Constitutive Relations for `ReactiveBond`

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`ReactiveBond_v1.1`
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Contents

1 Introduction

The following includes the specific constitutive relations for `ReactiveBond` software. The Constitutive relations should be passed to the function as structures, and the parameters should be passed in vectors, with the order presented in the following tables. As an example,

```
%Kinetics equation of bonds
kinetics.name          = 'nth_order';
kinetics.parameters    = [k,N];

%Intrinsic hyperelasticity
IntHyper.name          = 'neohookean';
IntHyper.parameters    = C1;

%Sliding
sliding.name           = 'sliding_power';
sliding.parameters     = [c,b,r0]

%Damage
damage.name            = 'weibull';
damage.parameters      = [k,l,r0]
```

2 Kinetics

2.1 First-order

First order kinetics relation for kinetics (snytax name: `first_order`)

Parameter	Physical dimension
τ	$[[t]]$

$$\dot{w} = -\frac{1}{\tau} w \tag{1}$$

2.2 First-order stretched

First order kinetics relation for kinetics with stretched time constant (snytax name: `first_order_stretched`)

Parameter	Physical dimension
τ	$[[t]]^{-\beta}$
β	$[[-]]$

$$\dot{w} = -\frac{\beta}{\tau} u^{\beta-1} w \tag{2}$$

in this equation, u is the time passed from the time the generation was imitated.

2.3 Second-order

Second order kinetics relation for kinetics (snytax name: `second_order`)

Parameter	Physical dimension
k	$[[t]]^{-1}$

$$\dot{w} = -kw^2 \tag{3}$$

2.4 Nth-order

Generalized nth-order kinetics relation for kinetics (snytax name: `nth_order`)

Parameter	Physical dimension
k	$[[t]]^{-1}$

$$\dot{w} = -kw^N \tag{4}$$

3 Intrinsic Hyperelasticity

3.1 Linear elastic

Linear elastic relation. Not recommended for large deformations (snytax name: `linear_elastic`)

Parameter	Physical dimension
E	$[[N.m^{-2}]]$

3.2 Exponential linear

Exponential-Linear elastic relation for toe-region and transition to linear region. Not recommended for large deformations (snytax name: `exp_lin`)

Parameter	Physical dimension
C_3	$[[N.m^{-2}]]$
C_4	$[[N.m^{-2}]]$
λ_s	$[[-]]$

$$\psi(\lambda) = \begin{cases} \dfrac{C_1}{C_2}\left[\exp\left(C_2(\lambda - 1)\right) - C_2\lambda - 1\right], & \lambda \le \lambda_s \\ \left[(\dfrac{C_3}{2}\lambda^2) + C_4\lambda\right] + C_6, & \lambda_s < \lambda \end{cases} \tag{5}$$

In the above relation $C_1, C_4, C_5,$ and C_6 are determined by the conditions of smoothness.

3.3 Fiber exponential

Exponential elastic relation with (snytax name: `fiber_exp`)

Parameter	Physical dimension
C_3	$[[N.m^{-2}]]$
C_2	$[[-]]$

$$\psi_{EF}(I_4) = C_2\left(\exp\left[C_3(I_4-1)^2\right]-1\right)u(I_4-1)\,, \tag{6}$$

Here, u is the Heaviside step function.

3.4 Fiber exponential- Jacobs

Exponential elastic relation with (snytax name: `fiber_exp_Jacobs`) From Jacobs 2013 JBME

Parameter	Physical dimension
C_5	$[[N.m^{-2}]]$
C_4	$[[-]]$

$$\psi_{EF}(I_4) = \frac{C_5}{2C_4}\left(\exp\left[C_4(I_4-1)^2\right]-1\right)u(I_4-1)\,, \tag{7}$$

Here, u is the Heaviside step function.

3.5 Fiber exponential simple

Exponential elastic relation (snytax name: `fiber_exp_simple`)

Parameter	Physical dimension
C_1	$[[N.m^{-2}]]$
C_2	$[[-]]$

$$\psi_{EFS}(\lambda) = \left(C_1\exp(-C_2)Ei(C_2\lambda) - C_1\log\lambda + C_3\right)u(\lambda-1)\,, \tag{8}$$

Here, u is the Heaviside step function and C_3 is:

$$C_3 = -C_1\exp\left(-C_2\right)Ei(C2) \tag{9}$$

This relation results in the simple stress response for toe-region as

$$T_{EFS} = \lambda\frac{\partial\psi_{EFS}}{\partial\lambda} = C_1\left(\exp\left[C_2(\lambda-1)\right]-1\right)u(\lambda-1) \tag{10}$$

3.6 Neo-Hookean

Neo-Hookean relation for isochoric uniaxial deformation (snytax name: `neohookean`)

Parameter	Physical dimension
C_1	$[[N.m^{-2}]]$

$$\psi_{NH}(I_1) = C_1(I_1-3)\,. \tag{11}$$

3.7 Holmes-Mow

Holmes-Mow material for isochoric uniaxial deformation (snytax name: `holmes_mow`)

Parameter	Physical dimension
E	$[[N.m^{-2}]]$
v	$[[-]]$
C_0	$[[N.m^{-2}]]$

$$\psi_{HM}(I_1, I_2, I_3) = \alpha_0 \left(I_3^{-\beta} \exp\left[\alpha_1(I_1 - 3) + \alpha_2(I_2 - 3)\right] - 1 \right) . \tag{12}$$

$$\alpha_0 = C0, \quad \alpha_1 = \frac{E/\alpha_0}{4(1+\nu)} - \alpha_2 \,, \text{ and } \quad \alpha_2 = \frac{(E/\alpha_0)\nu}{4(1+\nu)(1-2\nu)} = \beta - \alpha_1 \,, \tag{13}$$

4 Sliding

4.1 Power-law

The power law for sliding with three model parameters (snytax name: `sliding_power`)

Parameter	Physical dimension
c	$[[-]]$
b	$[[-]]$
$r0$	$[[-]]$

$$f_s(\Xi_s) = c\,(\Xi_s - (r_s)_0)^b \tag{14}$$

4.2 Modified Weibull

The modified Weibull (sliding exponential) for sliding with three model parameters (snytax name: `sliding_exp`)

Parameter	Physical dimension
k	$[[-]]$
l	$[[-]]$
$r0$	$[[-]]$

$$f_s(\Xi_s) = (\Xi_s - 1)\left(1 - \exp\left[-\left(\frac{\Xi_s - (r_s)_0}{l-1}\right)^k\right]\right) \tag{15}$$

5 Damage

5.1 Weibull

The Weibull CDF function with three model parameters for damage rule (snytax name: `weibull`)

Parameter	Physical dimension
k	$[[-]]$
l	$[[-]]$
$r0$	$[[-]]$

$$f_D(\Xi_D) = 1 - \exp\left[-\left(\frac{\Xi_D - (r_D)_0}{l-1}\right)^k\right] \tag{16}$$