

## Problem 1

- A) Optimal Speedup:  $S_p(n) = \theta(p)$  when  $p = O(T^*(n)/T(n))$   
 Then  $W(n) = \theta(T^*(n)) = O(n \log n)$ , we can get  $T^*(n) = n \log n$   
 And  $T(n) = O(\log^2 n)$   
 Therefore,  $p = O(T^*(n)/T(n)) = O((n \log n)/(\log^2 n))$   
 $p = O(n/\log n)$

- B) A floating point add << A floating point divide << exp(x) << OpenMP atomic  
 pragma << OpenMP critical region

## Problem 2

NOTE: I implemented a running shell to COMPILE and RUN this summation program with different values of N and p. More details, please see the files enclosed in the folder.

I choose  $N = 10^4, 10^5, 10^6, 10^7, 10^8$  because if  $N < 10^4$ , the running time is very small so that that cannot present anything reasonable, and if  $N > 10^8$ , it is too big to use.

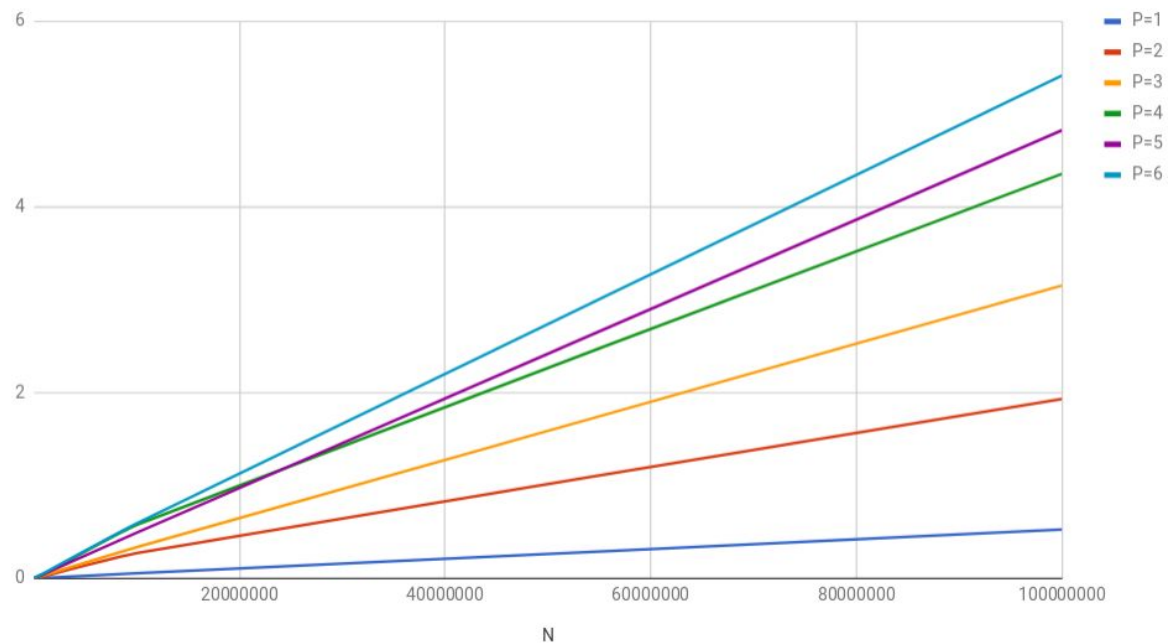
I choose  $p = 1, 2, 3, 4, 5, 6$  because I can compare running time between one thread and multi-thread, and I also compare the running time between the number of the thread of the optimal speedup and multiple threads.

If N increasing, the running time can clearly present the differences among different numbers of the processor. When p increases, the running time decreases in the same N until p reaches the value of optimal speedup. When p reaches the value of optimal speedup, the number of the processor is greater than the p-value of optimal speedup that has no significant change of running time.

