6. Wealth Inequality

Adv. Macro: Heterogenous Agent Models

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2024

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- We can switch to "economics" :-)
- But let's recap so far what we have done, and what you should remember

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- Good calibration trick to remember: β method

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- Fake-news algorithm speeds up the computation of Sequence Space Jacobian

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2. Permanent shocks: a transition between one-state to another: only with non-linear transitions

If you are unsure about those concepts, check the GEModelToolsNotebooks repository, especially the HANC folder.

Roadmap for the rest of the class

7. Today: Wealth Inequality
8. Next week: Secular Stagnation

After this, fiscal and monetary policy in HANK, SAM, I-HANK...

Introduction

Wealth inequality

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Wealth inequality in the data

Earnings and wealth inequality

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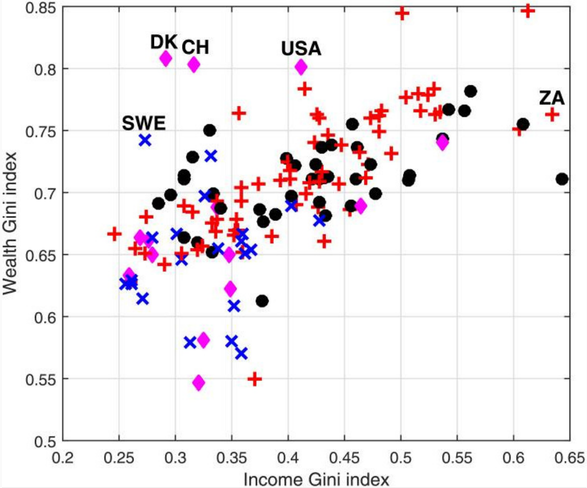
- Wealth more concentrated than earnings
- Skewed distributions with thick upper tails

Wealth more concentrated than earnings

Not only in the US, but also Denmark and almost all other countries



Top wealth shares in the US over time

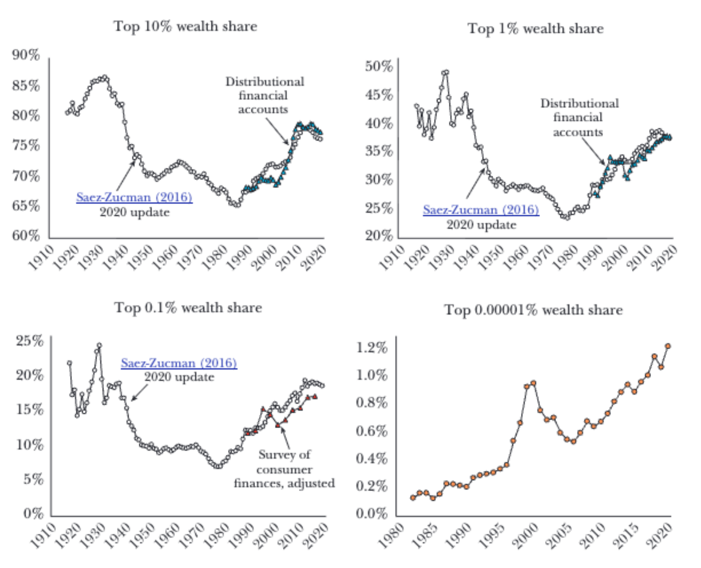


Figure 1: Figure 2 from Saez-Zucman (2020)

Income inequality has increased since the 70s (US)

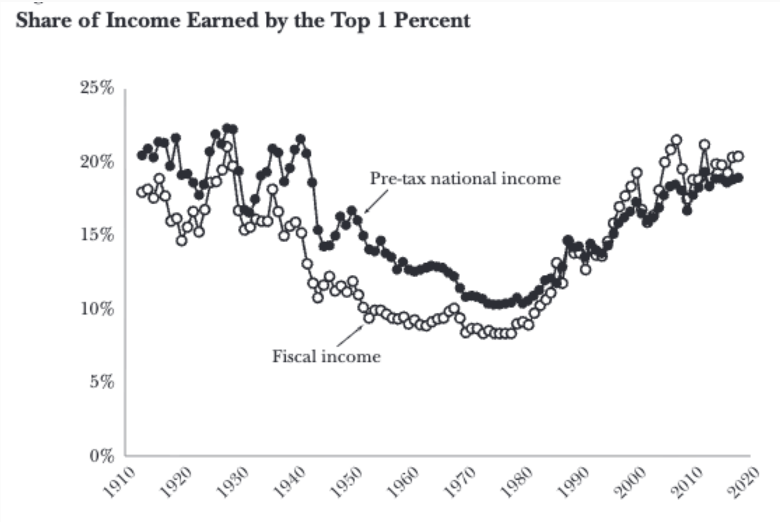


Figure 2: Figure 3 from Saez, Zucman (2020)

Income growth by decile in the U.S.

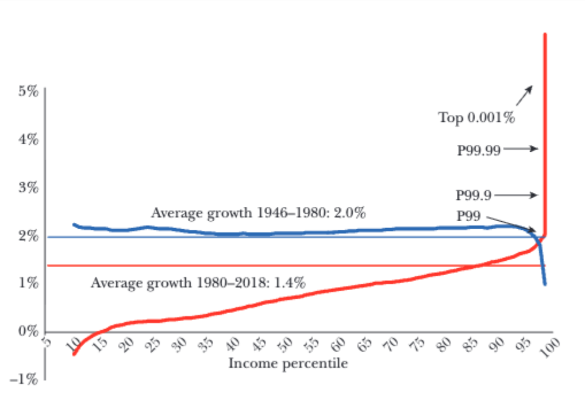


Figure 3: Figure 4 from Saez, Zucman (2020)

