UCSB Math 3 Series (from Fall 1988)

Chapter 1

Calculus with Analytical Geometry, 3rd. Edition, Robert Ellis & Denny Gulick

Mathematica Notes (2016-10-30)

I have written *s*(x) which wraps **Sign** so it can return text:

```
s[x_] := Switch[Sign[x], -1, "-", 0, "0", 1, "+"]
```

It appears to be the case that **Interval** does *not* distinguish between (half) open/closed intervals. **IntervalMemberQ** (and, strangely, **Between**) appears to *only* support closed intervals like [-8,3]. Nevertheless, the mathematical notation for intervals like [-8,3) *cannot* be supported by the bracket-based Wolfram Language.

1.1 Points and Lines in the Plane

pg. 7: the negative multiplicity rule and its relation to inequalities

($a < b$; $c < 0 \Rightarrow a \cdot c > b \cdot c$)

Given this function:

$$f[x\_] := \frac{(x-1)\ (x-3)}{x+2}$$

Solve the inequality $f > 0$.

Find zeroes from the factors of the numerator of *f*:

```
Solve[(x - 1) == 0, x]
```

$\{\{x \to 1\}\}$

```
Solve[(x - 3) == 0, x]
```

$\{\{x \to 3\}\}$

Zeros are *f*(1) and *f*(3) verified by:

```
zeroes = Map[f, {1, 3}];
VerificationTest[AllTrue[zeros, # == 0 &]]
```

TestResultObject[Outcome: Success Test ID: None]

Find infinity from the denominator:

```
Solve[x + 2 == 0, x]
```

{{x → -2}}

```
VerificationTest[Limit[f[x], Rule[x, -2]], ∞]
```

TestResultObject[Outcome: Success Test ID: None]

∴ intervals are (-2,1) ∪ (1,3) ∪ (3,+∞):

```
interval1 = Interval[{-2, 1}];
interval2 = Interval[{1, 3}];
interval3 = Interval[{3, +∞}];
```

For each interval we choose *f*(0), *f*(2) and *f*(4) respectively ∴

```
test1 = Min[interval1] < 0 < Max[interval1];
test2 = Min[interval2] < 2 < Max[interval2];
test3 = Min[interval3] < 4 < Max[interval3];
VerificationTest[test1 && test2 && test3]
```

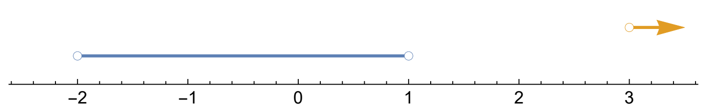
TestResultObject[Outcome: Success Test ID: None]

```
Map[s, Map[f, {0, 2, 4}]]
```

{+, -, +}

∴ *f* > 0 for (-2, 1) ∪ (3, +∞)

```
NumberLinePlot[{-2 < x < 1, x > 3}, x]
```




```
Clear[f, interval1, interval2, interval3, test1, test2, test3, zeroes]
```

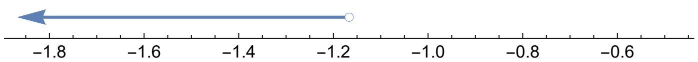
#19

$-6x - 2 > 5;\ -6x - 2 + 2 > 5 + 2;\ -6x > 7;\ \frac{-6x}{-6} > -\frac{7}{6}$

$x > -\frac{7}{6}$

$(-\infty, -\frac{7}{6})$

```
NumberLinePlot[x < -7/6, x]
```



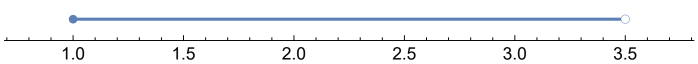
#21

$-1 \le 2x - 3 < 4;\ -1 + 3 \le 2x - 3 + 3 < 4 + 3;\ 2 \le 2x < 7;\ \frac{2}{2} \le \frac{2x}{2} < \frac{7}{2}$

$1 \le x < \frac{7}{2}$

$[1, \frac{7}{2})$

```
NumberLinePlot[1 ≤ x < 7/2, x]
```



#23

$f[x\_] := (x - 1)\left(x + \frac{1}{2}\right)$

Solve the inequality $f \geqslant 0$.

