Identifying Critical Infrastructure: The Median and Covering Facility Interdiction Problems

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Facilities and their services can be lost due to natural disasters as well as to intentional strikes, either by terrorism or an army. An intentional strike against a system is called interdiction. The geographical distribution of facilities in a supply or service system may be particularly vulnerable to interdiction, and the resulting impacts of the loss of one or more facilities may be substantial. Critical infrastructure can be defined as those elements of infrastructure that, if lost, could pose a significant threat to needed supplies (e.g., food, energy, medicines), services (e.g., police, fire, and EMS), and communication or a significant loss of service coverage or efficiency. In this article we introduce two new spatial optimization models called the *r*-interdiction median problem and the *r*-interdiction covering problem. Both models identify for a given service/supply system, that set of facilities that, if lost, would affect service delivery the most, depending upon the type of service protocol. These models can then be used to identify the most critical facility assets in a service/supply system. Results of both models applied to spatial data are also presented. Several solutions derived from these two interdiction models are presented in greater detail and demonstrate the degree to which the loss of one or more facilities disrupts system efficiencies or coverage. Recommendations for further research are also made. *Key Words: critical infrastructure, facility location, p-median problem, maximal covering, interdiction.*

Supply systems involve a set of manufacturing, storage, and transportation facilities and assets that accomplish the supply of goods and services. There is a risk of sudden loss of facilities and assets due to natural causes such as floods and fire and due to man-made causes such as terrorism and military action. Our concern in this article deals with the latter type of problem, that is, a loss of capacity due to some type of attack. Intentional disruption of a supply system is called interdiction. The problem of interdiction has received considerable attention due to the interest in interdicting supply lines during warfare (McMasters and Mustin 1970). More recently, interest has focused on what is termed "critical infrastructure." We define critical infrastructure as those elements of infrastructure that, if lost, could pose a significant threat to needed supplies (e.g., food, energy, medicines), services (e.g., police, fire, and EMS), and communication or a significant loss of service coverage or efficiency. These services and supplies are often termed "lifelines." Losing capacity of a lifeline could have a great impact on a population or army. Each lifeline system has certain elements that are more important than others. Those elements of infrastructure that are most important in a lifeline system are often called the "vital" links. For example, one element of a power transmission system may be a key link in providing power to a very large area, and alternate system routes may not have the capacity to provide adequate supply. The loss of that key link would be detrimental to the full operation of the system and is therefore vital to system supply. Cutter, Richardson, and Wilbanks (2003) have identified the need to develop methods for identifying critical infrastructure as one of several national research priorities.

In this article we introduce two new models called the *r*-interdiction median problem and the *r*-interdiction covering problem. Both models identify for a given service/supply system, that set of facilities that, if lost, would affect service delivery the most, depending upon the type of service protocol. These models can then be used to identify the most critical facility assets in a service/supply system. In the next section, we give a brief review of the literature on modeling interdiction. Then we define a general problem of facility interdiction that can be used to help identify "critical facilities," that is, which facilities to interdict or which facilities to protect. In a subsequent section, we define the *p*-median location model and its interdiction model counterpart. We then define the maximal covering location problem along with its interdiction model counterpart. Both of these interdiction models are new, innovative models that can be used to identify critical facility assets. We follow with some computational experience and present several example solutions to both interdiction models. We

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conclude with a summary and recommendations for future work.

Background

Loss of service or supply can be the result of a number of different types of factors, including system component failure, natural disaster (e.g., earthquake), catastrophic accident (e.g., Chernobyl nuclear power plant disaster), financial collapse, and intentional disruption (e.g., terrorism and military action). We exclude from our concern here issues of system reliability due to common component failure. System reliability is a well-defined research field that involves the design of a system to meet specified levels of reliability (based upon failure rates of specific components) and the modeling and simulation of system reliability (see, for example Tillman, Hwang, and Kuo 1977; Mohamed, Leemes, and Ravindran 1992). Reliability analysis has been applied to many different systems including water distribution systems (Kansal, Kumar, and Sharma 1995) and telecommunications (Premkumar, Chou, and Chou 2000). Our focus in this article is on intentional disruption of specific components of a system and not on analyzing system reliability due to known failure rates of system components.

In services or supply, there are two principal types of losses: (1) loss of transportation/communication system capacity, and (2) loss of supply, storage, or manufacturing capacity. For example, consider an integrated petroleum fuel system consisting of oil wells, crude storage facilities, crude transportation facilities (e.g., ships and pipelines), refineries, and a product distribution system of storage facilities, trucks, and pipelines. If a system operates at capacity in virtually every component, then any disruption of any part of the system will cause loss of supply or service. If excess capacity exists in many of the component parts, then it is possible that the destruction of some parts of the system may have little effect on the ability of that system to supply fuel to its customers. Thus, some components may be more important than others in maintaining system operation without reducing fuel deliveries.

Interdicting elements of a supply system has been the subject of interest by military planners (McMasters and Mustin 1970). For example, consider a logistics system that supplies an advancing army. The natural question is where along the supply routes are the locations where interdiction would be most effective. For example, what is the impact on a supply route if a bridge used in that supply route is destroyed by a bombing mission? If an alternate bridge crossing is nearby, the system functionality might be easily restored by forcing the supply route to detour to that nearby bridge. The overall impact of the loss of the bridge might therefore be minimal. Interdiction is most effective if it is focused on those parts of the supply system that, if disrupted, cause the greatest impact on system operation. Optimal interdiction involves identifying the most cost-effective, maximal way to disrupt a system.

