Water-Supply Operations during Drought: Continuous Hedging Rule

By Jhih-Shyang Shih[1] and Charles ReVelle[2]

Abstract: Demand-management policy rules are sought during drought and impending drought for a water system consisting of a reservoir dedicated only to water supply. The creation of such rules requires solution of a nonlinear, nonseparable mathematical programming problem. A polytope search algorithm using a combination of simulation and optimization is compared to an iterative mixed integer programming method to determine the parameters of continuous demand management rules. The signal used for calling rationing is a trigger volume given in terms of months of demand (as a volume) that are needed in storage. When the sum of actual storage plus anticipated inflow is less than the trigger volume, rationing is initiated. The extent of rationing or demand reduction that is required is determined by the ratio of the sum of storage plus inflow to the trigger volume. The two methodologies for parameter determination are compared using as a criteria the maximum shortage that occurs over some planning period.

INTRODUCTION AND BACKGROUND

Periods of water shortage may occur frequently in a fully utilized water resource system, especially when naturally variable surface waters are the main supply source. Operation/demand management is particularly critical during these periods of shortage. The predominant operating objective during such shortages should be to minimize the overall damages from the reduced ability to fulfill demands. However, because of the difficulty of measuring damages from shortages that have not occurred, other objectives, which are easier to measure, may be utilized. Other objectives that can be measured more easily may be to minimize the maximum expected shortage, or to minimize the total expected shortage, but these are surrogates for the underlying objective of minimizing damages.

In this research, we adopt a typical textbook definition of drought. Linsley et al. (1982) offer the following definition. They say that drought is a "period during which streamflows are inadequate to supply established uses under a given management system." Other definitions of drought are offered by Matalas (1963) and Whipple (1966). Although these other definitions are useful in specific circumstances, for purposes of this research, the Linsley et al. definition appears most useful. Our goal in this research is to create rules for demand management/operation that can carry a water-supply system through a drought with the least total damages or the least disruption of economic activities.

Although Rippl (1883) first suggested his graphical design procedure for reservoirs over 100 yr ago and despite significant progress in the field of water-resources planning and management over the last three decades, the planning for real-time operation during drought or impending drought of

[1]Res. Assoc., Ctr. for Energy and Envir. Studies, Dept. of Engrg. and Public Policy, Carnegie Mellon Univ., Pittsburgh, PA 15213.

[2]Prof., Dept. of Geography and Envir. Engrg., Johns Hopkins Univ., Baltimore, MD 21218.

Note. Discussion open until March 1, 1995. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this paper was submitted for review and possible publication on March 3, 1993. This paper is part of the ***Journal of Water Resources Planning and Management***, Vol. 120, No. 5, September/October, 1994. ©ASCE, ISSN 0733-9496/94/0005-0613/$2.00 + $.25 per page. Paper No. 5713.

J. Water Resour. Plann. Manage., 1994, 120(5): 613-629

the single reservoir, dedicated only to water supply, has not evolved significantly. Rippl's procedure gradually became known as the mass diagram and became a fundamental part of the education of water-supply engineers (Hazen 1914). The assumption of a constant rather than seasonal demand was recognized as a limiting feature of the methodology, and the procedure was modified by Babbit and Doland (1939) to account for seasonal as opposed to constant demands. In the early 1960s, Thomas (Fair et al. 1966; Thomas and Burden 1963) suggested another simple method, which he named "sequent peak," to handle reservoir design under seasonal demands. In addition, Fiering (1967) and others introduced the idea of synthetic sequences as a mean to supplement the historical record and to introduce the notion of reliability into the reservoir design procedure. In 1962, Dorfman structured the design/operation problem for a water-supply reservoir as a linear program. Hirsch (1979) approached the operation of the single water-supply reservoir via two methodologies. The first, a generalized risk-analysis model (GRAM) utilized simulation with reconstructed flows to evaluate the likelihood of emergency procedures being called into play. The second, position analysis, used reconstructed flows to calculate water-emergency probabilities based on current reservoir conditions. Design/operation procedures for single water-supply reservoirs have not been significantly extended since that time, but recently the linked notions of reliability, vulnerability, and resilience have been explored (Hashimoto et al. 1982; and Moy 1986).

In related studies, Palmer and Holmes (1988) utilize expert-systems methodology to develop for the city of Seattle a rule-based methodology for the determination of system yield and optimal operating policies based on past hydrologic regimes. Although they indicate utilization of a linear programming model to generate specific operating policies, the linear programming model itself is not discussed. Randall et al. (1986, 1990) in two papers discuss a multiobjective linear programming model both for long-term and real-time operation of a metropolitan water-supply system during drought. The model, which considers both the supply and distribution portions of an existing metropolitan water-supply system, uses stream flow and demand forecasts plus adaptive/recursive programming to update operating policy as each period's inflow occurs. No general rule for demand management is created. Wright et al. (1986) describe a simulation game to explore drought mitigation strategies as a means to prepare decision makers for a real drought. The game incorporates not only the physical water system, but also the political and institutional setting as well. Wilhite and Easterling (1989) review features of policies that should be central to the drought planning process and suggest appropriate objectives for developing such policies. A 10-step drought-planning process is recommended to facilitate drought preparedness including the development of a demand-reduction/reservoir-operating rule for drought situations.

Missing from most of the operating procedures for a single-water-supply-only reservoir is the notion of an allowable failure that might occur or the consequences of such a failure. Hashimoto et al. and Moy et al. do allow failures in their models, but they include no underlying rules for demand management to dampen these failures. That is, for reservoir of a particular size and inflow sequence, they determine demand-management/release decisions that optimize some objective. These rules, however, are specific for the size and inflow sequence and do not provide guidance on demand management decisions under other flow regimes, even though the models can certainly be solved for those altered conditions. What is offered by these models are decisions on demand management rather than demand-management policy.

