13. I-HANK

Adv. Macro: Heterogenous Agent Models

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Introduction

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- **Literature:**
  - Auclert, Rognlie, Souchier, & Straub (2024) »Exchange rates and monetary policy with heterogeneous agents: Sizing up the real income channel«
  - Druedahl, Ravn, Sunder-Plassmann, Sundram, & Waldstrøm (2024) »The Transmission of Foreign Demand Shocks«
  - Druedahl, Ravn, Sunder-Plassmann, Sundram, & Waldstrøm (2024) »Fiscal Multipliers in Small Open Economies With Heterogeneous Households«

IHANK Model

Small Open Economy HANK Model

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- IHANK model:
  - Take Gali-Monacelli
  - Add sticky wages
  - Add heterogeneous agents

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- **Central bank**
- **Foreign economy** (mostly exogenous)

Households

- **Household problem:**

$$
\begin{aligned}
v_t(z_{it}, a_{it-1}) &= \max_{c_{it}} \frac{c_{it}^{1-\sigma}}{1-\sigma} - \varphi \frac{\ell_{it}^{1+\nu}}{1+\nu} + \beta \mathbb{E}_t \left[v_{t+1}(z_{it+1}, a_{it}) \right] \\
\text{s.t. } a_{it} + c_{it} &= (1 + r_t^a) a_{t-1} + Z_t z_{it} \\
\log z_{it+1} &= \rho_z \log z_{it} + \psi_{it+1} \quad , \psi_t \sim \mathcal{N}(\mu_\psi, \sigma_\psi), \ \mathbb{E}[z_{it}] = 1 \\
a_{it} &\geq 0
\end{aligned}
$$

- **Active decisions:** Consumption-saving, c_{it} (and a_{it})
- **Union decision:** Labor supply, ℓ_t
- **Aggregate Consumption:** $C_t^{hh} = \int c_{it} d\mathcal{D}_{it}$
- **Consumption function:** $C_t^{hh} = C^{hh}\left(\{r_s^a, Z_s\}_{s=0}^{\infty}\right)$

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- with P_t =CPI:

$$P_t = \left[\alpha P_{F,t}^{1-\eta} + (1-\alpha) P_{H,t}^{1-\eta} \right]^{\frac{1}{1-\eta}}$$

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  - All households choose the same consumption basket: $\frac{c_{F,it}}{c_{it}} = \alpha \left(\frac{P_{F,t}}{P_t} \right)^{-\eta}$

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- If preferences are non-homothetic HH problem is more complicated

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- where P_{it} is the ideal price index associated with the consumption basket of individual i

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- Note: Usual macro convention, an increase in E_t corresponds to a **depreciation**
  - So $E_t = 7,46$ between DKK and EURO for instance

Firms

- **Production and profits:**

$$Y_t = L_t$$

$$\Pi_t = \frac{P_{H,t}}{P_t} Y_t - \frac{W_t}{P_t} L_t$$

- **First order condition:**

$$\frac{P_{H,t}}{P_t} \frac{1}{\mu} = w_t$$

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- where $r_t = r_{t+1}^a$ the ex-ante interest rate, r_t^* is the foreign real interest rate, $Q_t = \frac{E_t}{P_t} P_t^*$ is the **real exchange rate**

Union

- Everybody works the same:

$$\ell_t = L_t^{hh}$$

- Maximization subject to wage adjustment cost imply a **New Keynesian Wage (Phillips) Curve** (NKWPC or NKWC)

$$\pi_t^w = \kappa \left(\varphi \left(L_t^{hh} \right)^{\nu} - \frac{1}{\mu} w_t \left(C_t^{hh} \right)^{-\sigma} \right) + \beta \pi_{t+1}^w$$

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- with $\phi_E \to \infty$

Foreign Economy

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- Law of one price: $P_{H,t}^* = \frac{P_{H,t}}{E_t}$

Trade and current account

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- The current account is:

$$CA_t = NX_t + r_t^a NFA_{t-1}$$

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- Current account and net foreign asset position are related by [Walras]:

$$NFA_t - NFA_{t-1} = CA_t$$

Market clearing

1. Labor market: $L_t = L_t^{hh}$
2. Goods market (Version 1)

$$Y_t = C_{H,t} + C_{H,t}^*$$

3. Goods market (Version 2)

$$GDP_t = C_t^{hh} + NX_t$$

with $NX_t = \frac{P_{H,t}}{P_t} C_{H,t}^* - \frac{P_{F,t}}{P_t} C_{F,t}$

International Keynesian Cross

Sequence-space - goods market

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$$C^*_{H,t} = \alpha\left(\frac{P^*_{H,t}}{P^*_{F,t}}\right)^{-\eta^*} M^*_t \Rightarrow d\boldsymbol{C}^*_H = \alpha d\boldsymbol{M}^* - \eta^*\alpha d\left(\frac{\boldsymbol{P}^*_H}{\boldsymbol{P}^*_F}\right)$$

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- For now assume no change in foreign demand, $d\boldsymbol{M}^* = 0$

$$d\boldsymbol{Y} = (1-\alpha)\, d\boldsymbol{C}^{hh} - \eta\,(1-\alpha)\, d\left(\frac{\boldsymbol{P}_H}{\boldsymbol{P}}\right) - \eta^*\alpha d\left(\frac{\boldsymbol{P}^*_H}{\boldsymbol{P}^*_F}\right)$$