Assume that a logistics system is represented as a transportation network, representing sources, demands, and transport links or arcs. A feasible supply route can be represented as a series of connected arcs that can accommodate flow oriented from the source to the destination. Interdiction on a supply network can be defined at nodes or along arcs. There are two basic types of interdiction: (1) partial or incremental interdiction, and (2) complete interdiction. In partial interdiction, the capacity along an arc or at a facility can be lowered incrementally, by a series of attacks, to a lower bound (≥ 0). For incremental interdiction, it is usually assumed that the capacity reduction is a linear function of the levels of allocated interdiction such as bombing sorties (e.g., Wood 1993). Complete interdiction is based upon the assumption that an arc or facility loses all capacity upon interdiction. Thus, complete interdiction is an all or nothing proposition (i.e., 0 or 1). Several different types of interdiction models have been developed. Table 1 summarizes past work in this area, specifying the types of models that have been developed, based upon the objective used, the type of interdiction used, special constraints (e.g., interdiction budget), and the underlying network model. Three major models, borrowed from the field of network optimization theory, have been used as underlying models for supply network interdiction: (1) the capacitated network flow model, (2) the minimum cost flow model, and (3) the shortest path model. The first attempt to analyze the sensitivity of a transportation network to interdiction was undertaken by Wollmer (1964), who considered removing a fixed number of arcs so as to minimize the network flow capability (whenever the model requires a preset number of complete interdictions to be allocated across a network, we indicate the corresponding constraint in Table 1 as a "cardinality" constraint). The interdiction model introduced by McMasters and Mustin (1970) also aims at minimizing the maximal flow capacity from a source to a sink in a network but allows incremental interdiction on each arc and imposes a budget constraint on the cost of interdiction. Ghare, Montgomery, and Turner (1971) developed a similar model where the type of interdiction was 0 or 1 (complete). An examination of the past work demonstrates an emphasis on the development of

Table 1. Interdiction Problems: References and Structural Characteristics

Reference	Objective	Decision	Constraint	Underlying Model
Wollmer (1964)	Minimize network flow capacity	Complete interdiction on arcs	Cardinality	Maximum flow through planar networks
Wollmer (1970)	Maximize minimum-cost flow	Complete interdiction on arcs	Cardinality	Minimum cost flow through networks
McMasters and Mustin (1970)	Minimize network flow capacity	Interdiction on arc capacities by units	Budget	Maximum flow through planar networks
Ghare, Montgomery, and Turner (1971)	Minimize network flow capacity	Complete interdiction on arcs	Budget	Maximum flow through networks
Corley and Chang (1974)	Minimize network flow capacity	Complete interdiction on nodes and incident arcs	Cardinality	Maximum flow through networks
Ratliff, Sicilia, and Lubore (1975)	Minimize network flow capacity	Complete interdiction on arcs	Cardinality	Maximum flow through networks
Fulkerson and Harding (1977)	Maximize shortest source-sink path	Interdiction on arc lengths by units	Budget	Minimum cost flow through networks
Golden (1978)	Minimize interdiction costs	Interdiction on arc lengths by units	Disruption Level	Minimum cost flow through networks
Corley and Sha (1982) Ball, Golden, and Vohra (1989) Malik, Mittal, and Gupta (1989)	Maximize shortest source-sink path	Complete interdiction on arcs	Cardinality	Shortest path through networks
Phillips (1993)	Minimize network flow capacity	Interdiction on arc capacities by units	Budget	Maximum flow through outerplanar and planar networks
Wood (1993)	Minimize network flow capacity	Complete interdiction on arcs Interdiction on arc capacities by units	Budget Cardinality	Maximum flow through general networks and multi-commodity networks
Cormican, Morton, and Wood (1998)	Minimize expected maximum flow	Interdiction attempt on arcs	Budget	Maximum flow through networks
Whiteman (1999)	Minimize interdiction costs	Complete and partial interdiction on nodes	Disruption level	Maximum flow through multi-commodity networks
Israeli and Wood (2002)	Maximize shortest source-sink path	Complete interdiction on arcs	Budget	Shortest path through networks
Burch et al. (2003)	Minimize network flow capacity	Complete interdiction on arcs	Budget	Maximum flow through nonplanar networks
Hemmecke, Schultz, and Woodruff (2002) Held et al. (2003)	Maximize the probability of given disruption level	Complete interdiction on arcs	Budget	Shortest path through uncertain networks

models involving arc interdiction. Corley and Chang (1974) were the first to demonstrate that an interdiction model based on the disruption of a fixed number of nodes could be reduced to an analogous arc interdiction model (Ratliff, Sicilia, and Lubore 1975) by suitably augmenting the underlying flow network. Subsequently, interdiction on nodes was investigated by Whiteman (1999), who used a capacitated flow network model to identify least-cost target sets for military strikes. Both complete and partial interdictions were allowed, depending on the target point, in order to induce a given level of interdiction for each commodity moved on the network. Whiteman's model is an extension of one of the models introduced by Wood (1993) for arc interdiction.

Wollmer (1970) was the first to model interdiction on a network where the objective was to maximize the minimum cost flow. Interdiction reduced the capacity and increased the cost of flow along an arc. Wollmer also considered the time to repair the arc along with repair costs. Thus, interdiction was not permanent, but lasted until repairs could be made. Fulkerson and Harding (1977) and Golden (1978) also modeled interdiction by using a minimum cost flow problem. Fulkerson and Harding (1977) considered partial interdiction on arc lengths so as to maximize the shortest paths between supply and demand points subject to an interdiction budget. Golden (1978) investigated a least-cost partial interdiction strategy to ensure a predetermined increase in the shortest path length. Several authors (Corley and Sha 1982; Ball, Golden, and Vohra 1989; Malik, Mittal, and Gupta 1989) considered the problem of identifying a fixed number of arcs that, if removed (i.e., complete interdiction), would cause the greatest increase in shortest distance between two prespecified points. Israeli and Wood (2002) addressed a generalization of this problem by including an interdiction budget constraint. Finally, stochastic variants of network flow interdiction problems have been investigated by Cormican, Morton, and Wood (1998), Hemmecke, Schultz, and Woodruff (2002), and Held, Hemmecke, and Woodruff (2003).

