

22.9.30

作业

p.508(p.101). 13(1)(3)(5)

9
10

13.(1)

① $\alpha \rightarrow (\beta \rightarrow \gamma), \alpha \wedge \beta \vdash \alpha \rightarrow (\beta \rightarrow \gamma), \alpha \wedge \beta \quad (E)$

② $\alpha \rightarrow (\beta \rightarrow \gamma), \alpha \wedge \beta \vdash \alpha, \beta \quad (\wedge \rightarrow)$

③ $\alpha \rightarrow (\beta \rightarrow \gamma), \alpha \wedge \beta \vdash \beta \rightarrow \gamma \quad (\rightarrow -) \text{ ① ②}$

④ $\alpha \rightarrow (\beta \rightarrow \gamma), \alpha \wedge \beta \vdash \gamma \quad (\rightarrow -) \text{ ③}$

(2)

① $\alpha \rightarrow \beta, \alpha \vdash \alpha, \alpha \rightarrow \beta \quad (E)$

② $\alpha \rightarrow \beta, \alpha \vdash \beta \quad (\rightarrow -)$

③ $\alpha \rightarrow \beta, \alpha \vdash \alpha \wedge \beta \quad (\wedge +) \text{ ②}$

④ $\alpha \rightarrow \beta \vdash \alpha \rightarrow (\alpha \wedge \beta) \quad (\rightarrow +) \text{ ③}$

(5)

① $\alpha \rightarrow (\alpha \rightarrow \beta), \gamma \beta, \alpha \vdash \alpha, \alpha \rightarrow (\alpha \rightarrow \beta), \gamma \beta \quad (E)$

② $\alpha \rightarrow (\alpha \rightarrow \beta), \gamma \beta, \alpha \vdash \alpha \rightarrow \beta \quad (\rightarrow -) \text{ ①}$

③ $\alpha \rightarrow (\alpha \rightarrow \beta), \gamma \beta, \alpha \vdash \beta \quad (\rightarrow -) \text{ ②}$

④ $\alpha \rightarrow (\alpha \rightarrow \beta), \gamma \beta \vdash \beta \quad (\gamma +) \text{ ③}$

⑤ $\alpha \rightarrow (\alpha \rightarrow \beta) \vdash \beta \rightarrow \beta \quad (\gamma +) \text{ ④}$

9. (1)

① $\alpha \rightarrow \beta, \gamma \beta, \alpha \vdash \alpha \rightarrow \beta, \gamma \beta, \alpha \quad (E)$

② $\alpha \rightarrow \beta, \gamma \beta, \alpha \vdash \beta \quad (\rightarrow -) \text{ ①}$

③ $\alpha \rightarrow \beta, \gamma \beta \vdash \gamma \alpha \quad (\gamma +) \text{ ②}$

④ $\alpha \rightarrow \beta \vdash \gamma \beta \rightarrow \gamma \alpha \quad (\rightarrow +) \text{ ③}$

(3) ① $\alpha \rightarrow \beta, \gamma \beta, \alpha \vdash \alpha \rightarrow \beta, \gamma \beta, \alpha \quad (E)$

② $\alpha \rightarrow \beta, \gamma \beta, \alpha \vdash \beta \quad (\rightarrow -) \text{ ①}$

③ $\alpha \rightarrow \beta, \gamma \beta \vdash \gamma \alpha \quad (\gamma +) \text{ ②}$

④ $\alpha \rightarrow \beta \vdash \gamma \beta \rightarrow \gamma \alpha \quad (\rightarrow +) \text{ ③}$

10. (2)

① $\alpha \rightarrow \alpha, \alpha \vdash \alpha \rightarrow \alpha, \alpha \quad (E)$

② $\alpha \rightarrow \alpha, \alpha \vdash \alpha \quad (\rightarrow -) \text{ ①}$

③ $\alpha \rightarrow \alpha \vdash \gamma \alpha \quad (\gamma +) \text{ ②}$

(3)

