UCSB Math 3 Series (from Fall 1988)

Chapter 1

Calculus with Analytical Geometry, 3rd. Edition, Robert Ellis & Denny Gulick

1.3 Functions

Definition 1.4: A **function** consists of a domain and a rule. The domain is a set of real numbers, ℝ. The **rule** assigns to each number in the domain one and only one number.

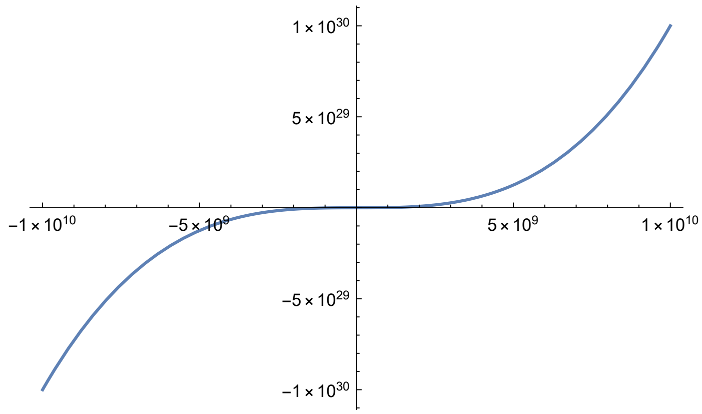
#11

```
f[x_] := x^3 - 4 x + 1
```

Find the domain of *f*.

The domain is all real numbers ℝ or (-∞,∞):

```
pseudoInfinity = 10^10;
interval = Interval[{-pseudoInfinity, pseudoInfinity}];
Plot[f[x], {x, Min[interval], Max[interval]}]
```




```
Clear[f, pseudoInfinity, interval]
```

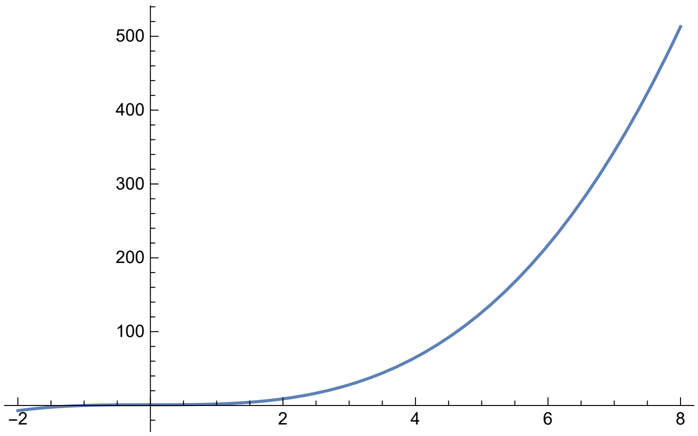
#13

```
k[x_] := 1 + x^3
```

Find the domain of *k* for $-2 \leqslant x \leqslant 8$.

The domain is [-2,-8]:

```
interval = Interval[{-2, 8}];
Plot[k[x], {x, Min[interval], Max[interval]}]
```




```
Clear[k, interval]
```

#15

```
f[x_] := √(x + 2)
```

Find the domain of *f*.

We see that x < -2 is not among ℝ:

```
test1 = Element[f[-2.1], Reals];
VerificationTest[test1 == False]
```

TestResultObject[Outcome: Success Test ID: None]

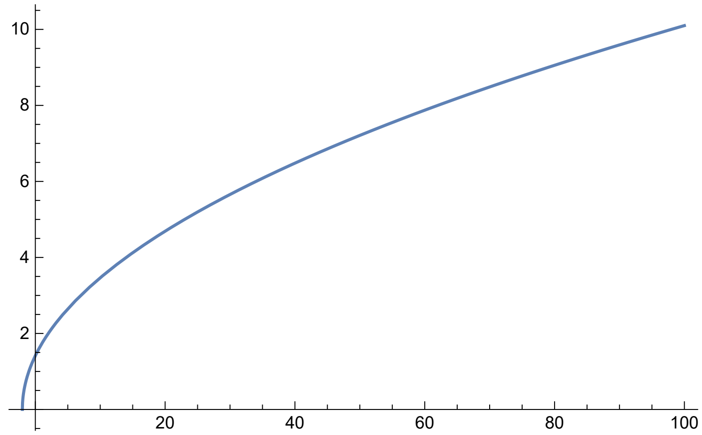
Alternatively and precisely, we can solve this inequality:

```
x + 2 ≥ 0; x + 2 - 2 ≥ 0 - 2
```

