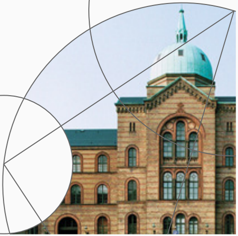
2. Consumption-Saving Models

Adv. Macro: Heterogenous Agent Models

Nicolai Waldstrøm

2024



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Cagé and Piketty at CSS

- Thomas Piketty and Julia Cagé will visit CSS and discuss **the history of policy conflict**
- October 10 at 17:00-18:00 in room 35.01.05
- Interview by editor at danish newspaper *Information*, Rune Lykkeberg
- The first 100 students who sign-up will be able to attend (signup link)

Introduction

Consumption-Saving Models

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  1. How do households consume out of transitory income shocks?
  2. How to design models that match the empirical evidence on the Marginal Propensity to Consume (MPC)?
  3. What is the effect of income risk on consumption dynamics?

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- Primarily **partial equilibrium** - leave general equilibrium for next lecture

MPC

The Marginal Propensity to Consume (MPC)

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- For a comprehensive overview, see Kaplan and Violante (2022)

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- Historically: Tension between data and models
- We need macro models that can reproduce the data on MPC

MPC in the Data: Methods

- Three strands of empirical evidence on the size of the MPC:

  1. Quasi-experimental evidence
     Johnson-Parker-Souleles (2006): Income tax rebates
     Gelman et al. (2020): government shutdown
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  3. Structural estimates
     Blundell-Pistaferri-Preston (2008), Commault (2019)

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- The quarterly aggregate MPC is between 15% and 25%
  - Annual MPCs are larger since spending responses are *persistent*
  - Size dependence: MPC larger for small income shocks
  - Sign asymmetry: MPC much larger for negative income shocks
- There is large heterogeneity in MPCs across households
  - Liquid wealth: MPC larger for low wealth households
  - Fixed individual characteristics: MPC larger for young, low-income households

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- Question: how can common macro models generate a large MPC?

MPCs in Macro Models

Model overview

1. Permanent income hypothesis
   Friedman (1957)
2. Buffer-stock consumption model
   Deaton (1991, 1992); Carroll (1992, 1997)
3. Multiple-asset buffer-stock consumption models
   Kaplan and Violante (2014)

Quick aside: General vs. partial equilibrium

- Today everything is gonna be set in **partial equilibrium**
  - No market clearing (labor market, goods market, asset market)
  - Prices w, r are therefore **exogenous**
  - Only endogenous variables are the choice variables and endo. states of households
  - Typically consumption c and savings a

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- General equilibrium
  - Households, firms and government interact through **market clearing**
  - Prices are endogenous and adjust to clear these markets
  - **Next lecture**

Representative Agent (RA) Model

- No idiosyncratic risk, no borrowing constraint
- Household problem:

$$\max_{\{c_t, a_t\}} \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}$$

s.t.

$$c_t + a_t = R a_{t-1} + y_t$$

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- Observation: The consumption function is linear in asset holdings
  $\rightarrow$ wealth distribution irrelevant for MPC
  $\Rightarrow$ Cannot reproduce empirical evidence on correlation between wealth and MPCs

Representative Agent (RA) Model

- Parameterization:
  1. Log utility ($\sigma = 1$): then we can simplify to: $\mathfrak{m} = 1 - \beta = r$
  2. Plausible (quarterly) calibrations: $\mathfrak{m} = 0.5\%$
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  - In the RA model there is nothing preventing excessive consumption smoothing
  - Household optimally spread out spending out of income gain across all periods $\Rightarrow$ low MPC

Main Takeaways for the MPC

Can macro models generate a high MPC, and if so, how?

