Phase A

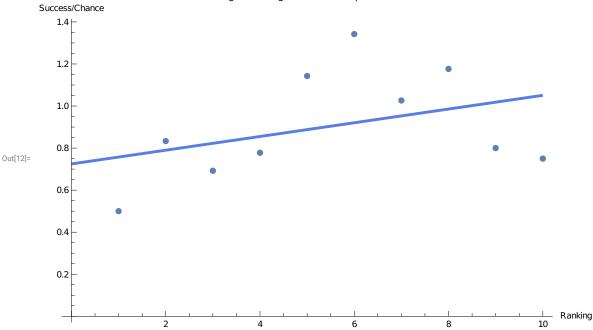
```
in[1]:= astrologerSuccess = {3, 10, 9, 7, 16, 17, 13, 20, 4, 1};
      expected = {6, 12, 13, 9, 14, 12.66666667, 12.66666667, 17, 5, 1.333333333};
 In[3]:= data = N[astrologerSuccess/expected]
\texttt{Out[3]=} \ \{0.5,\, 0.833333,\, 0.692308,\, 0.777778,\, 1.14286,\, 1.34211,\, 1.02632,\, 1.17647,\, 0.8,\, 0.75\}
 In[4]:= ListPlot[data]
      1.4
      1.2
      1.0
      0.8
Out[4]=
      0.6
      0.4
      0.2
                   2
                              4
                                                     8
                                                                10
 In[8]:= lmfA = LinearModelFit[data, x, x]
     FittedModel 0.724705+0.0326203x
Out[8]=
 In[9]:= Normal[lmfA]
Out[9]= 0.724705 + 0.0326203 x
In[10]:= ListPlot[lmfA["FitResiduals"]]
       0.4
       0.2
Out[10]=
                                                                10
                    2
                                                     8
                                          6
      -0.2
```

In[12]:= Show[ListPlot[data,

PlotLabel → "Are Astrologer Rankings Effective Compared to Chance?", AxesLabel → {"Ranking", "Success/Chance"}],

 $Plot[lmf[x], \{x, 0, 10\}, PlotTheme \rightarrow "Business"], ImageSize \rightarrow Large]$

Are Astrologer Rankings Effective Compared to Chance?



In[13]:= lmfA["ParameterTable"]

Estimate Standard Error t-Statistic P-Value Out[13]= 1 0.724705 0.173139 4.18569 0.0030557 x 0.0326203 0.0279038 1.16903 0.276045

In[14]:= lmfA["ParameterConfidenceIntervals"]

 $Out[14] = \{ \{0.325447, 1.12396\}, \{-0.031726, 0.0969666\} \}$

In[15]:= lmfA["RSquared"]

Out[15]= 0.145903

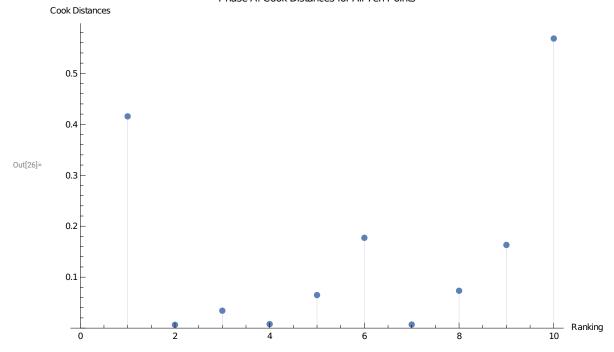
In[16]:= lmfA["ANOVATable"]

DF SS MS F-Statistic P-Value 1 0.0877867 0.0877867 1.36662 0.276045 Out[16]= Error 8 0.513891 0.0642364 Total 9 0.601678

Check Cook distances to identify highly influential points:

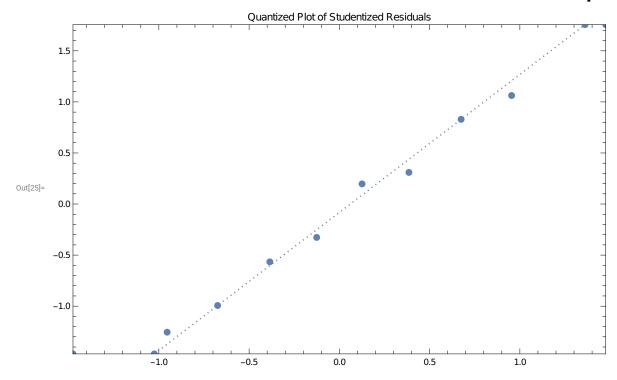
lo[26]:= ListPlot[cdA = lmfA["CookDistances"], PlotRange \rightarrow {0, All}, Filling \rightarrow 0, AxesLabel \rightarrow {"Ranking", "Cook Distances"}, PlotLabel → "Phase A: Cook Distances for All Ten Points", ImageSize → "Large"

Phase A: Cook Distances for All Ten Points



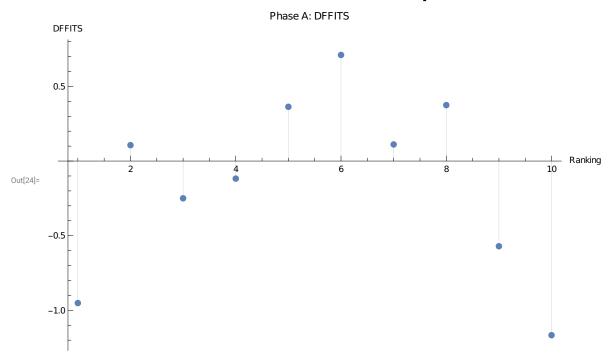
In[18]:= Position[cdA, $_$?(\sharp > .5 &)]

Out[18]= $\{\{10\}\}$



Use DFFITS values to assess the influence of each point on the fitted values:

ln[24]:= ListPlot[lmfA["FitDifferences"], PlotRange \rightarrow All, Filling → 0, "PlotLabel" → "Phase A: DFFITS", AxesLabel → {"Ranking", "DFFITS"}, ImageSize → Large



Use DFBETAS values to assess the influence of each point on each estimated parameter:

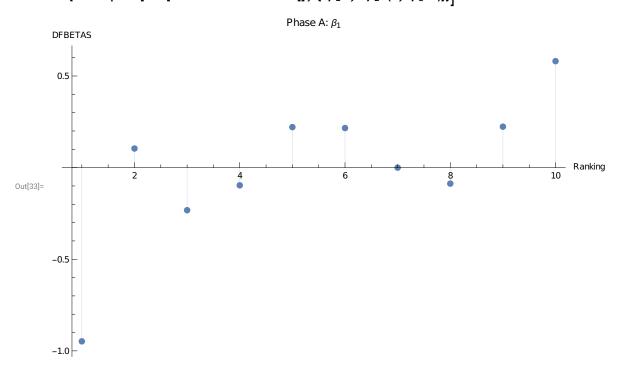
```
In[.]:= N[2/Sqrt[10]]
```

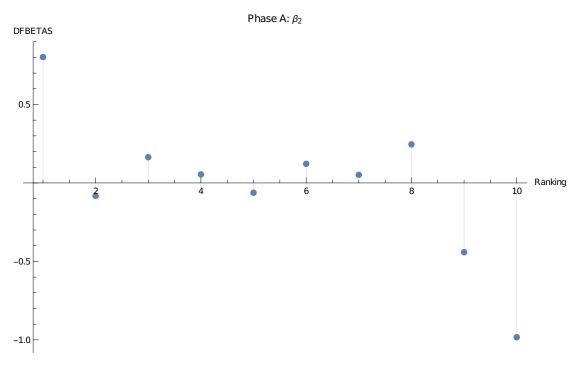
 $Out[\circ] = 0.632456$

```
In[28]:= dfbetasA = Transpose[lmfA["BetaDifferences"]]
```

```
Out[28]= {-0.94802, 0.104226, -0.232384, -0.0964876, 0.22102,}
       0.216049, -4.00511 \times 10^{-18}, -0.0872096, 0.223478, 0.581074
      {0.802133, -0.0823081, 0.163853, 0.0544264, -0.0623362,
       0.121868, 0.0514793, 0.245964, -0.441205, -0.98331
```