$x \geq -2$

The domain is [-2,∞):

```
pseudoInfinity = 10^2;
interval = Interval[{-2, pseudoInfinity}];
Plot[f[x], {x, Min[interval], Max[interval]}]
```




```
Clear[f, pseudoInfinity, interval, test1]
```

#17

```
f[x_] := Sqrt[x^2 + 4]
```

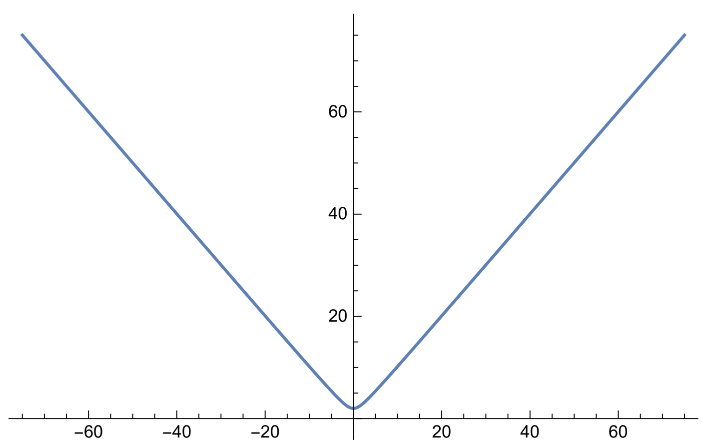
Find the domain of *f*.

```
x^2 + 4 ≥ 0; x^2 + 4 - 4 ≥ 0 - 4; x^2 ≥ -4
```

$x^2 \geq -4$

There are no real numbers for the solution to the inequality $x^2 \geq -4$. Without the benefit of complex (or imaginary) numbers, we *can* say that there are no real numbers bounding x. Alternatively, because $x^2 \geqslant 0$ for all real numbers $\mathbb{R}$ or $(-\infty,\infty)$, the domain of *f* is the same:

```
pseudoInfinity = 75;
interval = Interval[{-pseudoInfinity, pseudoInfinity}];
Plot[f[x], {x, Min[interval], Max[interval]}]
```




```
Clear[f, pseudoInfinity, interval]
```

#19

```
f[t_] := Sqrt[3 - 1/t^2]
```

$$f[t\_] := \sqrt{3 - \frac{1}{t^2}}$$

Find the domain of *f*.

$$3 - \frac{1}{t^2} \geq 0;\ -\frac{1}{t^2} \geq -3;\ \frac{1}{t^2} \leq 3;\ t^2 \leq \frac{1}{3};\ \sqrt{t^2} \leq \sqrt{\frac{1}{3}};\ \text{Reduce}\left[\sqrt{t^2} \leq \sqrt{\frac{1}{3}}\right]$$

$$-\frac{1}{\sqrt{3}} \leq t \leq \frac{1}{\sqrt{3}}$$

$$N\left[\frac{1}{\sqrt{3}}\right]$$

```
0.57735
```

```
test1 = Element[f[0.4], Reals];
VerificationTest[test1 == False]
```

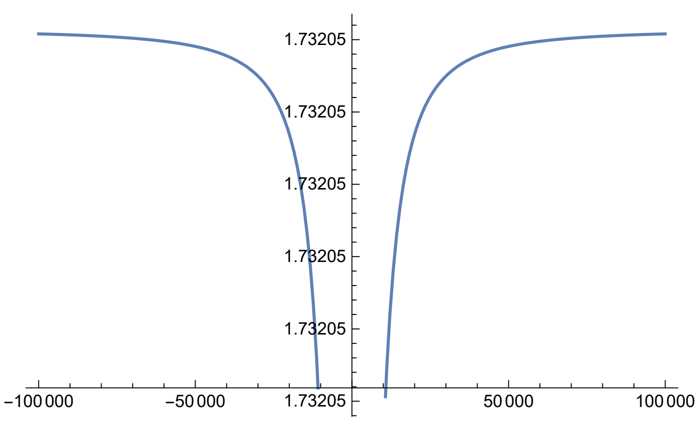
```
TestResultObject[ Outcome: Success
                  Test ID: None ]
```