Average tax rates by income groups

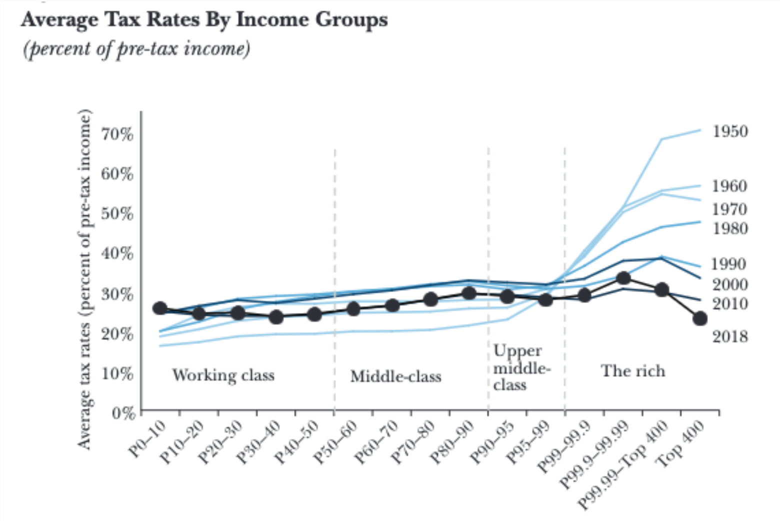


Figure 4: Figure 5 from Saez, Zucman (2020)

Richer households hold more risky assets

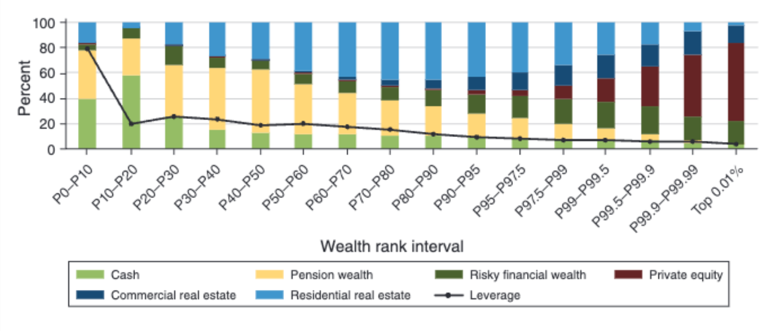


Figure 5: Figure 2 from Bach et al (2020)

Richer households have higher returns

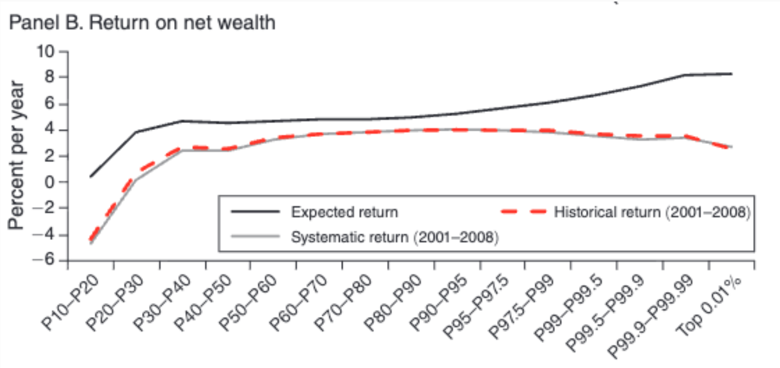


Figure 6: Figure 3 from Bach et al (2020)

→ But still a debate in the literature: is it because of higher risk or higher skill (Fagereng et al, 2020)?

The rich save more, because of capital gains

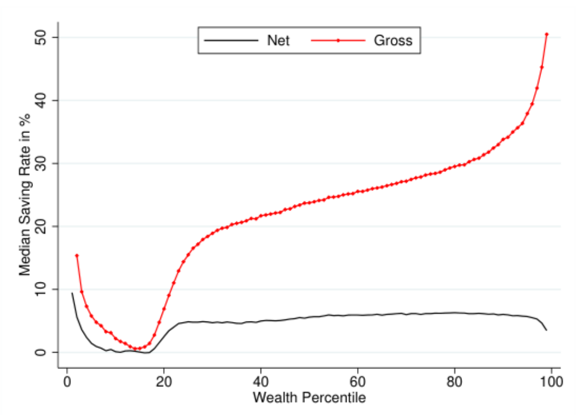


Figure 7: Figure 1 from Fagereng et al (2019)

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- Returns are heterogeneous, and higher for richer households
- The rich save more, mostly because of capital gains

Explaining wealth inequality

Aiyagari Model

- Infinitely lived agents with preferences

$$\max_{\{c_t\}_{t=0}^{\infty}} E \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}$$

- Budget constraint and borrowing constraint

$$a_t = y_t + (1+r)a_{t-1} - c_t, \quad a_t \geq \underline{a}$$

- Idiosyncratic earnings risk:

$$\ln y_t = \rho \ln y_{t-1} + \epsilon_t, \quad \epsilon_t \sim \mathcal{N}\left(0, \sigma_\epsilon^2\right)$$

- As usual, calibrate parameters in earnings process $\left(\rho, \sigma_\epsilon^2\right)$ based on estimates from panel data on earnings, i.e. Floden and Linde (2001)

Aiyagari Model - wealth inequality fit

	Wealth Gini	Wealth in top (%)		
		1%	5 %	20 %
U.S. data, 1989 SCF				
	.78	29	53	80
Aiyagari Baseline				
	.38	3.2	12.2	41.0