① $\alpha \rightarrow \beta, \alpha \rightarrow \gamma \beta, \alpha \vdash \alpha \rightarrow \beta, \alpha \rightarrow \gamma \beta, \alpha \quad (E)$

② $\alpha \rightarrow \beta, \alpha \rightarrow \gamma \beta, \alpha \vdash \beta, \gamma \beta \quad (\rightarrow -) \text{ ①}$

③ $\alpha \rightarrow \beta, \alpha \rightarrow \gamma \beta \vdash \gamma \alpha \quad (\gamma +) \text{ ②}$

(6)

① $\gamma (\alpha \rightarrow \beta), \beta, \alpha \vdash \beta \quad (E)$

② $\gamma (\alpha \rightarrow \beta), \beta \vdash \alpha \rightarrow \beta \quad (\rightarrow +) \text{ ①}$

③ $\gamma (\alpha \rightarrow \beta), \beta \vdash \gamma (\alpha \rightarrow \beta) \quad (E)$

④ $\gamma (\alpha \rightarrow \beta) \vdash \gamma \alpha \quad (\gamma +) \text{ ③}$

作业

p.508(p.101). 11(3), (6)
11(1), (2), (5)
12(4), (6), (8)
13(2), (7), (10), (12), (14)

80

11. (1)

(1) ① $\alpha \vdash \alpha \vee \beta, \alpha \vee \gamma (V+)$

② $\beta \wedge \gamma \vdash \beta, \gamma (1-)$

③ $\beta \vdash \alpha \vee \beta (V+)$

④ $\beta \wedge \gamma \vdash \alpha \vee \beta (Tr)$ ②③

⑤ $\alpha \vee (\beta \wedge \gamma) \vdash \alpha \vee \beta (V+)$ ④④

⑥ $\gamma \vdash \alpha \vee \gamma (V+)$

⑦ $\beta \wedge \gamma \vdash \alpha \vee \gamma (Tr)$ ⑥⑥

⑧ $\alpha \vee (\beta \wedge \gamma) \vdash \alpha \vee \gamma (V+)$ ⑥⑦

⑨ $\alpha \vee (\beta \wedge \gamma) \vdash (\alpha \vee \beta) \wedge (\alpha \vee \gamma) (1+)$ ⑤⑧

(1) ⑩ $(\alpha \vee \beta) \wedge (\alpha \vee \gamma), \neg(\alpha \vee \beta \wedge \gamma) \vdash \alpha \vee \beta, \alpha \vee \gamma,$

$\neg(\alpha \vee \beta \wedge \gamma) (E)(1-)$ (证)

① $\alpha \wedge \neg(\beta \wedge \gamma) \vdash \alpha, \neg(\beta \wedge \gamma) (1-)$ ①

② $\neg(\beta \wedge \gamma) \vdash \neg\beta \vee \neg\gamma$ (证) ②

③ $\alpha \vee \beta, \neg\alpha, \neg\beta \vdash \neg(\alpha \vee \beta) (E)(1+)$

④ $\alpha \vee \beta, \neg\alpha, \neg\beta \vdash \neg(\alpha \vee \beta)$ (证) (Tr) ③

⑤ $\alpha \vee \beta, \neg\alpha \vdash \neg\beta$ (1-) ④⑤

⑥ $\alpha \vee \beta, \neg\alpha \vdash \neg\beta$ (1-) ④⑤

⑦ $\alpha \vee \beta, \neg\alpha \vdash \neg\beta$ (1-) ④⑤

⑧ $\alpha \vee \beta, \neg\alpha \vdash \neg\beta$ (1-) ④⑤

⑨ $\beta \wedge \gamma \vdash \neg(\neg\beta \vee \neg\gamma)$ (证)

⑩ $(\alpha \vee \beta) \wedge (\alpha \vee \gamma) \vdash \alpha \vee (\beta \wedge \gamma) (1-)$

(2)