 $\label{eq:loss_problem} $$\inf_{x \in \mathbb{R}^2} \mathbb{E}[x] = \mathbb{E}[x] $$\inf_{x \in \mathbb{R}^2} \mathbb{E}[x] $$\inf_{x \in \mathbb{R}^2} \mathbb{E}[x] $$\inf_{x \in \mathbb{R}^2} \mathbb{E}[x] = \mathbb{E}[x] $$\inf_{x \in \mathbb{R}^2} \mathbb{E}[x] $$\inf_{x \in \mathbb{R}^2} \mathbb{E}[x] $$\inf_{x \in \mathbb{R}^2} \mathbb{E}[x] $$\inf_{x \in \mathbb{R}^2} \mathbb{E}[x] $$\inf_{x \in \mathbb{R}^2$ $ImageSize \rightarrow Large, AxesLabel \rightarrow \{"Ranking", "DFBETAS"\}] \&,$ {Transpose[lmf["BetaDifferences"]], {" β_1 ", " β_2 "(*, " β_3 "*)}}] // Row





Phase B

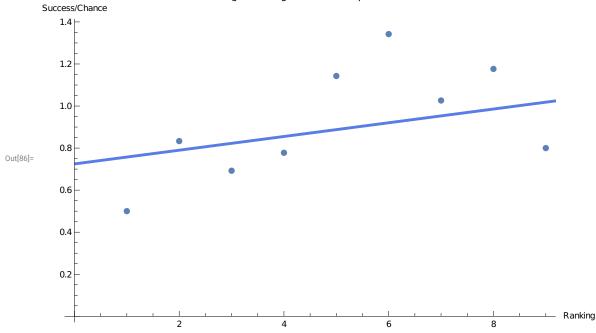
```
ln[80]:= astrologerSuccess = {3, 10, 9, 7, 16, 17, 13, 20, 4(*,1*)};
      expected = {6, 12, 13, 9, 14, 12.66666667, 12.66666667, 17, 5(*,1.333333333*)};
In[81]:= data = N[astrologerSuccess/expected]
Out[81]= {0.5, 0.833333, 0.692308, 0.777778, 1.14286, 1.34211, 1.02632, 1.17647, 0.8}
In[82]:= ListPlot[data]
     1.4
     1.2
     1.0
     0.8
Out[82]=
     0.6
     0.4
     0.2
                   2
                               4
                                                       8
In[83]:= lmfB = LinearModelFit[data, x, x]
Out[83]= FittedModel 0.632761+0.0576959x
In[84]:= Normal[lmfB]
Out[84]= 0.632761 + 0.0576959 x
In[85]:= ListPlot[lmfB["FitResiduals"]]
      0.4
      0.3
      0.2
      0.1
Out[85]=
                                4
                                           6
      -0.1
      -0.2
      -0.3
```

In[86]:= Show[ListPlot[data,

PlotLabel → "Are Astrologer Rankings Effective Compared to Chance?", AxesLabel → {"Ranking", "Success/Chance"}],

 $Plot[lmf[x], \{x, 0, 10\}, PlotTheme \rightarrow "Business"], ImageSize \rightarrow Large]$

Are Astrologer Rankings Effective Compared to Chance?



In[87]:= lmfB["ParameterTable"]

Estimate Standard Error t-Statistic P-Value Out[87]= 1 0.632761 0.168273 3.76032 0.00707189 x 0.0576959 0.0299029 1.92944 0.0950004

In[88]:= lmfB["ParameterConfidenceIntervals"]

 $\{\{0.234859, 1.03066\}, \{-0.0130132, 0.128405\}\}$

In[89]:= lmfB["RSquared"]

Out[89] = 0.347182

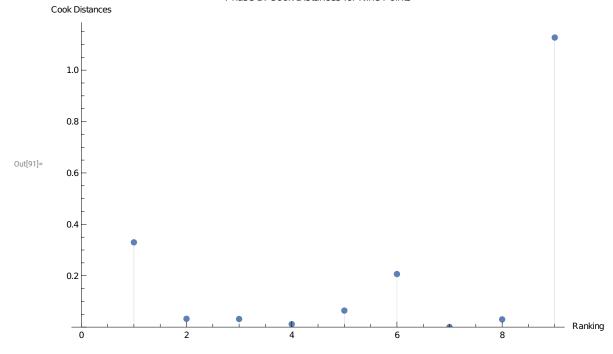
In[90]:= lmfB["ANOVATable"]