In summary, the major emphasis has been on arc interdiction, as opposed to facility interdiction. For the remainder of this article, we will focus on the loss of service or supply facilities and not on the loss of capacity of a transport link. In addition, interdiction will be modeled as all or nothing. Our reason for approaching this type of problem is to identify "critical service facilities" in a system. For example, we may have five supply facilities servicing a region of 100 different demand locations. A natural question is, which of the five facilities are the most important locations in providing efficient service? That is, which facilities, if lost, would lead to the greatest disruption of service provision? Let us say, for the sake of an example, that we had the resources to completely interdict two of the five facilities. Then, we would want to identify the two facility locations that, if taken out, have the greatest impact on the remaining system. Conversely, assume that we have resources to increase protection of two of the five facilities, making them less vulnerable to interdiction. Obviously, we would want to protect the two sites that, if lost, would result in the most negative consequences in service provision (or protect the two facilities which are the most important to efficient operation). To address either question, we would seek to find the worst-case scenario of losing two facilities. In the next few sections of this article, we will define two new facility interdiction problems, based upon the nature of how disruption to service can be measured.

The *p*-Median Problem and the *r*-Interdiction Median Problem

The p-median problem involves the location of p facilities in such a manner that the total weighted distance of supplying each demand from its closest facility is minimized (Hakimi 1964, 1965). The idea is to find the set of p sites that can supply all of the demands most efficiently as measured by weighted distance. Weighted distance represents the sum of all demand/facility interactions where each demand is assigned to its closest facility. For example, the distance to the closest facility is weighted by the number of trips needed to supply that demand from a facility utilizing some type of transport mode (e.g., truck). Thus, the objective might represent the total truck miles of travel needed to supply all of the demand from the set of located facilities. The problem is to find that set of p supply locations that yields the smallest needed amount of truck miles of transport. The p-median problem is based upon the assumption that the capacity of any facility will exceed the demands placed upon it. Given this assumption, each demand can be served by its closest facility. Numerous solution procedures and applications have been proposed for the p-median problem starting with the classic works of Teitz and Bart (1968) and ReVelle and Swain (1970).

We define the r-interdiction median problem as:

> Of the p different locations of supply, find the subset of r facilities, which when removed, yields the highest level of weighted distance.

The interdiction median problem is the antithesis of the p-median problem. Whereas the p-median location problem involves locating a set of facilities that can efficiently supply a set of demand points, the r-interdiction median problem involves finding the best subset of existing supply sites to remove in order to decrease the efficiency of the existing supply system the most. The r-interdiction median problem begins with an existing facility system and assignment of demand to supply points. For each facility that is subject to interdiction, the demand that was supplied by that facility must now be assigned to a facility farther away, thus increasing the sum of weighted distances.

Formulating the *r*-Interdiction Median Model

In order to formulate an optimization model for the r-interdiction median problem, consider the following notation:

i index representing places of demand

j index representing existing facility locations

$$s_j = \begin{cases} 1, & \text{if a facility located at } j \text{ is eliminated,} \\ & \text{i.e. interdicted} \\ 0, & \text{otherwise} \end{cases}.$$

F = the set of existing facilities j

$$x_{ij} = \begin{cases} 1, & \text{if demand } i \text{ assigns to a facility at } j \\ 0, & \text{otherwise} \end{cases}$$

a_i = a measure of demand (e.g., number of supply trips) needed at demand i

d_{ij} = the shortest distance between the supply/service facility at j and demand i

r = the number of facilities to be interdicted or eliminated

$T_{ij} = \{k \in F \mid k \neq j \text{ and } d_{ik} > d_{ij}\}$, the set of existing sites (not including j) that are as far or farther than j is from demand i.

We can now formulate the *r-i*nterdiction *m*edian (RIM) problem as the following integer-programming problem:

$$Max\ Z = \sum_i \sum_{j \in F} a_i d_{ij} x_{ij} \tag{1}$$

Subject to:

$$\sum_{j \in F} x_{ij} = 1 \quad \text{for each demand } i \tag{2}$$

$$\sum_{j \in F} s_j = r \tag{3}$$

$$\sum_{k \in T_{ij}} x_{ik} \le s_j \quad \text{for each } i \text{ and each } j \in F \tag{4}$$

$$\begin{aligned} x_{ij} &= 0, 1 \quad \text{for each } i \text{ and each } j \in F \\ s_j &= 0, 1 \quad \text{for each } j \in F \end{aligned} \tag{5}$$

The objective of this model (1) seeks to maximize the resulting weighted distance impact due to the interdiction of r-facilities. Constraint (2) maintains that each demand assigns to a facility after interdiction. Constraint (3) restricts the number of interdicted facilities to equal r. Constraints (4) ensure that the assignment from a given demand i is made to the closest remaining facility to i. Essentially, Constraint (4) prevents assignments from demand i to facilities farther than what j is from i, unless the facility at j has been subject to interdiction. Thus, demand i will be forced to assign to its closest remaining facility. Finally, the set of Constraints (5) establishes the integer restrictions on the variables. Note, if all of the site interdiction variables s_j are zero–one in value, then the demand assignment variables, x_{ij}, will be zero–one as well. The RIM model is quite different from the original p-median model formulation of ReVelle and Swain (1970), in that the p-median model locates facilities in order to minimize weighted distance and the RIM model eliminates facilities in order to maximize weighted distance.

The above RIM model is an integer-linear programming model. Possible solution procedures include general-purpose, integer-linear programming software (ReVelle and Swain 1970), Lagrangian relaxation with subgradient optimization (Narula, Ogbu, and Samuelsson 1977), heuristics such as Tabu search (Rolland, Schilling, and Current 1999) and vertex substitution (Teitz and Bart 1968), and in smaller cases, enumeration. In a later section, we will give some computational examples of solving the RIM model using general-purpose integer programming software.