Sequence-space - trade elasticity

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d\boldsymbol{Y} &= (1-\alpha)\, d\boldsymbol{C}^{hh} + \eta\,(1-\alpha)\,\frac{\alpha}{1-\alpha} dQ_t + \eta^*\,\frac{\alpha}{1-\alpha} d\boldsymbol{Q} \\
&= (1-\alpha)\, d\boldsymbol{C}^{hh} + \chi \frac{\alpha}{1-\alpha} d\boldsymbol{Q}
\end{aligned}
$$

- with $\chi = \eta\,(1-\alpha) + \eta^*$ being the *trade elasticity*
  - Captures the elasticity of net exports to changes in relative prices (the real EXR Q)
  - Typically called *expenditure switching*
  - If the DKK appreciates ($d\boldsymbol{Q} \downarrow$) against the USD both DK HHs and US HHs will **substitute toward** US goods ($d\boldsymbol{Y} \downarrow$)

Sequence-space - HHs

- What can we say about $d\boldsymbol{C}^{hh}$? Consumption function is $C^{hh}\left(\{r_s^a, Z_s\}_{s=0}^{\infty}\right)$:

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- Use firm FOC + production function $Z_t = w_t L_t = \frac{P_{H,t}}{P_t} Y_t \Rightarrow dZ_t = -\frac{\alpha}{1-\alpha} dQ_t + dY_t$ + small valuation effect $\boldsymbol{M}^{r^a} \approx \boldsymbol{M}^{\boldsymbol{r}}$:

$$d\boldsymbol{C}^{hh} = \boldsymbol{M}^{\boldsymbol{r}} d\boldsymbol{r} + \boldsymbol{M} d\boldsymbol{Y} - \frac{\alpha}{1-\alpha} \boldsymbol{M} d\boldsymbol{Q}$$

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- Putting it together:

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4. **Real income channel of EXR**: Appreciation ($Q \downarrow$) causes foreign goods to be cheaper in home currency $\Rightarrow$ Reduces PF, P, raises real income Z

Monetary Policy

Sequence-space - Keynesian Cross

- Use the Keynesian Cross to analyze monetary policy with heterogeneous agents
  - Reference: Auclert, Rognlie, Souchier, & Straub (2024) »Exchange rates and monetary policy with heterogeneous agents: Sizing up the real income channel«

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$$1 + r_t = \frac{Q_{t+1}}{Q_t}(1 + r^*)$$

- For given foreign rate r^* an increase in domestic real rate r_t will attract foreign capital flows $\Rightarrow$ Appreciation of Q_t
  - To first-order we have $dQ_t = -\sum_{s \geq t}^{\infty} dr_s$ for a constant r^*

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  - Decreases demand in HANK due to real income effect
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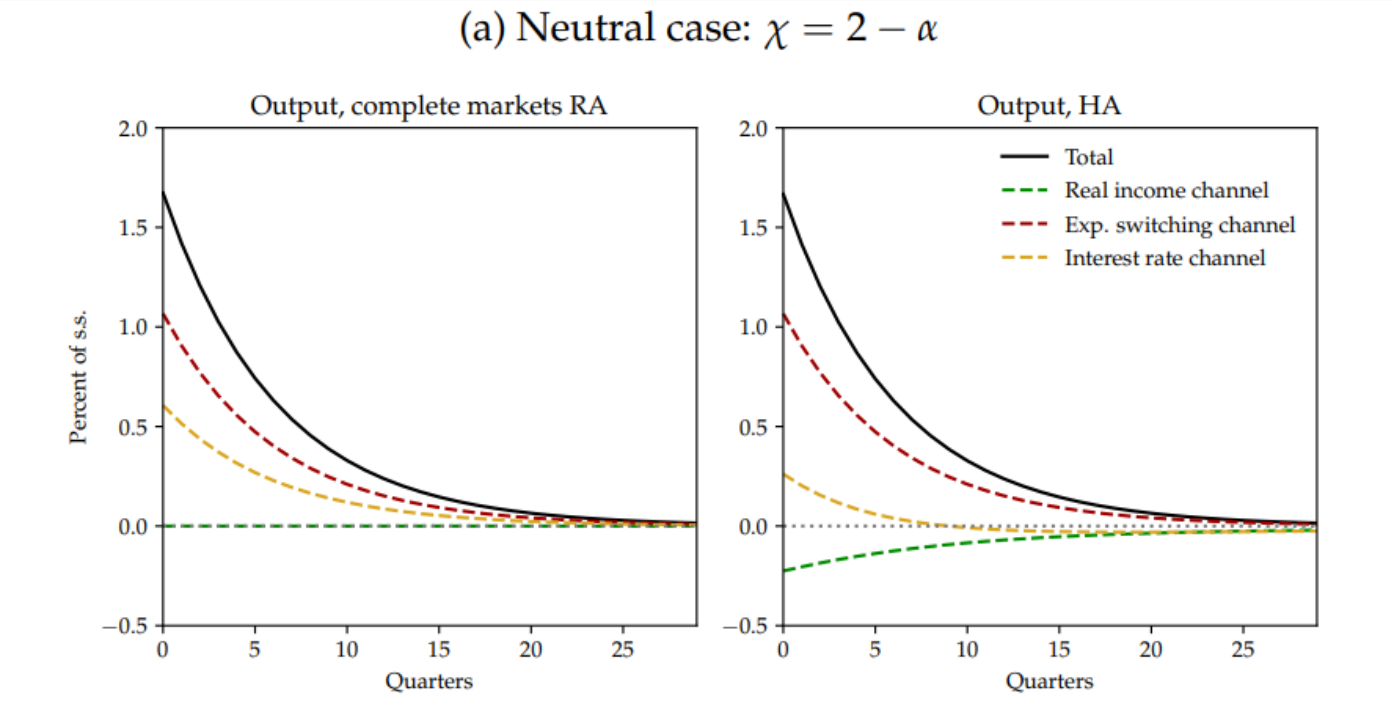
- Turns out that these effects **balance each other exactly** if trade elasticity $\chi = 2 - \alpha$
  - If $\chi < 2 - \alpha$ then the real income effect dominates and monetary policy *less* effective in HANK
  - If $\chi > 2 - \alpha$ then expenditure switching dominates and monetary policy is *more* effective in HANK

