Reliability and Availability Assessment Methods

Reliability of the system made of dependent, irreparable and irreplaceable components

Reliability of the system made of dependent, irreparable and irreplaceable components – serial connection of components

- Malfunction of certain components can significantly influence the reliability of other components by changing the parameters defining the reliability
- The system is made of two identical electrical resistors:

➢ $T_2 > T_1$; $\lambda_2 > \lambda_1$ (failure rate when two resistors are functional/when only one resistor is functional)

• The serial connection of components

➢ $$R(t) = e^{-2\lambda_2 t}$$

Reliability of the system made of dependent, irreparable and irreplaceable components – parallel connection of components

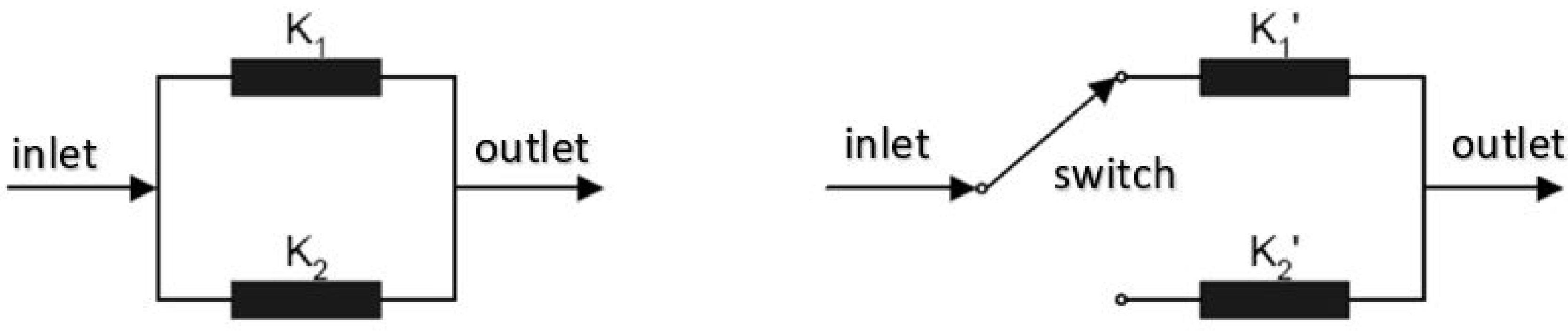
- The reliability of a parallel system equals the sum of the probability that both resistors are working and the probability of one resistor malfunction
  - The first probability is easily determined by $2e^{-\lambda_2 t}$ but the probability of one resistor malfunction cannot be determined by such a simple equation since the failure rate is changing
    - $R(t) = P(x_1 + x_2) = P(x_1) + P(x_2) - P(x_1 x_2) =$ $= P(x_1) + P(x_2) - P(x_1)\,P(x_2/x_1)$
- Interpretation of $P(x_1)$, $P(x_2)$ and $P(x_2/x_1)$ is unclear from the physics standpoint

Possible component dependencies

- Physical influence of the components
- Increase or decrease of the load (influences of the pressure, temperature, etc.)
- Dependence as „a state of knowledge”
- Common cause failures

➢ **Only a thorough knowledge of a problem can lead to an assessment of dependency/acceptable error of the independency assumption**

Passive (standby) redundancy –
comparison with a parallel system

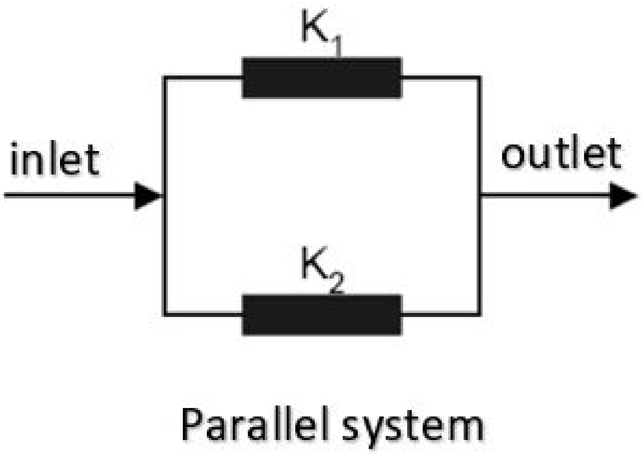


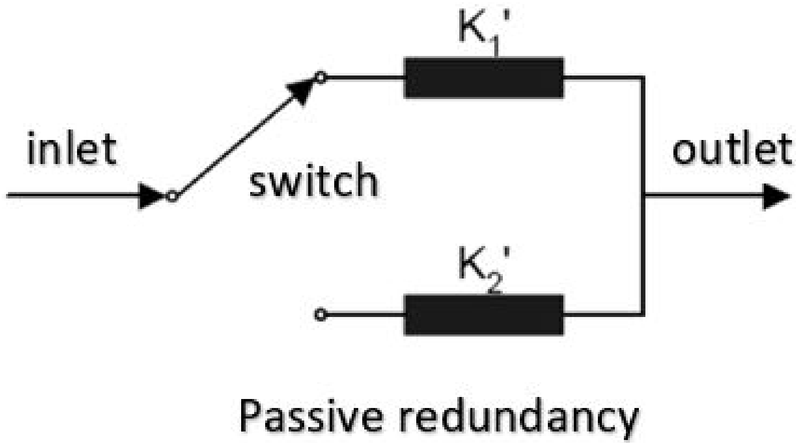
Parallel system

Passive redundancy

- Consider a system for which the redundancy component <u>cannot fail</u>
  - Previously we analyzed the system for which the redundancy component can fail

Passive (standby) redundancy – comparison with a parallel system





$$R_P(t) = 1 - P(\overline{x}_1 \overline{x}_2) = 1 - P(\overline{x}_1) P(\overline{x}_2 / \overline{x}_1)$$

$$R_r(t) = 1 - P\left(\overline{x}_1' \overline{x}_2'\right) = 1 - P\left(\overline{x}_1'\right) P\left(\overline{x}_2' / \overline{x}_1'\right)$$

$$P(\overline{x}_2 / \overline{x}_1) \quad P(\overline{x}_2' / \overline{x}_1')$$

At the time the first component fails, the second just starts to operate and so the failure rate/reliability significantly changes

Passive redundancy system – n identical components: one works, n-1 on standby

In case of a passive redundancy, the redundant components do not share any load with the operating component. The redundant components are put in use one at a time after failure of the currently operating component and the remaining components are kept in reserve. If the operating component fails, one of the components on standby is put into use through switching.