DF SS MS F-Statistic P-Value 1 0.199729 0.199729 3.72274 0.0950004 Out[90]= Error 7 0.375557 0.0536511 Total 8 0.575287

Check Cook distances to identify highly influential points:

 $\label{eq:loss_problem} $$ \inf[91] = $$ ListPlot[cdB = lmfB["CookDistances"], PlotRange $\to \{0, All\}, $$ $$ $\to \infty$. $$$ $\label{eq:filling} \textbf{Filling} \rightarrow \textbf{0}, \, \textbf{AxesLabel} \rightarrow \Big\{ \text{"Ranking"}, \, \text{"Cook Distances"} \Big\},$ PlotLabel → "Phase B: Cook Distances for Nine Points", ImageSize → "Large"

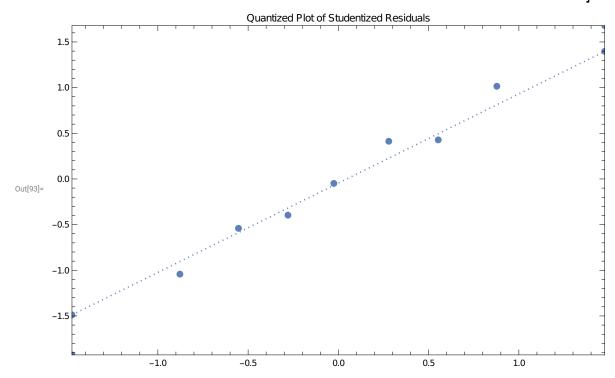
Phase B: Cook Distances for Nine Points



In[92]:= Position[cdB, $_$?(\sharp > .5 &)]

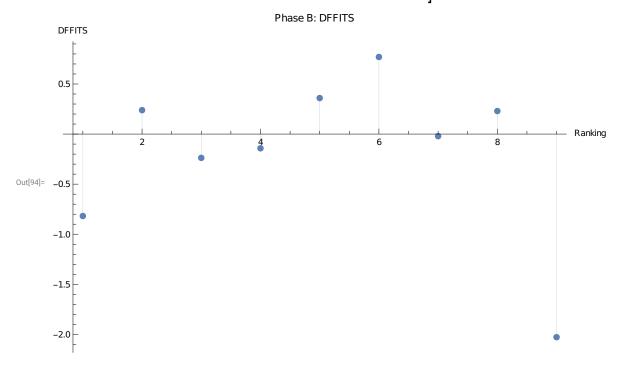
Out[92]= $\{\{9\}\}$

 $\verb| [93] = QuantilePlot[lmfB["StandardizedResiduals"], \\$ Table[InverseCDF[NormalDistribution[], q], {q, 1/100, 99/100, 1/50}], PlotLabel → "Quantized Plot of Studentized Residuals", ImageSize → Large



Use DFFITS values to assess the influence of each point on the fitted values:

ln[94]:= ListPlot[lmfB["FitDifferences"], PlotRange \rightarrow All, Filling → 0, "PlotLabel" → "Phase B: DFFITS", AxesLabel → {"Ranking", "DFFITS"}, ImageSize → Large



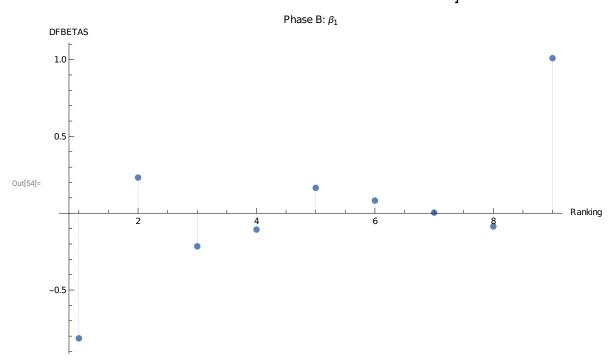
Use DFBETAS values to assess the influence of each point on each estimated parameter:

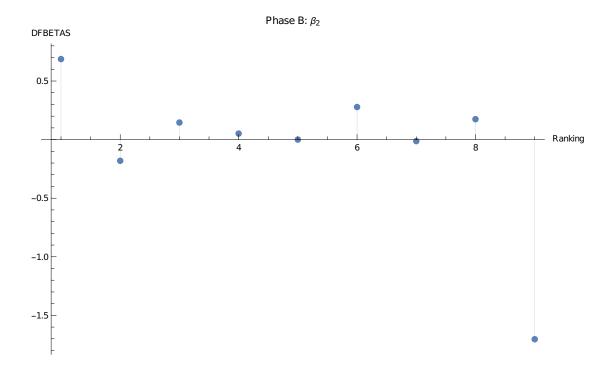
```
In[51]:= N[2/Sqrt[9]]
```

Out[51]= 0.666667

In[52]:= dfbetasB = Transpose[lmfB["BetaDifferences"]]

Out[52]= ${-0.814351, 0.232095, -0.215593, -0.106398, 0.165037, 0.0823324, }$ 0.00383596, -0.0860024, 1.00929, $\{0.687391, -0.180841, 0.145585,$ $0.0513201, -1.37421 \times 10^{-17}, 0.277986, -0.0129517, 0.174227, -1.70389$ [n[54]]= MapThread[ListPlot[#1, PlotRange \rightarrow All, Filling \rightarrow 0, PlotLabel \rightarrow "Phase B: " \iff #2, $ImageSize \rightarrow Large, AxesLabel \rightarrow \{"Ranking", "DFBETAS"\}] \&,$ {Transpose[lmfB["BetaDifferences"]], {" β_1 ", " β_2 "(*, " β_3 "*)}}] // Row





Phase C

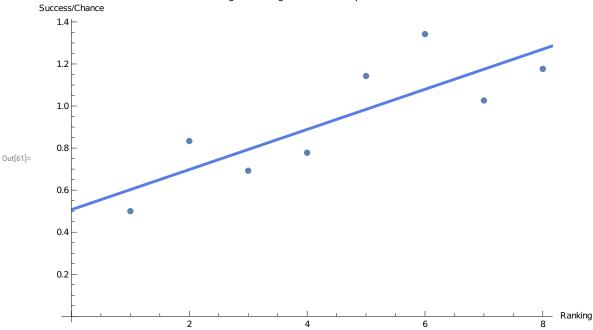
```
in[55]:= astrologerSuccess = {3, 10, 9, 7, 16, 17, 13, 20(*,4*)(*,1*)};
      expected = {6, 12, 13, 9, 14, 12.66666667, 12.66666667, 17(*,5*)(*,1.333333333*)};
In[56]:= data = N[astrologerSuccess/expected]
Out[56]= {0.5, 0.833333, 0.692308, 0.777778, 1.14286, 1.34211, 1.02632, 1.17647}
In[57]:= ListPlot[data]
     1.4
     1.2
     1.0
     0.8
Out[57]=
     0.6
     0.4
     0.2
                     2
In[58]:= lmfC = LinearModelFit[data, x, x]
Out[58]= FittedModel 0.507038+0.0954128x
In[59]:= Normal[lmfC]
Out[59]= 0.507038 + 0.0954128 x
In[60]:= ListPlot[lmfC["FitResiduals"]]
      0.2
      0.1
Out[60]=
                                                             8
      -0.1
```

In[61]:= Show[ListPlot[data,

PlotLabel → "Are Astrologer Rankings Effective Compared to Chance?", AxesLabel → {"Ranking", "Success/Chance"}],

 $Plot[lmfC[x], \{x, 0, 10\}, PlotTheme \rightarrow "Business"], ImageSize \rightarrow Large]$

Are Astrologer Rankings Effective Compared to Chance?



In[62]:= lmfC["ParameterTable"]

Estimate Standard Error t-Statistic P-Value Out[62]= 1 0.507038 0.133603 3.7951 0.00901931 x 0.0954128 0.0264574 3.60628 0.0112811

In[63]:= lmfC["ParameterConfidenceIntervals"]

 $Out[63] = \{ \{0.180123, 0.833953\}, \{0.030674, 0.160152\} \}$

In[64]:= lmfC["RSquared"]

Out[64]= 0.684298

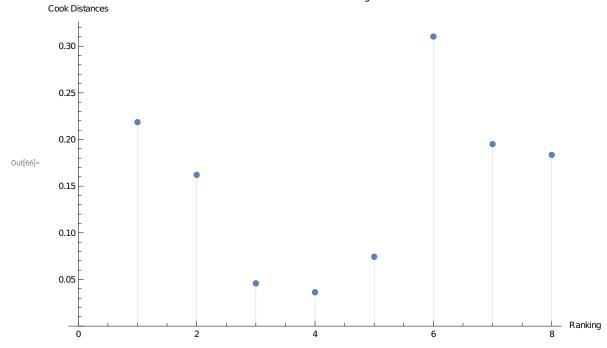
In[65]:= lmfC["ANOVATable"]

