The following equations were employed in the development of PlateVib in reference to a letter on the vibration analysis of rectangular clamped plates written by J. P. Arenas for the editor of Journal of Sound and Vibration (2003).

Let consider an isotropic, un-damped, rectangular, thin plate with dimension of a (m) by b (m) and h (m) thick. Based on the principle of virtual work, the steady state transverse displacement, $\xi_0(x,y)$, of a full clamped plate subjected to harmonic point excitation at (x',y'), is

$$\xi_0(x,y) = F_0 \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \frac{\Psi_{mn}(x,y)\Psi_{mn}(x',y')}{B(I_1 I_2 + 2I_3 I_4 + I_5 I_6) - \rho_s \omega^2 I_2 I_6}$$

Here,

F_0 : force amplitude,

$B = Eh^3/12(1-\upsilon^2)$: bending stiffness,

E : Young's modulus,

υ : Poisson ratio,

ρ_s : plate surface density, and

ω : angular frequency.

The shape function is given as,

$$\Psi_{mn}(x,y) = \vartheta_m(x)\zeta_n(y)$$

where

$$\vartheta_m(x) = \mathcal{J}(\beta_m x/a) - \frac{\mathcal{J}(\beta_m)}{\mathcal{H}(\beta_m)}\mathcal{H}\left(\frac{\beta_m x}{a}\right);\ \mathcal{J}(s) = \cosh(s) - \cos(s)\,;\ \mathcal{H}(s) = \sinh(s) - \sin\,(s)$$

and

$$\zeta_n(y) = \mathcal{J}(\beta_n y/b) - \frac{\mathcal{J}(\beta_n)}{\mathcal{H}(\beta_n)}\mathcal{H}(\beta_n y/b)$$

Note that β_i is the i -th root of $\cosh(\beta)\,cos(\beta) = 0$.

The denominator can be calculated using

$$I_2 I_6 = \frac{ab}{\beta_m \beta_n}\mathcal{L}_m\mathcal{L}_n;\ I_3 I_4 = \frac{\beta_m\beta_n}{ab}\mathcal{R}_m\mathcal{R}_n;\ I_1 = I_6(\beta_m/a)^4;\ I_5 = I_2(\beta_n/b)^4$$

where

$$\mathcal{L}_i = \frac{(1+\mathcal{D}_i^2)sinh(2\beta_i)}{4} + sinh(\beta_i)\left[2\mathcal{D}_i\sin(\beta_i) - (1-\mathcal{D}_i^2)\cos(\beta_i)\right] - (1+\mathcal{D}_i^2)sin(\beta_i)cosh(\beta_i)$$
$$+ (1-\mathcal{D}_i^2)sin(\beta_i)cos(\beta_i) + \beta_i - \frac{\mathcal{D}_i[1+cosh(2\beta_i)]}{2} + \mathcal{D}_i\cos^2(\beta_i)\,;\ \mathcal{D}_i = \frac{\mathcal{J}(\beta_i)}{\mathcal{H}(\beta_i)}$$

and

$$\mathcal{R}_i = \frac{(1+\mathcal{D}_i^2)sinh(2\beta_i)}{4} - \frac{\mathcal{D}_i cosh(2\beta_i)}{2} - \frac{(1-\mathcal{D}_i^2)sin(\beta_i)cos(\beta_i)}{2} - \mathcal{D}_i\cos^2(\beta_i) - \mathcal{D}_i^2\beta_i + \frac{3\mathcal{D}_i}{2}$$