The possibility exists that we can solve this type of problem for some applications by enumeration. For example, if there were 10 supply sources and the possibility of interdicting 3 of those sources, then there would be only 120 different combinations. This is small enough in scope that it would make sense to enumerate all of the possibilities and then study the pattern that results in the highest impact to weighted distance as well as the distribution of impacts for the 120 different combinations. It also would not be unreasonable to generate all of the combinations that 10 sources could be interdicted in 1, 2, 3, and so on, ways. However, there are also many systems in which a brute force enumeration would not be feasible. For example, the City of Los Angeles has 105 fire stations. When the number of facilities is relatively large, the impact of the loss of a few facilities on the operation of the system may be relatively low. For such problems there is safety in large numbers of facilities. On the other side of the issue is calculating the impact of interdiction of a given number of these facilities. Enumeration is not realistic for even relatively small problems, such as interdicting 10 facilities out of 105, where the number of possibilities is 28,848,458,598,960. The time it would take to enumerate all of these solutions, even on a fast computer, would be measured in decades and is clearly not a realistic approach. Consequently, developing models to study facility interdiction is indicated.

The Maximal Covering Problem and the *r*-Interdiction Covering Problem

The maximal covering problem involves identifying the best placement for a set of p facilities (Church and ReVelle 1974). The objective is to place the facilities in such a manner as to maximize the coverage of demand. A demand is said to be covered if a facility is placed within some maximal range, such as distance, time, or within line of sight. For example, in fire station location, it is necessary to locate stations so that neighborhoods are within a prespecified maximal service distance or time of travel (Toregas 1970). This ensures that a quick response to a neighborhood can be made from a station. A related problem involves locating the smallest number of facilities in such a manner that every demand area or point is covered by at least one station or facility. This second type of problem is called the Location Set Covering Problem (Toregas 1970). Many emergency services, such as fire, EMS, and hazardous materials spill response teams are located using covering objectives (see for example Eaton et al. 1985 and Plane and Hendrick 1977).

Interdiction of emergency services facilities, like hazardous materials spill response teams or bomb disposal equipment/teams, would involve eliminating the capability of one or more such facilities from being able to respond or operate in a timely manner. The most critical components would be the facilities that, if lost, would yield the highest drop in operational response capability, that is, coverage.

We define the *r*-Interdiction Covering (RIC) problem as:

> Of the *p* different service locations, find the subset of *r* facilities, which when removed, maximizes the resulting drop in coverage.

Suppose there is a set of facilities that currently provides a high level of coverage to a set of demands. Perhaps the facilities are hazardous materials response facilities. As long as a facility is within a defined coverage distance of a demand area, adequate response for a hazardous material spill recovery can be made. The solution to an RIC problem would represent the greatest disruption to service should *r* facilities be compromised or lost. One might think that, if such a loss happened, there would then be automatic redeployment of the assets of the other facilities. If the facilities were owned and operated by two different political jurisdictions (e.g., neighboring counties) then redeployment might not immediately occur as a county would want to keep equipment to protect its inhabitants. A second example would be an emergency radio system, communication system, or public safety call center. If one or more were destroyed, redeployment or redirecting calls might be nearly impossible for quite some time. Obviously, some systems, when disrupted, might be able to reorganize quickly so that the loss of service would be limited. RIM and RIC do not address the time in which a system might be able to replace lost services or reorganize, but only the level of disruption upon the loss of one or more facilities.

Formulating the *r*-Interdiction Covering Model

We can formulate the RIC model as an integer-programming model using the notation that was previously defined along with the following notation:

$$y_i = \begin{cases} 1, & \text{if demand } i \text{ is no longer covered} \\ 0, & \text{otherwise} \end{cases}$$

$$N_i = \{j \mid \text{site } j \text{ covers demand } i\}$$

The *r*-Interdiction Covering Problem can then be formulated in the following manner:

$$Max\ z = \sum_i a_i y_i \tag{6}$$

subject to:

$$y_i \le s_j \quad \text{for each } i \text{ and and for each } j \in N_i \cap F \tag{7}$$

$$\sum_{j \in F} s_j = r \tag{8}$$

$$y_i = 0,\ 1 \quad \text{for each } i \tag{9}$$

$$s_j = 0,\ 1 \quad \text{for each } j$$

The objective of the RIC model (6) involves maximizing the amount of demand that is no longer covered after interdiction. A demand i is no longer covered if $y_i = 1$. The value of y_i can equal one only when all of the facilities that currently cover i have been eliminated. This property is ensured in Constraints (7). Constraint (7) limits $y_i = 0$ unless each facility site in F that covers i has been eliminated (i.e., $s_j = 1$). Constraint (8) limits the number of facilities to be eliminated to equal r. The last set of constraints (9) represent the integer restrictions on the decision variables y_i and s_j. It should be noted that one can solve this problem with only integer restrictions on the s_j. It can be easily proven that in any optimal solution to the model where the s_j values are binary integer, the values of y_i are binary integer as well.

The model has one facility elimination variable, s_j, for each existing facility as well as one variable for each demand, y_i, that represents if coverage has been lost for a given demand. The number of constraints equals the total number of times demands are covered by existing facilities plus one. This is a relatively small problem as most demands are usually covered only a few times at most. The constraints of type (7) can be condensed into the following set:

$$K_i y_i - \sum_{j \in N_i \cap F} s_j \le 0 \quad \text{for each } i, \tag{10}$$

where K_i is the number of times demand i is covered by existing facilities. Using Constraints (10) instead of Constraints (7) yields a model that has one constraint for each demand plus one constraint involving the elimination of exactly r facilities.

