANK and TANK models

In the RANK and the TANK models we replace the heterogeneous agent household block with a simpler structure that features only two types of households: Unconstrained (R) and constrained (C). Unconstrained agents are on their Euler-equation at all times, while constrained agents are up against the borrowing constraint implying they have no savings. The total measure of households is 1, and constrained households make up a share λ of these households. The budget constraints are:

$$c_t^R + \frac{A_t}{1-\lambda} = (1+r_t)\frac{A_{t-1}}{1-\lambda} + (1-\tau)L_t^{hh} + \chi_t \tag{13}$$

$$c_t^C = (1-\tau)L_t^{hh} + \chi_t \tag{14}$$

with the Euler-equation for unconstrained households being:

$$\left(c_t^R\right)^{-\sigma} = \beta\left(1+r_{t+1}\right)\left(c_{t+1}^R\right)^{-\sigma} \tag{15}$$

Aggregate consumption is given by:

$$C_t^{hh} = (1-\lambda)\, c_t^R + \lambda c_t^c \tag{16}$$

and aggregate savings is simply $A_t^{hh} = A_t$. The aggregate marginal utility of consumption is $MUC_t^{hh} = (1-\lambda)\left(c_t^R\right)^{-\sigma} + \lambda\left(c_t^c\right)^{-\sigma}$.

The model is calibrated to the same aggregate level of bonds and assets as the HANK model.