

## Programming project 2 — Coordinate Descent

Name: Xuanyu Chen

PID: A53306915

### I. Description

Coordinate descent is a simple and useful approach to solve an unconstrained optimization problem. Coordinate descent algorithms solve optimization problems by successively performing approximate minimization along coordinate directions. If  $f$  is a  $k$ -dimensional function, we can minimize  $f$  by successively minimizing each of the individual dimensions of  $f$  while holding the values of  $f$  in the other dimensions fixed. The primary advantage of this approach is that it takes an arbitrarily complex  $k$ -dimensional problem and reduces it to a collection of  $k$  one-dimensional problems.

## II. Pseudocode

### Algorithm 1 Coordinate Descent

1. **Input:**
  - $\alpha$ : a positive constant, the step-size for every update in one coordinate
  - $\theta$ : threshold
  - $N$ : maximum number of iterations
  - $F$ : bool, if False means the loss value is not optimal, True means optimal
2. **Output:** vector  $w$  and optimal loss value  $f(w^t)$
3. **Procedure** COORDINATEDDESCENT
4. set  $t \leftarrow 0$  and choose  $w^0 \in \mathbb{R}^d$
5.  $loss \leftarrow f(w^0)$
6. **while** ( $F == False$ ) & ( $t < N$ ):
  7. choose index  $i_t \in \{1, 2, \dots, d\}$  which is  $argmax_{i_t} abs([\nabla f(w^t)]_{i_t})$
  8.  $I.append(i_t)$
  9.  $w^{t+1} \leftarrow w^t - \alpha [\nabla f(w^t)]_{i_t};$
  10.  $newloss \leftarrow f(w^{t+1})$
  11. **while** ( $loss - newloss < \theta$ ):
    12. choose index  $i_t \in \{1, 2, \dots, d\} \& i_t \notin I$  which is  $argmax_{i_t} abs([\nabla f(w^t)]_{i_t})$
    13.  $I.append(i_t)$
    14.  $w^{t+1} \leftarrow w^t - \alpha_t [\nabla f(w^t)]_{i_t};$
    15.  $newloss \leftarrow f(w^{t+1})$
    16. **if**  $len(I) == len(w)$ :
      17.  $F \leftarrow True$
      18. **break**
  19.  $loss \leftarrow newloss$
  20.  $t \leftarrow t + 1;$

### III. Experimental results

The optimal log-loss calculated based on the LogisticRegression function (with parameter  $C = 50$ ) from scikit-learn package is 0.02102.

Here are the results calculated by my Coordinate Descent algorithm:

Table 1.

Max-Iteration	500	1500	5000	7000	9000
alpha	0.1	0.1	0.1	0.1	0.1
threshold	1e-3	1e-4	1e-5	1e-6	1e-6
Log-loss	0.475	0.158	0.0589	0.048	0.0386
iterations	403	1500	4622	7000	9000

Figure 1. (First 100 iterations)