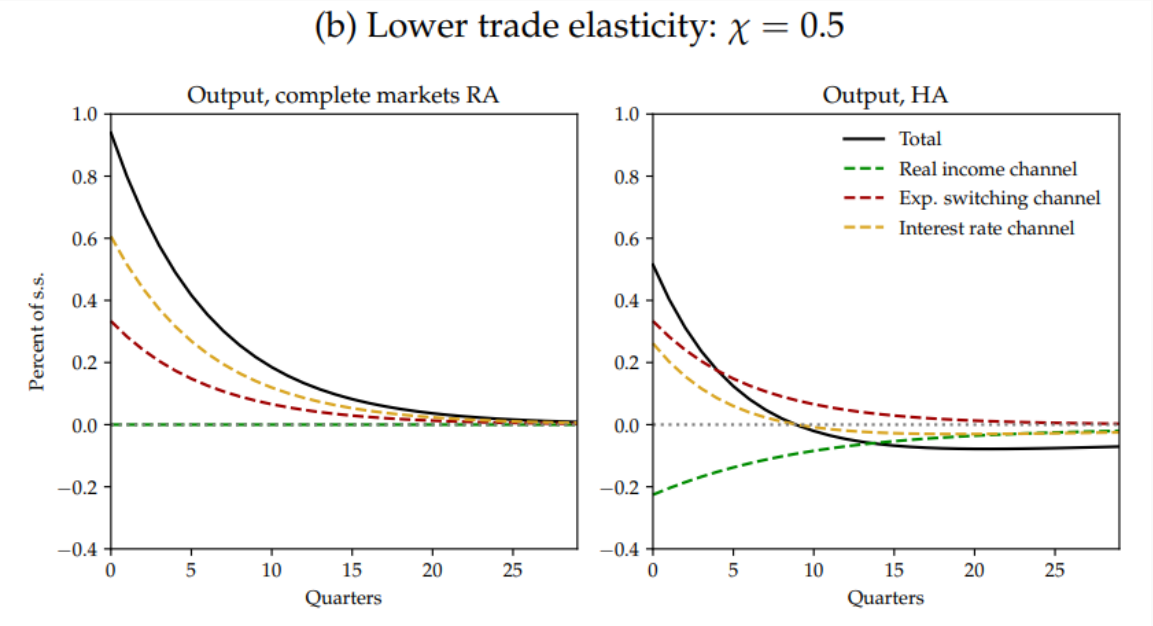
Monetary policy - $\chi = 2 - \alpha$

- Output response in HANK/RANK with neutrality, $\chi = 2 - \alpha$



Monetary policy - $\chi < 2 - \alpha$

- Empirically we expect the trade elasticity to be **low** in the short run
  - Takes time for firms/households to respond to changes in relative prices
  - But *probably* larger in the long run ($\chi > 2 - \alpha$)
- Output response in HANK/RANK with $\chi = 0.5 < 2 - \alpha$
  - Monetary policy **less** effective in HANK



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- Take model from before, add government + Taylor rule

Keynesian cross with G

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- In fact **isomorphic** to closed economy Cross with $\tilde{\boldsymbol{M}} \equiv (1-\alpha)\boldsymbol{M}$

$$d\boldsymbol{Y} = d\boldsymbol{G} - \tilde{\boldsymbol{M}}d\boldsymbol{T} + \tilde{\boldsymbol{M}}d\boldsymbol{Y}$$