(1) ① $\alpha \wedge (\beta \vee \gamma) \vdash \alpha, \beta \vee \gamma (E)$

② $\alpha, \beta \vdash \alpha \wedge \beta (1+)$

③ $\alpha, \beta \vdash (\alpha \wedge \beta) \vee (\alpha \wedge \gamma) (V+)$

④ $\alpha, \gamma \vdash \alpha \wedge \gamma (1+)$

⑤ $\alpha, \gamma \vdash (\alpha \wedge \beta) \vee (\alpha \wedge \gamma) (V+)$

⑥ $\alpha, \beta \vee \gamma \vdash (\alpha \wedge \beta) \vee (\alpha \wedge \gamma) (V-)$

⑦ $\alpha \wedge (\beta \vee \gamma) \vdash (\alpha \wedge \beta) \vee (\alpha \wedge \gamma) (Tr)$

(1) ① $(\alpha \wedge \beta) \vee (\alpha \wedge \gamma) \vdash \alpha \wedge \beta, \alpha \wedge \gamma (E)$

② $\alpha \wedge \beta \vdash \alpha, \beta (1-)$

③ $\beta \vdash \beta \vee \gamma (V+)$

④ $\alpha \wedge \beta \vdash \alpha \wedge (\beta \vee \gamma) (Tr)(1+)$

⑤ $\alpha \wedge \gamma \vdash \alpha, \gamma (1-)$

⑥ $\gamma \vdash \beta \vee \gamma (V+)$

⑦ $\alpha \wedge \gamma \vdash \alpha \wedge (\beta \vee \gamma) (Tr)(1+)$

⑧ $(\alpha \wedge \beta) \vee (\alpha \wedge \gamma) \vdash \alpha \wedge (\beta \vee \gamma) (V-)$

(a)

(1) $\mathcal{Q} \rightarrow (\mathcal{P} \wedge \mathcal{X}), \mathcal{Q} \vdash \mathcal{P} \wedge \mathcal{X} (e) (\rightarrow -)$

② $\mathcal{Q} \rightarrow (\mathcal{P} \wedge \mathcal{X}), \mathcal{Q} \vdash \mathcal{P}, \mathcal{X} (\wedge -) (Tr)$

③ $\mathcal{Q} \rightarrow (\mathcal{P} \wedge \mathcal{X}) \vdash \mathcal{Q} \rightarrow \mathcal{P} (\rightarrow +)$

④ $\mathcal{Q} \rightarrow (\mathcal{P} \wedge \mathcal{X}) \vdash \mathcal{Q} \rightarrow \mathcal{X} (\rightarrow +)$

⑤ $\mathcal{Q} \rightarrow (\mathcal{P} \wedge \mathcal{X}) \vdash (\mathcal{Q} \rightarrow \mathcal{P}) \wedge (\mathcal{Q} \rightarrow \mathcal{X}) (\wedge +)$

(1) ① $(\mathcal{Q} \rightarrow \mathcal{P}) \wedge (\mathcal{Q} \rightarrow \mathcal{X}), \mathcal{Q} \vdash \mathcal{Q} \rightarrow \mathcal{P}, \mathcal{Q} \rightarrow \mathcal{X}, \mathcal{Q} (e)$

② $(\mathcal{Q} \rightarrow \mathcal{P}) \wedge (\mathcal{Q} \rightarrow \mathcal{X}), \mathcal{Q} \vdash \mathcal{P}, \mathcal{X} (\rightarrow -) (Tr)$

③ $(\mathcal{Q} \rightarrow \mathcal{P}) \wedge (\mathcal{Q} \rightarrow \mathcal{X}), \mathcal{Q} \vdash \mathcal{P} \wedge \mathcal{X} (\wedge +)$

④ $(\mathcal{Q} \rightarrow \mathcal{P}) \wedge (\mathcal{Q} \rightarrow \mathcal{X}) \vdash \mathcal{Q} \rightarrow (\mathcal{P} \wedge \mathcal{X}) (\rightarrow +)$