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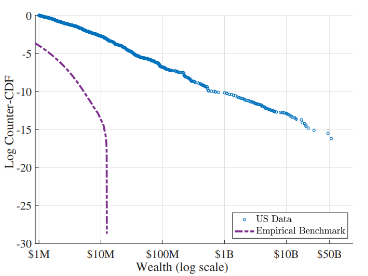
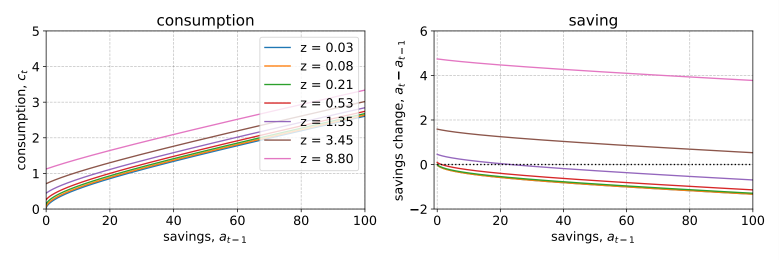


Policies in the Buffer-Stock model



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- Note also: Only driver of wealth inequality is earnings risk
  - Income inequality in data typically lower than *wealth* inequality
  - In reality multiple drivers such as entrepreneurship, preferences, bequests, return heterogeneity

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- Return heterogeneity
    - Hubmer, Krusell, Smith (2021), Ozkan et al. (2023), Guvenen et al. (2023)

Bequests

$$\max_{\{c_t\}_{t=0}^{T}} E\sum_{t=0}^{T}\beta^t\left(s_t\frac{c_t^{1-\sigma}}{1-\sigma}+(1-s_t)\,\phi\,(a_{t-1})\right)$$

$$c_t+a_t=y_t+(1+r)a_{t-1}+b_t,\quad a_t\geq\underline{a}$$

1. Bequests and human capital transmission across generations (*warm glow*)

Explaining wealth inequality

$$\max_{\{c_t\}_{t=0}^{T}} E \sum_{t=0}^{T} {\beta_i}^t s_t \frac{c_t^{1-\sigma_i}}{1-\sigma_i}$$

$$c_t + a_t = y_t + (1+r)a_{t-1}, \quad a_t \geq \underline{a}$$

1.
2. Heterogeneous preferences

Explaining wealth inequality

$$\max_{\{c_t\}_{t=0}^{T}} E \sum_{t=0}^{T} \beta^t s_t \frac{c_t^{1-\sigma}}{1-\sigma}$$

$$c_t + a_t = \left[l_e f\left(\theta_t, k_{t-1}\right) + \left(1 - l_e\right) y_t \right] + \left(1 + r\right)\left(a_{t-1} - k_{t-1}\right), \quad a_t \geq \underline{a}$$

1.
2.
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$$c_t + a_t = y_t + \left(1 + r_t^i\right) a_{t-1}, \quad a_t \geq \underline{a}$$

1.
2.
3.
4. Idiosyncratic rates of return

Gaillard, Hellwig, Wanger and Werguin (2024)

Ranking of Pareto tails in the Data

Empirical ranking of Pareto tails (US):

capital income $<$ wealth $<$ labor income $<$ consumption

Table 1. Top consumption, income, and wealth ineq

Data	Variable	Best fit Pareto[b] $\underline{\hat{x}}^{OLS}$	$\hat{\zeta}^{OLS}$	$\underline{\hat{x}}^{MLE}$	$\hat{\zeta}^{MLE}$
PSID	Capital income	0.96	**1.22**	0.96	**1.21**
		(0.02)	(0.15)	(0.02)	(0.14)
	Wealth	0.93	**1.48**	0.92	**1.47**
		(0.03)	(0.09)	(0.03)	(0.09)
	Labor income	0.88	**2.42**	0.89	**2.50**
		(0.04)	(0.15)	(0.04)	(0.13)
	Consumption	0.89	**3.11**	0.90	**3.13**
		(0.04)	(0.28)	(0.04)	(0.20)
	Food consumption	0.93	**4.40**	0.93	**4.26**
		(0.05)	(0.33)	(0.05)	(0.43)

Theoretical results

Using a continuous-time HA model, they show that we need two key features to match this ranking:

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(they also allow for random returns, death probability, and progressive taxes)

Quantitative model overview

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On the supply side, a classic Cobb-Douglas production function with perfect competition.

Quantitative results - Bellman equation

$$
\begin{aligned}
V(y,z,a) = \max_{c,a' \geq a} \frac{c^{1-\gamma}}{1-\gamma} + \kappa \frac{(a/A)^{1-\nu}}{1-\nu} \\
+ \beta(1-\xi) \sum_{y' \in \mathcal{Y}} \sum_{z' \in \mathcal{Z}} P\left(y' \mid y\right) P\left(z' \mid z\right) V\left(y', z', a'\right) \\
\text{s.t. } c + a' = wy - T(wy) + \left(1 - \tau_K\right) rzS(a)a + a
\end{aligned}
$$

with

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- $S(a) = 1 + \psi a^{\eta}$ (scale dependence)

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$\Rightarrow$ Can explain why the rich save so much!

A note on death probability

Many HA models with a Pareto tail have a death probability:

- Main assumption: every period, households face a constant probability to die ξ

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$\rightarrow$ Especially important in a non-homothetic model to have a non-degenerate distribution of wealth: we need a force to stop them from accumulating infinite amounts of wealth.

Quantitative results - details on heterogeneous returns

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$\rightarrow$ as an entrepreneur, you want to save a lot because you get temporarily very high returns on your wealth.

Quantitative results - supply side and market clearing

The rest of the model is standard:

- $Y = K^{\alpha} L^{1-\alpha}$, factors paid their marginal productivity
- Asset market clearing: $K = A = \int zS(a) a dF(y, z, a)$
- Government budget balances (government fully taxes accidental bequests).