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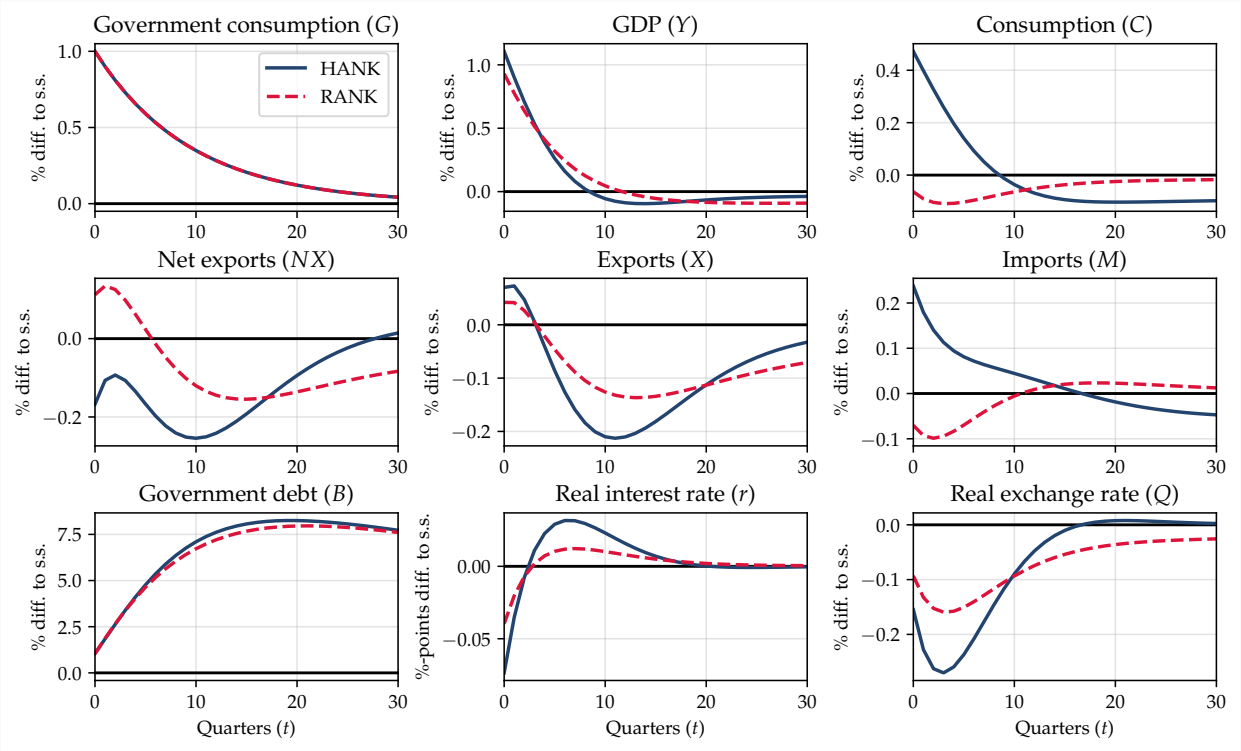
- ... but some analytical results in paper, for instance:
  - In limit $\alpha \to 1$ (fully open economy) HANK/RANK equivalence since multiplier effects do not matter

Fiscal spending shocks

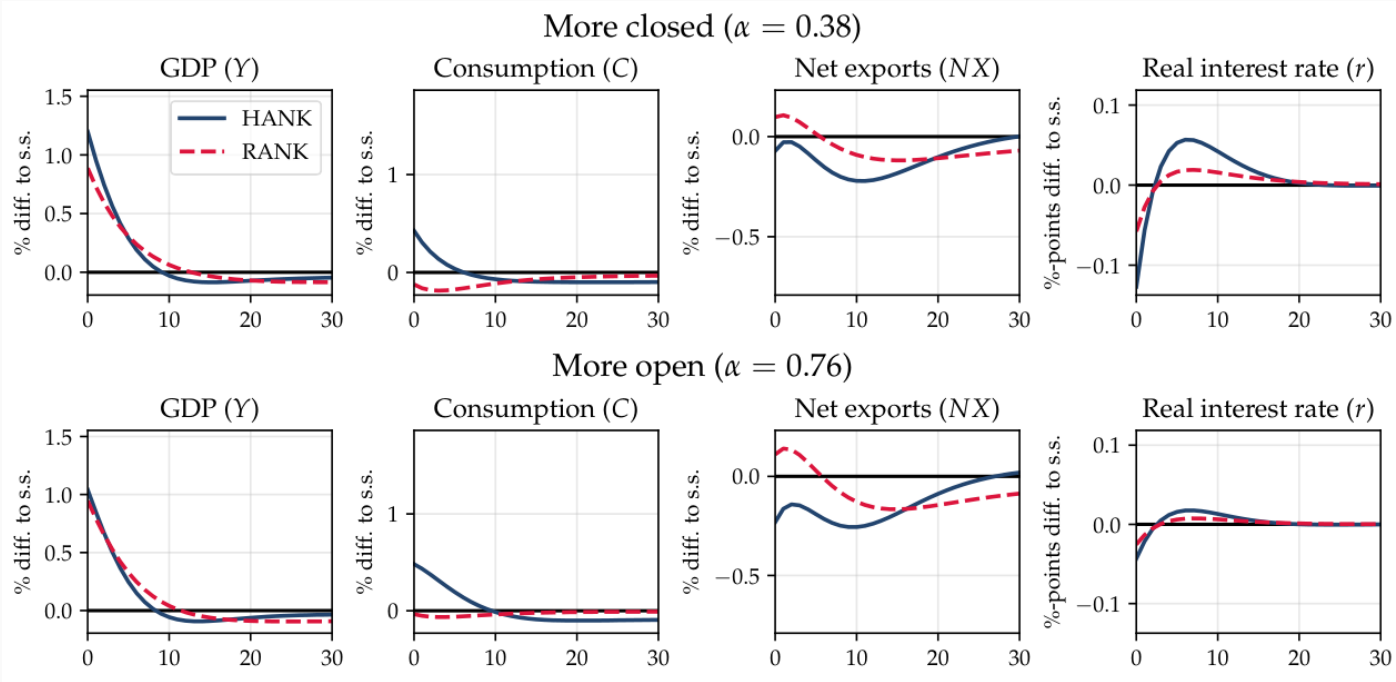
- Main result with deficit financed G shock:



- Relatively similar fiscal multiplier
  - HANK produces **much** larger C response
  - ... But this gets counteracted by larger drop in net exports

Fiscal spending shocks - openness

- How does fiscal multiplier vary with openness α? (plot IRFs for first and third quartile of $\frac{Imports}{GDP}$ across sample of OECD countries.)



