9-10. Fiscal Policy in HANK

Adv. Macro: Heterogenous Agent Models

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2025

Introduction

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  1. Economic content: Long run trends and outcomes
  2. Methods: Stationary eq., Non-linear transition path and perfect foresight

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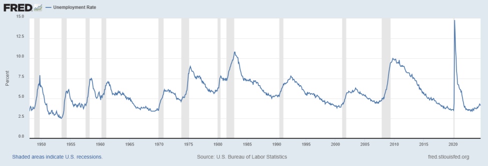
- **Today:**
  1. Business cycles in the New Keynesian model
  2. Linearized solution in models with aggregate risk
- **Literature:**
  - NK:
    1. Gali textbook ch. 3-4
    2. *Macroeconomics* textbook ch. 16
  - Solution methods:
    1. Auclert et. al. (2021), »Using the Sequence-Space Jacobian to Solve and Estimate Heterogeneous-Agent Models«
    2. Boppart et. al. (2018), »Exploiting MIT shocks in heterogeneous-agent economies: The impulse response as a numerical derivative«
    3. Documentation for GEModelTools

Business cycles

- Macro variables relatively volatile around long-run trends



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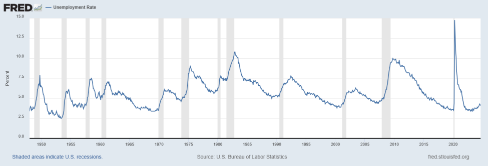

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  - Study how aggregate shocks cause business cycles
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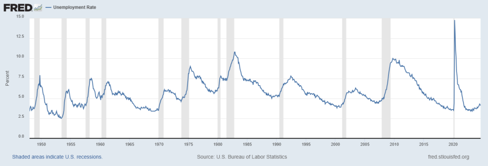

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  - Study how aggregate shocks cause business cycles
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- First point on agenda: Need **role** for monetary policy
  - Models so far in the course have featured **monetary-non neutrality**
  - Monetary policy cannot affect real quantities (unemployment, GDP)

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  - Monopolistic competition (price-setting)
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- The New Keynesian (NK) model addresses these two concerns by adding to the standard model:
    - Monopolistic competition (price-setting)
    - Price rigidities
- The basic NK model is simple (can be reduced to 3 equations) but **extremely influential**

Quick Recap: The IS-LM model

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- **Core intuition**: <u>all of income is somebody's else spendings!</u>

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- What's missing?
  - A supply side, micro-foundations, expectations, etc.

The New Keynesian model

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- The model consists of the following agents:
    1. A representative household who consumes, saves and supplies labor
    2. Firms with market power who produce output using labor and sets prices subject to nominal rigidities
    3. A central bank which conduct monetary policy

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- **Central bank**: Sets nominal interest rate

Households

- The **representative household** solves the following problem:

$$\max_{C_t, A_t, L_t} E_0 \sum_{t=0}^{\infty} \beta^t \left[u(C_t) - \nu\left(L_t^{hh}\right) \right]$$

$$s.t.$$

$$C_t + A_t = (1 + r_t) A_{t-1} + \left(w_t L_t^{hh} + \Pi_t \right)$$

- Note: Expectation taken w.r.t **aggregate shocks** (TFP, monetary policy, markup etc.)
- Standard first-order conditions:

$$u'(C_t) = E_t \beta (1 + r_{t+1}) u'(C_{t+1})$$
$$\nu'\left(L_t^{hh}\right) = w_t u'(C_t)$$

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- Ricardian equivalence holds: fiscal policy has no impact on aggregate demand

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$$\max_{y_{jt}\ \forall j} P_t Y_t - \int_0^1 p_{jt} y_{jt} dj \text{ s.t. } Y_t = \left(\int_0^1 y_{jt}^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}$$

for given output price, P_t, and input prices, p_{jt}

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- **Note:** Zero profits (can be used to derive price index)

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- **Implied dividends:** $\Pi_t = Y_t - w_t L_t - \frac{\theta}{2} \left[\frac{p_{jt}}{p_{jt-1}} - 1 \right]^2 Y_t$

Derivation of NKPC

- **FOC wrt. p_{jt}:**

$$0 = (1-\epsilon)\left(\frac{p_{jt}}{P_t}\right)^{-\epsilon}\frac{Y_t}{P_t} + \epsilon\frac{w_t}{\Gamma_t}\left(\frac{p_{jt}}{P_t}\right)^{-\epsilon-1}\frac{Y_t}{P_t} - \theta\left[\frac{p_{jt}}{p_{jt-1}} - 1\right]\frac{Y_t}{p_{jt-1}} + E_t\frac{J'_{t+1}(p_{jt})}{1+r_{t+1}}$$

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- **FOC + Envelope + Symmetry +** $\pi_t = P_t/P_{t-1}-1$

$$0 = \left[(1-\epsilon)+\epsilon\frac{w_t}{\Gamma_t}\right]\frac{Y_t}{P_t}$$
$$-\theta\left[\frac{P_t}{P_{t-1}}-1\right]\frac{Y_t}{P_{t-1}} - E_t\frac{\theta\left[\frac{P_{t+1}}{P_t}-1\right]\left(\frac{P_{t+1}}{P_t^2}\right)Y_{t+1}}{1+r_{t+1}}$$

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   $\pi_t = 0 \rightarrow w_t = \frac{\Gamma_t}{\mu} \rightarrow$ wage is mark-downed relative to MPL $(\mu > 1)$

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4. Note:
   - Sometimes a β^{firm} is used instead of $\frac{1}{1+r_{t+1}}$
   - $\pi_t(1+\pi_t) \approx \pi_t$ for small π_t

