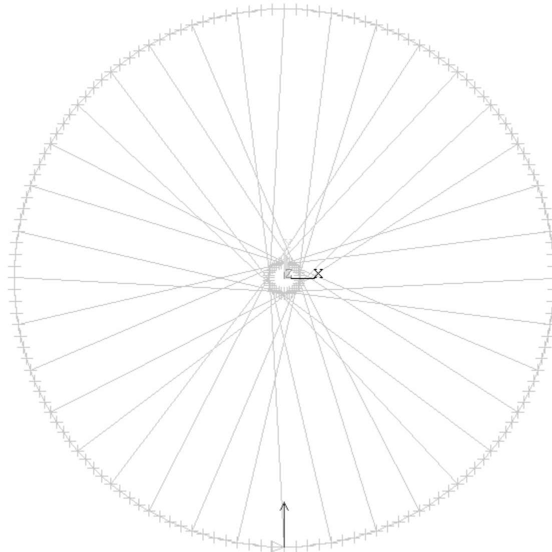

Finite Element Analysis of the Classic Bicycle Wheel

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Finite Element Analysis of the Classic Bicycle Wheel

1.0 Introduction

In the traditional realm of Finite Elements, there are very few practical applications where the finite code can be compared to direct measurements. At best, Engineers can usually only compare their computer analysis with the results predicted by established mechanical formulae. At worst, Engineers must rely on their experience and intuition to guide them towards a workable "right" answer.

In this exercise, we consider the classical bicycle wheel. This is a familiar structure whose geometrical design has remained virtually unchanged for the past several decades. However, although the design is familiar, and the design effective, it is still not very well understood from a "textbook analysis" standpoint. The optimal design was converged upon by trail and error – not through thorough calculation.

For that reason, C.J. Burgoyne and R. Dilmaghanian prepared and published a paper on this very subject in the *Journal of Engineering Mechanics* in September of 1991. In this paper, the authors conduct a series of experiments to measure the strain and bending moment inside the wheel's structure, and then compare their results to those predicted by accepted pencil and paper analysis methods. The purpose of this analysis is to formulate a finite element model of the classical bicycle wheel and compare published results with those revealed by ANSYS.

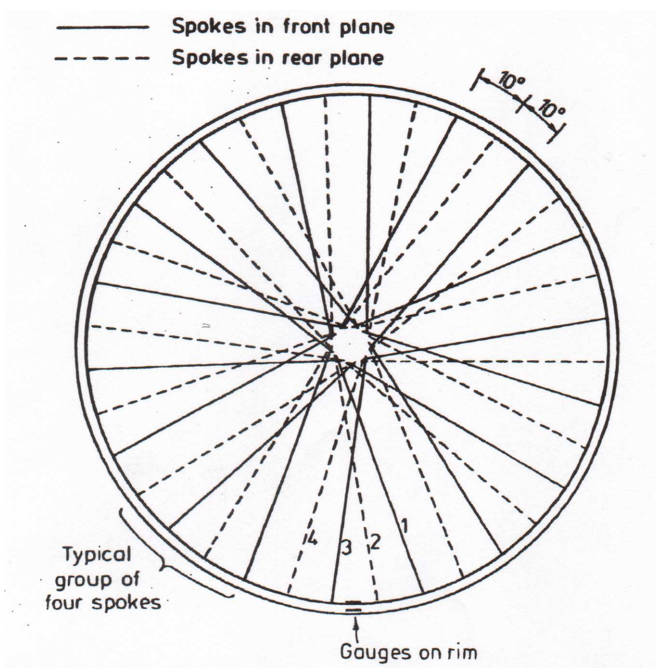


Figure 1 - Classical Bicycle Wheel Geometry[i]

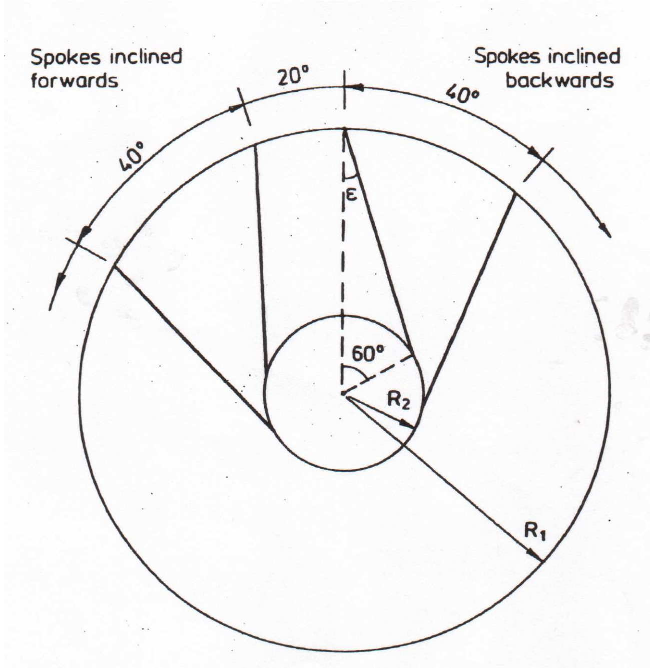


Figure 2 - Spoke Geometry[ii]

2.0 Measured Results

The Journal of Engineering Mechanics paper describes in detail the geometry of the classical bicycle wheel. Figures 1 and 2 show a portion of the spoke geometry provided by the paper. In addition, the authors provide data on moments of inertia, cross sectional areas, and other pertinent material properties.