(5)

(1) ① $\neg(\mathcal{Q} \rightarrow \mathcal{X}) \vee (\mathcal{P} \rightarrow \mathcal{X}) \vdash \neg(\mathcal{Q} \rightarrow \mathcal{X}), \neg(\mathcal{P} \rightarrow \mathcal{X}) (E\neg) (\wedge -)$

② $\neg(\mathcal{Q} \rightarrow \mathcal{X}), \neg(\mathcal{P} \rightarrow \mathcal{X}) \vdash \mathcal{Q}, \mathcal{P}, \neg \mathcal{X} (E\neg) (+)$

③ $\mathcal{Q} \wedge \mathcal{P} \rightarrow \mathcal{X} \vdash \neg(\mathcal{Q} \wedge \mathcal{P}) \vee \mathcal{X} (E\neg)$

④ $\mathcal{Q}, \mathcal{P}, \neg \mathcal{X} \vdash (\mathcal{Q} \wedge \mathcal{P}) \wedge \neg \mathcal{X} (\wedge +)$

⑤ $(\mathcal{Q} \wedge \mathcal{P}) \wedge \neg \mathcal{X} \vdash \neg(\neg(\mathcal{Q} \wedge \mathcal{P}) \vee \neg \mathcal{X}) (E\neg)$

⑥ $(\mathcal{Q} \wedge \mathcal{P}) \rightarrow \mathcal{X} \vdash (\mathcal{Q} \rightarrow \mathcal{X}) \vee (\mathcal{P} \rightarrow \mathcal{X}) (\neg -) (Tr)$

(1) ① $\mathcal{Q} \rightarrow \mathcal{X}, \mathcal{Q} \wedge \mathcal{P} \vdash \mathcal{Q} \rightarrow \mathcal{X}, \mathcal{Q}, \mathcal{P} (E\wedge) (\wedge -)$

② $\mathcal{Q} \rightarrow \mathcal{X}, \mathcal{Q} \wedge \mathcal{P} \vdash \mathcal{X} (\rightarrow -) (Tr)$

③ $(\mathcal{P} \rightarrow \mathcal{X}), \mathcal{Q} \wedge \mathcal{P} \vdash \mathcal{P} \rightarrow \mathcal{X}, \mathcal{Q}, \mathcal{P} (e) (\wedge -)$

④ $(\mathcal{P} \rightarrow \mathcal{X}), \mathcal{Q} \wedge \mathcal{P} \vdash \mathcal{X} (\rightarrow -) (Tr)$

⑤ $(\mathcal{Q} \rightarrow \mathcal{X}) \vee (\mathcal{P} \rightarrow \mathcal{X}), \mathcal{Q} \wedge \mathcal{P} \vdash \mathcal{X} (\vee +)$

⑥ $(\mathcal{Q} \rightarrow \mathcal{X}) \vee (\mathcal{P} \rightarrow \mathcal{X}) \vdash (\mathcal{Q} \wedge \mathcal{P}) \rightarrow \mathcal{X} (\rightarrow +)$

(b)

(1) ① $\mathcal{Q} \vee \mathcal{P} \rightarrow \mathcal{X}, \mathcal{Q} \vdash \mathcal{Q} \vee \mathcal{P}, (\mathcal{Q} \vee \mathcal{P}) \rightarrow \mathcal{X} (e) (\vee +)$

② $\mathcal{Q} \vee \mathcal{P} \rightarrow \mathcal{X}, \mathcal{Q} \vdash \mathcal{X} (\rightarrow -) (Tr)$

③ $(\mathcal{Q} \vee \mathcal{P}) \rightarrow \mathcal{X} \vdash \mathcal{Q} \rightarrow \mathcal{X} (\rightarrow +)$

④ $\mathcal{Q} \vee \mathcal{P} \rightarrow \mathcal{X}, \mathcal{P} \vdash \mathcal{Q} \vee \mathcal{P}, (\mathcal{Q} \vee \mathcal{P}) \rightarrow \mathcal{X} (e) (\vee +)$