### Version 1:

	N	P=1	P=2	P=3	P=4	P=5	P=6
	10000	0.00006	0.000312	0.000327	0.000471	0.0005	0.000575
	100000	0.000549	0.002752	0.003008	0.004304	0.005096	0.005872
Running Time	1000000	0.005247	0.021818	0.034127	0.04325	0.047442	0.056697
	10000000	0.052829	0.270903	0.334626	0.575486	0.491692	0.590409
	100000000	0.52546	1.932309	3.15414	4.358463	4.827309	5.416135

Version 1: N vs. p

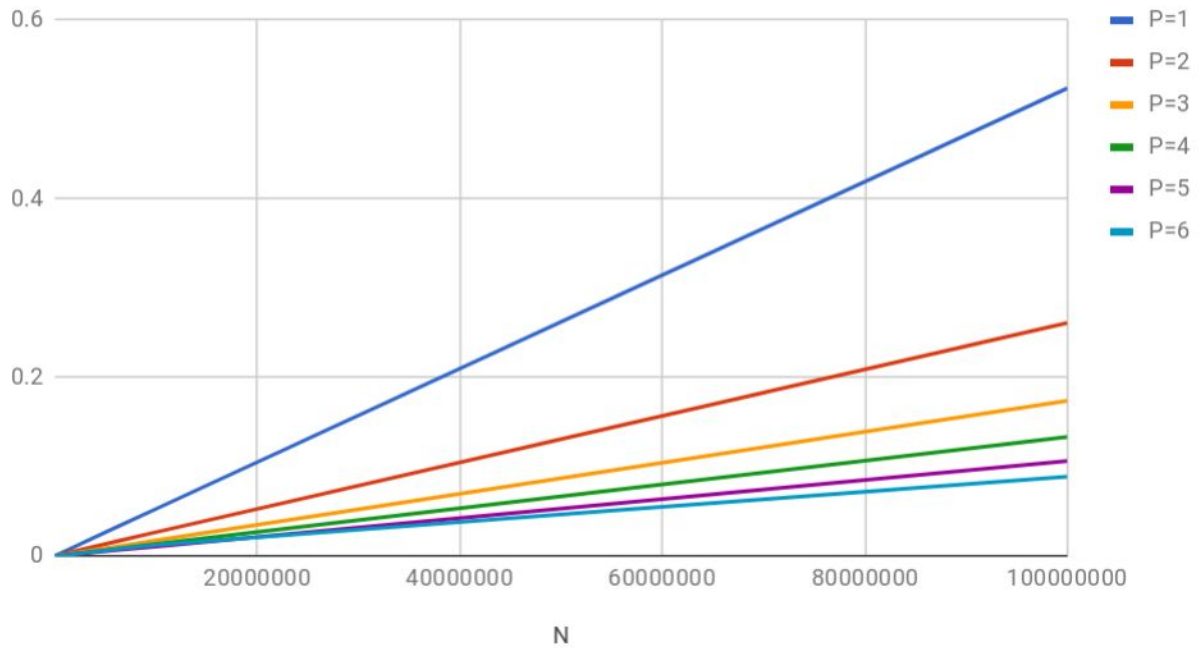


This result can adapt Brent's theorem. The PRAM algorithm runs in  $T(n)$  time while performing  $W(n)$  work can be adapted to run on a  $p$ -processor PRAM in at most  $\lceil W(n)/p \rceil + T(n)$  parallel steps. More number of threads cannot help the code run faster after  $p$  reaches the value of optimal speedup. By Amdahl's law, when the speedup is higher, the theoretical speedup of the execution of the whole task will be increased. That also impacts the running task.

## Version 2:

	N	P=1	P=2	P=3	P=4	P=5	P=6
	10000	0.000069	0.000107	0.000122	0.000118	0.000168	0.000162
	100000	0.000546	0.000339	0.000248	0.000226	0.000227	0.000249
Running Time	1000000	0.005373	0.002742	0.001916	0.00143	0.001226	0.001042
	10000000	0.052693	0.026709	0.01753	0.013507	0.010802	0.012495
	100000000	0.523932	0.261161	0.173948	0.133561	0.10646	0.089047

Version 2: N vs. p



This result can adapt Brent's Theorem too. The p-value of optimal speed is 4 threads. More than 4 threads such as 5 and 6, that shows the running times of 5 and 6 are very similar to the time of 4-thread. More threads cannot speed up when it reaches the optimal speed up. The p-value of optimal speedup also makes the theoretical speedup of the execution of the whole task have the highest value in Amdahl's law. They both affect the running task.