Find zeros from the factors of *f*:

```
Solve[(x - 1) == 0, x]
```

{{x → 1}}

```
Solve[(x + 1/2) == 0, x]
```

$\{\{x \to -\frac{1}{2}\}\}$

Zeros are $f(1)$ and $f(-\frac{1}{2})$ verified by:

```
zeroes = Map[f, {-1/2, 1}];
VerificationTest[AllTrue[zeros, # == 0 &]]
```

TestResultObject[Outcome: Success Test ID: None]

∴ intervals are $(-\infty,-\frac{1}{2}]$, $[-\frac{1}{2},1]$, $[1,+\infty)$.

```
interval1 = Interval[{-∞, -1/2}];
interval2 = Interval[{-1/2, 1}];
interval3 = Interval[{1, +∞}];
```

For each interval we choose *f*(-2), *f*(0) and *f*(2) respectively:

```
test1 = Min[interval1] < -2 <= Max[interval1];
test2 = Min[interval2] <= 0 <= Max[interval2];
test3 = Min[interval3] <= 2 < Max[interval3];
VerificationTest[test1 && test2 && test3]
```

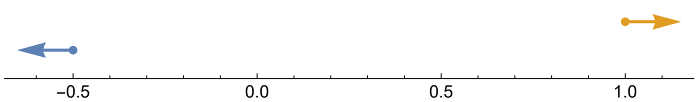
```
TestResultObject[ Outcome: Success
                  Test ID: None ]
```

```
zeroes = Map[s, Map[f, {-2, 0, 2}]];
VerificationTest[AllTrue[zeros, # == 0 &]]
```

```
TestResultObject[ Outcome: Success
                  Test ID: None ]
```

$\therefore f \geqslant 0$ for $(-\infty, -\frac{1}{2}] \cup [1, +\infty)$

```
NumberLinePlot[{interval1, interval3}]
```




```
Clear[f, interval1, interval2, interval3, test1, test2, test3, zeroes]
```

#25

```
f[x_] := x (x - 2/3) (x + 1/3)
```

Solve the inequality $f < 0$.

Find zeroes from the factors of *f*:

```
Solve[(x + 1/3) == 0, x]
```

$\{\{x \to -\frac{1}{3}\}\}$

```
Solve[x == 0, x]
```

$\{\{x \to 0\}\}$

```
Solve[(x - 2/3) == 0, x]
```

$\{\{x \to \frac{2}{3}\}\}$

Zeros are $f(-\frac{1}{3})$, $f(0)$ and $f(\frac{2}{3})$ verified by:

```
zeroes = Map[f, {-1/3, 0, 2/3}];
VerificationTest[AllTrue[zeros, # == 0 &]]
```

TestResultObject[Outcome: Success Test ID: None]

∴ intervals are $(-\infty, -\frac{1}{3}] \cup [-\frac{1}{3}, 0] \cup [0, \frac{2}{3}] \cup [\frac{2}{3}, +\infty)$:

```
interval1 = Interval[{-∞, -1/3}];
interval2 = Interval[{-1/3, 0}];
interval3 = Interval[{0, 2/3}];
interval4 = Interval[{2/3, +∞}];
```

For each interval we choose $f(-\frac{6}{3})$, $f(-\frac{1}{10})$, $f(\frac{1}{3})$, $f(\frac{6}{3})$ respectively:

```
test1 = Min[interval1] < -6/3 <= Max[interval1];
test2 = Min[interval2] <= -1/10 <= Max[interval2];
test3 = Min[interval3] <= 1/3 <= Max[interval3];
test4 = Min[interval4] <= 6/3 < Max[interval4];
VerificationTest[test1 && test2 && test3 && test4]
```

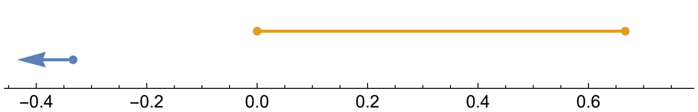
TestResultObject[Outcome: Success Test ID: None]

```
Map[s, Map[f, {-6/3, -1/10, 1/3, 6/3}]]
```

```
{-, +, -, +}
```

∴ $f < 0$ for $(-\infty, -\frac{1}{3}] \cup [0, \frac{2}{3}]$

```
NumberLinePlot[{interval1, interval3}]
```




```
Clear[f, interval1, interval2, interval3, interval4, test1, test2, test3, test4, zeroes]
```

#27

$$f[x\_] := \frac{(2x-1)^2}{(x+1)(x+3)}$$

Solve the inequality $f \geqslant 0$.