DF SS MS F-Statistic P-Value 1 0.382352 0.382352 13.0053 0.0112811 Out[65]= Error 6 0.176398 0.0293997 Total 7 0.55875

Check Cook distances to identify highly influential points:

lo[66]:= ListPlot[cdC = lmfC["CookDistances"], PlotRange \rightarrow {0, All}, Filling \rightarrow 0, AxesLabel \rightarrow {"Ranking", "Cook Distances"}, PlotLabel → "Phase C: Cook Distances for Eight Points", ImageSize → "Large"

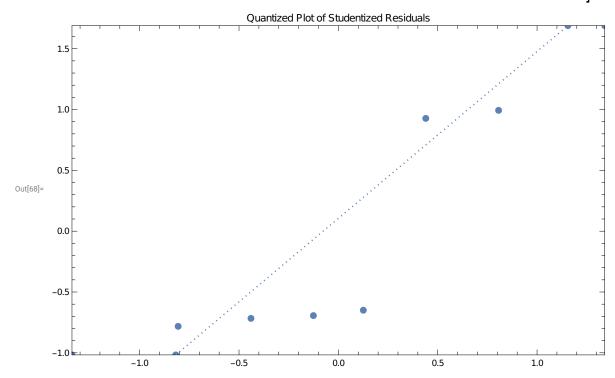
Phase C: Cook Distances for Eight Points



In[67]:= Position[cdC, _?(\sharp > .5 &)]

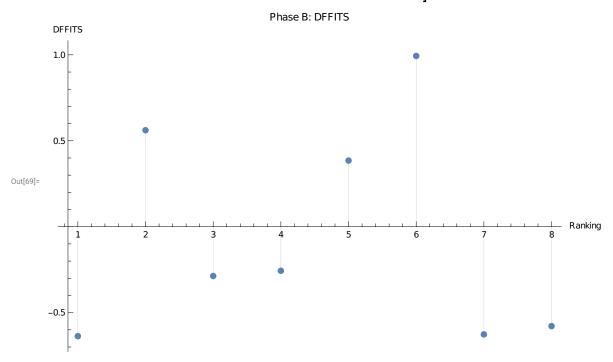
Out[67]= {}

 $\verb||n[68]|= ||QuantilePlot[lmfC["StandardizedResiduals"], ||$ Table[InverseCDF[NormalDistribution[], q], {q, 1/100, 99/100, 1/50}], PlotLabel → "Quantized Plot of Studentized Residuals", ImageSize → Large



Use DFFITS values to assess the influence of each point on the fitted values:

ln[69]:= ListPlot[lmfC["FitDifferences"], PlotRange \rightarrow All, Filling → 0, "PlotLabel" → "Phase B: DFFITS", AxesLabel → {"Ranking", "DFFITS"}, ImageSize → Large



Use DFBETAS values to assess the influence of each point on each estimated parameter:

```
In[70]:= N[2/Sqrt[8]]
```

Out[70]= 0.707107

In[72]:= dfbetasC = Transpose[lmfC["BetaDifferences"]]

 $Out[72] = \{ \{-0.633177, 0.540997, -0.248877, -0.162369, \} \}$

0.0975326, -0.107752, 0.219579, 0.287464, $\{0.532898, -0.413924,$

0.157096, 0.0546615, 0.0820859, 0.544121, -0.462009, -0.483874

 $\label{eq:loss_problem} $$\inf_{\theta \in \mathbb{R}^+}$ MapThread[ListPlot[\#1, PlotRange \rightarrow All, Filling $\rightarrow 0$, PlotLabel \rightarrow "Phase C: " $<> $$$$$$$$$$$$$$$$$$$$$$$ ImageSize → Large, AxesLabel → {"Ranking", "DFBETAS"}] &, {Transpose[lmfC["BetaDifferences"]], {" β_1 ", " β_2 "(*, " β_3 "*)}}] // Row