The RIC model is classified as an integer-linear programming problem. As with the RIM model, a number of different solution approaches are possible candidates for application. They include techniques used in the solution of maximal covering problems such as general-purpose integer programming software (Church and ReVelle 1974), Lagrangian relaxation with subgradient optimization (Galvao 1993), and heuristics (Murray and Church 1996). As in the case of applying the RIM model, enumeration might also be used when problem sizes are small. In the next section we give some example computational experience with solving RIC, along with discussion of two solutions to the RIC problem.

Example Solutions to RIM and RIC

Example solutions to the r-interdiction median and r-interdiction covering problems (RIM and RIC) are presented in this section. Our main objective is to give some example computational experience and discuss the impact that results from interdiction. To solve these two problems, we utilized AMPL (Fourer, Gay, and Kernighan 1993) in conjunction with CPLEX (a general purpose linear-integer programming solution software package distributed by the ILOG corporation). This software was executed on a Sun Ultra Sparc 10 computer using the SunOS 5.7 operating system (approx. 470 megahertz speed). The models were set up using AMPL and solved using CPLEX.

We chose to use the Swain data set of 55 demand points as a sample data set for solving the RIM and RIC models (Swain 1971). For this data set, each demand point also represents a potential facility site. Thus, the Swain data comprises a problem of 55 demand points and 55 potential facility sites. This data set does not include a network, so distances are measured as Euclidean distances. Demand weights vary from a low of 2 to a high of 72, with a concentration of high-demand points in the center of the demand region. Swain originally generated this data set from postal zones in the Washington, DC, area. The points are displayed in Figure 1. We started with solving for optimal solutions to the p-median problem and the maximal covering location problem for various values of p. These solutions are given in Tables 2 and 3. For each of the optimal p-median solutions and maximal covering solutions, we solved several RIM or RIC problems. The RIM and RIC model results are also given in Tables 2 and 3.

Table 2 presents results of the p-median and RIM models. We solved three different median problems

Figure 1. The layout of the Swain data where points are numbered in order of decreasing demand.

associated with values of $p = 5$, 7, and 9. For each of the p-median solutions we solved several RIM models. For example, the optimal 5 median solution had a weighted distance of 2950.41 and utilized sites 1, 3, 10, 22, and 36. The RIM model for this five-site configuration when $r = 2$ involved interdicting sites 1 and 3 yielding a solution with a weighted distance of 6124.53. Thus, weighted distance increased by over 100 percent upon interdiction. Optimal interdiction of three sites for this configuration increased weighted distance to 8769.33 or by 197 percent. Solution times for the p-median and RIM models are given in the table as well. For example, the five-facility median problem took 0.49 seconds to solve whereas the RIM model where $r = 2$ applied to this problem took 1.45 seconds. Finally, the last two columns in the table provide the number of nodes in the branch and bound tree generated by CPLEX. This information is indicative of the problem difficulty.

Table 3 presents results of the Maximal Covering Location model and the RIC model applied to the Swain data. We solved three different maximal cover problems associated with values of $p = 5$, 7, and 9 and where the maximal coverage distance was set at 10.00. For each of the maximal covering solutions we solved several RIC models. For example, the optimal five-site maximal

Table 2. Results of the r-Interdiction Median Problem for the Swain Data Set

		Obj. Values		Solutions		Time (sec.)		B&B Nodes	
p	r	PMP	RIM	PMP	RIM	PMP	RIM	PMP	RIM
5	2	2950.41	6124.53	1 3 10 22 36	1 3	0.49	1.45	0	10
5	3	2950.41	8769.33	1 3 10 22 36	1 3 10	0.49	1.20	0	0
7	2	2427.41	3870.20	1 2 3 10 16 17 36	1 2	0.39	1.87	0	37
7	3	2427.41	5859.01	1 2 3 10 16 17 36	1 2 3	0.39	2.60	0	16
7	4	2427.41	8526.50	1 2 3 10 16 17 36	1 2 3 10	0.39	2.36	0	11
9	2	2067.97	3361.83	1 2 3 6 12 16 21 24 29	1 2	0.37	3.05	0	35
9	3	2067.97	4425.91	1 2 3 6 12 16 21 24 29	1 2 3	0.37	3.70	0	14
9	4	2067.97	6744.13	1 2 3 6 12 16 21 24 29	1 2 3 6	0.37	3.92	0	11

covering solution covered demand totaling 609 and utilized sites 8, 10, 17, 27, and 36. The RIC model for this five-site configuration when $r = 2$ involved interdicting sites 8 and 10 yielding a solution with a coverage of 170. Thus, coverage decreased by 72 percent upon the interdiction of two of the five facilities. Optimal interdiction of three sites for this configuration decreased coverage to 97 or by 84 percent. Solution times for the maximal cover and RIC models are given in Table 3 as well. For example, the nine-facility maximal covering problem took approximately 0.01 seconds to solve and the RIC model applied to the nine-facility location pattern where $r = 2$ needed less than 0.01 seconds to solve.

For the problems solved, the RIC model took comparatively less time to solve than the corresponding Maximal Covering Location Problem model. In contrast the RIM model was considerably more difficult to solve than a corresponding p-median problem and often required a number of branches to solve optimally, even though these problems were relatively modest in size. In both cases, the interdiction models demonstrate that the interdiction of a modest number of facilities may cause significant impacts on efficient operation (in terms of weighted distance) or extent of service (in terms of desired coverage). To depict impacts of interdiction, two different p-median solutions and two different maximal covering solutions are presented in Figures 2, 3, 4, and 5, along with patterns resulting from interdiction. In each figure, points selected for facilities are displayed slightly larger the remaining points. Interdicted facilities are displayed as squares. For example, Figure 2a depicts the optimal five-facility median solution and Figure 2b depicts the optimal $r = 2$ RIM solution associated with the same five-facility pattern. The lines in each figure depict assignments of demand to specific facilities. For example, in Figure 2a each point is connected to the closest of the five located facilities. This five-site pattern after an optimal $r = 2$ interdiction resulted in a three-site pattern, which is depicted with assignments in Figure 2b. Each demand that had been served by a facility that was interdicted is depicted as a small square and is shown being reassigned to a closest remaining facility. These two solutions are presented on the first line of Table 2. We might think of the five-facility pattern depicted in Figure 2a as being somewhat fragile, as interdiction causes such a significant impact on overall weighted distance. One might wonder if other five-facility patterns exist that are less fragile. To demonstrate that this is a distinct possibility, consider the five-median solution given in Figure 3a. This solution is a near optimal five-median solution.

