Reliability and Availability Assessment Methods

Reliability and availability of the systems with repairable components

Systems with reparable (replaceable) components

So far we have assumed that the **component failures were irreversible**; **failed components were removed from the system thus changing the system's structure.**

In further considerations we will deal with the system with **reparable or replaceable** components and we will investigate the influence of these actions on the increase of the system reliability.

About component reliability

When describing reparable systems, along with the reliability, we consider the properties which determine **the degree of usefulness of the repairs in the system** which are specific for all the systems that allow the failure to last for a while.

In addition to reliability, the most important feature of the system is the function of the system called **"availability".**

Availability function A(t) is defined as the probability that the system will work properly at an arbitrary time t in the future.

Component reliability and availability

- **R(t) ≤ A(t)**
- **A(t) = R(t)** (irreparable component)
- **R(t) < A(t)** (reparable component)
- **The component repair does not change the reliability of the series system, but changes its availability**
- **In redundancy systems (parallel system, system with a backup) the repair significantly changes both reliability and availability functions, R(t) and A(t)**

Condition for determining the availability function

Let’s analyze the influence of the component repair on the system reliability and availability.

Recall the condition that has to be met in order to use the **Markov process** for system availability determination:

- **The repair probability density function has to be exponential, which means that <u>the repair rate function is constant</u> (model without memory)**

Component availability

$$P_{so}(t + \Delta t) = P_{so}(t)(1 - \lambda \Delta t) +$$

$$+ P_{s1}(t)\ (\mu \Delta t)$$

$$P_{s1}(t + \Delta t) = P_{o}(t)(\lambda \Delta t) + P_{s1}(t)$$

$$(1 - \mu \Delta t)$$

The probability that a component will be repaired in time **Δt** is equal to **μΔt**, and the probability that it will remain broken is **(1-μΔt)**.

(**μ** is **a constant repair rate**)

Component availability– transient probability matrix

	$s_0 + \Delta t$	$s_1 + \Delta t$
s_0	$1 - \lambda \Delta t$	$\lambda \Delta t$
s_1	$\mu \Delta t$	$1 - \mu \Delta t$

Without repair

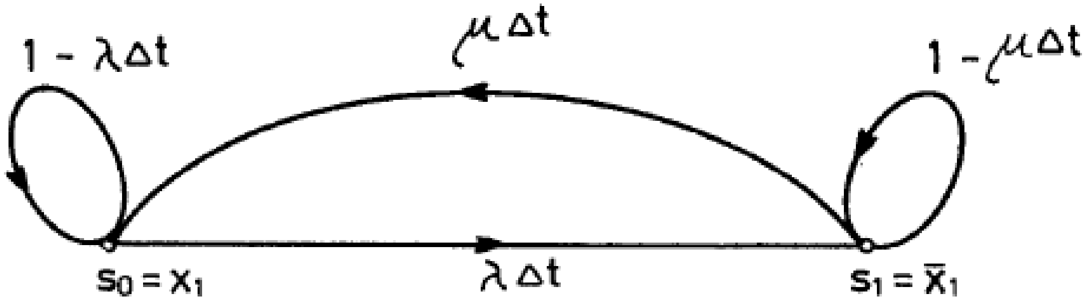
	Final states	
Initial states	$s_0(t+\Delta t)$	$s_1(t+\Delta t)$
$s_0(t)$	$1 - \lambda \Delta t$	$\lambda \Delta t$
$s_1(t)$	0	1

Rearranging previous equations we obtain the following differential equations:

$$P'_{so}(t) = -\lambda P_{so}(t) + \mu P_{s1}(t)$$

$$P'_{s1}(t) = -\mu P_{s1}(t) + \lambda P_{so}(t)$$

Component availability – Markov graph



$$P'_{s0}(t) = -\lambda P_{s0}(t) + \mu P_{s1}(t)$$

$$P'_{s1}(t) = -\mu P_{s1}(t) + \lambda P_{s0}(t)$$

$$P_{s0}(0) = 1; \quad P_{s1}(0) = 0$$

Component availability and unavailability

The probabilities of system states **s_0** and **s_1** are determined by solving the system of equations, ie., since the availability is by its definition **“the probability that the system works properly at a certain time t”**, **the availability and unavailability functions are determined:**

