

22.11.18

作业

p.559(p.184)

14. (2),(4),(5)

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作业

p.559(p.184)

6. 若 y 对 x 在 α 中自由, 且 y 不在 α 中自由出现, 则
 $\forall x\alpha \vdash \forall y\alpha(x/y)$.

7.

9. 证明例12的(2),(4)

12. 证明: 若 $\Gamma, \alpha \vdash_{N_E} \beta$, 且 x 不在 Γ 的任何公式中自由出现, 则 $\Gamma, \exists x\alpha \vdash_{N_E} \exists x\beta$.

13. 证明: 若 $\Gamma, \alpha \vdash_{N_E} \beta$, 且 x 不在 $\Gamma \cup \{\beta\}$ 的任何公式中自由出现, 则 $\Gamma, \exists x\alpha \vdash_{N_E} \forall x\beta$.

14. (2),(4),(5)

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14. (2)

① $\forall x(\alpha \rightarrow \beta), \alpha \vdash \forall x\alpha \rightarrow \beta$

② $\forall x(\alpha \rightarrow \beta), \alpha \vdash \alpha \rightarrow \beta$ (V-) ①

③ $\forall x(\alpha \rightarrow \beta), \alpha \vdash \alpha$ (E)

④ $\forall x(\alpha \rightarrow \beta), \alpha \vdash \beta$ (→-) ②③

⑤ $\forall x(\alpha \rightarrow \beta), \alpha \vdash \alpha \wedge \beta$ (∧+) ③④

⑥ $\forall x(\alpha \rightarrow \beta), \alpha \vdash \exists x(\alpha \wedge \beta)$ (∃+)

⑦ $\forall x(\alpha \rightarrow \beta), \exists x\alpha \vdash \exists x(\alpha \wedge \beta)$ (∃-)

(4)

① $\alpha \vdash \alpha$ (E)

② $\alpha \vdash \exists x\alpha$ (∃+) ①

③ $\alpha \vdash \exists y\alpha$ (∃+) ②

④ $\exists y\alpha \vdash \exists y\alpha$ (∃-) ③

⑤ $\exists x\alpha \vdash \exists y\alpha$ (∃-) ④

15) ① $\forall y\alpha \vdash \forall y\alpha$ (E)

② $\forall y\alpha \vdash \alpha$ (V-) ①

③ $\forall y\alpha \vdash \exists x\alpha$ (∃+) ②

④ $\forall y\alpha \vdash \forall y\exists x\alpha$ (V+) ③

⑤ $\exists x\forall y\alpha \vdash \forall y\exists x\alpha$ (∃-) ④

6. $\alpha = \alpha(x/y) \text{ (U/x)}$

(1) ① $\forall y\alpha(x/y) \vdash \forall y\alpha(x/y)$ (E)

② $\forall y\alpha(x/y) \vdash \alpha(x/y) \text{ (U/x)}$ (V-)

③ $\alpha(x/y) \text{ (U/x)}$ (E)

④ $\forall y\alpha(x/y) \vdash \alpha$ (Tr)

⑤ $\forall y\alpha(x/y) \vdash \forall x\alpha$ (V+)

(1) ① $\forall x\alpha(x/y) \vdash \forall y\alpha(x/y)$ (V-)

② $\forall x\alpha \vdash \forall x\alpha(x/y) \text{ (U/x)}$ (禁律定理)

③ $\forall x\alpha \vdash \forall y\alpha(x/y)$

7. ① $\forall x^2 \vdash \exists x^2$ (E)

② $\forall x^2 \rightarrow \exists x^2 \vdash \exists x^2 \rightarrow \forall x^2$ (E)

③ $\exists x^2 \vdash \forall x^2$ (E) ②

④ $\alpha \vdash \alpha$ (E)

⑤ $\exists x^2 \vdash \exists x^2$ (禁律定理)

⑥ $\exists x\alpha \vdash \forall x\alpha$ (Tr) ⑤

9. (2)