Find zeroes from the factors of the numerator of *f*:

```
Solve[(2 x - 1)^2 == 0, x]
```

$\{\{x \to \frac{1}{2}\}, \{x \to \frac{1}{2}\}\}$

Zero is $f\left(\frac{1}{2}\right)$ verified by:

```
zeroes = Map[f, {1/2}];
VerificationTest[AllTrue[zeros, # == 0 &]]
```

TestResultObject[Outcome: Success Test ID: None]

Find infinity from the denominator:

```
Solve[(x + 1) (x + 3) == 0, x]
```

$\{\{x \to -3\}, \{x \to -1\}\}$

```
VerificationTest[Limit[f[x], Rule[x, -3]], -∞]
```

TestResultObject[Outcome: Success Test ID: None]

```
VerificationTest[Limit[f[x], Rule[x, -1]], ∞]
```

TestResultObject[Outcome: Success Test ID: None]

∴ intervals are $(-\infty,-3) \cup (-3,-1) \cup (-1,\frac{1}{2}] \cup [\frac{1}{2},+\infty)$:

```
interval1 = Interval[{-∞, -3}];
interval2 = Interval[{-3, -1}];
interval3 = Interval[{-1, 1/2}];
interval4 = Interval[1/2, +∞];
```

For each interval we choose $f(-4)$, $f(-2)$, $f(0)$, $f(1)$ respectively:

```
test1 = Min[interval1] < -4 < Max[interval1];
test2 = Min[interval2] < -2 < Max[interval2];
test3 = Min[interval3] < 0 <= Max[interval3];
test4 = Min[interval4] ≤ 1 < Max[interval4];
VerificationTest[test1 && test2 && test3 && test4]
```

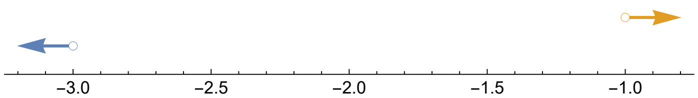
TestResultObject[Outcome: Success Test ID: None]

```
Map[s, Map[f, {-4, -2, 0, 1}]]
```

{+, -, +, +}

∴ $f \geqslant 0$ for (-∞,-3) ∪ (-1,+∞)

```
NumberLinePlot[{x < -3, -1 < x}, x]
```




```
Clear[f, interval1, interval2, interval3, interval4, test1, test2, test3, test4, zeroes]
```

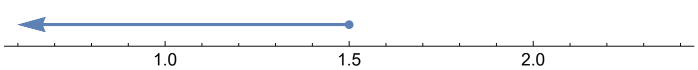
#29

$$4x^3 - 6x^2 \leqslant 0;$$
$$4x^3 - 6x^2 + 6x^2 \leqslant 0 + 6x^2$$
$$4x^3 \leq 6x^2$$

$$4x^3\left(\frac{1}{x^2}\right) \leq 6x^2\left(\frac{1}{x^2}\right)$$
$$4x \leq 6$$

$$4x\left(\frac{1}{4}\right) \leq 6\left(\frac{1}{4}\right)$$
$$x \leq \frac{3}{2}$$

$$\text{NumberLinePlot}\left[x \leq \frac{3}{2}, x\right]$$



#31

$$8x - \frac{1}{x^2} > 0;$$

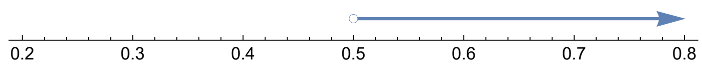
$$8x - \frac{1}{x^2} + \frac{1}{x^2} > 0 + \frac{1}{x^2}$$

$$8x > \frac{1}{x^2}$$

$$8x\left(\frac{1}{8}\right) > \frac{1}{x^2}\left(\frac{1}{8}\right)$$

$$x > \frac{1}{8x^2}$$

```
NumberLinePlot[x > 1/(8 x^2), x]
```



#33

```
f[x_] := (4 x (x^2 - 6))/(x^2 - 4)
```

Solve the inequality $f < 0$.