1. RA model: No

One-Asset Heterogeneous Agent (HA) Model

- Add idiosyncratic income risk, realistic borrowing constraint
- Household problem:

$$\max_{\{c_t, a_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}$$

s.t.

$$c_t + a_t = Ra_{t-1} + y_t$$

$$y_{t+1} \sim \mathcal{F}(y_t)$$

$$a_t \geq 0$$

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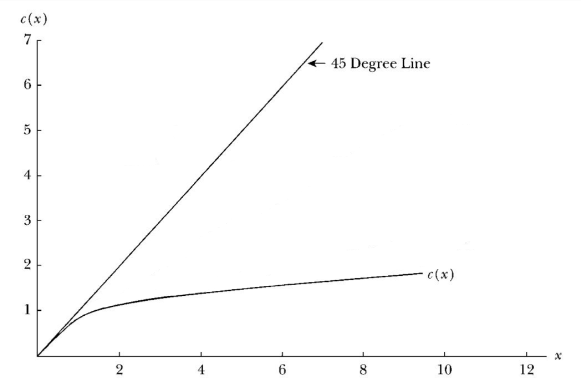
- Main takeaways:
  1. Consumption function $c(a)$ is concave due to precautionary motive
  2. There is an optimal buffer stock of assets that HHs want to achieve

Consumption function is concave



- $x = {}^a/_y$ is the share of assets to permanent income (Carroll 2001)
- Concavity: Slope of consumption (=MPC) increases as $x \to 0$
- But approximately linear for large x (as in representative agent model)

Households try to achieve an optimal buffer stock

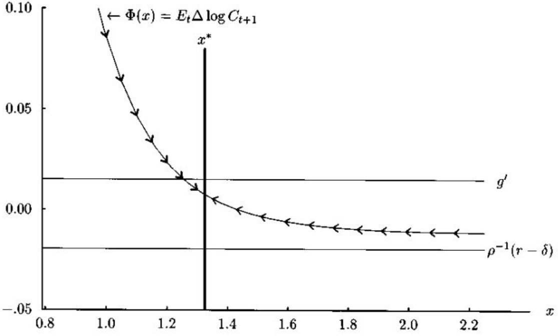


FIGURE Ia
Expected Consumption Growth as a Function of Cash on Hand

- If $x_t < x^*$: Expected consumption growth decreases (precautionary saving motive)
- If $x_t > x^*$: Expected consumption growth increases (impatience, $\beta R < 1$)

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Takeaways:

1. As $x \to \infty$, the expected growth rate of consumption (and the MPC) converge to their values in the RA model
2. As $x \to 0$ the MPC approaches due to binding borrowing constraint
3. If the consumer is impatient, there exists a unique target assets-to-permanent-income ratio (x^*)

From the inidividual to the aggregate MPC

- Individual MPC for a household with state (a, y):

$$m(a, y) = \frac{c(a + x, y) - c(a, y)}{x} \simeq \frac{\partial c(a, y)}{\partial a}$$

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- Two key determinants:
  1. Consumption function $c(a, y) \Longrightarrow$ MPC function $m(a, y)$
  2. Wealth distribution $D(a, y)$

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  - As wealth grows, the MPC $\rightarrow$ MPC in the RA model
- Shape of the wealth distribution $D(a, y)$
  - Bigger mass at bottom, where c function is concave $\rightarrow$ large MPC

What is a reasonable calibration of such a model?

- **Calibration Strategy:**
  1. As before, we set $\sigma = 1$, so that we have log utility
  2. Set the interest rate r to be 1% per year
  3. Choose β so that the model matches some target of mean wealth

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  1. Target US data: wealth to income ratio of 4.1
  2. This gives an MPC of 4.6%

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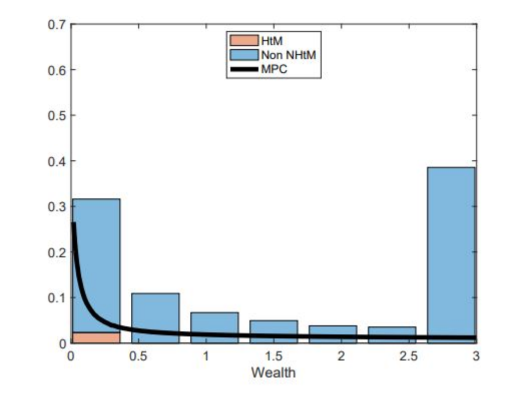

