Drought Management of Existing Water Supply System

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Abstract: A multi-objective linear program is developed to study the operation of a metropolitan water supply system during drought. The Indianapolis Water Company is used as a case study. Twelve pressure districts, three reservoirs, and eight supply sources are included in the model. Four noncommensurate and conflicting objectives are identified and built into the linear program. The model is used to analyze operations during the drought of record and to develop trade-off curves among the objectives. Experience with the model suggests that this methodology can provide information that can aid operational decision making.

Introduction

The purpose of this paper is to describe the development and use of a multi-objective optimization model to analyze the operation of a metropolitan water supply system during a drought. The model's principal use would be to screen potential operating policies on the basis of multiple objectives to facilitate the selection of the best operating strategy.

The model is comprehensive enough to provide useful operational information while being simple enough to solve efficiently. The explicit consideration of multiple objectives is appropriate, as no single objective function can be formulated to satisfy all concerns. This is particularly true during a time of drought. This model has been used to study both long-term and real-time operation, although the focus of this paper is on model development and long-term operation. Real-time operation is the primary topic of another paper (Randall et al. 1986).

The Indianapolis water supply system was selected as a case study because it includes multiple surface and ground water sources and has a complex distribution system. Therefore, many problems that will be encountered in other systems are considered. Furthermore, the Indianapolis Water Company (IWC) generously provided data that were essential to the research.

IWC is a private utility and the primary supplier of water to the Indianapolis area. A map of the system is shown in Fig. 1. IWC's major source of supply is surface water. Water is withdrawn from White River at Broad Ripple Dam, Fall Creek at Keystone Dam, and Eagle Creek Reservoir. Morse and Geist Reservoirs are company-owned, single-purpose reservoirs that are used to augment low flows at Broad Ripple and Keystone Dams. Eagle Creek Reservoir is a multi-owner, multipurpose reservoir used primarily for recreation and secondarily for supply and flood control. Four well fields provide a small additional supply.

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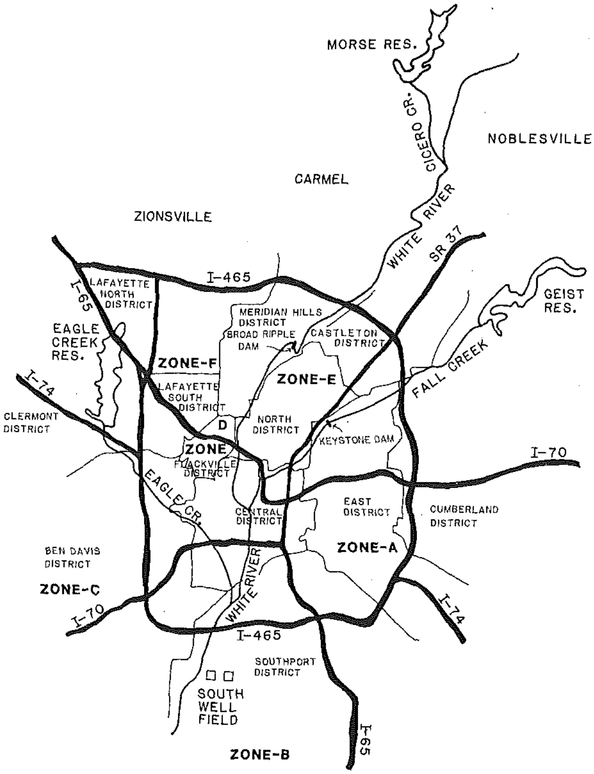

FIG. 1. Map of Indianapolis Water Company System

At the time of this study, IWC estimated its "safe yield" to be 118 million gallons per day (mgd) or 446 million liters per day (MLd), of which 112 mgd (423 MLd) is from surface sources. This safe yield is based on the 1940–42 drought, the most severe on record. Another recent study has estimated the system safe yield at 139 mgd (525 MLd), assuming a 50-yr return period drought and system-wide coordinated operation (Lampe and Bakken 1985). The current annual mean daily demand for the system is 100 mgd (378 MLd). The development of a new well field in southern Indianapolis, the "South Well Field," is in the planning stage. IWC estimates that this well field will increase its safe yield by 17%.

No water shortages have been experienced in the IWC system in over 50 yr. However, per capita demand is growing, and IWC is aggressively seeking new customers by expanding its service area. Thus, the probability that a water shortage will be experienced in the foreseeable future is increasing.

Model

Two principal goals guided the development of the mathemtical model. The first was to describe accurately each of the components of the IWC system and the interactions among these components. The second goal was

J. Water Resour. Plann. Manage., 1990, 116(1): 1-20

to produce a model that was efficient enough to be of use by researchers and practitioners in both long-term studies and real-time operation of the system.

Many different approaches to the use of mathematical models to optimize the operation of existing metropolitan water supply systems have been proposed. For example, Joeres et al. (1971) used chance-constrained linear programming to develop operating rules for the Baltimore water supply system, which draws water from three rivers, two of which are regulated by dams. Collins (1977) used a dynamic programming model to find operating rules for the least-cost operation of four reservoirs supplying water to Dallas, Texas. In each of these papers, the authors focused on the supply system without considering the operation of the distribution system. This separation of the supply system from the distribution system is typical of the optimization models described in the literature.

Philosophy Behind Model

The IWC system includes three major reservoirs, significant ground water sources, three treatment plants, 12 pressure districts, and more than 700,000 customers. It would be unrealistic to construct a model that considered individual customers: it would be too large to solve. A model that considers pressure districts as the smallest unit, however, would be small enough to be efficiently solved and still capture important operational considerations. Some justification for using a model at this scale can be found in papers by Coskunoglu and Shetty (1981), Maknoon and Burges (1978), and Flores et al. (1978). These authors all point out that increasing the complexity of the hydraulic/hydrologic methods of a management model may result in only slight improvement of the results.

Because South Well Field will add significantly to the safe yield of the system, two separate versions of the optimization model are developed, one for the existing condition and one for the future conditions. In both cases, it is possible to formulate the models as linear programs.

Several definitions are useful in understanding the model.