Find zeroes from the factors of the numerator of f:

```
Solve[4 x (x^2 - 6) == 0, x]
```

$\{\{x \to 0\}, \{x \to -\sqrt{6}\}, \{x \to \sqrt{6}\}\}$

Zeros are $f(0)$ and $f(\pm\sqrt{6})$ verified by:

```
zeroes = Map[f, {-√6, 0, √6}];
VerificationTest[AllTrue[zeros, # == 0 &]]
```

TestResultObject[Outcome: Success Test ID: None]

Find infinity from the denominator:

```
Solve[x^2 - 4 == 0, x]
```

$\{\{x \to -2\}, \{x \to 2\}\}$

```
VerificationTest[Limit[f[x], Rule[x, -2]], -∞]
VerificationTest[Limit[f[x], Rule[x, 2]], -∞]
```

TestResultObject[Outcome: Success Test ID: None]

TestResultObject[Outcome: Success Test ID: None]

∴ intervals are $(-\infty, -\sqrt{6}] \cup [-\sqrt{6}, -2) \cup (-2, 0] \cup [0, 2) \cup (2, \sqrt{6}] \cup [\sqrt{6}, +\infty)$:

```
interval1 = Interval[{-∞, -√6}];
interval2 = Interval[{-√6, -2}];
interval3 = Interval[{-2, 0}];
interval4 = Interval[{0, 2}];
interval5 = Interval[{2, √6}];
interval6 = Interval[{√6, +∞}];
```

For each interval we have *f*(-4), *f*(-2.1), *f*(-1), *f*(1), *f*(2.1), *f*(4) respectively:

```
test1 = Min[interval1] < -4 <= Max[interval1];
test2 = Min[interval2] <= -2.1 < Max[interval2];
test3 = Min[interval3] < -1 <= Max[interval3];
test4 = Min[interval4] ≤ 1 < Max[interval4];
test5 = Min[interval5] < 2.1 <= Max[interval5];
test6 = Min[interval6] ≤ 4 < Max[interval6];
VerificationTest[test1 && test2 && test3 && test4 && test5 && test6]
```

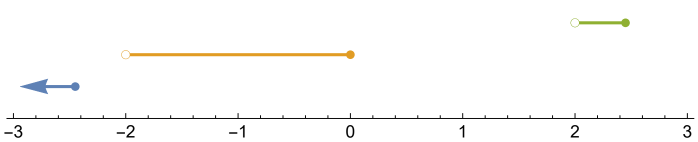
```
TestResultObject[ Outcome: Success
                  Test ID: None ]
```

```
Map[s, Map[f, {-4, -2.1, -1, 1, 2.1, 4}]]
```

```
{-, +, -, +, -, +}
```

∴ $f \geqslant 0$ for $(-\infty,-\sqrt{6}] \cup (-2,0] \cup (2,\sqrt{6}]$

```
NumberLinePlot[{x <= -√6, -2 < x ≤ 0, 2 < x <= √6}, x]
```




```
Clear[f, interval1, interval2, interval3, interval4,
 interval5, interval6, test1, test2, test3, test4, test5, test6, zeroes]
```

#35

$$\frac{t^2+t-2}{(t^2-1)^3} \Rightarrow \frac{(t+2)(t-1)}{(t^2-1)^3};$$

$$f[t\_] := \frac{(t+2)(t-1)}{(t^2-1)^3}$$

Solve the inequality $f \geqslant 0$.

