11. Monetary Policy in HANK

Adv. Macro: Heterogenous Agent Models

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Summing Up What We Did So Far:

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- **Dedicate last hour to the 2nd assignment.**

Introduction

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- **Today:**
  - Other pillar of stabilization policy: **Monetary policy**
  - Will use as example to study alternatives to **rational expecations** (RE) in HANK
- **Literature:**
  - *Seminal paper*: Kaplan, Moll, Violante (2018) »Monetary policy according to HANK«
  - Auclert Rognlie, Straub (2020) »Micro jumps, macro humps«
  - Alves, Kaplan, Moll, Violante (2020) »A further look at the propagation of monetary policy shocks in HANK«

Monetary Policy in HANK

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- Introducing heterogeneous agents into the standard NK model **fundamentally** changes the transmission of Fiscal Policy
  - *Potentially* more effective
  - Important whether policy is deficit financed or tax financed

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- What about monetary policy?

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    - **Solution:** Firm equity

Households

- **Household problem:**

$$
\begin{aligned}
v_t(z_t, a_{t-1}) &= \max_{c_t} \frac{c_t^{1-\sigma}}{1-\sigma} - \varphi \frac{\ell_t^{1+\nu}}{1+\nu} + \beta \mathbb{E}_t \left[v_{t+1}(z_{t+1}, a_t) \right] \\
\text{s.t. } \ a_t + c_t &= (1 + r_t^a) a_{t-1} + Z_t z_t + \chi_t \\
\log z_{t+1} &= \rho_z \log z_t + \psi_{t+1} \ \ , \psi_t \sim \mathcal{N}(\mu_\psi, \sigma_\psi), \ \mathbb{E}[z_t] = 1 \\
a_t &\geq 0
\end{aligned}
$$

- with $Z_t = w_t \ell_t$ - real labor income
- **decisions:** Consumption-saving, c_t (and a_t)
- **Union decision:** Labor supply, ℓ_t
- **Aggregate Consumption:** $C_t^{hh} = \int c_t d\mathcal{D}_t$
- **Consumption function:** $C_t^{hh} = C^{hh} \left(\{ r_s^a, Z_s, \chi_s \}_{s=0}^{\infty} \right)$

Firms

- **Production and profits:**

$$Y_t = L_t$$

$$\Pi_t = Y_t - w_t L_t$$

- Optimize subject to demand curve (monopolistic competition)
- **First order condition:**

$$w_t = \frac{1}{\mu}$$

- where $\mu > 1 =$ markup - firms make positive profits in equilibrium

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- Total value of firm equity is then $\int p^D_t \upsilon_{j,t} dj = p^D_t$

Mutual fund II

- **Problem**:

$$\max_{\upsilon_{j,t}} \int \left(\Pi_{j,t+1} + p^{D}_{j,t+1}\right) \upsilon_{j,t} - \left(1 + r^{a}_{t+1}\right) A_t$$

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- Valuation effects: As with nominal gov bonds:

$$1 + r^a_t = \begin{cases} \frac{\Pi_0 + p^D_0}{p^D_{ss}} & t = 0 \\ 1 + r_{t-1} & t > 0 \end{cases}$$

Union

- Everybody works the same:

$$\ell_t = L_t^{hh}$$

- Maximization subject to wage adjustment cost imply a **New Keynesian Wage (Phillips) Curve** (NKWPC or NKWC)

$$\pi_t^w = \kappa \left(\varphi \left(L_t^{hh} \right)^{\nu} - \frac{1}{\mu} \left(1 - \tau_t \right) w_t \left(C_t^{hh} \right)^{-\sigma} \right) + \beta \pi_{t+1}^w$$

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- **2. Alternative:** Real rate rule. CB chooses real rate r_t directly

$$r_t = r_{ss} + (\phi - 1)\,\pi_t$$

Market clearing

1. Asset market: $p_t^D = A_t^{hh}$
2. Labor market: $L_t = L_t^{hh}$
3. Goods market: $Y_t = C_t^{hh}$

The consumption function

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$$C_t^{hh} = C_t^{hh}\left(\{r_s^a, Z_s\}_{s=0}^{\infty}\right) \Rightarrow \boldsymbol{C}^{hh} = C^{hh}\left(\boldsymbol{r}^a, \boldsymbol{Z}\right)$$