1. Supply system—that portion of the system upstream from the primary pumping stations (White River, Riverside Creek, Fall Creek, and Eagle Creek). This includes streams, reservoirs, aquifers, and treatment plants. Fig. 2 depicts the IWC supply system.
2. Distribution system—that portion of the system downstream from the primary pump stations. Fig. 3 depicts the IWC distribution system.
3. Demand—the amount of water that would be used given an unlimited supply.
4. Consumption—the amount of water that is actually consumed. Consumption equals demand when sufficient water is available.
5. Shortage—the difference between demand and consumption.
6. Net revenue—the difference between income from selling water and the electrical cost of pumping water. Only electricity costs are used because other costs do not vary by pressure district.

In constructing the model, five major assumptions about the physical system were made. First, the volume of storage in elevated and underground tanks within the distribution system is too small to have an impact on op-

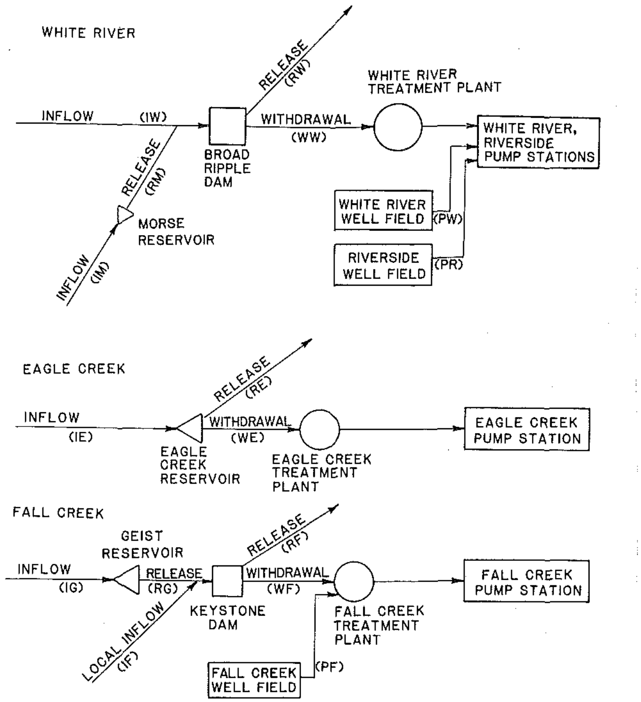

FIG. 2. Schematic of Supply System as Modeled

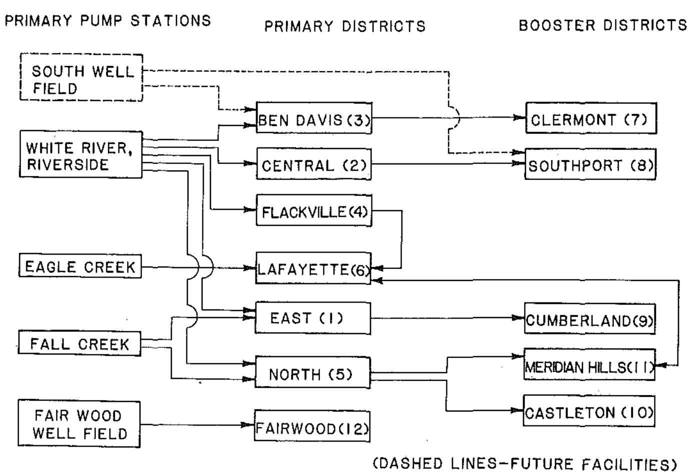

FIG. 3. Schematic of Distribution System as Modeled

eration during a long period, so it is not included in the model. Second, Broad Ripple and Keystone Dams merely act as intake structures with no appreciable storage. Third, during a drought it is assumed that Eagle Creek Reservoir will be operated as a supply facility with no recreational uses considered. Fourth, because they are company-owned, theoretically Morse and Geist Reservoirs can be drawn down to zero storage. Fifth, the IWC intake in Eagle Creek Reservoir is at an elevation corresponding to a storage of 2,120 million gal; storage is constrained not to go below this minimal level.

The model optimizes the system operation for a stated time horizon, given a series of forecasted streamflows and demands. A time increment ranging from one to several days is intended. The constraints are primarily continuity equations to insure that water is routed properly through the system; restrictions on maximum and minimum storage; and pipe, pump station, and treatment plant capacities.

One important aspect of this study is the formulation of objectives articulated by IWC officials. Four objectives were finally incorporated in the optimization model. The objectives are: maximize net revenue, maximize reliability, maximize storage at the end of the optimization horizon, and maximize streamflows.

The *maximize net revenue* objective seeks to operate the system such that the difference between income and electrical cost of pumping is maximized. No data that will allow the estimation of unit water revenue (dollars per million gallons or $/mg) by pressure district were available when this model was developed. Therefore, the average unit revenue for the system, about $1,280/mg ($339/ML), is used for each district. The unit cost of delivering water for each district varies from $26.4/mg to $90.0/mg ($6.99/ML to $23.8/ML).

The *maximize reliability* objective seeks to treat all districts equally by maximizing the minimum ratio of consumption to demand for each district during each time period. For the purposes of this study, this is called "reliability." This insures that no district will be amply supplied while another is severely shorted, which is likely under the revenue objective.

The *maximize end-of-optimization-horizon reservoir storage*, or *ending storage*, objective seeks to keep the ending storage high, thus providing more insurance against running short of water during the remainder of the drought. This objective is in direct conflict with both the revenue and reliability objectives.

It is desirable to maintain as large a minimum flow as possible in the streams, at least up to the 10-yr, 7-day low flow. Therefore a *maximize streamflow* objective that attempts to maximize the minimum flow in the streams is included. This objective is in direct conflict with all three of the other objectives.