∴ the domain is $(-\infty,-\sqrt{\frac{1}{3}}] \cup [\sqrt{\frac{1}{3}},\infty)$.

$$\text{interval1} = \text{Interval}\left[\left\{-10^5, -\sqrt{\frac{1}{3}}\right\}\right];$$

$$\text{interval2} = \text{Interval}\left[\left\{\sqrt{\frac{1}{3}}, 10^5\right\}\right];$$

```
Show[
 Plot[f[t], {t, Min[interval1], Max[interval1]}],
 Plot[f[t], {t, Min[interval2], Max[interval2]}],
 PlotRange -> All
]
```




```
Clear[f, test1, interval1, interval2]
```

#21

```
f[t_] := (1 - t^2)^(1/3)
```

$f[t\_] := \sqrt[3]{1 - t^2}$

Find the domain of *f*.

$1 - t^2 \geq 0;\ -t^2 \geq -1;\ t^2 \leq 1;\ \sqrt{t^2} \leq \sqrt{1}$

$\sqrt{t^2} \leq 1$

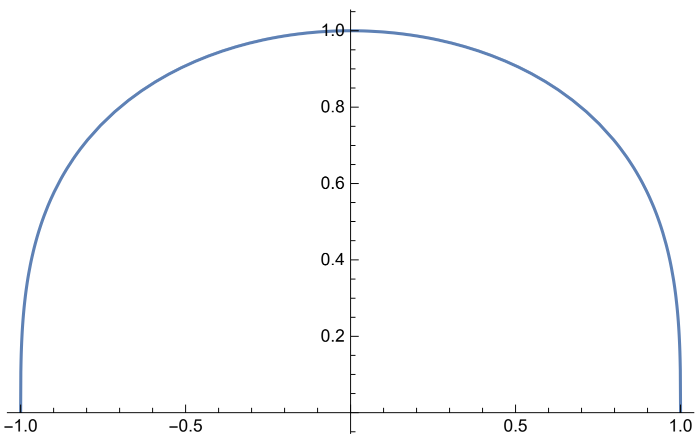
$\text{Reduce}\left[\sqrt{t^2} \leq 1\right]$

$-1 \leq t \leq 1$

```
test1 = Element[f[2], Reals];
VerificationTest[test1 == False]
```

TestResultObject[Outcome: Success Test ID: None]

```
interval = Interval[{-1, 1}];
Plot[f[x], {x, Min[interval], Max[interval]}]
```



∴ the domain is [-1,1].

```
Clear[f, interval, test1]
```

#23

Find the domain of *g*.

Find the infinity values excluded from the domain:

```
Solve[x - 1 == 0, x]
```

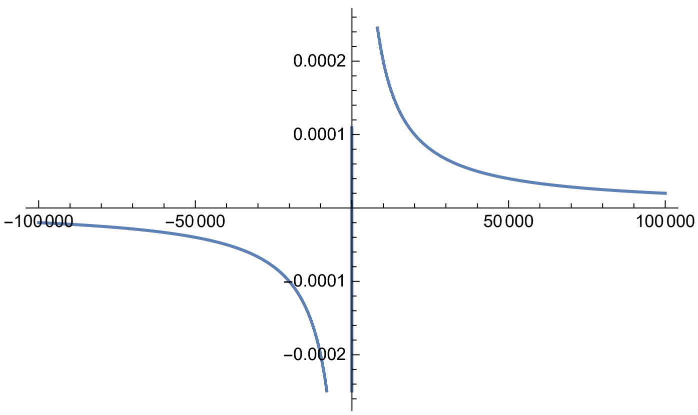
{{x → 1}}

∴ the domain is $(-\infty,1) \cup (1,\infty)$.

```
VerificationTest[Limit[g[x], Rule[x, 1]], ∞]
```

```
TestResultObject[ Outcome: Failure
                  Test ID: None ]
```

```
pseudoInfinity = 10^5;
interval1 = Interval[{-pseudoInfinity, 1}];
interval2 = Interval[{1, pseudoInfinity}];
Show[
 Plot[g[x], {x, Min[interval1], Max[interval1]}],
 Plot[g[x], {x, Min[interval2], Max[interval2]}],
 PlotRange -> All
]
```




```
Clear[g, pseudoInfinity, interval1, interval2]
```