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- Other shocks often studied in open economy context:
  - Foreign monetary policy shocks
  - Capital flow shocks (»sudden stops«)
  - Import price shocks

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- Motivation: Go back to international Kenyesian Cross with foreign demand $\boldsymbol{M}^*$

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- What is sign of $cov\,(d\boldsymbol{C}, d\boldsymbol{M}^*)$ empirically?

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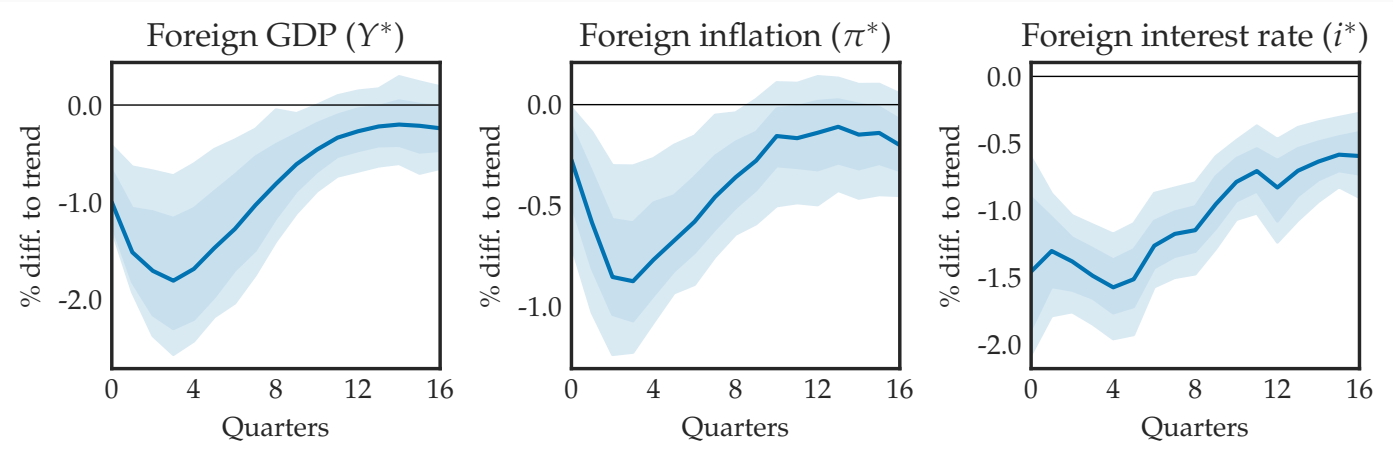
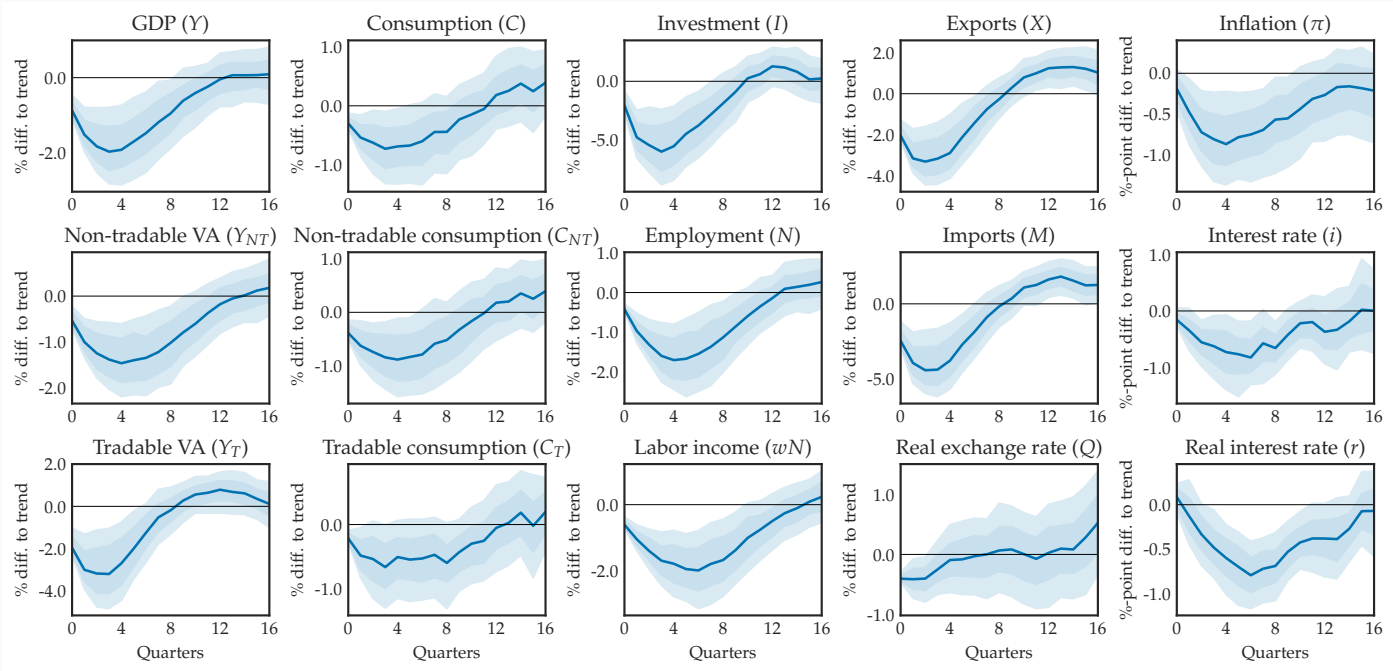
- Estimated foreign shock