Passive redundancy system – n identical components: one works, n-1 on standby

- Passive redundancy system consists of n identical components:
  - One component is in operation and n-1 components are on standby
- **Assessing the reliability means assessing the probability of n-1 malfunctions in the time t**
  - The system will work properly during time t even if n-1 failures occurred during that time, which means that the last, n-th, component has worked correctly until time t

Passive redundancy system – n identical components: one works, n-1 on standby

When determining the mathematical model of the system, we should take into account the following relations:

1. The number of malfunctions in a system determines separate system states. Eg., n-th system state means there have been n malfunctions in the system, n+1 system state means there have been n+1 malfunctions in the system, etc.;

2. The probability that the system has „moved" from the state n to the state n+1, i.e. the probability that after n malfunctions the malfunction n+1 occurs within the time interval Δt equals $\boldsymbol{\lambda\Delta t}$. The parameter λ is the constant failure rate;

3. The probability of two or more failures occurring in the time interval Δt can be neglected

$\longrightarrow$ $\boldsymbol{(\lambda\Delta t)\cdot(\lambda\Delta t) \approx 0}$

Reliability of a passive redundancy system – the probabilities of failures in a system

- n-th system state - $\mathbf{P_n(t)}$ is the probability that **n failures** have occurred in the time **t**
- the probability that no failure has occurred in the time frame $\mathbf{t+\Delta t}$ will be $\mathbf{P_0(t+\Delta t)}$ and that probability (the probability of a complex event) will be expressed by the following differential equation:

$$P_0(t+\Delta t) = P_0(t)\cdot(1-\lambda\Delta t)$$

The probability that one malfunction occurs in the time frame $\mathbf{t+\Delta t}$ will then be:

$$P_1(t+\Delta t) = P_0(t)\cdot(\lambda\Delta t) + P_1(t)\cdot(1-\lambda\Delta t)$$

Reliability of a passive redundancy system – the probabilities of failures in a system

If we generalize the previous equation, we get:

$$P_n(t+\Delta t) = P_{n-1}(t)\cdot(\lambda\Delta t) + P_n(t)\cdot(1-\lambda\Delta t)$$

i.e. **n malfunctions** in the time frame **t+Δt** can occur in two, mutually exclusive ways:

- **n-1 malfunctions** could have occurred in time **t**, and then one more in time **Δt** or, all **n failures** could have occurred in time **t** and no failure (malfunction) in time **Δt**.

Reliability of a passive redundancy system – the probabilities of failures in a system

But the **malfunctions occur in the continuous and not the discrete time period:**

$$P_0(t+\Delta t) = P_0(t)\cdot(1 - \lambda\Delta t)$$

$$\lim_{\Delta t\to 0}\frac{P_0(t+\Delta t) - P_0(t)}{\Delta t} = \lim_{\Delta t\to 0}\left(-\lambda P_0(t)\right)$$

$$\frac{dP_0(t)}{dt} = P_0(t)' = -\lambda P_0(t)$$

Reliability of a passive redundancy system – the probabilities of failures in a system

Also:

$$\lim_{\Delta t \to 0} \frac{P_n(t+\Delta t) - P_n(t)}{\Delta t} = \lim_{\Delta t \to 0} \lambda P_{n-1}(t) - \lim_{\Delta t \to 0} \lambda P_n(t)$$

$$\frac{dP_n(t)}{dt} = P_n(t)' = \lambda P_{n-1}(t) - \lambda P_n(t)$$

The differential equations along with initial conditions describe the behaviour of a passive redundancy system.

Reliability of a passive redundancy system – the probabilities of failures in a system

Probability of occurrence 0, 1, 2, ..., n malfunctions in a passive redundancy system in a time interval [0,t] is determined by differential equations.

Initial conditions can be:

$P_0(0) = 1$, $P_1(0) = P_2(0) = \ldots = P_n(0) = 0$, i.e. if there is no malfunction at the beginning of the operation, $t = 0$, $n \neq 0$, or

$P_0(0) = P_1(0) = P_2(0) = 0$, $P_3(0) = 1$, $P_4(0) = P_5(0) = \ldots = P_n(0) = 0$ if we start the operation with three malfunctions etc.

Reliability of a passive redundancy system – identical components – probability of zero malfunctions

The equations are linear differential equations with the constant coefficients so they can be solved in many ways. If we use the undefined coefficient technique, we obtain these solutions:

$$P_0(t)' + \lambda P_0(t) = 0,$$

$$r + \lambda = 0 \quad \Rightarrow \quad r = -\lambda,$$

$$P_0(t) = Ce^{-\lambda t}.$$

Reliability of a passive redundancy system – identical components – probability of zero malfunctions

If we introduce initial condition $P_0(0) = 1$

$$P_0(0) = Ce^{-\lambda 0} = C = 1$$

we get:

$$P_0(t) = e^{-\lambda t},$$

i.e. the probability that no malfunction will occur is exactly the component reliability (only one component works properly), so if the failure rate is constant we have the exponential reliability law.

Reliability of a passive redundancy system – identical components – probability of one malfunction

For **n=1** we get:

$$P_1(t)' + \lambda P_1(t) = \lambda P_0(t)$$

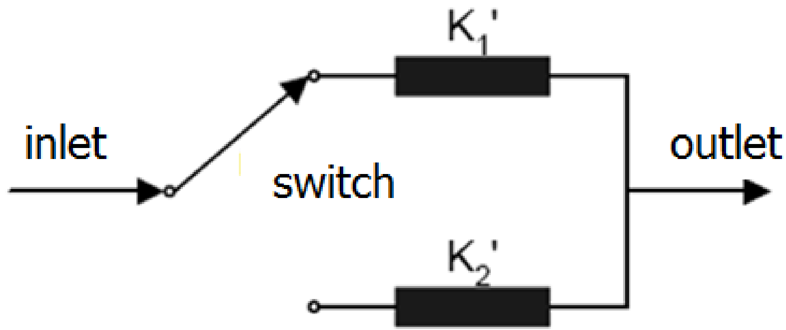
If we introduce expression for $P_0(t)$:

$$P_1(t)' + \lambda P_1(t) = \lambda e^{-\lambda t}$$

We obtain the solution of the differential equation:

$$P_1(t) = \lambda t e^{-\lambda t}$$

Reliability of a passive redundancy system – two identical components



The reliability of such system is then:

$$R_{r_1}(t) = P_0(t) + P_1(t) = e^{-\lambda t} + \lambda t e^{-\lambda t},$$

i.e. it is equal to the sum of probability that there are no malfunctions and the probability of one malfunction. In other words, the system will be operational if there are no malfunctions and also if there is only one malfunction.

Reliability of a passive redundancy system – identical components – probability of two malfunctions

If the system has two components on standby, then **n=2**, and we get the probability of two malfunctions occurring in time t [($P_1(t) = \lambda t e^{-\lambda t}$]:

$$P_2(t)' + \lambda P_2(t) = \lambda P_1(t) = \lambda^2 t e^{-\lambda t}$$

$$P_2(t) = \frac{(\lambda t)^2}{2} e^{-\lambda t}$$

Reliability of a passive redundancy system – three identical components – comparison with a parallel system

The system reliability is then:

$$R_{r_2}(t) = e^{-\lambda t} + \lambda t e^{-\lambda t} + \frac{(\lambda t)^2}{2} e^{-\lambda t}.$$

We can now compare such a system with corresponding parallel system: a system with three components in parallel. Since the reliability of each component is $e^{-\lambda t}$, the reliability of a the parallel system is:

$$R_p(t) = e^{-3\lambda t} - 3e^{-2\lambda t} + 3e^{-\lambda t}$$

Reliability of a passive redundancy system – three identical components – comparison with a parallel system

To make the comparison easier, we will assume that each component reliability in a certain time is **0,9** (eg. one year).

Then we obtain that the reliability of the redundancy system is **0,99982**, and of the parallel system **0,99900,** because $\mathbf{0{,}9 = e^{-\lambda t}}$ and $\boldsymbol{\lambda t = 0{,}10536}$.

