$$
\begin{aligned}
\dot{v}^X - v^Y \dot{\psi} &= \frac{1}{m}(F_f^x \cos(\delta) + F_r^x - F_f^y \sin(\delta)), \\
\dot{v}^Y + v^X \dot{\psi} &= \frac{1}{m}(F_f^y \cos(\delta) + F_r^y + F_f^x \sin(\delta)), \\
I_{ZZ} \ddot{\psi} &= l_f F_f^y \cos(\delta) - l_r F_r^y + l_f F_f^x \sin(\delta),
\end{aligned}
\tag{1}
$$

where m is the vehicle mass, I_{ZZ} is the vehicle inertia about the Z-axis, $\dot{\psi}$ is the yaw rate, δ is the steer angle, $[v^X, v^Y]$ are the longitudinal and lateral velocities at the center of gravity, $[l_f, l_r]$ are the distances from the mass center to the front and rear wheel base, and $[F^x, F^y]$ are the longitudinal and lateral tire forces acting at the front and rear wheels.

Fig. **??** provides a schematic of the double-track model that is used in the low-level formulation. It has five degrees of freedom: two translational (v^X and v^Y) and three rotational (the roll-pitch-yaw angles (ϕ, θ, ψ)). The suspension model is a rotational spring-damper system, and longitudinal and lateral load transfer is included. The derivation and details of both models are found in [**?**, **?**].

The nominal tire forces F_0^x and F_0^y for the longitudinal and lateral directions under pure slip conditions are computed with the Magic formula [**?**], given by

$$
\begin{aligned}
F_0^x &= \mu_x F^z \sin\Big(C_x \arctan\big(B_x \lambda_i \\
&\qquad - E_x(B_x \lambda - \arctan B_x \lambda)\big)\Big), \\
F_0^y &= \mu_y F^z \sin\Big(C_y \arctan\big(B_y \alpha \\
&\qquad - E_y(B_y \alpha - \arctan B_y \alpha)\big)\Big),
\end{aligned}
\tag{2}
$$

with lateral slip α_i and longitudinal slip λ_i defined as

$$
\dot{\alpha}_i \frac{\sigma}{v_i^x} + \alpha_i := -\arctan\left(\frac{v_i^y}{v_i^x}\right), \tag{3a}
$$

$$
\lambda_i := \frac{R_w \omega_i - v_i^x}{v_i^x}, \tag{3b}
$$

where σ is the relaxation length, R_w is the wheel radius, ω_i is the wheel angular velocity for wheel $i \in \{f, r\}$ or $\{1,2,3,4\}$, and $[v_i^y, v_i^x]$ are the lateral and longitudinal wheel velocities for wheel i. In the following we suppress the index i for brevity. In (2), μ_x and μ_y are friction coefficients and B, C, and E are parameters. The nominal normal force acting on each wheel axle is given by

$$
F_{0,f}^z = mg\frac{l_r}{l}, \quad F_{0,r}^z = mg\frac{l_f}{l},
$$

where g is the gravitational acceleration and $l = l_f + l_r$. In the single-track model $F^z = F_0^z$ in (2). This is not true for the double-track model, because of load transfer.

An experimentally verified approach to tire modeling under combined slip constraints is to scale the nominal forces (2) with a weighting function G for each direction, which depends on α and λ

[**?**]. The relations are

$$\begin{aligned} F^{x,y} &= F_0^{x,y} G_m, \\ G_m &= \cos(C_m \arctan(H_m m)), \\ H_m &= B_{m1} \cos(\arctan(B_{m2} m)), \end{aligned} \tag{4}$$

where m is either α or λ. Moreover, since it is the torques that can be controlled in a physical setup, we introduce a model for the wheel dynamics, namely

$$\tau = I_w \dot{\omega} - R_w F^x,$$

where I_w is the wheel inertia and τ is the input torque. To account for that commanded steer angle and brake/drive torques are not achieved instantenously, we incorporate first-order models from reference to achieved value according to

$$T \dot{\delta} = -\delta + \delta_{\text{ref}}, \tag{5}$$

and similarly for the torques, where T in (5) is the time constant of the control loop. The parameter values used here correspond to a medium-sized passenger car on dry asphalt.