Task 1

Consider a set of elements containing three systems. System A starts operating at time t=0 h, system B at time t=220 h, and system C at time t=400 h. After time t=600 h six failures have been recorded. What is the failure rate, mean time between failures, and the probability that the failure is not going to occur within the next 100 h of operation?

- System A worked in total t_1 = 600 h,
- System B worked in total t_2 = 380 h,
- System C worked in total time t_3 = 200 h,

- The total time of operation of all the systems is T = 1180 h.

Task 1

- Failure rate is:

$$\lambda = \frac{number\ of\ failures}{total\ time} = \frac{6}{1180} = 0.0051\ h^{-1}$$

- Mean time to failure is:

$$MTTF = T_0 = \frac{1}{\lambda} = \frac{1}{0.0051} = \ 197\ h$$

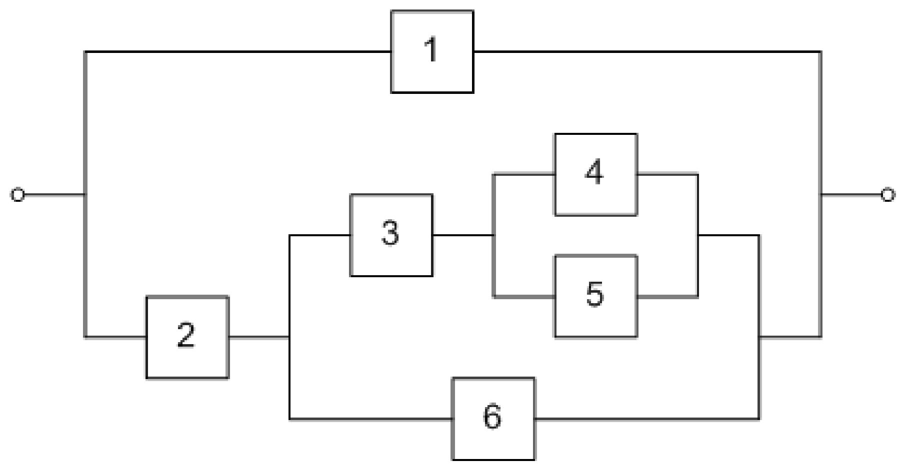
- The probability that in the next 100 h there will be no failures is:

$$R(t, t + \Delta t) = \frac{R(t + \Delta t)}{R(t)} = \frac{e^{-\lambda(t+\Delta t)}}{e^{-\lambda t}} = e^{-\lambda \Delta t}$$

$$R(600{,}700) = e^{-\lambda \cdot 100} = e^{-0.0051 \cdot 100} = 0.6$$

Task 2

Calculate the reliability of the system shown in the figure after one year of operation.



Given the values of MTTF:

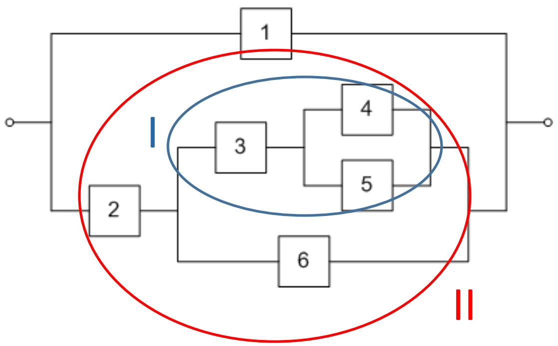
Comp. 1, 6: 20000 h   Comp. 2, 3: 30000 h

Comp. 4: 50000 h   Comp. 5: 60000 h

Task 2

First we calculate the reliability of each component using the expression:

$$R(t) = e^{-\lambda t} = e^{-\frac{t}{T_0}}$$



t = 8760 h

R_1=R_6=0.6453 R_2=R_3=0.7468

R_4=0.8393 R_5=0.8642

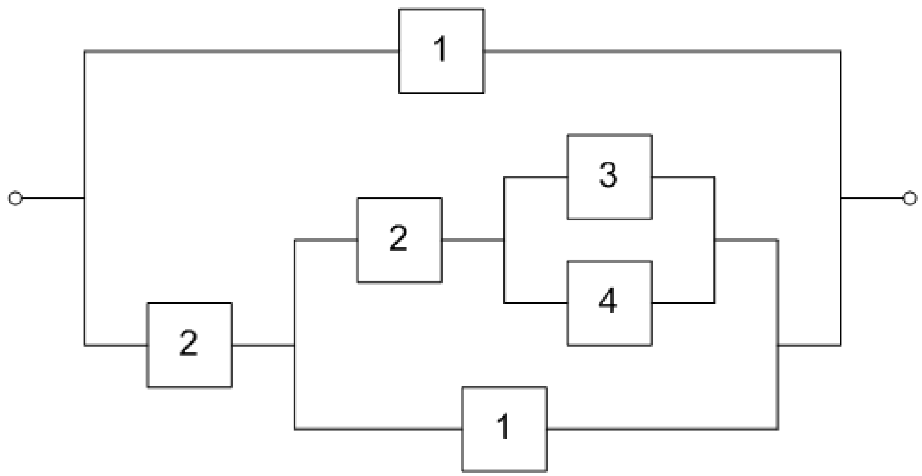
$R_I = R_3 \cdot [1-(1-R_4)\cdot(1-R_5)] = 0.7468 \cdot 0.9782 = 0.7305$

$R_{II} = R_2 \cdot [1-(1-R_I)\cdot(1-R_6)] = 0.7468 \cdot 0.9044 = 0.6754$

$R_{tot} = 1-(1-R_1)\cdot(1-R_{II}) = 0.885$

Task 3

Calculate the reliability of the system shown in the figure after one year of operation.



Given the values of MTTF:

Comp. 1: 20.000 h   Comp. 2: 30.000 h

Comp. 3: 50.000 h   Comp. 4: 60.000 h

Task 3

First, we calculate the reliabilities of each component using the expression:

$$R(t) = e^{-\lambda t} = e^{-\frac{t}{T_0}}$$

t = 8760 h

R_1=0.6453

R_2= 0.7468

R_3= 0.8393

R_4= 0.8642

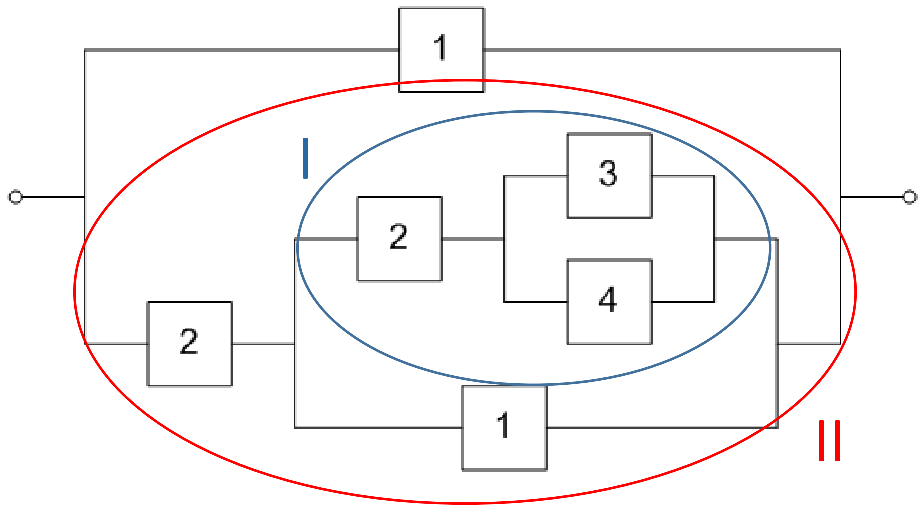
In this case, to derive the expression for system reliability we have to use Boolean algebra because some components can be found in more than one path.

For some event A:

- A·A = A ≠ A^2 (the intersection of an event with itself is still the same event)

  P(A·A) = P(A) ≠ $P(A)^2$, P(A) = R (reliability of the component)

Task 3

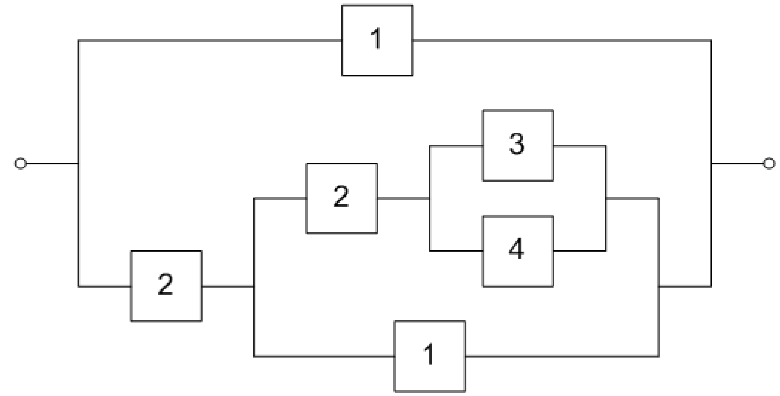


$$R_I = R_2 \cdot [1 - (1 - R_3) \cdot (1 - R_4)] = R_2R_3 + R_2R_4 - R_2R_3R_4$$