Therefore, the probability of a malfunction of a parallel system is $\mathbf{100 \cdot 10^{-5}}$, and of a redundancy system $\mathbf{18 \cdot 10^{-5}}$.

Constructing the system with the same components in a different manner, we achieved about five times increase in reliability.

Reliability of a passive redundancy system – identical components – probability of *n* malfunctions

While determining the probability of a system state, i.e. the probability of number of failures in the system, we obtained first-order differential equations because only the previous system state was included in the calculations. In a general case, for n failures we will get by analogous solving:

$$P_n(t) = \frac{(\lambda t)^n}{n!} e^{-\lambda t}$$

The probability of n failures in an observed passive redundancy system.

Reliability of a passive redundancy system – n identical components: one in operation, n-1 on standby

The number of failures is in accordance with the **Poisson distribution.**

If the system is composed of n identical components, out of which n-1 are on standby, the system reliability is given by the expression:

$$R_{r\,n-1}(t) = e^{-\lambda t}\left(1+\lambda t+\frac{(\lambda t)^2}{2}+\frac{(\lambda t)^3}{3!}+\ldots+\frac{(\lambda t)^{n-1}}{(n-1)!}\right)$$

Reliability of a passive redundancy system – two different components

- The reliability of a redundancy system with two different components with different failure rates:

$$R_{r1}(t) = P(x_1 + \overline{x_1} \cdot x_2)$$

$$R_{r1}(t) = P(x_1) + P(\overline{x_1}) \cdot P(x_2)$$

$P(x_2)$ is probability of second component operation in a time interval $[t_1, t]$ (otherwise we would have a conditional probability $\underline{P(x_2/\overline{x_1})}$). (the first component fails in $t = t_1$)

Reliability of a passive redundancy system – two different components

We can now determine the probabilities of events, ie. the reliabilities (unreliabilities):

$$P(x_1),\ P(\overline{x}_1)\ \text{i}\ P(x_2).$$

To do that we will use the **failure probability density function q(t):**

$$q(t) = \frac{dQ(t)}{dt} = \frac{-\,de^{-\lambda t}}{dt} = \lambda e^{-\lambda t}.$$

By integrating that function over the total time interval, from 0 to ∞, we will obtain that the probability of component failure is then equal to one, i.e. that this is a certain event which will be valid for any failure probability density function.

Reliability of a passive redundancy system – two different components

In a special case of the **constant failure rate of a component** [$\mathbf{R(t)=e^{-\lambda t}}$], we obtain:

$$\int_0^\infty \lambda e^{-\lambda t} dt = -\left|e^{-\lambda t}\right|_0^\infty = -(0-1) \equiv 1\,.$$

By integrating the function q(t) from 0 to t, the failure probability function Q(t) is obtained, and since the sum of the probability of a malfunction and probability of a correct operation as a complementary event equals one, **integral from time t to infinity of the function q(t) is equal to the probability that the component will not fail before the time t, i.e. the component reliability:**

Reliability of a passive redundancy system – two different components

$$\int_0^\infty q(t)dt \equiv \int_0^t q(t)dt + \int_t^\infty q(t)dt = Q(t) + R(t) \equiv 1\ ;\ R(t) = \int_t^\infty q(t)dt$$

The expression $R_{r1}(t) = P(x_1) + P(\overline{x}_1) \cdot P(x_2)$ can be written as:

$$R_{r1}(t) = \int q_1(t)dt + \int \left[q_1(t_1) \cdot \left(\int q_2(t)dt \right) \right] dt_1$$

integrating the corresponding variables over the associated time intervals.

Reliability of a passive redundancy system – two different components

To determine the probability of a correct operation (reliability) of the first component, the interval of the first term in the equation is set from time t to ∞, and **the variable is t**. (Because the component works until time t.)

The probability density function is $\lambda_1 e^{-\lambda_1 t}$

Thus, we obtain the expected result:

$$P(x_1) = \int_t^{\infty} \lambda_1 e^{-\lambda_1 \xi} d\xi = e^{-\lambda_1 t}$$

Reliability of a passive redundancy system – two different components

- Integration interval for the first integral of the second term is from 0 to t, because it represents probability for failure of the first component from time zero to the time t but the integration variable is now $\mathbf{t_1}$ because the component fails in time $t_1<t$. This is pointed out by putting the variable t_1 outside of the brackets.
- Regarding the last interval; the second component has to work from time t_1 (when the first component failed) until time t. If we want to calculate the probability of its malfunction, we have to set the limits from time t_1 to time t because the component has started to work from time t_1.

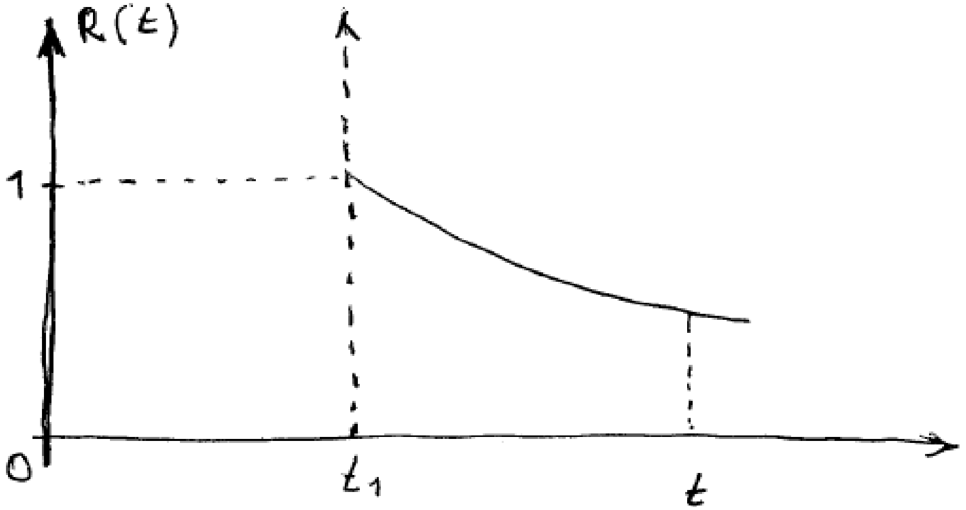
$$Q_2(t)=\int_{t_1}^{t} q_2(u)du$$

Reliability of a passive redundancy system – two different components

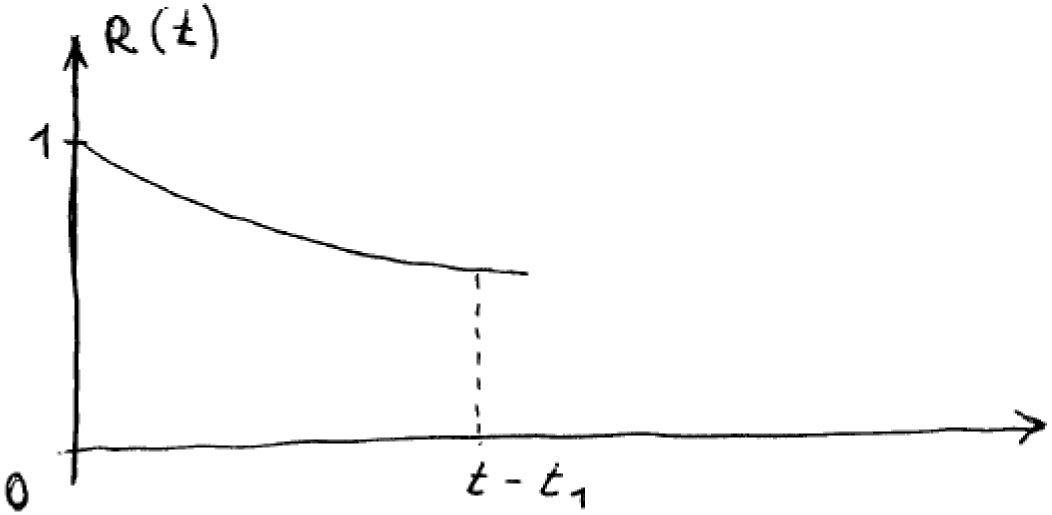
Thus, in order to calculate probability of operation of the second component from time t_1 until time t, the right integration interval of the second equation term will be from **t** to $\boldsymbol{\infty - t_1}$.