#25

$$g[w\_] := \frac{2w-8}{w^2-16}$$

Find the domain of *g*.

$$\frac{2w-8}{w^2-16} \Rightarrow \frac{2(w-4)}{(w+4)(w-4)} == \frac{2}{4+w};$$

$$g[w\_] := \frac{2}{4+w}$$

Find the infinity values excluded from the domain:

```
Solve[4 + w == 0, w]
```

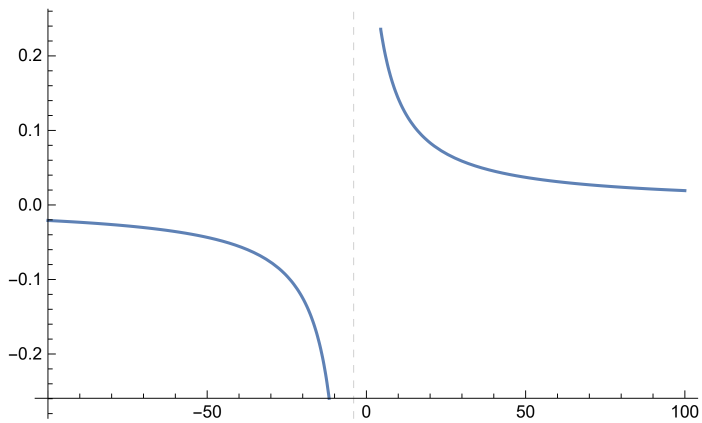
{{w → −4}}

∴ the domain is (-∞,-4) ∪ (-4,∞).

```
VerificationTest[Limit[g[w], Rule[w, -4]], ∞]
```

```
TestResultObject[ Outcome: Success
                  Test ID: None ]
```

```
pseudoInfinity = 10^2;
interval1 = Interval[{-pseudoInfinity, -4}];
interval2 = Interval[{-4, pseudoInfinity}];
Show[
 Plot[g[w], {w, Min[interval1], Max[interval1]}],
 Plot[g[w], {w, Min[interval2], Max[interval2]}],
 GridLines → {{{-4, Dashed}}, None },
 PlotRange -> All
]
```




```
Clear[g, pseudoInfinity, interval1, interval2]
```

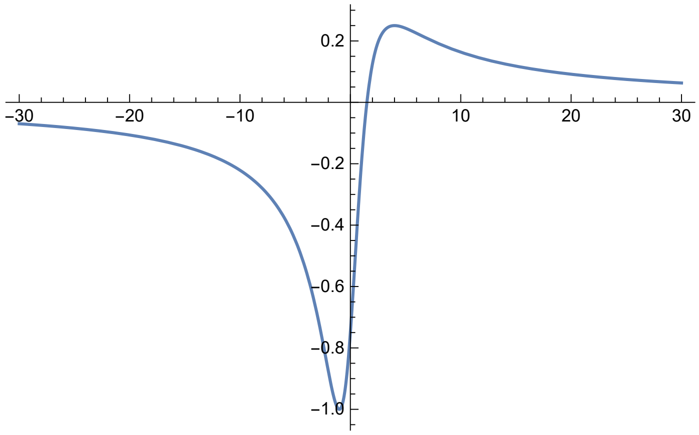
#27

$$k[x\_] := \frac{2x - 3}{x^2 + 4}$$

Find the domain of *k*.

The domain is all real numbers ℝ or (-∞,∞):

```
pseudoInfinity = 30;
interval = Interval[{-pseudoInfinity, pseudoInfinity}];
Plot[k[x], {x, Min[interval], Max[interval]}]
```




```
Clear[k, pseudoInfinity, interval]
```