Model Structure

A generalized version of the model is provided here. A complete list of the constraints and objective functions for the model of the IWC system can be found in Randall (1984) and Randall et al. (1986). To permit the formulation of the generalized model, it is necessary to define critical points of the system. These include any points in the system where the flow of water is controlled, diverted, combined, or divided (dams, pumping stations, pipe connections); stored (reservoirs); restricted (pipes, treatment plants); or where it enters (ground and surface water sources) or leaves (consumed in the districts, evaporation) the system. The mass balance or continuity restriction at these points is defined in Eq. 1. In this and all of the other constraints, the known quantities are placed on the right-hand sides of the equations, and the unknowns are on the left-hand sides

$$S_{it+1} - S_{it} - \sum_{h \in U_i} F_{hit} + \sum_{j \in D_i} F_{ijt} = 0$$

$$i = 1, 2, \ldots, C \qquad t = 1, 2, \ldots, T \qquad (1)$$

where S_{it} = storage volume at the beginning of period t at critical point i; F_{hit} = flow volume from critical point h to critical point i during period t; F_{ijt} = flow volume from critical point i to critical point j during period t; U_i = set of critical points directly upstream of critical point i; D_i = set of critical points directly downstream of critical point i; C = total number of critical points—streams, reservoirs, well fields, pumping stations, pipe connections, treatment plants, consumption districts, etc., and T = number of time periods in the operating horizon.

The storage volumes are restricted to acceptable ranges at each critical point

$$\begin{Bmatrix} S_{it} \leq SMAX_i \\ S_{it} \geq SMIN_i \end{Bmatrix} \qquad i = 2, 3, \ldots, C; \qquad t = 1, 2, \ldots, T \qquad (2)$$

where $SMAX_i$ = maximum acceptable storage volume at critical point i; and $SMIN_i$ = minimum acceptable storage volume at critical point i. The values of $SMIN_i$ and $SMAX_i$ vary according to the type of critical point. If the critical point is a reservoir or storage facility in the system, then they represent the minimum and maximum volumes of water that can be stored at the facility. If the critical point does not actually have any storage capacity—for example, a pipe connection or a pump station or a treatment plant—then $SMIN_i$ and $SMAX_i$ both equal zero, indicating that no water can be stored there ($S_{it} = 0$; $t = 1, 2, \ldots, T$). Eq. 1 then implies that whatever enters the critical point during a period must also leave during the period.

Capacity restrictions will constrain the movement of water through some critical points in the system. For example, the flow through a treatment plant cannot exceed the plant's capacity (Eq. 3); the flow through a pipe cannot exceed the pipe's capacity (Eq. 4); and the flow through a pump station cannot exceed the pump's capacity (Eq. 5).

$$\sum_{h \in U_i} F_{hit} \leq TCAP_i \; i \in TPS \qquad t = 1, 2, \ldots, T \qquad (3)$$

$$F_{hit} \leq PIPE_{hi} \; (h, i) \in PIPES \qquad t = 1, 2, \ldots, T \qquad (4)$$

$$\sum_{h \in U_i} F_{hit} \leq PCAP_i \; i \in PUMPS \qquad t = 1, 2, \ldots, T \qquad (5)$$

where $TCAP_i$ = capacity of treatment plant i; $PIPE_{hi}$ = capacity of the pipe connecting critical points h and i; $PCAP_i$ = pumping capacity at critical point i; TPS = set of critical points that are treatment plants; $PIPES$ = set of critical point pairs (h, i) that are connected with pipes; and $PUMPS$ = set of critical points that are pump stations.

There are other restrictions on some of the flows through the system. For example, inflows to a reservoir or evaporation losses from a reservoir may be assumed to be known. Withdrawals from a ground water supply are limited by the aquifer and the pumping capacity of the wells. There may also exist minimum values for some flows, such as low flow releases from a reservoir. And in no case can any flows be negative. All of these represent additional constraints on the values of F_{hit} and F_{ijt} in the model.

One last constraint that is typically needed in an operations model concerned with droughts defines the shortages that may occur in the system. Eq. 6 defines the amount of water delivered to a district as equaling the consumption. Eq. 7 ensures that the consumption does not exceed the demand and computes the value of any shortage

$$\sum_{h \in U_i} F_{hit} - C_{it} = 0 \qquad i \in DIST \qquad t = 1, 2, \ldots, T \quad (6)$$

$$C_{it} + SHORT_{it} = D_{it} \qquad i \in DIST \qquad t = 1, 2, \ldots, T \quad (7)$$

where C_{it} = volume of water consumed in district i in period t; $DIST$ = set of critical points that are districts in which water is consumed; $SHORT_{it}$ = volume of water shortage in district i in period t; and D_{it} = water volume demanded in district i in period t.

There are four objectives that are incorporated in the model. The first of these is to maximize net revenue

$$\text{Maximize } NR = \sum_{i \in DIST} \sum_{t=1}^{T} (rev_i - cost_i)C_{it} \quad (8)$$

where NR = total net revenue from sale of water in all districts over T periods; rev_i = revenue per unit of water consumed in district i; in the IWC model, this was assumed to equal \$1,280/mg (\$339/ML) for each district; and $cost_i$ = unit cost of delivering water to district i; for the 12 districts in the IWC model this varied from \$26.4/mg (\$6.98/ML) to \$90.0/mg (\$23.81/ML).

The second objective is to maximize reliability and may be described mathematically as

$$\text{Maximize } REL = r \quad (9)$$

subject to

$$(D_{it}^{-1})C_{it} - r \geq 0 \qquad i \in DIST \qquad t = 1, 2, \ldots, T \quad (10)$$

where REL = minimum ratio of consumption to demand in any district during any time period.

The third objective is to maximize the storage at the end of the forecast or operating horizon T periods in the future. For the IWC model, this was accomplished by summing the storages in the three main reservoirs. In other cases, it may be desirable to weight the storage in one storage facility differently from another. The mathematical formulation of the objective, without including weights, is

$$\text{Maximize } ENDS = \sum_{i \in STOR} S_{iT+1} \quad (11)$$

where $ENDS$ = total volume of water in all storage facilities at the end of the T period forecast or operating horizon; and $STOR$ = set of critical points that are storage facilities (e.g., reservoirs).

The fourth objective to maximize the minimum streamflow is to satisfy environmental and/or legal constraints. The objective is formulated to maximize the minimum ratio of actual flow at some site to the 10-yr, 7-day low flow at the site over the forecast or operating horizon. Mathematically this is formulated as

$$\text{Maximize } MINFLO = m \quad \ldots\ldots \quad (12)$$

subject to

$$(RMIN_i^{-1})\left(\sum_{h \in U_i} F_{hit}\right) - m \geq 0 \qquad i \in LOW \qquad t = 1, 2, \ldots, T$$

where $MINFLOW$ = minimum ratio of actual flow at any critical flow point to the 10-yr, 7-day low flow at those sites over the T period forecast or operating horizon; $RMIN_i$ = 10-yr, 7-day low flow at critical point i; and LOW = set of critical points for low flow measurement; for the IWC model, this set only included sites directly below reservoirs and dams.