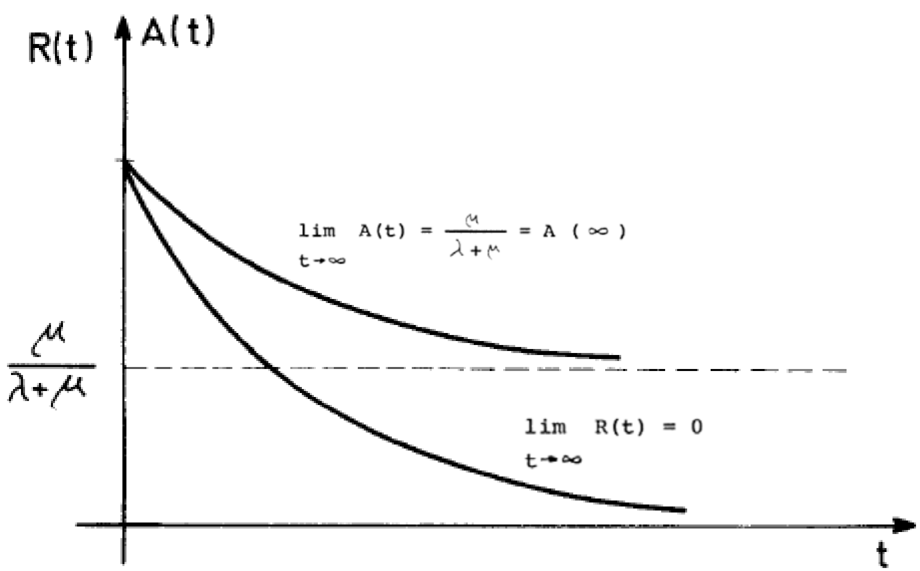
$$A(t) = P_{so}(t) = \frac{\mu}{\lambda + \mu} + \frac{\lambda}{\lambda + \mu} e^{-(\lambda + \mu)t}$$

$$N(t) = P_{s1}(t) = \frac{\lambda}{\lambda + \mu} - \frac{\lambda}{\lambda + \mu} e^{-(\lambda + \mu)t}$$

Certainly, **A(t) + N(t) = 1** because there are only two possible system states regarding availability, ie., unavailability

Availability and reliability functions

If we draw the function graphs, we will notice a significant difference in behavior between the reliability and availability functions**: if t is approaching infinity**, **the reliability function approaches zero** and **the availability function approaches a certain constant, stationary value**:



$$\lim_{t\to\infty} A(t) = \frac{\mu}{\lambda+\mu} = A(\infty)$$

$$\lim_{t\to\infty} N(t) = \frac{\lambda}{\lambda+\mu} = N(\infty)$$

$$A(\infty) + N(\infty) = 1$$

Mean time to repair

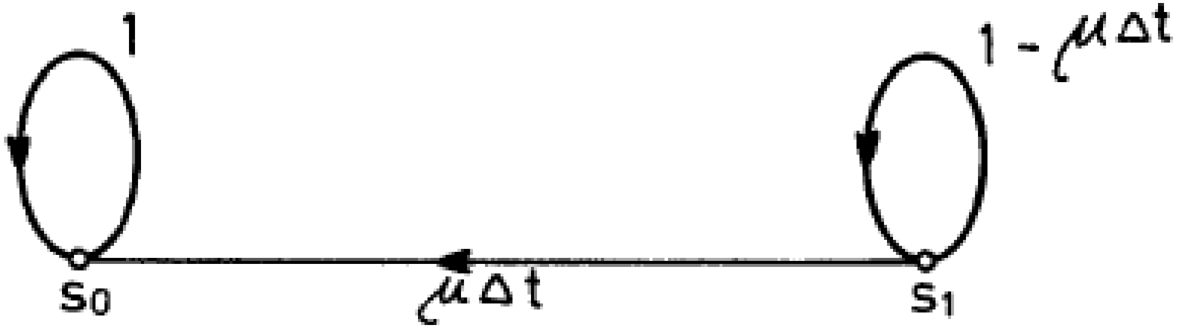
Consider some other important properties related to the component availability.

It has already been mentioned that the component repair probability in time **Δt** is equal to **$\mu\Delta t$**, where **μ is the repair rate**. The probability that a broken component will not be repaired during time **Δt** is **$1-\mu\Delta t$**.

Using Markov analysis we will find the probability P(T) that the component will be repaired during time T – <u>Maintainability M(T)</u>.

If we assume that once repaired component will not fail again (the initial state is now s_1 and the final state s_0), Markov equations are the following:

Mean time to repair



$$P_0'(T) = \mu P_1(T) \qquad P_1'(T) = -\mu P_1(T)$$

At time **t=0** a component is broken; thus **$P_0(0)=0$,** and **$P_1(0)=1$**. Solving the equations we obtain:

$$P_0(T) = 1 - e^{-\mu T} \quad \text{i} \quad P_1(T) = e^{-\mu T}$$

Mean time to repair

It follows that the probability that a component will remain broken during time **T** is equal to **$e^{-\mu T}$**, and the probability that a failed component will be repaired during time **T** (maintainability) is equal to **$1- e^{-\mu T}$**.