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- **Government:** In standard model Government simply supplies bonds that are in net-zero supply, $B = 0$
  - Note: HHs still make consumption-saving decisions (so cannot impose $A = 0$ in budget), but in equilibrium prices will adjust such that $A = B = 0$
  - Simplifying assumption, can easily incorporate more reaslitic government $\tau_t = r_t B_{ss} + G_t$ with $B_{ss} > 0$ (see HANK later)

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- As usual, in practice we will only impose market clearing in two of the markets when solving the model

Aggregate shocks

- In the standard NK model business cycles arise due to fluctuations in aggregate shocks:

1. TFP (supply)

$$\ln \Gamma_t = \overline{\Gamma} + \ln \Gamma_{t-1} + \epsilon_t^{\Gamma}, \quad \epsilon_t^{\Gamma} \sim \mathcal{N}\left(0, \sigma_{\Gamma}^2\right)$$

2. Discount factor (demand)

$$\ln \beta_t = \overline{\beta} + \ln \beta_{t-1} + \epsilon_t^{\beta}, \quad \epsilon_t^{\beta} \sim \mathcal{N}\left(0, \sigma_{\beta}^2\right)$$

3. Monetary policy

$$i_t^* = \overline{i^*} + \ln i_{t-1}^* + \epsilon_t^{i^*}, \quad \epsilon_t^{i^*} \sim \mathcal{N}\left(0, \sigma_{i^*}^2\right)$$

The 3 equation NK model

- Consider the deterministic, zero-inflation steady state of the model (with TFP and prices normalized to 1):

$$\pi_{ss} = 0, \quad Y_{ss} = C_{ss} = 1$$
$$r_{ss} = i_{ss} = \frac{1}{\beta} - 1, \quad w_{ss} = \frac{1}{\mu}$$

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$$\begin{aligned}
\hat{Y}_t &= -\sigma\left(i_t - \pi_{t+1}\right) + \hat{Y}_{t+1} + \epsilon_t^D && \text{(Euler/demand curve)} \\
\hat{\pi}_t &= \tilde{\kappa}\hat{Y}_t + \beta\hat{\pi}_{t+1} + \epsilon_t^S && \text{(NKPC/supply curve)} \\
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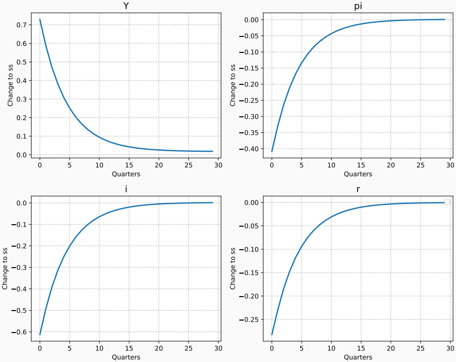
- With three unknowns (per period) $\hat{Y}_t, \hat{\pi}_t, \hat{i}_t$

Technology shocks in the NK model

- Effects of a positive TFP shock (increase Γ_t)



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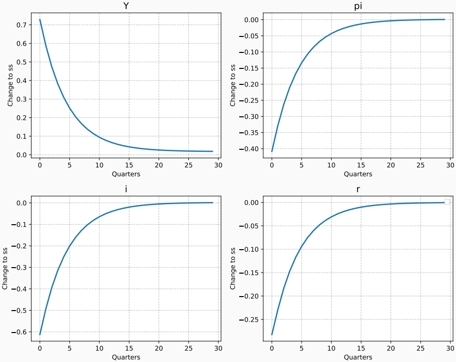

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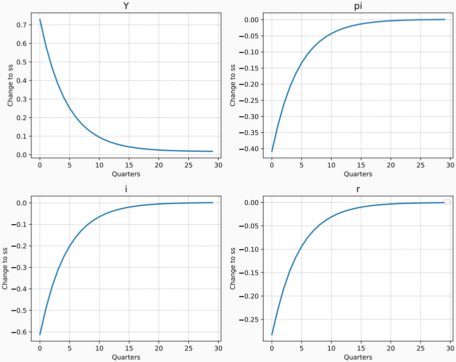

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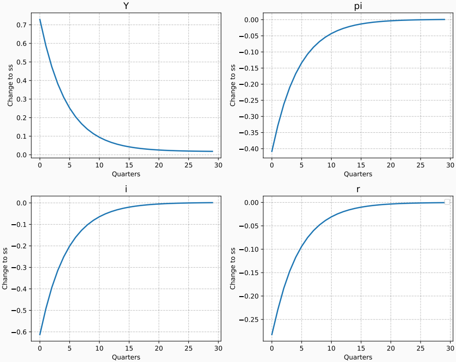

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- Intertemporal sub. $\Rightarrow C, Y \uparrow$

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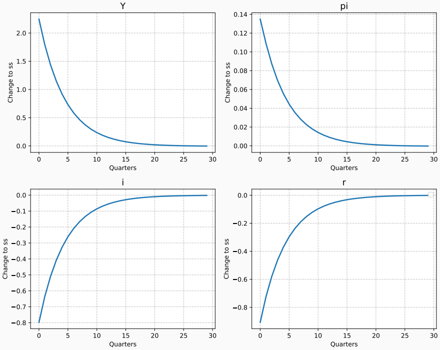
- Effects of accomodating monetary policy (easing) with persistent decline in i_t^*



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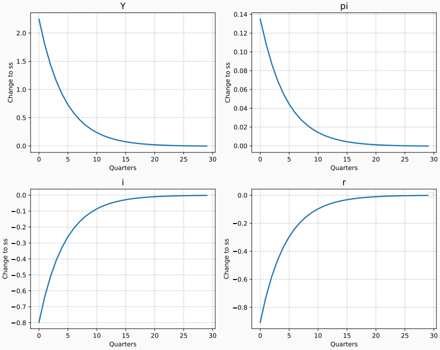

