23.2.24

习题一

4. $P(AB)=P(A)+P(B)-P(A\cup B)$

$=p+q-r$

$P(A\bar{B})=P(A\cup B)-P(B)$

$=r-q$

$P(\bar{A}\bar{B})=1-P(A\cup B)$

$=1-r$

7. $P(\cdots)=\dfrac{A_7^5}{7^5}$

≈ 0.1499

9. $P(\cdots)=\dfrac{C_4^1 C_5^1}{C_{10}^3}=\dfrac{1}{6}$

12. 将52张牌分别标号为1, 2, …, 52

$\Omega=\{(x_1,x_2,x_3,x_4,x_5,x_6)\mid$

$1\leq x_i\leq 52,\ \forall i\in\{1,2,3,4,5,6\}$

且 $x_i\neq x_j,\ \forall i,j\in\{1,2,3,4,5,6\},\ i\neq j\}$

(1) $P(\cdots)=\dfrac{C_{51}^5}{C_{52}^6}=\dfrac{3}{26}$

(2) $P(\cdots)=\dfrac{1}{C_{52}^6}(C_4^1C_{13}^3C_{13}^1C_{13}^1C_{13}^1+C_4^2C_{13}^2C_{13}^2C_{13}^1C_{13}^1)$

≈ 0.426

(3) $P(\cdots)=1-\dfrac{C_{13}^6\cdot 4^6}{C_{52}^6}$

≈ 0.655

20. $P(\text{最大号码等于}m)=\dfrac{C_{m-1}^{k-1}}{C_n^k}$

$P(\text{号码均不超过}m)=\dfrac{C_m^k}{C_n^k}$

26. $P(\text{丙中}|\text{两中})=\dfrac{P(\text{甲丙中})+P(\text{乙丙中})}{P(\text{两中})}$

$=\dfrac{0.2}{P(\text{两中})}$

$P(\text{丙不中}|\text{两中})=\dfrac{P(\text{甲乙中})}{P(\text{两中})}$

$=\dfrac{0.12}{P(\text{两中})}$

综上，丙中靶可能性更大

35. $P(\cdots)=\dfrac{C_5^2}{C_6^3}=\dfrac{1}{2}$

23.3.3

习题一

31. $P(\text{甲胜}) = \sum_{i=0}^{n-1} P(\text{甲胜且共输}i\text{次})$

$$= p\sum_{i=0}^{n-1} C_{m+i-1}^{i}\, p^{m-1}(1-p)^{i}$$

$$= p^{m}\sum_{i=0}^{n-1} C_{m+i-1}^{i}(1-p)^{i}$$

32. $P(\text{先摸者胜}) = \sum_{i=1}^{N} P(\text{先摸者在第}i\text{次摸中首得白球})$

$$= \sum_{i=1}^{N} \frac{n(n-1)}{(m+n)(m+n-1)} \cdot \frac{(n-2)(n-3)}{(m+n-2)(m+n-3)} \cdots \frac{(n-2i+4)(n-2i+3)}{(m+n-2i+4)(m+n-2i+3)} \cdot \frac{m}{m+n-2i+2}$$

$$= \sum_{i=1}^{N} \frac{(m+n-2i+1)!}{(m+n)!} \cdot \frac{n!}{(n-2i+2)!} \cdot m$$

$$= \frac{1}{C_{m+n}^{m}} \sum_{i=1}^{N} \frac{(m+n-2i+1)!}{(n-2i+2)!(m-1)!}$$

$$= \frac{1}{C_{m+n}^{m}} \sum_{i=1}^{N} C_{m+n-2i+1}^{m-1}$$

其中 $N = \lceil \frac{n}{2} \rceil$

34. $P(\text{在抽屉}8 \mid \text{不在抽屉}1\text{-}7) = \dfrac{P(\text{在抽屉}8\text{且不在}1\text{-}7)}{P(\text{不在抽屉}1\text{-}7)}$

$$= \frac{p}{8} \Big/ \left(1 - \frac{7}{8}p\right)$$

$$= \frac{p}{8-7p}$$

38. $P(m\text{条下一代}) = \sum_{i=m}^{\infty} P(\text{产}i\text{卵且其中孵化}m\text{个})$

$$= \sum_{i=m}^{\infty} \frac{\lambda^i}{i!} e^{-\lambda} \cdot C_i^m p^m (1-p)^{i-m}$$

$$= \sum_{i=m}^{\infty} \frac{\lambda^i e^{-\lambda}}{m!(i-m)!} p^m (1-p)^{i-m}$$

$$= \sum_{i=0}^{\infty} \frac{\lambda^{i+m} e^{-\lambda}}{m!\, i!} p^m (1-p)^i$$

$$= \frac{(\lambda p)^m}{m!} e^{-\lambda} \sum_{i=0}^{\infty} \frac{[\lambda(1-p)]^i}{i!}$$

$$= \frac{(\lambda p)^m}{m!} e^{-\lambda} \cdot e^{\lambda(1-p)}$$

$$= \frac{(\lambda p)^m}{m!} e^{-\lambda p}$$

42. 先证 AB，C 独立

$$P(ABC) = P(C)P(AB|C)$$

$$= P(C)P(B|C)P(A|BC)$$

$$= P(C)P(B)P(A)$$

$$= P(C)P(AB)$$

再证 $A\cup B$，C 独立

$$P((A\cup B)C) = P(C)P(A\cup B|C)$$

$$= P(C)[P(A|C)+P(B|C)-P(AB|C)]$$

$$= P(C)[P(A)+P(B)-P(AB)]$$

$$= P(C)P(A\cup B)$$

再证 $A-B$，C 独立

$$P((A-B)C) = P(A\bar{B}C)$$

$$= P(A)P(\bar{B})P(C)$$

$$= P(A\bar{B})P(C)$$

$$= P(A-B)P(C)$$

44. P(A在B前发生) $= P$(首次掷出2,4前未掷过3,6)

$$= \sum_{i=1}^{\infty} P(\text{首次掷出2,4前未掷过3,6 且首次掷出2,4在第}i\text{次})$$

$$= \sum_{i=1}^{\infty} \left(\frac{1}{3}\right)^{i-1} \frac{1}{3}$$

$$= \frac{1}{2}$$

46. $P(A) = P$(死亡至少6人)

$$= 1 - P(\text{死亡至多5人})$$

$$= 1 - \sum_{i=0}^{5} P(\text{死亡}i\text{人})$$

$$= 1 - \sum_{i=0}^{5} C_{2500}^{i} (10^{-4})^{i} (1-10^{-4})^{2500-i}$$

$$\approx 3 \times 10^{-7}$$

$P(B) = P$(死亡至多2人)

$$= \sum_{i=0}^{2} P(\text{死亡}i\text{人})$$

$$= \sum_{i=0}^{2} C_{2500}^{i} (10^{-4})^{i} (1-10^{-4})^{2500-i}$$

$$\approx 0.9978$$

23.3.10

习题二

2. $P(X=1)=P(X=2)$

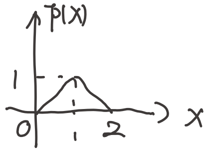
$\Rightarrow \lambda e^{-\lambda} = \frac{1}{2}\lambda^2 e^{-\lambda}$