In that case „*mean time to repair - MTTR*”, *in accordance with* $T_0 = \int_0^\infty R(t)dt$, is equal to:

$$T_p = \int_0^\infty P_1(T)\ dT = \int_0^\infty e^{-\mu T}\ dt = \frac{1}{\mu}$$

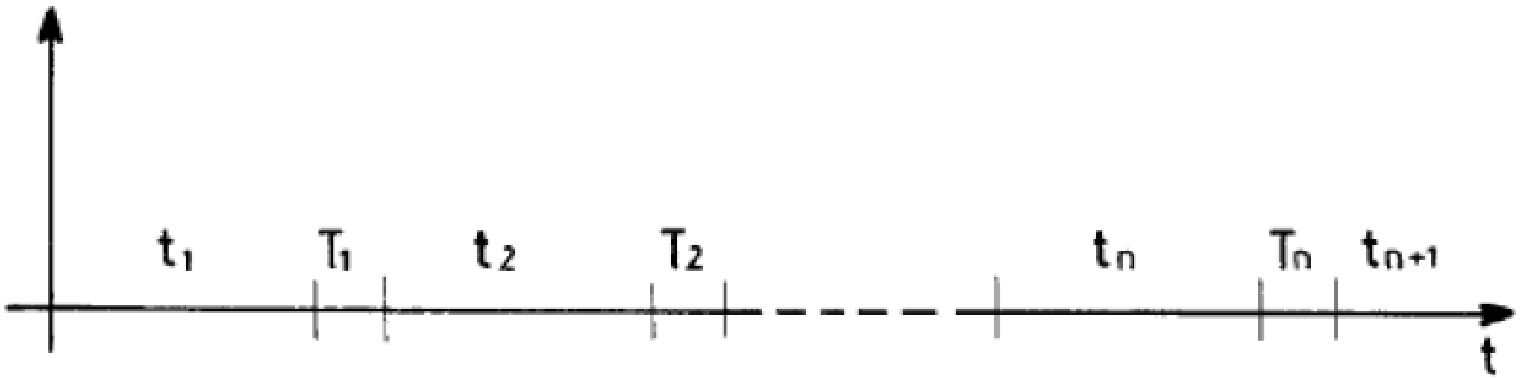
Thus, the obtained relation is analogous to the relation for component mean time to failure: $\frac{1}{\lambda}$

Mean time to repair

In normal cases, **mean time to repair** $\frac{1}{\mu}$, ie. **mean down time of the component is much lower than the component mean time to failure**, ie., **mean time of the component correct operation ($\mu >> \lambda$).**

Therefore, if we consider the typical failure history of the component, we can observe **a series of periods of correct component operation interrupted by periods of component failure ($T_o >> T_p$)**, figure.

Proper work and failure distribution



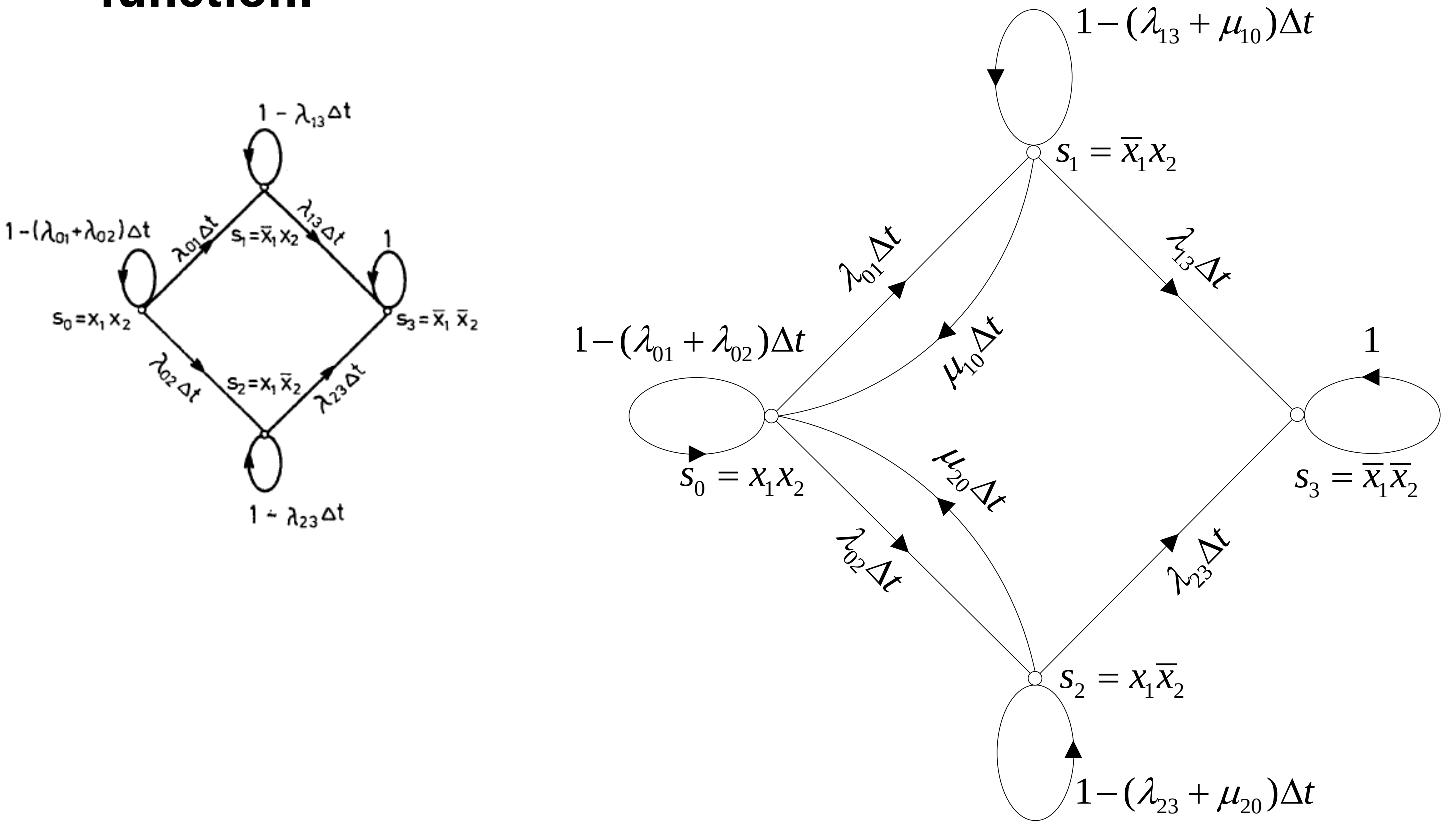
$t_1, t_2, \ldots t_i, \ldots, t_n \rightarrow T_0$ (mean time of the component correct operation or mean time to failure)