2.1 Test Apparatus

Figure 3 shows the test apparatus used to take measurements on the actual bicycle wheel. The wheel is loaded directly through its axle, and data is taken via strain gauges. This setup can be duplicated in ANSYS though the model's boundary conditions.

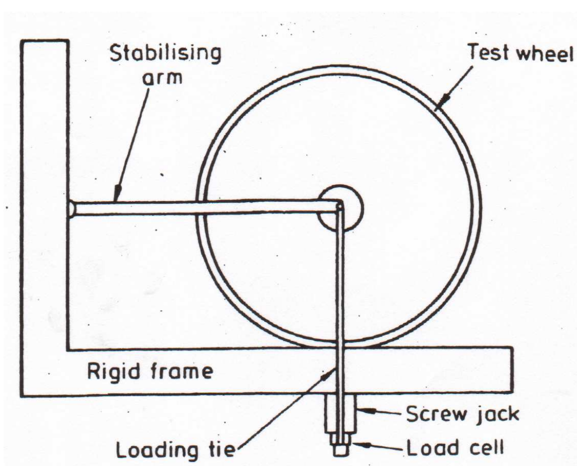


Figure 3 - Measured Results Test Apparatus[iii]

2.2 Spoke Strain

The published results included Figure 4, which details spoke strain as a function of wheel rotation. These results show that the maximum strain is approximately 6.00×10^{-4} but the actual value varies by about 10% depending on the spoke that is considered. Note that the strain in Spoke #3 is considerably less than the other spokes.

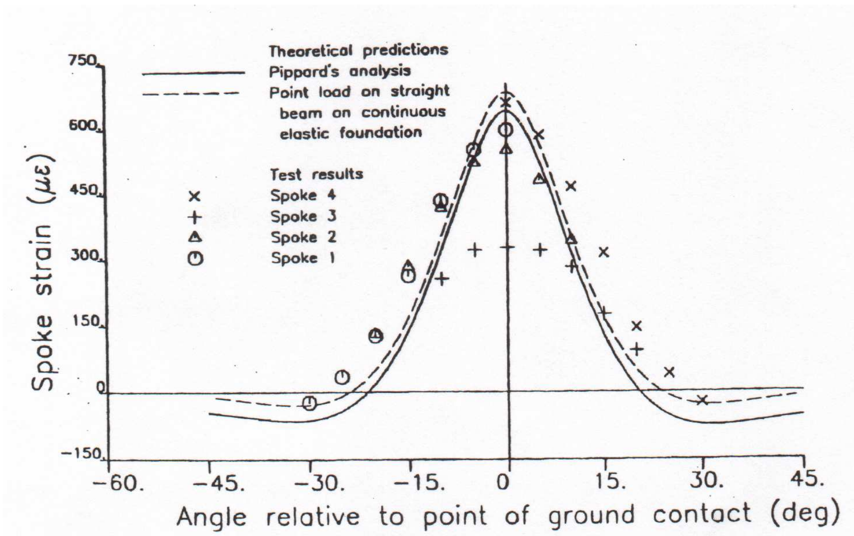


Figure 4 - Measured Spoke Strain[iv]

2.3 Spoke Bending

Figure 5 shows the measured results for bending in the rim. Note that the moment shown is scaled by the factor $1/PR_1$, where P is the applied load and R_1 is the radius of the rim. The measured results show a maximum bending stress of approximately 0.03 for the bare rim.

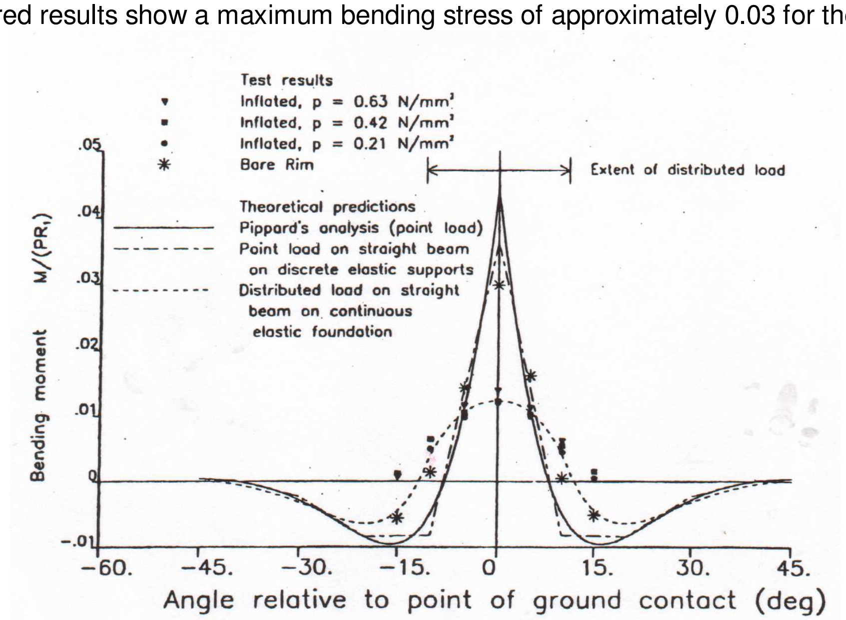


Figure 5 - Measured Spoke Bending[v]

3.0 Analysis in ANSYSed 5.4

ANSYSed 5.4 was used for all computer modeling. This educational version of ANSYS has full 3D capability, but is limited in the number of nodes and elements it can generate. Thus, a simple structure, such as a bicycle wheel, is a good candidate for analysis.