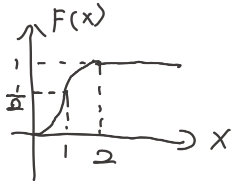
$\Rightarrow \lambda = 2$

$P(X=4) = \frac{1}{24}\lambda^4 e^{-\lambda}$

$= \frac{2}{3e^2}$

11.





$$F(x)=\int_{-\infty}^{x} p(t)\,dt = \begin{cases} 0, & x<0, \\ \frac{1}{2}x^2, & 0\le x<1, \\ -\frac{1}{2}x^2+2x-1, & 1\le x<2, \\ 1, & x\ge 2. \end{cases}$$

5. 由 $\{a\} = \bigcap_{n=1}^{\infty}(a-\frac{1}{n}, a] \triangleq \bigcap_{n=1}^{\infty} A_n$,

有 $P(X=a) = \lim_{n\to\infty} P(A_n)$

$= \lim_{n\to\infty}(F(a)-F(a-\frac{1}{n}))$

$= F(a)-F(a-0)$

由 $(a,b) = \bigcup_{n=1}^{\infty}(a, b-\frac{1}{n}] \triangleq \bigcup_{n=1}^{\infty} B_n$,

有 $P(a<X<b) = \lim_{n\to\infty} P(B_n)$

$= \lim_{n\to\infty}(F(b-\frac{1}{n})-F(a))$

$= F(b-0)-F(a)$

12. $P(120\le X\le 200)$

$= P(\mu-\frac{40}{\sigma} < X < \mu+\frac{40}{\sigma})$

$= 2\Phi(\frac{40}{\sigma})-1$

≥ 0.8

$\Rightarrow \Phi(\frac{40}{\sigma}) \ge 0.9$

查表知其近似解为

$\frac{40}{\sigma} \ge 1.28$

即 $\sigma \le 31.25$

21. 由于 X 服从 $[0,5]$ 的均匀分布，

有 $F_X(1)=\frac{1}{5}$, $F_X(3)=\frac{3}{5}$,

从而 $F_Y(x)=\begin{cases} 0, & x<0, \\ \frac{1}{5}, & 0\le x\le 1, \\ \frac{1}{5}x, & 1<x<3, \\ \frac{3}{5}, & 3\le x<5, \\ 1, & x\ge 5. \end{cases}$

23. $x>0$时，

$$p(x)=F'(x)=e^{-\frac{x^{\alpha}}{\eta^{\alpha}}}\cdot\frac{\alpha x^{\alpha-1}}{\eta^{\alpha}}$$

从而 $q(y)=p(x)\cdot e^{y}$

$$=\frac{\alpha}{\eta^{\alpha}}e^{\alpha y}\cdot e^{-\frac{e^{\alpha y}}{\eta^{\alpha}}}$$

$$=\frac{\alpha}{\eta^{\alpha}}e^{\alpha y-\frac{1}{\eta^{\alpha}}e^{\alpha y}}$$

其中$q(y)$为Y的分布密度，$y\in\mathbb{R}$

25. $q(y)=p(x)\cdot\left|\left(\frac{y-b}{a}\right)'\right|$

$$=\frac{1}{|a|}p\left(\frac{y-b}{a}\right)$$

23.3.17

习题二

7. $1=\int_{-\infty}^{+\infty}p(x)dx$

$=\int_0^{+\infty}Axe^{-\frac{x^2}{2\sigma^2}}dx$

$=\sigma^2A\int_0^{+\infty}e^{-\frac{x^2}{2\sigma^2}}d\frac{x^2}{2\sigma^2}$

$=\sigma^2A\int_0^{+\infty}e^{-t}dt$

$=\sigma^2A$

$\Rightarrow A=\frac{1}{\sigma^2}$

$E(X)=\int_{-\infty}^{+\infty}xp(x)dx$

$=\frac{1}{\sigma^2}\int_0^{+\infty}x^2e^{-\frac{x^2}{2\sigma^2}}dx$

由 $\int_0^{+\infty}x^2e^{-\frac{a}{2}x^2}dx=\frac{1}{2a}\sqrt{\frac{2\pi}{a}}$ 知

$E(X)=\frac{\sqrt{2\pi}}{2}\sigma$

$E(X^2)=\frac{1}{\sigma^2}\int_0^{+\infty}x^3e^{-\frac{x^2}{2\sigma^2}}dx$

由 $\int_0^{+\infty}x^3e^{-ax^2}dx=\frac{1}{2a^2}$ 知

$E(X^2)=2\sigma^2$

故 $var(X)=E(X^2)-E^2(X)$

$=2\sigma^2-\frac{1}{2}\pi\sigma^2$

$=\frac{4-\pi}{2}\sigma^2$

$P(X>EX)=\frac{1}{\sigma^2}\int_{\sqrt{\frac{\pi}{2}}\sigma}^{+\infty}xe^{-\frac{x^2}{2\sigma^2}}dx$

$=\int_{\frac{\pi}{4}}^{+\infty}e^{-t}dt$

$=e^{-\frac{\pi}{4}}$

13. 由 $p(x)$ 是偶函数，立知 $E(X)=0$

16. $p(x)=F'(x)=\begin{cases}\frac{1}{2}e^x, & x<0,\\ 0, & 0\le x<1,\\ \frac{1}{4}e^{-\frac{x-1}{2}}, & x\ge 1\end{cases}$

$E(X)=\int_{-\infty}^{+\infty}xp(x)dx$

$=\int_{-\infty}^{0}\frac{1}{2}xe^xdx+\int_1^{+\infty}\frac{x}{4}e^{-\frac{x-1}{2}}dx$