$T_1, T_2, \ldots T_i, \ldots, T_n \rightarrow T_p$ (mean down time or mean time to repair)

$$T_0 = \frac{1}{\lambda}, \quad T_p = \frac{1}{\mu} \qquad A(\infty) = \frac{T_0}{T_0 + T_p}, \qquad N(\infty) = \frac{T_p}{T_0 + T_p}$$

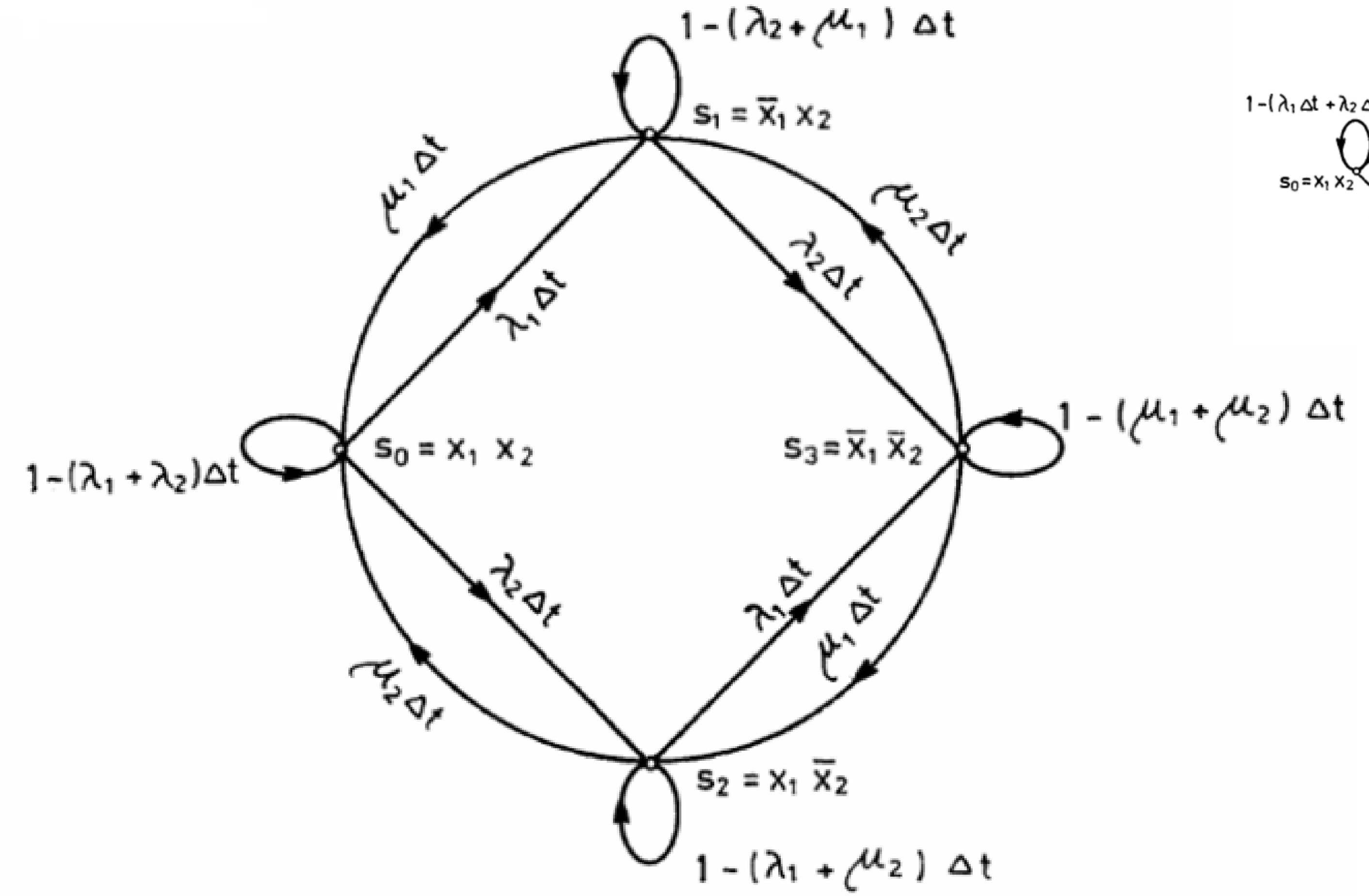
The reliability of the system with two different reparable components

