

22.10.17

作业

p.508(p.101). 14(1), (3), (4)

15(1), (2)

14. (1)

$$① (\neg p_1 \rightarrow \neg p_2) \rightarrow (p_2 \rightarrow p_1) \quad (A_3)$$

$$② ((\neg(\neg p_1 \rightarrow p_2) \rightarrow (\neg p_1 \rightarrow p_1)) \rightarrow ((\neg p_1 \rightarrow p_2) \rightarrow (\neg p_1 \rightarrow p_1))) \quad (A_1)$$

$$③ (\neg p_1 \rightarrow p_2) \rightarrow ((\neg p_1 \rightarrow p_2) \rightarrow (p_2 \rightarrow p_1)) \quad (M) ① ②$$

$$④ \text{ 证: } \vdash p_1 \rightarrow (\neg p_1 \rightarrow p_2)$$

$$\text{证: } \vdash p_1, p_1 \rightarrow p_2 \vdash p_2 \quad (M)$$

$$p_1 \vdash (\neg p_1 \rightarrow p_2) \rightarrow p_2$$

$$\vdash p_1 \rightarrow ((\neg p_1 \rightarrow p_2) \rightarrow p_2)$$

$$⑤ \dots$$

$$\vdots$$

$$\text{---}$$

$$⑥ p_1 \rightarrow ((\neg p_1 \rightarrow p_2) \rightarrow p_2)$$

$$⑦ (\neg p_1 \rightarrow ((\neg p_1 \rightarrow p_2) \rightarrow p_2)) \rightarrow ((\neg p_1 \rightarrow p_2) \rightarrow (p_2 \rightarrow p_1)) \quad (A_2)$$

$$⑧ (\neg p_1 \rightarrow ((\neg p_1 \rightarrow p_2) \rightarrow p_2)) \rightarrow (p_2 \rightarrow p_1) \quad (M) ⑥ ⑦$$

$$(4) ① p_2 \rightarrow (p_1 \rightarrow p_2) \quad (A_1)$$

$$② (p_2 \rightarrow (p_1 \rightarrow p_2)) \rightarrow (p_1 \rightarrow (p_2 \rightarrow (p_1 \rightarrow p_2))) \quad (A_1)$$

$$③ p_1 \rightarrow (p_2 \rightarrow (p_1 \rightarrow p_2)) \quad (M) ① ②$$

15. (1)

$$① \alpha \rightarrow \beta, \alpha, \neg(\beta \rightarrow \gamma) \vdash \alpha \rightarrow \beta, \alpha, \neg(\beta \rightarrow \gamma) \quad (\text{证})$$

$$② \beta \quad (M) ①$$

$$③ \alpha \rightarrow (\beta \rightarrow \gamma) \quad (A_3) ①$$

$$④ \beta \rightarrow \gamma \quad (M) ① ③$$

$$⑤ \gamma \quad (M) ② ④$$

$$⑥ \alpha \rightarrow \beta, \alpha, \neg(\beta \rightarrow \gamma) \vdash \gamma \quad (\text{证}) ① ② ③ ④ ⑤$$

$$⑦ \alpha \rightarrow \beta, \neg(\beta \rightarrow \gamma) \vdash \alpha \rightarrow \gamma \quad (\text{证}) ⑥$$

(2)

$$① \alpha \rightarrow (\beta \rightarrow \gamma), \beta, \alpha \vdash \alpha \rightarrow (\beta \rightarrow \gamma), \beta, \alpha \quad (\text{证})$$

$$② \beta \rightarrow \gamma \quad (M) ①$$

$$③ \gamma \quad (M) ① ②$$

$$④ \alpha \rightarrow (\beta \rightarrow \gamma), \beta, \alpha \vdash \gamma \quad (\text{证}) ① ② ③$$

$$⑤ \alpha \rightarrow (\beta \rightarrow \gamma), (\beta \rightarrow \alpha) \vdash \gamma \quad (\text{证}) ④$$

$$⑥ \alpha \rightarrow (\beta \rightarrow \gamma) \vdash \beta \rightarrow (\alpha \rightarrow \gamma) \quad (\text{证}) ⑤$$

# 作业

p.508(p.101). 14(2)  
15(3), (4)  
16(2), (3)

[补充] 在P中证明:

- (1)  $(\alpha \rightarrow \neg \alpha) \rightarrow \neg \alpha$ .
- (2)  $\{\alpha \rightarrow \beta, \alpha \rightarrow \neg \beta\} \vdash \neg \alpha$