$=-\frac{1}{2}+\frac{3}{2}$

$=1$

24. X：停止时经历的局数

$$P(X=2n-1)=0$$

$$P(X=2n)=[2p(1-p)]^{n-1}[1-2p(1-p)]$$

$$=[2p(1-p)]^{n-1}-[2p(1-p)]^{n}$$

$n\in N^*$

$$E(X)=\sum_{n=1}^{\infty}(2n-1)P(X=2n-1)+\sum_{n=1}^{\infty}2nP(X=2n)$$

$$=2\sum_{n=1}^{\infty}\left(n[2p(1-p)]^{n-1}-n[2p(1-p)]^{n}\right)$$

令 $t=2p(1-p)$

$$E(X)=2\sum_{n=1}^{\infty}(nt^{n-1}-nt^{n})$$

$$=2\sum_{n=1}^{\infty}(t^{n-1}+(n-1)t^{n-1}-nt^{n})$$

$$=2\sum_{n=1}^{\infty}t^{n-1}-2\lim_{n\to\infty}nt^{n}$$

$$=2\cdot\frac{1}{1-t}$$

$$=\frac{2}{1-2p(1-p)}$$

$$=\frac{2}{p^2+q^2}$$

30. $$E(d)=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{6}{\pi}\cdot\frac{v^2}{g}\sin2\theta\,d\theta$$

$$=\frac{6v^2}{\pi g}\left(-\frac{1}{2}\cos2\theta\right)\Big|_{\frac{\pi}{6}}^{\frac{\pi}{3}}$$

$$=-\frac{3v^2}{\pi g}\left(-\frac{1}{2}-\frac{1}{2}\right)$$

$$=\frac{3v^2}{\pi g}$$

$$E(d^2)=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{6}{\pi}\cdot\frac{v^4}{g^2}\sin^2 2\theta\,d\theta$$

$$=\frac{3v^4}{\pi g^2}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}(1-\cos4\theta)\,d\theta$$

$$=\frac{3v^4}{\pi g^2}\left(\theta-\frac{1}{4}\sin4\theta\right)\Big|_{\frac{\pi}{6}}^{\frac{\pi}{3}}$$

$$=\frac{3v^4}{\pi g^2}\left(\frac{\pi}{6}+\frac{\sqrt3}{8}+\frac{\sqrt3}{8}\right)$$

$$=\frac{v^4}{2g^2}+\frac{3\sqrt3v^4}{4\pi g^2}$$

$$\mathrm{var}(d)=E(d^2)-E^2(d)$$

$$=\frac{v^4}{2g^2}+\frac{3\sqrt3v^4}{4\pi g^2}-\frac{9v^4}{\pi^2g^2}$$

$$=\frac{v^4}{g^2}\left(\frac{1}{2}+\frac{3\sqrt3}{4\pi}-\frac{9}{\pi^2}\right)$$

33. 由期望的单调性，立刻有

$$a\le E(X)\le b$$

注意到 $(X-a)(X-b)\le0$

故 $E((X-a)(X-b))\le0$

$$\Rightarrow E(X^2-(a+b)X+ab)\le0$$

$$\Rightarrow E(X^2)\le(a+b)E(X)-ab$$

$$\mathrm{var}(X)=E(X^2)-E^2(X)$$

$$\le -E^2(X)+(a+b)E(X)-ab$$

$$\le\frac{1}{4}(b-a)^2$$

23.3.24

习题三

1. X的分布: $P(X=0)=\frac{1}{4}$

$P(X=1)=\frac{1}{2}$

$P(X=2)=\frac{1}{4}$

Y的分布: $P(Y=-1)=\frac{1}{4}$

$P(Y=0)=\frac{1}{2}$

$P(Y=1)=\frac{1}{4}$

不独立, $P(Y=0)\neq P(Y=0|X=1)$

3.(1) $1=\iint_{x^2+y^2\leq R^2} c(R-\sqrt{x^2+y^2})\,dx\,dy$

$=\int_0^R\int_0^{2\pi} c(R-r)r\,d\theta\,dr$

$=2\pi c\int_0^R (Rr-r^2)\,dr$

$=\frac{\pi}{3}R^3c$

$\Rightarrow c=\frac{3}{\pi R^3}$

(2) $P(X^2+Y^2\leq r^2)=\iint_{x^2+y^2\leq r^2} c(R-\sqrt{x^2+y^2})\,dx\,dy$

$=\int_0^r\int_0^{2\pi} c(R-s)s\,d\theta\,ds$

$=2\pi c(\frac{1}{2}Rr^2-\frac{1}{3}r^3)$

$=3(\frac{r}{R})^2-2(\frac{r}{R})^3$

7. $p_{X+Y}(t)=\int_{-\infty}^{+\infty}p_X(u)p_Y(t-u)\,du$

$=\int_0^1 p_Y(t-u)\,du$

$=\int_{t-1}^{t} p_Y(u)\,du$

$=\int_{\max\{0,t-1\}}^{\max\{0,t\}} e^{-u}\,du$

$=e^{-\max\{0,t-1\}}-e^{-\max\{0,t\}}$

$=\begin{cases}0, & t<0,\\ 1-e^{-t}, & 0\leq t\leq 1,\\ e^{-t}(e-1), & t>1\end{cases}$