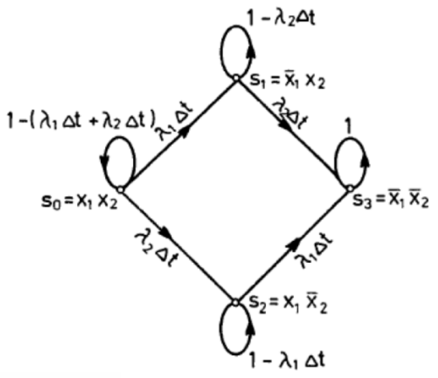
If we consider a system whose components are not connected in series, the repairs will affect both availability function and reliability function.



Markov **reliability** graph of the system with two reparable components

The availability of the system with two different reparable components





Markov **availability** graph of the system with two reparable components

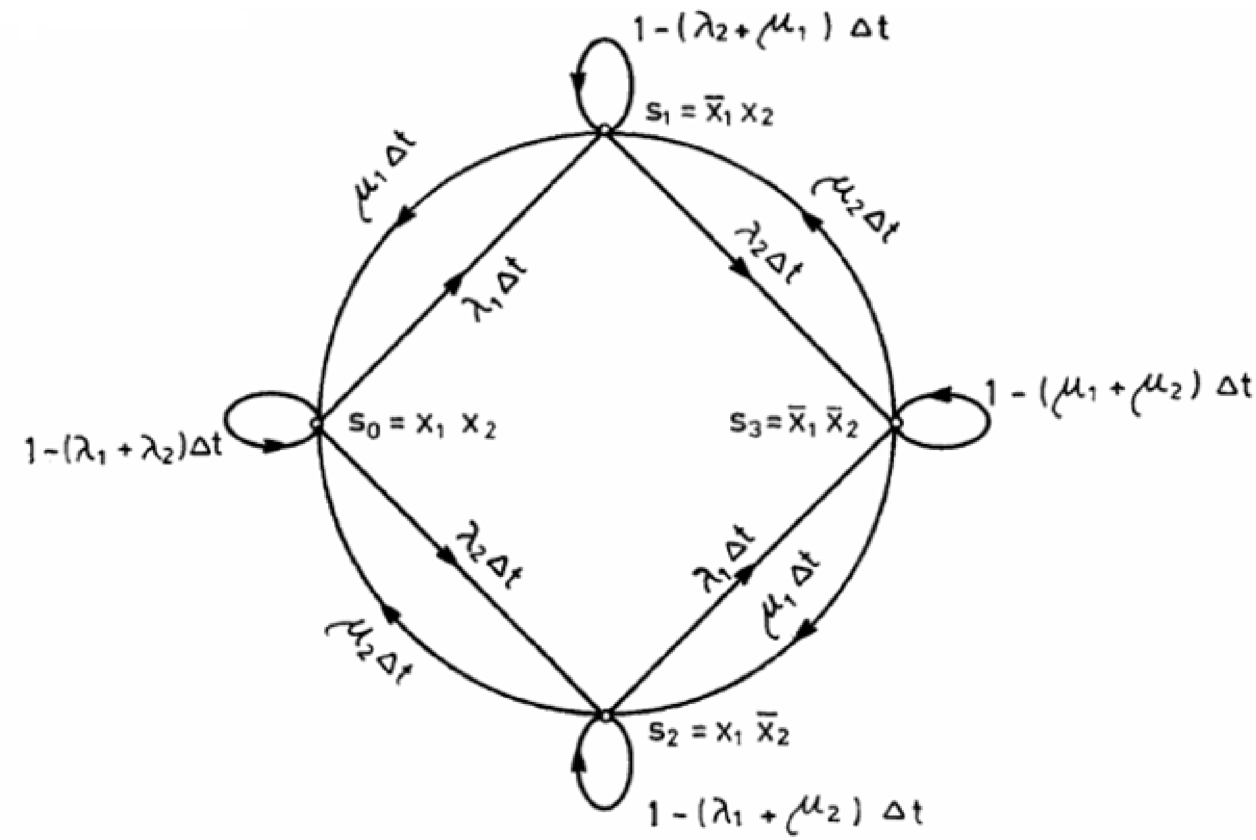
Steady-state availability determination

Let's demonstrate how to determine the **steady-state availability** of the system.

In the case of a complex system, the determination of time dependent availability function is a very demanding task so we will be **satisfied with knowing the steady-state availability**. This is acceptable because in most practical cases **the transient part of the availability function is negligible due to short duration.**

Steady-state availability determination

Consider the **system with two different reparable components.** Based on the Markov graph we can write the steady-state equations.



Steady-state availability determination

In the following equations all the probabilities are constant due to steady-state condition:

$$P'_{so}(t) = P'_{s1}(t) = P'_{s2}(t) = P'_{s3}(t) = 0$$

$$0 = -(\lambda_1 + \lambda_2) P_{so}(\infty) + \mu_1 P_{s1}(\infty) + \mu_2 P_{s2}(\infty)$$

$$0 = \lambda_1 P_{so}(\infty) - (\lambda_2 + \mu_1) P_{s1}(\infty) + \mu_2 P_{s3}(\infty)$$

$$0 = \lambda_2 P_{so}(\infty) - (\lambda_1 + \mu_2) P_{s2}(\infty) + \mu_1 P_{s3}(\infty)$$

$$0 = \lambda_2 P_{s1}(\infty) + \lambda_1 P_{s2}(\infty) - (\mu_1 + \mu_2) P_{s3}(\infty)$$

Steady-state availability determination