It is appropriate to change the time axis by shifting the time axis considering the fact that the zero time (start of the operation) is set at time t_1. The new value for t is equal to $t-t_1$, thus the integration interval for the failure of the second component will be from 0 to $t-t_1$. Finally, the interval for calculating the probability that the second component will correctly operate is from time $\boldsymbol{t-t_1}$ **to** ∞. Since the second component is working until the time t, the integration variable is t.

Reliability of a passive redundancy system – two different components



t to ∞ - t_1



$t-t_1$ to ∞

Reliability of a passive redundancy system – two different components

Since the probability of a failure of the first component is multiplied with the probability of correct operation of the second component in each time interval, the right (second) integration is performed first. The probability density function of the of the failure of the second component $q_2(t)$ is equal to $\lambda_2 e^{-\lambda_2 t}$.

We can now calculate the second term as:

$$\int_0^t \left(q_1(t_1) \int_{t-t_1}^{\infty} q_2(u)du \right) dt_1 = \int_0^t \left(\lambda_1 e^{-\lambda_1 t_1} \int_{t-t_1}^{\infty} \lambda_2 e^{-\lambda_2 u} du \right) dt_1 = \frac{\lambda_1}{\lambda_2 - \lambda_1} \left(e^{-\lambda_1 t} - e^{-\lambda_2 t} \right)$$

and the system reliability is

$$R_{r_1}(t) = e^{-\lambda_1 t} + \frac{\lambda_1}{\lambda_2 - \lambda_1} \left(e^{-\lambda_1 t} - e^{-\lambda_2 t} \right)$$

Reliability of a passive redundancy system – n different components

In order to generalize the formula for the case of two, three or more backup components, the expression is rearranged as following:

$$R_{r1}(t) = \frac{\lambda_2}{\lambda_2 - \lambda_1} e^{-\lambda_1 t} + \frac{\lambda_1}{\lambda_1 - \lambda_2} e^{-\lambda_2 t}.$$

For example, the reliability of the system with two backup components is:

$$R_{r2}(t) = \frac{\lambda_2}{\lambda_2 - \lambda_1} \frac{\lambda_3}{\lambda_3 - \lambda_1} e^{-\lambda_1 t} + \frac{\lambda_1}{\lambda_1 - \lambda_2} \frac{\lambda_3}{\lambda_3 - \lambda_2} e^{-\lambda_2 t} + \frac{\lambda_1}{\lambda_1 - \lambda_3} \frac{\lambda_2}{\lambda_2 - \lambda_3} e^{-\lambda_3 t}$$

What if the components are identical?

Reliability of a passive redundancy system

In case that $\boldsymbol{\lambda_1=\lambda_2=\lambda}$, ie. if the components are identical, the result of the equation is undefined. However, using the **L'Hospital's rule**, derivating the denominator and numerator with respect to $\boldsymbol{\lambda_2}$ and assuming $\boldsymbol{\lambda_1=\lambda_2=\lambda}$, we obtain:

$$R(t) = e^{-\lambda t} + \lambda t\, e^{-\lambda t}$$

Reliability of a passive redundancy system – n different components: one works, n-1 on standby

The reliability of a system that has n-1 components on standby is:

$$R_{r_{n-1}}(t) = \frac{\lambda_2}{\lambda_2 - \lambda_1}\frac{\lambda_3}{\lambda_3 - \lambda_1}\cdots\frac{\lambda_n}{\lambda_n - \lambda_1}e^{-\lambda_1 t} + \frac{\lambda_1}{\lambda_1 - \lambda_2}\frac{\lambda_3}{\lambda_3 - \lambda_2}\cdots\frac{\lambda_n}{\lambda_n - \lambda_2}e^{-\lambda_2 t} + \ldots + \frac{\lambda_1}{\lambda_1 - \lambda_n}\frac{\lambda_2}{\lambda_2 - \lambda_n}\cdots\frac{\lambda_{n-1}}{\lambda_{n-1} - \lambda_n}e^{-\lambda_n t}$$

Reliability of a passive redundancy system – n+k identical components: k components in operation, n on standby

Let's consider the system consisting of more than one component in operation (**k**) and one or more components on standby (**n**). In this case, the components are usually identical for practical reasons: eg. two identical transformers are in operation and the third is on standby.

$$R_{r\,n}(t) = e^{-k\lambda t}\left(1 + k\lambda t + \frac{(k\lambda t)^2}{2} + \frac{(k\lambda t)^3}{3!} + \ldots + \frac{(k\lambda t)^n}{n!}\right)$$

1 component in operation:

$$R_{r\,n-1}(t) = e^{-\lambda t}\left(1 + \lambda t + \frac{(\lambda t)^2}{2} + \frac{(\lambda t)^3}{3!} + \ldots + \frac{(\lambda t)^{n-1}}{(n-1)!}\right)$$

Reliability and availability calculations using Markov processes

- **Markov processes** – a type of **Markov models**
- **Markov models** – functions of two random variables:
  - **System state and**
  - **Observation time**
- **Four model types, 2 most significant**
  - **Markov chain**
    - **discrete system state, discrete observation time**
  - **Markov process**
    - **discrete system state, continuous observation time**

Reliability of a system made of dependent, irreparable and irreplaceable components

Method of modelling Markov processes (differential eq.) is similar to calculation of reliability of passive redundancy systems.

The first step is determination of **all mutually exclusive discrete separate system states regarding the number of malfunctions and failed components.**

The system state is uniformly determined by the number of component malfunctions in the system (e.g. **n-th** system state means there have been **n** malfunctions in the system, **n+1-st** state, **n+1** malfunctions, etc.) and components that failed; obviously, these states are discrete.

Reliability of a system made of dependent, irreparable and irreplaceable components

In that case, Markov process will determine the probability of finding the system in specific states which will, regarding the components' correct operation or malfunction, represent system operation or failure.

For example, if the system contains only **one** component, it can have only two states: $\mathbf{s_0 = x}$, the component is correct, and $s_1 = \bar{x}$, the component failed.

The states of the system at the moment of observing are called the **initial** states, and at any given moment in the future, the **final** states.