11. 分布密度 $p(x,y)=2,\ 0<x<1,\ 0<y<x$

$$p_X(x)=\int_0^x 2\,dy=2x,\ 0<x<1$$

$$E(X)=\int_0^1 2x^2\,dx=\frac{2}{3}$$

$$E(X^2)=\int_0^1 2x^3\,dx=\frac{1}{2}$$

$$var(X)=E(X^2)-E^2(X)=\frac{1}{18}$$

$$p_Y(y)=\int_y^1 2\,dx=2-2y,\ 0<y<1$$

$$E(Y)=\int_0^1(2-2y)y\,dy=\frac{1}{3}$$

$$E(Y^2)=\int_0^1(2-2y)y^2\,dy=\frac{1}{6}$$

$$var(Y)=\frac{1}{18}$$

$$E(XY)=\int_0^1\int_0^x 2xy\,dy\,dx$$

$$=\int_0^1 x^3\,dx$$

$$=\frac{1}{4}$$

$$cov(X,Y)=E(XY)-E(X)E(Y)$$

$$=\frac{1}{36}$$

$$\rho_{X,Y}=\frac{cov(X,Y)}{\sqrt{var(X)var(Y)}}=\frac{1}{2}$$

13. $E(X_1)=\int_0^1 2x^2\,dx=\frac{2}{3}$

$$E(X_2)=\int_5^{+\infty}xe^{-(x-5)}\,dx=6$$

$$E(X_1X_2)=E(X_1)E(X_2)=4$$

16. $p_X(x)=\int_0^{+\infty}\int_0^{+\infty}e^{-(x+y+z)}\,dy\,dz$

$$=e^{-x},\ x>0$$

同理 $p_Y(y)=e^{-y},\ y>0$

$p_Z(z)=e^{-z},\ z>0$

由于 $p(x,y,z)=p_X(x)p_Y(y)p_Z(z)$，

故 X,Y,Z 相互独立

18. $P(\xi<t)=1-P(\xi>t)$

$$=1-\prod_{i=1}^{n}P(X_i>t)$$

$$=1-\prod_{i=1}^{n}(1-F(t))$$

$$=1-\prod_{i=1}^{n}e^{-(\frac{x}{\eta})^m}$$

$$=1-e^{-n(\frac{x}{\eta})^m}$$

$$=1-e^{-\left(\frac{x}{\eta(\frac{1}{n})^{\frac{1}{m}}}\right)^m}$$

故 ξ 也服从威布尔分布

21. 事实上，这正是作了一个极坐标变换，由课堂例题即知 U,V 独立

25. $E\left(\frac{x_1+\cdots+x_k}{x_1+\cdots+x_n}\right)=E\left(\sum_{i=1}^{k}\frac{x_i}{x_1+\cdots+x_n}\right)$

$$=\sum_{i=1}^{k}E\left(\frac{x_i}{x_1+\cdots+x_n}\right)$$

由对称性，$E\left(\frac{x_i}{x_1+\cdots+x_n}\right)$彼此相等，$\forall 1\le i\le n$

又 $\sum_{i=1}^{n}E\left(\frac{x_i}{x_1+\cdots+x_n}\right)=E\left(\sum_{i=1}^{n}\frac{x_i}{x_1+\cdots+x_n}\right)$

$$=E(1)$$

$$=1$$

故 $E\left(\frac{x_i}{x_1+\cdots+x_n}\right)=\frac{1}{n}$，$\forall 1\le i\le n$，

即有 $E\left(\frac{x_1+\cdots+x_k}{x_1+\cdots+x_n}\right)=\frac{k}{n}$

30. 由$p(x)$为偶函数之知 $E(X)=0$，$E(X|X|)=0$

故 $\mathrm{cov}(|X|,X)=E(X|X|)-E(X)E(|X|)=0$，

即$|X|$，X不相关

但显然$|X|$，X不独立

令 $W=\mathrm{var}\left(\sum_{i=1}^{n}a_ix_i\right)$

$$=E\left(\sum_{i=1}^{n}a_ix_i-E\left(\sum_{i=1}^{n}a_ix_i\right)\right)^2$$

$$=E\left(\sum_{i=1}^{n}a_i(X_i-E(X_i)\right)^2$$

$$=\sum_{i=1}^{n}a_i^2E(X_i-E(X_i))^2+2\sum_{i<j}a_ia_jE(X_i-E(X_i)(X_j-E(X_j))$$

$$=\sum_{i=1}^{n}a_i^2\sigma_i^2$$

若$\exists\sigma_m=0$，取$a_i=\delta_{im}$（Kronecker记号），即有$W=0$

若$\forall\sigma_i>0$，$\left(\sum_{i=1}^{n}a_i^2\sigma_i^2\right)\left(\sum_{i=1}^{n}\frac{1}{\sigma_i^2}\right)\ge\left(\sum_{i=1}^{n}a_i\right)^2=1$

故$W_{min}=1/\sum_{i=1}^{n}\frac{1}{\sigma_i^2}$，当且仅当$a_i=\frac{1}{\sigma_i^2}/\sum_{k=1}^{n}\frac{1}{\sigma_k^2}$时取等

41. $1=\int_0^2\int_0^1 cy^2\,dy\,dx$

$=\frac{2}{3}c$

$\Rightarrow c=\frac{3}{2}$

$P(X\le 1)=\int_0^1\int_0^1 \frac{3}{2}y^2\,dy\,dx$

$=\frac{1}{2}$

$P(X+Y>2)=\int_1^2 dx\int_{2-x}^1 \frac{3}{2}y^2\,dy$

$=\frac{1}{2}\int_1^2 (x^3-6x^2+12x-7)\,dx$

$=\frac{3}{8}$

$P(X=3Y)=0$

23.3.31

习题四

5. $E(X_n)=0$

$E(X_n^2)=n^{2\alpha}$

$var(X_n)=E(X_n^2)-E^2(X_n)=n^{2\alpha}$

$\sum_{n=1}^{\infty}\frac{var(X_n)}{n^2}=\sum_{n=1}^{\infty}\frac{1}{n^{2-2\alpha}}$ 收敛

由Kolmogorov强大数律知

$X_1,\cdots,X_n,\cdots$服从大数律

6. $\forall\varepsilon>0$, $P(\xi_n\leq a-\varepsilon)=\prod_{i=1}^{n}P(X_n\leq a-\varepsilon)$

$=(\frac{a-\varepsilon}{a})^n$

$\lim_{n\to\infty}P(|\xi_n-a|\geq\varepsilon)=\lim_{n\to\infty}(\frac{a-\varepsilon}{a})^n=0$