⑤ $\mathcal{Q} \vee \mathcal{P} \rightarrow \mathcal{X}, \mathcal{P} \vdash \mathcal{X} (\rightarrow -) (Tr)$

⑥ $(\mathcal{Q} \vee \mathcal{P}) \rightarrow \mathcal{X} \vdash \mathcal{P} \rightarrow \mathcal{X} (\rightarrow +)$

⑦ $(\mathcal{Q} \vee \mathcal{P}) \rightarrow \mathcal{X} \vdash (\mathcal{Q} \rightarrow \mathcal{X}) \wedge (\mathcal{P} \rightarrow \mathcal{X}) (\wedge +)$

(1) ① $(\mathcal{Q} \rightarrow \mathcal{X}) \wedge (\mathcal{P} \rightarrow \mathcal{X}) \vdash \mathcal{Q} \rightarrow \mathcal{X}, \mathcal{P} \rightarrow \mathcal{X} (e)$

② $(\mathcal{Q} \rightarrow \mathcal{X}) \wedge (\mathcal{P} \rightarrow \mathcal{X}), \mathcal{Q} \vdash \mathcal{X} (e) (+) (\rightarrow -)$

③ $(\mathcal{Q} \rightarrow \mathcal{X}) \wedge (\mathcal{P} \rightarrow \mathcal{X}), \mathcal{P} \vdash \mathcal{X} (e) (+) (\rightarrow -)$

④ $(\mathcal{Q} \rightarrow \mathcal{X}) \wedge (\mathcal{P} \rightarrow \mathcal{X}), \mathcal{Q} \vee \mathcal{P} \vdash \mathcal{X} (\vee -)$

⑤ $(\mathcal{Q} \rightarrow \mathcal{X}) \wedge (\mathcal{P} \rightarrow \mathcal{X}) \vdash (\mathcal{Q} \vee \mathcal{P}) \rightarrow \mathcal{X} (\rightarrow +)$

12. (4)

(H) ③ $\alpha \rightarrow \beta \vdash (\alpha \rightarrow \beta) \wedge (\beta \rightarrow \alpha)$ (E₁)

④ $\neg(\alpha \rightarrow \beta) \wedge (\beta \rightarrow \alpha) \vdash \neg(\alpha \rightarrow \beta) \vee \neg(\beta \rightarrow \alpha)$ (E₁)

⑤ $\alpha \rightarrow \beta \vdash \neg \neg \alpha \vee \beta$ (E₁)

⑥ $\neg(\neg \alpha \vee \beta) \vdash \neg \alpha \wedge \neg \beta$ (E₁)

⑦ $\neg(\beta \rightarrow \alpha) \vdash \neg \alpha \wedge \beta$ (E₁)

⑧ $\neg(\alpha \rightarrow \beta) \vdash (\neg \alpha \wedge \beta) \vee (\neg \alpha \wedge \neg \beta)$ (Tr)

⑨ $\neg \alpha \wedge \beta, \neg \alpha \vdash \beta$ (A₁) (E₁)

⑩ $\neg \alpha \wedge \beta, \beta \vdash \beta$ (A₁) (E₁)

⑪ $\beta, \neg \beta \vdash \neg \alpha$ (E₁)

⑫ $\neg \alpha \wedge \beta, \beta \vdash \neg \alpha$ (Tr)

⑬ $\neg \alpha \wedge \beta \vdash \neg \alpha \vee \beta$ (A₁)

⑭ $\neg \alpha \wedge \beta, \beta \vdash \neg \alpha$ (A₁)

⑮ $\neg \alpha \wedge \beta \vdash \neg \alpha \vee \beta$ (A₁)

⑯ $(\neg \alpha \wedge \beta) \vee (\neg \alpha \wedge \neg \beta) \vdash \neg \alpha \vee \beta$ (Tr)