We know that in drought situations, which may be more severe than those planned for, reservoirs do sometimes fail to deliver their "safe" yields. In such situations, we know that water-supply managers would rather incur a sequence of smaller shortages than one catastrophic shortage. As a consequence, in order to mitigate the consequences of potential failures, water restrictions or rationing may be instituted for a city as a means to reduce temporarily the level of demand and to preserve storage and inflows for future use (Water Science and Technology Board 1986). Indeed, if a reservoir has been designed for a lower safe yield than the yield it is currently being used to provide, we know that rationing could become a common experience. If the reservoir is being used to deliver more than the safe yield, or the drought appears to be worse than any planned for, we need to determine the quantitative value(s) of the signal(s) that should be used to trigger rationing to prevent larger shortages later.

One mode of operation assumed in water-supply simulation procedures is referred to as the standard operating policy (SOP) and sometimes as the S-shaped curve of operation (Maass et al. 1962). The curve applies to reservoirs used for water supply only and is illustrated in Fig. 1. It is a rule suggested to simulate reservoir operation, not a rule that is necessarily suggested for actual operation of a reservoir in a practical situation. The SOP calls for the draft in each period of the demand, if possible. If insufficient water is available to meet the target, the reservoir releases all the water available and becomes empty; if too much water is available, the reservoir will fill and spill its excess water. Mathematically this release rule is expressed as

$$R_t = S_{t-1} + I_t \qquad \text{if } S_{t-1} + I_t \leq D_t \tag{1a}$$

$$R_t = D_t \qquad \text{if } D_t \leq S_{t-1} + I_t \leq C \tag{1b}$$

$$R_t = S_{t-1} + I_t - C \qquad \text{if } S_{t-1} + I_t - D_t > C \tag{1c}$$

Hashimoto et al. (1982) concluded that the standard operating policy is optimal if one's objective is to minimize the total release shortfall. The

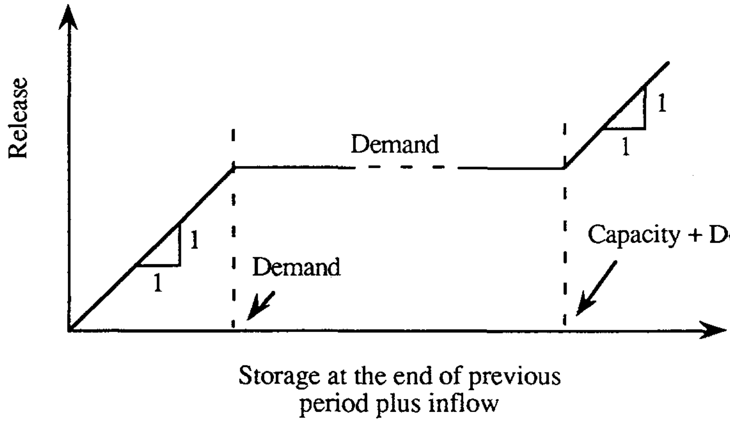

FIG. 1. S-shaped Curve

J. Water Resour. Plann. Manage., 1994, 120(5): 613-629

shortcoming of the rule is that it does not provide a mechanism for rationing supplies when there may be insufficient water, nor does it suggest a mechanism for releasing more water when there is a surplus available (Stedinger 1984). The S-shaped curve is never fully realized in the design procedure of sequent peak or in the linear programming design formulation because failure never occurs in these procedures because it is not allowed. Only those portions of the S-shaped curve are utilized in the design procedure that: (1) Deliver the demand if it is available; and (2) spill when capacity is exceeded. Realistic operating rules would suggest that, during periods of impending drought, reductions be made in demand even when the usual demand can be delivered from storage and current inflow. Such reductions would serve to prevent larger shortages in later periods. Our goal is to show how to create rules for demand and draft reduction along with their associated triggering mechanism; these are the rules that can carry a reservoir through a drought without producing severe shortfalls from demand.

Economically, a hedging rule can be justified only if a loss or damage function convex in shortage quantity is associated with the proposed uses of water. If the marginal values of additional water for the specific uses are constant, the economic losses from shortage must be linear. Since stream flow is stochastic, it follows that it is optimal to postpone the shortage as long as possible. The following example supports this conclusion. Suppose five units of shortage in time period one and five units of shortage in time period two are economically equivalent to 10 units of shortage in a single time period. This is the linearity assumption. In such a case, a water manager would like to distribute all the shortage to the second time period. Such a decision follows because a smaller shortage could occur in the second time period if a higher than anticipated inflow occurs. However, if severe deficits cause damages more-than-proportionately greater than mild deficits, it may be advantageous to reduce the probabilities of suffering heavy deficits. Thus, it may be economical to accept a small current deficit in order to decrease the magnitude of a severe shortage in the future. That is the premise of this work and the foundation upon which hedging rules are built.

Rationing rules could utilize either a value of storage or a value of storage plus projected inflow as the mechanism that triggers demand reductions. Storage has been commonly utilized (ReVelle et al. 1969; Houck et al. 1980) as the signal in operating rules for multipurpose reservoirs, but storage augmented by expected flows promises potentially more reasonable behavior. Indeed, the standard operating policy utilizes storage plus the current rather than the projected inflow, but again, this is in water-supply simulations. The SOP can use such a rule because it does not need to declare rationing until after the operating period is over, a rather unrealistic mode of operation. In this research, we explore triggers that consist of values of current storage plus the conditional expected value of inflow in the upcoming period. However, the state variables themselves and the values of those variables that might trigger demand reductions are open to further investigation.