由定义知 $\xi_n\xrightarrow{P}a$

7. $E(X_i)=\int_1^{+\infty}\frac{2}{x^3}dx$ 收敛

由Kolmogorov强大数律,

$X_1,X_2,\cdots$服从大数律

8. $E(\rho_i)=P(f(\xi_i)\geq\eta_i)$

$=\int_0^1dx\int_0^{f(x)}dy$

$=\int_0^1f(x)dx$

由Kolmogorov强大数律,

$\frac{1}{n}\sum_{i=1}^{n}\rho_i\xrightarrow{a.s.}E(\rho_i)=\int_0^1f(x)dx$

23.4.7

12. 第i次的误差 $X_i \sim U(-\frac{1}{2},\frac{1}{2})$

令 $S_n=\sum_{i=1}^{n}X_i$

$E(S_n)=0$

$var(S_n)=n\,var(X_1)=\frac{n}{12}$

$S_n^*=\frac{S_n}{\sqrt{var(S_n)}}\xrightarrow{d}Z\sim N(0,1)$

(1) $P(|S_{1500}|\geq 15)$

$=1-P(-15\leq S_{1500}\leq 15)$

$=1-P(-\frac{3}{\sqrt{5}}\leq S_{1500}^*\leq\frac{3}{\sqrt{5}})$

$\approx 2-2\phi(\frac{3}{\sqrt{5}})$

≈ 0.177

(2) $P(|S_n|\leq 10)$

$=P(-10\leq S_n\leq 10)$

$=P(-\frac{10}{\sqrt{n/12}}\leq S_n^*\leq\frac{10}{\sqrt{n/12}})$

$\approx 2\phi(\frac{10}{\sqrt{n/12}})-1$

$=0.90$

$\Rightarrow\phi(\frac{10}{\sqrt{n/12}})=0.95$

查表知 $\frac{10}{\sqrt{n/12}}=1.65$

$\Rightarrow n\approx 441$

14. D_i的寿命 $X_i\sim Exp(0.1)$

总寿命 $S_{30}=\sum_{i=1}^{n}X_i$

$E(S_{30})=30E(X_1)=300$

$var(S_{30})=30\,var(X_1)=3000$

$S^*=\frac{S_{30}-300}{10\sqrt{30}}\xrightarrow{d}Z\sim N(0,1)$

$P(S_{30}\geq 350)=1-P(S_{30}\leq 350)$

$=1-P(S^*\leq 0.92)$

$\approx 1-\phi(0.92)$

≈ 0.18

15. 第i名选民的选票 $X_i\sim B(1,p)$

总赞成数 $S_n=\sum_{i=1}^{n}X_i$

$E(S_n)=np$

$var(S_n)=np(1-p)$

$S_n^*=\frac{S_n-np}{\sqrt{np(1-p)}}\xrightarrow{d}Z\sim N(0,1)$

$P(|\frac{S_n}{n}-p|\leq 0.045)$

$=P(|S_n^*|\leq\frac{0.045n}{\sqrt{np(1-p)}})$

$\approx 2\phi(\frac{0.045n}{\sqrt{np(1-p)}})-1$

$=0.95$

$\Rightarrow\phi(\frac{0.045n}{\sqrt{np(1-p)}})=0.975$

查表知 $\frac{0.045n}{\sqrt{np(1-p)}}=1.96$

$\Rightarrow n=[1897p(1-p)]_{max}\approx 475$

16. 总索赔费 $S=\sum_{k=1}^{4} S_k$

其中 $S_k=\sum_{i=1}^{n_k} X_i$，$\frac{X_i}{m_k} \sim B(1, q_k)$，$m_k$为索赔额

$$E(S)=E(S_1)+E(S_2)+E(S_3)+E(S_4)$$

$$=10+20+30+100$$

$$=160$$

$$var(S)=9.8+39.2+27+180$$

$$=256$$

$$S^*=\frac{S-160}{16} \overset{d}{\approx} Z \sim N(0,1)$$

总保费 $\pi=(1+\theta)160$

$$P(S \geq \pi)=P(S^* \geq 10\theta)$$

$$\approx 1-\phi(10\theta)$$

$$=0.05$$

$$\Rightarrow \phi(10\theta)=0.95$$

查表知 $10\theta=1.65$

$$\Rightarrow \theta=0.165$$

23.4.21

习题七

1. (1) $L(p)=p^n(1-p)^{(\sum_{i=1}^n x_i)-n}$

(2) $\hat{p}=\arg\max_p L(p)$

记 $a=\sum_{i=1}^n x_i-n$

$L(p)=p^n(1-p)^a$

$=\frac{1}{a^n n^a}(ap)^n(n-np)^a$

$\le \frac{1}{a^n n^a}\cdot\left(\frac{na}{n+a}\right)^{n+a}$

$=\frac{n^n a^a}{(n+a)^{n+a}}$

当且仅当 $ap=n-np$，即 $p=\frac{n}{a+n}$ 时

取等

故 $\hat{p}=n/\sum_{i=1}^n x_i$

2. (1) $L(\sigma)=\prod_{i=1}^n\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}$

$=\frac{1}{(2\pi)^{\frac{n}{2}}}\cdot\frac{1}{\sigma^n}\cdot e^{-\frac{\sum_{i=1}^n(x_i-\mu)^2}{2\sigma^2}}$

$\hat{\sigma}=\arg\max_\sigma L(\sigma)$

$=\arg\max_\sigma \frac{1}{\sigma^n}e^{-\frac{a}{2\sigma^2}}$ (其中 $a\triangleq\sum_{i=1}^n(x_i-\mu)^2$)

$=\arg\max_\sigma\left(-\frac{a}{2\sigma^2}-n\ln\sigma\right)$

$\triangleq\arg\max_\sigma f(\sigma)$

令 $f'(\sigma)=\frac{a}{\sigma^3}-\frac{n}{\sigma}=0$

$\Rightarrow\sigma^2=\frac{a}{n}$

可验证此时 $f(\sigma)$ 有最大值，故 $\hat{\sigma}^2=\frac{1}{n}\sum_{i=1}^n(x_i-\mu)^2$

(2) $L(\mu)=\frac{1}{(2\pi)^{\frac{n}{2}}}\cdot\frac{1}{\sigma^n}\cdot e^{-\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{2\sigma^2}}$

$\hat{\mu}=\underset{\mu}{\operatorname{argmax}}\, L(\mu)$

$=\underset{\mu}{\operatorname{argmin}}\sum_{i=1}^{n}(x_i-\mu)^2$

$\triangleq\underset{\mu}{\operatorname{argmin}}\, f(\mu)$

$f(\mu)=n\mu^2-(\sum_{i=1}^{n}x_i)\mu+\sum_{i=1}^{n}x_i^2$

易知$f(\mu)$的最小值点为$\frac{1}{n}\sum_{i=1}^{n}x_i$，

即$\hat{\mu}=\frac{1}{n}\sum_{i=1}^{n}x_i$

3. 记$I(x)=\begin{cases}1, & x\geq\theta,\\ 0, & x<\theta\end{cases}$

$L(\theta)=\prod_{i=1}^{n}I(x_i)e^{-(x_i-\theta)}$

易观察知，当$\theta=\min\{x_1,\cdots,x_n\}$时，

$L(\theta)$有最大值，故

$\hat{\theta}=\min\{x_1,\cdots,x_n\}$

4. (1) $p(1-p)$

(2) $L(p)=p^{\sum_{i=1}^{n}x_i}(1-p)^{n-\sum_{i=1}^{n}x_i}$

$=p^{n\bar{x}}(1-p)^{n-n\bar{x}}$

$=\frac{1}{(n\bar{x})^{n-n\bar{x}}(n-n\bar{x})^{n\bar{x}}}[(n-n\bar{x})p]^{n\bar{x}}[n\bar{x}(1-p)]^{n-n\bar{x}}$