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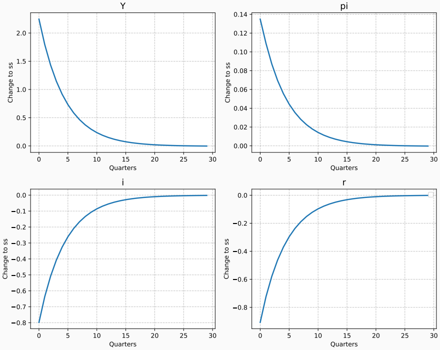

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- Increase in employment pushes up wages (marginal costs), so inflation increases

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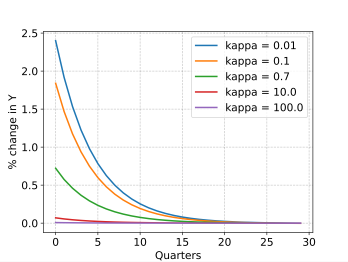

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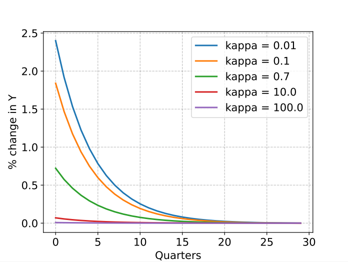

- Why? With completely flexible prices monetary policy just increases inflation 1-1 without affecting r

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- **No role for fiscal policy:** Ricardian equivalence holds (impact and cumulative multipliers of 1)
- **Not Keynesian?:** aggregate demand moves essentially because of intertemporal substitution rather than income effects (aggregate demand plays a very small role)
- **Can't speak to redistribution effects:** representative agent framework, no inequality
- **Calibration:** cannot take the model to micro-data

Review questions

- Consider the standard New Keynesian model

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  1. How does a positive demand shock ϵ_t^β (which decrease β) affect output Y, inflation π, and interest rates i, r?
  2. Are firm markups pro-cyclical or counter-cyclical (w.r.t Y) in response to the demand shock?
  3. Consider an extension with a government that spends G and raises lumpsum taxes τ
     - What is the effect of a shock to G? Is the fiscal multiplier $\frac{dY}{dG}$ above or below one?
     - Does the effects of the shock dependent on the method of financing (debt vs taxes)?

Exercise

Exercise - NK model with government

1. Familiarize yourself with the model equations in *blocks.py*. Do you understand all the equations?
2. Compute the non-linear response to a temporary increase in government spending
   2.1 Use *model.find_transition_path()* for the non-linear response (results are in *model.path*)
   2.2 Use *model.find_IRFs()* for the linear response (results are in *model.IRF*)
3. Add a zero lower bound to the model:

   $$i_t = \max\{i_{ss} + \phi\pi_t, 0\}$$

   Compute linear and non-linear responses to a β-shock of size 0.05 and compare.
4. Assume that the government tries to stabilize the economy after the demand shock. Compute linear and non-linear responses to a simultaneous shock to β ($d\beta_0 = 0.05$) and G ($dG_0 = 0.03$).
5. Is stabilization policy more or less efficient once we take the ZLB into account? Hint: Compare the multipliers $\frac{dY^{\beta,G} - dY^{\beta}}{dG}$ for the linear and non-linear responses and compare.

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    - **Application**: Fiscal policy in HANK

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  - The canonical HANK model
    - Model with sticky wages
  - **Application**: Fiscal policy in HANK
- **Literature:** Auclert et. al. (2023), "The Intertemporal Keynesian Cross"
  - Long paper with many (technical) details
  - We will focus on the main results

Sticky Wages

Detour

- Early HANK papers formulated *Heterogeneous Agent New Keynesian* Models by:
  - Take standard NK model from last lecture
  - Replace Representative agent HH block with Heterogeneous Agents HH block
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- Need to make one adjustment to standard NK model before adding HA

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- If HHs have little saving, $c_i \approx w l_i \Rightarrow MPC_i = MPE_i$!

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- Tension between model and data $\Rightarrow$ Standard model with high MPCs imply too large wealth effects on labor supply

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- Will also use this a method of introducing **sticky wages**
    - Empirical evidence typically show that wages adjust more sluggishly than prices w.r.t aggregate shocks

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- where $\frac{\theta^W}{2} \left(W_t^j / W_{t-1}^j - 1 \right)^2$ is a quadratic utility cost from changing wages $\Rightarrow$ sticky wages

New Keynesian Wage Phillips curve

- Solving the union's maximization problem in a symmetric equilibrium $L_t^j = L_t,\ W_t^j = W_t$:

$$\pi_t^w \left(1 + \pi_t^w\right) = \kappa^w \left\{ \nu' \left(L_t\right) - \frac{w_t}{\mu^w} \int z_t u' \left(c_{i,t}\right) d\mathcal{D}_{i,t} \right\} L_t + \beta \pi_{t+1}^w \left(1 + \pi_{t+1}^w\right)$$

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- Breaks wealth effect on labor supply through two mechanisms:
  - Sticky wages (low κ^w)
  - Wealth effect on agg. L depends on agg. marginal utility $\int z_t u'\left(c_{i,t}\right) d\mathcal{D}_{i,t}$

More tractable version

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- $u'\left(C_t\right)$ instead of $\int z_t u'\left(c_{i,t}\right) d\mathcal{D}_{i,t}$
- Slightly easier to implement in code and when working analytically

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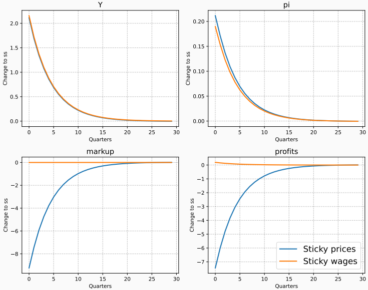

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- The Heterogeneous Agent New Keynesian model
  - Similar to standard NK model but substitute sticky prices with wages + HA instead of RA
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Households