3.1 Analysis Goals

The goals of this analysis were to:

1. Determine the accuracy that the published results can be duplicated using ANSYS.
2. Determine the benefit of adding additional nodes to the wheel rim.
3. Analyze the impact spoke geometry has on the effectiveness of the wheel structure.

3.2 Assumptions

With known software limitations, we choose the following assumptions:

- Because the spokes were tensioned prior to the installation of the strain gauges, our ANSYS model does not need to pretension the spokes.
- While the published results included data using a bicycle tire inflated to various pressures, an ANSYS analysis that includes the bicycle tire is beyond the scope of this project. As such, all models shall include the rim and spokes only, and ignore the effects of the tire.
- As the bicycle wheel is primarily a 2D structure, all ANSYS models will be done in 2D.
- The published results included some discussion of a distributed load. Due to the limitations of the number of nodes available in ANSYSed, all models will use only point loads.

3.3 ANSYS Approach

Four models were generated using ANSYS scripts (See Table 1). Two of these models used "simple" geometry, in that all spokes were linked to a common center point (see Figure 6). The two other models used "complex" spoke geometry that more closely resembled the classical spoke designs described by Burgoyne and Dilmaghanian (see Figure 7). To explore Goal #2 described in section 3.1, scripts were developed for both the simple and complex models to place rim nodes at 2.5 and 10 degrees.

Beam elements (BEAM4) for the rim and truss elements (LINK8) were used to model the spokes. Although these elements are fully capable of modeling 3D geometry, they also work quite well in a 2D application. The material properties and dimensions used in the published results are used for all ANSYS modeling.

Dr. Jerry Fine provided an ANSYS script that "rotated" the bicycle wheel model to better duplicate the figures presented in the published results. This script was integrated into the models using 2.5° of rim node separation. Samples of the ANSYS scripts can be found in Appendix A and B.

In all models, all nodes at the wheel hub were constrained in the UX and UY directions. Additionally, all nodes were constrained the UZ direction to eliminate out of plane bending.

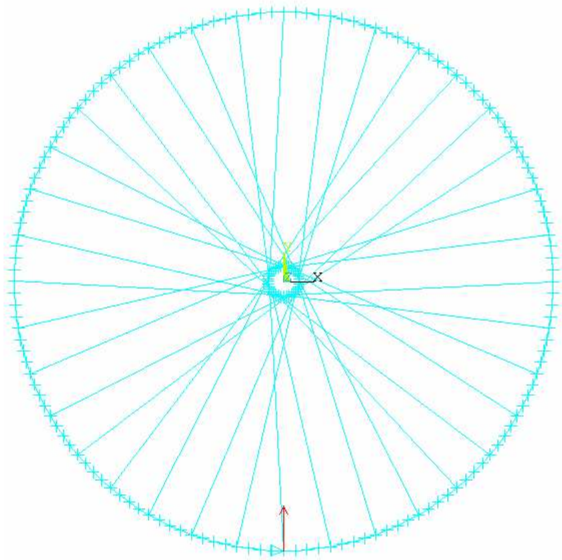


Figure 6 - Simple Geometry Boundary Conditions

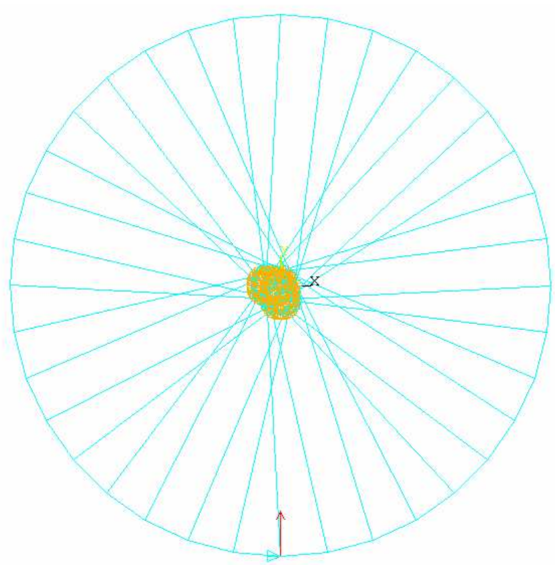


Figure 7 - Complex Geometry

Table 1 - Models Used in Analysis

#	Model Geometry Type	# of Rim Nodes	# of Spokes	Degrees Between Rim Nodes	Rotate Script Used
1	Simple	36	36	10.0	No
2	Complex	36	36	10.0	No
3	Simple	144	36	2.5	Yes
4	Complex	144	36	2.5	Yes

4.0 Results

Using the above approach, our scripts were run in ANSYSed for the following results.

4.1 Displacement

Figures 8 and 9 show the typical displacement (UY) calculated for the simple and complex geometry types. The maximum displacements remained rather constant for all four model types. ***This similarity in the displacement values is an important "sanity check" to be reviewed before further analysis is completed.*** Table 2 shows the calculated displacements of all four models.