Table 3. Results of the r-Interdiction Covering Problem for the Swain Data Set

		Obj. Values		Solutions		Time (sec.)		B&B Nodes	
p	r	MCP	RIC	MCP	RIC	MCP	RIC	MCP	RIC
5	2	609	170	8 10 17 27 36	8 10	0.00	0.00	0	0
5	3	609	97	8 10 17 27 36	8 10 17	0.00	0.00	0	0
7	2	633	198	2 17 20 27 36 53 55	2 17	0.01	0.00	0	0
7	3	633	148	2 17 20 27 36 53 55	2 17 36	0.01	0.01	0	0
7	4	633	95	2 17 20 27 36 53 55	2 17 20 36	0.01	0.01	0	0
9	2	640	321	5 17 21 27 30 36 41 50 55	5 30	0.02	0.00	1	0
9	3	640	237	5 17 21 27 30 36 41 50 55	5 30 41	0.02	0.01	1	0
9	4	640	164	5 17 21 27 30 36 41 50 55	5 17 30 41	0.02	0.01	1	0

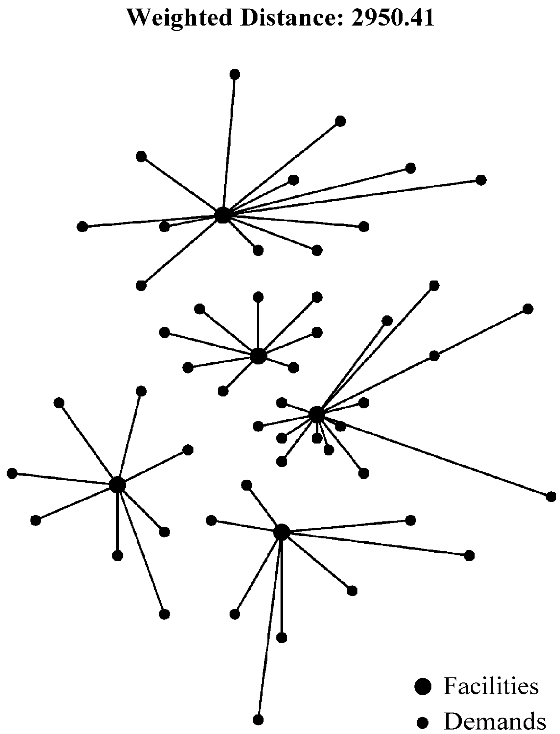

Figure 2a. Optimal solution to the p-median problem ($p = 5$).

Optimal interdiction, where $r = 2$, of this pattern results in the three-median solution depicted in Figure 3b. The near-optimal five-median solution in Figure 3b compares favorably to the optimal solution in Figure 2b in terms of weighted distance. The associated worst-case interdiction solutions presented in Figures 2b and 3b differ

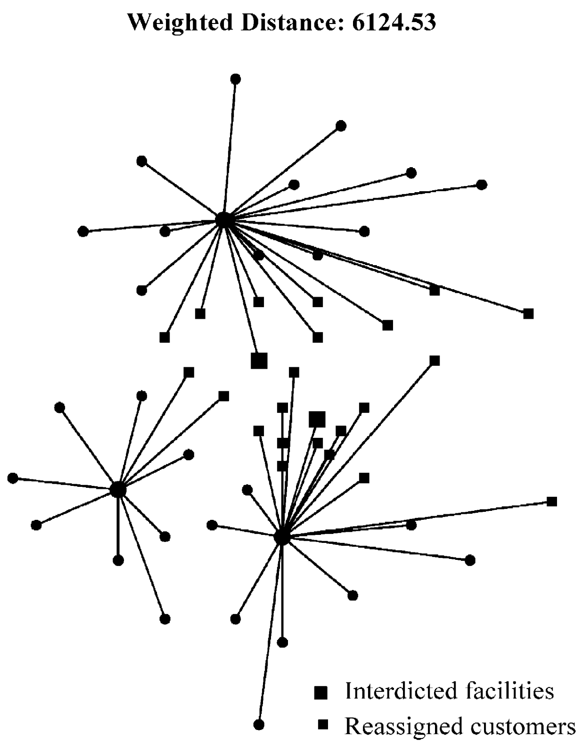

Figure 2b. Optimal interdiction of the p-median solution given in Figure 2a ($r = 2$).