- **Household problem:**

$$
\begin{aligned}
v_t(z_t, a_{t-1}) &= \max_{c_t} \frac{c_t^{1-\sigma}}{1-\sigma} - \varphi \frac{\ell_t^{1+\nu}}{1+\nu} + \beta \mathbb{E}_t \left[v_{t+1}(z_{t+1}, a_t) \right] \\
\text{s.t. } a_t + c_t &= (1 + r_t^a) a_{t-1} + (1 - \tau_t) w_t \ell_t z_t + \chi_t \\
\log z_{t+1} &= \rho_z \log z_t + \psi_{t+1} \ , \psi_t \sim \mathcal{N}(\mu_\psi, \sigma_\psi), \, \mathbb{E}[z_t] = 1 \\
a_t &\geq 0
\end{aligned}
$$

- **Active decisions:** Consumption-saving, c_t (and a_t)
- **Union decision:** Labor supply, ℓ_t
- **Aggregate Consumption:** $C_t^{hh} = \int c_t d\mathcal{D}_t$
- **Consumption function:** $C_t^{hh} = C^{hh} \left(\{ r_s^a, (1 - \tau_s) w_s \ell_s, \chi_s \}_{s=0}^{\infty} \right)$

Firms

- **Production and profits:**

$$Y_t = \Gamma_t L_t$$

$$\Pi_t = Y_t - \frac{W_t}{p} L_t$$

- **First order condition:**

$$w_t = \Gamma_t$$

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- **FOCs** (no arbitrage conditions):

$$1 + r_t = 1 + r^a_t$$

Union

- Everybody works the same:

$$\ell_t = L_t^{hh}$$

- Maximization subject to wage adjustment cost imply a **New Keynesian Wage (Phillips) Curve** (NKWPC or NKWC)

$$\pi_t^w = \kappa \left(\varphi \left(L_t^{hh} \right)^{\nu} - \frac{1}{\mu} \left(1 - \tau_t \right) w_t \left(C_t^{hh} \right)^{\text{-}\sigma} \right) + \beta \pi_{t+1}^w$$

Government

- **Spending:** G_t

$$B_t + T_t + = (1 + r_t)B_{t-1} + G_t + \chi_t$$

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- In which case taxes T_t adjust fully every period to ensure that the budget holds

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- Indeterminacy: Consider limit for nominal rule $i_t = i_{ss} + \phi \pi_{t+1}$ or assume future tightening

Market clearing

1. Asset market: $B_t = A_t^{hh}$
2. Labor market: $L_t = L_t^{hh}$
3. Goods market: $Y_t = C_t^{hh} + G_t$

Fiscal Policy

Simpler consumption function

- **Assumptions:**
    1. One-period real bond
    2. No lump-sum transfers, $\chi_t = 0$
    3. Fiscal policy in terms of dG_t and dT_t satisfying IBC (e.g. government needs to repay excess debt dB eventually)

$$\sum_{t=0}^{\infty}(1+r_{ss})^{-t}(dG_t - dT_t) = 0$$

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- **Household income:** $(1 - \tau_t) w_t \ell_t z_t = \underbrace{(Y_t - T_t)}_{\equiv Z_t} z_t = Z_t z_t$
- **Consumption function in sequence-space:** Simplifies to

$$C_t^{hh} = C^{hh}\left(\{Y_s - T_s, r_s\}_{s \geq 0}\right) \Rightarrow \boldsymbol{C}^{hh} = C^{hh}(\boldsymbol{Y} - \boldsymbol{T}, \boldsymbol{r}) = C^{hh}(\boldsymbol{Z}, \boldsymbol{r})$$

Side-note: Two-equation version in $\boldsymbol{Y}$ and $\boldsymbol{r}$

$$\boldsymbol{Y} = \boldsymbol{G} + C^{hh}(\boldsymbol{r}, \boldsymbol{Y} - \boldsymbol{T})$$
$$\boldsymbol{r} = \mathcal{R}(\boldsymbol{Y})$$

- **First equation:** Goods market clearing
- **Second equation (**Firms + NKWPC**):**
  1. Given output $\boldsymbol{Y}$, can compute $\boldsymbol{L}$
  2. Firm behavior I: $\boldsymbol{\Gamma}, \boldsymbol{Y} \to \boldsymbol{L}, \boldsymbol{w}$
  3. NKWC: $\boldsymbol{L}, \boldsymbol{C}, \boldsymbol{w}, \boldsymbol{\tau} \to \boldsymbol{\pi}^{\boldsymbol{w}}$
  4. Firm behavior II: $\boldsymbol{\pi}^{\boldsymbol{w}}, \boldsymbol{\Gamma} \to \boldsymbol{\pi}$
  5. Central bank: $\boldsymbol{\pi} \to \boldsymbol{i}$
  6. Fisher: $\boldsymbol{i}, \boldsymbol{\pi} \to \boldsymbol{r}$
- Final assumption for today: **Constant $\boldsymbol{r}$**
  - Can replace $\boldsymbol{r} = \mathcal{R}(\boldsymbol{Y})$ with $\boldsymbol{r} = r_{ss}$
  - Entire model boils down to 1 equation

Intertemporal Keynesian Cross

$$Y_t = G_t + C_t^{hh}\left(\{Y_s - T_s\}_{s=0}^{\infty}\right) \qquad \text{Static}$$
$$\boldsymbol{Y} = \boldsymbol{G} + C^{hh}(\boldsymbol{Y} - \boldsymbol{T}) \qquad \text{Sequence-space/vector}$$

- **Total differentiation/linearize around ss:**

$$dY_t = dG_t + \sum_{s=0}^{\infty} \frac{\partial C_t^{hh}}{\partial Z_s} dZ_s = dG_t + \sum_{s=0}^{\infty} \frac{\partial C_t^{hh}}{\partial Z_s} \left(dY_s - dT_s\right)$$