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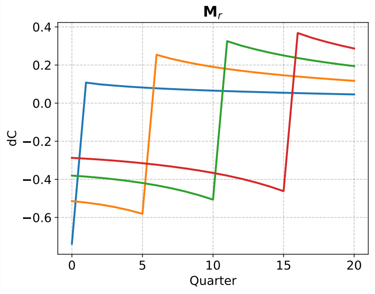
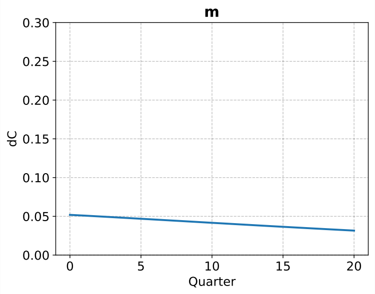
- Note: $\boldsymbol{m}$ is a vector not matrix (multiplies onto scalar $dcap_0$, not vector)

Interest rate Jacobians





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- **Q2**: Do you expect the effects of monetary policy on output to be larger in HANK than RANK?

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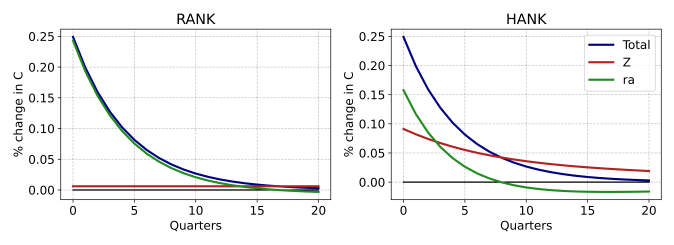
- **... but transmission channel is different**
- Decompose $d\boldsymbol{Y}$ into direct and indrect effect using $d\boldsymbol{Y}^j = \boldsymbol{M}^j_{r^a} d\boldsymbol{r^a} + \boldsymbol{M}^j d\boldsymbol{Z}$ for $j \in \{HA, RA\}$



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- How does the effectiveness of monetary policy look in more realistic models?
  - Kaplan, Moll, Violante (2018) »Monetary policy according to HANK«
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KMV 2018

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- They study the transmission of monetary policy in medium scale HANK model
- Follows Kaplan & Violante (2014) closely
  - See lecture 2
  - Household can hold both liquid and illiquid assets
  - Model features both **poor** and **wealthy** Hand-to-mouth households

Household problem

- Households solve (here converted to discrete time, paper in cont. time):

$$V_t\left(a_{t-1}, b_{t-1}, z_t\right) = \max_{c_t, a_t, b_t} u\left(c_t, \ell_t\right) + \beta E_t V_{t+1}\left(a_t, b_t, z_{t+1}\right)$$

$$b_t + c_t = (1-\tau_t) w_t z_t \ell_t + \left(1 + r_t^b\right) b_{t-1} - d_t - \chi(d_t, a_{t-1})$$

$$a_t = \left(1 + r_t^a\right) a_{t-1} + d_t$$

$$b_t \geq -\bar{b} \quad a_t \geq 0.$$

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- with b_t=liquid asset, a_t=illiquid assets, d_t=deposits into illiquid asset, $\chi(d_t, a_{t-1})$ a convex cost

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- Households solve (here converted to discrete time, paper in cont. time):

$$V_t\left(a_{t-1}, b_{t-1}, z_t\right) = \max_{c_t, a_t, b_t} u\left(c_t, \ell_t\right) + \beta E_t V_{t+1}\left(a_t, b_t, z_{t+1}\right)$$

$$b_t + c_t = (1-\tau_t) w_t z_t \ell_t + \left(1 + r_t^b\right) b_{t-1} - d_t - \chi(d_t, a_{t-1})$$

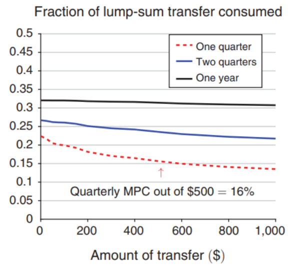
$$a_t = \left(1 + r_t^a\right) a_{t-1} + d_t$$

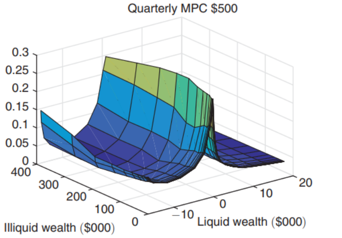
$$b_t \geq -\bar{b} \quad a_t \geq 0.$$

- with b_t=liquid asset, a_t=illiquid assets, d_t=deposits into illiquid asset, $\chi(d_t, a_{t-1})$ a convex cost
- Return on illiquid asset r_t^a Return on liquid asset r_t^b
    - Household will prefer to hold a_t due to superior return
    - But not good for consumption smoothing as they have to pay adjustment cost to use a_t for smoothing against shocks
    - Some HHs will be wealthy hand-to-mouth

