FACULDADE DE ENGENHARIA DA UNIVERSIDADE DO PORTO

# Quantum Algorithms for Optimizing Urban Transportation

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# Abstract

Transportation is a fundamental aspect of modern urban life, profoundly influencing the daily experiences of countless individuals residing in major cities. Its pivotal role extends beyond mere convenience, as transportation systems significantly shape energy consumption patterns and substantially impact the environment. Our choices in optimising transportation affect the efficiency of our daily commutes and play a critical role in determining the sustainability of our cities and the planet's well-being. As cities grow and face escalating congestion, energy usage, and environmental sustainability challenges, exploring innovative solutions becomes imperative.

Within this context, the convergence of quantum computing and transportation optimisation stands out as a compelling pathway toward minimising commuting times, energy consumption, and carbon emissions and enhancing the efficient utilisation of vehicles. In a hypothetical urban environment without private vehicles, the focus lies in designing a quantum algorithm capable of optimising public transportation systems to offer citizens seamless, eco-friendly, and energy-efficient travel experiences. Additionally, exploration extends to assessing the viability of integrating such a system with vehicles owned by external entities.

Building upon prior research that explores the quantum iteration of the vehicle routing problem, the objective is to devise an algorithm adept at dynamically allocating resources, such as large buses and small cars, in response to real-time demand and passenger distribution across geographical locations. Notably, existing studies in this realm indicate progress, yet they underscore the persistent limitations, leaving considerable scope for further advancements and refinements.

We intend to use the Qiskit framework, a robust quantum computing platform, to implement an algorithm to optimise public transportation. We want to simulate real-world complexities by employing mock data representative of a major city's public transport system. Furthermore, we intend to enhance authenticity by exploring the integration of actual data from a major city. This approach aims to validate the algorithm's efficacy and demonstrates its relevance to the intricacies of urban mobility.

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# Abbreviations and Symbols

| | |
|---|---|
| NISQ | Noisy Intermediate-Scale Quantum |
| QC | Quantum Computing |
| Qubit | Quantum Bit |

# Chapter 1

# Literature Review

## 1.1 Introduction

This chapter aims to present a thorough and contemporary understanding of the theme explored in this dissertation. It is organized into 2 main sections.

The first one, Background, is structured into distinct topics, providing insights into the key concepts of the dissertation's theme. Specifically, it is divided as follows:

- **Urban Transportation**: Explores the functioning of contemporary urban transportation in major cities and outlines existing challenges in these systems.
- **Quantum Computing**: Describes the principles of quantum computing, its operational mechanisms, advantages over classical computers, exploration of hybrid systems, and introduces relevant quantum algorithms.
- **Quantum Hardware**: Discusses the current state of hardware for quantum computers in the NISQ era, available technologies, their limitations, and offers prospects for future developments.

The second section, Related Work, examines current methodologies addressing the issues outlined in the dissertation. This analysis is grounded in recent papers published over the past few years.

## 1.2 Background

### 1.2.1 Urban Transportation

FALAR COMO O TRANSPORTE URBANO / INDIVIDUAL HOJE EM DIA FUNCIONA EM GRANDES CIDADES, EXPLICAR OS PROBLEMAS QUE TEM E O QUE PODIA SER MELHORADO. TENTAR ENCONTRAR PAPERS PARA DAR BACKUP.

### 1.2.2 Quantum Computing

ESTA SECTION VAI SER GRANDE, É PRECISO EXPLICAR O QUE É QUANTUM COMPUTING, COMO FUNCIONA, AS VANTAGENS COM O CLASSICO, SISTEMAS HIBRIDOS, SUPERPOSITION, ALGORITMOS QUANTICOS RELEVANTES, ETC.

Quantum computing represents a novel paradigm in computation, drawing on the principles of quantum mechanics. Unlike classical computing, which relies on bits as binary units, quantum computers utilize qubits capable of existing in multiple states simultaneously. This unique property, known as superposition, enables quantum computers to conduct parallel computations. Additionally, the phenomenon of entanglement allows qubits to be interconnected, facilitating more efficient problem-solving. This section provides an in-depth exploration of quantum computing, clarifying its essential attributes, potential applications, and inherent limitations.