- **Intertemporal Keynesian Cross** in vector form

$$d\boldsymbol{Y} = d\boldsymbol{G} + \boldsymbol{M}\left(d\boldsymbol{Y} - d\boldsymbol{T}\right) \Leftrightarrow$$
$$(\boldsymbol{I} - \boldsymbol{M})d\boldsymbol{Y} = d\boldsymbol{G} - \boldsymbol{M}d\boldsymbol{T}$$

where $M_{t,s} = \frac{\partial C_t^{hh}}{\partial Z_s}$ encodes the entire *complexity of HH behavior*

Illustration

- Writing out the IKC:

$$\begin{bmatrix} dY_0 \\ dY_1 \\ dY_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} dG_0 \\ dG_1 \\ dG_2 \\ \vdots \end{bmatrix} + \begin{bmatrix} \frac{\partial C_0^{hh}}{\partial Z_0} & \frac{\partial C_0^{hh}}{\partial Z_1} & \frac{\partial C_0^{hh}}{\partial Z_2} & \cdots \\ \frac{\partial C_1^{hh}}{\partial Z_0} & \frac{\partial C_1^{hh}}{\partial Z_1} & \frac{\partial C_1^{hh}}{\partial Z_2} & \cdots \\ \frac{\partial C_2^{hh}}{\partial Z_0} & \frac{\partial C_2^{hh}}{\partial Z_1} & \frac{\partial C_2^{hh}}{\partial Z_2} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \left(\begin{bmatrix} dY_0 \\ dY_1 \\ dY_2 \\ \vdots \end{bmatrix} - \begin{bmatrix} dT_0 \\ dT_1 \\ dT_2 \\ \vdots \end{bmatrix} \right)$$

iMPC matrix

- $\boldsymbol{M}$ is the Jacobian of aggregate C w.r.t (post-tax) labor income
  - Column s: Response of C at different dates to unit change in Z at date s (IRF)
  - Row s: Change in C at date s to change in income Z at different dates

$$\boldsymbol{M} = \begin{bmatrix} \frac{\partial C_0^{hh}}{\partial Z_0} & \frac{\partial C_0^{hh}}{\partial Z_1} & \cdots \\ \frac{\partial C_1^{hh}}{\partial Z_0} & \frac{\partial C_1^{hh}}{\partial Z_1} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix}$$

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- In a quarterly model $\frac{\partial C_0^{hh}}{\partial Z_0}$ is essentially the quarterly MPC
  - Note: Typically define MPCs following change in lump-sum transfer and not labor income, so not quite MPC

iMPCs in the data

- What can we say about the iMPC matrix $\boldsymbol{M}$?

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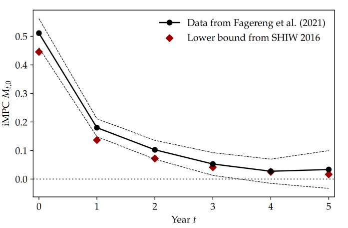
- Fagareng et al (2016) estimate dynamic response of C to income shocks (lottery winnings)
  - Info on first column $\left[\frac{\partial C_0^{hh}}{\partial Z_0}, \frac{\partial C_1^{hh}}{\partial Z_0}, \frac{\partial C_2^{hh}}{\partial Z_0}, \frac{\partial C_3^{hh}}{\partial Z_0}, \ldots\right]'$

Figure 1: iMPCs in the Norwegian and Italian data



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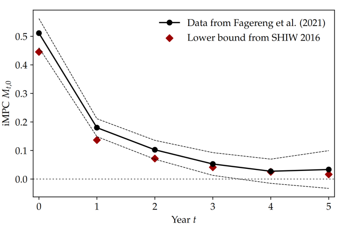

- Very hard to say something about remaining columns (announcement effects)
    - Best we can do: Calibrate model to first column, get rest of $\boldsymbol{M}$ from model

Perspective: Static Keynesian Cross

- **Old Keynesians:** Consumption only depends on current income

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- **Solution**

$$dY_t = dG_t + \frac{\text{mpc}}{1 - \text{mpc}}\left(dG_t - dT_t\right)$$

from multiplier-process $\text{mpc} \times \left(1 + \text{mpc} + \text{mpc}^2 \ldots\right) = \frac{\text{mpc}}{1-\text{mpc}}$

NPV-vector

- **NPV-vector:** $\boldsymbol{q} \equiv [1, (1 + r_{ss})^{-1}, (1 + r_{ss})^{-2}, \ldots]'$ - implies $\sum_{t=0}^{\infty}(1 + r_{ss})^{-t} x_t = \boldsymbol{q}'\boldsymbol{x}$

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$$\sum_{t=0}^{\infty}(1+r_{ss})^{-t}M_{t,s} = \frac{1}{(1+r)^s} \Rightarrow \boldsymbol{q}'\boldsymbol{M} = \boldsymbol{q}' \Leftrightarrow \boldsymbol{q}'(\boldsymbol{I} - \boldsymbol{M}) = \boldsymbol{0}$$

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- Present value of $\boldsymbol{M}$ columns is 1 - HHs must *eventually* spend income they receive
  - $\sum_{t=0}^{\infty}\frac{MPC_{t,s}}{(1+r_{ss})^{t-s}} = 1$

Form of unique solution

- **Back to IKC**:

$$(\boldsymbol{I} - \boldsymbol{M})d\boldsymbol{Y} = d\boldsymbol{G} - \boldsymbol{M}d\boldsymbol{T}$$

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- **Result:** If unique solution then on the form

$$d\boldsymbol{Y} = \mathcal{M}(d\boldsymbol{G} - \boldsymbol{M}d\boldsymbol{T})$$
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- **Note:** This is only an issue in infinite horizon
    - When solving numerically we truncate at horizon T, implying that columns in $\boldsymbol{M}$ do not exactly add to 1
    - Can then invert $(\boldsymbol{I}-\boldsymbol{M})$ (but precision becomes worse as horizon T increases)