OPTIMIZATION MODEL FOR NEW HEDGING RULE

We describe here a linear hedging rule for demand management during drought or impending drought. According to the rule, once demand reductions have been mandated, demand (and hence release) is to be a function of the sum of reservoir storage at the end of the previous period plus the projected inflow in the current period. The parameters of the new rule,

J. Water Resour. Plann. Manage., 1994, 120(5): 613-629

which is, in fact, guidance for how to temper demand, are obtained from the application of mathematical programming techniques. A search procedure, the polytope algorithm, has been utilized and an iterative mixed-integer programming method has been created to determine the parameters. These two methods are compared.

Rules for demand management for water supply-only reservoirs are needed both in the planning phase for new reservoirs and for the real-time management of existing reservoirs. The development of a realistic rule for reducing demand and hence draft from the reservoir during drought and predrought conditions raises at least two questions:

1. How early in time and at what levels of storage and projected inflow should the rationing begin in order to reduce later shortfalls?
2. How much should demand and hence draft be reduced during each of the intervals of the rationing period?

In order to approach answers to these questions, mathematical-optimization models are built to find the parameters of realistic hedging rules based on historical data. These rules can then be used as guidance for the management of the water-supply system.

The basic reservoir design problem that is equivalent to the sequent peak procedure, can be stated in several ways as a linear program. In nonstandard form, it is often seen as

$$\min Z = C \tag{2}$$

subject to

$$S_t - S_{t-1} + W_t = I_t - D_t \qquad \forall t \tag{3}$$

$$S_t \leq C \qquad \forall t \tag{4}$$

$$S_n \geq S_0 \tag{5}$$

$$C, S_t, W_t \geq 0 \qquad \forall t \tag{6}$$

This basic model has been used by many reservoir management investigators, and it is this model that serves as the base from which new forms are derived.

The operating rule implicit in this formulation is this: If the storage at the end of the previous period plus the inflow is less than the demand of the current period, then there will be no release, since storage would be forced to be negative. If the condition occurred, the mathematical program would show that operation is infeasible at the level of demand specified since release of the demand would lead to a negative storage. This infeasible region corresponds to the shaded area in Fig. 2. If storage plus inflow is both less than the capacity plus demand and greater than demand, the reservoir will release exactly the demand. If the storage plus inflow minus demand exceeds capacity, release of demand plus spill will occur. Such an operating rule does not correspond to actual operations preceding or during drought, however, since at these times hedging is a common practice.

A new type of rule designed for drought and impending drought conditions suggests that demand and consequently release should be manipulated to decline gradually as reservoir contents and projected inflow fall (Fig. 3). This rule would also include an additional segment, i.e., most of the flat portion of the S-shaped curve, but would allow any level of draft below

J. Water Resour. Plann. Manage., 1994, 120(5): 613-629

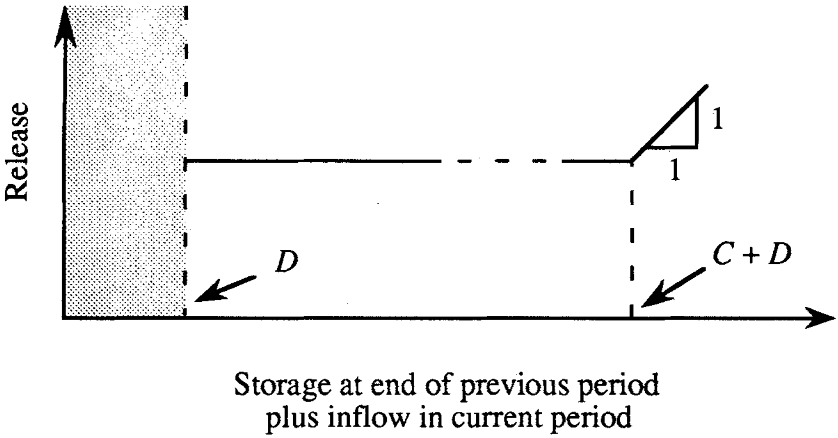

FIG. 2. Basic Operating Rule

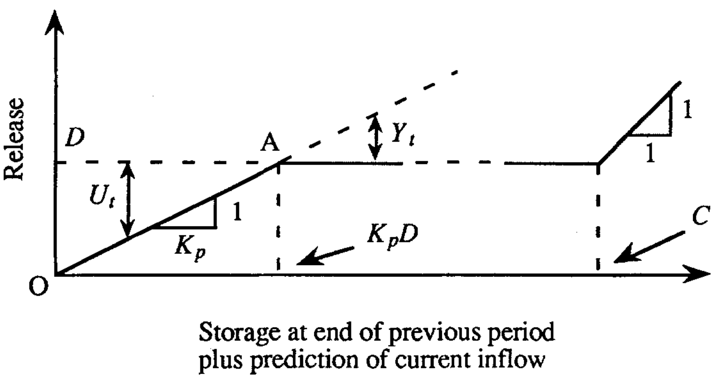

FIG. 3. Hedging Rule

demand D. In this version of the model, we assume that demand is the same for the entire horizon of operations. It is not difficult, however, to extend the model to consider demand levels that are specific for each month of the year. The value of storage plus the anticipated value of current inflow that triggers use of the rule might be arbitrarily set at a particular number of months of demand or, as it is in this study, can be a decision variable in an optimization model.