The probabilities of systems transition from initial to final states are described by **Markov equations.**

Reliability of a system made of dependent, irreparable and irreplaceable components

The following assumptions must be valid in order to develop mathematical models:

1. The probability of system transition in time **Δt** from one state to another is equal to **$\lambda\Delta t$**; this is the probability of occurrence n+1-st system malfunction in time **Δt** after occurrence of n malfunctions
2. The probability of two or more transitions in time interval **Δt** is zero, that is, the probability of occurrence of two or more malfunctions in time **Δt** is zero **$[\lambda \Delta t \cdot \lambda \Delta t \approx 0]$**
3. The probability of transition between states **i** and **j** depends only on states **i** and **j** and is independent of all other previous states **(model without memory)**

Reliability of a system made of dependent, irreparable and irreplaceable components

If we specify with

$$\mathbf{P_{S0}(t)}$$

the probability that in time **t** the system is in the state $\mathbf{s_0}$, i.e. that in the time period from **zero to t** no malfunction has occurred, the probability

$$P_{S0}(t+\Delta t)\,,$$

that the system in the time **t+Δt** is in the state $\mathbf{s_0}$ will be equal to the product of the probability that the system in time **t** is in the state $\mathbf{s_0}$, $\mathbf{P_{S0}(t)}$, and the probability that during the time interval **Δt** the system did not change its state to $\mathbf{s_1}$, (**1-λΔt**):

Reliability of a system made of dependent, irreparable and irreplaceable components

$$P_{S0}(t+\Delta t)=P_{S0}(t)\cdot(1-\lambda\Delta t)$$

The equation is still not complete because of the possibility of repair:

$$P_{S0}(t+\Delta t)=P_{S0}(t)\cdot(1-\lambda\Delta t)+P_{S1}(t)\cdot 0$$

Reliability of a system made of dependent, irreparable and irreplaceable components

Malfunctions in the system, that is, system transitions from one state to another, occur in the continuous and not in the discrete time.

Thus, we have to set the limit **$\Delta t \to 0$** to obtain the process with the continues time. If we also divide the equations with **Δt** and we set the limits, we will obtain:

$$\lim_{\Delta t \to 0} \frac{P_{s_0}(t+\Delta t) - P_{s_0}(t)}{\Delta t} = \lim_{\Delta t \to 0} -\lambda P_{s_0}(t).$$

Reliability of a system made of dependent, irreparable and irreplaceable components

According to the definition, the left side of the equation is the derivative of $\mathbf{P_{s0}(t)}$ with respect to time, and the right side is independent of $\mathbf{\Delta t}$, thus:

$$\frac{d\,P_{s_0}(t)}{d\,t} = P_{s_0}(t)' = -\lambda P_{s_0}(t)$$

Reliability of a system made of dependent, irreparable and irreplaceable components

Similar to discussion of deriving equations for the system state S0, we obtain equations for the system state S1:

$$P_{S1}(t+\Delta t) = P_{S0}(t)\cdot\lambda\Delta t + P_{S1}(t)\cdot 1$$

$$\lim_{\Delta t \to 0} \frac{P_{s_1}(t+\Delta t) - P_{s_1}(t)}{\Delta t} = \lim_{\Delta t \to 0} \lambda P_{s_o}(t)$$

$$\frac{d\, P_{s_1}(t)}{d\, t} = P_{s_1}(t)' = \lambda P_{s_o}(t)$$

Reliability of a system made of dependent, irreparable and irreplaceable components

Differential equations, along with initial conditions, describe the system events, ie. determine the probabilities of occurrence of malfunctions in the system, **the probabilities of finding the system in certain states.**

The most common initial conditions are:

$\mathbf{P_{s0}(0)=1}$ and $\mathbf{P_{s1}(0)=0}$, ie. in time $\mathbf{t=0}$ the system was operational.

The differential equations are easily solved since they are linear equations of the first order (including only the previous state in the calculations). Using Laplace technique we obtain:

$$P_{s_0}(t) = e^{-\lambda t} \qquad P_{s_1}(t) = 1 - e^{-\lambda t}$$

Reliability of a system made of dependent, irreparable and irreplaceable components

the probability that the system will be in the state $\mathbf{s_0}$ after time t, $\mathbf{P_{s0}(t)}$ (the component is correct). ie., the probability that the system will be in the state $\mathbf{s_1}$ after time t, $\mathbf{P_{s1}(t)}$ (component failed).

The probability $\mathbf{P_{s0}(t)}$ represents the reliability of the component:

$$R(t) = P_{S0}(t) = e^{-\lambda t}$$

Reliability of a system made of dependent, irreparable and irreplaceable components

The equations are general and we can assume any kind of initial conditions. For example:

- $P_{S0}(0)=0$, $P_{S0}(t)=0$, $R(t)=0$
- $P_{S0}(0)=0{,}5$, $R(t)=0{,}5e^{-\lambda t}$

The initial conditions allow to include the probability of an initial failure even before the start of the system operation.

Reliability of a system made of dependent, irreparable and irreplaceable components

The equation coefficients can be obtained using the **matrix of transitional probabilities**:

	Final states	
Initial states	$s_0(t+\Delta t)$	$s_1(t+\Delta t)$
$s_0(t)$	$1-\lambda\Delta t$	$\lambda\Delta t$
$s_1(t)$	0	1

$\Sigma = 1$

The probability of transition from the initial to the final state

The probability of the final state s1:

$$P_{s1}(t+\Delta t) = (\lambda \Delta t)\, P_{s0}(t) + 1 \cdot P_{s1}(t)$$

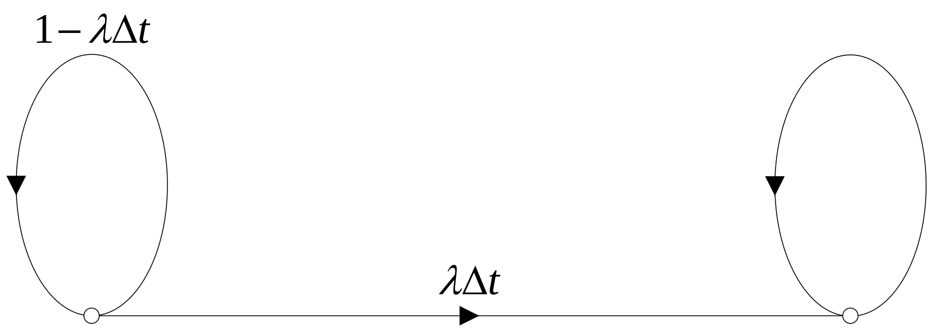
Reliability of a system made of dependent, irreparable and irreplaceable components

$$P_{s1}(t+\Delta t) = (\lambda \Delta t)\, P_{s0}(t) + 1 \cdot P_{s1}(t)$$

We can easily determine the differential equations by using the graphical display of the Markov process. The graph is composed of:

- **Nodes which represent system states**
- **Oriented lines which represent transitional probabilities**.