Figure 8 - Simple Geometry Max Displacement

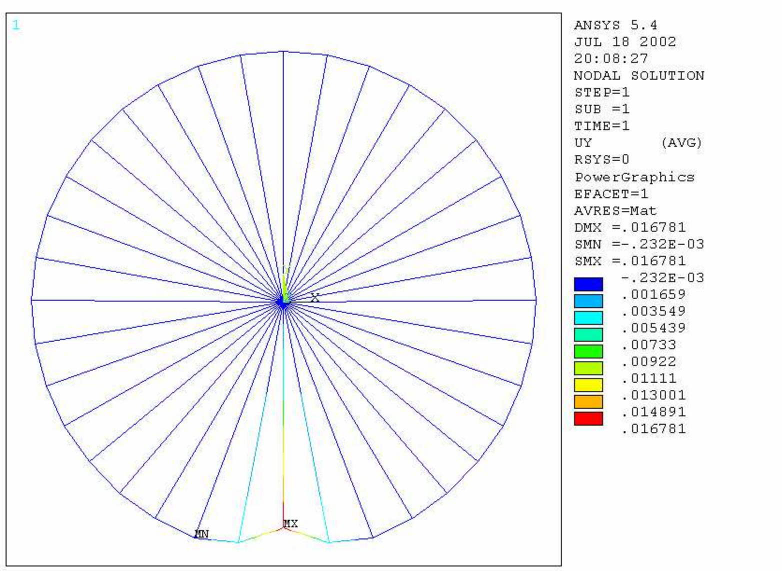


Figure 9 - Complex Geometry Max Displacement

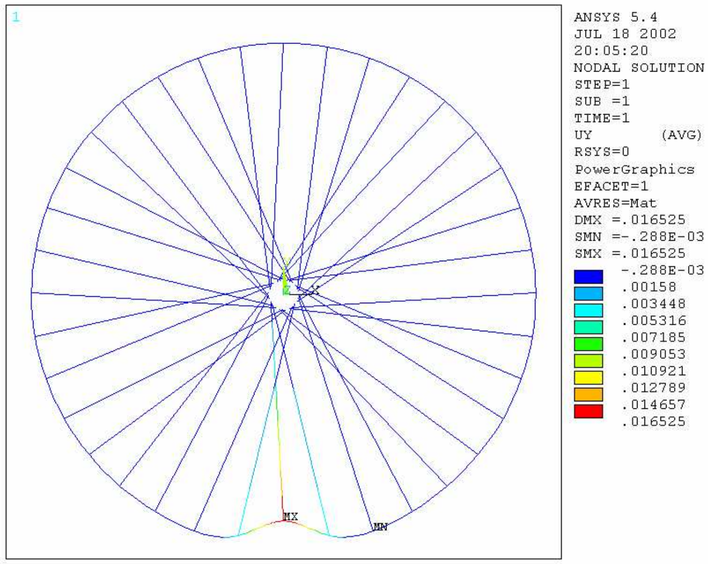


Table 2 - Maximum Displacement Results

Model Geometry Type	Rim Node Separation (deg)	Max Displacement (mm)
Simple	10	0.0168
Simple	2.5	0.0168
Complex	10	0.0165
Complex	2.5	0.0165

4.2 Strain

Figures 10 and 11 show the resulting strain should we rotate the wheel as done in the paper. Plots are shown for both the Simple and Complex models. Here the results are puzzling. The overall shape of the plot compares favorably to what was expected, though the maximum strain of about 5.5×10^{-5} at angle 0 is an order of magnitude less than the published result of about 6.00×10^{-4}. The differences between the simple and complex spoke models are rather slight – they are shown in separate plots because the lines overlap too well to differentiate one curve from the other.