The demand reduction/operating rule shown in Fig. 3 (line OA) uses a gradually declining draft as reservoir contents plus projected inflow fall. For the example rule shown in Fig. 3, suppose $K_p = 4$. When storage plus projected inflow falls below four months of demand (i.e., $4D$), demand/draft is cut back according to the linear schedule shown, i.e., at storage plus inflow values of three months of demand, only three-fourths of usual demand is required, and so forth. That is

$$\text{draft} = 1/K_p(\text{storage} + \text{projected inflow}) \quad (7)$$

In this case, the level of storage plus inflow at which to start rationing and the portion of water demand to be met are determined at the same time by knowledge of the trigger value, K_p, assigned to each month p. When storage plus inflow amount to less than K_p months of demand, rationing is begun. When this quantity is greater than K_p months of demand, the full demand can be drafted from the reservoir.

The trigger value is easy to modify to explore its impact on shortages. Obviously the larger the trigger value, the smaller the maximum shortfall that will occur, but the more frequently rationing will be necessary. The declining curve could also be a general concave function or a piecewise concave function, but these shapes are not explored here. Such shapes may, in fact, have desirable properties and we plan to investigate them in future work.

From an optimization point of view, we can develop the following model that incorporates a general rule of the form shown in Fig. 3. In this model we seek that set of linear rationing schedules and their associated K_p values that minimize the maximum of the monthly shortfalls. We choose to minimize the maximum shortfall because losses are convex in shortfall, and, as a consequence, minimization of total shortfall is unlikely to minimize losses. That is

$$\min Z = M \tag{8}$$

subject to

$$R_t + Y_t = (1/K_p)(S_{t-1} + \hat{I}_t) \qquad \forall t \tag{9}$$

$$R_t + U_t = D \qquad \forall t \tag{10}$$

$$Y_t \times U_t = 0 \qquad \forall t \tag{11}$$

$$U_t \leq M \qquad \forall t \tag{12}$$

$$S_t \leq C \qquad \forall t \tag{13}$$

$$S_n \geq S_0 \tag{14}$$

$$b_t \leq S_t/C \qquad \forall t \tag{15}$$

$$W_t \leq b_t \times B \qquad \forall t \tag{16}$$

$$S_t - S_{t-1} + R_t + W_t = I_t \qquad \forall t \tag{17}$$

$$b_t = 0 \text{ or } 1 \qquad \forall t \tag{18}$$

$$S_t, W_t, R_t, Y_t, U_t, K_p \geq 0 \qquad \forall t, \forall p \tag{19}$$

In this model, the first three constraint types describe the operating rule. If storage plus projected inflow is greater than or equal to the trigger volume K_pD, the full demand is drafted from the reservoir. As a consequence, the shortage, U_t, will be equal to zero and the slack variable, Y_t, equal to a positive number. Otherwise, we will release only $(1/K_p)(S_{t-1} + \hat{I}_t)$, in which case the slack variable, Y_t, will be zero and the shortage, U_t, will be a positive number as shown in Fig. 3. The third constraint type assures that the shortage and slack variable cannot be positive numbers simultaneously for period t. These constraints insure that operation will follow the curve we have indicated in Fig. 3.

Eq. (12) is a definitional constraint and says that shortage in every period should be less than or equal to the maximum shortage, M. Eq. (13) is the physical limitation on storage, i.e., at the end of any operating period, the storage of the reservoir should be less than its capacity. Eq. (14) does not allow us to borrow water from initial storage during the interval of operation.

Eqs. (15) and (16) prevent the spill of water if the reservoir is not full since it does not make sense to spill water at such a time. Without these constraints, spill can occur in the model even though the reservoir is not at capacity. We wish to prevent such spill since if spill occurred this month, then next month instead of releasing full demand, we might only be able to release part of the demand. Eq. (15) says if storage at period t is smaller than the reservoir capacity then b_t, which is a 0 or 1 integer variable, will be equal to zero. Eq. (16) forces the spill, W_t to be zero if storage is less than capacity. If, on the other hand, storage, S_t, equals the capacity, then b_t is bounded above by 1, and must be equal to one. Spill, through (16), is then bounded only by an arbitrarily big number B.

Eq. (17) is the continuity constraint relating reservoir draft in any period to the inflow, spill, and storage volume. In this model, the storage level indicators, the b_t, are the only 0,1 variables; all other decision and state variables are continuous.

Moreover, in (9), both trigger values, K_p and storage at the end of previous period, S_{t-1}, are unknown variables, as are the slack variables, Y_t, and shortages, U_t in (11). This mathematical model constitutes a nonlinear, nonseparable mixed-integer programming problem.

SOLUTION METHODS

This mathematical programming problem is not straightforward to solve. It is: (1) A mixed-integer problem; and (2) the feasible region is not a convex set. As a consequence, the solution we find cannot be guaranteed to be a global optimum. Further, there appear to be weak local optimal solutions, so that a number of starting points ought to be utilized.

POLYTOPE SEARCH PROCEDURE

One way to deal with this problem is to create individual values of the objective function through the multiple computer simulations, each simulation using prespecified values of the rationing triggers. The objective function is then searched for minima over various combinations of the trigger values. Such an algorithm works effectively in the absence of analytic derivatives. Thus, one of the methods we explored here was the simulation-optimization process.

The polytope search algorithm operates on a grid of points in n space, using functional comparisons to reduce the region. At each stage of the algorithm, $n + 1$ points are retained, together with the functional values at these points. The method derived its name because these points can be considered to be the vertices of a polytope in n space. At each iteration, a new polytope will be generated by producing a new point to replace the worst point in those $n + 1$ points. A new point is generated by selecting a set of trigger values; a simulation of reservoir contents through time is then conducted, yielding a value of M. Sometimes, the most recent polytope is discarded and replaced by a regular polytope; this procedure is called restarting. Restarting can be used to prevent the polytope from becoming unbalanced after several successive steps are made; it can also be used to check the validity of a solution. The details of generating a new point as well as stopping criteria for the algorithm can be found in Gill (1981). An

IMSL (1987) subroutine, UMPOL, was utilized to implement the polytope algorithm.