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14.(2)

$$\textcircled{1} (\beta \rightarrow (\beta \rightarrow \beta)) \rightarrow ((\beta \rightarrow \beta) \rightarrow (\beta \rightarrow \beta)) \text{ (A2)}$$

$$\textcircled{2} ((\beta \rightarrow (\beta \rightarrow \beta)) \rightarrow ((\beta \rightarrow \beta) \rightarrow (\beta \rightarrow \beta))) \rightarrow$$

$$((\beta \rightarrow (\beta \rightarrow \beta)) \rightarrow (\beta \rightarrow \beta)) \rightarrow ((\beta \rightarrow (\beta \rightarrow \beta)) \rightarrow (\beta \rightarrow \beta)) \text{ (A2)}$$

$$\textcircled{3} ((\beta \rightarrow (\beta \rightarrow \beta)) \rightarrow (\beta \rightarrow \beta)) \rightarrow ((\beta \rightarrow (\beta \rightarrow \beta)) \rightarrow (\beta \rightarrow \beta)) \text{ (M) } \textcircled{1} \textcircled{2}$$

15.(3)

$$\textcircled{1} \alpha \rightarrow (\alpha \rightarrow \beta) \text{ (E1)}$$

$$\textcircled{2} (\alpha \rightarrow (\alpha \rightarrow \beta)) \rightarrow (\alpha \rightarrow \beta) \text{ (E2)}$$

$$\textcircled{3} \neg (\alpha \rightarrow \beta) \rightarrow \alpha \text{ (M) } \textcircled{1} \textcircled{2}$$

$$\textcircled{4} \neg (\alpha \rightarrow \beta)$$

$$\textcircled{5} \alpha$$

$$\textcircled{14} \alpha \rightarrow (\alpha \rightarrow \beta) \text{ (A1)}$$

$$\textcircled{2} (\beta \rightarrow (\alpha \rightarrow \beta)) \rightarrow (\alpha \rightarrow \beta) \text{ (E2)}$$

$$\textcircled{3} \neg (\alpha \rightarrow \beta) \rightarrow \neg \beta \text{ (M) } \textcircled{1} \textcircled{2}$$

$$\textcircled{4} \neg (\alpha \rightarrow \beta)$$

$$\textcircled{5} \neg \beta$$

16.(2)

证明  $\alpha \rightarrow (\beta \rightarrow \alpha), \alpha \vdash \beta$

$$\textcircled{1} \alpha \rightarrow (\beta \rightarrow \alpha)$$

$$\textcircled{2} \alpha$$

$$\textcircled{3} \beta \rightarrow \alpha \text{ (M) } \textcircled{1} \textcircled{2}$$

$$\textcircled{4} (\beta \rightarrow \alpha) \rightarrow (\neg \alpha \rightarrow \neg \beta) \text{ (E1)}$$

$$\textcircled{5} \neg \alpha \rightarrow \neg \beta \text{ (M) } \textcircled{3} \textcircled{4}$$

$$\textcircled{6} \alpha \rightarrow \neg \alpha \text{ (E1)}$$

$$\textcircled{7} \neg \alpha \text{ (M) } \textcircled{6}$$

$$\textcircled{8} \neg \beta \text{ (M) } \textcircled{5} \textcircled{7}$$

(3) 证明  $(\alpha \rightarrow \beta) \rightarrow \gamma, \alpha \vdash \gamma$

$$\textcircled{1} (\alpha \rightarrow \beta) \rightarrow \gamma$$

$$\textcircled{2} \alpha$$

$$\textcircled{3} \alpha \rightarrow \beta \text{ (E1)}$$

$$\textcircled{4} \neg \alpha \text{ (M) } \textcircled{2} \textcircled{3}$$

$$\textcircled{5} \neg \alpha \rightarrow (\alpha \rightarrow \beta) \text{ (E1)}$$

$$\textcircled{6} \neg \alpha \rightarrow \beta \text{ (M) } \textcircled{5}$$

$$\textcircled{7} \gamma \text{ (M) } \textcircled{6}$$

补充:

(1) 引理: 若  $\alpha, \beta \vdash \gamma$ , 且  $\neg \beta \vdash \gamma$ , 则  $\alpha \vdash \gamma$

证: 由  $\alpha, \beta \vdash \gamma$  有  $\alpha \vdash \beta \rightarrow \gamma$

由  $\alpha, \neg \beta \vdash \gamma$  有  $\alpha \vdash \neg \beta \rightarrow \gamma$

而  $\beta \rightarrow \gamma, \neg \beta \rightarrow \gamma \vdash \gamma$  (E<sub>1</sub>)

故  $\alpha \vdash \gamma$

回到本证, 显然  $\alpha \rightarrow \beta, \beta \vdash \gamma$

$\alpha \rightarrow \beta, \alpha \vdash \beta$

故  $\alpha \rightarrow \beta \vdash \beta$ , 即  $\vdash (\alpha \rightarrow \beta) \rightarrow \beta$

(2) 引理:  $\alpha, \neg \alpha \vdash \beta$

由  $\neg \alpha \rightarrow \alpha$  可得.

回到本证, 显然  $\alpha \rightarrow \beta, \alpha \rightarrow \neg \beta, \alpha \vdash \alpha$

$\alpha \rightarrow \beta, \alpha \rightarrow \neg \beta, \alpha \vdash \beta$  (M) (引理)

故由 (1) 引理知  $\alpha \rightarrow \beta, \alpha \rightarrow \neg \beta \vdash \alpha$