- ① $\alpha \wedge \beta \vdash \neg(\alpha \rightarrow \neg \beta)$ (E₁)
- ② $\exists x(\alpha \wedge \beta) \vdash \exists x \neg(\alpha \rightarrow \neg \beta)$ (E₂)
- ③ $\neg \exists x(\alpha \wedge \beta) \vdash \neg \exists x \neg(\alpha \rightarrow \neg \beta)$ (E₃)
- ④ $\neg \exists x \neg(\alpha \rightarrow \neg \beta) \vdash \forall x(\alpha \rightarrow \neg \beta)$ (E₄)
- ⑤ $\forall x(\alpha \rightarrow \neg \beta) \vdash \alpha \rightarrow \forall x \neg \beta$ (E₅)
- ⑥ $\neg \exists x(\alpha \wedge \beta) \vdash \alpha \rightarrow \forall x \neg \beta$ (Tr) ③④⑤
- ⑦ $\exists x(\alpha \wedge \beta) \vdash \neg(\alpha \rightarrow \forall x \neg \beta)$ (E₇)
- ⑧ $\neg(\alpha \rightarrow \forall x \neg \beta) \vdash \alpha \wedge \neg \forall x \neg \beta$ (E₈)
- ⑨ $\exists x \beta \vdash \neg \forall x \neg \beta$ (E₉)
- ⑩ $\alpha \wedge \neg \forall x \neg \beta \vdash \alpha \wedge \exists x \beta$ (E₁₀)
- ⑪ $\exists x(\alpha \wedge \beta) \vdash \alpha \wedge \exists x \beta$ (Tr) ③⑧⑩

(4) ① $\alpha \vee \beta \vdash \alpha \vee \beta$ (E₁)

- ② $\exists x(\alpha \vee \beta) \vdash \exists x(\alpha \vee \beta)$ (E₂)
- ③ $\exists x(\alpha \vee \beta) \vdash \alpha \vee \exists x \beta$ (E₃)
- ④ $\alpha \vee \exists x \beta \vdash \alpha \vee \exists x \beta$ (E₄)
- ⑤ $\alpha \vee \exists x \beta \vdash \exists x(\alpha \vee \beta)$ (Tr) ②③④

12.

$\Gamma, \alpha \vdash \beta$

$\Gamma, \alpha \vdash \exists x \beta$ (E₊)

$\Gamma, \exists x \alpha \vdash \exists x \beta$ (E₋)

13.

$\Gamma, \alpha \vdash \beta$

$\Gamma, \exists x \alpha \vdash \beta$ (E₋)

$\Gamma, \exists x \alpha \vdash \forall x \beta$ (E₊)

作业

p.560(p.185)

17. (1), (2), (3)

18. (1), (3)

19. (2), (3)

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$$17. (1) \forall x_1 F_1(x_1, x_2) \wedge \forall x_2 F_1(x_1, x_2)$$

$$\vdash \forall x_3 F_1(x_3, x_2) \wedge \forall x_4 F_1(x_1, x_4)$$

$$\vdash \forall x_3 x_4 (F_1(x_3, x_2) \wedge F_1(x_1, x_4))$$

$$(2) \forall x_2 F_1(x_1) \rightarrow \exists x_1 F_1(x_2)$$

$$\vdash \forall x_3 F_1(x_1) \rightarrow \exists x_4 F_1(x_2)$$

$$\vdash \exists x_3 x_4 (F_1(x_1) \rightarrow F_1(x_2))$$

$$(3) \forall x_1 \forall x_2 F_1(x_1, x_2) \rightarrow \forall x_1 \forall x_2 F_1(x_1, x_2)$$

$$\vdash \forall x_1 \forall x_2 F_1(x_1, x_2) \rightarrow \forall x_3 \forall x_4 F_1(x_3, x_4)$$

$$\vdash \exists x_1 x_2 \forall x_3 x_4 (F_1(x_1, x_2) \rightarrow F_1(x_3, x_4))$$

$$18. (1) \exists x_1 \exists x_2 F_1(x_1, x_2)$$

$$\vdash \forall x_3 \exists x_1 \exists x_2 F_1(x_1, x_2)$$

$$(3) \exists x_1 F_1(x_1) \rightarrow \exists x_2 F_1(x_2)$$

$$\vdash \forall x_1 \exists x_2 (F_1(x_1) \rightarrow F_1(x_2))$$

$$19. (2) \exists x_1 F_1(x_1) \rightarrow \forall x_1 \exists x_2 F_1(x_1, x_2)$$

$$\vdash \exists x_1 F_1(x_1) \rightarrow \forall x_3 \exists x_2 F_1(x_3, x_2)$$

$$\vdash \exists x_3 \forall x_1 x_2 (F_1(x_1) \rightarrow F_1(x_3, x_2))$$

$$(3) \forall x_1 x_2 F_1(x_1, x_2) \wedge \forall x_2 F_1(x_1, x_2)$$

$$\vdash \forall x_4 x_2 F_1(x_4, x_2) \wedge \forall x_3 F_1(x_1, x_3)$$

$$\vdash \exists x_5 \forall x_4 x_3 (F_1(x_4, x_2) \wedge F_1(x_1, x_3))$$