In this process, if the final reservoir storage S_n was smaller than the initial storage S_0, then a large number was assigned to the objective, and the next iteration began. The procedure avoids movement in the search process in an infeasible direction. Because of the nonconvexity of the constraint set, the solutions obtained by this technique are not guaranteed to be globally optimal, and a number of different starting points are used to make sure that a good solution is found.

MIXED-INTEGER NONLINEAR METHOD

There are two nonlinear parts in the model presented in (9)–(19). The first involves the product term in (20), namely,

$$Y_t \times U_t = 0 \tag{20}$$

The second part consists of the right-hand side of (21)

$$R_t + Y_t = (1/K_p)(S_{t-1} + \hat{I}_t) \tag{21}$$

In this section, we show how to transform the nonlinear model into a mixed-integer programming formulation. Once this is done, the problem can be solved by a linear programming solver with integer capability, e.g., MPSX with MIP (*IBM* 1979). First, we focus on how to transform the first nonlinear constraint set into a mixed-integer constraint.

The equation $U_t \times Y_t = 0$, belongs to an important class of problems that have "either-or" conditions, which can be formulated by introducing an integer variable (Dantzig 1965). With many sets of such conditions, the problem is acknowledged to be computationally difficult. We consider one of the constraints [(20)].

For constraint t, we introduce a 0,1 variable, Z_t, and a large positive number, B. We then replace the constraint equation $U_t \times Y_t = 0$ with two constraints

$$U_t \leq B \times Z_t \tag{22}$$

and

$$Y_t \leq B \times (1 - Z_t) \tag{23}$$

When Z_t is 0, only Y_t can be positive. When Z_t is 1, only U_t can be positive. A variable Z_t and the two equations can be introduced for every constraint of the form $U_t \times Y_t = 0$. We call the formulation up to here the "mixed-integer programming formulation"; it will be used as the basis for an iterative procedure to be described next.

We then focus on the nonlinear nonseparable portion on the right-hand side of (21). We define a new variable, H_p, to simplify manipulations

$$H_p = (1/K_p) \tag{24}$$

Next, the right-hand side is separated into two terms

$$R_t + Y_t = H_p \times S_{t-1} + H_p \times \hat{I}_t \tag{25}$$

We treat the H_p in the first term of the right-hand side as a given number, naming it as H_p^k, and the second as H_p^{k+1}. The variable H_p^{k+1} is an input in the $(k + 1)$th iteration. The equation is rewritten with the H_p terms as sequentially determined variables. Thus, in the kth iteration, we have

$$R_t + Y_t = H_p^k \times S_{t-1} + H_p^{k+1} \times \hat{I}_t \tag{26}$$

The values of the H_p^k are known prior to the kth iteration—from a preceding iteration or as initial values in the first iteration. The values of the H_p^{k+1} are determined on the kth iteration and constitute the input to the $(k + 1)$th iteration with modification by a scaling factor.

The solution procedure can be summarized as follows. On the kth iteration with input of the H_p^k, the mixed-integer programming problem is solved yielding values for the H_p^{k+1}. These values modified by a scaling factor are input to the $(k + 1)$th iteration in which the mixed integer programming formulation is solved again. The procedure terminates when the absolute value of the differences, $|H_p^{k+1} - H_p^k|$, is sufficiently small. On the kth iteration of the mixed integer programming model, the formulation is

$$\min Z = M \tag{27}$$

subject to

$$R_t + Y_t = H_p^k \times S_{t-1} + H_p^{k+1} \times \hat{I}_t \quad \forall t \tag{28}$$

$$R_t + U_t = D \quad \forall t \tag{29}$$

$$U_t - B \times Z_t \leq 0 \quad \forall t \tag{30}$$

$$Y_t + B \times Z_t \leq B \quad \forall t \tag{31}$$

$$U_t \leq M \quad \forall t \tag{32}$$

$$S_t \leq C \quad \forall t \tag{33}$$

$$S_n \geq S_0 \tag{34}$$

$$b_t \times C - S_t \leq 0 \quad \forall t \tag{35}$$

$$W_t - b_t \times B \leq 0 \quad \forall t \tag{36}$$

$$S_t - S_{t-1} + R_t + W_t = I_t \quad \forall t \tag{37}$$

$$Z_t, b_t = 0 \text{ or } 1 \quad \forall t \tag{38}$$

$$S_t, W_t, R_t, Y_t, U_t, H_p \geq 0 \quad \forall t \tag{39}$$

The following algorithm was employed to move toward a solution of this problem. We display the kth iteration only.

1. Input a set of H_p^k, a scaling factor, α, and precision, ε.
2. Run a linear programming code with the mixed-integer option and obtain the decision variables H_p^{k+1}. If $|H_p^{k+1} - H_p^k| < \varepsilon$, go to step 3. If the difference is greater than some ε, let $H_p^{k+1} = \alpha H_p^k + (1 - \alpha)H_p^{k+1}$ for use in the $(k + 1)$th iteration. Return to step 1.
3. Print out all the decision and state variables and stop.

We have thus transformed the nonlinear, nonseparable problem into an equivalent iteratively solved mixed-integer programming model. Reasonable size models (2 or 3 years or more of drought duration) can be solved in this way.