Response of consumption

$$
\begin{aligned}
d\boldsymbol{Y} &= d\boldsymbol{G} + \boldsymbol{M}(d\boldsymbol{Y} - d\boldsymbol{T}) \Leftrightarrow \\
d\boldsymbol{Y} - d\boldsymbol{G} &= \boldsymbol{M}(d\boldsymbol{G} - d\boldsymbol{T}) + \boldsymbol{M}(d\boldsymbol{Y} - d\boldsymbol{G}) \Leftrightarrow \\
(I - \boldsymbol{M})(d\boldsymbol{Y} - d\boldsymbol{G}) &= \boldsymbol{M}(d\boldsymbol{G} - d\boldsymbol{T}) \Leftrightarrow \\
d\boldsymbol{Y} - d\boldsymbol{G} &= \mathcal{M}\boldsymbol{M}(d\boldsymbol{G} - d\boldsymbol{T}) \Leftrightarrow \\
d\boldsymbol{C} &= \mathcal{M}\boldsymbol{M}(d\boldsymbol{G} - d\boldsymbol{T})
\end{aligned}
$$

Fiscal multipliers

$$d\boldsymbol{Y} = d\boldsymbol{G} + \underbrace{\mathcal{M}\boldsymbol{M}(d\boldsymbol{G} - d\boldsymbol{T})}_{d\boldsymbol{C}}$$

- **Balanced budget multiplier:**

$$d\boldsymbol{G} = d\boldsymbol{T} \Rightarrow d\boldsymbol{Y} = d\boldsymbol{G}, d\boldsymbol{C} = 0$$

  Note: Central that income and taxes affect household income proportionally in exactly the same way = no redistribution

- **Deficit multiplier:** $d\boldsymbol{G} \neq d\boldsymbol{T}$
  1. Potentially fiscal multiplier **above** 1
  2. Larger effect of $d\boldsymbol{G}$ than $d\boldsymbol{T}$
  3. *Numerical results needed*

Fiscal multiplier

Impact-multiplier:

$$\frac{\partial Y_0}{\partial G_0}$$

Cumulative-multiplier:

$$\frac{\sum_{t=0}^{\infty}(1+r_{ss})^{-t}dY_t}{\sum_{t=0}^{\infty}(1+r_{ss})^{-t}dG_t}$$

Comparison with RA model

- **From lecture 1:** $\beta(1+r_{ss}) = 1$ implies

$$C_t = (1-\beta)\sum_{s=0}^{\infty} \beta^s Y_{t+s}^{hh} + r_{ss} a_{-1}$$

- The **iMPC-matrix** becomes ($\mathbf{1}$ is a square matrix of 1's)

$$\boldsymbol{M}^{RA} = \begin{bmatrix} (1-\beta) & (1-\beta)\beta & (1-\beta)\beta^2 & \cdots \\ (1-\beta) & (1-\beta)\beta & (1-\beta)\beta^2 & \cdots \\ (1-\beta) & (1-\beta)\beta & (1-\beta)\beta^2 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} = (1-\beta)\mathbf{1}\boldsymbol{q}'$$

- **Consumption response** is zero

$$\begin{aligned} d\boldsymbol{C}^{\boldsymbol{RA}} &= \mathcal{M}\boldsymbol{M}^{\boldsymbol{RA}}(d\boldsymbol{G} - d\boldsymbol{T}) \\ &= \mathcal{M}(1-\beta)\mathbf{1}\boldsymbol{q}'(d\boldsymbol{G} - d\boldsymbol{T}) \\ &= \mathbf{0} \Leftrightarrow d\boldsymbol{Y} = d\boldsymbol{G} \end{aligned}$$

- **Fiscal multiplier is 1**
  - Note: Lower when real rate responds

Details on matrix formulation

$$
\begin{aligned}
(1-\beta)\mathbf{1}\boldsymbol{q}' &= \begin{bmatrix} (1-\beta) & (1-\beta) & (1-\beta) & \cdots \\ (1-\beta) & (1-\beta) & (1-\beta) & \cdots \\ (1-\beta) & (1-\beta) & (1-\beta) & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} 1 & (1+r_{ss})^{-1} & (1+r_{ss})^{-2} & \cdots \end{bmatrix} \\
&= \begin{bmatrix} (1-\beta) & (1-\beta) & (1-\beta) & \cdots \\ (1-\beta) & (1-\beta) & (1-\beta) & \cdots \\ (1-\beta) & (1-\beta) & (1-\beta) & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} 1 & \beta & \beta^2 & \cdots \end{bmatrix} \\
&= \begin{bmatrix} (1-\beta) & (1-\beta)\beta & (1-\beta)\beta^2 & \cdots \\ (1-\beta) & (1-\beta)\beta & (1-\beta)\beta^2 & \cdots \\ (1-\beta) & (1-\beta)\beta & (1-\beta)\beta^2 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}
\end{aligned}
$$

TANK

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- Large litterature on Two Agent New Keyniesan models (TANK) starting with Campbell and Mankiw (1989)
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  - Share λ is constrained (no savings) and consume entire income each period, $MPC = 1$ (hand to mouth)

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- Simple to implement+tractable, but some drawbacks
  - No intertemporal MPCs
  - Extremely stylized level of inequality
  - Hard to connect to micro data
  - No precautionary saving

Comparison with TANK model

- **Hand-to-Mouth (HtM) households:** λ share have $C_t = Y_t^{hh}$

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$$\begin{aligned} d\boldsymbol{Y} &= d\boldsymbol{G} + \boldsymbol{M}^{TA}(d\boldsymbol{Y} - d\boldsymbol{T}) \\ (\boldsymbol{I} - \boldsymbol{M}^{TA})d\boldsymbol{Y} &= d\boldsymbol{G} - \boldsymbol{M}^{TA}d\boldsymbol{T} \\ (\boldsymbol{I} - \boldsymbol{M}^{RA})d\boldsymbol{Y} &= \frac{1}{1-\lambda}\left[d\boldsymbol{G} - \lambda d\boldsymbol{T}\right] - \boldsymbol{M}^{RA}d\boldsymbol{T} \end{aligned}$$