To solve this set of equations, let's introduce the fifth equation:

$$P_{s0}(\infty) + P_{s1}(\infty) + P_{s2}(\infty) + P_{s3}(\infty) = 1$$

Then we get the steady-state probabilities:

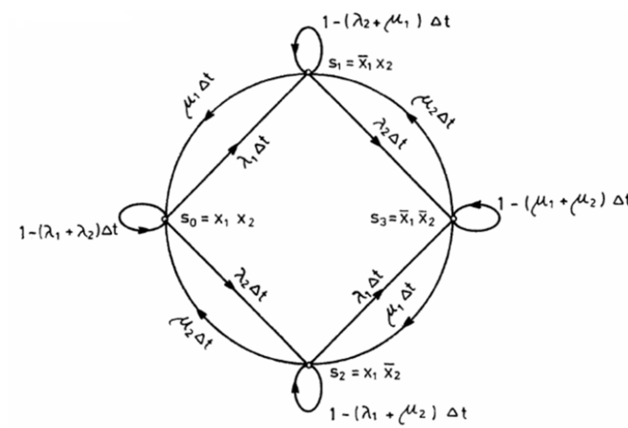
$$P_{s0}(\infty) = \frac{\mu_1 \mu_2}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)}$$

Steady-state availability determination

$$P_{s1}(\infty) = \frac{\lambda_1 \mu_2}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)}$$

$$P_{s_2}(\infty) = \frac{\mu_1 \lambda_2}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)}$$

$$P_{s3}(\infty) = \frac{\lambda_1 \lambda_2}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)}$$



Taking into account the expressions for steady-state availability and unavailability of the component, we get the steady-state values of each system state:

Steady-state availability determination

$$P_{so}(\infty) = A_1(\infty) \cdot A_2(\infty); \quad P_{s1}(\infty) = N_1(\infty) \cdot A_2(\infty)$$

$$P_{s2}(\infty) = A_1(\infty) \cdot N_2(\infty); \quad P_{s3}(\infty) = N_1(\infty) \cdot N_2(\infty)$$

Thus, knowing the general expressions for availability, ie., unavailability of the component in a steady-state, we can derive the expressions for steady-state probabilities of the system states, ie., the steady-state availability of the system. Eg., the expression of a parallel system is the following:

$$A(\infty) = P_{so}(\infty) + P_{s1}(\infty) + P_{s2}(\infty)$$

Steady-state availability determination

Analogously, we can derive similar expressions for any number of components in the system, while the expression for steady-state availability of the system will depend on the specific structure of the system.