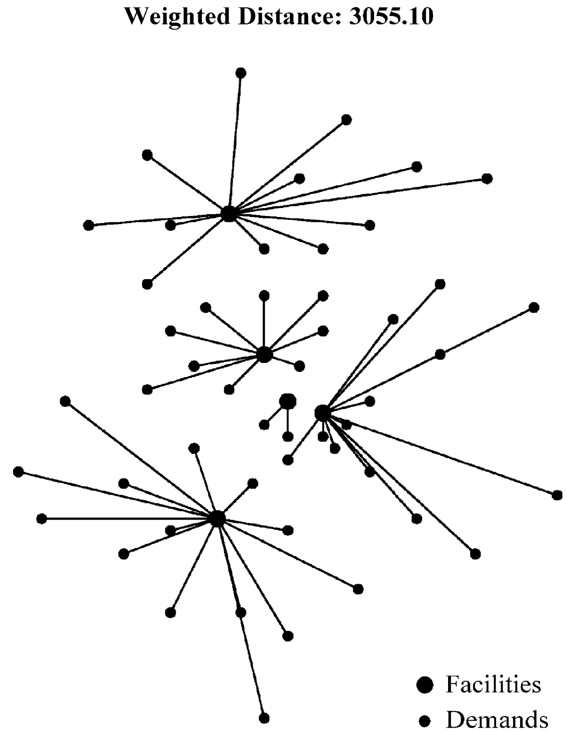

Figure 3a. A close-to-optimal solution to the p-median problem ($p = 5$).

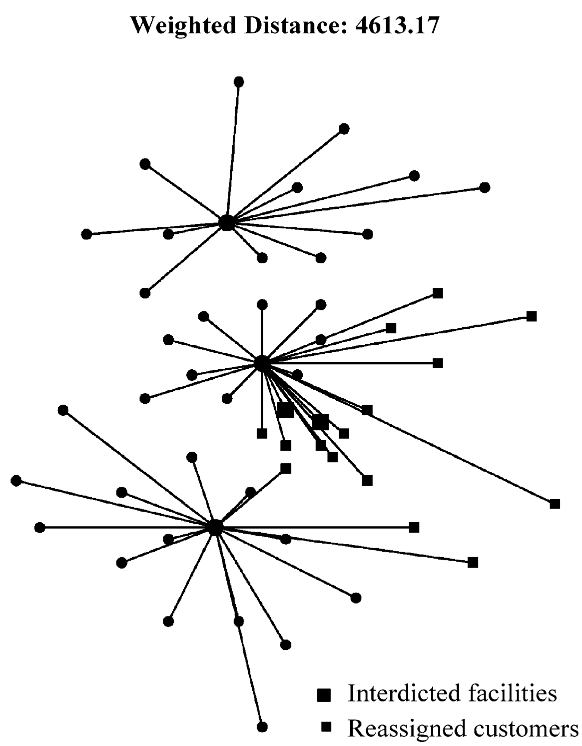

Figure 3b. Optimal interdiction of the p-median solution given in Figure 3a ($r = 2$).

considerably, however. The solution in Figure 2b involves an increase in weighted distance by 107 percent as compared to the optimal five-median solution. The solution in Figure 3b has a weighted distance of only 56 percent over that of the optimal five-median solution. Thus, there is the possibility that good facility patterns can exist that are less subject to disruption, as measured by weighted distance. It makes sense to consider the search for such solutions when making facility location decisions involving vital services.

Figures 4a and 5a depict two different covering solutions, each using five facilities. Coverage is depicted by the gray circles drawn about the chosen facility locations. In Figure 4a, the optimal five-facility maximal covering solution is given. Optimal interdiction, where $r = 2$, involving the optimal maximal covering solution results in the three-site solution depicted in Figure 4b. Coverage falls to 170, or a drop of 72 percent. Points no longer covered after interdiction are depicted with small squares. Figure 5a depicts a covering solution that covers 567 (versus 609 for the maximal covering solution). Optimal $r = 2$ interdiction on this solution results in the three-facility pattern depicted in Figure 5b. This solution maintains a coverage of 475, which is 270 percent higher than the coverage of the interdicted maximal covering solution. Again, this demonstrates that the impacts of interdiction can be reduced by the appropriate placement of facilities and that models need to be developed to identify such solutions.

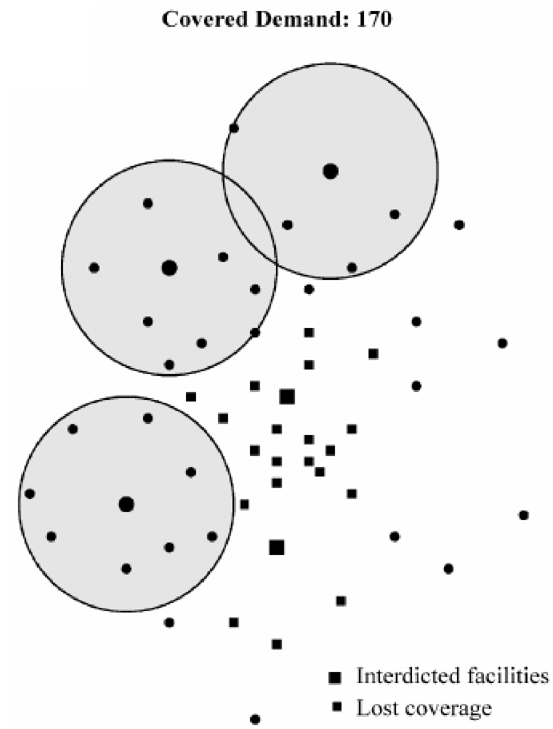

Figure 4b. Optimal interdiction of the maximal covering solution given in Figure 4a ($r = 2$).

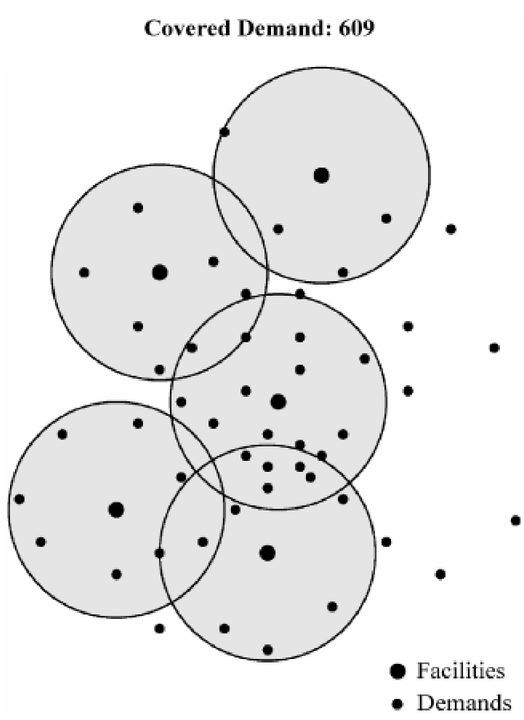

Figure 4a. Optimal solution to the maximal covering problem ($p = 5$).