Quantitative results - main exercise

They calibrate most of the model parameters, and estimate:

1. κ: strength of taste for wealth
2. ν: exponent of the taste for wealth
3. z_h: excess returns of high-return type
4. ψ: strength of scale dependence
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And they target the following moments (at the steady-state)

1. Ratio of capital income/wealth Pareto coefficients
2. Ratio of consumption to wealth Pareto coefficients
3. Ratio of wealth to labour income Pareto coefficients

+ Capital income to wealth tail for the top 1% + W/Y=3.8 + Top 1% wealth share

Results

Table 4. Counterfactual models and selected moments.

	Data/Model	Pareto tails: mean MLE estimate[a]					Top 1% wealth	$\frac{\text{Wealth}}{\text{income}}$ ratio
		ζ_c	ζ_y	ζ_y^{net}	ζ_a	ζ_{ra}		
	Adjusted PSID (2005–2021)	3.06	2.25	2.57	1.38	1.20	0.35	3.8
Homothetic preferences								
(1)	Homogeneous returns	3.04	2.25	2.57	2.42	2.42	0.09	3.8
(2)	Type-dependence	2.65	2.25	2.57	1.32	1.02	0.29	3.7
(3)	Scale-dependence	2.56	2.25	2.57	1.30	1.16	0.32	3.7
(4)	Type- and scale-dependence	2.65	2.25	2.57	1.34	1.08	0.35	3.6
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$\rightarrow$ many models can generate high degree of wealth inequality.
But both non-homothetic and type-dependence are key to match relative ranking of Pareto tails!

Hubmer, Krussel and Smith (2021)

Explaining wealth inequality

- Hubmer, Krussel and Smith (2021): *Sources of US wealth inequality: Past, present, and future*
  - Model which matches key features of US wealth inequality in 1967
  - Can we account for changes in wealth inequality going forward from 1967 based on observables?
    - I.e. changes in income inequality, taxes, asset returns

Model

- Household problem features non-linear tax schedules, heterogeneous returns and β-het.

$$V_t(a_{t-1}, p_t, \beta_t) = \max_{a_{t+1} \geq 0} \{u(c_t) + \beta_t \mathbb{E}[V_{t+1}(a_t, p_{t+1}, \beta_{t+1}) | p_t, \beta_t]\}$$

$$\begin{aligned} c_t + a_t =& y_t - \tau_t(y_t) + (1 - \tilde{\tau}_t)\tilde{y}_t + T_t \\ y_t =& (\underline{r_t} + r_t^X(a_{t-1}))a_{t-1} + w_t l_t(p_t) \\ \tilde{y}_t =& \sigma_t^X(a_{t-1})\eta_t a_{t-1} \end{aligned}$$

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  - How does return uncertainty vary with wealth?
- Example: If rich HHs primarily invest in stocks, poorer HHs in bonds. Would expect both $r_t^X(a_{t-1}), \sigma_t^X(a_{t-1})$ to be increasing in a_{t-1}

Facts

- Fagereng, Guiso, Malacrino, Pistaferri (2020) find that rates of returns are:
  - Heterogeneous across households (over 200 basis points between 10th and 90th percentile of the distribution of returns)
  - Also heterogenous within asset classes
    - So return differences cannot be explained only by poorer HHs holding banket deposits and rich HHs investing in stocks
  - Persistent
  - Correlated with household wealth and across generations

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- Plus goods market clearing, but redundant given other 2

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1. Calibrate earnings process, tax rates, return process, social safety net to observables
2. Choose randomness in discount factor β residually so as to replicate the wealth distribution in the initial steady state (1967)
3. Then feed in exogenous changes in tax rates, earnings inequality, etc. between 1967 and 2015 to understand the role of these different factors

Return heterogeneity

- Overall return given asset holdings a_{t-1} equals

$$\underline{r}_t + r_t^X(a_{t-1}) + \sigma^X(a_{t-1})\eta_t$$

- $\underline{r}_t$ is endogenous
- $r_t^X(\cdot)$ and $\sigma^X(\cdot)$ are exogenous excess return schedules (mean and st.dev.), taken from the data
- η_t is an i.i.d. standard normal shock
- Reduced form portfolio choice

Calibration: return process

$$r_t^X(a_t) = \sum_{c \in C} w_c(a_t) \left(\bar{r}_{c,t} + \tilde{r}_c^X(a_t) \right)$$

$$\sigma^X(a_t)^2 = \sum_{c \in C} \left(w_c(a_t) \tilde{\sigma}_c^X(a_t) \right)^2$$

- Asset classes C: risk-free, public equity, private equity, housing
- $\bar{r}_{c,t}$: aggregate return on asset class c (U.S. data), time-varying
- Fixed over time, based on Swedish administrative data from Bach, Calvet, Sodini (2016):
  - $w_c(\cdot)$: portfolio weights
  - $\tilde{r}_c^X(\cdot)$: within asset class return heterogeneity
  - $\tilde{\sigma}_c^X(\cdot)$: asset c idiosyncratic return standard deviation

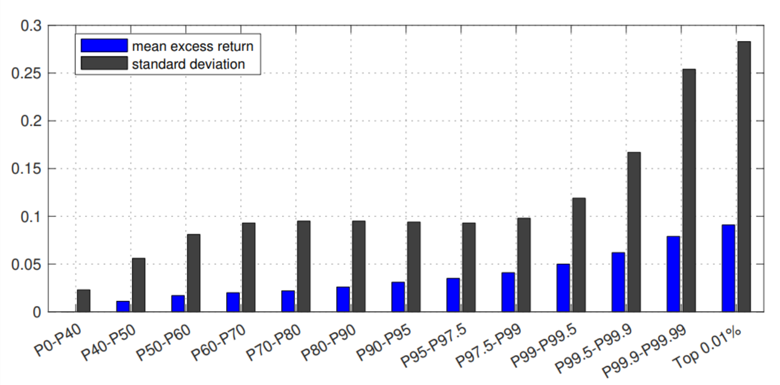
Excess return schedule details

- Aggregate Excess Returns in 1967 steady state:
  - public equity 0.067 (U.S., Kartashova 2014)
  - private equity 0.129 (U.S., Kartashova 2014)
  - housing 0.037 (incl. imputed rent; Jorda, et al, 2017)

