Voronoi Treemaps in D3

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ABSTRACT
Blah blah blah

1. INTRODUCTION

Treemaps are a category of visualizations used for displaying hierarchical data (see Fig. ?? as an example). While node-and-edge diagrams are often used for visualizing hierachical structures, treemaps offer some significant advantages. Primarily, treemaps are space-filling, and therefore allow each node in a hierarchy to have more viewing area devoted to it than in a node-and-edge diagram. This allows both larger hierarchies to be visualized, as well as more detail to be shown on each node, such as additional text, colors, or glyphs to show attributes of the node.

The majority of treemap layouts used are variants of rectangular treemaps. These have the advantage of being relatively fast to layout, and in cases of limited scale produce reasonably understandable treemaps. However, there are three drawbacks to rectangular treemaps.

First, as hierarchies become deeper, the treemap cells can become increasingly extreme in aspect ratio, resulting in narrow rectangles more difficult to see than if their area was distributed in a more square-like space. This problem is mostly mitigated by various tweaks to the treemapping algorithm to try to keep the aspect ratio of regions close to one.

Second, the borders between different regions in the hierarchy can become difficult to see. In particular, two cells neighboring one another in the treemap but not siblings in the hierarchy can appear to share a common edge delineating the same inner node as their parent, when this is in fact not the case. Finally, rectangular treemap algorithms only fill rectangular regions, which could be undesirable for aesthetic or practical reasons.

Voronoi treemaps eliminate these problems. Firstly, Voronoi

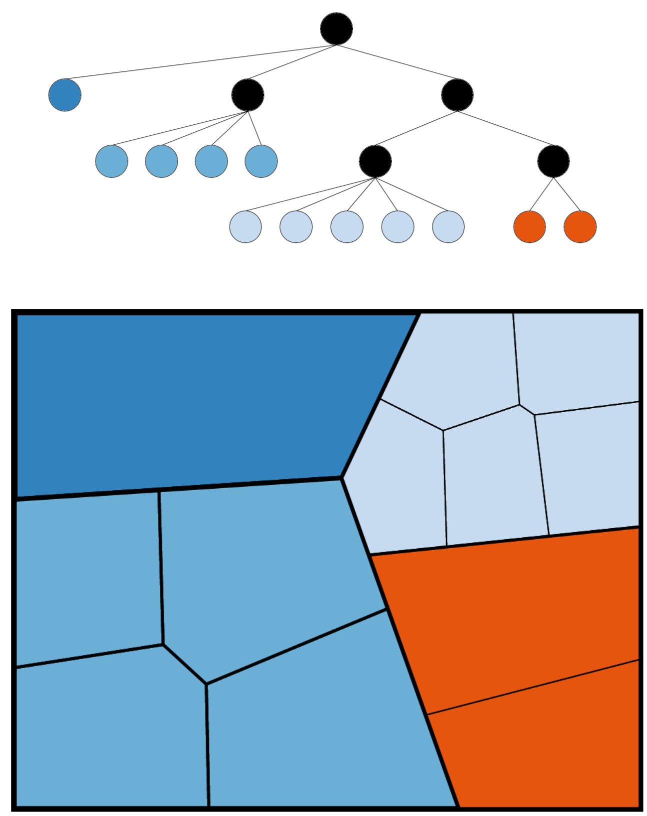


Figure 1: Example of the same tree represented by a colored node-and-edge diagram and as a Voronoi treemap.

treemap cells are arbitrary polygons, and as will be discussed later, the generation algorithm results in generally low aspect ratio cells. Secondly, the fact that Voronoi treemap cells are arbitrary polygons means edges between cells will fall at any angle, rather than only vertical or horizontal, and thus two neighboring cells will rarely have a continuous-looking edge unless they are in fact siblings in the hierarchy and thus share the edge of their parent node's cell. Finally, Voronoi treemaps can be produced for any arbitrary polygonal region, and so do not restrict the shape to be filled by the treemap.

Multiple Voronoi treemap algorithms have been created in recent years [?, ?, ?]. However, none are available for use in a web framework. Our work has been to implement one of the fastest algorithms for use in the D3 web framework [?]. Despite the optimizations employed by the algorithm creators, generation of a Voronoi treemap is still a computationally intensive task. Therefore, we have additionally written the D3 module with features to try to allow Voronoi treemaps to be used for web visualizations without causing a poor user experience even on complex datasets.

