23.3.14

习题十六

11.

$*'$	$[a]$	$[b]$
$[a]$	$[a]$	$[b]$
$[b]$	$[b]$	$[a]$

12. (1) 先证明R是等价关系

自反性、对称性显然，只证传递性

设 xRy, yRz，即

$x=y \vee (x\in I \wedge y\in I)$

$y=z \vee (y\in I \wedge z\in I)$

欲证 xRz，即 $x=z \vee (x\in I \wedge z\in I)$

① $x=y, y=z \Rightarrow x=z$

② $x=y, y\in I, z\in I \Rightarrow x\in I, z\in I$

③ $y=z, x\in I, y\in I \Rightarrow x\in I, z\in I$

④ $x\in I, y\in I, z\in I \Rightarrow x\in I, z\in I$

传递性得证

再证R是同余关系，即证

$x_1Rx_2, y_1Ry_2 \Rightarrow (x_1y_1)R(x_2y_2)$

即证 $x_1=x_2 \vee (x_1\in I \wedge x_2\in I)$

$y_1=y_2 \vee (y_1\in I \wedge y_2\in I)$

$\Rightarrow x_1y_1=x_2y_2 \vee (x_1y_1\in I \wedge x_2y_2\in I)$

① $x_1=x_2, y_1=y_2 \Rightarrow x_1y_1=x_2y_2$

② $x_1=x_2, y_1\in I, y_2\in I \Rightarrow x_1y_1\in I, x_2y_2\in I$

③ $y_1=y_2, x_1\in I, x_2\in I \Rightarrow x_1y_1\in I, x_2y_2\in I$

④ $x_1\in I, x_2\in I, y_1\in I, y_2\in I \Rightarrow x_1y_1\in I, x_2y_2\in I$

(2) $S/R=\{I\}\cup\{\{x\} \mid x\in S-I\}$

$\forall x, y\notin I$,

$[x][y]=[xy]$,

$[x]I=I[x]=I$,

$II=I$

习题十七

2. 封闭：显然

结合性：$(a\circ b)\circ c = au^{-1}bu^{-1}c = a\circ(b\circ c)$

单位元：$a\circ u = u\circ a = a$，故u为单位元

逆元：$a\circ(ua^{-1}u) = (ua^{-1}u)\circ a = u$，故$ua^{-1}u$为$a$的逆元

6. $(ab)^2 = a^2b^2$

$\Rightarrow abab = a^2b^2$

$\Rightarrow a^{-1}ababb^{-1} = a^{-1}a^2b^2b^{-1}$

$\Rightarrow ba = ab$

9. (2) 设$(ab)^k = e$

则 $a^{-1}(ab)^k a = e$

即 $(ba)^k = e$

同理 $(ba)^k = e \Rightarrow (ab)^k = e$

故必有$|ab| = |ba|$

(3) 设$(abc)^k = e$

则 $a^{-1}(abc)a = e$

即$(bca)^k = e$

同理易得$(abc)^k = e$

$\Leftrightarrow (bca)^k = e$

$\Leftrightarrow (cab)^k = e$

故必有$|abc| = |bca| = |cab|$

11. 反之，若$\forall a\neq b$，$a,b\neq e$，都有$ab\neq ba$

取$b = a^{-1}$，即知$\forall a\neq e$，$a = a^{-1}$，即$a^2 = e$

由例题知这说明G是交换群，矛盾