- and cross-sectional data from Bach, Calvet, Sodini (2019) implies

	P0-P40	P40-P50	P50-P60	P60-P70	P70-P80	P80-P90	P90-P95	P95-P97.5	P97.5-P99	P99-P99.5	P99.5-P99.9	P99.9-P99.99	Top 0.01%
fixed portfolio weights													
risk-free	0.722	0.412	0.248	0.182	0.156	0.134	0.115	0.102	0.090	0.079	0.071	0.051	0.029
housing	0.162	0.394	0.580	0.662	0.678	0.674	0.658	0.626	0.572	0.482	0.363	0.253	0.155
public equity	0.113	0.189	0.165	0.147	0.153	0.170	0.189	0.207	0.219	0.232	0.230	0.185	0.179
private equity	0.002	0.005	0.007	0.009	0.013	0.021	0.038	0.065	0.118	0.207	0.336	0.511	0.637

Schedule of excess returns



Data sources: Bach, Calvet, Sodini (2019); Kartashova (2014); Jorda, Knoll, Kuvshinov, Schularick, Taylor (2019); Case-Shiller.

Hubmer, Krussel and Smith (2021)

Results

Results, I: Steady state (1967)

- Steady state fit (with and without β-het)

	Top 10%	Top 1%	Top 0.1%	Top 0.01%
Data	70.8%	27.8%	9.4%	3.1%
Single-β Model	66.6%	23.7%	11.2%	7.2%
Benchmark Model	73.8%	27.4%	8.4%	3.2%
	Bottom 50%	Fraction $a < 0$		
Data	4.0%	8.0%		
Single-β Model	3.5%	7.3%		
Benchmark Model	3.0%	6.6%		

Results, I: steady state (1967)

#		top 10%	top 1%	top 0.1%	top 0.01%	Gini
1	β-heterogeneity	8.8%	7.7%	3.8%	2.0%	0.050
2	earnings heterogeneity	−27.5%	−17.8%	−9.5%	−6.4%	−0.173
3	persistent	−5.0%	−7.5%	−4.2%	−2.9%	0.009
4	transitory	−11.6%	−4.3%	−1.7%	−0.9%	−0.109
5	tax progressivity	−21.3%	−61.8%	−71.2%	−67.1%	−0.148
6	return heterogeneity	29.5%	18.4%	6.6%	2.8%	0.192
7	mean differences	25.8%	16.7%	6.0%	2.6%	0.174
8	return risk	0.7%	2.2%	3.3%	2.5%	0.004

- How to read: Shutting of β-het reduces top 10% wealth share by 8.8%
- Model matches wealth distribution well on its entire domain
  - return heterogeneity is key ingredient
  - wealth concentration is mitigated by progressive taxation and labor income risk

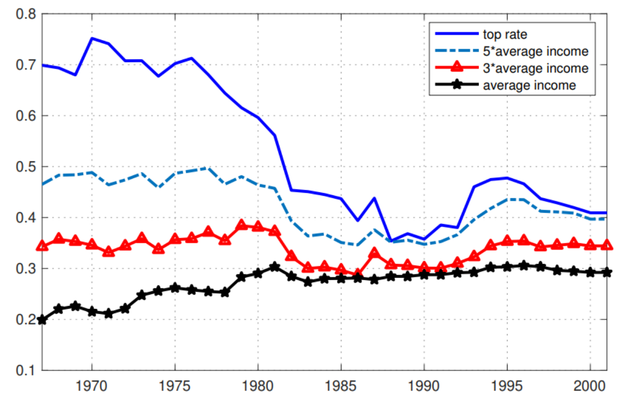
Next step: transition

The authors feed in four different factors that have changed during the past 50 years

- Decrease in tax progressivity
- Increase in labor income risk
- Increase in income going to the top
- Changing return premia to different asset classes

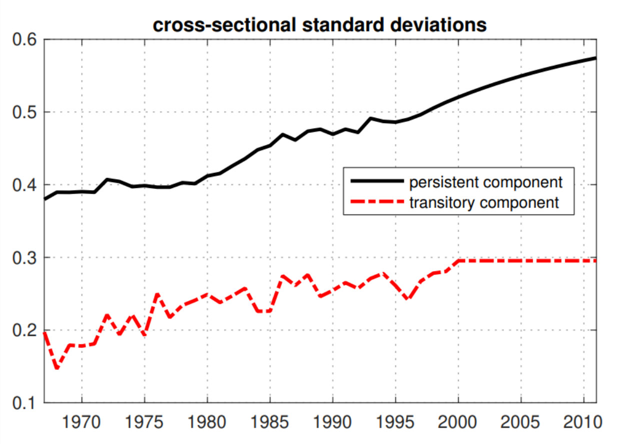
Observed change 1: Decrease in tax progressivity

- Federal effective tax rates (Piketty & Saez 2007): income, payroll, corporate and estate taxes