Figure 10 - Strain Results, Simple Spoke Geometry

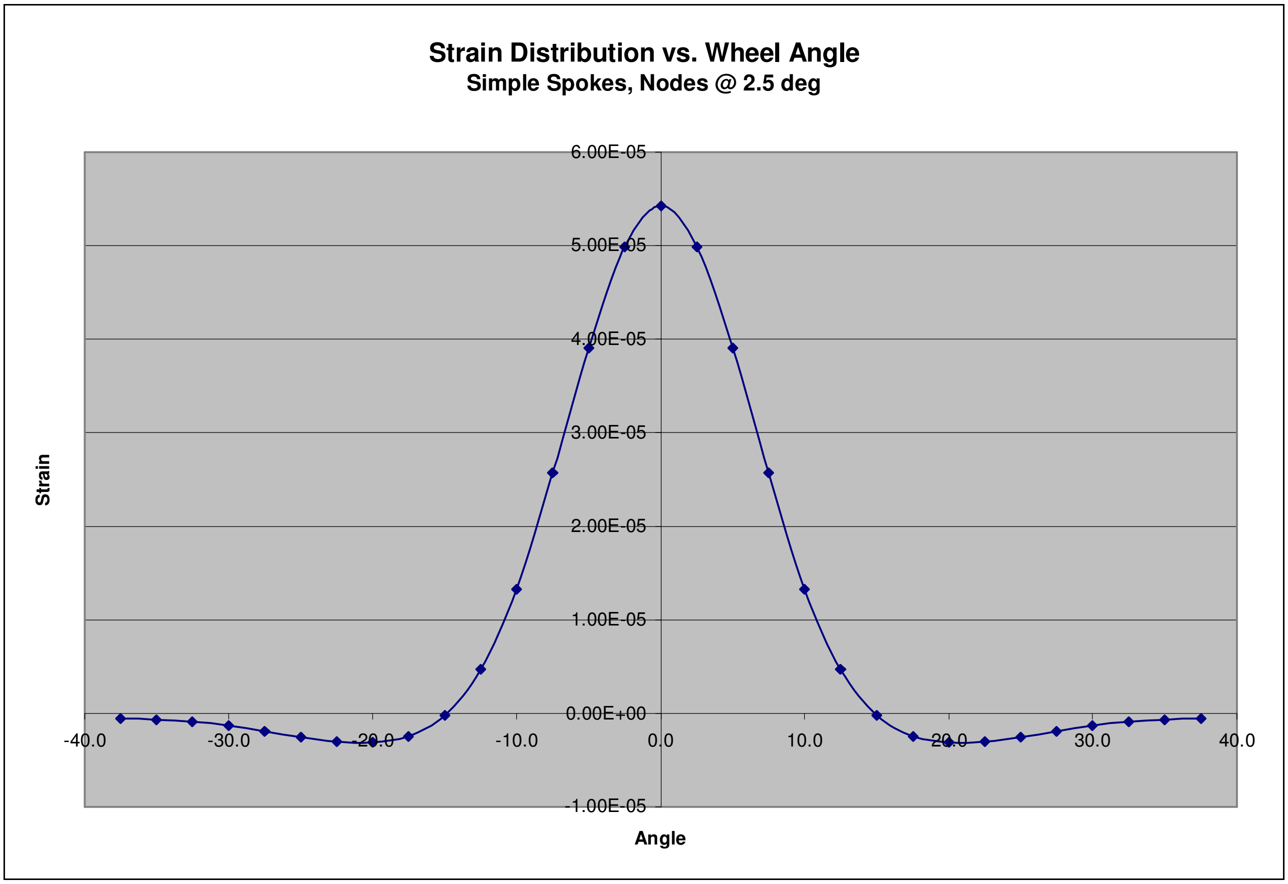
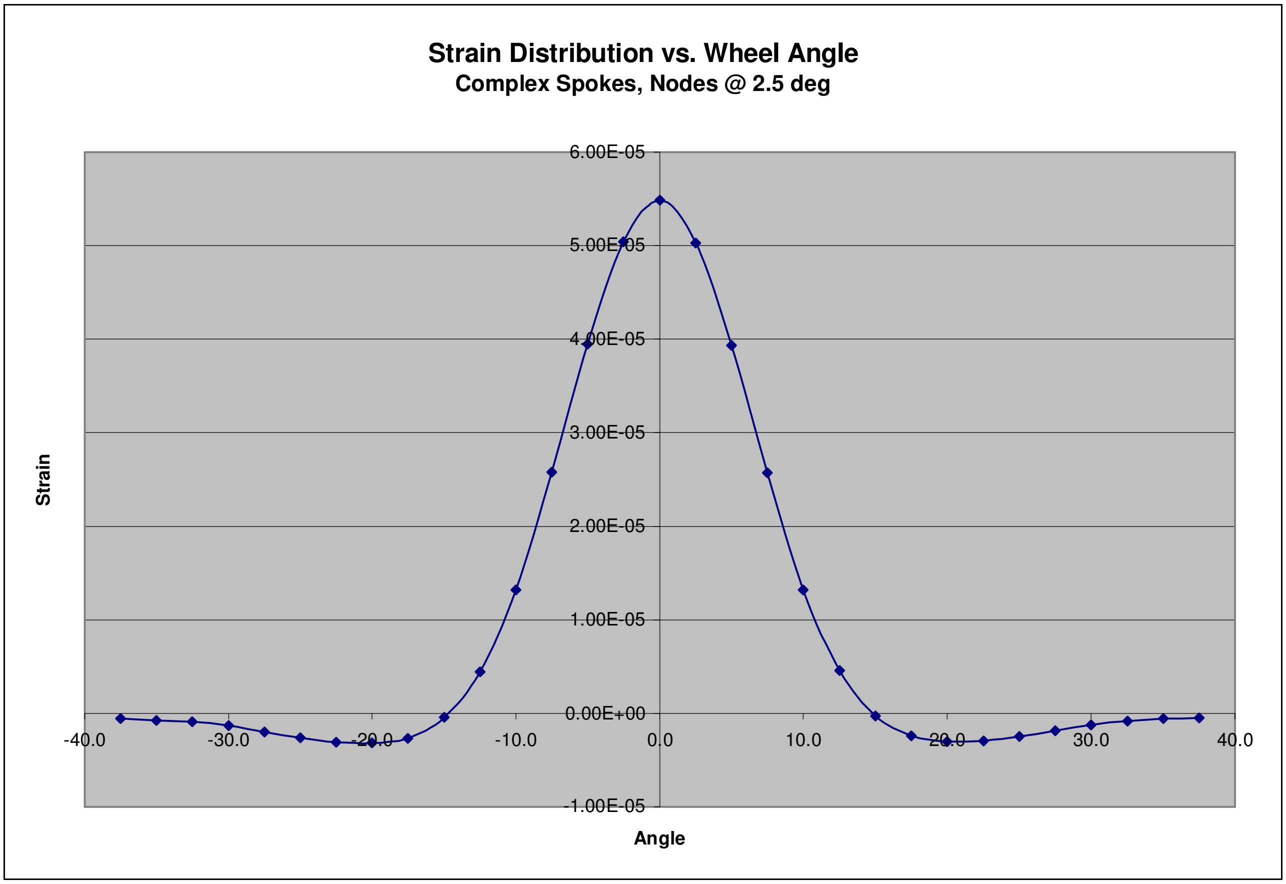