MPCs

- 1) MPCs for different sizes of stimulus checks, 2) MPCs across the wealth distribution





Direct vs indirect effects

- Amplification in HANK (elasticity of $C^{HANK} = -2.9$ vs $C^{RANK} = -2.07$)

Direct vs indirect effects

- Amplification in HANK (elasticity of $C^{HANK} = -2.9$ vs $C^{RANK} = -2.07$)
- Baseline HANK: Indirect effects account for majority of transmission ($\approx$ 80%)

TABLE 7—DECOMPOSITION OF THE EFFECT OF MONETARY SHOCK ON AGGREGATE CONSUMPTION

	Baseline (1)	$\omega = 1$ (2)	$\omega = 0.1$ (3)	$\frac{\varepsilon}{\theta} = 0.2$ (4)	$\phi = 2.0$ (5)	$\frac{1}{\nu} = 0.5$ (6)
Change in r^b (pp)	−0.28	−0.34	−0.16	−0.21	−0.14	−0.25
Elasticity of Y	−3.96	−0.13	−24.9	−4.11	−3.94	−4.30
Elasticity of I	−9.43	7.83	−105	−9.47	−9.72	−9.79
Elasticity of C	−2.93	−2.06	−6.50	−2.96	−3.00	−2.87
Partial eq. elasticity of C	−0.55	−0.45	−0.99	−0.57	−0.59	−0.62
Component of percent change in C due to						
Direct effect: r^b	19	22	15	19	20	22
Indirect effect: w	51	56	51	51	51	38
Indirect effect: T	32	38	19	31	31	45
Indirect effect: r^a and q	−2	−16	15	−2	−2	−4

Expectations

Micro Jumps, Macro Humps

- Auclert, Rognlie and Straub (2020) »Micro jumps, macro humps«

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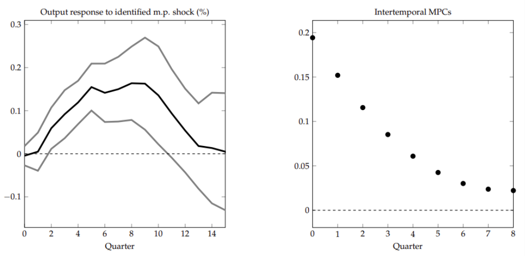
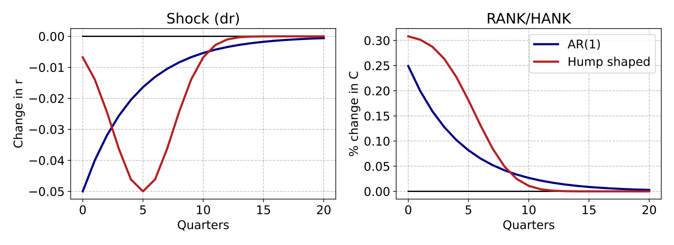
Micro Jumps, Macro Humps

- Auclert, Rognlie and Straub (2020) »Micro jumps, macro humps«
- Estimate parameters in quantitative HANK model to match estimated effects of causal monetary policy shock
- Main hurdle: Empirical response of C, Y is hump-shaped to monetary policy shock.
- Want a model that simultaneously match hump-shaped agg. response to r and iMPC moment



The problem

- Standard model does **not** give hump shaped for C to standard shock
- Does not matter if **shock** is hump shaped or not



The solution: RANK

- Solution in RANK litterature: Habits in utility function:

$$\sum_{t=0}^{\infty} \beta^t u\left(C_t - \gamma C_{t-1}\right)$$
$$\Rightarrow u'\left(C_t - \gamma C_{t-1}\right) = \beta R_{t+1} u'\left(C_{t+1} - \gamma C_t\right)$$

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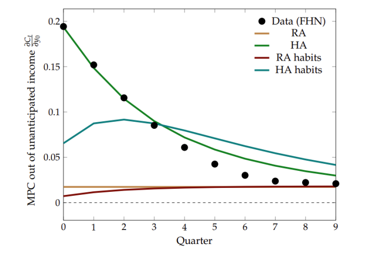
- Generates persistence in C response to shocks because household don't want to deviate too much from last periods consumption level
- **However**: Does not work in HANK because it kills iMPCs:



Deviations from alternative expecations

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Deviations from alternative expecations

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  - Implies that steady state is unaffected
  - Still rational expecations w.r.t idiosyncratic income shocks
- Will only implement this to first-order (e.g. linear approximations)
  - Much more difficult if we want full non-linear solution

Income Jacobian

- Example: Response of aggregate consumption $\boldsymbol{C}$ to change in agg. income $\boldsymbol{Z}$

$$d\boldsymbol{C} = \boldsymbol{M} d\boldsymbol{Z}$$

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- Note that elements above diagonal are affected by expectations (i.e. they concern the **future**)
    - Elements on and below diagonal reflect changes in income **today** or in the **past** (known by HHs)

Expectations matrix

- Introduce expectations matrix $\boldsymbol{E}$:

$$\boldsymbol{E} = \begin{bmatrix} 1 & * & * & * & \cdots \\ 1 & 1 & * & * & \cdots \\ 1 & 1 & 1 & * & \cdots \\ 1 & 1 & 1 & 1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

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- How to get jacobian $\hat{\boldsymbol{M}}$ associated with $\boldsymbol{E}$?

Stylized Example I

- Expectations matrix:

$$\boldsymbol{E} = \begin{bmatrix} 1 & 0.4 & 0.3 & \cdots \\ 1 & 1 & 0.6 & \cdots \\ 1 & 1 & 1 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

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- **Fast** and **easy** to implement

Examples

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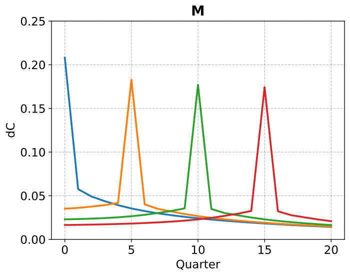
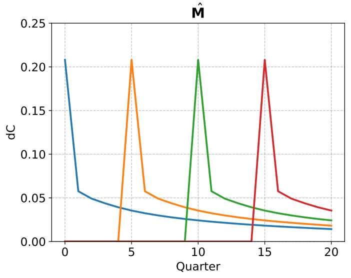
$$\hat{\boldsymbol{M}}^{\text{RE}} = \boldsymbol{M} = \begin{bmatrix} \frac{\partial C_0}{\partial Z_0} & \frac{\partial C_0}{\partial Z_1} & \frac{\partial C_0}{\partial Z_2} & \cdots \\ \frac{\partial C_1}{\partial Z_0} & \frac{\partial C_1}{\partial Z_1} & \frac{\partial C_1}{\partial Z_2} & \cdots \\ \frac{\partial C_2}{\partial Z_0} & \frac{\partial C_2}{\partial Z_1} & \frac{\partial C_2}{\partial Z_2} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}, \qquad \hat{\boldsymbol{M}}^{\text{Myopic}} = \begin{bmatrix} \frac{\partial C_0}{\partial Z_0} & 0 & 0 & \cdots \\ \frac{\partial C_1}{\partial Z_0} & \frac{\partial C_0}{\partial Z_0} & 0 & \cdots \\ \frac{\partial C_2}{\partial Z_0} & \frac{\partial C_1}{\partial Z_0} & \frac{\partial C_0}{\partial Z_0} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

Jacobians

- Jacobian of C w.r.t Z





Solving GE with non-RE expecations

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- Given some exp. matrix $\boldsymbol{E}$ we can construct alternative jacobians $\hat{\boldsymbol{M}}, \hat{\boldsymbol{M}}_r$
- Solve for GE using these jacobians instead of RE jacobians $\boldsymbol{M}, \boldsymbol{M}_r$ ($\boldsymbol{X}$=shock):

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  5. Solve for IRFs:
     - `model.find_IRFs(shocks=[x])`

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- Expectations matrix:

$$\boldsymbol{E} = \begin{pmatrix} 1 & 1-\theta & 1-\theta & \dots \\ 1 & 1 & 1-\theta^2 & \dots \\ 1 & 1 & 1 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Sticky expectations

- Properties:
  - Response of consumption at 0 to Z_1 is $(1-\theta)\frac{\partial C_0}{\partial Z_1}$
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- $\theta=0$ gives us RE, $\theta=1$ gives us myopic behavior.
- Since households perfectly observe income changes **today and in past** iMPCs are preserved
    - **Unlike** habit formation