#29

The function *f* is

```
f1[x_] := 2 x
```

for $-4 \leqslant x \leqslant -1$ and

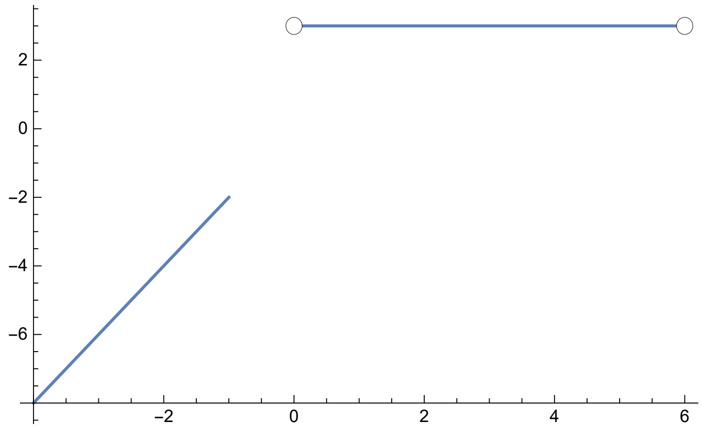
```
f2[x_] := 3
```

for $0 < x < 6$.

Find the domain of *f*.

The domain is $[-4,-1] \cup (0,6)$:

```
interval1 = Interval[{-4, -1}];
interval2 = Interval[{0, 6}];
Show[
 Plot[f1[x], {x, Min[interval1], Max[interval1]}],
 Plot[f2[x], {x, Min[interval2], Max[interval2]}],
 PlotRange -> All,
 Epilog → {EdgeForm[Thin], White, Disk[{0, 3}, {.125, .25}], Disk[{6, 3}, {.125, .25}]}
]
```




```
Clear[f1, f2, interval1, interval2]
```

#31

$$f[x\_] := \sqrt{1 - \sqrt{9 - x^2}}$$

Find the domain of *f*.

$9 - x^2 \leq 0;\ 9 - x^2 - 9 \leq 0 - 9;\ -x^2 \leq -9;\ \sqrt{x^2} \geq \sqrt{9}$

$\sqrt{x^2} \geq 3$

We see that x ⩽ 3 and x ⩾ -3.

```
VerificationTest[Element[f[3.1], Reals] == False]
```

TestResultObject[Outcome: Success Test ID: None]

$\sqrt{9 - x^2} \leq 1;\ \left(\sqrt{9 - x^2}\right)^2 \leq 1^2;\ 9 - x^2 \leq 1;\ 9 - x^2 - 9 \leq 1 - 9;\ -x^2 \leq -8;\ \sqrt{x^2} \geq \sqrt{8}$

$\sqrt{x^2} \geq 2\sqrt{2}$

```
N[2 √2]
```

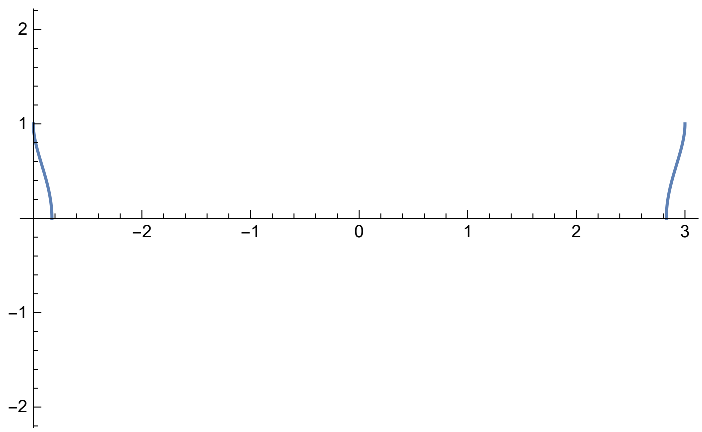
2.82843

We see that $2\sqrt{2} \leqslant x \leqslant 3$ and $-2\sqrt{2} \geqslant x \geqslant -3$.

```
VerificationTest[Element[f[2.7], Reals] == False]
```

TestResultObject[Outcome: Success Test ID: None]

```
interval1 = Interval[{-2 √2, -3}];
interval2 = Interval[{2 √2, 3}];
Show[
 Plot[f[x], {x, Min[interval1], Max[interval1]}],
 Plot[f[x], {x, Min[interval2], Max[interval2]}],
 PlotRange → {-2, 2}
]
```



∴ the domain is $[-2\sqrt{2}, -3] \cup [2\sqrt{2}, 3]$

```
Clear[f, pseudoInfinity, interval1, interval2]
```