- High wealth target imply high β -> HHs are very patient and save a lot
- Very few high MPC households

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  2. This gives an MPC of 4.6%
- **Calibration 2:**
  1. Target a counterfactual wealth-to-income ratio of 0.5
  2. This gives an MPC of 14%

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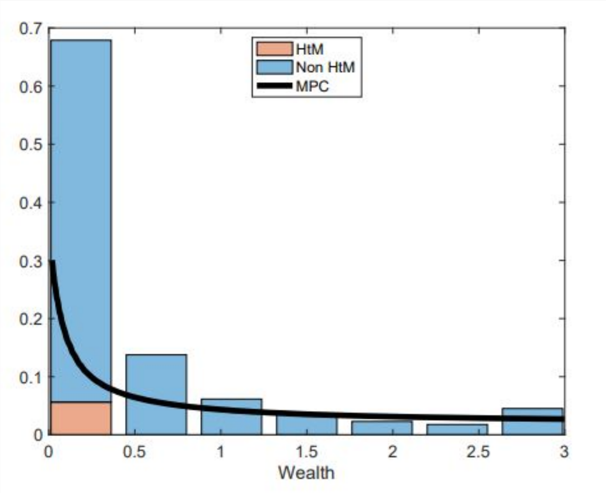

- Now we have a lot more high MPC households (hand-to-mouth HHs)

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  2. Important difference between liquid (i.e. bank deposits) and illiquid wealth (i.e. housing, retirement accounts)
  3. $\Rightarrow$ Third generation of consumption-saving models: Multiple-asset buffer-stock consumption models

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  - Liquid: cash + deposits + directly held stock - unsecured debt
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- **Q:** Why do HHs want to hold liquid or illiquid assets in this model? Why would you want to hold both assets?

Two-Asset HA Model

- Value function in period j is the max of the value if you do not (N) or do adjust (A) illiquid assets

$$V_j\left(a_{j-1}, m_{j-1}, z_j\right) = max\left\{V_j^N\left(a_{j-1}, m_{j-1}, z_j\right),\ V_j^A\left(a_{j-1}, m_{j-1}, z_j\right)\right\}$$

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- Value function if you do not adjust:

$$\begin{aligned} V_j^N\left(a_{j-1}, m_{j-1}, z_j\right) = & \max_{c_j, m_j} u\left(c_j\right) + \beta \mathbb{E}_j\left[V_{j+1}\left(a_j, m_j, z_{j+1}\right)\right] \\ & \text{subject to} \\ & c_j + m_j \leq m_{j-1}\left(1 + r^m\right) + y_j\left(z_j\right) \\ & a_j = a_{j-1}(1 + r^a) \\ & m_j \geq \underline{m} \end{aligned}$$

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$$\begin{aligned} V_j^N\left(a_{j-1}, m_{j-1}, z_j\right) = & \max_{c_j, m_j} u\left(c_j\right) + \beta \mathbb{E}_j\left[V_{j+1}\left(a_j, m_j, z_{j+1}\right)\right] \\ & \text{subject to} \\ & c_j + m_j \leq m_{j-1}\left(1 + r^m\right) + y_j\left(z_j\right) \\ & a_j = a_{j-1}(1 + r^a) \\ & m_j \geq \underline{m} \end{aligned}$$

- States: $\left(a_{j-1}, m_{j-1}, z_j\right) =$ illiquid assets, liquid assets, productivity

Two-Asset HA Model

- Value function in period j is the max of the value if you do not (N) or do adjust (A) illiquid assets

