

# Exercise sheet 5: Local Sequence Alignment

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## Exercise 1 - Smith-Waterman

Consider the following sequences  $S1$  and  $S2$ , a similarity scoring via  $s(x, y)$  and a gap cost function  $g(k)$ .

$$S1 = TCCGA \quad (1)$$

$$S2 = TACGCAGA \quad (2)$$

$$s(x, y) = \begin{cases} +1 & \text{if } x = y \\ 0 & \text{else} \end{cases} \quad (3)$$

$$g(k) = -k \quad (4)$$

**Question 1A** Compute the local alignment matrix  $S_{i,j}$  for the given sequence.

**Matrix**

**Hint: Formulae**

$$H_{0,0} = 0 \quad (5)$$

$$H_{i,0} = 0 \quad (6)$$

$$H_{0,j} = 0 \quad (7)$$

$$H_{i,j} = \max \begin{cases} H_{i-1,j-1} + s(a_i, b_j) \\ H_{i-1,j} + s(a_i, -) \\ H_{i,j-1} + s(-, b_j) \\ 0 \end{cases} \quad (8)$$

**Solution**

**Question 1B** Give all optimal local alignments and the according score.

**Solution** score: 3

TCCG	TCCGA	TCCG-A	CCGA
:	: :	:	:
TACG	TACGC	TACGCA	CAGA

## Exercise 2 - Affine gap costs

**Question 2A** Which of these statements are correct?

- ☐ Distance and similarity scores are equally useful for local alignment scoring
- ☐ Similarity scores are not suited for local alignment scoring
- ☐ Distance scores are not suited for local alignment scoring

**Solution**

- ☐ Distance and similarity scores are equally useful for local alignment scoring
- ☐ Similarity scores are not suited for local alignment scoring
- ☒ Distance scores are not suited for local alignment scoring

**Question 2B** You want to extend the Smith-Waterman algorithm for local alignment to more general gap scoring functions  $g(k)$  (where  $k$  denotes the gap length).

The following recursions were created analogously to the Waterman-Smith-Beyer algorithm. Which of these (if any) represents a variant of the Smith-Waterman algorithm that allows for an arbitrary gap scoring function?

$$D_{0,0} = 0, D_{i,0} = 0, D_{0,j} = 0$$

$$D_{i,j} \stackrel{(i)}{=} \max \begin{cases} Di - 1, j - 1 + s(a_i, b_j) \\ \max_{1 \leq k \leq j} D_{i,j-k} + g(1) \\ \max_{1 \leq k \leq i} D_{i-k,j} + g(1) \\ 0 \end{cases} \quad D_{i,j} \stackrel{(ii)}{=} \max \begin{cases} Di - 1, j - 1 + s(a_i, b_j) \\ \max_{1 \leq k \leq j} D_{i,j-k} + g(k) \\ \max_{1 \leq k \leq i} D_{i-k,j} + g(k) \\ 0 \end{cases} \quad (9)$$

(10)

$$D_{i,j} \stackrel{(iii)}{=} \min \begin{cases} Di - 1, j - 1 + s(a_i, b_j) \\ \max_{1 \leq k \leq j} D_{i,j-k} + g(k) \\ \max_{1 \leq k \leq i} D_{i-k,j} + g(k) \\ 0 \end{cases} \quad D_{i,j} \stackrel{(iv)}{=} \min \begin{cases} Di - 1, j - 1 + s(a_i, b_j) \\ \max_{1 \leq k \leq j} D_{i,j-k} + g(1) \\ \max_{1 \leq k \leq i} D_{i-k,j} + g(1) \\ 0 \end{cases} \quad (11)$$

(12)

$$D_{i,j} \stackrel{(v)}{=} \max \begin{cases} Di - 1, j - 1 + s(a_i, b_j) \\ \max_{1 \leq k \leq j} D_{i,j-k} + g(1) \\ \max_{1 \leq k \leq i} D_{i-k,j} + g(1) \\ -1 \end{cases} \quad (13)$$

**Solution** Only Formula *ii* allows for an arbitrary gap scoring.

**Note**

Distance scores are not suited for local alignment scoring, thus all minimization approaches are not suitable. Further  $g(1)$  does not represent an affine scoring.

**Question 2C** The following recursions were created analogously to the Gotoh algorithm. Which of these (if any) represents a variant of the Smith-Waterman algorithm that allows for an affine gap scoring function?

$$D_{0,0} = 0, D_{i,0} = 0, D_{0,j} = 0$$

$$D_{i,j} \stackrel{(i)}{=} \max \begin{cases} D_{i-1,j-1} + s(a_i, b_j) \\ P_{i,j} \\ Q_{i,j} \\ 0 \end{cases} \quad \begin{aligned} P_{i,j} &= \max \begin{cases} D_{i-1,j} + g(1) \\ P_{i-1,j} + \beta \end{cases} \\ Q_{i,j} &= \max \begin{cases} D_{i,j-1} + g(1) \\ Q_{i,j-1} + \beta \end{cases} \end{aligned} \quad (14)$$

$$D_{i,j} \stackrel{(ii)}{=} \max \begin{cases} D_{i-1,j-1} + s(a_i, b_j) \\ P_{i,j} \\ Q_{i,j} \\ 0 \end{cases} \quad \begin{aligned} P_{i,j} &= \max \begin{cases} D_{i-1,j} + g(k) \\ P_{i-1,j} + \beta \end{cases} \\ Q_{i,j} &= \max \begin{cases} D_{i,j-1} + g(k) \\ Q_{i,j-1} + \beta \end{cases} \end{aligned} \quad (15)$$

$$D_{i,j} \stackrel{(iii)}{=} \min \begin{cases} D_{i-1,j-1} + s(a_i, b_j) \\ P_{i,j} \\ Q_{i,j} \\ 0 \end{cases} \quad \begin{aligned} P_{i,j} &= \min \begin{cases} D_{i-1,j} + g(k) \\ P_{i-1,j} + \beta \end{cases} \\ Q_{i,j} &= \min \begin{cases} D_{i,j-1} + g(k) \\ Q_{i,j-1} + \beta \end{cases} \end{aligned} \quad (16)$$

$$D_{i,j} \stackrel{(iv)}{=} \min \begin{cases} D_{i-1,j-1} + s(a_i, b_j) \\ P_{i,j} \\ Q_{i,j} \\ 0 \end{cases} \quad \begin{aligned} P_{i,j} &= \min \begin{cases} D_{i-1,j} + g(1) \\ P_{i-1,j} + \beta \end{cases} \\ Q_{i,j} &= \min \begin{cases} D_{i,j-1} + g(1) \\ Q_{i,j-1} + \beta \end{cases} \end{aligned} \quad (17)$$

**Solution** Only Formula *i* allows for an affine gap scoring.

**Note**

Distance scores are not suited for local alignment scoring, thus all minimization approaches are not suitable. Further  $g(1)$  does not represent an affine scoring.

## Exercise 3 - Programming assignment

Programming assignments are available via Github Classroom and contain automatic tests.

We recommend doing these assignments since they will help you to further understand this topic.

Access the Github Classroom link: [Programming Assignment: Sheet 05](#).

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