COMPUTATIONAL RESULTS

In this study, we solved for the rationing rules for a 36-month horizon. The polytope search procedure discussed earlier was run on a VAXstation 3500 under the operating system VAX/VMS V5.0-1. The iteratively solved mixed integer programming model was run using MPSX/MIP on the Cornell supercomputer, IBM 3090-600J, under the operating system VM/XA CMS version 5.6.

Monthly streamflow data from the Gunpowder River in Maryland were utilized as the reservoir inputs. Specifically, a 36-month sequence representing the worst drought on record was abstracted from over 80 yr of flow history. A conditional expected streamflow was used in calculating the sum of storage plus inflow for use in evaluating whether rationing should be called.

For the iterative mixed-integer programming model, two strategies were employed to achieve fast convergence: The first is the careful selection of the scaling factor α. The scaling factor, as a positive number between 0 and 1, makes the input to the next iteration a linear combination of the input and output of the current iteration. When the scaling factor was set to one, that is, when the output of the current iteration was used as an input to the next iteration, the output was scattered and convergence was poor. When α was close to zero, the iteration moved only slowly from previous input in the direction of the output, but the slow movement of the input seemed to help the problem converge more quickly.

In another strategy to hasten convergence, definitional constraints were added

$$H_p^{k+1} - H_p^k = H_p^+ - H_p^- \tag{40}$$

and a new term was added to the objective with a small weight

$$\min Z = H_p^+ + H_p^- \tag{41}$$

This strategy improved the speed of convergence of the problem, probably because the new objective reduced the number of alternatives and hence reduced the execution time of branch and bound.

In Table 1, we compare the two algorithmic approaches to this problem. The first and second rows display the maximum shortages and times to the termination of th two approaches when both begin at the same starting point. The third row displays the maximum shortage and time when the polytope algorithm was initiated at 1,000 different starting points, again using the VAXstation 3500.

While these results are shown in the same table, no firm conclusions can be drawn as to the most cost efficient means of solution. The nearly equal times of the polytope algorithm run from 1,000 starting points using the VAXstation 3500 as compared to the iterative algorithm run on the Cornell 3090-600J are not really comparable. The appropriate comparison would involve multiple runs of the polytope algorithm on the 3090-600J to provide run times that could be displayed against the run time of the iterative algorithm. Only two firm statements can be made. First, the polytope algorithm never found a solution as good as the mixed-integer iterative method. And second, although the instance does not show in the table, the polytope algorithm when run with the solution from the iterative mixed-integer programming method as the seed never found an improved solution. This suggests, but does not prove, that the iterative mixed integer programming

J. Water Resour. Plann. Manage., 1994, 120(5): 613-629

TABLE 1. Comparison of Results of Polytope (Poly) Algorithm and Iterative (Iter) Mixed Integer Programming Method

Maximum shortage (1)	Algorithm (2)	CPU time (min:s) (3)	K_1 (4)	K_2 (5)	K_3 (6)	K_4 (7)	K_5 (8)	K_6 (9)	K_7 (10)	K_8 (11)	K_9 (12)	K_{10} (13)	K_{11} (14)	K_{12} (15)
(*a*) Demand = 18.93														
3.63	Poly	1.77[b]	3.50	4.07	3.82	3.88	4.17	4.52	4.07	3.33	2.93	2.28	3.27	3.08
1.36	Iter	8:12	2.91	3.24	2.91	2.86	2.97	3.22	2.68	2.02	1.51	1.00	1.65	2.67
1.78	Poly (1,000)[a]	7:02	3.00	1.57	2.94	2.92	3.06	3.33	1.40	2.02	1.50	1.01	1.62	2.73
(*b*) Demand = 19.30														
4.28	Poly	1.80[b]	3.49	3.89	3.69	3.89	4.05	4.40	4.07	3.49	3.04	2.54	3.50	2.98
2.12	Inter	7:02	2.75	3.12	2.80	2.77	2.91	3.18	2.65	2.00	1.50	1.00	1.51	2.49
2.57	Poly (1,000)[a]	5:04	2.85	3.24	2.94	2.94	3.01	2.73	2.71	2.07	1.58	1.02	1.45	2.56

Note: This is a 12-season and 36-month case; initial storage and reservoir capacity both equal 114,000,000 m^3; maximum shortage and demand are in million cubic meter per month.

[a]Polytope algorithm was initiated at 1,000 different starting points.

[b]CPU time is in seconds.

method finds a solution that the polytope search procedure cannot improve upon.

Comparison of these operating policies on other criteria such as average shortfall was not done, so that conclusions are really restricted to the criterion of maximum shortfall. Nonetheless, since damages appear to be convex in shortages, the criterion of minimizing maximum shortfall seems an adequate measure against which to compare.

CONVERTING TO DISCRETE RULE

The continuous hedging rule that was created here was derived in a way that minimized the maximum shortage over the drought record. The inclined line, however, does not correspond to actual operational behavior of water managers in an impending drought situation—because water managers do not have a continuous gradation of options available to them. In fact, rationing is declared in discrete steps. As an example, alternate-day lawn watering may be called for first, then no lawn watering or filling of swimming pools, then penalties for excess volumetric consumption, and so on. Hence, the continuous rule needs to be converted to the best possible discrete hedging rule. That is, we need to determine trigger volumes that as accurately as possible transform continuous hedging rules into discrete hedging rules. The following derivation shows that transformation.