There are several methods to solve multi-objective linear programs (Cohon 1978). The two general techniques are called generating and preference. Generating techniques provide an approximation of the non-inferior set (also called the trade-off curve because it illustrates the trade-offs among the objectives) while preference methods do not. Also, preference techniques require the decision-makers—in this case, IWC officials—to make value judgments prior to analysis, while generating techniques allow the decision-makers to make value judgments only after the trade-off curves have been generated. Thus decisions are made with full knowledge of the trade-offs among the objectives. For this reason, a generating technique for this problem is preferred. The drawback is that a heavier computational cost must be paid.

From the set of generating techniques, the constraint method was chosen. This method requires solving the problem several times. Each time one objective is optimized with the other objectives included as constraints at user-specified levels (objective constraints). The constraint method is appropriate for this problem because the end storage and streamflow objectives are easily handled in this way.

Model Building Blocks

Five fundamental components—a pumping-cost-by-pressure-district model, a streamflow generation model, a reservoir loss model, a daily district demand model, and a well field model—are required in the mathematical program described in the previous section. Each of these components requires a separate analysis.

District Pumping Costs

IWC does not record pumping cost by district. However, 13 monthly pumpage reports and consumption reports from June 1982 through June 1983 were available. The pumpage reports contain total discharge, energy consumption, and cost for the month for each pump station. The consumption

J. Water Resour. Plann. Manage., 1990, 116(1): 1-20

reports contain total discharge for each district and bleeder. (Several mains leave White River Pump Station, for example, carrying water to different locations. These mains are referred to as "bleeders.").

In order to compute unit cost ($/mg) by district, the pump station costs must be converted into district costs. A method was devised in which unit energies (kWh/mg) are computed for each bleeder based on bleeder energy weighting factors. The bleeder energy weighting factors are in turn based on estimates of the head required to deliver a unit of water to the customer via that bleeder. These bleeder unit energies are then multiplied by energy price ($/kWh) to get bleeder unit costs. The average district unit costs are then computed from the bleeder unit costs.

The only nonlinearity in expressing pump energy as a function of discharge is in the friction loss. This analysis shows that much more energy is required to pressurize the lines and to overcome gravity than to overcome friction, which makes the friction loss insignificant. Thus the assumption that energy is a linear function of discharge is justified, and the use of linear programming as the solution procedure is reasonable.

Streamflow Generation

IWC uses the 1940–42 drought (June 18, 1940–February 19, 1942) as the benchmark for all its drought studies. Because of this, no hydrologic model was developed to predict streamflows. Rather, the historical flow data for the 1940–42 drought, obtained from U.S. Geological Survey (USGS) records (*Surface* 1940–42), were used in this analysis.

Daily stream discharges are needed at five locations in the system: Morse Reservoir, Geist Reservoir, Eagle Creek Reservoir, Broad Ripple Dam, and Keystone Dam. Although no gauges existed at these locations during the drought, the USGS did operate several gauges in the area. Analysis of these streamflows during the drought period showed that the flows were highly correlated (r^2 values ranged from 0.76 to 0.90) and the discharges per unit drainage area were similar. None of the three reservoirs existed during the drought, so the gauges are not affected by storage. To estimate these streamflows at the five needed locations, flows were "transferred" from the nearest hydrologically appropriate stream gauge that existed during the drought to each of the five locations. This was done by multiplying the gauged discharge per drainage area by the drainage area at the point in question.

Reservoir Loss

Reservoir loss, which is expected to be primarily for evaporation, is included in the model to insure that the amount of water available is not overpredicted. The loss function, which is essentially an estimate of the difference between reservoir evaporation and precipitation on the reservoir, is built into the reservoir continuity equations. This provides an expedient way of handling the losses.

An important assumption is required to define the loss function. The average evaporation (0.7 × pan evaporation) is about 31 in./yr (790 mm/yr), while the average rainfall for the area is about 40 in./yr (1,020 mm/yr). Further, the minimum recorded 2-yr rainfall is 28 in./yr (711 mm/yr). Unfortunately, no evaporation records were found for the 1940–42 drought. If one assumes that the weather during a drought is hotter and drier than normal, thereby causing increased evaporation (for example, about 40 in./yr or 1,020 mm/yr); and the rainfall will be low (for example, about 25 in./yr or 635 mm/yr); then the difference between drought rainfall and drought evaporation will be about one-half of the normal evaporation. So, the net resevoir losses for the 1940–42 drought are estimated to be one-half of the normal evaporation or 15.5 in./yr (395 mm/yr).

This net reservoir loss can be directly included in the model by expressing evaporation as a function of reservoir storage instead of surface area. To do this, reservoir surface area (A) is regressed linearly with reservoir storage (S) for each of the three reservoirs to estimate values of θ and η in Eq. 13

$$A = \theta \times S + \eta \qquad (13)$$

The r^2 values for these regressions are all greater than 0.95. One of the terms in Eq. 1 for a critical point that is a reservoir is the net change of volume due to evaporation and precipitation on the reservoir. This term can now be defined by the product of A from Eq. 13 and the net reservoir loss rate estimated as 15.5 in./yr (395 mm/yr).

Demand Analysis

The model requires the use of daily district demands, which have been assumed to be known in the linear program (LP). These routinely fluctuate in a seemingly random fashion. Conversations with IWC officials revealed that the system experiences no "wash-day." In other words, the variation in district demands does not seem to follow any periodic cycle other than an annual one.

At the time of the development of this model, no daily district demand data were available. Data were available from 1971–1982 that showed monthly system demands for each year, and mean daily demand and maximum day demand for the system and for each district for each of the 12 yr. These data were analyzed to develop the input to a model to generate synthetic daily district demands that approximated the variability of the actual demands while preserving the annual cycle and the percentages of the system demand for each district. This is a rather lengthy derivation and can be found in Appendix I.