$\leq\frac{(n\bar{x})^{n\bar{x}}(n-n\bar{x})^{n-n\bar{x}}}{n^n}$

当且仅当$(n-n\bar{x})p=n\bar{x}(1-p)$，即$p=\bar{x}$时取等

故$\hat{p}=\bar{x}$，$\widehat{var}(X_1)=\bar{x}(1-\bar{x})$

(3) $E(T(X_1,\cdots,X_n))=E(\overline{X}(1-\overline{X}))$

$$=\frac{1}{n^2}E\left(\sum_{i=1}^{n}X_i\right)\left(n-\sum_{i=1}^{n}X_i\right)$$

$$=\frac{1}{n^2}E\left(n\sum_{i=1}^{n}X_i\right)-\frac{1}{n^2}E\left(\sum_{i,j}X_iX_j\right)$$

$$=p-\frac{1}{n^2}\left(nE(X_1^2)+(n^2-n)E(X_1X_2)\right)$$

$$=p-\frac{1}{n^2}\left(np+(n^2-n)p^2\right)$$

$$=(p-p^2)\left(1-\frac{1}{n}\right)$$

5. (1) $var(\hat{p})=var\left(\frac{1}{n}\sum_{i=1}^{n}X_i\right)$

$$=\frac{1}{n^2}\sum_{i=1}^{n}var(X_i)$$

$$=\frac{p(1-p)}{n}$$

(2) $\widehat{var}(\hat{p})=\frac{\hat{p}(1-\hat{p})}{n}=\frac{s}{n^2}\left(1-\frac{s}{n}\right)$

12. 由前面题目的结果知

$\hat{\mu}=\min\{0,\frac{1}{n}\overline{x}\}$

$\hat{\sigma}^2=\min\{2,\frac{1}{n}\sum_{i=1}^{n}(x_i-\overline{x})^2\}$

故 $\widehat{\left(\frac{\sigma}{\mu}\right)}=\hat{\sigma}/\hat{\mu}$

将上式代入即可

23.4.28

习题七

1. (3) $E(X)=\frac{1}{p}=\overline{X}$

$\Rightarrow \hat{p}=\frac{1}{\overline{X}}$

6. (1) $E(X)=\int_{\theta}^{+\infty} x e^{-(x-\theta)} dx$

$=\int_{0}^{+\infty}(x+\theta)e^{-x}dx$

$=\theta+\int_{0}^{+\infty}xe^{-x}dx$

$=\theta+1$

$E(X)=\overline{X}\Rightarrow T_2=\overline{X}-1$

(2) $T_1=\min\limits_{i} X_i$

$t\geq\theta$时,

$F_{T_1}(t)=P(\min X_i\leq t)$

$=1-\prod_{i=1}^{n}P(X_i>t)$

$=1-\left(\int_{t}^{+\infty}e^{-(x-\theta)}dx\right)^n$

$=1-e^{-n(t-\theta)}$

故 $p_{T_1}(t)=\begin{cases}0, & t<\theta,\\ ne^{-n(t-\theta)}, & t\geq\theta\end{cases}$

$E(T_1-\theta)^2=\int_{\theta}^{+\infty}(t-\theta)^2ne^{-n(t-\theta)}dt$

$=\int_{0}^{+\infty}t^2ne^{-nt}dt$

$=\frac{2}{n^2}$

$E(T_2-\theta)^2=E(\overline{X}-(\theta+1))^2$

$=var(\overline{X})$

$=E(\overline{X}^2)-E^2(\overline{X})$

$=E(\overline{X}^2)-(\theta+1)^2$

$=\frac{1}{n^2}E(x_1+\cdots+x_n)^2-(\theta+1)^2$

$=\frac{1}{n^2}(nE(X_1^2)+n(n-1)E(X_1X_2))-(\theta+1)^2$

$E(X_1^2)=\int_{\theta}^{+\infty}x^2e^{-(x-\theta)}dx$

$=\int_{0}^{+\infty}(x^2+2\theta x+\theta^2)e^{-x}dx$

$=\theta^2+2\theta+2$

$E(X_1X_2)=E(X_1)E(X_2)=(\theta+1)^2$

$E(T_2-\theta)^2=\frac{1}{n^2}(n(\theta^2+2\theta+2)+n(n-1)(\theta+1)^2)-(\theta+1)^2$

$=\frac{1}{n}$

8. (1) $E(X)=\int_0^1 \theta x^{\theta} dx$

$$= \frac{\theta}{\theta+1}$$

$E(X)=\bar{X} \Rightarrow \hat{\theta} = \frac{\bar{X}}{1-\bar{X}}$

(2) 不妨设 $0\le x_i \le 1$，否则样本无意义

$L(\theta)=\prod_{i=1}^{n} \theta x_i^{\theta-1}$

$$= \theta^n \left(\prod_{i=1}^{n} x_i\right)^{\theta-1}$$

$\hat{\theta} = \underset{\theta}{\arg\max}\, L(\theta)$

$$= \underset{\theta}{\arg\max}\, \theta^n a^{\theta-1} \quad \left(a \triangleq \prod_{i=1}^{n} x_i \in [0,1]\right)$$

由于 $P(a=0)=0$，不妨设 $a\in(0,1]$

设 $f(\theta)=\theta^n a^{\theta-1}$

令 $f'(\theta)= n\theta^{n-1}a^{\theta-1}+\theta^n \cdot a^{\theta-1}\ln a=0$

$$\Rightarrow \quad n+\theta\ln a=0$$

$$\Rightarrow \quad \theta = -\frac{n}{\ln a}$$

易验证此时 $f(\theta)$ 有最大值

故 $\hat{\theta} = -\frac{n}{\ln a} = -\frac{n}{\sum_{i=1}^{n}\ln x_i}$

13. 记 $u_i=\frac{x_i-\mu_1}{\sigma_1}$, $v_i=\frac{y_i-\mu_2}{\sigma_2}$

表达式化为

$$-\frac{1}{2(1-\rho^2)}\sum_{i=1}^{n}(u_i^2+v_i^2-2\rho u_iv_i)$$

$$=-\frac{1}{2(1-\rho^2)}\left(\sum_{i=1}^{n}u_i^2+\sum_{i=1}^{n}v_i^2-2\rho\sum_{i=1}^{n}u_iv_i\right)\ (*)$$

$$\sum_{i=1}^{n}u_i^2=\frac{1}{\sigma_1^2}\sum_{i=1}^{n}(x_i-\bar{x}+\bar{x}-\mu_1)^2$$

$$=\frac{1}{\sigma_1^2}\Big(\sum_{i=1}^{n}(x_i-\bar{x})^2+n(\bar{x}-\mu_1)^2$$
$$+2(\bar{x}-\mu_1)\sum_{i=1}^{n}(x_i-\bar{x})\Big)$$

$$=\frac{1}{\sigma_1^2}\left(\sum_{i=1}^{n}(x_i-\bar{x})^2+n(\bar{x}-\mu_1)^2\right)$$