Spillover effects

- Use estimated shock in foreign trading parters to estimate effects on domestic, SOE economy
- Estimate dyanmic OLS/LP $y_{c,t+h} = \beta_h i^*_{c,t} + \alpha_h \pi^*_{c,t} + \Theta_h M^*_{c,t}$ where y = domestic outcomes (GDP,C ...)



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- Why focus specifically on a **foreign demand shock?**
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- What about domestic demand shock (G)?
  - Identification more difficult
  - Literature ambiguous on whether C increases or decreases

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- Feed in estimated foreign demand shock, compare with empirics

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- Household problem:

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$$a_t \geq 0$$

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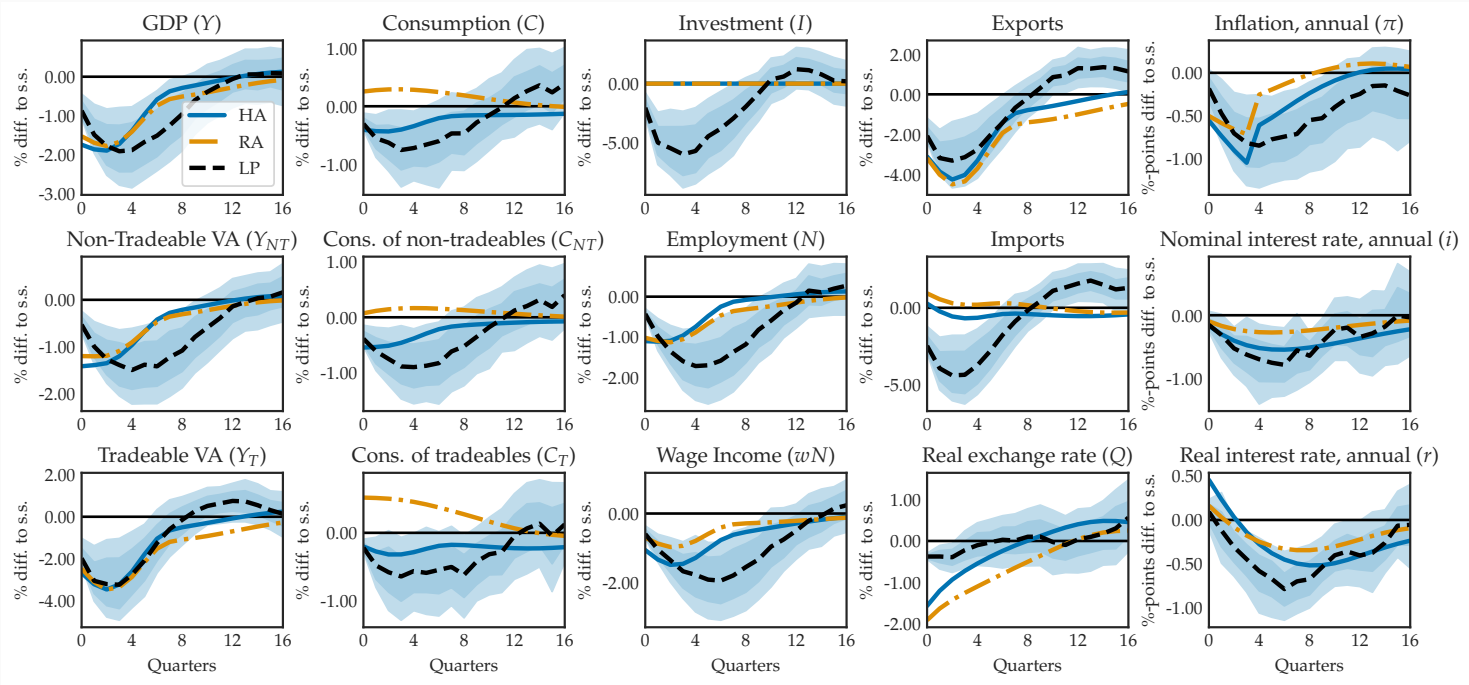
- Markov matrix for s is $P^s = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
  - HHs cannot move sectors. Harsh assumption, but consistent with short-run dynamics. Can alleviate by changing P^s
  - Could also have endogoues sector choice at HH level