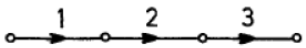
Therefore, the probabilities of the system states for three components are (for simplicity the infinity subscript is omitted):

$$P_{so} = A_1A_2A_3; \quad P_{s1} = N_1A_2A_3; \quad P_{s2} = A_1N_2A_3;$$

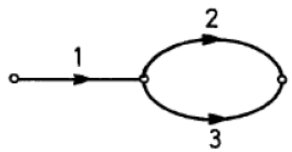
$$P_{s3} = A_1A_2N_3; \quad P_{s4} = N_1N_2A_3; \quad P_{s5} = N_1A_2N_3;$$

$$P_{s6} = A_1N_2N_3 \quad i \quad P_{s7} = N_1N_2N_3$$

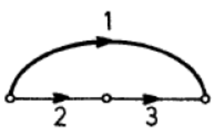
Steady-state availability determination



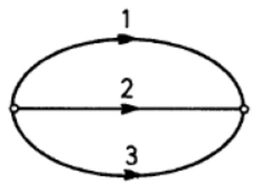
System A



System B



System C



System D

$P_{so} = A_1A_2A_3$; $P_{s1} = N_1A_2A_3$; $P_{s2} = A_1N_2A_3$;

$P_{s3} = A_1A_2N_3$; $P_{s4} = N_1N_2A_3$; $P_{s5} = N_1A_2N_3$;

$P_{s6} = A_1N_2N_3$ i $P_{s7} = N_1N_2N_3$

$$A_A = P_{S_0}; A_B = P_{S_0} + P_{S_2} + P_{S_3};$$

$$A_C = P_{S_0} + P_{S_1} + P_{S_2} + P_{S_3} + P_{S_6};$$

$$A_D = P_{S_0} + P_{S_1} + P_{S_2} + P_{S_3} + P_{S_4} + P_{S_5} + P_{S_6};$$

Steady-state availability determination

Let us demonstrate the **application of the methods described in the previous considerations of the reliability of the systems with independent components when determining (steady-state) system availability.**

In this case, we have to know the availability functions or steady-state availabilities of the components.

For example, system **B** will be available if component **1** is available and, component **2** or component **3** are available.

The expression for the steady-state availability will be obtained by replacing the reliability of the components in the expression for the reliability of the system with independent, irreversible and irreplaceable components with the steady-state availability of the components.

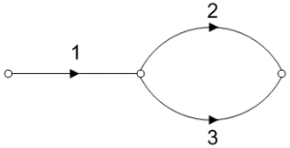
Let us prove the validity of such a procedure.

Steady-state availability determination

$$R(t) = P\left[x_1(x_2+x_3)\right] = P(x_1x_2 + x_1x_3) = P(x_1x_2)+$$

$$+ P(x_1x_3) - P(x_1x_2x_3) = P(x_1)\, P(x_2) + P(x_1)P(x_3)-$$

$$P(x_1)\, P(x_2)\, P(x_3)$$



where $P(x_i) = e^{-\lambda_i t}$ is the system reliability.

We get:

$$R(t) = e^{-\lambda_1 t} \cdot e^{-\lambda_2 t} + e^{-\lambda_1 t} \cdot e^{-\lambda_3 t}$$

$$- e^{-\lambda_1 t} \cdot e^{-\lambda_2 t} \cdot e^{-\lambda_3 t} \Rightarrow$$

Steady-state availability determination

$$A(\infty) = \frac{\mu_1\mu_2}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)} + \frac{\mu_1\mu_3}{(\lambda_1+\mu_1)(\lambda_3+\mu_3)} - \frac{\mu_1\mu_2\mu_3}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)(\lambda_3+\mu_3)}$$

We would have obtained the same result if we used the expression previously derived for availability of the system B:

Steady-state availability determination

$P_{so} = A_1A_2A_3$; $P_{s1} = N_1A_2A_3$; $P_{s2} = A_1N_2A_3$;

$P_{s3} = A_1A_2N_3$; $P_{s4} = N_1N_2A_3$; $P_{s5} = N_1A_2N_3$;

$P_{s6} = A_1N_2N_3$ i $P_{s7} = N_1N_2N_3$

$$A_B(\infty) = P_{so}(\infty) + P_{s2}(\infty) + P_{s3}(\infty) = A_1(\infty)$$

$$A_2(\infty)\ A_3(\infty) + A_1(\infty)\ N_2(\infty)\ A_3(\infty) + A_1(\infty)$$

$$A_2(\infty)\ N_3(\infty) = \frac{\mu_1\mu_2\mu_3}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)(\lambda_3+\mu_3)} +$$

$$+ \frac{\mu_1\lambda_2\mu_3}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)(\lambda_3+\mu_3)} + \frac{\mu_1\mu_2\lambda_3}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)(\lambda_3+\mu_3)}$$

Steady-state availability determination

The expressions are identical, which can be demonstrated by reducing to a common denominator; the first expression becomes the second one.