The remainder of the paper is structured as follows: Section **??** has a discussion of related work including a brief introduction to Weighted Voronoi diagrams and a discussion of the algorithms created for Voronoi treemaps. Section **??** describes the implementation of our work in D3 and optimizations added for client-side web usability. Section **??** shows the use of our framework on several datasets and an evaluation of the computational burden of our system. Section **??** discusses the potential applications of our system. Section **??** concludes with proposals of future work to be done in this space.

2. RELATED WORK

2.1 Voronoi Diagrams

Voronoi diagrams are a technique for dividing a region containing sites into cells to satisfy the following condition: given a distance function $d(p, q)$ where p and q are points, any point p is labeled as belonging to the site q that results in the lowest distance, $d(p, q)$. In this case to be labeled means to be inside a bounding polygon formed for each site. In the case of a simple euclidean distance function, $d(p, q) = \sqrt{(}dx^2 + dy^2)$ this results in a cell border being equidistant between the two closest sites.

For Voronoi treemaps, two extensions are made to the basic Voronoi diagram. First, sites are given initially random positions, a diagram is generated, and then sites are moved to the centroidal positions in their cell and then the diagram is re-generated. This is repeated until a relatively stable set of positions is found [?]. The effect of this iterative process is to create lower aspect ratio cells. Second, rather than using a standard euclidean distance function the generation algorithm uses a weighted distance function, where each site is assigned a weight that corresponds to generating a larger or smaller cell. This allows the sizes of cells to be adjusted to reflect the relative size or number of children of a specific node in the hierarchy being displayed.

After these extensions are made, the Voronoi treemap algorithm proceeds to compute the Voronoi diagram for each level of the hierarchy: it starts at the highest level, generates the Voronoi diagram of the first level of nodes, and then recursively descends into each cell and generates the Voronoi diagram for the children of that node using the cell as the new bounding region. The computational burden of this can be high; several different algorithms for computing the Voronoi diagram have been developed and are briefly summarized below.

2.2 Previous Approaches

Voronoi treemaps have been implemented previously [?] using both additively weighted and geometrically weighted Voronoi diagram algorithms. This initial system used the iterative algorithm for creating centroidal Voronoi diagrams described above. To create the weighted diagrams, however, it used a sampling algorithm wherein points were sampled in the space and distances to nearest sites calculated, to give an approximation of the correct weighted Voronoi diagram. This results in an algorithm on the order of $O(n^2)$ where n is the number of sites. The benefit of this algorithm is that it the sampling process is the bottleneck and is easily parallelized to achieve linear speedups with additional CPU cores.

This algorithm implementation was improved by Sud et al. [?] by using GPU programming to significantly speedup computation by parallelizing across graphics hardware. However, the algorithm remained $O(n^2)$ for the number of sites. Further, this approach is not feasible for web programming because consumer devices are not commonly equipped with powerful graphics cards and do not all support the use of the graphics card by a website [?].

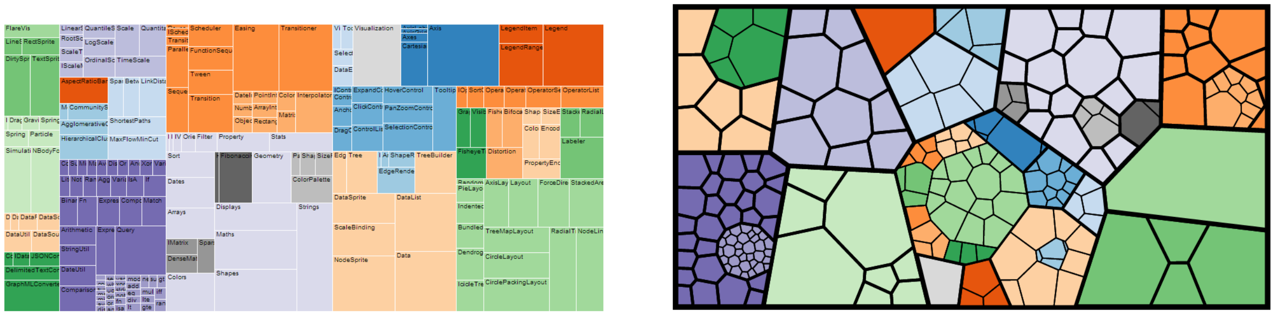
The algorithm proposed by Nocaj & Brandes [?] offers a significant asymptotic improvement on these previous designs. Rather than a sampling-based approach, this implementation uses the algorithm for computing arbitrary-dimension Power Diagrams proposed by Aurenhammer [?]. In this approach the 2D points representing sites are lifted into 3-dimensional dual space based on their weights. The convex hull made by these 3D points is then computed, and projected back down to 2D to produce the Voronoi diagram. This method is on the order of $O(n \log n)$ and so can provide a significant speedup for generating treemaps of larger datasets. A second benefit is that by computing the Power Diagram analytically, the resulting Voronoi treemap is resolution-independent.