Ground Water

At present, IWC uses a small amount of ground water; only about 5% of its safe yield comes from the four existing well fields. Only Fairwood Well Field is used continuously, as it is the primary source of supply for Fairwood District. Flow from South Well Field will go to Ben Davis and Southport Districts.

The characteristics of the existing well fields vary widely, based on information obtained from Indiana Department of Natural Resources, Division of Water (DNR). The well diameters for Fairwood, Riverside, and White River Fields vary within each field. White River and Fairwood Fields pump from only the unconfined aquifer near the surface. Fall Creek and Riverside wells penetrate into the underlying confined aquifer, apparently pumping from both the unconfined and confined aquifers. Also, the storage coefficient and transmissivities for these fields vary widely. Because of these factors, accurately modeling these fields would be difficult.

The well fields were modeled using IWC's values of well field capacity and safe yield to constrain the flows. Eq. 14 insures that the withdrawals from well i do not exceed the pumping capacity of the well

$$\sum_{j \in D_i} F_{ijt} \leq WCAP_i \qquad i \in WELLS \qquad t = 1, 2, \ldots, T \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \quad (14)$$

where $WCAP_i$ = pumping capacity of well i; and $WELLS$ = set of critical points that are wells. This restriction does not preclude significant mining of the aquifer if the well is producing at capacity over a long time period. To prevent this from occurring, the output of the well over the forecast or operating horizon is restricted not to exceed the safe yield of the aquifer

$$\sum_{t=1}^{T} \sum_{j \in D_i} F_{ijt} \leq T \times S_i \qquad i \in WELLS \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \quad (15)$$

where S_i = safe yield of well i per time period.

Sixteen wells are planned for South Well Field. All of these are 2 ft in diameter; their capacities are either 2 or 3 mgd (7.56 or 11.3 MLd), and they each penetrate only to the bottom of the unconfined aquifer that lies near the surface. IWC had estimates of the transmissivities and storage coefficients of the aquifer.

Given the amount of data and the uniformity of the wells, the linear response function model developed by Maddock (1974) and used by Bathala et al. (1980) is ideal for this situation, as it can be easily incorporated into an LP, and because the drawdown can be constrained. Unfortunately, with 16 wells and many time periods, this model increases the size and solution time of the LP enormously. Therefore, South Well Field was modeled the same way the other fields were modeled. Once the LP has been solved, the linear response functions are used with the well flows from the LP solution to compute the drawdowns. The drawback with this method is that it does not allow the drawdowns to be constrained, but they can be computed. If they are found to violate the allowable drawdowns, the LP can be rerun with more restrictive constraints on pumping.

Pumping costs for Fall Creek, Riverside, and White River Well Fields are based on estimates of the average depth to the water surface and the average electricity price. Pumping costs for South Well Field are based on the estimated average depth to the water surface, estimated heads required to pump to Ben Davis and Southport Districts, and the electricity price. The estimated pumping heads and pumping costs can be checked using the linear response function models; if significant deviations are found, the linear program can be rerun with better estimates.

Summary

The current-conditions linear program, when maximizing revenue, has $32T + 7$ constraints and $11T$ simple upper bounds (SUB), plus $12T$ SUBs if reliability is used as an objective constraint. The maximize reliability objective requires an additional $12T$ constraints. The future-conditions LP needs one additional constraint and $16T$ more SUBs than the current conditions LP. The LP is solved using the linear, interactive, discrete optimizer—LINDO (Schrage 1981).

Using Model

The entire 600-day drought was modeled using 60 10-day time periods ($T = 60$). Using shorter time periods became cost-prohibitive due to the ex-

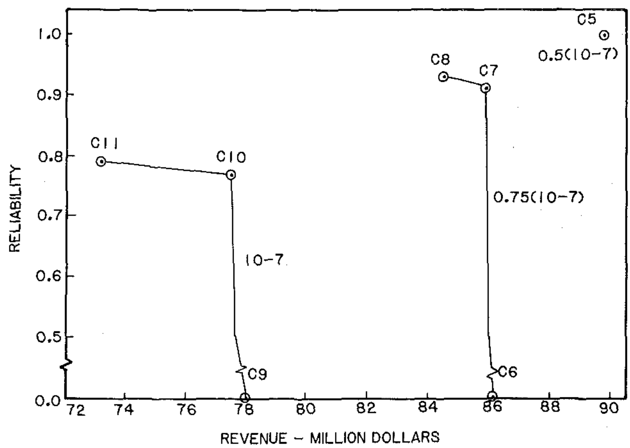

FIG. 4. Trade-Off Curves for Demand = 120 mgd (456 MLd)

ponential increase in computer time to solve LPs with more constraints. The current-conditions model was solved with demand levels of 100, 120, and 140 mgd (378, 454, and 529 MLd, respectively) with flowby levels in increments of one-fourth of the 10-yr, 7-day (10-7) low flow. The future-conditions model was used with the 120 and 140 mgd (454 and 529 MLd) demand levels. The 120-mgd demand level is expected to be reached sometime between 1992 and 2015, while the 140-mgd demand level is expected sometime after 2005.

The inherent assumption in the long-term optimization is that the streamflows and district demands are known perfectly for the entire period of the drought. This is obviously not the case in a real situation. However, the long-term operation has been optimized with the expectation that it may lend some insight into the real-time operation. Also, the peak operating efficiency can be determined by solving the model for long-term operation. Because the long-term optimization is done with only one LP, the ending storage objective is inappropriate; so there are only three objectives to consider in the trade-offs. For short-term optimization, the ending storage objective remains important.

Trade-Off Curves

The most important conceptual result of the long-term model runs is a set of trade-off curves; one example of these for the 120-mgd (454-MLd) demand level is provided in Fig. 4. The revenue-reliability trade-off is plotted for flowby requirements of one-fourth, one-half, three-quarters, and the full 10-7 low flow. These results are for the current system conditions without South Well Field. The single point at revenue equals $\$89.8 \times 10^6$ and reliability equals 1.0 indicates that all the demand is supplied during the entire drought when only one-half of the 10-7 low flow is required as flowby. These data and model results for other system conditions and demand levels are also listed in Table 1. They may be used to construct trade-off curves for higher and lower demand levels and for the water supply system including South Well Field.