Find zeroes from the factors of the numerator of *f*:

```
Solve[(t + 2) (t - 1) == 0, t]
```

```
{{t → -2}, {t → 1}}
```

Zero is *f*(-2) (because *f*(1) goes to infinity) verified by:

```
zeroes = Map[f, {-2}];
VerificationTest[AllTrue[zeros, # == 0 &]]
```

TestResultObject[Outcome: Success Test ID: None]

Find infinity from the denominator:

```
DeleteDuplicates[Solve[(t^2 - 1)^3 == 0, t]]
```

{{t → -1}, {t → 1}}

```
VerificationTest[Limit[f[t], Rule[t, -1]], ∞]
```

TestResultObject[Outcome: Success Test ID: None]

```
VerificationTest[Limit[f[t], Rule[t, 1]], ∞]
```

TestResultObject[Outcome: Success Test ID: None]

∴ intervals are (-∞,-2] ∪ [-2,-1) ∪ (-1,1) ∪ (1,2] ∪ [2,+∞):

```
interval1 = Interval[{-∞, -2}];
interval2 = Interval[{-2, -1}];
interval3 = Interval[{-1, 1}];
interval4 = Interval[{1, 2}];
interval5 = Interval[{2, +∞}];
```

For each interval, we have $f(-3)$, $f(-1.1)$, $f(0)$, $f(1.1)$, $f(3)$ respectively:

```
test1 = Min[interval1] < -3 <= Max[interval1];
test2 = Min[interval2] <= -1.1 < Max[interval2];
test3 = Min[interval3] < 0 < Max[interval3];
test4 = Min[interval4] < 1.1 <= Max[interval4];
test5 = Min[interval5] <= 3 < Max[interval5];
VerificationTest[test1 && test2 && test3 && test4 && test5]
```

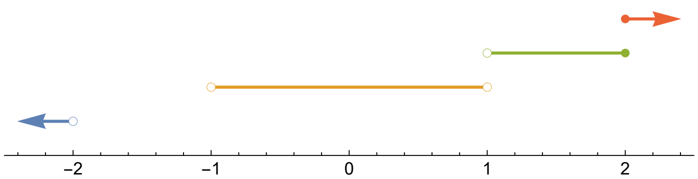
TestResultObject[Outcome: Success Test ID: None]

```
Map[s, Map[f, {-3, -1.1, 0, 1.1, 3}]]
```

{+, -, +, +, +}

∴ $f \geqslant 0$ for (-∞,-2] ∪ (-1,1) ∪ (1,2] ∪ [2,+∞)

```
NumberLinePlot[{x < -2, -1 < x < 1, 1 < x ≤ 2, x >= 2}, x]
```




```
Clear[f, interval1, interval2, interval3, interval4,
 interval5, test1, test2, test3, test4, test5, zeroes]
```

#37

$$f[x\_] := \frac{2 - x}{\sqrt{9 - 6x}}$$

Solve the inequality $f > 0$.

Find zeroes from the factors of the numerator of *f*:

```
Solve[2 - x == 0, x]
```

$\{\{x \to 2\}\}$

Zero is *f*(2) verified by:

```
zeroes = Map[f, {2}];
VerificationTest[AllTrue[zeros, # == 0 &]]
```

TestResultObject[Outcome: Success Test ID: None]

Find infinity from the denominator:

$$\text{Solve}\left[\sqrt{9 - 6x} == 0, x\right]$$

$\left\{\left\{x \to \frac{3}{2}\right\}\right\}$

Find the imaginary from the quantity of the square root:

```
9 - 6 x < 0;
9 - 6 x - 9 < 0 - 9
```

$-6x < -9$

$$-6x\left(-\frac{1}{6}\right) < -9\left(-\frac{1}{6}\right)$$

$x < \frac{3}{2}$

```
VerificationTest[Limit[f[x], Rule[x, 3/2]], (-ⅈ) ∞]
```

TestResultObject[Outcome: Success Test ID: None]

∴ interval is $(-\infty, \frac{3}{2})$:

```
interval = Interval[{-∞, 3/2}];
```

For the interval, we have *f*(1):

```
test = Min[interval] < 1 < Max[interval];
VerificationTest[test]
```

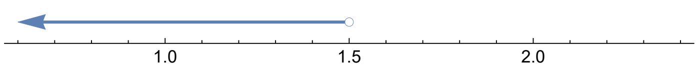
TestResultObject[Outcome: Success Test ID: None]

```
Map[s, Map[f, {1}]]
```