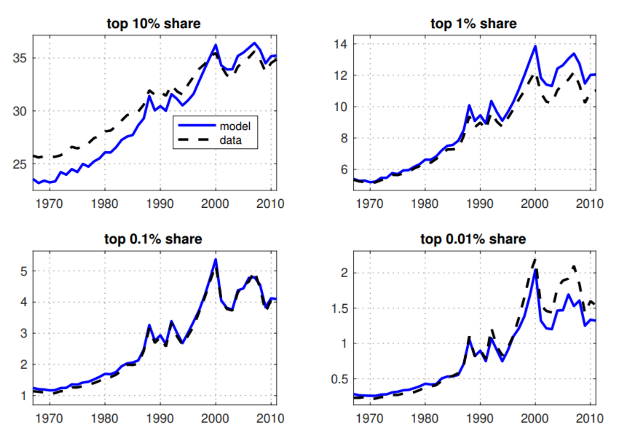
Observed change 2: Increase in labor income risk

- Estimates for variance of persistent and temporary components 1967-2000 (Heathcote, Storesletten & Violante 2010)



Observed change 3: Increase in top labor income shares

- Adjust standard AR(1) in idiosyncratic productivity by imposing a Pareto tail for the top 10% earners: calibrated tail coefficient decreases from 2.8 to 1.9 (updated Piketty & Saez 2003 series)



Observed change 4: return premia

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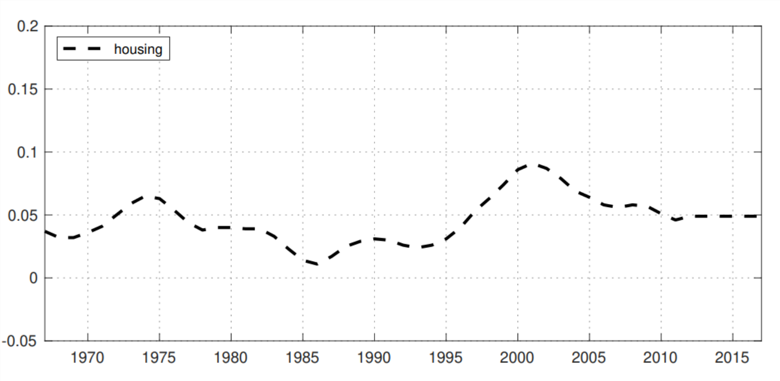

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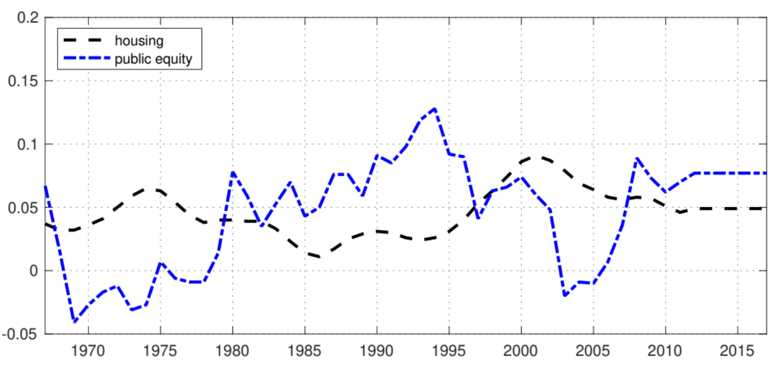

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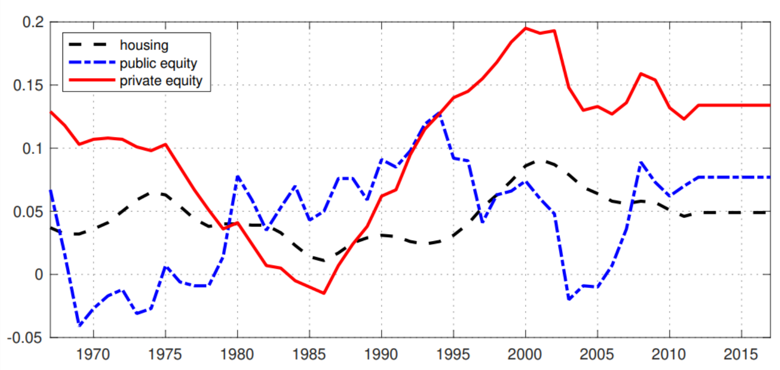
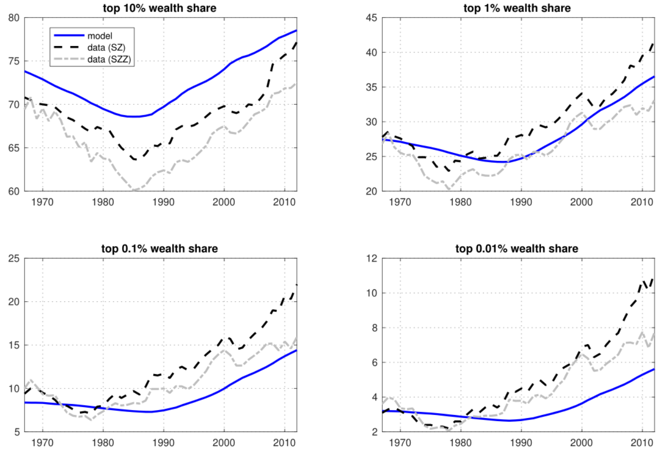
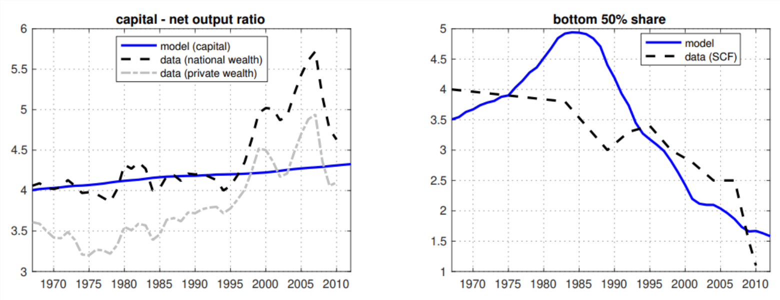