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- States: $\left(a_{j-1}, m_{j-1}, z_j\right) =$ illiquid assets, liquid assets, productivity
- Choices: $\left(c_j, m_j\right) =$ consumption, liquid asset tmrw

Two-Asset HA Model

- Value function if you adjust:

$$V_j^A(a_j, m_{j-1}, z_j) = \max_{c_j, a_j, m_j} u(c_j) + \beta \mathbb{E}_j [V_{j+1}(a_j, m_j, z_{j+1})]$$

$$\text{subject to}$$

$$c_j + a_j + m_j \leq a_{j-1}(1 + r^a) + m_{j-1}(1 + r^m) - \kappa + y_j(z_j)$$

$$a_j \geq 0, m_j \geq \underline{m}$$

- Choices: (c_j, a_j, m_j) = consumption, illiquid asset tmrw, liquid asset tmrw

Result: Two different Euler equations

- Short-Run Euler Equation - governed by saving vs dissaving in the liquid asset (HHs adjust liquid assets every period)

$$u'(c_j) = \beta(1 + r^m)u'(c_{j+1})$$

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$$u'(c_j) = \beta(1 + r^m)u'(c_{j+1})$$

- Long-Run Euler Equation - governed by saving vs dissaving in the illiquid assets (only adjust illiquid asset infrequently)

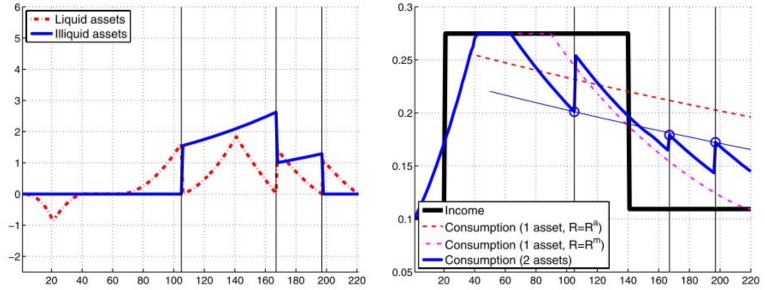
$$u'(c_j) = \beta(1 + r^a)^N u'(c_{j+N})$$

  - where N is the number of periods between adjustment

Stylized example 1 - policy function

- Zoom in on life-cycle dynamics of savings and portefolio choice in simplified model with:
  - Coarse hump-shaped earnings profile over life
  - Large transaction cost κ

Stylized example 1

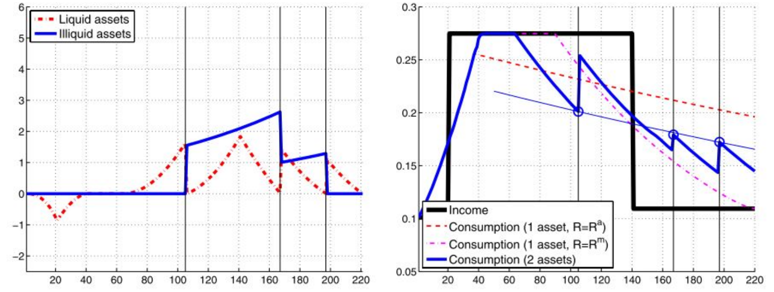


(a) Life-cycle asset accumulation (b) Life-cycle income and consumption path

FIGURE 1.—Example of life-cycle of a poor hand-to-mouth agent in the model.

- Income profile: High earnings while working, lower after retirement

Stylized example 1



(a) Life-cycle asset accumulation
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FIGURE 1.—Example of life-cycle of a poor hand-to-mouth agent in the model.