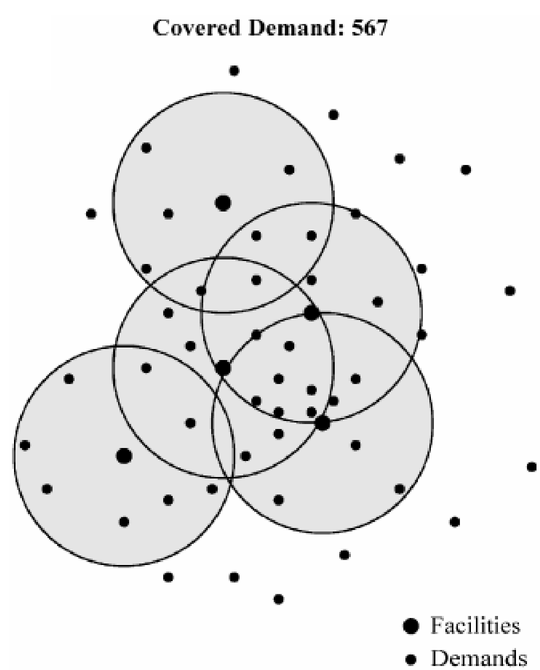

Figure 5a. A close-to-optimal solution to the maximal covering problem ($p = 5$).

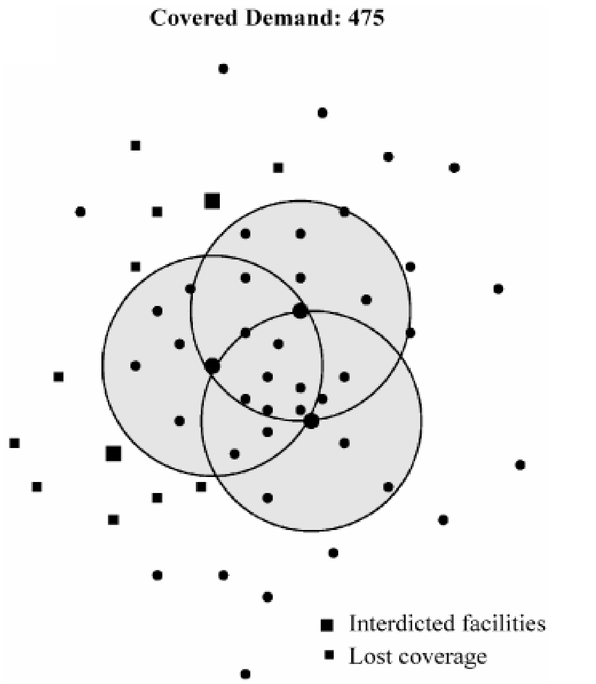

Figure 5b. Optimal interdiction of the maximal covering solution given in Figure 5a ($r = 2$).

Summary and Conclusions

Facilities and their services can be lost due to natural disasters as well as to intentional strikes, by either terrorism or an army. The geographical distribution of facilities in a supply or service system may be particularly vulnerable to interdiction, and the resulting impacts of the loss of one or more facilities may be substantial. For this reason it is desirable to identify elements that are critical components of such systems. Identifying critical elements within a supply or service system can be useful both in natural disaster planning and in planning to protect such assets from acts of terrorism. To understand the possible impacts of the loss of critical infrastructure as well as plan for possible acts of terrorism, it is important to develop geographical models that can help evaluate and identify critical infrastructure in lifeline systems. In this article we have presented two models of interdiction. These two interdiction models focus on the worst possible events of losing one or more facilities in a system, where the system is characterized by closest assignment or by coverage. Our focus has been to identify the most critical facilities in a configuration.

Two models of facility location that have received widespread attention are the p-median and maximal, covering location models. These two models have been widely applied, and software has been developed to solve such problems easily with some GIS packages. We have developed in this article two interdiction models, one based upon the p-median problem and one based upon the maximal covering location problem. Each of these models seeks to find a subset of facilities that, when eliminated by interdiction, cause the greatest loss of efficiency or coverage. Such interdiction solutions represent those facility locations that are the most important for efficient system operation and therefore can be considered critical or vital. We have also presented example solutions for several applications to a hypothetical data set. The models are relatively easy to solve using existing general-purpose software. More research is needed to test the models on larger problems, preferably problems of vital interest.

We have shown that location patterns may be fragile, in that interdiction can cause significant disruption of service coverage or efficiency (as measured by weighted distance). The two models have focused on the role that facility location plays as a vital asset in a supply or service system. These two models do not, however, focus on lost capacity in a capacity-constrained system. Both problems are based upon the assumption that, after interdiction, enough supply capacity remains to serve each demand by the closest remaining facility. Research is needed to model the case where such an assumption does not hold. Research is also needed to identify location patterns that maintain high levels of efficient service, even in the event of interdiction.

We believe that a suite of geographical models needs to be developed that can be used to identify critical infrastructure. Cutter, Richardson, and Wilbanks (2003) suggest a number of research priorities for understanding and planning for homeland security needs, of which this is listed as one of the priority research areas. It is hoped that this research is indicative of the role that geographers can play in the development of such models. Security and risk to lifelines can be modeled at a number of scales from global to local. We hope that the research reported here will inspire further work.

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