Results, II: historical evolution



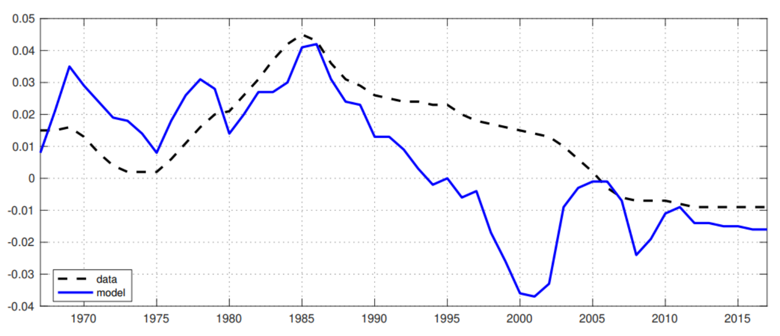
Data sources: dashed black lines refer to Saez & Zucman (2016); dash-dotted gray lines refer to Smith et al. (2020).

Results: Capital-output ratio and bottom 50 %

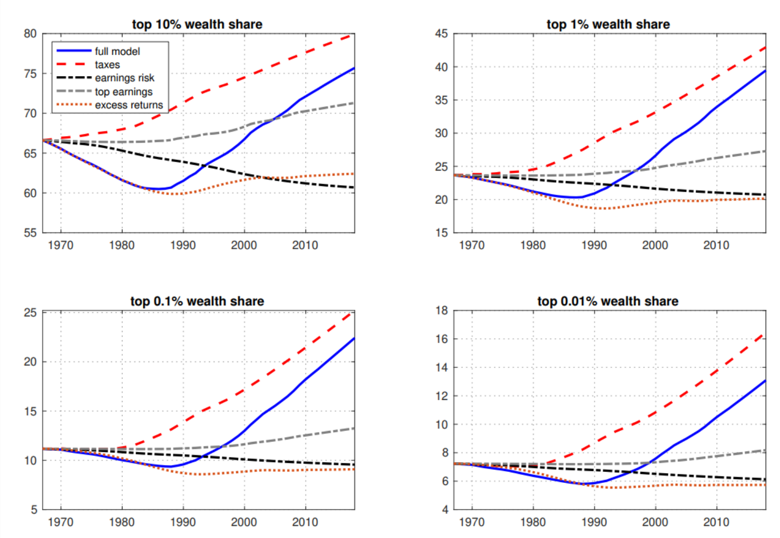


Results: Risk-free rate

- Return premia are matched in model by construction
- Risk-free rate r is endogenous: comparable level and decline



Decomposition of transitional dynamics



Decomposition of transitional dynamics

- Overall increase in wealth inequality (more than) fully explained by declining tax progressivity
  - Primarily due to direct effect on resource distribution and not due to changing savings behavior
- Time-varying return premia account for U-shape in wealth inequality
- Subtle role of increasing earnings dispersion
  - Thickening Pareto tail in labor income contributes slightly positively to wealth inequality
  - Increase in overall earnings risk *decreases* wealth inequality because precautionary savings motive is stronger for poorer HHs

Summary

- **Hubmer, Krussel and Smith (2021)**
- HANC with:
  - Income risk
  - Return heterogeneity
  - β-heterogeneity
  - Tax system
- Main finding:
  - Return heterogeneity key in matching initial (1967) wealth inequality
  - Can roughly explain evolution in US wealth inequality with observable changes in tax systems

Ozkan et al. (2024)

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  - Why do some people become wealther than others?
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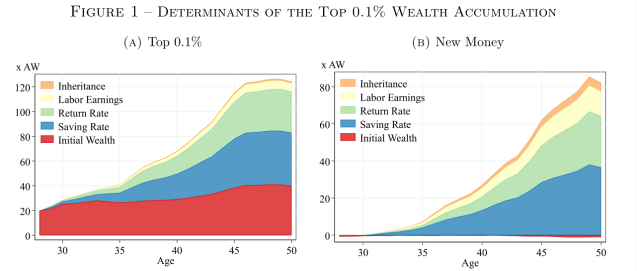
  5. Initial wealth a_{i0}
- Using budget constraint:

$$a_{it} = a_{it-1} + \left(L_{it} + H_{it} + r_{it} a_{it-1}\right) \times s_{it}$$

Results from Ozkan et al. (2024)

- Left panel: Decomposition of wealth for top 0.1%
- Right panel: »Poorest« HHs *within* top 0.1% *(New Money)*



Application to Wealth Taxation

Wealth taxation I

- Spend a lot of time understanding *what* drives wealth inequality
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        - Efficiency concern 2: In the Ramsey model aggregate K is generally below the golden rule level

Wealth taxation II

- Guvenen et al. (2023): *Use It or Lose It: Efficiency and Redistributional Effects of Wealth Taxation*
- Study optimal taxation in two tax systems:
  - Wealth tax: a_i
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- Social planner can implement same allocation using these two different instruments by setting $\tau_a = r\tau_k$

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  - Shifts tax base towards unproductive agents
- Note: We say HHs with high r_i are more **productive**
  - Think in terms of *entrepreneurial* models
  - High productivity HHs have better technology (i.e. are better entrepreneurs) and can make their wealth growth faster (high r_i)

