Set A

1. A maximum spanning tree (i.e. a spanning tree that maximizes the sum of the weights) can be constructed in

   (a) O(E log V)

   (b) O(E)

   (c) O(E log E)

   (d) None of these

2. A union find data-structure is commonly applied while implementing:

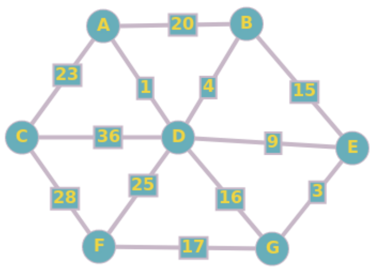
   (a) A depth-first search traversal of a graph.

   (b) Computing the minimum spanning tree of a graph using the Prim's algorithm.

   (c) Computing the minimum spanning tree of a graph using the Kruskal algorithm.

   (d) Computing the all-pairs shortest path in a graph.

3. Which is the correct order for Kruskals minimum spanning tree algorithm to add edges to the minimum spanning tree for the figure shown Below:



   (a) (A, D) then (E, G) then (B, D) then (D, E) then (F, G) then (A, C).

   (b) (A, B) then (A, C) then (A, D) then (D, E) then (C, F) then (D, G)

   (c) Both A and B

   (d) None of above

4. Select correct statements:

   (a) Prims and Kruskals algorithm can also produce a MST for a graph whose weights can be positive or negative.

(b) Consider a reversed Kruskals algorithm for computing a MST. Initialize T to be the set of all edges in the graph. Now, consider edges from largest to smallest cost. For each edge, delete it from T if that edge belongs to a cycle in T. (Never mind how to implement this. Just note that union-find does not allow deletions, so an inefficient implementation of this reversed Kruskal is not obvious.) Assuming all the edge costs are distinct, This new algorithm correctly computes MST.

(c) Suppose we have computed a minimum spanning tree of a graph and its weight. If we make a new graph by doubling the weight of every edge in original graph, we still need additional $\Omega(E)/$ time to compute the cost of the MST of new graph.

(d) All of above

5. Select correct statements:

    (a) Given a graph G = (V, E) with positive edge weights, the Bellman-Ford algorithm and Dijkstras algorithm can produce different shortest-path trees despite always producing the same shortest-path weights.

    (b) Every directed acyclic graph has exactly one topological ordering.

    (c) The depth of a breadth-first search tree on an undirected graph G = (V, E) from an arbitrary vertex v $\in$ V is the diameter of the graph G. (The diameter of a binary tree is the length of the longest path between any two nodes in a tree.)

    (d) None of above

6. Select correct statements for Bellman-Ford algorithm:

    (a) Bellman-Ford detects negative cycles.

    (b) If the graph has no negative cycles, then the distance estimates on the last iteration are equal to the true shortest distances. (for graph with n nodes, $(n-1)^{th}$ iteration is the last.)

    (c) d[v] $\geq \delta$(s,v) (where d[v] is distance maintained by Bellman-Ford).

    (d) After i iterations of relaxing on all edges, if the shortest path to v has i edges, then d[v] = δ(s, v) (where d[v] is distance maintained by Bellman-Ford).

7. Consider an undirected graph G. Let T be a depth first search traversal tree. Let u be a vertex and v be the first unvisited vertex after visiting u. Which of the following statements is always true ?

(a) (u,v) must be an edge in G.

(b) (u,v) must be an edge and v is a descendant of u in T.

(c) if (u,v) is not an edge, u and v have the same parent.

(d) if (u,v) is not an edge, then u is a leaf.

(e) None of the above

8. Which of the following are true for a segment tree built over N elements.

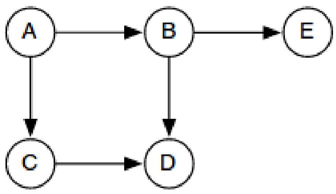
   (a) Time taken to build the tree is θ(N)

   (b) Time taken to build the tree is θ(NlogN)

   (c) Space complexity of the tree is θ(N)

   (d) Space complexity of the tree is θ(NlogN)

9. For a DFS starting at some node in the following graph, fill in the blank with



   True/False for each of the statement given below.

   (a) Start time of D could be less than start time of E

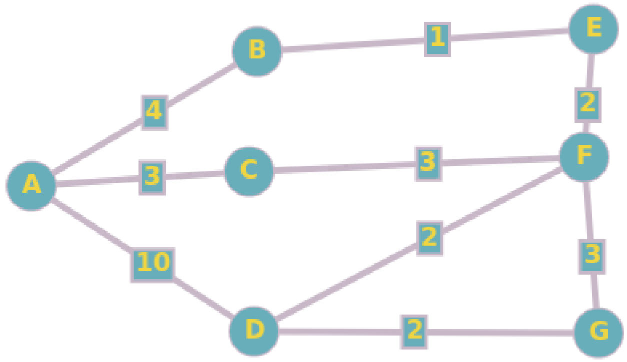
   (b) start time of E could be less than start time of D

   (c) Start time of D could be less than start time of C

   (d) Finish time of A could be less than finish time of B

   (e) Finish time of D could be less than finish time of B

10. Fill in the blank with the order in which vertices are visited by dijkstra when node A is considered as the source. In case of multiple answers possible, you



   may write any correct solution.