$$R_{II} = R_2 \cdot [1 - (1 - R_I) \cdot (1 - R_1)] =$$
$$= R_1R_2 + R_2R_3 + R_2R_4 - R_1R_2R_3 - R_1R_2R_4 - R_2R_3R_4 + R_1R_2R_3R_4$$

$$R_{tot} = 1 - (1 - R_1) \cdot (1 - R_{II}) =$$
$$= R_1 + R_2R_3 + R_2R_4 - R_1R_2R_3 - R_1R_2R_4 - R_2R_3R_4 + R_1R_2R_3R_4 = 0.9044$$

Task 3



$R_{tot} = P(x_1 + x_2(x_1 + x_2(x_3 + x_4))) =$

$= P(x_1 + x_2x_1 + x_2(x_3 + x_4)) =$

$= P(x_1 + x_1x_2 + x_2x_3 + x_2x_4) =$

$=$ *($2^4 - 1$ terms)* $=$

$= P(x_1) + P(x_1x_2) + P(x_2x_3) + P(x_2x_4) -$

$- P(x_1x_2) - P(x_1x_2x_3) - P(x_1x_2x_4) - P(x_1x_2x_3) - P(x_1x_2x_4) - P(x_2x_3x_4) +$

$+ P(x_1x_2x_3) + P(x_1x_2x_4) + P(x_1x_2x_3x_4) + P(x_1x_2x_3x_4) -$

$- P(x_1x_2x_3x_4) =$

$=$ *(given $P(x) = R$)* $=$

$= R_1 + R_2R_3 + R_2R_4 - R_1R_2R_3 - R_1R_2R_4 - R_2R_3R_4 + R_1R_2R_3R_4 = 0.9044$

Task 4

Calculate the reliability of the system with a **parallel** reliability model consisting of three **dependent** components. The probability that component 1 fails is 0.1; the probability that component 2 fails given the component 1 failed is 0.3; and the probability that component 3 fails given both component 1 and 2 failed is 0.5.

$$R_p(t) = 1 - Q_p(t) = 1 - P(\bar{x}_1\bar{x}_2\bar{x}_3)$$

$$R_p(t) = 1 - Q_p(t) = 1 - P(\bar{x}_1)P(\bar{x}_2|\bar{x}_1)P(\bar{x}_3|\bar{x}_1\bar{x}_2)$$

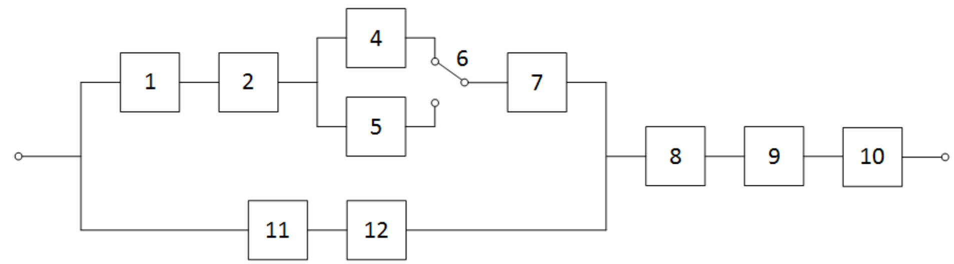
$$R_p(t) = 1 - Q_p(t) = 1 - 0.1 \cdot 0.3 \cdot 0.5 = 0.985$$

Task 5

What is the reliability of aircraft installations used for startup of the commands during a 1.5 h flight given the exponential distribution.

Input data:

Tank: $\lambda_1 = 66.67 \cdot 10^{-6}\ h^{-1}$



Hydraulic pump: $\lambda_2 = 9.0 \cdot 10^{-6}\ h^{-1}$

Filter: $\lambda_4 = 666.7 \cdot 10^{-6}\ h^{-1}$

Overflow valve: $\lambda_5 = 5.7 \cdot 10^{-6}\ h^{-1}$

Switch: $\lambda_6 = 0.12 \cdot 10^{-6}\ h^{-1}$, $Q_6' = 0.0000015$

Pressure reg.: $\lambda_7 = 0.054 \cdot 10^{-6}\ h^{-1}$

Pipe: $R_8 = 0.99935$

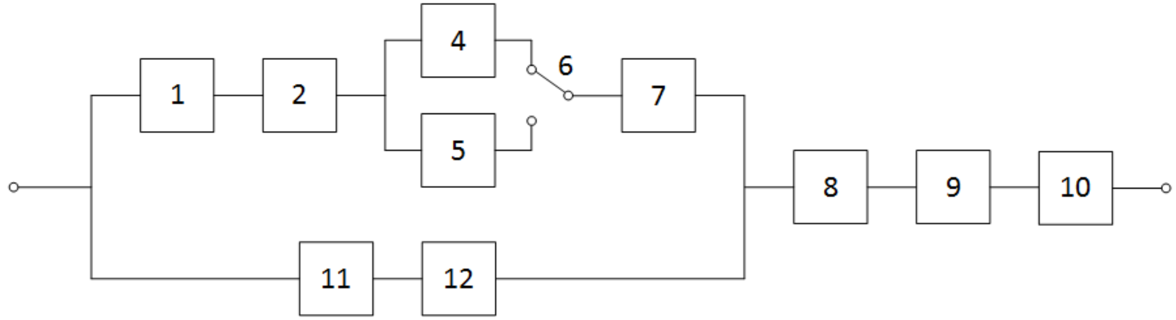
Connectors: $\lambda_9 = 100.0 \cdot 10^{-6}\ h^{-1}$

Servo device: $\lambda_{10} = 100.0 \cdot 10^{-6}\ h^{-1}$

Gas reg.: $\lambda_{11} = 0.001\ h^{-1}$

Accumulator: $\lambda_{12} = 8.3 \cdot 10^{-5}\ h^{-1}$

Task 5



$R_1 = e^{-\lambda 1t} = 0.9999$

$R_2 = e^{-\lambda 2t} = 0.99998650$

$R_4 = e^{-\lambda 4t} = 0.99900045$

$R_5 = e^{-\lambda 5t} = 0.99999145$

$R_6 = e^{-\lambda 6t} = 0.99999982$ *(Switch activated on a demand)*

$R_6' = 1 - Q_6' = 0.9999985$ *(Switch not activated too early)*

$R_7 = e^{-\lambda 7t} = 0.99999992$

$R_8 = 0.99935$

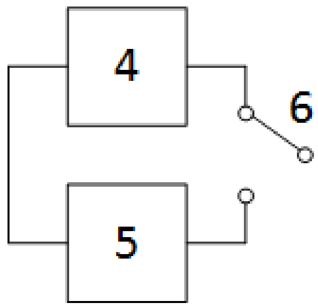
$R_9 = e^{-\lambda 9t} = 0.99985001$

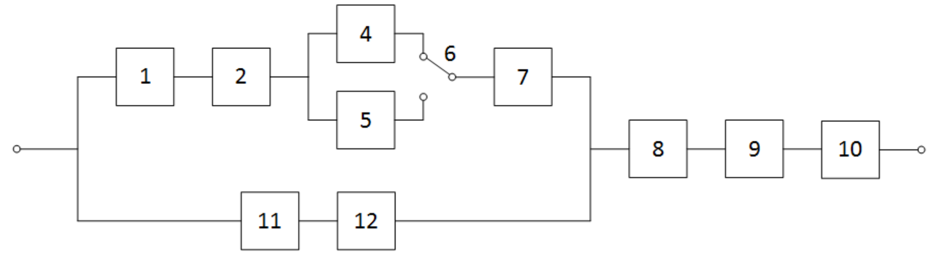
$R_{10} = e^{-\lambda 10t} = 0.99985001$

$R_{11} = e^{-\lambda 11t} = 0.99850112$

$R_{12} = e^{-\lambda 12t} = 0.99987551$

Task 5





$R_{456} = R_4\,R_6' + (1 - R_4)\,R_6\,R_5 + R_4\,(1 - R_6')\,R_5 =$
$= 0.99900045 \cdot 0.9999985 +$
$+ (1 - 0.99900045) \cdot 0.99999982 \cdot 0.99999145 +$
$+ 0.99900045 \cdot (1 - 0.9999985) \cdot 0.99999145 =$
$= 0.99999999$

Upper row: $R_G = R_1\,R_2\,R_{456}\,R_7 = 0.999886411$
Lower row: $R_D = R_{11}\,R_{12} = 0.998376816$
Parallel: $R_{DG} = R_D + R_G - R_D\,R_G = 0.999999815$
Total: $R = R_{DG}\,R_8\,R_9\,R_{10} = 0.999$

Task 1

– The mean time to failure is described with the exponential distribution and its value is 25 h. The mean time to repair is also described with the exponential distribution and its value is 40 h. Draw the availability function graph and calculate the steady state availability.