同理 $\sum_{i=1}^{n}v_i^2=\frac{1}{\sigma_2^2}\left(\sum_{i=1}^{n}(y_i-\bar{y})^2+n(\bar{y}-\mu_2)^2\right)$

$$\sum_{i=1}^{n}u_iv_i=\frac{1}{\sigma_1\sigma_2}\sum_{i=1}^{n}(x_i-\bar{x}+\bar{x}-\mu_1)(y_i-\bar{y}+\bar{y}-\mu_2)$$

$$=\frac{1}{\sigma_1\sigma_2}\left(\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})+n(\bar{x}-\mu_1)(\bar{y}-\mu_2)\right)$$

记 $\left(\bar{x},\bar{y},\sum_{i=1}^{n}(x_i-\bar{x})^2,\sum_{i=1}^{n}(y_i-\bar{y})^2,\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})\right)$

$=(a_1,a_2,a_3,a_4,a_5)$

代入 $(*)$ 得

$$-\frac{1}{2(1-\rho^2)}\left[\frac{a_3+n(a_1-\mu_1)^2}{\sigma_1^2}+\frac{a_4+n(a_2-\mu_2)^2}{\sigma_2^2}-2\rho\frac{a_5+n(a_1-\mu_1)(a_2-\mu_2)}{\sigma_1\sigma_2}\right]$$

16. 我们知道$\bar{X}$是完全充分统计量

由$E(\bar{X})=\mu$知$\bar{X}$是μ的UMVU估计

由$\mu^2=E^2(\bar{X})$

$$=E(\bar{X}^2)-var(\bar{X})$$

$$=E(\bar{X}^2)-\frac{1}{n^2}var(\sum_{i=1}^{n}X_i)$$

$$=E(\bar{X}^2)-\frac{1}{n^2}\sum_{i=1}^{n}var(X_i)$$

$$=E(\bar{X}^2)-\frac{1}{n}$$

$$=E(\bar{X}^2-\frac{1}{n})$$

知$\bar{X}^2-\frac{1}{n}$是μ^2的UMVU估计

$E(X)=\mu$

$E(X^2)=E^2(X)+var(X)=\mu^2+1$

$$E(\bar{X}^3)=\frac{1}{n^3}E((\sum_{i=1}^{n}X_i)^3)$$

$$=\frac{1}{n^3}[nE(X^3)+3n(n-1)E(X^2)E(X)+n(n-1)(n-2)E^3(X)]$$

$$=\frac{1}{n^2}E(X^3)+\frac{3(n-1)}{n^2}(\mu^3+\mu)+\frac{(n-1)(n-2)}{n^2}\mu^3$$

$$=\frac{1}{n^2}E(X^3)+(1-\frac{1}{n^2})\mu^3+\frac{3n-3}{n^2}\mu$$

$$E(X^3)=\int_{-\infty}^{+\infty}x^3\frac{1}{\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2}}dx=\mu^3+3\mu$$

故$E(\bar{X}^3)=\mu^3+\frac{3}{n}\mu=\mu^3+\frac{3}{n}E(\bar{X})$

$\Rightarrow \mu^3=E(\bar{X}^3-\frac{3}{n}\bar{X})$

即$\bar{X}^3-\frac{3}{n}\bar{X}$是$\mu^3$的UMVU估计

21. $p(x,\theta)=\theta e^{\ln(1-\theta)\cdot(x-1)}$

$$\prod_{i=1}^{n} p(x_i,\theta)=\theta^n e^{\ln(1-\theta)(n\bar{x}-n)}$$

由该形式即知 $(x_1,\cdots,x_n)$ 满足指数族分布，$\bar{x}$ 为完全充分统计量

23.5.12

习题七

25. $\hat{\lambda} = \frac{n-1}{n\bar{X}} = \frac{n-1}{X_1+\cdots+X_n} \triangleq \frac{n-1}{y}$

$y \sim \Gamma(n, \lambda)$

$p(y) = \frac{\lambda^n}{(n-1)!} y^{n-1} e^{-\lambda y}, \ y>0$

(1) $E(\frac{1}{y^2}) = \int_0^{+\infty} \frac{\lambda^n}{(n-1)!} y^{n-3} e^{-\lambda y} dy$

$= \frac{\lambda^2}{(n-2)(n-1)} \int_0^{+\infty} \frac{\lambda^{n-2}}{(n-3)!} y^{n-3} e^{-\lambda y} dy$

$= \frac{\lambda^2}{(n-2)(n-1)}$

$\mathrm{var}(\frac{n-1}{y}) = E(\frac{(n-1)^2}{y^2}) - E^2(\frac{n-1}{y})$

$= \frac{(n-1)^2\lambda^2}{(n-2)(n-1)} - \lambda^2$

$= \frac{\lambda^2}{n-2}$

(2) 由CLT. $\frac{\lambda}{\sqrt{n}}(y-\frac{n}{\lambda}) \xrightarrow{d} N(0,1)$

$\Rightarrow \sqrt{n}(\frac{y}{n}-\frac{1}{\lambda}) \xrightarrow{d} N(0,\frac{1}{\lambda^2})$