- Liquid assets adjust more throughout lifecycle since they are suitable for consumption smoothing

Stylized example 1

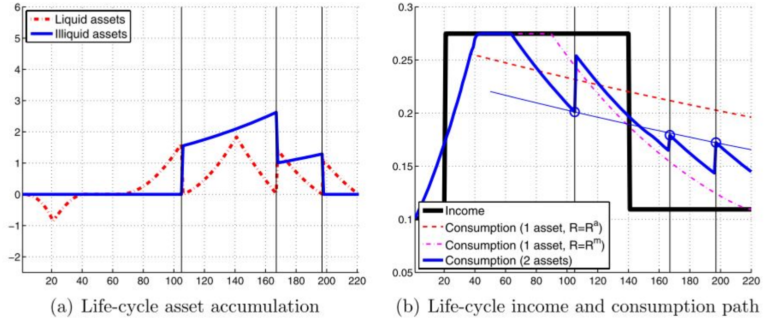
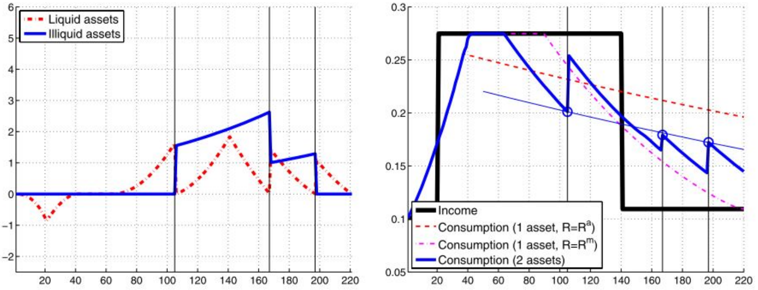


FIGURE 1.—Example of life-cycle of a poor hand-to-mouth agent in the model.

- Illiquid assets adjust only 3 times

Stylized example 1

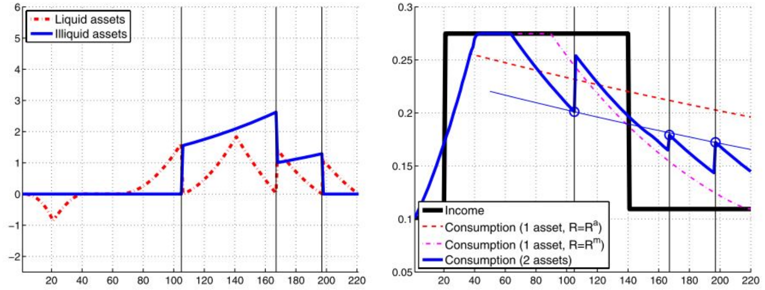


(a) Life-cycle asset accumulation (b) Life-cycle income and consumption path

FIGURE 1.—Example of life-cycle of a poor hand-to-mouth agent in the model.

- Slope of consumption function *between* adj. dates obey short-run Euler, slope across adj. dates obey long-run euler

Stylized example 1

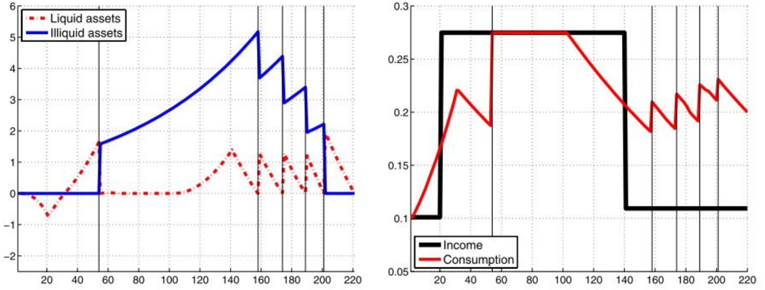


(a) Life-cycle asset accumulation (b) Life-cycle income and consumption path

FIGURE 1.—Example of life-cycle of a poor hand-to-mouth agent in the model.