Reliability of a system made of dependent, irreparable and irreplaceable components



- Derivation of probability associated with the node is equal to the sum of information which are coming back to the node
- Factor **1** on the line of the graph that returns to the same node is replaced with **zero**
- Factor **Δt** is replaced with **one**

$$P'_{s0} = -\lambda P_{s0} \qquad P'_{s1} = \lambda P_{s0}$$

Reliability of a system made of dependent, irreparable and irreplaceable components

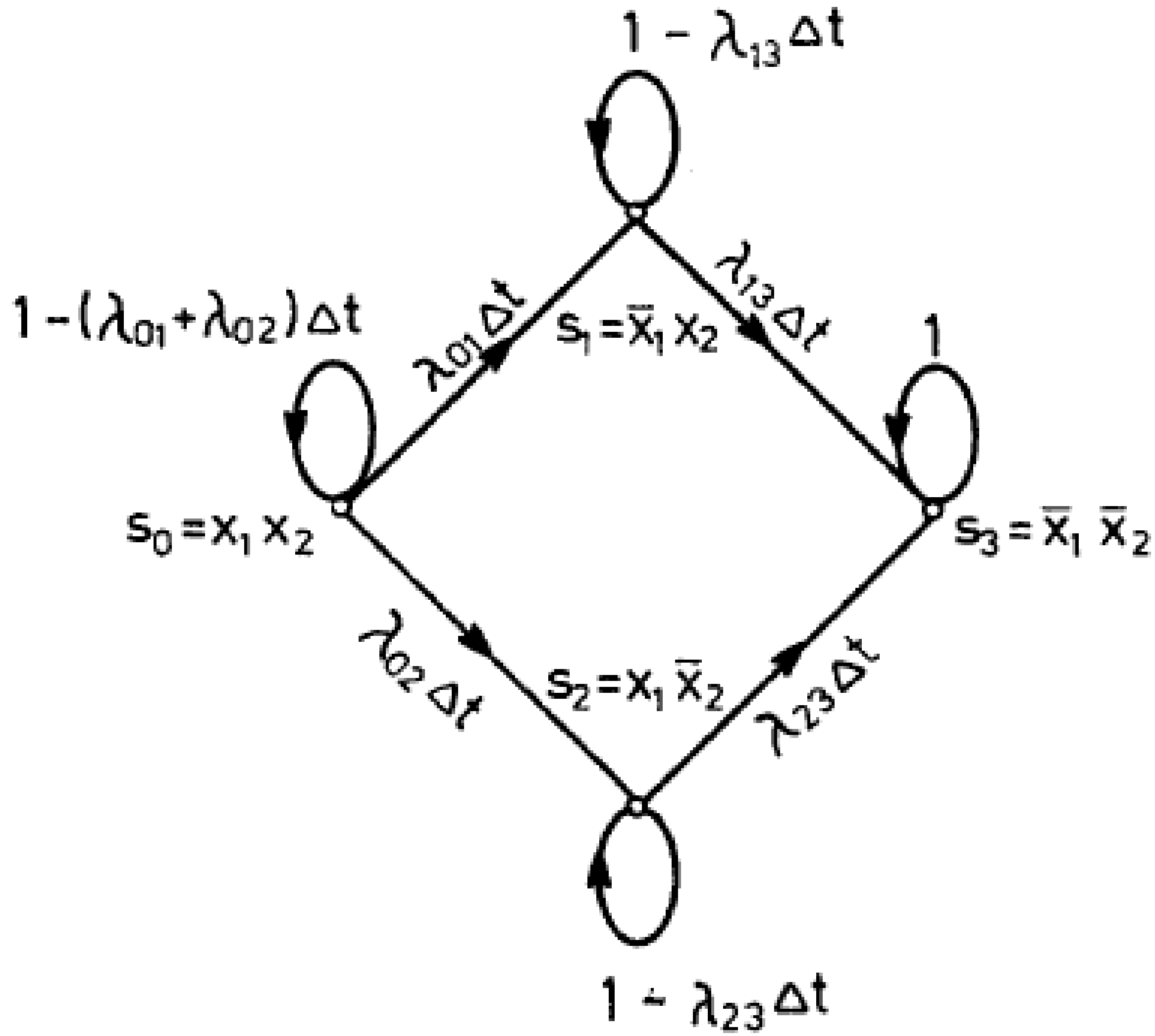
Using the Markov process we will determine reliability of the system with dependent components and the redundancy system.

We will assume that the system is made of two components that cannot be repaired or replaced.

In that case four system states are possible:

$$s_0 = x_1 x_2,\ s_1 = \bar{x}_1 x_2,\ s_2 = x_1 \bar{x}_2 \text{ i } s_3 = \bar{x}_1 \bar{x}_2$$

Reliability of a system made of dependent, irreparable and irreplaceable components



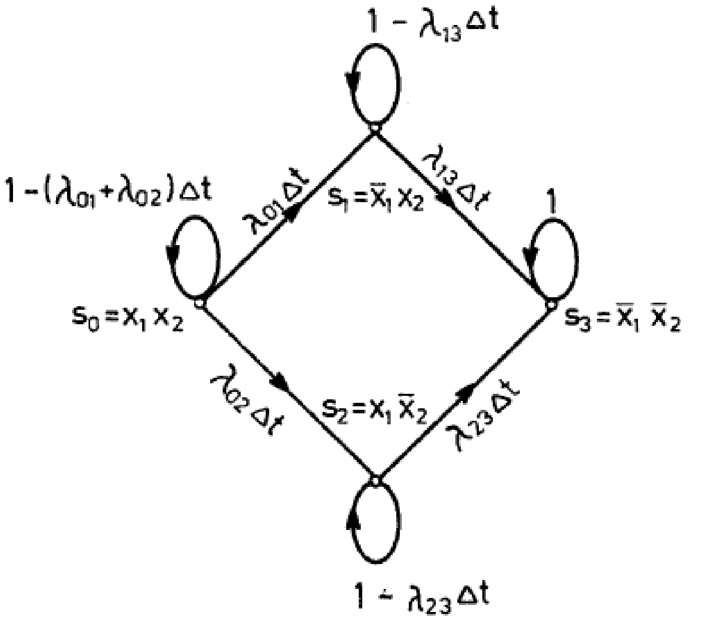
Reliability of a system made of dependent, irreparable and irreplaceable components

$$P_{S_0}{}'(t) = -(\lambda_{01} + \lambda_{02}) P_{S_0}(t)$$

$$P_{S_1}{}'(t) = -\lambda_{13} P_{S_1}(t) + \lambda_{01} P_{S_0}(t)$$

$$P_{S_2}{}'(t) = -\lambda_{23} P_{S_2}(t) + \lambda_{02} P_{S_0}(t)$$

$$P_{S_3}{}'(t) = \lambda_{13} P_{S_1}(t) + \lambda_{23} P_{S_2}(t)$$



Reliability of a system made of dependent, irreparable and irreplaceable components

By solving the system of differential equations we obtain the probabilities of **zero, one and two** failures of the system, ie. the probabilities of finding the system in the states $\mathbf{s_0}$, $\mathbf{s_1}$, $\mathbf{s_2}$ and $\mathbf{s_3}$ at any time **t** in the future:

$$P_{S_0}(t) = e^{-(\lambda_{01}+\lambda_{02})t}$$

$$P_{S_1}(t) = \frac{\lambda_{01}}{\lambda_{01}+\lambda_{02}-\lambda_{13}} / \, e^{-\lambda_{13}t} - e^{-(\lambda_{01}+\lambda_{02})t} \, /$$

$$P_{S_2}(t) = \frac{\lambda_{02}}{\lambda_{01}+\lambda_{02}-\lambda_{23}} / \, e^{-\lambda_{23}t} - e^{-(\lambda_{01}+\lambda_{02})t} \, /$$

Reliability of a system made of dependent, irreparable and irreplaceable components

The probability $\mathbf{P_{S3}(t)}$, ie. the probability that the system is in state $\mathbf{s_3}$, which also means that two failures have occurred in the system, is derived from the fact that the system must be in one of the possible 4 states at any given moment:

$$P_{S_3}(t) = 1 - / P_{S_0}(t) + P_{S_1}(t) + P_{S_2}(t) /$$

Reliability of a system made of dependent, irreparable and irreplaceable components

Notice that so far we have determined the reliability of the system based on the number of components and transitional probabilities, without saying anything about the system structure. In that way, we obtained the reliability of the system consisting of two components by finding the solutions for $\mathbf{P_{s0}(t)}$, $\mathbf{P_{s1}(t)}$ **and** $\mathbf{P_{s2}(t)}$, independent of the structure or type of the system.