⑰ $\neg(\alpha \rightarrow \beta) \vdash \neg \alpha \vee \beta$ (Tr)

(H) ① $\neg \alpha \vee \beta, \alpha \rightarrow \beta, \alpha \vdash \neg \alpha \vee \beta$ (A₁)

② $\neg \alpha \vee \beta, \alpha \rightarrow \beta, \alpha \vdash \neg \alpha$ (E₁)

③ $\neg \alpha \vee \beta, \alpha \rightarrow \beta \vdash \neg \alpha$ (E₁)

④ $\neg \alpha \vee \beta \vdash \neg(\alpha \rightarrow \beta)$ (A₁)

16) ① $\alpha \rightarrow \beta \vdash (\alpha \rightarrow \beta) \wedge (\beta \rightarrow \alpha)$ (E₁)

② $\alpha \rightarrow \beta \vdash \neg \neg \alpha \vee \beta$ (E₁)

③ $\beta \rightarrow \alpha \vdash \neg \neg \beta \vee \alpha$ (E₁)

④ $\alpha \rightarrow \beta \vdash (\neg \alpha \vee \beta) \wedge (\neg \beta \vee \alpha)$ (Tr)

(8)

(H) ③ E₁:

Lemma 1: $a, b \vdash a \leftrightarrow b$ Lemma 3: $\neg a, \neg b \vdash a \leftrightarrow b$

Lemma 2: $a \leftrightarrow b, \neg a \vdash \neg b$

Lemma 4: $a \leftrightarrow b, \neg a \vdash \neg b$

① $\alpha \leftrightarrow \beta \leftrightarrow \gamma, \alpha, \beta \vdash \alpha \leftrightarrow \beta, \alpha \leftrightarrow \beta \leftrightarrow \gamma$ (E₁) (Lemma 1)

② $\alpha \leftrightarrow \beta \leftrightarrow \gamma, \alpha, \beta \vdash \gamma$ (A₁) (Tr)

③ $\alpha \leftrightarrow \beta \leftrightarrow \gamma, \alpha, \beta \vdash \beta \leftrightarrow \gamma$ (E₁) (Lemma 1)

④ $\alpha \leftrightarrow \beta \leftrightarrow \gamma, \alpha, \beta \vdash \alpha \leftrightarrow (\beta \leftrightarrow \gamma)$ (E₁) (Lemma 1)

⑤ $\alpha \leftrightarrow \beta \leftrightarrow \gamma, \alpha, \neg \beta \vdash \neg \alpha$ (E₁) (A₁)

⑥ $\alpha \leftrightarrow \beta \leftrightarrow \gamma, \alpha, \neg \beta \vdash \neg(\alpha \leftrightarrow \beta)$ (E₁) (Tr)

⑦ $\alpha \leftrightarrow \beta \leftrightarrow \gamma, \alpha, \neg \beta \vdash \neg \gamma$ (Lemma 2) (Tr)

⑧ $\alpha \leftrightarrow \beta \leftrightarrow \gamma, \alpha, \neg \beta \vdash \beta \leftrightarrow \gamma$ (Lemma 3) (Tr)

⑨ $\alpha \leftrightarrow \beta \leftrightarrow \gamma, \alpha, \neg \beta \vdash \alpha \leftrightarrow (\beta \leftrightarrow \gamma)$ (Lemma 1) (Tr)

⑩ $\alpha \leftrightarrow \beta \leftrightarrow \gamma, \alpha \vdash \alpha \leftrightarrow (\beta \leftrightarrow \gamma)$ (E₁)

⑪ $\alpha \leftrightarrow \beta \leftrightarrow \gamma, \alpha \vdash \alpha \leftrightarrow (\beta \leftrightarrow \gamma)$ (E₁)

⑫ $\alpha \leftrightarrow \beta \leftrightarrow \gamma \vdash \alpha \leftrightarrow (\beta \leftrightarrow \gamma)$ (E₁)