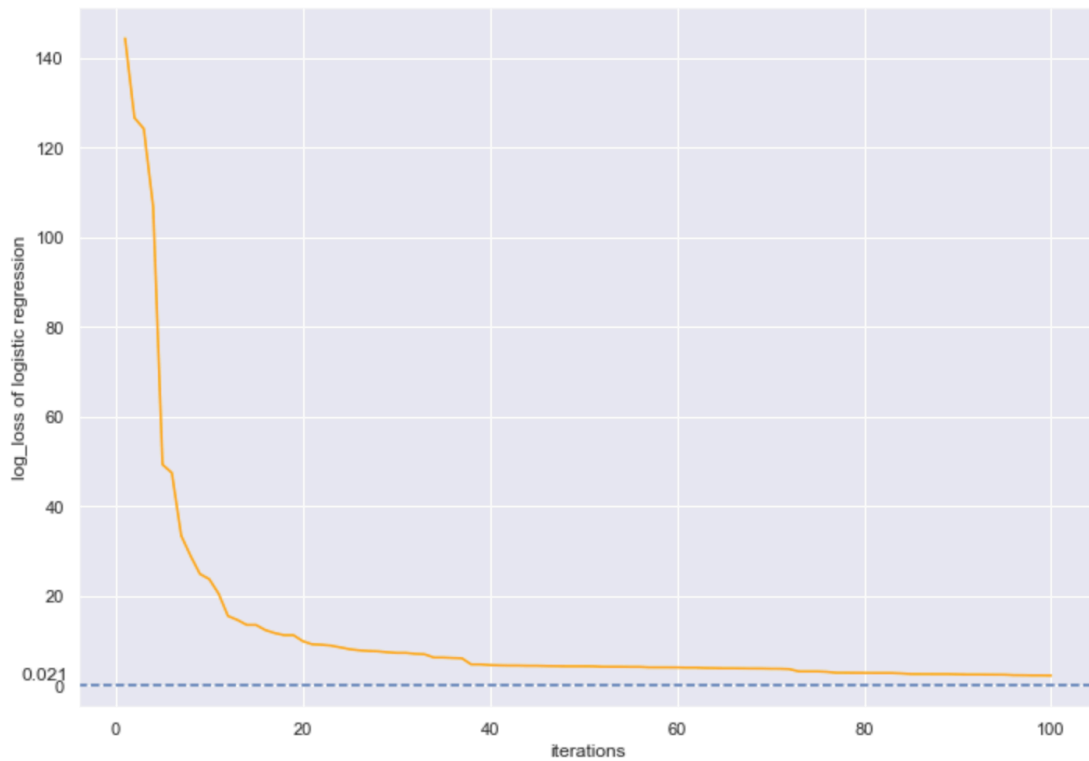
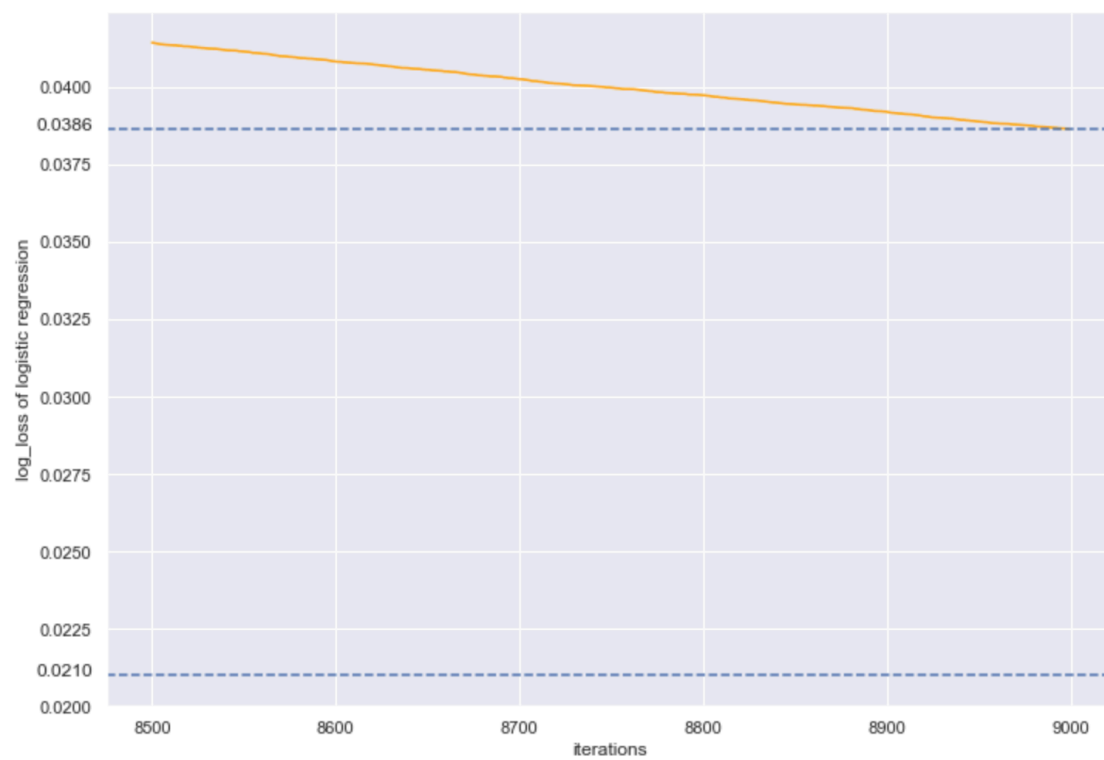


Figure 2. (Last 500 iterations)



#### IV. Critical evaluation

- (1) In my Coordinate Descent algorithm, I choose the index of coordinate which maximizes the first derivative of function  $f$ . I also tried to use the method of choosing the index of coordinate randomly. It turns out that the difference between these two methods is not such big like I imagined before. In the future, I may try to use other smarter method to select the coordinate index so that  $L(w^t)$  can approach to  $L^*$  more quickly and better.
- (2) Through my experiment, the “initial starting point” and the original data influence my algorithm a lot. If I use a vector containing all zeros as my initial weight, then it turns out that my final outcome is pretty bad. Moreover, I use MinMaxScaler() function to standardize the original data so that  $L(w^t)$  can approach to  $L^*$  better.
- (3) For every coordinate-dimension update, my step size is constant. Intuitively, the update step size will decrease as iteration goes on. I have read a paper about *Adam Optimization Algorithm* before and it inspired me that my future improvements could include updating step size in real time.