Task 1

– The mean time to failure is described with the exponential distribution and its value is 25 h. The mean time to repair is also described with the exponential distribution and its value is 40 h. Draw the availability function graph and calculate the steady state availability.

$$\lambda = \frac{1}{MTTF} = \frac{1}{25} = 0,04\ h^{-1}$$

$$\mu = \frac{1}{MTTR} = \frac{1}{40} = 0,025\ h^{-1}$$

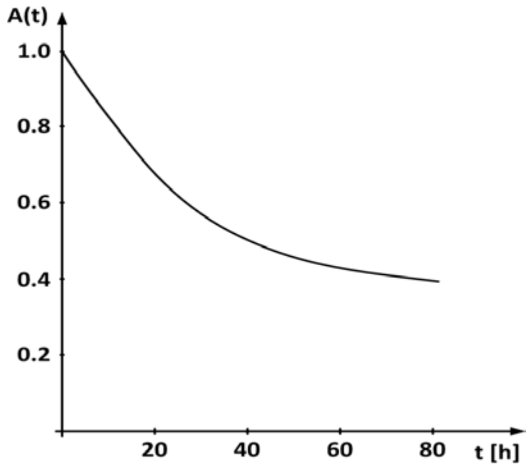
Steady state availability:

$$A_0 = \frac{\mu}{\lambda + \mu} = \frac{0,025}{0,04 + 0,025} = 0,385$$

Task 1

– Availability function:

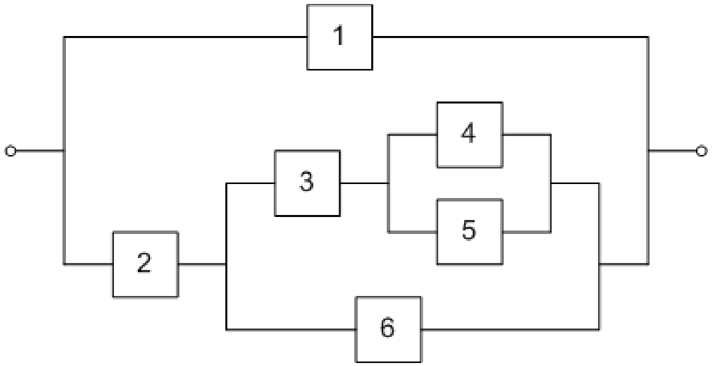
$$A(t) = \frac{\mu}{\lambda+\mu} + \frac{\lambda}{\lambda+\mu} e^{-(\lambda+\mu)t} = \frac{0,025}{0,04+0,025} + \frac{0,04}{0,04+0,025} e^{-(0,04+0,025)t}$$



Task 2

- Calculate the availability of the system shown in the figure after one year of operation, given the data in the table:

Component	MTTF [h]	MTTR [h]
1	2.000	10
2	3.000	12
3	3.000	12
4	5.000	14
5	6.000	15
6	2.000	10



Task 2

- Firstly, we calculate the availability of the components according to the relation:

$$A(t) = \frac{\mu}{\lambda + \mu} + \frac{\lambda}{\lambda + \mu} e^{-(\lambda+\mu)t}$$

t = 8760 h

$$\lambda = \frac{1}{MTTF}$$

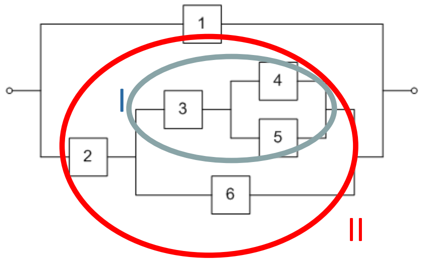
$$\mu = \frac{1}{MTTR}$$

Component	λ [h^{-1}]	μ [h^{-1}]	$A(8760)=A_0$
1	$5 \cdot 10^{-4}$	0.1	0.995
2	$3.333 \cdot 10^{-4}$	0.08333	0.996
3	$3.333 \cdot 10^{-4}$	0.08333	0.996
4	$2 \cdot 10^{-4}$	0.0714	0.997
5	$1.666 \cdot 10^{-4}$	0.0667	0.998
6	$5 \cdot 10^{-4}$	0.1	0.995

Task 2

The transient part is exponentially approaching zero ($e^{-5} = 0.007$, what happens after ~$50\ h$), as well as the the ratio $\lambda/(\lambda+\mu) \approx 10^{-3} - 10^{-2}$:

- To calculate the availability, we use the same rules as for calculating the reliability



$A_I = A_3 \cdot [1-(1-A_4)\cdot(1-A_5)] = 0.996 \cdot 0.999994 = 0.996$

$A_{II} = A_2 \cdot [1-(1-A_I)\cdot(1-A_6)] = 0.996 \cdot 0.99998 = 0.996$

$A_{uk} = 1-(1-A_1)\cdot(1-A_{II}) = 0.99998$