Model

- HH problem:

$$\max_{\{c_t\}_{t=0}^{T}} E \sum_{t=0}^{T} \beta^t \left(s_t \frac{c_t^{1-\sigma}}{1-\sigma} + (1-s_t)\,\phi(a_t) \right)$$

$$a_t + c_t = \mathcal{W}(a_{t-1}, z_{t-1}) + w_t(e_t)\,\ell_t, \quad a_t \geq \underline{a}$$

$$\mathcal{W}(a_{t-1}, z_t) = \begin{cases} a_{t-1} + (\pi(a_{t-1}, z_t) + r a_{t-1})(1-\tau_k) & \text{if CI tax} \\ a_{t-1}(1-\tau_a) + (\pi(a_{t-1}, z_t) + r a_{t-1}) & \text{if wealth tax} \end{cases}$$

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- Entrepreneurial abilitiy z follow markov chain with values $z = [0, z_L, z_H]'$ and transition matrix Π_z
  - HHs with $z = 0$ are normal workers
  - HHs with $z = z_L$ are »unproductive« entrepreneurs
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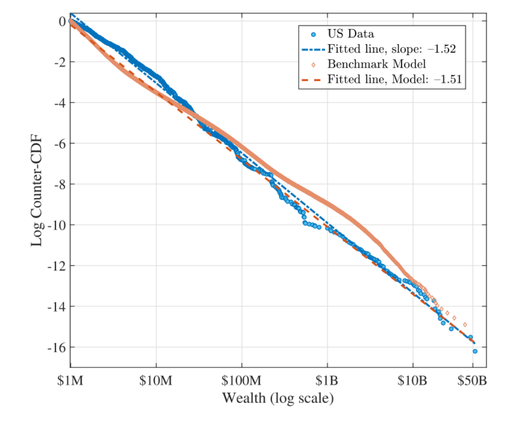
- Entrepreneurial profit $\pi\,(a_{t-1}, z_{t-1})$ given by:

$$\pi\,(a_{t-1}, z_t) = \max_{k_t < \kappa a_{t-1}} \{p_t z_t k_t - (r+\delta)\,k_t\}$$

Empirical fit

- Calibrate model to US. Model reproduces wealth inequality in the data, also for the extremely rich



Results

- Exercise: Replace capital income tax $\tau_k = 25\%$ with wealth tax $\tau_a > 0$ in a government revenue-neutral way (requires $\tau_a = 1.2\%$)

TABLE V

TAX REFORM: CHANGE IN MACRO VARIABLES FROM CURRENT U.S. BENCHMARK

	Quantities (% change)						Prices (change)			
	K	Q	TFP_Q	L	Y	C	$\overline{w}$	$\overline{w}$ (net)	$\Delta r^{\dagger}$	$\Delta r^{\dagger}$ (net)
RN reform	16.4	22.6	5.3	1.2	9.2	9.5	8.0	8.0	0.21	–0.36
BB reform	9.2	16.0	6.2	1.2	6.9	7.7	5.6	5.6	0.67	–0.38

- Capital, productivity output, consumption, wages increases
  - Efficency gain from shifting tax base away from productive agents
- Also generates large welfare gain (around 7% consumption equivalent gains)

Results - optimal taxation

- Now find tax rates that maximize aggregate welfare
  - Wealth taxation (OWT) vs. capital income taxation (OKIT)
- Results:

OPTIMAL TAXATION: TAX RATES AND AVERAGE WELFARE EFFECTS

	Benchmark U.S. economy	RN reform (1)	OWT (2)	OWT L-INEQ (3)	OWT-X (4)	WTE-X (5)	OKIT (6)
Tax rates							
τ_k	25.0	—	—	—	—	—	–13.6
τ_a	—	1.19	3.03	2.54	3.80†	3.30	—
τ_ℓ	22.4	22.4	15.4	18.1	14.4	17.7	31.2
ΔWelfare							
$\overline{CE}_1$	—	6.8	9.0	6.0	9.1	8.4	4.2
$\overline{CE}_2$	—	7.2	8.7	5.2	8.8	8.6	5.1

- Wealth taxation: Positive taxation $\tau_a = 3.03\%$, large welfare gain of 9%
- Capital income taxation: *Subsidy* $\tau_K = -13.6\%$ and smaller welfare gain of 4.2%

Summary

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- Guvenen et al. (2023) study optimal wealth taxation
- Source of wealth inequality matters for optimal taxation
- If driven by return heterogeneity **wealth tax** strongly preffered to **capital income tax**
  - Why? It distorts investment decisions of high productivity HHs less than a capital income tax

Exercise

Standard HANC model with return heterogeneity

- HH problem:

$$v_t(, e_{it} r^x_{it}, , a_{it-1}) = \max_{c_t} u(c_t) + \beta \underline{v}_{t+1}(e_{it+1}, r^x_{it+1}, a_{it})$$

s.t.

$$\begin{aligned}
a_{it} &= (1 + r_t + r^x_{it}) a_{it-1} + w_t e_{it} - c_{it} \\
\log e_{it+1} &= \rho_e \log e_{it} + \psi^e_{it+1}, \quad \psi^e_{it+1} \sim \mathcal{N}\left(0, \sigma^2_e\right) \\
r^x_{it+1} &= \overline{r}^x + \rho_z r^x_{it} + \psi^{r^x}_{it+1}, \quad \psi^{r^x}_{it+1} \sim \mathcal{N}\left(0, \sigma^2_{r^x}\right) \\
a_{it} &\geq 0
\end{aligned}$$

- **Q1**: Solve the PE HA model with return heterogeneity
- **Q2**: Calibrate the HANC model such that average returns are 4%
- **Q3**: Calibrate a standard HA model without return heterogeneity. Compare the wealth distributions obtained in the two models.

Summary

Summary and next week

- **Today:** Various explanations of wealth inequality
  1. Preferences
  2. Bequests
  3. Returns
- **Next week:** Secular stagnation
- **Midterm evaluation:** Don't forget to fill out questionnaire
- **Homework exercise**: Solve model with return heterogeneity
  - See Github repo