We assume for simplicity that the discrete hedging rule has two rationing phases. In the first phase, only a fraction α_1 of the normal demand is delivered. In the second phase, only a fraction α_2 of normal demand is made available. Once the inclined rules have been determined by mathematical programming, we know the trigger volumes V_1 for each month; these are the levels of storage plus inflow which initiated the first phase of rationing, or $V_1 = K_pD$. We also know the minimum volume, V_3, for each month. This is the lowest level allowable of storage plus inflow. In addition, we have the proportions of demand α_1 and α_2 that are delivered in the two assumed rationing phases. We want to find V_2, the best trigger volume to distinguish the two phases of rationing. This derivation can be generalized to more than two phases of rationing. The first phase of rationing delivers the demand α_1D; the second phase delivers α_2D.

Now, given V_1, V_3, α_1, and α_2, we want to locate V_2 between V_1 and V_3 such that the sum of the areas of the two shaded triangles in Fig. 4 is

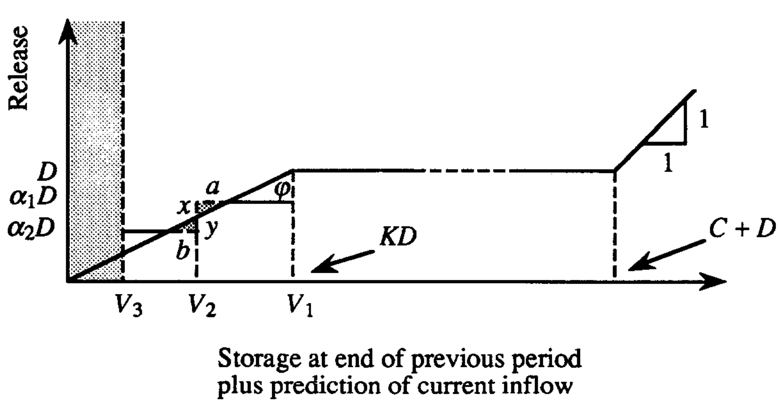

FIG. 4. Fitting Best Discrete Rule to Continuous Rule

J. Water Resour. Plann. Manage., 1994, 120(5): 613-629

minimized. The unshaded triangles are fixed in size and are not influenced by the choice of V_2. We can formulate this problem as a simple optimization problem. The problem is to

$$\min Z = \frac{1}{2}(ax + by) \tag{42}$$

subject to

$$\frac{(\alpha_1 - \alpha_2)D}{(a + b)} = \frac{D}{V_1} \tag{43}$$

Since the objective is always a positive number, we can ignore the 1/2 in the objective function. From simple geometric considerations, we have

$$x = a \tan \varphi \tag{44}$$

$$y = b \tan \varphi \tag{45}$$

$$\tan \varphi = \frac{D}{V_1} \tag{46}$$

so the optimization problem can be reformulated as

$$\min Z = \frac{D}{V_1}(a^2 + b^2) \tag{47}$$

subject to

$$(a + b) = (\alpha_1 - \alpha_2)V_1 \tag{48}$$

To solve this optimization problem with its equality constraint, we can use the method of Lagrange multipliers or we can use variable substitution. Both methods give the result

$$V_2 = \frac{1}{2}(\alpha_1 + \alpha_2)V_1 \tag{49}$$

This equation can be generalized to more than two phases of rationing. If you have n^* phases of rationing then you need to find $V_2, \ldots, V_{n^*-1}$. Any V_k between V_1 and V_{n^*} is given by the following equation, which provides the discrete operating rule which best fits the inclined line

$$V_k = \frac{1}{2}(\alpha_{k-1} + \alpha_k)V_1 \tag{50}$$

$$k = 2, \ldots, n^* - 1 \tag{51}$$

This approximation of the continuous operating rule provides a translation of the rule into a form usable by a water manager. That is, when storage plus expected inflow is between V_1 and V_2, first phase rationing is begun. When storage plus inflow is between V_2 and V_3, second-phasing rationing is undertaken, and so on.

The authors did not conduct simulations that use a discrete hedging rule derived in this way from the continuous rule. It would, nonetheless, be of interest to compare the operation of the reservoir with operation using the continuous demand-management rule. Comparison would have to be visual, however, because the maximum shortage that comes from suing the continuous rule is not strictly comparable with the frequency of various levels of shortage that come from using the discrete rule.

CONCLUSION

In this research, we have explored and comapred two methods to determine the parameters of continuous rationing rules for water-supply reservoir operation. The first method, the polytope search procedure, appears to be efficient and relatively effective in reducing the maximum value of shortage over a drought record. The second method, a mixed integer iterative method for a nonlinear program, provides smaller values of the maximum storage at the expense of more time on a more expensive machine.

The research reported is in its initial stages as it deals with only a single drought, albeit the worst on record. It will be important to find methodologies that extend the procedure to multiple droughts as well as to stochastic inputs. Further, the research opens up a methodology only for the single reservoir, dedicated to water supply, but we have begun as well to extend the methodology to multiple reservoirs.

ACKNOWLEDGMENTS

This research was conducted in part using the Cornell National Supercomputer Facility, a resource of the Center for Theory and Simulation in Science and Engineering at Cornell University, which is funded in part by the National Science Foundation, New York State, the IBM Corporation and members of the Corporate Research Institute. The writers are grateful for the access to the Cornell facility. We also like to thank two anonymous reviewers for their insightful comments.

APPENDIX I. REFERENCES

Babbit, H. E., and Doland, J. J. (1939). *Water supply engineering*. McGraw-Hill, New York, N.Y.

Dantzig, G. B. (1965). *Linear programming and extensions*, Princeton University Press, Princeton, N.J.

Dorfman, R. (1962). "Mathematical models: the multi-structure approach." *Design of water resources systems*, A. Maass, ed., Harvard University Press, Cambridge, Mass.

Fair, G. M., Geyer, J. C., and Okun, D. A. (1966). *Water and wastewater engineering: Volume 1, Water supply and wastewater removal*. John Wiley and Sons, New York, N.Y.