- Agent exhibits poor hand-to-mouth behavior between periods 40-60, when she consumes all of her income and holds zero liquid assets

Example 2



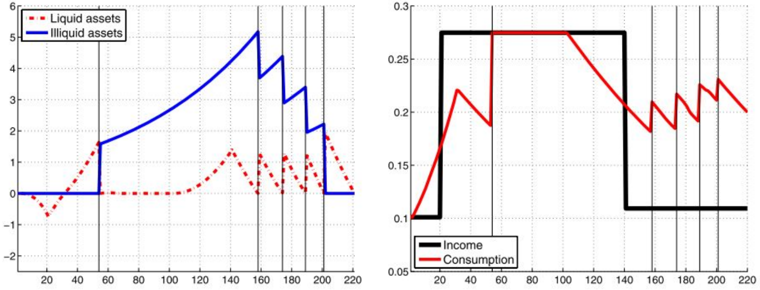
(a) Life-cycle asset accumulation

(b) Life-cycle income and consumption path

FIGURE 2.—Example of life-cycle of a wealthy hand-to-mouth agent in the model.

- Same example as before, but increase the return on the illiquid asset r^a. This incentivizes HHs to substitute from the liquid to illiquid asset

Example 2



(a) Life-cycle asset accumulation

(b) Life-cycle income and consumption path

FIGURE 2.—Example of life-cycle of a wealthy hand-to-mouth agent in the model.

- Agent exhibits wealthy hand-to-mouth behavior between periods 55 to 100, when she owns illiquid wealth, but zero liquid wealth

Result: Emergence of Wealthy HtM Households

- Three types of households in the model:
  - Unconstrained (60%) (positive liquid and illiquid wealth)
  - Poor HtM: zero net worth (14%) (zero liquid and illiquid wealth)
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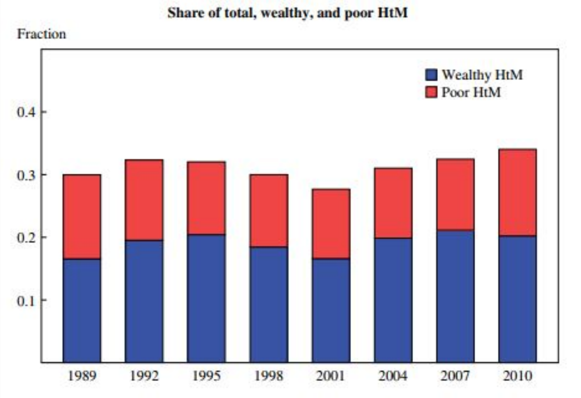
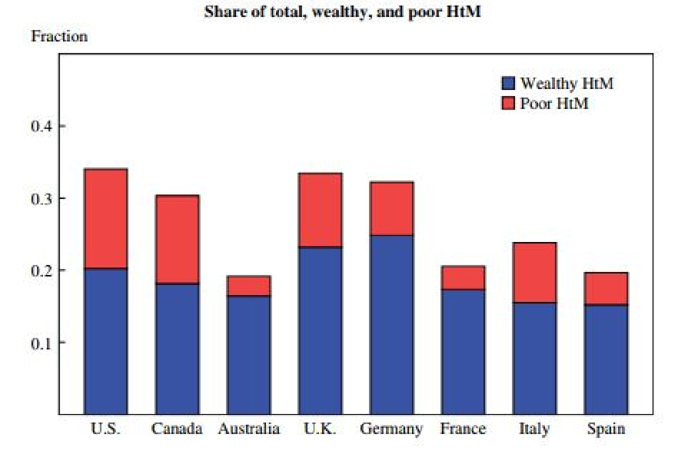
- If gains exceeds costs $\Longrightarrow$ Wealthy HtM

Wealthy HtM households in the data



- Share of US population that are Hand-to-mouth in *Survey of Consumer Finances*

Wealthy HtM households in the data



What is a reasonable calibration of such a model?