TABLE 1. Results of Long-Term Drought Management Model for Indianapolis Water Supply System under Various Demands and System Conditions

Run[a] (1)	Demand, q_{day} (mgd) (2)	Flowby in fraction of 10-yr, 7-day low flow (3)	Revenue (millions of dollars) (4)	Reliability (5)
C1	100	0.75	75.04	1.00
C2	100	1.00	72.48	0.00
C3	100	1.00	72.31	0.92
C4	100	1.00	71.43	0.94
C5	120	0.50	89.84	1.00
C6	120	0.75	86.12	0.00
C7	120	0.75	85.93	0.91
C8	120	0.75	84.55	0.93
C9	120	1.00	77.97	0.00
C10	120	1.00	77.50	0.77
C11	120	1.00	73.16	0.79
C12	140	0.25	105.24	1.00
C13	140	0.50	98.05	0.00
C14	140	0.50	97.74	0.88
C15	140	0.50	95.67	0.90
C16	140	0.75	90.38	0.00
C17	140	0.75	89.85	0.77
C18	140	0.75	84.96	0.79
C19	140	1.00	81.08	0.00
C20	140	1.00	80.47	0.65
C21	140	1.00	73.00	0.67
F1	120	1.00	89.85	1.00
F2	140	0.75	104.79	0.90
F3	140	1.00	95.82	0.00
F4	140	1.00	95.33	0.85
F5	140	1.00	93.22	0.87

[a]The runs labeled with C are for current conditions system, without the South Well Field. The runs labeled with F are for future conditions system with South Well Field.
Note: SI conversion = 1 mgd = 3.78 MLd.

Of particular importance is the rectangular nature of the trade-off between revenue and reliability. Using the curve for the 10-7 flowby as an example, the upper end of the curve indicates that a 2% reduction in reliability buys a 6% increase in revenue (and therefore, a 6% increase in consumption). The lower end shows that a large decrease in reliability buys almost no increase in revenue. Experiments with the model show that the break in slope of the curve becomes sharper under heavier demands and/or flowbys.

There are many ways this type of information may be used. For example, one can argue that a 2% reduction in reliability is slight in the sense that the consumer will not be likely to feel it, while a 6% increase in revenue (and consumption) is significant because it will be noticed by any organization that keeps financial records. This discussion of a water utility's profits during a drought may seem rather mercenary. But this kind of knowledge may help the utility operate efficiently enough to forestall a rate increase, which has happened in at least one case (*The 1976–1977 California Drought*

1978), thereby reducing the overall financial impact on the consumers.

It is important to remember that the preceding conclusions have been based on only one realization of a stochastic process. Because the stochastic nature of the demand model yields a distribution of daily district demands around some mean values, there is a distribution of points on the trade-off curves about some mean value. The model was solved for four other realizations of the stochastic process for demand of 120 mgd (454 MLd) and a 10-7 flowby. Clearly, to be statistically significant, additional realizations should be considered. However, these five realizations resulted in trade-off curves whose shapes were very similar to the one shown. Further, these realizations indicate that the variance of the distribution of points on the curve is small. There is about a 0.5% difference in revenue between the lowest and highest values for the runs that maximize revenue with no reliability constraint (Randall 1984).

The noninferior set has been approximated by only three points. This is due to the high cost of the computer runs. Experimenting with the model in the region of the break in slope shows that this is a close approximation.

Reservoir Operation

Figs. 5 and 6 are reservoir storage plots for runs C9 and C11, respectively. These indicate how differently the reservoirs are operated, and how the operation of Geist and particularly Morse Reservoir varies with objective. Morse Reservoir is drawn down faster during dry periods and recovers faster during wet periods.

Fig. 5 indicates that Eagle Creek Reservoir never approaches its minimum storage, while Fig. 6 shows that during the second time period Eagle Creek Reservoir drops nearly 3,000 mg (11,300 ML) of storage. Fig. 6 was included in part to illustrate the concept of alternative optimal solutions. In this solution the LP recognized that the 3,000 mg (11,300 ML) of storage was not necessary to meet the constraints and did not affect the optimal reliability. So the LP arbitrarily released enough water to meet the minimum storage requirement at the end of the run. Obviously, the operators would not operate the system in this manner, and the LP could be constrained to prevent this result. However, this kind of result is useful as it underscores the fact the Eagle Creek Reservoir is not being used to the fullest extent.

Improved Supply Systems Integration

Studying the model results brought some "bottlenecks" to light. Eagle Creek Reservoir could provide much more water, but is constrained by limits on the physical system. As the system exists, Eagle Creek can only supply Lafayette District and part of Meridian Hills District (see Fig. 3). The physical system could be changed to allow Eagle Creek to be more heavily used. This could be done in two ways. One is to increase the capacity of the Meridian Hills Bleeder connecting Lafayette and Meridian Hills Districts to allow all of Meridian Hills to be supplied from Eagle Creek. The other is to add a link to connect Eagle Creek with one of larger districts, nearby, such as Ben Davis.

An analysis was done for run C9 to determine the effect of changing the system. Increasing the capacity of the Meridian Hills Bleeder so that it is not restrictive would allow an additional 600 mg (2,270 ML) to be supplied, for a revenue increase of about $740,000. A link between Eagle Creek and