Fiering, M. B. (1967). *Stream flow synthesis*, Harvard University Press, Cambridge, Mass.

Gill, P. E., Murray, W., and Wright, M. (1981). *Practical optimization*. Academic Press, New York, N.Y.

Hashimoto, T., Stedinger, J. R., and Loucks, D. P. (1982). "Reliability, resilience and vulnerability criteria for water resources system performance evaluation." *Water Resour. Res.*, 18(1), 14–20.

Hazen, A. (1914). "Storage to be provided in impounding reservoirs for municipal water supply." *Trans.*, ASCE, 77, 1539.

Hirsch, R. M. (1979). "Risk analysis for a water supply system: Occoquan rservoir, Fairfax and Prince William Counties, Virginia, USA." *Hydrologic Sci. Bulletin*, 23(4), 475–505.

Houck, M. H., Cohon, J. L., and ReVelle, C. S. (1980). "Linear decision rule in reservoir design and management, 6, incorporation of economic efficiency benefits and hydroelectric energy generation." *Water Resour. Res.*, 16(1), 196–200.

IBM mathematical programming system extended/370 program reference manual. (1979). IBM, New York, N.Y.
Linsley, Jr., R. K., Kohler, M. A., and Paulus, J. L. H. (1982). *Hydrology for Engineers*. McGraw Hill, New York, N.Y.
Maass, A., Hufschmidt, M. M., Dorfman, R., Thomas, Jr., H. A., Marglin, S. A., and Fair, G. M. (1962). *Design of water-resource systems*, Harvard University Press, Cambridge, Mass.
Matalas, N. C. (1963). "Probability distribution of low flows." *Professional Paper 434-A*, U.S. Government Printing Office, Washington, D.C.
Moy, W-S., Cohon, J. L., and ReVelle, C. S. (1986). "A programming model for analysis of there reliability, resilience, and vulnerability of a water supply reservoir." *Water Resour. Res.*, 22(4), 489–498, 1986.
Palmer, R., and Holmes, K. J. (1988). "Operational guidance during droughts: expert system approach." *J. Water Res. Mgmt. and Plng.*, 114(6), 647–666.
Randall, D., Houck, M. H., and Wright, J. R. (1990). "Drought management of existing water supply system." *J. Water Res. Plng. and Mgmt.*, 116(1), 1–20.
Randall, D., Houck, M. H., and Wright, J. R. (1986). "Simulating water system operation during periods of drought." *J. AWWA*, 53–60.
ReVelle, C. S., Joeres, E., and Kirby, W. (1969). "The linear decision rule in reservoir management and design, 1, development of the stochastic model." *Water Resour. Res.*, 5(4), 767–777.
Rippl, W. (1883). "The capacity of storage reservoirs for water supply." *Proc. Inst. Civ. Eng.*, 71, 270–278.
Stedinger, J. R. (1984). "The performance of LDR models for preliminary design and reservoir operation." *Water Resour. Res.*, 20(2), 215–224.
Thomas, H. A., and Burden, R. P. (1963). *Operations research in water quality management*. Harvard University, Cambridge, Mass.
User's manual, math/library. FORTRAN subroutines for mathematical applications. (1987). IMSL, Inc., Houston, Tex.
Water Science and Technology Board. (1986). "Drought management and its impact on public water systems." *Colloquium Report*, National Research Council, National Academy Press, Washington, D.C.
Whipple, W., Jr. (1966). "Regional drought frequency analysis." *J. Irrig. and Drain. Engrg.*, ASCE, 92(2), 11–31.
Wilhite, D. A., and Easterling, W. E. (1989). "Coping With drought: toward a plan of action." *Eos*, Feb.
Wright, J. R., Houck, M. H., Diamond, J. T., and Randall, D. (1986). "Drought contingency planning." *Civ. Eng. Syst.*, 3, 210–221.

APPENDIX II. NOTATION

The following symbols are used in this paper:

a, b = bases of two shaded triangles, unknown;
B = arbitrarily large number;
b_t = storage level index, 0 or 1 variable;
C = reservoir capacity;
D = water demand, known;
D_t = water demand in period t, known;
H_p = $1/K_p$
I_t = inflow in period t, known;
$\hat{I}_t$ = stream-flow prediction that can be expected value or conditional expected flow, known by prior calculation, known;

K_p = trigger value, number of months of demand that must be available in storage plus projected inflow for full demand to be released; below this value, linear schedule is to be followed; Parameter is unknown;
K_pD = trigger volume, value of storage plus inflow that triggers rationing, unknown;
M = maximum shortage, unknown;
n = index of last month in sequence;
n^* = total number of phases of rationing;
p = $t - 12([t - 1]/12)$, $[\mu]$ = integer part of μ, and $p \in (1, 2, \ldots, 12)$;
R_t = release in period $t = D$ if $S_{t-1} + \hat{I}_t \geq K_pD$; $(1/K_p)(S_{t-1} + \hat{I}_t)$ otherwise;
S_0 = initial storage;
S_t = storage at end of period t, unknown;
t = index of months in sequence;
U_t = shortage in period t, difference between demand and draft, unknown;
V_1 = trigger volume ($V_1 = KD$), known;
V_K = trigger that initiates Kth phase of rationing;
W_t = spill in period t, unknown;
x, y = heights of two shaded triangles, unknown;
Y_t = slack variable, unknown;
Z = objective function;
Z_t = 0 or 1 variable for period t;
α_1 = fraction of demand that obtains during restriction phase one, known;
α_2 = fraction of demand that obtains during restriction phase two, known;
φ = angle of inclined line, known.