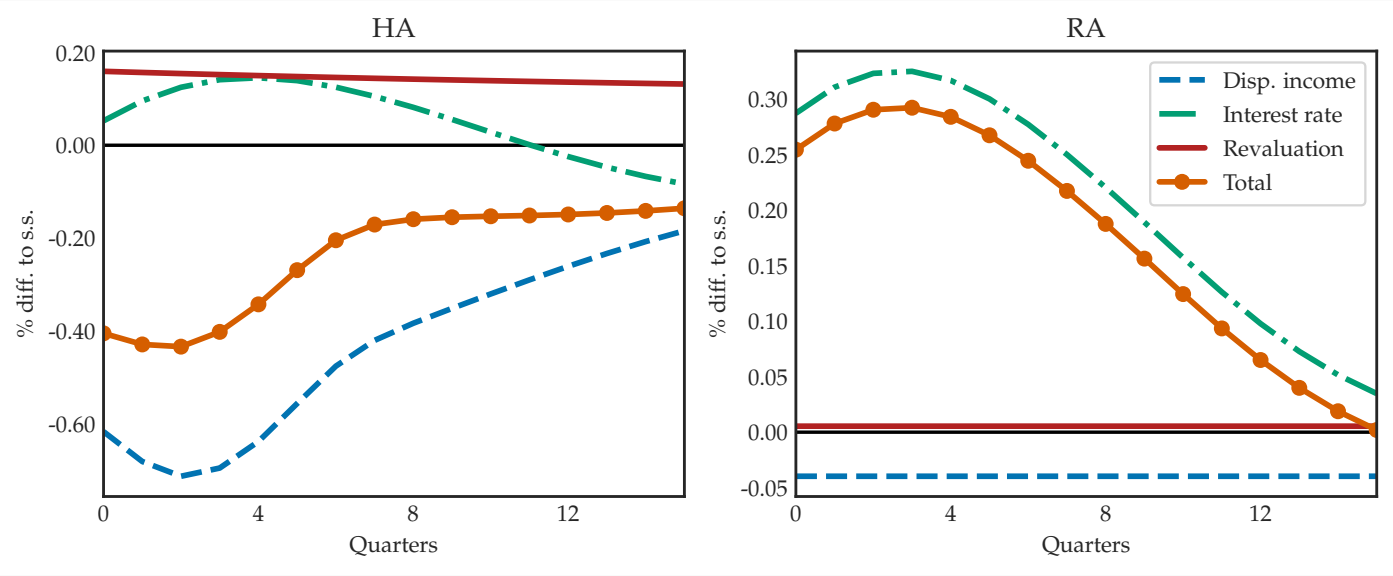
Model fit - floating

- Effects of foreign demand shock with a floating EXR



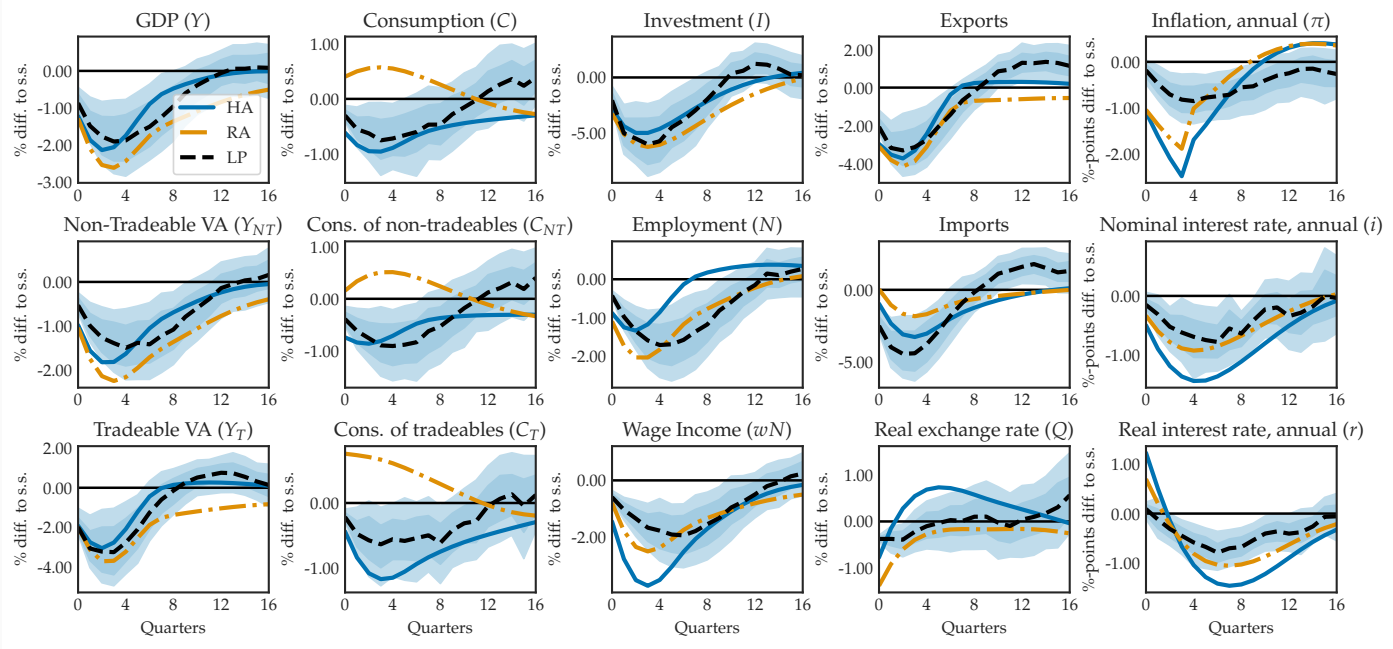
Decomposition

- Decompose $d\boldsymbol{C}$ into effects from interest rate, labor income and capital gain effects