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- **Solution:**

$$d\boldsymbol{Y} = d\boldsymbol{G} + \frac{\lambda}{1-\lambda}\left[d\boldsymbol{G} - d\boldsymbol{T}\right]$$

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- **Solution:**

$$d\boldsymbol{Y} = d\boldsymbol{G} + \frac{\lambda}{1-\lambda}\left[d\boldsymbol{G} - d\boldsymbol{T}\right]$$

- **Amplification** of fiscal policy with deficit financing $d\boldsymbol{G} > d\boldsymbol{T}$
  - Size of amplification increasing in share of constrained agents λ
  - Solution **very** similar to static, old Keynesian cross (multiplier: $\frac{\text{mpc}}{1-\text{mpc}}$)

TANK Proof

- In TANK we have:

$$
\begin{aligned}
d\boldsymbol{Y} &= d\boldsymbol{G} + \boldsymbol{M}^{TA}(d\boldsymbol{Y} - d\boldsymbol{T}) \\
(\boldsymbol{I} - \boldsymbol{M}^{TA})d\boldsymbol{Y} &= d\boldsymbol{G} - \boldsymbol{M}^{TA}d\boldsymbol{T} \\
(\boldsymbol{I} - \boldsymbol{M}^{RA})d\boldsymbol{Y} &= \frac{1}{1-\lambda}[d\boldsymbol{G} - \lambda d\boldsymbol{T}] - \boldsymbol{M}^{RA}d\boldsymbol{T} \\
d\boldsymbol{Y} &= \boldsymbol{M}^{RA}d\boldsymbol{Y} - \boldsymbol{M}^{RA}d\boldsymbol{T} + \frac{1}{1-\lambda}[d\boldsymbol{G} - \lambda d\boldsymbol{T}] \\
d\boldsymbol{Y} &= d\tilde{\boldsymbol{G}} + \boldsymbol{M}^{RA}(d\boldsymbol{Y} - d\boldsymbol{T})
\end{aligned}
$$

- where $d\tilde{\boldsymbol{G}} = \frac{1}{1-\lambda}[d\boldsymbol{G} - \lambda d\boldsymbol{T}]$
- Recall that in RANK $d\boldsymbol{Y} = d\boldsymbol{G} + \boldsymbol{M}^{RA}(d\boldsymbol{Y} - d\boldsymbol{T})$ with solution $d\boldsymbol{Y} = d\boldsymbol{G}$ thus implying $d\boldsymbol{Y} = d\tilde{\boldsymbol{G}}$ in TANK:

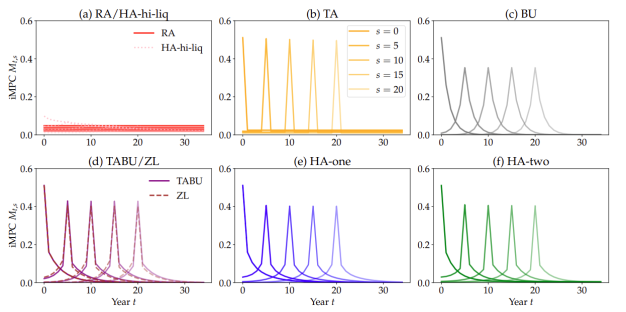
$$
\begin{aligned}
d\boldsymbol{Y} &= \frac{1}{1-\lambda}[d\boldsymbol{G} - \lambda d\boldsymbol{T}] \\
d\boldsymbol{Y} &= d\boldsymbol{G} - d\boldsymbol{G} + \frac{1}{1-\lambda}[d\boldsymbol{G} - \lambda d\boldsymbol{T}] \\
d\boldsymbol{Y} &= d\boldsymbol{G} + \frac{\lambda}{1-\lambda}[d\boldsymbol{G} - \lambda d\boldsymbol{T}]
\end{aligned}
$$

Cumulative multiplier still one

$$\frac{\boldsymbol{q}' d\boldsymbol{Y}}{\boldsymbol{q}' d\boldsymbol{G}} = \frac{\boldsymbol{q}' d\boldsymbol{G}_t + \frac{\lambda}{1-\lambda}\boldsymbol{q}' \left[d\boldsymbol{G} - d\boldsymbol{T}\right]}{\boldsymbol{q}' d\boldsymbol{G}}$$
$$= 1$$

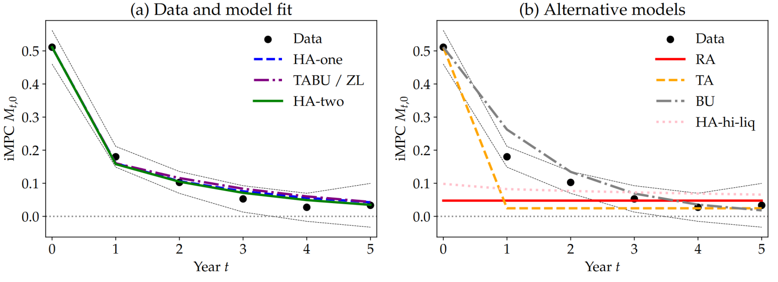
Jacobian columns

- TANK produces positive C response, fiscal multiplier above 1 - do we need HANK?
- Plot columns of $\boldsymbol{M}$ in TANK, HANK + other models
  - Recall columns: dynamic C response to change in Z at various dates



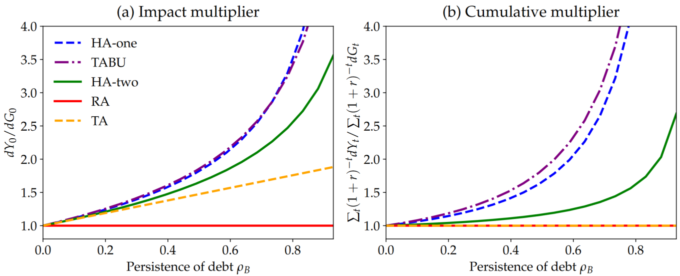
iMPCs in models

Figure 2: iMPCs in the Norwegian data and several models



Multipliers and debt-financing

Figure 5: Multipliers according to the IKC



Note. These figures assume a persistence of government spending equal to $\rho_G = 0.76$, and vary ρ_B in $dB_t = \rho_B(dB_{t-1} + dG_t)$. See section 7.1 for details on calibration choices.