3. METHODS

The core computational components of our implementation were adapted from a Java implementation[1] of the Nocaj & Brandes algorithm [?] using a lift into 3-dimensions followed by computation of the convex hull and projection back into 2-dimensions to create the Voronoi diagram. As with other implementations, we use Lloyd's algorithm to iteratively adjust the site locations to be the centroids of their cells and then adjust the weights of the sites to fit the area of each cell to within an error threshold.

4. RESULTS

[1]`https://github.com/ArlindNocaj/Voronoi-Treemap-Library`



(a) D3 Standard Treemap (b) Our Voronoi Treemap

Figure 2: A comparison of standard and Voronoi treemaps on the same dataset.

Dataset	Nodes	Breadth	Depth	JS Time	Java Time
Flare	251	10	4	3.913	1.588
A	178	7	5	3.112	1.160
B	130	3	5	2.765	1.063
C	73	5	3	1.277	0.946
D	584	8	3	8.623	2.124
E	110	10	2	1.733	1.067

Table 1: This shows the data for how the javascript and java implementations performed on a set of hiearchical datasets.

We first present a visual comparison between a standard rectangular treemap and the same data visualized using our system (Figure **??**). Notice that the hierarchical structure is more readily apparent in the Voronoi treemap.

To evaluate our implementation we used a 251 node 4-level example hierarchical dataset used for the example implementation of the rectangular treemap in D3[2] and five other randomly generated hierarchical datasets of varying depth and breadth to test the limits of the dataset sizes our system could handle. We timed the time required to fully compute the Voronoi treemaps of these datasets, using a limit of 100 iterations which generally yielded error rates below 1% between optimal cell areas and generated cell areas. We additionally ran these examples through a Java implementation of the same algorithm by Nocaj & Brandes to compare speed differences between implementations. All tests were run on a Macbook Air 2012 running an Inte Core i5 1.8 GHz processor with 4 GB of 1600 MHz DDR3 RAM. The javascript was run in Google Chrome.

As can be seen in **??** the javascript implementation was consistantly slower, as was to be expected since javascript is typically a slower language and was being run within a browser. However, the difference is within well within an order of magnitude, and quite insignificant for the smaller datasets.

Of course, our javascript implementation is also meant to be used on websites, which are much more sensitive to latency than native applications. Even given this, the performance on the first four datasets is within reasonable limits for users to wait for a page to load.

[2] `http://bl.ocks.org/mbostock/4063582`

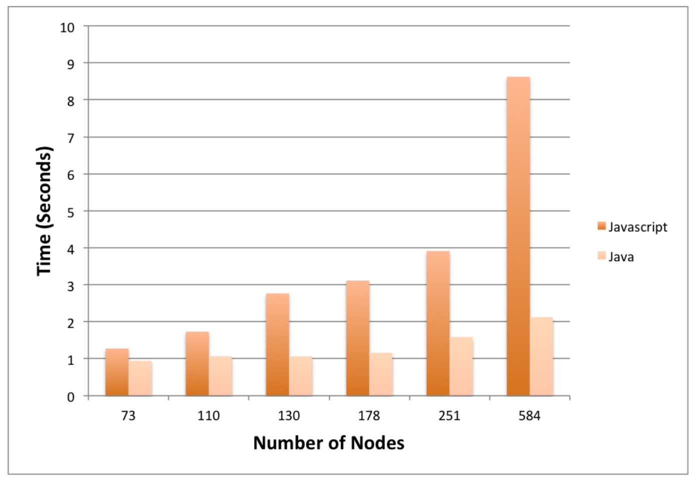


Figure 3: A chart showing the computation time required relative to the number of nodes in the dataset.