11. Which of the following statements are true.

    (a) Given a graph G with unique positive edges, MST will be the same on squaring all edges .

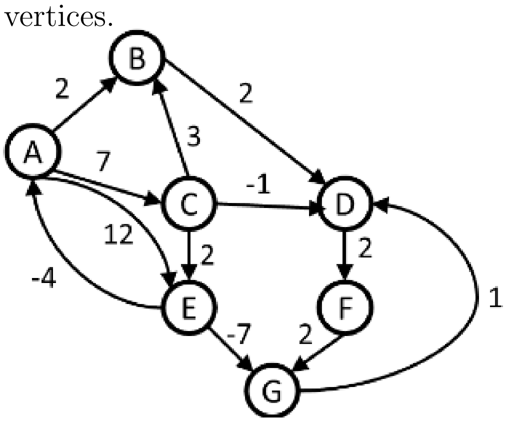
    (b) Given a graph G with positive edges, with a spanning tree T, MST will be the same on squaring all edges .

    (c) Given that graph G has positive as well as negative weight edges, the set of edges with minimum spanning all nodes can have a cycle.

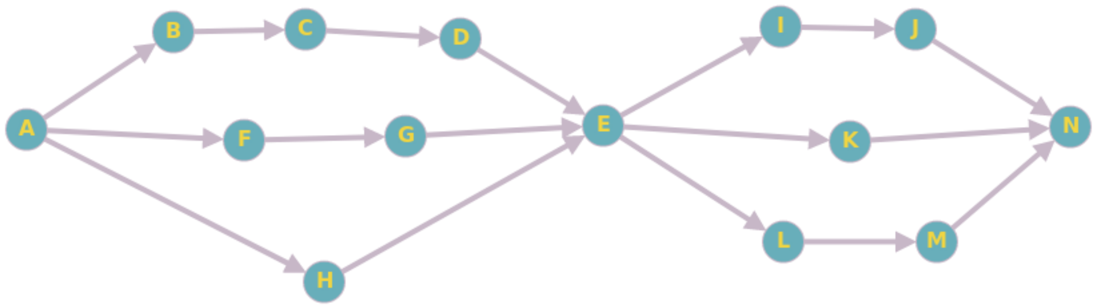
    (d) Given that graph G has positive as well as negative weight edges, the set of edges with minimum spanning all nodes cannot have a cycle.

12. Consider a complete graph (All pair of vertices connected) G with 4 nodes as shown below. Given that all edges are positive weight with one edge having weight 4, if the Minimum spanning tree has weight 10 fill in the blank with the minimum possible value of the sum of all edges of graph G.

13. Even though graph G shown below has negative edges, fill in the blank with the shortest path to each of the node as given by Dijkstra's algorithm with vertex A as source vertex. Fill the shortest path weight in alphabetical order of vertices.



14. Fill in the blank with the number of valid topological sortings of the graph given below.



15. Which of the following statements are true

    (a) Changing the ordering in which edges are relaxed in the Bellman-ford algorithm, never changes the correctness of the algorithm.

    (b) If all edges of a graph G are negative, we can modify Dijkstras Algorithm to find the Shortest path in this case.

    (c) If all edges of a graph G are negative, we can modify Dijkstras Algorithm to find the Longest path in this case.

    (d) There exists a graph G with positive as well as negative edges, such that output of Dijkstra's algorithm gives valid shortest path.

16. State true/false if each of the modified MST algorithm gives a valid MST of a given graph G.

a. MAYBE-MST-A(G, w)

```
sort the edges into nonincreasing order of edge weights w
T = E
for each edge e, taken in nonincreasing order by weight
    if T − {e} is a connected graph
        T = T − {e}
return T
```

b. MAYBE-MST-B(G, w)

```
T = ∅
for each edge e, taken in arbitrary order
    if T ∪ {e} has no cycles
        T = T ∪ {e}
return T
```

c. MAYBE-MST-C(G, w)

```
T = ∅
for each edge e, taken in arbitrary order
    T = T ∪ {e}
    if T has a cycle c
        let e′ be a maximum-weight edge on c
        T = T − {e′}
return T
```

Roll No : ______________________________

Answers:

	A	B	C	D	E
1	①	②	③	④	⑤
2	①	②	③	④	⑤
3	①	②	③	④	⑤
4	①	②	③	④	⑤
5	①	②	③	④	⑤
6	①	②	③	④	⑤
7	①	②	③	④	⑤
8	①	②	③	④	⑤
9	①	②	③	④	⑤
11	①	②	③	④	⑤
15	①	②	③	④	⑤

10 ____

12 ____

13 ____

14 ____

16 ____