Summary in table

- Summary:

Table 1: Government spending multipliers in the intertemporal Keynesian cross

Fiscal rule	Multiplier	Rep. agent (RA) doesn't match MPC	Two agents (TA) matches MPC	Het. agents (HA) matches iMPCs
balanced budget	impact	1	1	1
	cumulative	1	1	1
deficit financing	impact	1	> 1	> 1
	cumulative	1	1	> 1

Interest rate effects

- We assumed real bonds for tractablity - In reality, bonds are typically **nominal:**
- $$(1 + r_t)\, B_{t-1} \quad \textbf{versus} \quad \frac{1 + i_{t-1}}{1 + \pi_t} B_{t-1}$$

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- Period 0:
$$\frac{1+i_{ss}}{1+\pi_0}B_{ss}$$
- Period 1:
$$\frac{1+i_0}{1+\pi_1}B_0=(1+r)\,B_0$$

Interest rate effects

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-
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- Period 0:
$$\frac{1 + i_{ss}}{1 + \pi_0} B_{ss}$$
- Period 1:
$$\frac{1 + i_0}{1 + \pi_1} B_0 = (1 + r)\, B_0$$
- With nominal bonds **surprise inflation** affects returns - implies capital for households in period 0
  - Positive fiscal shock generates inflation, so **negative** effect on return

Generalized IKC

- **Budget constraint** can be written with initial capital gain

$$a_t + c_t = (Y_t - T_t)z_t + \chi_t + \begin{cases} (1 + r^a_{t-1})a_{t-1} & \text{if } t > 0 \\ (1 + r_{ss} + \text{cap}_0)a_{t-1} & \text{if } t = 0 \end{cases}$$

1. Real bond: $\text{cap}_0 = 0$
2. Nominal bond:

$$\text{cap}_0 = \frac{(1 + r_{ss})(1 + \pi_{ss})}{1 + \pi_0} - (1 + r_{ss})$$

Generalized IKC

- **Consumption-function** $\boldsymbol{C}^{hh} = C^{hh}(\boldsymbol{r}, \boldsymbol{Y} - \boldsymbol{T}, \textbf{cap}_0)$ implies

$$d\boldsymbol{C}^{hh} = \boldsymbol{M}^r d\boldsymbol{r} + \boldsymbol{M}(d\boldsymbol{Y} - d\boldsymbol{T}) + \boldsymbol{m}^{\text{cap}}\text{cap}_0$$

  where

$$\boldsymbol{M}^r_{t,s} = \left[\frac{\partial C^{hh}_t}{\partial r_s}\right], \boldsymbol{m}^{\text{cap}}_t = \left[\frac{\partial C^{hh}_t}{\partial \text{cap}_0}\right]$$

- Capital return effect is negative - can this overturn fiscal multiplier > 1?
  - No - entries in $\boldsymbol{M}$ are large
  - Entries in $\boldsymbol{m}^{\text{cap}}$ are **small**
- MPC out of capital gains approximatly 1-4% per year

Fiscal policy in HANK - litterature

- Seminal paper: **Intertemporal Keynesian Cross**
- **Many** other interesting papers:
- McKay and Reis - *The role of automatic stabilizers in the US business cycle* (2016)
  - Analyze the role of automatic stabilizers in a HANK model
- Bayer, Born, Luetticke - *The liquidity channel of fiscal policy* (2023)
  - Role of liquid and illiquid effects of fiscal policy
- Hagedorn, Manovskii, and Mitman - *The fiscal multiplier* (2019)
  - Systematic evaluation of multiplier in HANK + ZLB
- Druedahl, Ravn, Sunder-Plassmann, Sundram, & Waldstrøm - *Fiscal Multipliers in Small Open Economies With Heterogeneous Households (2024)*
  - Generalize to small open economies

Exercise

Exercise

Consider the standard HANK model outlined in section 2

1. Compute and plot selected columns of the household Jacobian of C w.r.t Z, r, χ
   Hint: use model._compute_jac_hh()to compute the jacobians. You can find the results in model.jac_hh
   1.1 Are the MPCs out of labor income Z and a transfer χ different? why?
2. Compute IRFs to a deficit financed ($\omega = 0.1$) and tax financed (ω large) fiscal spending shock. Compare the responses.
3. Compute the output IRF $d\boldsymbol{Y}$ using the fomula $d\boldsymbol{Y} = d\boldsymbol{G} + \mathcal{M}\boldsymbol{M}\left[d\boldsymbol{G} - d\boldsymbol{T}\right]$ where $\mathcal{M} = (\boldsymbol{I} - \boldsymbol{M})^{-1}$. Check that you get the same as when using model.find_IRFs()
4. Redo Q2 with active monetary policy, $\phi_\pi = 1.5$. How does the fiscal multiplier change?
5. Redo Q2 with active monetary policy *and* a flatter NKWPC, $\kappa = 0.01$. How does the fiscal multiplier change?

Summary

Summary and next week

- **Today:** Fiscal policy in a HANK model with sticky wages
- **Next week:** Assignment workshop
- **Homework:**
  1. Work on exercise
  2. Work on assignment