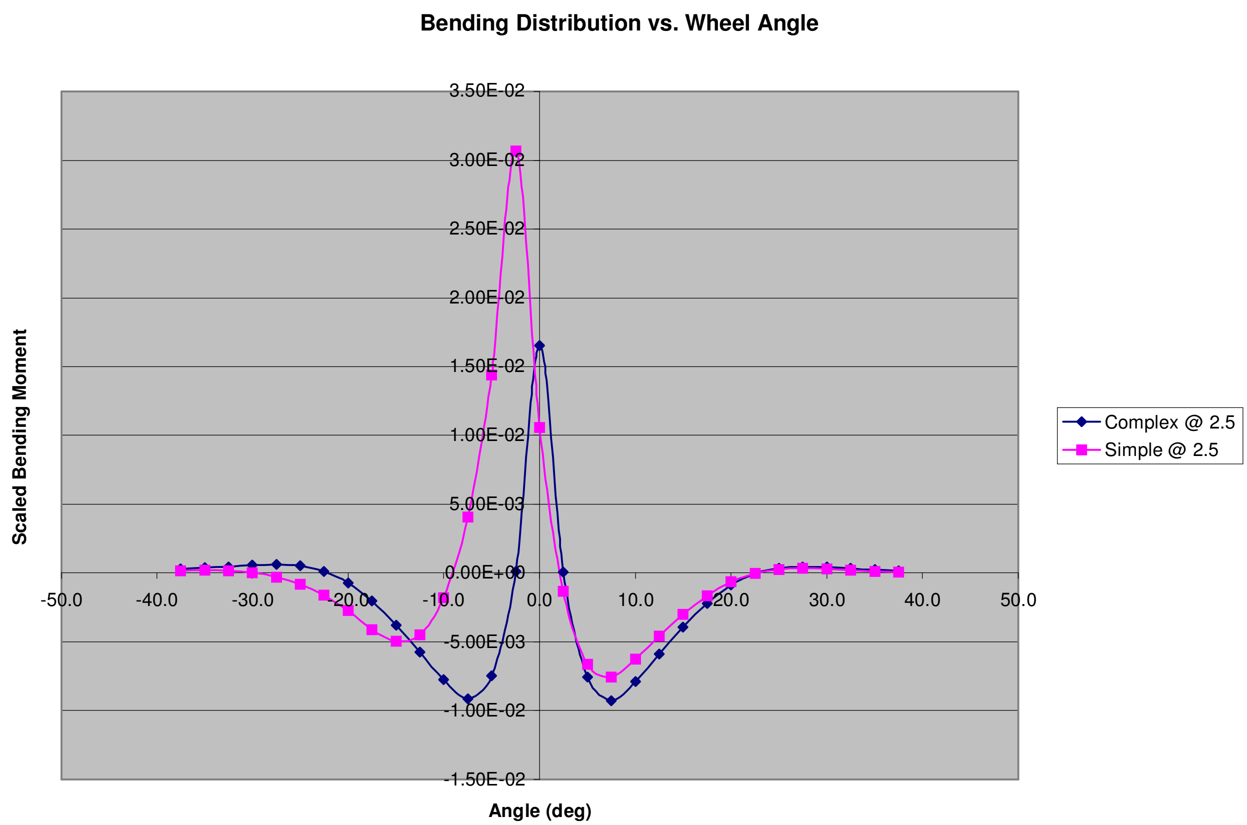
Figure 11 - Strain Results, Complex Spoke Geometry



4.3 Rim Bending

Figures 13 and 14 show the results of the ANSYS bending calculation, scaled by $1/PR_1$ for direct comparison to the published results. Here the results are more favorable, as the maximum bending for the complex model shows 0.016, compared to the published result of 0.03. While not an exact match in magnitude, the shape of the complex curve follows the published trajectory, and the maximum value lies within the range calculated for a partially inflated tire. The simple spoke model shows similar (though higher) magnitude, but the results are asymmetric.

Figure 12 - Bending Results, Complex and Simple Spoke Geometry



5.0 Conclusion

This analysis shows that ANSYS modeling can be a useful tool for analyzing simple structures such as the classical bicycle wheel. Although the exact magnitudes published by Burgoyne and Dilmaghanian were not found, the results track closely with what was expected. The author feels that with some tinkering, the ANSYS scripts could be made more accurate than they currently are – there seems to be some hidden nuance that is being overlooked by the current approach.

Appendix A – Script Output Data

Models using 2.5^{o} between rim nodes.

