

# Constitutive Relations for ReactiveBond

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ReactiveBond\_v1.1

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# 1 Introduction

The following includes the specific constitutive relations for **ReactiveBond** software. The Constitutive relations should be passed to the function as structures, and the parameters should be passed in vectors, with the order presented in the following tables. As an example,

```
%Kinetics equation of bonds
kinetics.name      = 'nth_order';
kinetics.parameters = [k,N];

%Intrinsic hyperelasticity
IntHyper.name      = 'neohookean';
IntHyper.parameters = C1;

%Sliding
sliding.name       = 'sliding_power';
sliding.parameters = [c,b,r0]

%Damage
IntHyper.name      = 'weibull';
IntHyper.parameters = [k,l,r0]
```

## 2 Kinetics

### 2.1 First-order

First order kinetics relation for kinetics (syntax name: **first\_order**)

Parameter	Physical dimension
$\tau$	$[[t]]$

$$\dot{w} = -\frac{1}{\tau}w \quad (1)$$

### 2.2 First-order stretched

First order kinetics relation for kinetics with stretched time constant (syntax name: **first\_order\_stretched**)

Parameter	Physical dimension
$\tau$	$[[t]]^{-\beta}$
$\beta$	$[[ - ]]$

$$\dot{w} = -\frac{\beta}{\tau}u^{\beta-1}w \quad (2)$$

in this equation,  $u$  is the time passed from the time the generation was initiated.

### 2.3 Second-order

Second order kinetics relation for kinetics (snytax name: **second\_order**)

Parameter	Physical dimension
$k$	$[[t]]^{-1}$

$$\dot{w} = -kw^2 \quad (3)$$

### 2.4 Nth-order

Generalized nth-order kinetics relation for kinetics (snytax name: **nth\_order**)

Parameter	Physical dimension
$k$	$[[t]]^{-1}$

$$\dot{w} = -kw^N \quad (4)$$

## 3 Intrinsic Hyperelasticity

### 3.1 Linear elastic

Linear elastic relation. Not recommended for large deformations (snytax name: **linear\_elastic**)

Parameter	Physical dimension
$E$	$[[N.m^{-2}]]$

### 3.2 Exponential linear

Exponential-Linear elastic relation for toe-region and transition to linear region. Not recommended for large deformations (snytax name: **exp\_lin**)

Parameter	Physical dimension
$C_3$	$[[N.m^{-2}]]$
$C_4$	$[[N.m^{-2}]]$
$\lambda_s$	$[[ - ]]$

$$\psi(\lambda) = \begin{cases} \frac{C_1}{C_2} [\exp(C_2(\lambda - 1)) - C_2\lambda - 1], & \lambda \leq \lambda_s \\ \left[ \left( \frac{C_3}{2} \lambda^2 \right) + C_4 \lambda \right] + C_6, & \lambda_s < \lambda \end{cases} \quad (5)$$

In the above relation  $C_1, C_4, C_5$ , and  $C_6$  are determined by the conditions of smoothness.

### 3.3 Fiber exponential

Exponential elastic relation with (snytax name: **fiber\_exp**)

Parameter	Physical dimension
$C_3$	$[[N.m^{-2}]]$
$C_2$	$[[ - ]]$

$$\psi_{EF}(I_4) = C_2 \left( \exp [C_3(I_4 - 1)^2] - 1 \right) u(I_4 - 1), \quad (6)$$

Here,  $u$  is the Heaviside step function.

### 3.4 Fiber exponential simple

Exponential elastic relation (snytax name: **exp\_lin\_simple**)

Parameter	Physical dimension
$C_1$	$[[N.m^{-2}]]$
$C_2$	$[[ - ]]$

$$\psi_{EFS}(\lambda) = \frac{C_1}{C_2} \left( \exp [C_2(\lambda - 1)] - C_2\lambda - 1 \right) u(I_4 - 1), \quad (7)$$

Here,  $u$  is the Heaviside step function.

### 3.5 Neo-Hookean

Neo-Hookean relation for isochoric uniaxial deformation (snytax name: **neohookean**)

Parameter	Physical dimension
$C_1$	$[[N.m^{-2}]]$

$$\psi_{NH}(I_1) = C_1(I_1 - 3). \quad (8)$$

### 3.6 Holmes-Mow

Holmes-Mow material for isochoric uniaxial deformation (snytax name: **holmes\_mow**)

Parameter	Physical dimension
$E$	$[[N.m^{-2}]]$
$\nu$	$[[ - ]]$
$C_0$	$[[N.m^{-2}]]$

$$\psi_{HM}(I_1, I_2, I_3) = \alpha_0 \left( I_3^{-\beta} \exp [\alpha_1(I_1 - 3) + \alpha_2(I_2 - 3)] - 1 \right). \quad (9)$$

$$\alpha_0 = C_0, \quad \alpha_1 = \frac{E/\alpha_0}{4(1+\nu)} - \alpha_2, \text{ and } \alpha_2 = \frac{(E/\alpha_0)\nu}{4(1+\nu)(1-2\nu)} = \beta - \alpha_1, \quad (10)$$

## 4 Sliding

### 4.1 Power-law

The power law for sliding with three model parameters (snytax name: **sliding\_power**)

Parameter	Physical dimension
$c$	$[-]$
$b$	$[-]$
$r0$	$[-]$

$$f_s(\Xi_s) = c(\Xi_s - (r_s)_0)^b \quad (11)$$

### 4.2 Modified Weibull

The modified Weibull (sliding exponential) for sliding with three model parameters (snytax name: **sliding\_exp**)

Parameter	Physical dimension
$k$	$[-]$
$l$	$[-]$
$r0$	$[-]$

$$f_s(\Xi_s) = (\Xi_s - 1) \left( 1 - \exp \left[ - \left( \frac{\Xi_s - (r_s)_0}{l - 1} \right)^k \right] \right) \quad (12)$$

## 5 Damage

### 5.1 Weibull

The Weibull CDF function with three model parameters for damage rule (snytax name: **weibull**)

Parameter	Physical dimension
$k$	$[-]$
$l$	$[-]$
$r0$	$[-]$

$$f_D(\Xi_D) = 1 - \exp \left[ - \left( \frac{\Xi_D - (r_D)_0}{l - 1} \right)^k \right] \quad (13)$$