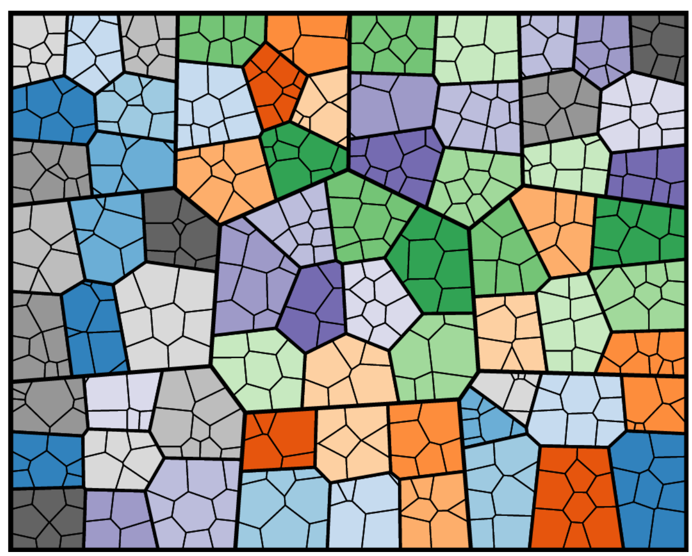


Figure 4

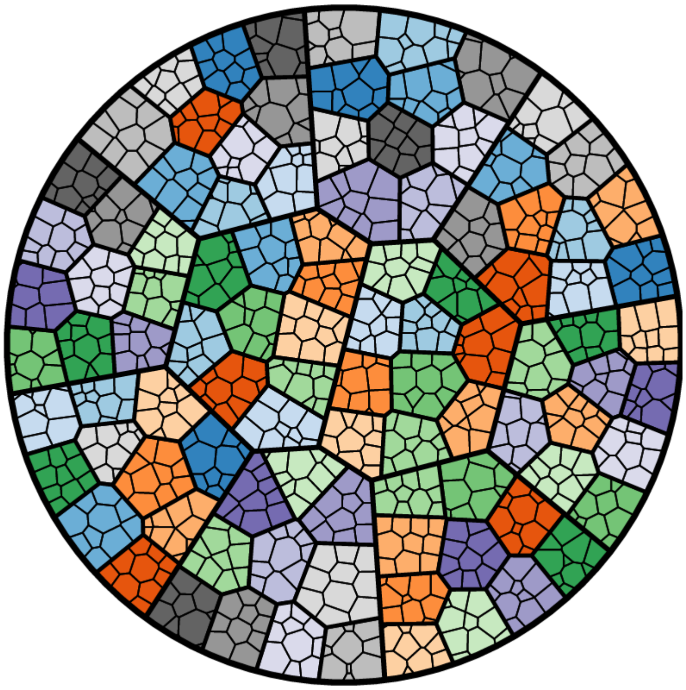


Figure 5

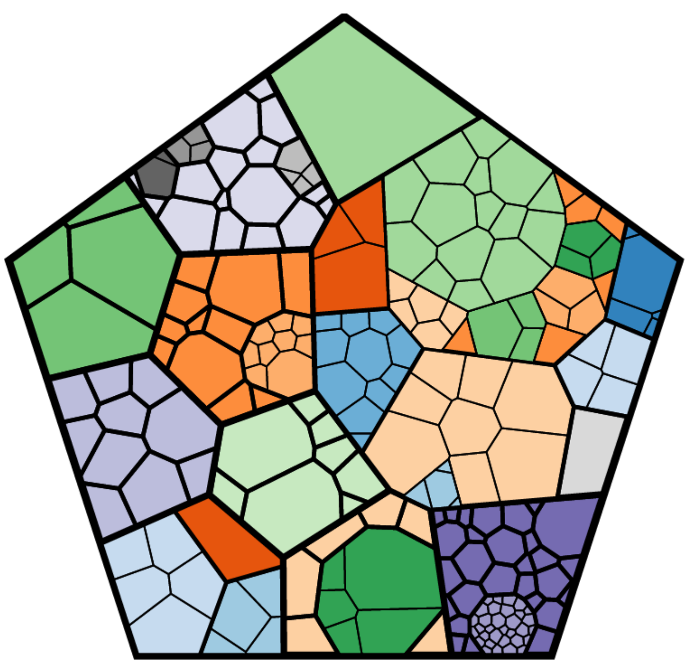


Figure 6

5. DISCUSSION

6. FUTURE WORK

7. ACKNOWLEDGMENTS

8. REFERENCES

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