

### Exercise 4

$x \backslash y$	0	1
0	$1/4$	$1/4$
1	0	$1/2$

$$a) P(X=0) = \frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$$

$$P(X=1) = 0 + \frac{1}{2} = \frac{1}{2}$$

$$\begin{aligned} H(X) &= - \sum_{x \in X} p(x) \cdot \log_2 p(x) = \\ &= - \left( \frac{1}{2} \cdot \log_2 \frac{1}{2} + \frac{1}{2} \cdot \log_2 \frac{1}{2} \right) = \\ &= - \left( \frac{1}{2} \cdot 0 - \frac{1}{2} \cdot 1 + \frac{1}{2} \cdot 0 - \frac{1}{2} \cdot 1 \right) = \\ &= - \left( -\frac{1}{2} - \frac{1}{2} \right) = -(-1) = 1 \end{aligned}$$

$$P(Y=0) = \frac{1}{4} + 0 = \frac{1}{4}$$

$$P(Y=1) = \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$$

$$\begin{aligned} H(Y) &= - \left( \frac{1}{4} \cdot \log_2 \frac{1}{4} + \frac{3}{4} \cdot \log_2 \frac{3}{4} \right) = \\ &= - \left( -\frac{1}{4} \cdot 2 + \frac{3}{4} \cdot \log_2 3 - \frac{3}{4} \cdot 2 \right) = \\ &= - \left( -\frac{1}{2} + \frac{3}{4} \cdot 1,585 - \frac{3}{2} \right) = \\ &= - \left( -2 + 1,18875 \right) = -(-0,81125) = 0,81125 \end{aligned}$$

$$\begin{aligned} b) H(Y|X) &= \sum_{x \in X} \sum_{y \in Y} p(x,y) \cdot \log_2 \frac{p(x)}{p(x,y)} = \\ &= \frac{1}{4} \cdot \log_2 \frac{\frac{1}{2}}{\frac{1}{4}} + \frac{1}{4} \cdot \log_2 \frac{\frac{1}{2}}{\frac{1}{4}} + 0 \cdot \log_2 \frac{\frac{1}{2}}{0} + \frac{1}{2} \log_2 \frac{\frac{1}{2}}{\frac{1}{2}} = \\ &= \frac{1}{4} \cdot 1 + \frac{1}{4} \cdot 1 + 0 + 0 = \frac{2}{4} = \frac{1}{2} = 0,5 \end{aligned}$$

$$\begin{aligned}
 H(X|Y) &= \sum_{y \in Y} \sum_{x \in X} p(y, x) \cdot \log_2 \frac{p(y)}{p(y, x)} = \\
 &= \frac{1}{4} \cdot \log_2 \frac{\frac{1}{4}}{\frac{1}{4}} + 0 \cdot \log_2 \frac{\frac{1}{4}}{0} + \frac{1}{4} \cdot \log_2 \frac{\frac{3}{4}}{\frac{1}{4}} + \frac{1}{2} \cdot \log_2 \frac{\frac{3}{4}}{\frac{1}{2}} = \\
 &= \frac{1}{4} \cdot 0 + 0 + \frac{1}{4} \cdot \log_2 3 + \frac{1}{2} \cdot \log_2 \frac{3}{2} = \\
 &= \frac{1}{4} \cdot 1,585 + \frac{1}{2} \cdot 1,585 - \frac{1}{2} \cdot 1 = \\
 &= 0,38625 + 0,7925 - 0,5 = 0,67875
 \end{aligned}$$

$$\begin{aligned}
 c) H(X, Y) &= H(X) + H(Y|X) \\
 &= 1 + 0,5 = \\
 &= 1,5
 \end{aligned}$$

$$\begin{aligned}
 d) i(X, Y) &= H(X) - H(X|Y) = \\
 &= 1 - 0,67875 = 0,32125 \\
 &= H(Y) - H(Y|X) = \\
 &= 0,81125 - 0,5 = 0,31125
 \end{aligned}
 \quad \approx$$