$$\lambda = \frac{1}{MTTF} = \frac{1}{25} = 0{,}04\ h^{-1}$$

$$\mu = \frac{1}{MTTR} = \frac{1}{40} = 0{,}025\ h^{-1}$$

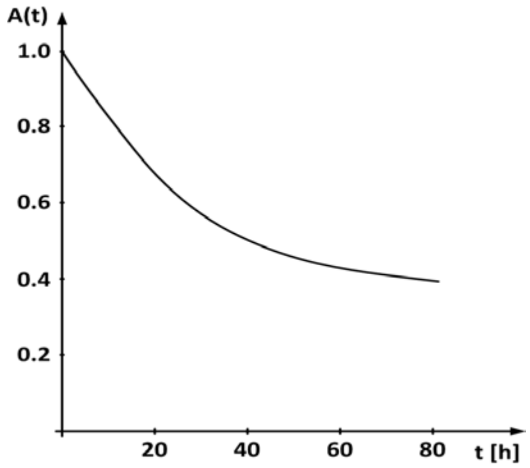
Steady state availability:

$$A_0 = \frac{\mu}{\lambda + \mu} = \frac{0{,}025}{0{,}04 + 0{,}025} = 0{,}385$$

Task 1

– Availability function:

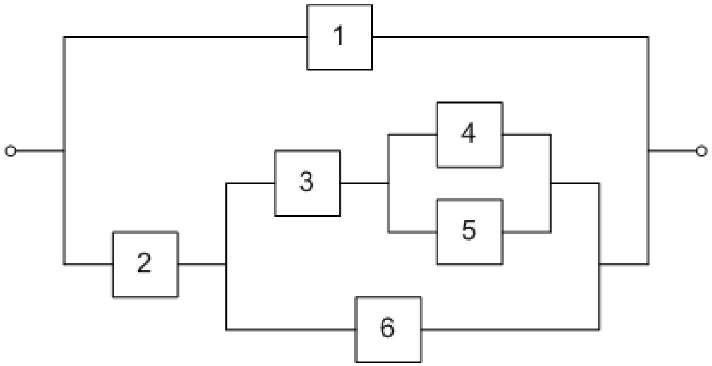
$$A(t)=\frac{\mu}{\lambda+\mu}+\frac{\lambda}{\lambda+\mu}e^{-(\lambda+\mu)t}=\frac{0,025}{0,04+0,025}+\frac{0,04}{0,04+0,025}e^{-(0,04+0,025)t}$$



Task 2

- Calculate the availability of the system shown in the figure after one year of operation, given the data in the table:

Component	MTTF [h]	MTTR [h]
1	2.000	10
2	3.000	12
3	3.000	12
4	5.000	14
5	6.000	15
6	2.000	10



Task 2

- Firstly, we calculate the availability of the components according to the relation:

$$A(t) = \frac{\mu}{\lambda + \mu} + \frac{\lambda}{\lambda + \mu} e^{-(\lambda+\mu)t}$$

t = 8760 h

$$\lambda = \frac{1}{MTTF}$$

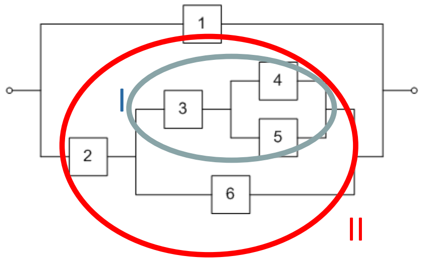
$$\mu = \frac{1}{MTTR}$$

Component	λ [h^{-1}]	μ [h^{-1}]	$A(8760)=A_0$
1	$5 \cdot 10^{-4}$	0.1	0.995
2	$3.333 \cdot 10^{-4}$	0.08333	0.996
3	$3.333 \cdot 10^{-4}$	0.08333	0.996
4	$2 \cdot 10^{-4}$	0.0714	0.997
5	$1.666 \cdot 10^{-4}$	0.0667	0.998
6	$5 \cdot 10^{-4}$	0.1	0.995

Task 2

The transient part is exponentially approaching zero ($e^{-5} = 0.007$, what happens after $\sim 50\ h$), as well as the the ratio $\lambda/(\lambda+\mu) \approx 10^{-3} - 10^{-2}$:

- To calculate the availability, we use the same rules as for calculating the reliability



$$A_I = A_3 \cdot [1-(1-A_4)\cdot(1-A_5)] = 0.996 \cdot 0.999994 = 0.996$$

$$A_{II} = A_2 \cdot [1-(1-A_I)\cdot(1-A_6)] = 0.996 \cdot 0.99998 = 0.996$$

$$A_{uk} = 1-(1-A_1)\cdot(1-A_{II}) = 0.99998$$