	Strain		Bending	
Angle	**Simple**	**Complex**	**Simple**	**Complex**
-37.5	-5.36E-07	-5.98E-07	1.83E-04	3.06E-04
-35.0	-6.73E-07	-7.21E-07	2.17E-04	3.89E-04
-32.5	-9.09E-07	-9.39E-07	1.80E-04	4.69E-04
-30.0	-1.32E-06	-1.34E-06	2.83E-05	5.52E-04
-27.5	-1.94E-06	-1.97E-06	-2.85E-04	6.13E-04
-25.0	-2.59E-06	-2.62E-06	-8.13E-04	5.13E-04
-22.5	-3.03E-06	-3.08E-06	-1.61E-03	1.13E-04
-20.0	-3.08E-06	-3.20E-06	-2.74E-03	-7.10E-04
-17.5	-2.46E-06	-2.66E-06	-4.09E-03	-2.04E-03
-15.0	-2.27E-07	-4.75E-07	-4.96E-03	-3.78E-03
-12.5	4.68E-06	4.45E-06	-4.47E-03	-5.75E-03
-10.0	1.33E-05	1.32E-05	-1.78E-03	-7.75E-03
-7.5	2.57E-05	2.58E-05	4.07E-03	-9.14E-03
-5.0	3.91E-05	3.95E-05	1.44E-02	-7.48E-03
-2.5	4.99E-05	5.04E-05	3.07E-02	1.35E-04
0.0	5.43E-05	5.49E-05	1.06E-02	1.66E-02
2.5	4.99E-05	5.03E-05	-1.32E-03	5.80E-05
5.0	3.91E-05	3.94E-05	-6.64E-03	-7.59E-03
7.5	2.57E-05	2.58E-05	-7.54E-03	-9.26E-03
10.0	1.33E-05	1.32E-05	-6.23E-03	-7.87E-03
12.5	4.68E-06	4.61E-06	-4.57E-03	-5.87E-03
15.0	-2.27E-07	-2.58E-07	-3.01E-03	-3.92E-03
17.5	-2.46E-06	-2.44E-06	-1.66E-03	-2.20E-03
20.0	-3.08E-06	-3.01E-06	-6.44E-04	-8.59E-04
22.5	-3.03E-06	-2.93E-06	-2.53E-05	-2.98E-05
25.0	-2.59E-06	-2.49E-06	2.64E-04	3.63E-04
27.5	-1.94E-06	-1.85E-06	3.32E-04	4.64E-04
30.0	-1.32E-06	-1.24E-06	2.84E-04	4.19E-04
32.5	-9.09E-07	-8.44E-07	2.11E-04	3.41E-04
35.0	-6.73E-07	-6.15E-07	1.38E-04	2.50E-04
37.5	-5.36E-07	-4.82E-07	7.31E-05	1.63E-04

Appendix B – Sample ANSYS Script

ANSYS Script, Simple Geometry, Nodes at 10 Degrees

```
!-----------------------------------------------------------------
!  Bicycle Wheel Problem
!  Cook Problem 2.24
!  Using data from "Bicycle Wheel as Presetressed Structure"
!    by C. J. Burgoyne and R. Dilmaghanian
!
!  SIMPLE RIM - RADIAL SPOKES
!  Rim Nodes at 10 degrees
!
!  ME522
!  Project
!  Andrew Hartz
!  7/7/02
!----------------------------------------------------------------
!
!
! *** Parameters ***
! All units in mm's & N's.
!
Arim   = 138.4
Aspoke = 62.34
Ixxrim = 1469
Iyyrim = 9366
Rrim   = 309.4
Rhub   = 18.0
Erim   = 70E3
Espoke = 210E3
rim_load = 1000
rim_d  = 13.22
rim_w  = 23.22
!
!----------------------------------------------------------------
! *** Preprocessor ***
/PREP7
CSYS,1  ! Cyl Coordinates
!
! Setup Spoke Elements
ET,1,LINK8
MP,EX,1,Espoke
R,1,Aspoke, ,
!
!
! Setup Rim Elements
!
ET,2,BEAM4
MP,EX,2,Erim
R,2,Arim,Ixxrim,Iyyrim,rim_d,rim_w, ,
!
!
!
! Simple Rim - Create Nodes and Elements
!
nnodes=36
theta = 360/nnodes
*DO,I,1,nnodes
   N,I,Rrim,(I-1)*theta
*ENDDO
N,,0,0
!
! Spoke Element Creation
!
TYPE,1
REAL,1
*DO,I,1,nnodes
   E,nnodes+1,I
*ENDDO
!
```

```
! Rim Element Creation
!
TYPE,2
REAL,2
*DO,I,1,nnodes-1
   E,I,I+1
*ENDDO
E,1,nnodes
!
! Apply Boundary Conditions
!
F,28,FY,rim_load,
D,28,,,,,,UX
D,nnodes+1,,,,,,ALL
D,ALL,,,,,,UZ
!
FINISH
!
/SOLU
SOLVE
FINISH
!
/post1
etable,strain,lepel,1          ! strain into element table
etable,mom_z,smisc,6           ! bending moment into element table
FINISH
!
```