#### 1.2.2.1 Quantum Bits

A quantum bit, or qubit, serves as a fundamental unit of information in quantum computing. It is depicted as a unit vector within a two-dimensional complex vector space, with a specific basis denoted by $\{|0\rangle, |1\rangle\}$. In the domain of quantum computation, the basis states $|0\rangle$ and $|1\rangle$ are utilized to represent classical bit values 0 and 1, respectively (Eleanor G. Rieffel (2000)). Nevertheless, quantum bits can also be expressed in an alternative arbitrary basis, for example, $\{|+\rangle, |-\rangle\}$.

The qubit's basis representation relies on the **bracket notation**, a concept introduced by Dirac for expressing a Hilbert space vector in quantum mechanics (Dirac (1939)). Alternatively, the basis can be conveyed in matrix form:

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{1.1}$$

#### 1.2.2.2 Superposition

In contrast to classical bits, which can only be in states 0 or 1, qubits have the unique ability to exist in a superposition of $|0\rangle$ and $|1\rangle$. This characteristic enables qubits to concurrently occupy multiple states, represented as a linear combination of all possible states.

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \equiv \alpha \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 0 \\ 1 \end{bmatrix} \equiv \begin{bmatrix} \alpha \\ \beta \end{bmatrix}, \tag{1.2}$$

where $\alpha$ and $\beta$ are complex numbers such that $|\alpha|^2 + |\beta|^2 = 1$ (Eleanor G. Rieffel (2000)). Intuitively, $\alpha^2$ and $\beta^2$ denote the probabilities of the quantum position being in that state after measurement, as further explained in section 1.2.2.3.

As only two numbers are required to represent a qubit, it can be mapped onto an arrow originating from the origin and extending to the three-dimensional sphere in $\mathbb{R}^3$ with a radius of 1, commonly referred to as the **Bloch sphere**. Each qubit can be depicted using two angles that define the orientation of this arrow, as shown in Figure 1.1.

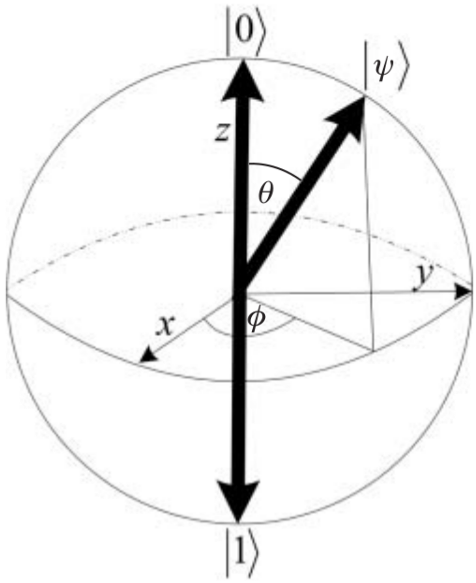


Figure 1.1: Bloch sphere representation, Noson S. Yanofsky (2008)

where $0 \leq \phi < 2\pi$ and $0 \leq \theta < \frac{\pi}{2}$. That said, the superposition can be written as shown in equation 1.2.2.2, from W. Dür (2013).

$$|\psi\rangle = cos\frac{\theta}{2}|0\rangle + e^{i\phi}sin\frac{\theta}{2}|1\rangle \tag{1.3}$$

#### 1.2.2.3 Measurement

#### 1.2.2.4 Quantum Gates

#### 1.2.2.5 Entanglement

#### 1.2.2.6 Why use Quantum Computing?

### 1.2.3 Quantum Hardware

SEPARADO DA SECÇAO ANTERIOR PORQUE NAO ESTA NO SCOPE DA TESE. NO ENTANTO, A REALIZAÇÃO DA SOLUÇAO VAI DEPENDER DO DESENVOLVIMENTO DO HARDWARE. EXPLICAR O ESTADO ATUAL DO HARDWARE (NISQ), TECNOLOGIAS (e.g. QisKit, IBMQ) E TALVEZ EXPETATIVAS PARA O FUTURO.

## 1.3 Related Work

FALAR DOS PAPERS QUE JA EXISTEM SOBRE ISTO. O PAPER PRINCIPAL É O DE QUANTUM COMPUTING FOR TRANSPORT OPTIMIZATION MAS TENHO QUE INCLUIR MAIS.

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