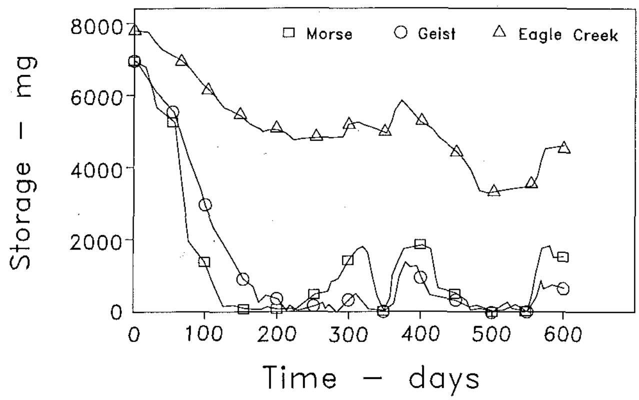

FIG. 5. Storage Plots for Run C9

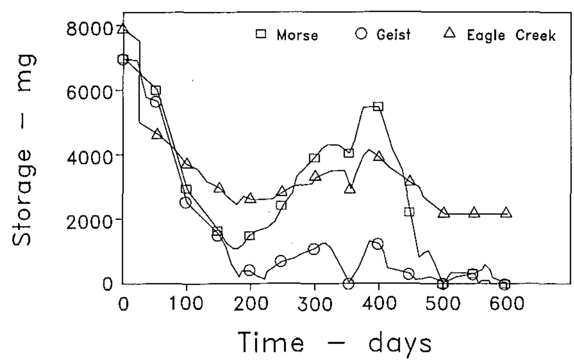

FIG. 6. Storage Plots for Run C11

Ben Davis would supply an additional 1,180 mg (4,460 ML), for a revenue increase of $1,470,000, and would draw the reservior down to the limit of 2,120 mg (8,010 ML). This increases system efficiency substantially.

System Safe Yield

One of the secondary benefits of this modeling process is an estimate of the system safe yield. As mentioned earlier, IWC estimated their safe yield at 118 mgd (446 MLd). That safe yield is computed using 5 mgd (18.9 MLd) (by agreement with another utility) and 0 mgd flowbys at Keystone Dam and Broad Ripple Dam, respectively. The USGS estimate (1972) of the 10-7 low flows at these locations is 17 (64 MLd) and 49 mgd (85 MLd), respectively. [Eagle Creek Reservoir's 10-7 low flow is less than 1 mgd (3.78 MLd), but the contractual flowby is 5.6 mgd (21 MLd).] Table 1 indicates that run C12 experienced no shortages with a mean demand of 140 mgd (529 MLd) and 0.25(10-7) flowby. Adjusting the results of this run for IWC's flowbys, the safe yield of the system would be about 150 mgd (567 MLd), rather than 118 mgd (446 MLd).

Summary

A multi-objective linear program has been developed and tested for the purpose of studying the drought operation of a major metropolitan water supply system. This work demonstrates that an optimization model that is readily solvable, yet provides information that will aid in operational decision-making, can be applied to a large-scale system. With some modifications, the model can be used to analyze short-term operation (Randall et al. 1986). Also, the model can be used to find "bottlenecks" in the system, and determine the effect of modifications to the system.

Acknowledgments and Disclaimer

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Appendix I. Demand Analysis

Historical values of daily district demands would be the ideal way to handle demands, given that the streamflows are historical. However, as stated in the text, no historical daily district demand data existed at the time of the study. The next best solution would be a complete stochastic model to generate daily district demands. Such a model would include autocorrelations within each district and cross-correlations among the districts. However, without historical data it was impossible to develop a complete model. Instead, a stochastic disaggregation model was used to generate daily district demands that represent the variability of the actual demands and preserve the annual cycle. The model requires only that data that were available. It splits the value of q_{day}, the annual mean daily system demand, into daily district demands for the duration of drought being studied. A complete set of daily district demands (d_{day}, which are the same as D_{it} in Eq. 7) will be computed based on one user-specified value, q_{day}, the mean daily system demand for a year. The available data are summarized in Table 2.

The following variables are used in the development of the demand model: q_{day} = mean daily system demand for a year (mgd or MLd); q_{sys} = total volume of system demand for a year (mg or ML), 365 × q_{day}; q_{mon} = mean daily system demand for a month (mgd or MLd); $\bar{x}_{mon}$ = mean fraction of annual system demand for a given month—one value for each month (unitless); $\bar{x}_{dis}$ = mean fraction of annual district demand to annual system demand—one value for each district (unitless); d_{dis} = mean daily district demand for a month (mgd or MLd); s_{dis} = standard deviation of district fraction of annual system demand (unitless); r_n = quasi-random number normally distributed with mean of zero and standard deviation of one, $N(0, 1)$ (unitless); d_{day} = district demand for a particular day (mg or ML); S_{day} estimated standard deviation of daily demand for a district, scaled to be unitless based on r_{mm} and $\alpha = 0.003$; m_{dd} = annual mean daily demand for a district (mgd or MLd); r_{mm} = the mean ratio of annual maximum daily district demand to m_{dd} (unitless).

TABLE 2. Demand Generation Data

Month (1)	$\bar{x}_{mon}$ (2)	District (3)	$\bar{x}_{dis}$ (4)	S_{dis} (5)	Mean[a] (6)	Maximum[b] (7)	r_{mm} (8)	s_{day} (9)
January	0.0780	East	0.206	0.0190	21.5	30.2	1.41	0.15
February	0.0762	Central	0.238	0.0280	23.1	31.6	1.48	0.17
March	0.0744	Ben Davis	0.0900	0.0113	9.51	11.9	1.31	0.11
April	0.0772	Flackville	0.0165	0.0020	1.70	3.03	2.22	0.44
May	0.0792	North	0.0840	0.0138	6.79	11.5	1.57	0.21
June	0.0890	Lafayette	0.0497	0.0310	5.04	7.31	1.47	0.17
July	0.0966	Cumberland	0.0943	0.0078	9.43	15.0	1.44	0.16
August	0.0967	Southport	0.125	0.0059	12.4	16.1	1.37	0.13
September	0.0942	Clermont	0.0028	0.0005	0.32	0.51	2.06	0.38
October	0.0837	Castleton	0.0300	0.0092	3.02	5.04	2.02	0.37
November	0.0780	Meridian Hills	0.0592	0.0036	6.43	9.89	1.74	0.27
December	0.0768	Fairwood	0.0045	0.0007	0.57	0.095	2.38	0.50
Total	1.0000		—	—	99.3	135.0	—	0.14

[a]Mean daily demand.
[b]Mean maximum day demand.