{+}

∴ $f > 0$ for $(-\infty, \frac{3}{2})$.

```
NumberLinePlot[x < 3/2, x]
```




```
Clear[f, interval, test, zeroes]
```

#47

```
f[x_] := Abs[x]
```

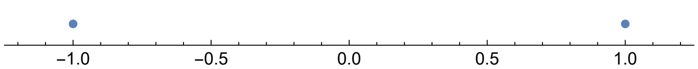
Solve the equation $f = |x| = 1$.

```
VerificationTest[f[-1] == f[1]]
```

TestResultObject[Outcome: Success Test ID: None]

∴ x = -1 or x = 1.

```
NumberLinePlot[{-1, 1}]
```



#49

```
f[x_] := Abs[x - 1]
```

Solve the equation $f = |x-1| = 2$.

```
(x - 1) == 2; x - 1 + 1 == 2 + 1
```

$x == 3$

```
- (x - 1) == 2; -1 (-x + 1 - 1) == -1 (2 - 1)
```

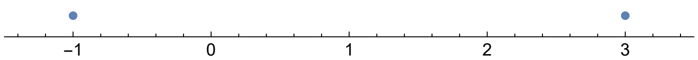
$x == -1$

```
VerificationTest[f[3] == f[-1]]
```

TestResultObject[Outcome: Success Test ID: None]

∴ x = -1 or x = 3.

```
NumberLinePlot[{-1, 3}]
```



#51

```
f[x_] := Abs[6 x + 5]
```

Solve the equation $f = |6x + 5| = 0$.

```
6 x + 5 == 0; 6 x + 5 - 5 == 0 - 5
```

$6\,x == -5$

$$6\,x\left(\frac{1}{6}\right) == -5\left(\frac{1}{6}\right)$$

$$x == -\frac{5}{6}$$

```
- (6 x + 5) == 0; -6 x - 5 + 5 == 0 + 5
```

$-6\,x == 5$

$$-6\,x\left(-\frac{1}{6}\right) == 5\left(-\frac{1}{6}\right)$$

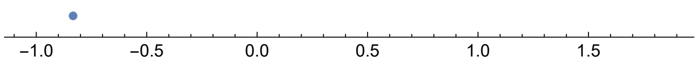
$$x == -\frac{5}{6}$$

$$\texttt{VerificationTest}\left[f\left[-\frac{5}{6}\right] == 0\right]$$

TestResultObject[Outcome: Success Test ID: None]

∴ $x = -\frac{5}{6}$.

$\texttt{NumberLinePlot}[\{-\frac{5}{6}\}]$



#53

```
f1[x_] := Abs[x]
```

$\texttt{f2[x\_] := Abs[x]}^2$

Solve the equation $f_1 = f_2 = |x| = |x|^2$.

$-x == x^2;\ \frac{-x}{-x} == -\frac{x^2}{x};\ \texttt{Solve}\left[\frac{-x}{-x} == -\frac{x^2}{x}, x\right]$

{{x → -1}}

$x == x^2;\ \frac{x}{x} == \frac{x^2}{x};\ \texttt{Solve}\left[\frac{x}{x} == \frac{x^2}{x}, x\right]$

{{x → 1}}

Recall that $x^2 == (-x)^2$.

```
VerificationTest[f1[1] == f2[1]]
VerificationTest[f1[-1] == f2[1]]
VerificationTest[f1[1] == f2[-1]]
VerificationTest[f1[-1] == f2[-1]]
```

TestResultObject[Outcome: Success Test ID: None]

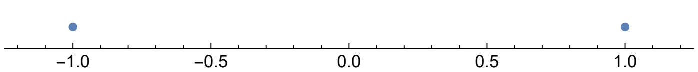
TestResultObject[Outcome: Success Test ID: None]

TestResultObject[Outcome: Success Test ID: None]

TestResultObject[Outcome: Success Test ID: None]

∴ x = -1 or x = 1.

```
NumberLinePlot[{-1, 1}]
```



```
Clear[f1, f2]
```

#55

```
f[x_] := Abs[x + 1]^2 + 3 Abs[x + 1] - 4
```

Solve the equation $f = 0 = |x + 1|^2 + 3|x + 1| - 4$.