(Δ方法) $\Rightarrow \sqrt{n}(\frac{n}{y}-\lambda) \xrightarrow{d} N(0,\lambda^2)$

$\Rightarrow \sqrt{n}(\frac{n-1}{y}-\lambda) \xrightarrow{d} N(0,\lambda^2)$

26. $\frac{\hat{\sigma}^2}{S_n^2} = \frac{n-1}{n} \to 1$

故渐近分布相同

28. $\bar{X} = 12.25$

$S^2 = 13.12$

$$\frac{\sqrt{n}(\bar{X}-EX)}{S} \overset{d}{\approx} Z \sim N(0,1)$$

$P(Z \le z) = 0.95 \Rightarrow z = 1.65$

$\frac{\sqrt{n}(\bar{X}-EX)}{S} \le 1.65 \Rightarrow EX \ge 10.59$

29. $\bar{X} = 276.9$

$S^2 = 733.4$

记 $Z = \frac{\sqrt{n}(\bar{X}-EX)}{S} \sim t(n-1)$

$P(|Z| \le z) = 0.95 \Rightarrow z = 2.11$

$|Z| \le 2.11 \Rightarrow EX \in [263.8, 290.0]$

40. 治疗前减治疗后的血糖差

$X \sim N(\mu_1 - \mu_2, 2\sigma^2)$

$\bar{X} = 8$

$S^2 = 614.7$

记 $Z = \frac{\sqrt{n}(\bar{X}-EX)}{S} \sim t(n-1)$

$P(|Z| \le z) = 0.95 \Rightarrow z = 2.045$

$|Z| \le 2.045 \Rightarrow EX = \mu_1 - \mu_2 \in [-1.25, 17.25]$

23.5.26

习题八

3. $\lambda(x)=\frac{L_1(x)}{L_0(x)}=3x^2$

$W_\lambda=\{x\mid 3x^2>\lambda\}$

$P_0(x\in W_\lambda)=1-\sqrt{\frac{\lambda}{3}}=\alpha$

$\Rightarrow \lambda=3(1-\alpha)^2$

由N-P引理，$W=\{x\mid 1-\alpha<x\leq 1\}$

是水平为α的UMP否定域

6. 应采用(1)

假设检验的结果是强烈肯定H_1或微弱肯定H_0，此处我们希望强烈肯定"安全"

7. $H_0:\sigma^2\geq\sigma_0^2 \longleftrightarrow H_1:\sigma^2<\sigma_0^2$

$T(\vec{x})=\frac{n\hat{\sigma}^2}{\sigma_0^2}=\frac{6\times 0.007^2}{0.005^2}=11.76$

$W=\{\vec{x}\mid T(\vec{x})<c\}$

其中$P(\chi^2_{n-1}<c)=\alpha \Rightarrow c=1.64$

故应肯定H_0，即认为标准差偏大

$P(\chi^2_{n-1}<T(\vec{x}))=p \Rightarrow P(\chi^2_6<11.76)=p$

利用该公式可计算出p

8. 设甲乙疗效差为 $X \sim N(\mu, \sigma^2)$

$H_0: \mu = 0 \longleftrightarrow H_1: \mu \neq 0$

$\bar{x} = 1.43,\ S = 1.21$

$T(\vec{x}) = \dfrac{\sqrt{n}(\bar{x} - 0)}{S} = 3.74$

$W = \{\vec{x} \mid |T(\vec{x})| > c\}$

其中 $P(|t_{n-1}| > c) = \alpha \Rightarrow c = 2.26$

故应否定 H_0，即认为疗效有差异

10. (1) $p(x) = \dfrac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}\cdot(x-\mu)^2}$

$W = \{\vec{x} \mid \sum_{i=1}^{n}(x_i - \mu)^2 > \tilde{c}\}$

$= \{\vec{x} \mid \frac{1}{\sigma_0^2}\sum_{i=1}^{n}(x_i - \mu)^2 > c\}$

$P_{\sigma_0}(\vec{x} \in W) \Rightarrow c = \chi^2_{1-\alpha}(n)$

(2) $\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu_0)^2$

$\hat{\sigma}_0^2 = \begin{cases} \hat{\sigma}^2, & \hat{\sigma}^2 \geq \sigma_0^2, \\ \sigma_0^2, & \hat{\sigma}^2 < \sigma_0^2 \end{cases}$

令 $T(\vec{x}) = \dfrac{n\hat{\sigma}^2}{\sigma_0^2}$

$\lambda(\vec{x}) = \begin{cases} 1, & \hat{\sigma}^2 < \sigma_0^2, \\ \left(\frac{T(\vec{x})}{n}\right)^{-\frac{n}{2}} e^{\frac{1}{2}T(\vec{x}) - \frac{n}{2}}, & \hat{\sigma}^2 \geq \sigma_0^2 \end{cases}$

$W = \{\vec{x} \mid T(\vec{x}) > c\}$

$\forall \sigma^2 \leq \sigma_0^2,\ P(T(\vec{x}) > c)$

$= P\left(\frac{n\hat{\sigma}^2}{\sigma^2} > c\frac{\sigma_0^2}{\sigma^2}\right)$

$\leq P(\chi^2_n > c)$

$= \alpha$

$\Rightarrow c = \chi^2_{1-\alpha}(n)$

14. 近似地，纸张长度 $X \sim N(\mu, \sigma^2)$

$H_0: \mu = 155 \longleftrightarrow H_1: \mu \neq 155$

$\bar{x} = 0.13$, $S = 0.49$

$T(\vec{x}) = \frac{\sqrt{n}(\bar{x} - \mu)}{S} = 0.84$

$W = \{\vec{x} \mid |T(\vec{x})| > c\}$

其中 $P(|t_{n-1}| > c) = \alpha \Rightarrow c = 2.26$

故应肯定 H_0，即认为设计长度是 $155mm$

23.6.2

习题九

1. $\sum_{i=1}^{n}(y_i-\hat{y}_i)$

$=\sum_{i=1}^{n}(y_i-\hat{b}_0-\hat{b}x_i)$

$=n(\bar{y}-\hat{b}_0-\hat{b}\bar{x})$

$=0$

$\sum_{i=1}^{n}x_i(y_i-\hat{y}_i)$

$=\sum_{i=1}^{n}(x_iy_i-\hat{b}_0x_i-\hat{b}x_i^2)$

$=\sum_{i=1}^{n}x_iy_i-n\hat{b}_0\bar{x}-\hat{b}\sum_{i=1}^{n}x_i^2$

$=\sum_{i=1}^{n}x_iy_i-n(\bar{y}-\hat{b}\bar{x})\bar{x}-\hat{b}\sum_{i=1}^{n}x_i^2$

代入 $\hat{b}=(\sum_{i=1}^{n}x_iy_i-n\bar{x}\bar{y})/(\sum_{i=1}^{n}x_i^2-n\bar{x}^2)$

即得 $\sum_{i=1}^{n}x_i(y_i-\hat{y}_i)=0$

2. $y=\frac{L}{1+e^{ax+b}}$

$\Rightarrow \ln(\frac{L}{y}-1)=ax+b$

再用最小二乘估计即可