For example, it is easy to show why the systems with the components connected in parallel are more reliable than the ones connected in series.

Reliability of a system made of dependent, irreparable and irreplaceable components

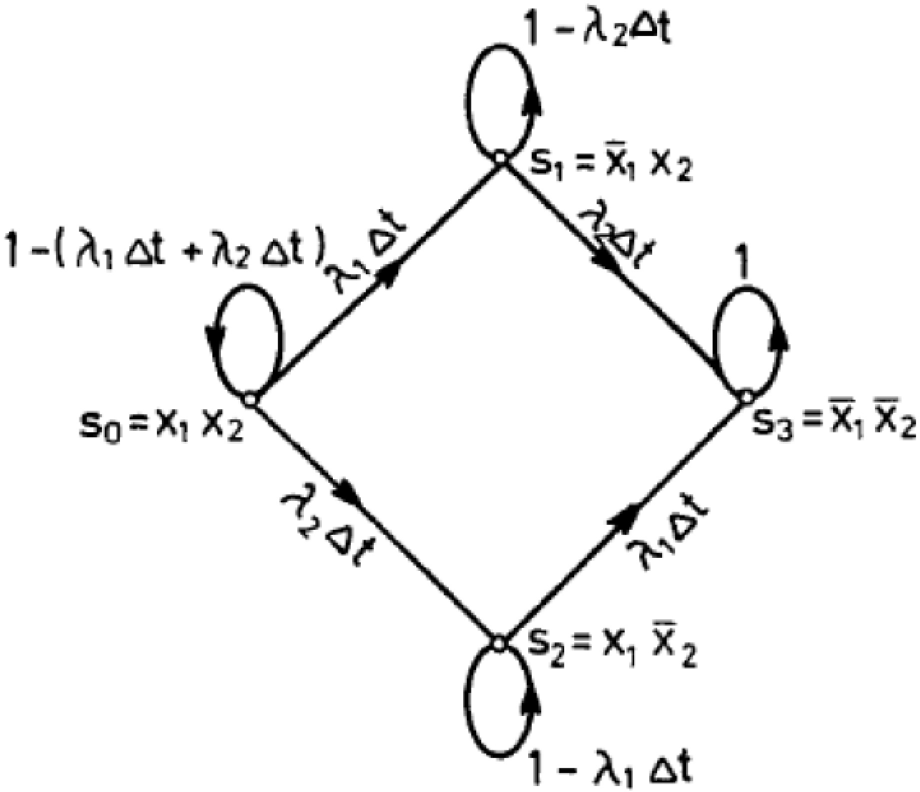
$$R_s(t) = P_{s0}(t) = e^{-(\lambda_{01} + \lambda_{02})t}$$

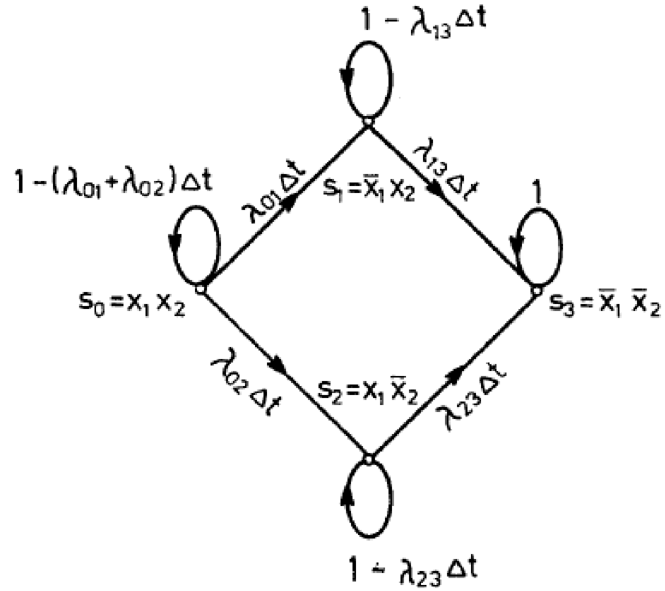
$$R_p(t) = P_{s0}(t) + P_{s1}(t) + P_{s2}(t) = e^{-(\lambda_{01} + \lambda_{02})t}$$

$$+ \frac{\lambda_{01}}{\lambda_{01} + \lambda_{02} - \lambda_{13}} \, / \, e^{-\lambda_{13}t} - e^{-(\lambda_{01} + \lambda_{02})t} / +$$

$$+ \frac{\lambda_{02}}{\lambda_{01} + \lambda_{02} - \lambda_{23}} \, / e^{-\lambda_{23}t} - e^{-(\lambda_{01} + \lambda_{02})t} /$$

Reliability of a system made of independent, irreparable and irreplaceable components