Let p = $|x + 1|$ ⇒

```
p^2 + 3 p - 4 == 0; (p + 4) (p - 1) == 0; Solve[(p + 4) (p - 1) == 0, p]
```

```
{{p → -4}, {p → 1}}
```

|x+1| cannot be -4 by definition of absolute value.

It follows that |x+1| = 1:

```
(x + 1) == 1; (x + 1) - 1 == 1 - 1
```

```
x == 0
```

```
- (x + 1) == 1; -x - 1 + 1 == 1 + 1; -x/-1 == -2/1
```

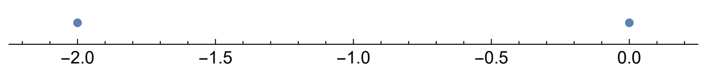
```
x == -2
```

```
VerificationTest[f[-2] == f[0] == 0]
```

```
TestResultObject[ Outcome: Success
                  Test ID: None ]
```

∴ x = -2 or x = 0.

```
NumberLinePlot[{-2, 0}]
```




```
Clear[f]
```

#57

```
f1[x_] := Abs[x + 4]
```

```
f2[x_] := Abs[x - 4]
```

Solve the equation $f_1 = f_2 = |x + 4| = |x - 4|$.

+(x + 4) ≠ +(x - 4)

-(x + 4) ≠ -(x - 4)

$+(x+4) == -(x-4);\ x+x+4-4 == -x+x+4-4;\ 2x == 0;\ \frac{2x}{2} == \frac{0}{2}$

$x == 0$

$-(x+4) == +(x-4);\ -x-x-4+4 == x-x-4+4;\ -2x == 0;\ \frac{-2x}{-2} == -\frac{0}{2}$

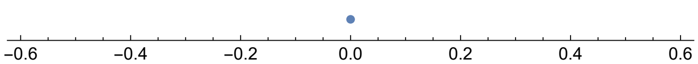
$x == 0$

```
VerificationTest[f1[0] == f2[0] == 4]
```

```
TestResultObject[ Outcome: Success
                  Test ID: None ]
```

∴ x = 0.

```
NumberLinePlot[0]
```




```
Clear[f1, f2]
```

#59

```
f[x_] := Abs[x - 2]
```

Solve the inequality $f < 1$.

$+(x-2) < 1;\ x-2+2 < 1+2$

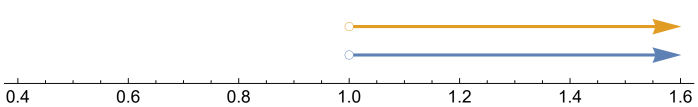
$x < 3$

$-(x-2) < 1;\ -x+2-2 < 1-2$

$-x < -1$

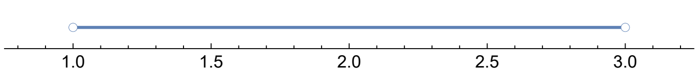
and -x < -1 is the additive inverse of x > 1:

```
NumberLinePlot[{-x < -1, x > 1}, x]
```



∴ 1 < x < 3 or (1,3).

```
NumberLinePlot[1 < x < 3, x]
```




```
Clear[f]
```

#61

```
f[x_] := Abs[x + 1]
```

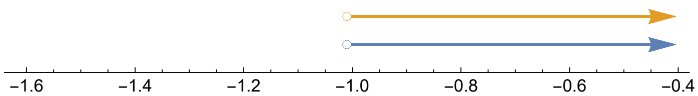
Solve the inequality $f < 0.01$.

```
+ (x + 1) < 0.01; x + 1 - 1 < 0.01 - 1
x < -0.99

- (x + 1) < 0.01; -x - 1 + 1 < 0.01 + 1
-x < 1.01
```

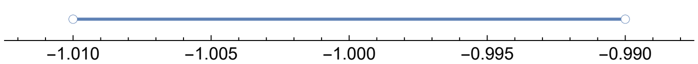
and -x < 1.01 is the additive inverse of x > -1.01:

```
NumberLinePlot[{-x < 1.01, x > -1.01}, x]
```