(H) Lemma 4: $a \leftrightarrow b \vdash b \leftrightarrow a$

Lemma 4: $a \leftrightarrow b \vdash b \leftrightarrow a$

① $\alpha \leftrightarrow (\beta \leftrightarrow \gamma) \vdash (\beta \leftrightarrow \gamma) \leftrightarrow \alpha$ (Lemma 4)

② $\alpha \leftrightarrow (\beta \leftrightarrow \gamma) \vdash \gamma \leftrightarrow (\beta \leftrightarrow \alpha)$ (Tr)

③ $\alpha \leftrightarrow (\beta \leftrightarrow \gamma) \vdash (\alpha \leftrightarrow \beta) \leftrightarrow \gamma$ (Lemma 4) (Tr)

13. (2)

lemma: $a \vee b, \neg a \vdash b$

i.e: ① $a \vee b, \neg a, \neg b \vdash a \vee b, \neg a \wedge \neg b (E) (1+)$

② $\neg a \wedge \neg b \vdash \neg (a \vee b) (E i.e)$

③ $a \vee b, \neg a, \neg b \vdash a \vee b, \neg (a \vee b) (Tr)$

④ $a \vee b, \neg a \vdash b$

⑤ $\neg \neg \neg \beta, \neg (\neg \neg \neg \beta), \gamma \vdash \neg \neg \neg \beta, \neg \neg \neg \gamma (E i.e) (E)$

⑥ $\neg \neg \neg \beta, \neg (\neg \neg \neg \beta), \gamma \vdash \neg \neg \neg \beta, \neg \beta (lemma)$

⑦ $\neg \neg \neg \beta, \neg (\neg \neg \neg \beta), \gamma \vdash \neg \neg (lemma)$

(7) ① $\neg \neg \neg \beta \vdash \neg \neg \neg \beta (E i.e)$

② $\neg \neg \neg \beta \vdash \neg \neg \neg \beta (\rightarrow +)$

③ $(\neg \neg \neg \beta) \rightarrow \neg \neg \neg \beta, \neg \neg \neg \beta \vdash \neg \neg \neg \beta (\rightarrow -) (E)$

④ $(\neg \neg \neg \beta) \rightarrow \neg \neg \neg \beta \vdash \neg \neg \neg \beta (\neg -)$

(10) ① $\neg (\neg \neg \neg \beta) \vdash \neg \neg \neg \beta (E i.e)$

② $\neg (\neg \neg \neg \beta), \neg \neg \neg \beta \vdash \neg \neg \neg \beta (+)$

③ $\neg (\neg \neg \neg \beta) \vdash \neg \neg \neg \beta (\rightarrow +)$

(12) ① $\neg \neg \neg \beta, \neg \neg \neg \gamma \vdash \neg \neg \neg \beta (E) (E i.e)$

② $\neg \neg \neg \beta, \neg \neg \neg \gamma \vdash \neg \neg \neg \gamma (13.6) (lemma) (Tr)$

③ $\neg \neg \neg \beta, \neg \neg \neg \gamma \vdash \neg \neg \neg \gamma (\neg +)$

(14) ① $\neg \neg \neg \beta, \neg \neg \neg \gamma \vdash \neg \neg \neg \gamma (E) (\rightarrow -)$

② $\neg \neg \neg \beta, \neg \neg \neg \gamma \vdash \neg \neg \neg \gamma \vee \neg \neg \neg \gamma (V+) (Tr)$

③ $\neg \neg \neg \beta, \neg \neg \neg \gamma \vdash \neg \neg \neg \gamma (E) (\rightarrow -)$

④ $\neg \neg \neg \beta, \neg \neg \neg \gamma \vdash \neg \neg \neg \gamma \vee \neg \neg \neg \gamma (V+) (Tr)$

⑤ $\neg \neg \neg \beta, \neg \neg \neg \gamma \vdash \neg \neg \neg \gamma \vee \neg \neg \neg \gamma (V-)$