Model fit - floating /w investment

- HANK response amplified by investment
- Note: Getting investment response right requires exogenous shock to investment



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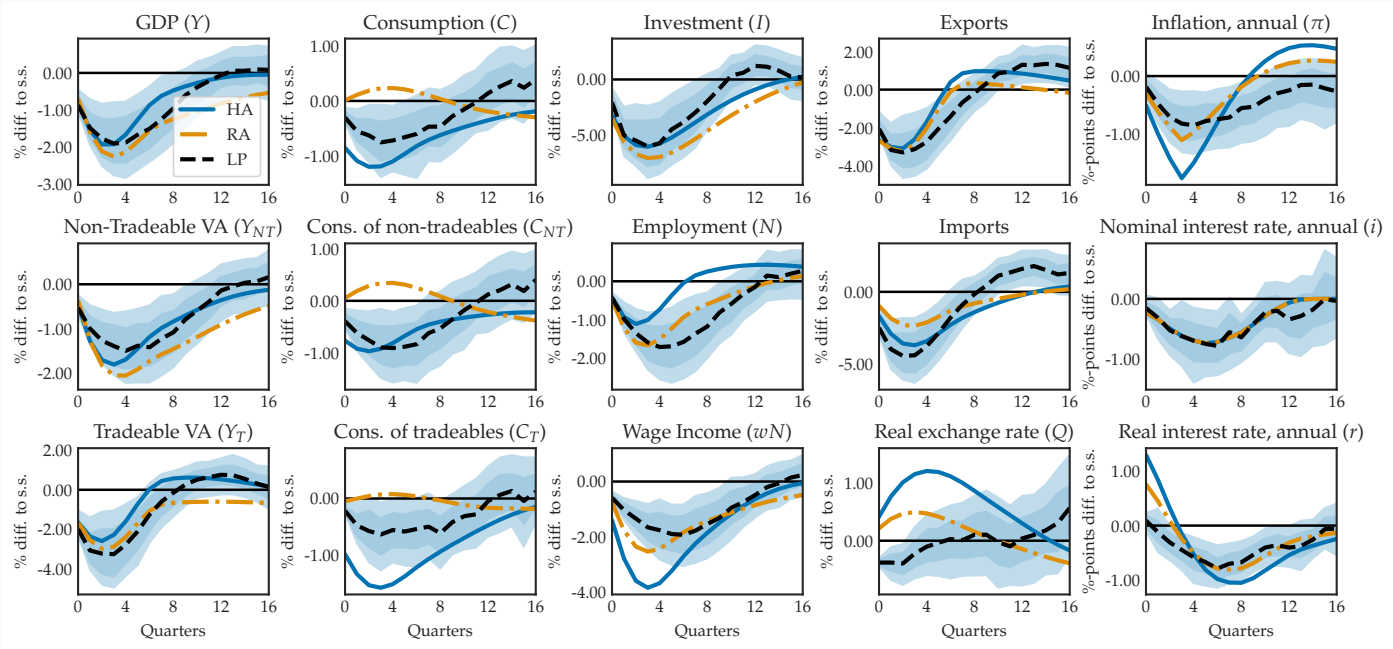
- UIP forces central bank in SOE to reduce i_t, so $r_t \downarrow$ (unless $\pi_{t+1} \downarrow\downarrow$)

Model fit - fixed

- Similar outcomes with fixed EXR



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- Fiscal policy loads on NT sector $\Rightarrow$ Very asymmetric effects, barely helps HHs in T sector
  - Issues for countries fixed EXRs or in monetary unions
  - Need targeted transfers

Conclusion

- How does heterogeneity affect transmission of shocks and policies in SOEs?
  - Monetary policy - Likely to be less effective due to real income channel of EXR
  - Fiscal policy - Closer to RANK multipliers due to crowding out of NX
  - Foreign demand shocks - larger transmission to domestic spending

IHANK - litterature

- Covered 3 papers here: Other papers in the litterature on HANK in open economies include:

1. Guo, X., Ottonello, P., & Perez, D. J. (2023) *Monetary policy and redistribution in open economies*
   - Redistributional effects of monetary policy in SOEs
2. Aggarwal, R., Auclert, A., Rognlie, M., & Straub, L. (2023). *Excess savings and twin deficits: The transmission of fiscal stimulus in open economies*
   - Fiscal stimulus in a multi-country model
3. De Ferra, S., Mitman, K., & Romei, F. (2020). *Household heterogeneity and the transmission of foreign shocks*
   - Effects of exchange rate depreciations when HHs have foreign currency debt
4. Bayer, C., Kriwoluzky, A., Müller, G. J., & Seyrich, F. (2024). *A HANK^2 model of monetary unions*. Journal of Monetary Economics
   - A 2-country HANK model

Summary

Summary and next week

- **Today:** Small open economy HANK models
- **Next week:**
  - Advanced HANK topics (**research frontier**)
  - Q&A
  - Exam
- **Homework:**
  1. Work on assignment