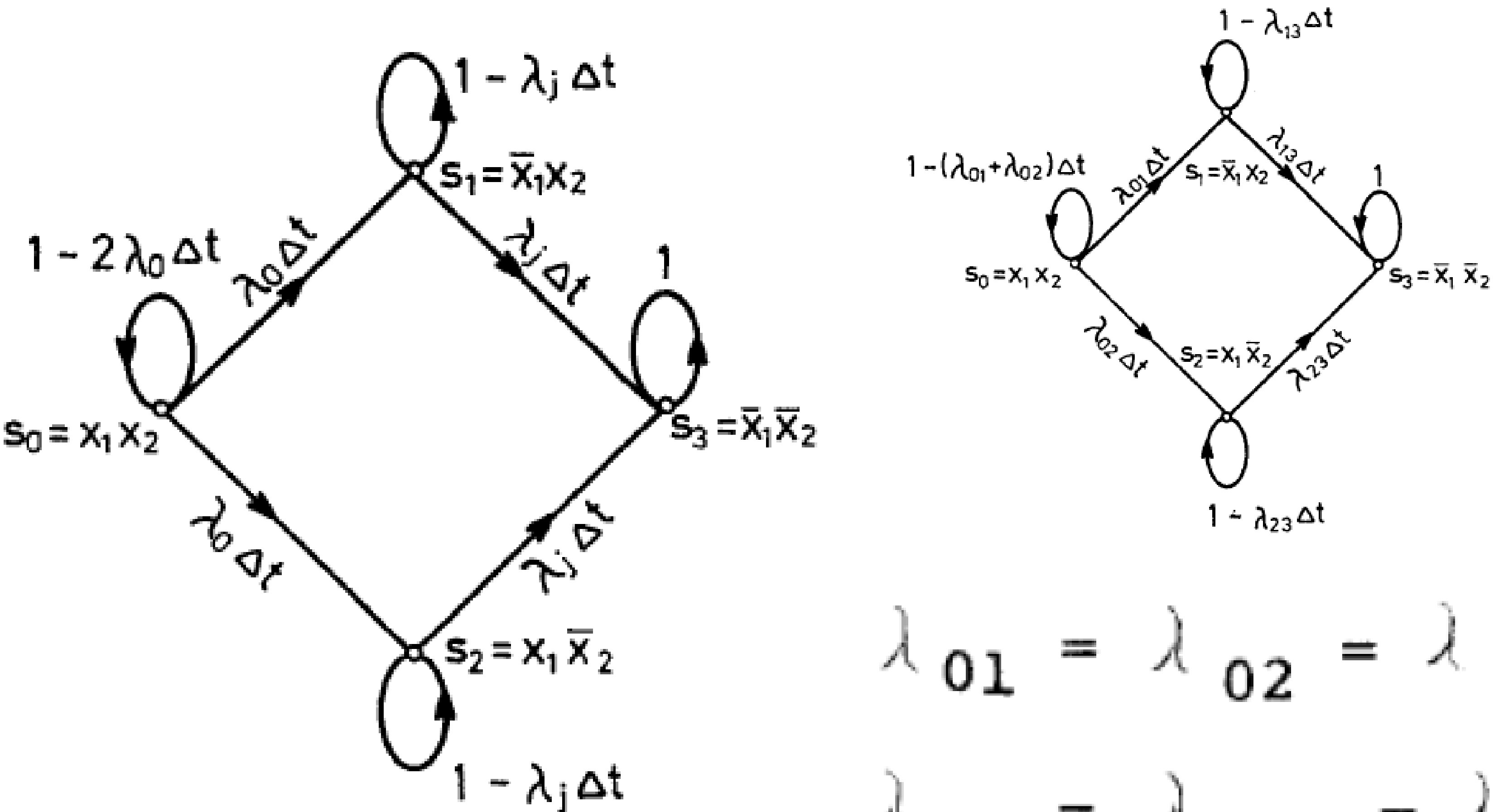


$$\lambda_{o1} = \lambda_{23} = \lambda_1$$

$$\lambda_{o2} = \lambda_{13} = \lambda_2$$

$$R_p(t) = e^{-\lambda_1 t} + e^{-\lambda_2 t} - e^{-(\lambda_1 + \lambda_2)t}$$

Reliability of the system with temperature dependent resistors



$$\lambda_{01} = \lambda_{02} = \lambda_0$$

$$\lambda_{13} = \lambda_{23} = \lambda_j$$

Reliability of the system with temperature dependent resistors

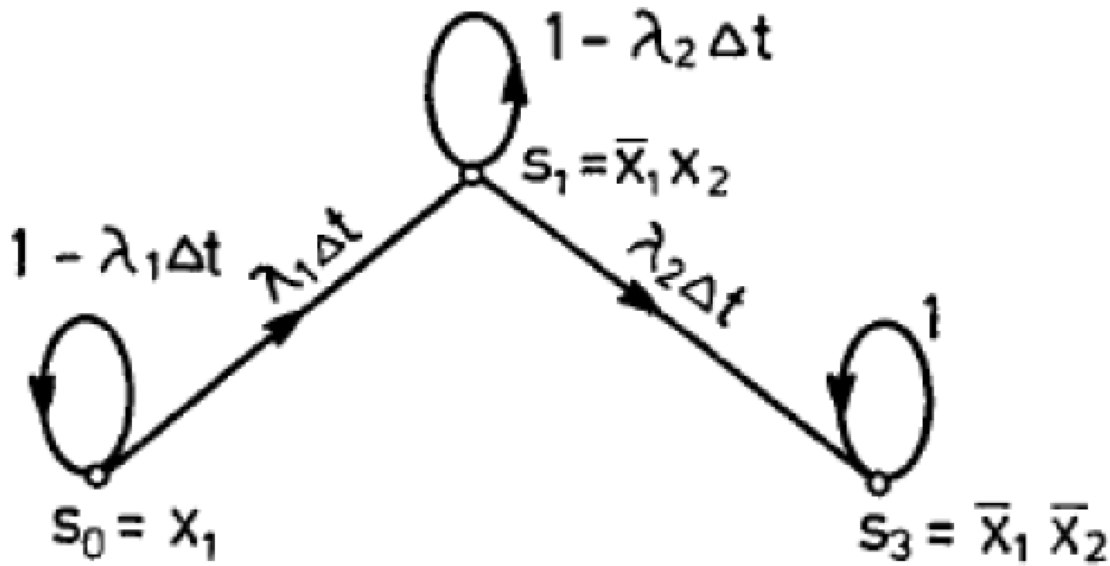
Replacing the failure rates in general equation, which represents the reliability function for any parallel system made of two components, we obtain reliability for a system with two temperature dependent resistors:

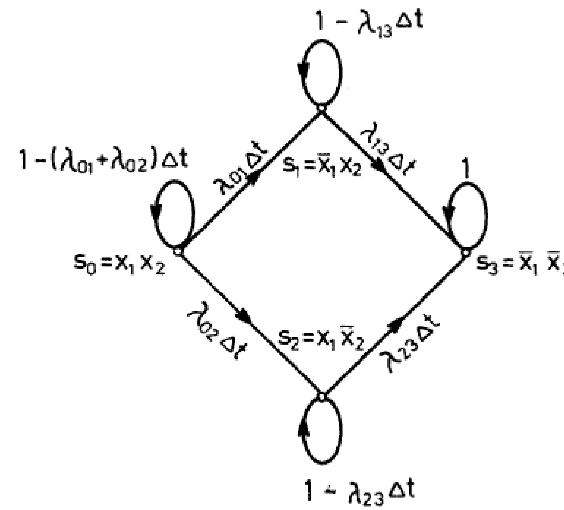
$$R(t) = \frac{2\lambda_o e^{-\lambda_j t} - \lambda_j e^{-2\lambda_o t}}{2\lambda_o - \lambda_j}$$

Reliability of a passive redundancy system

Similarly, if we apply general equation on a system with two components where one component is on standby, ie. the passive redundancy system which was in the initial state $\mathbf{s_0}$ (state with no failure), then $\boldsymbol{\lambda_{01}=\lambda_1}$ and $\boldsymbol{\lambda_{02}=0}$ since the second component cannot fail until the first component fails. In addition, $\boldsymbol{\lambda_{13}=\lambda_2}$, whereas the failure rate $\boldsymbol{\lambda_{23}}$ is not possible to define because the probability of the state $\mathbf{s_2}$ is zero, since $\mathbf{P_{so}(0)=1}$ and $\boldsymbol{\lambda_{02}=0}$:

Reliability of a passive redundancy system





$$R(t)=\frac{\lambda_1}{\lambda_1-\lambda_2}e^{-\lambda_2 t}-\frac{\lambda_2}{\lambda_1-\lambda_2}e^{-\lambda_1 t}$$