This is the rationale behind the model. Over the period of record (13 yr) the total system demand for each month is a certain percentage of the annual system demand, noted in Table 2. This value is $\bar{x}_{mon}$. This determines the value of the mean daily system demand for a month, q_{mon}. Next, over the course of a year, each district has a certain percentage of the system demand, which is $\bar{x}_{dis}$. $\bar{x}_{dis}$ has a distribution (assumed normal) with mean $\bar{x}_{dis}$ and standard deviation s_{dis}. The mean daily district demand d_{dis}, is calculated from $\bar{x}_{dis}$ and s_{dis}, with some random fluctuation about $\bar{x}_{dis}$, and q_{mon}. The probability of a daily district demand value, d_{day}, equaling or exeeding the annual maximum day district demand is 1/365, about 0.003, because it occurs only once per year. The procedure is illustrated with an example shown for East District during June, with q_{day} = 100 mgd (378 MLd). The value of r_{mm} is 1.41, which means that 1.41 $\times$ m_{dd} is about 2.75 standard deviations (based on α = 0.003 and a normal distribution) above m_{dd}. Thus the standard deviation for m_{dd} (this variable is s_{day}) is 0.41/2.75 (or 0.15) times m_{dd}. Once the values of s_{day} have been estimated, Eqs. 16–18 can be used to compute d_{day} values.

First compute q_{mon}

$$q_{mon} = \frac{\bar{x}_{mon} \times q_{sys}}{\#\text{ of days in month}} \quad (16)$$

which gives the mean daily system demand for the current month. The value of $\bar{x}_{jun}$ is 0.089, and q_{jun} = 0.089 $\times$ 36,500/30 = 108 mgd (409 MLd). Then Eq. 17 is used to distribute q_{mon} among the districts. Note that there is an element of randomness here

$$d_{dis} = (\bar{x}_{dis} + s_{dis} \times r_n) \times q_{mon} \quad (17)$$

The values of $\bar{x}_{eas}$ and s_{eas} are 0.206 and 0.019, respectively; the subscript *eas* denotes the East District. If the realization of the random number gen-

J. Water Resour. Plann. Manage., 1990, 116(1): 1-20

erator is 0.66 (i.e., $r_n = 0.66$), then $d_{eas} = (0.206 + 0.019 \times 0.66) \times 108 = 23.6$ mgd (89.2 MLd). Finally, d_{dis} is converted into daily demands for that district, again using an element of randomness

$$d_{day} = d_{dis} \times (1 + s_{day} \times r_n) \quad (18)$$

The value of s_{day} for East District is 0.15. Assuming that the next realization of the random number generator gives a value of $r_n = -0.82$, this gives $d_{day} = 23.6 \times [1.0 + 0.15 \times (-0.82)] = 20.7$ mgd (78.2 MLd). This achieves the original goal of computing daily district demands. The appropriate number of d_{day} values are summed for time steps longer than 1 day.

The use of the random number generator makes this a stochastic model. The sequence of daily district demands will change if a different seed value for the random number generator is used. Therefore, any results obtained using this demand model will be partly dependent on the seed used.

There is nothing in this method that assures that the total of all the d_{day} values for a year will equal the specified value of $q_{day} \times 365$ days. There will be some distribution of the total of the d_{day} about the mean value, which is $q_{day} \times 365$ days. However, experimentation with the model indicates that the error will typically be less than 0.5%.

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Appendix III. Notation

The following symbols are used in this paper:

A = reservoir surface area;
C = total number of critical points in system;
C_{it} = volume of water consumed in district i during period t;
$cost_i$ = unit cost of delivering water to district i;
D_i = set of critical points directly downstream of critical point i;
D_{it} = volume of water demanded in district i during period t;
$DIST$ = set of critical points that are districts in which water is consumed;
d_{day} = district demand for a particular day;
d_{dis} = mean daily district demand for month;
$ENDS$ = total volume of water in all storage facilities at end of optimization horizon;
F_{ijt} = flow volume from critical point i to critical point j during period t;
F_{hit} = flow volume from critical point h to critical point i during period t;
h = subscript referring to a critical point;
i = subscript referring to a critical point;
j = subscript referring to a critical point;
LOW = set of critical points for low flow measurement;
M = total number of wells;
$MINFLO$ = minimum ratio of actual flow at any critical point to 10-yr, 7-day low flow at those sites over optimization horizon;
m = same as *MINFLO*;
m_{dd} = annual mean daily demand for a district;
NR = total net revenue from sale of water in system during optimization horizon;
$PCAP_i$ = pumping capacity at critical point i;
$PIPE_{ji}$ = capacity of pipe connecting critical points j and i;
$PIPES$ = set of critical point pairs (j, i) that are connected with pipes;
$PUMPS$ = set of critical points that are pump stations;
q_{day} = mean daily demand for water in system for a year;
q_{mon} = mean daily system demand for a month;
q_{sys} = total volume of system demand for a year;
REL = minimum ratio of consumption to demand in any district during any time period;
$RMIN_i$ = 10-yr, 7-day low flow at critical point i;
r = same as *REL*;
r_{mm} = mean ratio of annual maximum daily district demand to m_{dd};
r_n = quasi-random number [$N(0, 1)$];
rev_i = revenue per unit of water consumed in district i;
S = storage volume in reservoir;
S_{it} = storage at critical point i during period t;
S_j = safe yield of well j per time period;
$SHORT_{it}$ = volume of shortage in district i during period t;
$SMAX_i$ = maximum acceptable storage volume at critical point i;
$SMIN_i$ = minimum acceptable storage volume at critical point i;

J. Water Resour. Plann. Manage., 1990, 116(1): 1-20

$STOR$ = set of critical points that are reservoirs;
s_{day} = estimated standard deviation of daily demand for a district;
s_{dis} = standard deviation of district fraction of annual system demand;
T = total number of time periods in optimization horizon;
$TCAP_i$ = capacity of treatment plant i;
TPS = set of critical points that are treatment plants;
t = subscript referring to a time period;
U_i = set of critical points directly upstream of critical point i;
$WCAP_j$ = pumping capacity of well j;
$WELLS$ = set of critical points that are wells;
$\bar{x}_{dis}$ = mean fraction of annual district demand to annual system demand;
$\bar{x}_{mon}$ = mean fraction of annual system demand for a given month;
θ = linear regression coefficient; and
η = linear regression coefficient.

J. Water Resour. Plann. Manage., 1990, 116(1): 1-20