- **Calibration Strategy:**
  - As before, we set $\gamma = 1$, so that we have log utility
  - Set the interest rate r^{liq} on liquid assets to -2% per year (cash or bonds)

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  - There remains three parameters:
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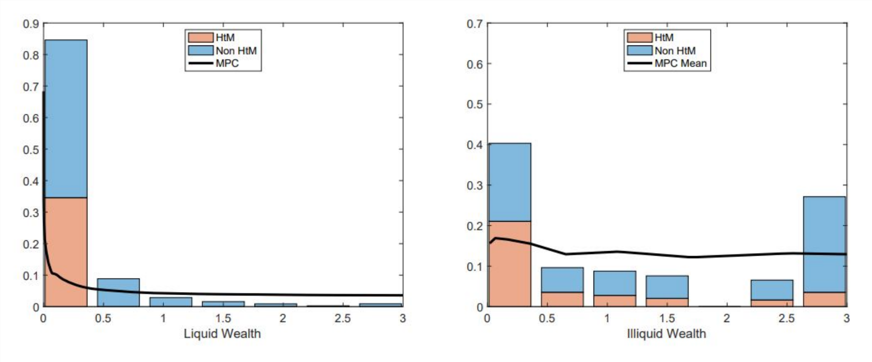
    - Transaction cost κ
  - Choose these three parameters so the model matches three targets:
    - Mean wealth-to-income ratio (4.1)
    - Share of HtM households (34%)
    - Share of wealthy HtM households (25%)

Results from the two-asset model



- What matters most for the MPC is liquid wealth, not total wealth
- MPC remains high even for households with sizeable illiquid wealth
- We can match both MPC and aggregate stock of wealth in the two-asset model

One-asset model with β-heterogeneity

- Two-asset models a la Kaplan & Violante (2014) are computationally intensive to solve due to:
  - Large state space (two endogenous states)
  - Non-convexities
- Simpler model that still matches 1) aggregate wealth, 2) aggregate MPC: **Heterogeneous β model**

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- Simpler model that still matches 1) aggregate wealth, 2) aggregate MPC: **Heterogeneous β model**
- Other options:
  - Wealth-in-utility (Michaillat and Saez 2021)
  - Behavoiral models (Present Bias, Maxted et al. 2014)

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- Standard one-asset model with *ex-ante* (=permanent) preference heterogeneity

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$$V\left(a_{t-1},z_t,\beta\right)=\max_{c_t} u\left(c_t\right)+\beta\mathbb{E}\left[V\left(a_t,z_{t+1},\beta\right)\right]$$

subject to

$$c_t+a_t\leq a_{t-1}\left(1+r\right)+z_t$$

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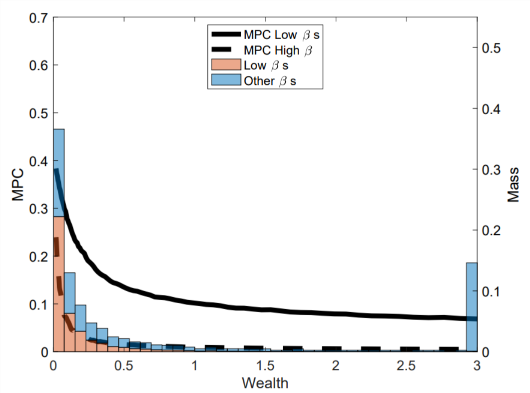
- Calibrate average β and dispersion Δ to match aggregate wealth and aggregate MPC
- Can match
  - Aggregate wealth since high β households hold a lot of wealth
  - Aggregate MPC since low β households have high MPC

One-asset model with β-heterogeneity



- Patient (high β) households have low MPCs but hold a lot of wealth
- Impatient (low β) households have high MPCs but hold a little wealth

Main Takeaways for the MPC

- Can macro models generate a high MPC, and if so, how?
  - RA model: No.
    - MPC $\sim =$ 0.5%
  - One-asset HA model:
    - Realistic wealth calibration: MPC $=$ 4.6%
    - Low wealth calibration or β-het: MPC $=$ 15%
  - Two-asset HA model:
    - Realistic wealth calibration: MPC $=$ 15%