Appendix B – Sample ANSYS Script

Complex Geometry, Nodes at 2.5 Degrees

```
!-----------------------------------------------------------------
!  Bicycle Wheel Problem
!  Cook Problem 2.24
!  Using data from "Bicycle Wheel as Presetressed Structure"
!    by C. J. Burgoyne and R. Dilmaghanian
!
!  COMPLEX RIM - GEOMETRY #2  (2.5 deg between rim nodes)
!
!  ME522
!  Project
!  Andrew Hartz
!  7/13/02
!-----------------------------------------------------------------
!
!
! *** Parameters ***
! All units in mm's & N's.
Arim   = 138.4
Aspoke = 62.34
Ixxrim = 1469
Iyyrim = 9366
Rrim   = 309.4
Rhub   = 18.0
Erim   = 70E3
Espoke = 210E3
F      = 1000
rim_d  = 13.22
rim_w  = 23.22
!
!
!-----------------------------------------------------------------
! *** Preprocessor ***
/PREP7
CSYS,1  ! Cyl Coordinates
!
! Setup Spoke Elements
ET,1,LINK8
MP,EX,1,Espoke
R,1,Aspoke, ,
!
! Rim Elements
ET,2,BEAM4
MP,EX,2,Erim
R,2,Arim,Ixxrim,Iyyrim,rim_d,rim_w, ,
!
!
! Create Nodes and Elements
!
theta = 2.5
!
! Rim Nodes
nnodes=17
*DO,I,1,nnodes
   N,I,Rrim,-1*(I-1)*theta
*ENDDO
!
! Spoke Nodes
N,18,Rhub,-60
N,19,Rhub,-70
N,20,Rhub,40
N,21,Rhub,30
!
!
! Rim Element Creation
!
TYPE,2
REAL,2
```

```
*DO,I,1,nnodes-1
   E,I,I+1
*ENDDO
!
! Spoke Element Creation
!
TYPE,1
REAL,1
E,1,18
E,5,19
E,9,20
E,13,21
!
!
! Copy Nodes and Elements and Merge the overlap
NGEN,9,21,ALL, , , ,40, ,1,
EGEN,9,21,ALL, , , , , , ,
NUMMRG,ALL, , ,
!
!
! Apply Boundary Conditions
!
F,152,FY,1000,
D,152,,,,,,UX
D,ALL,,,,,,UZ
!
NSEL,S,LOC,X,Rhub
D,ALL,,,,,,ALL
ALLSEL
!
FINISH
!
/SOLU
SOLVE
FINISH
!
!
!  --- Start of Output Script by Dr. Fine ----
!
!  Commands to analyze bicycle wheel.  Bare rim.  Force applied at ground.
!
!  Prints strain and bending moment vs. angle to a file for comparison with
!  bicycle paper by Burgoyne and Dalmaghanian.
!
!  This software is not guaranteed.  Use it at your peril.  Etc.
!
!===========================================================================
/prep7
!
!  To use this macro your model must be producing verified results in all test cases.
!  It assumes you have nodes at 2.5 deg. intervals along rim including a 6 o'clock node.
!  You should have already applied bc to the hub end of the spokes.
!  Remove your code which applies the load and bc at the contact point.  Macro will do
it.
!  You will have to edit two lines in the post processor to insert element numbers
!  from your own model.  Those lines are marked EDIT THIS LINE.
!  In this example element 17 is the 6 o'clock spoke.  Element 1 is rim beam at its foot.
!
rim_rad = 309.4   ! the rim radius
!
!  Rotate nodal coordinates of all nodes to global cylindrical.  This is needed
!  to apply radial force and tangential displacement bc.
!
allsel
csys,1        !  shift coordinate system  to global cylindrical
nrotat,all    !  rotate all nodal coordinate systems to global cyl.
!
!-------------End of model set up, Leave preprocessor-----------------------
!
finish
!
```

```
!-------------------Verify model---------------------------------------------
!
!   For comparison with paper move radial force between
!   -37.5 and 37.5 degrees around node originally at 6 o'clock.
!   This is equivalent to rotating the wheel and measuring the
!   strain on spoke, and bending moment next to it.
!
!----------------------------------------------------------------------------
!
! NOTES from Andy Hartz:
! Changes were required to get this script to run on AnsysEd 5.4
! Changes were:
! 1.  Rename variable 'iter' to 'andy' (Catchy name, eh?)
! 2.  Move *CFOPEN and *CFCLOS outside of the *do loop to avoid overwriting
!     existing data.
!
*CFOPEN,'results','txt'    ! file write stuff
!
rim_load    = -1000
*do,andy,1,31
    ang=-127.5+2.5*(andy-1)
    !
    !  run solver, set up force & displc. bc. and solve the load step
    !
    /solu
       nsel,s,loc,x,rim_rad,rim_rad  ! Select node to which
       nsel,r,loc,y,ang,ang          ! rim load is applied
       cm,bnode,node                 ! make it a component
       f,all,fx,rim_load             ! Apply the load
       d,all,uy,0                    ! Constrain against angular motion
       allsel                        ! Select everything before solve
       solve
       fdele,bnode,all               ! gets rid of the force, done with it
       ddele,bnode,all               ! gets rid of old displacement constraint
       allsel                        ! clean up
       cmdele,bnode
    finish
    !
    !  run post processor, extract strain and bending moment, write them to file
    !
    /post1
       etable,strain,lepel,1         ! strain into element table
       etable,mom_z,smisc,6          ! bending moment into element table
       *get,eps,etab,1,elem,158       ! EDIT THIS LINE. strain in elem #17 called eps
       *get,mom,etab,2,elem,145       ! EDIT THIS LINE. b.moment in elem #1 called mom
       mom=-mom/(rim_rad*rim_load)   ! Scale bending moment as per reference
       apos=-37.5+2.5*(andy-1)       ! angular position for the plot
       *VWRITE,apos,eps,mom
       (3e15.6)
       etable,erase
   finish
   !
   !  end post processor run, ready for another angle
   !
*enddo
*CFCLOS
!
!
```

References

[i] Burgoyne, C.J. and R. Dilmaghanian. "Bicycle Wheel as Prestressed Structure." *Journal of Engineering Mechanics*. 119(3), 442.

[ii] Ibid., 443.

[iii] Ibid., 444.

[iv] Ibid., 447.

[v] Ibid., 449.