∴ -1.01 < x < -0.99 or (-1.01,-0.99).

```
NumberLinePlot[-1.01 < x < -0.99, x]
```



```
Clear[f]
```

#63

```
f[x_] := Abs[x + 3]
```

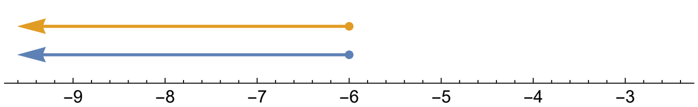
Solve the inequality $f \geqslant 3$.

```
+ (x + 3) ≥ 3; x + 3 - 3 ≥ 3 - 3
x ≥ 0

- (x + 3) ≥ 3; -x - 3 + 3 ≥ 3 + 3
-x ≥ 6
```

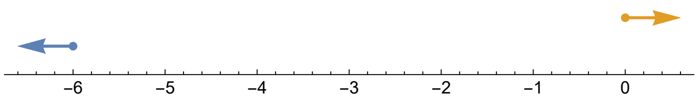
and -x ⩾ 6 is the additive inverse of x ⩽ -6:

```
NumberLinePlot[{-x ⩾ 6, x ⩽ -6}, x]
```



∴ (-∞,-6] ∪ [0,∞).

```
NumberLinePlot[{x ⩽ -6, x ≥ 0}, x]
```




```
VerificationTest[f[-7] ≥ 3]
VerificationTest[f[3] ≥ 3]
```

TestResultObject[Outcome: Success Test ID: None]

TestResultObject[Outcome: Success Test ID: None]

```
Clear[f]
```

#65

```
f[x_] := Abs[2 x + 1]
```

Solve the inequality $f \geq 1$.

$$+ (2x+1) \geq 1;\ 2x+1-1 \geq 1-1;\ \frac{2x}{2} \geq \frac{0}{2}$$

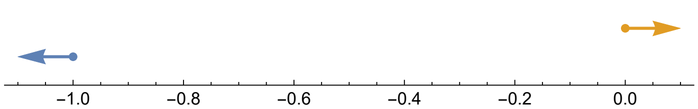
$x \geq 0$

$$-(2x+1) \geq 1;\ -2x-1+1 \geq 1+1;\ \frac{-2x}{-2} \leq -\frac{2}{2}$$

$x \leq -1$

$\therefore (-\infty,-1] \cup [0,\infty)$.

```
NumberLinePlot[{x ⩽ -1, x ≥ 0}, x]
```




```
VerificationTest[f[-2] ≥ 1]
VerificationTest[f[1] ≥ 1]
```

TestResultObject[Outcome: Success Test ID: None]

TestResultObject[Outcome: Success Test ID: None]

```
Clear[f]
```

#69

```
f[x_] := Abs[4 - 2 x]
```

Solve the inequality -1 < *f* < 1.

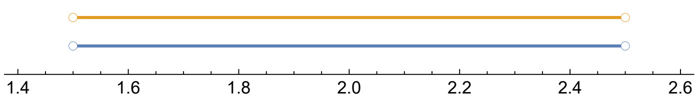
$-1 < (4 - 2x) < 1;\ -1 - 4 < 4 - 2x - 4 < 1 - 4;\ -5 < -2x < -3;\ \frac{-5}{-2} > \frac{-2x}{-2} > \frac{-3}{-2}$

$\frac{5}{2} > x > \frac{3}{2}$

$-1 < -(4 - 2x) < 1;\ -1 + 4 < -4 + 2x + 4 < 1 + 4;\ 3 < 2x < 5;\ \frac{3}{2} < \frac{2x}{2} < \frac{5}{2}$

$\frac{3}{2} < x < \frac{5}{2}$

$\text{NumberLinePlot}\left[\left\{\frac{5}{2} > x > \frac{3}{2}, \frac{3}{2} < x < \frac{5}{2}\right\}, x\right]$



$\therefore \left(\frac{3}{2}, \frac{5}{2}\right)$.

```
VerificationTest[-1 < f[1.6] < 1]
VerificationTest[-1 < f[2.2] < 1]
```

TestResultObject[Outcome: Success Test ID: None]

TestResultObject[Outcome: Success Test ID: None]

```
Clear[f]
```