22.12.9

22. (1) $\sigma(x_i)=1$

(3) $\sigma(x_i)=0$

(4) $\sigma(x_i)=i-2$

(5) $\sigma(x_i)=-i$

23. (1) 真

(7) 假

24. (1) 由可靠性定理，只需证明

$\vdash_{K_2} \exists x_1 \forall x_2 F(x_1,x_2) \to \forall x_2 \forall x_1 \exists x_1 F(x_1,x_2)$.

由演绎定理及 $N_2$、$K_2$ 等价性，只需证明

$\exists x \forall x_2 F(x_1,x_2) \vdash_{N_2} \forall x_2 \forall x_1 \exists x_1 F(x_1,x_2)$.

① $\forall x_2 F(x_1,x_2) \vdash F(x_1,x_2)$ $(\forall -)$

② $\forall x_2 F(x_1,x_2) \vdash \exists x_1 F(x_1,x_2)$ $(\exists +)$ ①

③ $\exists x_1 \forall x_2 F(x_1,x_2) \vdash \exists x_1 F(x_1,x_2)$ $(\exists -)$ ②

④ $\exists x_1 \forall x_2 F(x_1,x_2) \vdash \forall x_1 \exists x_1 F(x_1,x_2)$ $(\forall +)$ ③

⑤ $\exists x_1 \forall x_2 F(x_1,x_2) \vdash \forall x_2 \forall x_1 \exists x_1 F(x_1,x_2)$ $(\forall +)$ ④

(2) 同(1)，只需证明 $\forall x_1 F_2(x_1), \forall x_1 F_3(x_2) \vdash_{N_2^=} \forall x_2 F_2(x_2)$.

① $\forall x_1 F_2(x_1) \vdash \forall x_2 F_2(x_2)$ (换名)

② $\forall x_1 F_2(x_1), \forall x_1 F_3(x_2) \vdash \forall x_2 F_2(x_2)$ $(+)$ ①

25. 我们在 $N_2$ 中已证明过这两个公式是内定理，由 $N_2$、$K_2$ 的等价性及 $K_2$ 的可靠性，立得它们都是永真式.