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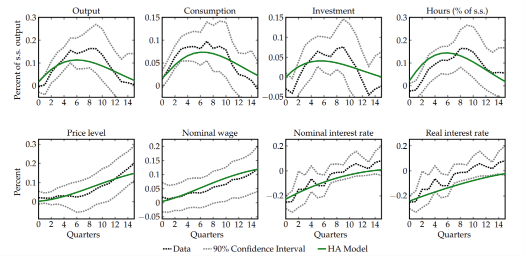

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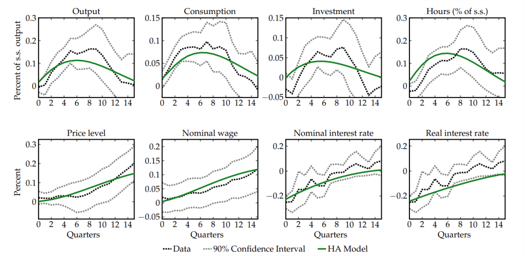
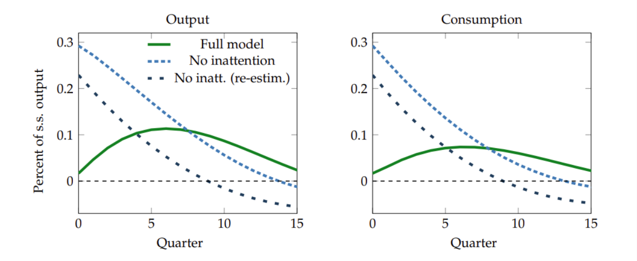


- Estimate $\theta = 0.935 \Rightarrow$ **Large** deviation from RE

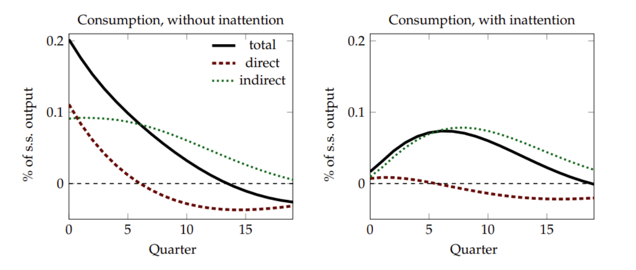
RE vs. Non-RE

- Why we need sticky expectations in order to match empirical response



Direct and indirect effects

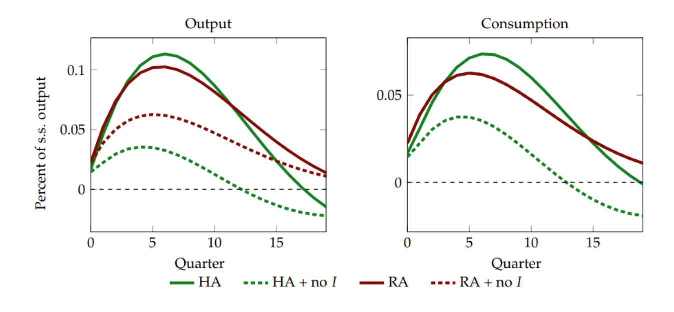
- Can decompose C into direct and indirect as before



- In the estimated model with sticky expectations indirect effect is by far the most important driver of consumption

Importance of Investment

- Importance of indirect effects in HANK partly comes from *investment*



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2. To do so, we just need define an expectation matrix, and recompute the Jacobian accordingly
3. This allows HANK models to match the observed "hump-shape" in the data

Exercise

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Consider the HANK model described in section 2

1. Compare a monetary policy shock in HANK and RANK. Decompose the response in HANK into direct and indirect effects using the household Jacobians
2. Solve for a monetary policy shock in HANK and RANK with myopic expectations w.r.t $\boldsymbol{r}$, $\boldsymbol{Z}$,only $\boldsymbol{r}$ and only $\boldsymbol{Z}$
3. Solve for a monetary policy shock in HANK and RANK with sticky expectations w.r.t $\boldsymbol{r}$, $\boldsymbol{Z}$,only $\boldsymbol{r}$ and only $\boldsymbol{Z}$
4. Consider a model where household hold nominal government debt instead. Relax the borrowing constraint to -1, $\underline{a} = -1$ and solve for a monetary policy shock (assume rational expectations). Does the presence of household debt amplify or dampen the effects of monetary policy?

Summary

Summary and next week

- **Today:**
  - Monetary policy in HANK
  - Alternatives to rational expecations, and how to implement them using jacobians
- **Next week:** HANK + unemployment risk in GE (**JD**)
- **Homework:**
  1. Work on exercise