Unemployment Risk

Unemployment Risk and Consumption Dynamics

- **Question:** How does unemployment risk affect household spending?
  - During recessions, unemployment risk increases
  - This may induce HHs to increase their buffer stock of assets (precautionary savings)
  - The resulting fall in consumption may increase output volatility (note: general equilibrium, so not today)
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  - This channel has been difficult (if not impossible) to capture with RA models
- **Our goal:** Study a HA model that can capture this channel
  - We will closely follow Harmenberg and Öberg (2021)

Model

- Start with a standard buffer stock model, expanded to have:
  1. Durable (d) and nondurable consumption (c)
     - Durable consumption: Car, fridge, furniture etc.
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  2. Time varying unemployment risk

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$$\begin{aligned} c_t + d_t + a_t &\leq \Upsilon\left(z_t, n_t\right) + (1-\delta)d_{t-1} + Ra_{t-1} - F\left(d_t, d_{t-1}\right), \\ a_t &\geq 0. \end{aligned}$$

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- Adjustment costs to durable consumption

$$F\left(d_t, d_{t-1}\right) = \begin{cases} 0 & \text{if } d_t = (1-\delta) d_{t-1}, \\ h d_{t-1} & \text{if } d_t \neq (1-\delta) d_{t-1} \end{cases}$$

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- Income depends on both productivity and employment status

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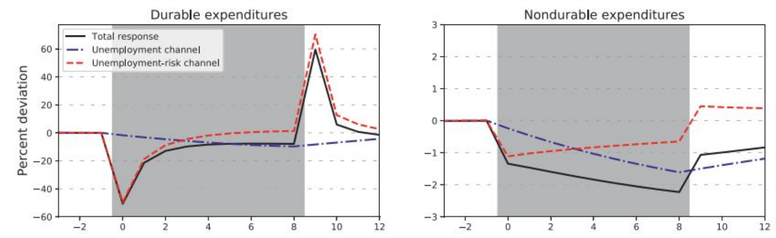
- Which of these channels is more important?

Results



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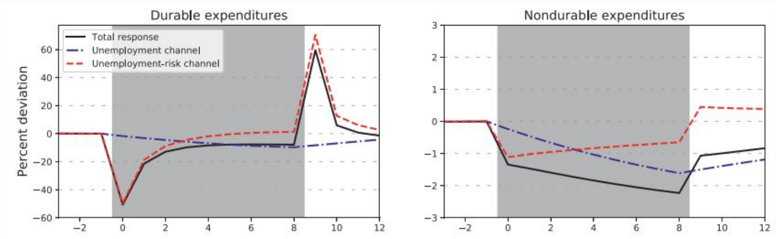

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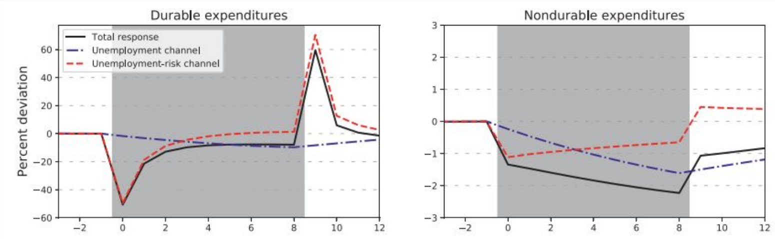

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- For durables: unemployment-risk channel is most important (*wait-and-see* effect)
- For nondurables: unemployment-risk matters initially, but unemployment accounts for the majority in the long-term

Summary

Summary and next week

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  1. The role of credit constraints
  2. Modeling the large average MPC to income shocks
  3. Consumption dynamics with time-varying unemployment risk

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- **Next week:** General equilibrium

- **Homework exercises:** (see notebook in Github repo)
  1. Adjust the discount factor, β, to target different levels of average wealth. How does the average MPC change across calibrations?
  2. Extend the model with permanent discount factor heterogeneity. Can you find a level of dispersion that allows you to both match a high level of liquidty and a higher MPC?