Basic Electronic Circuits (IEC-103)

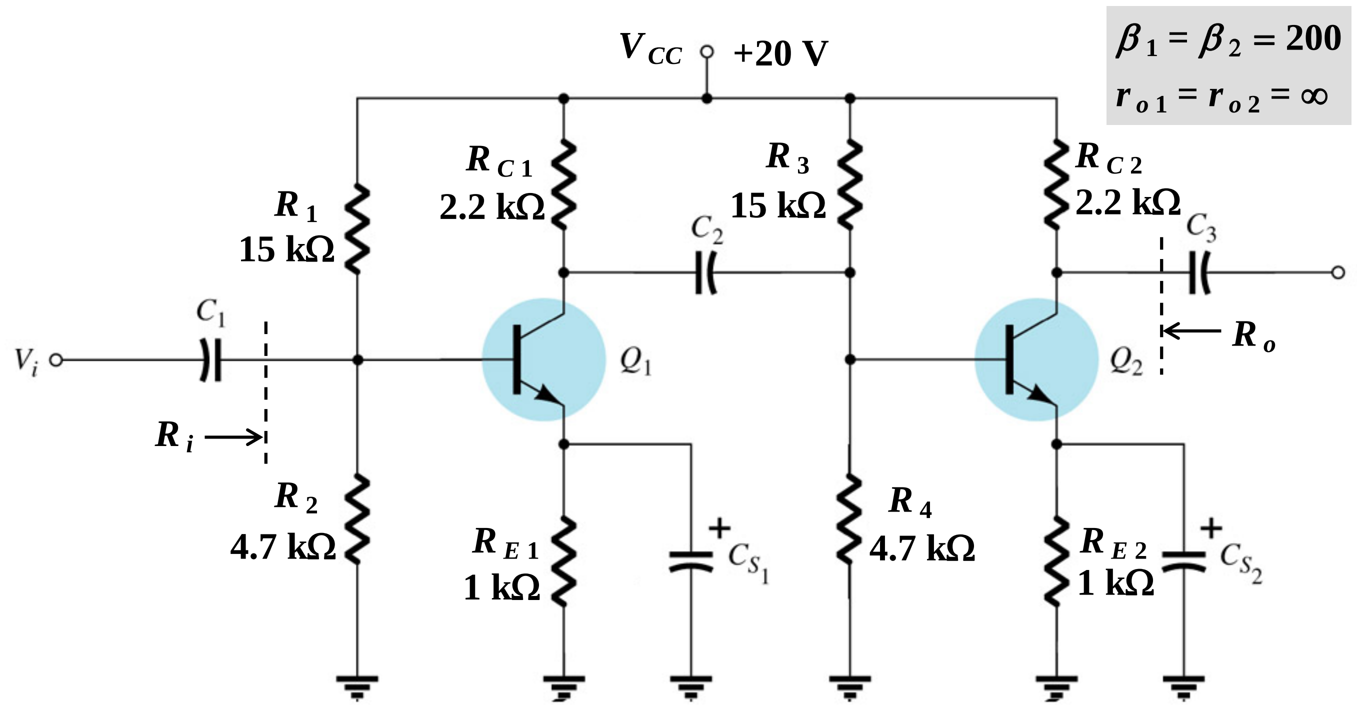
Lecture-20

Multistage Amplifiers

Cascade Connection

Example

Draw the AC equivalent circuit and calculate A_v, R_i, and R_o.



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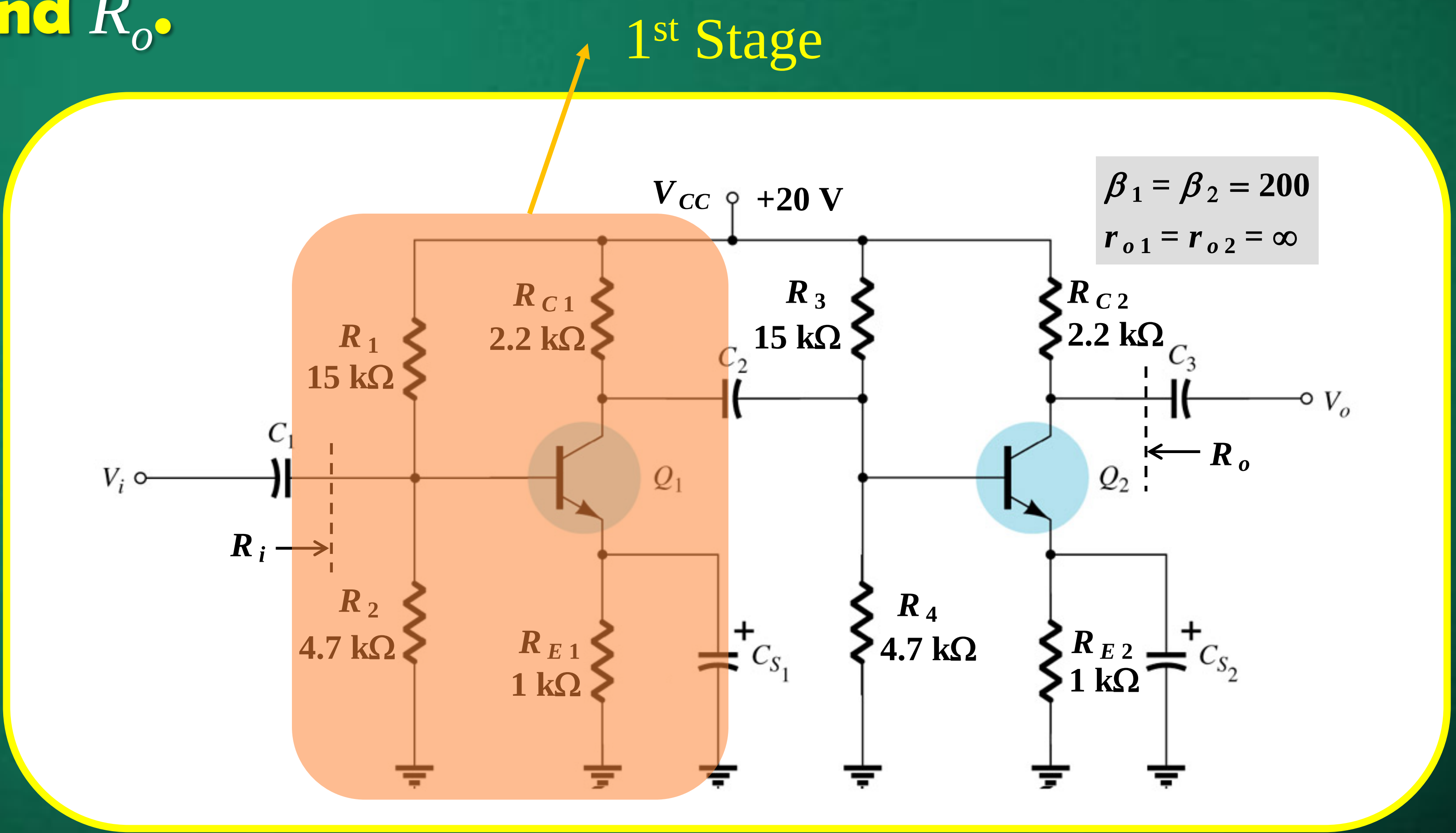

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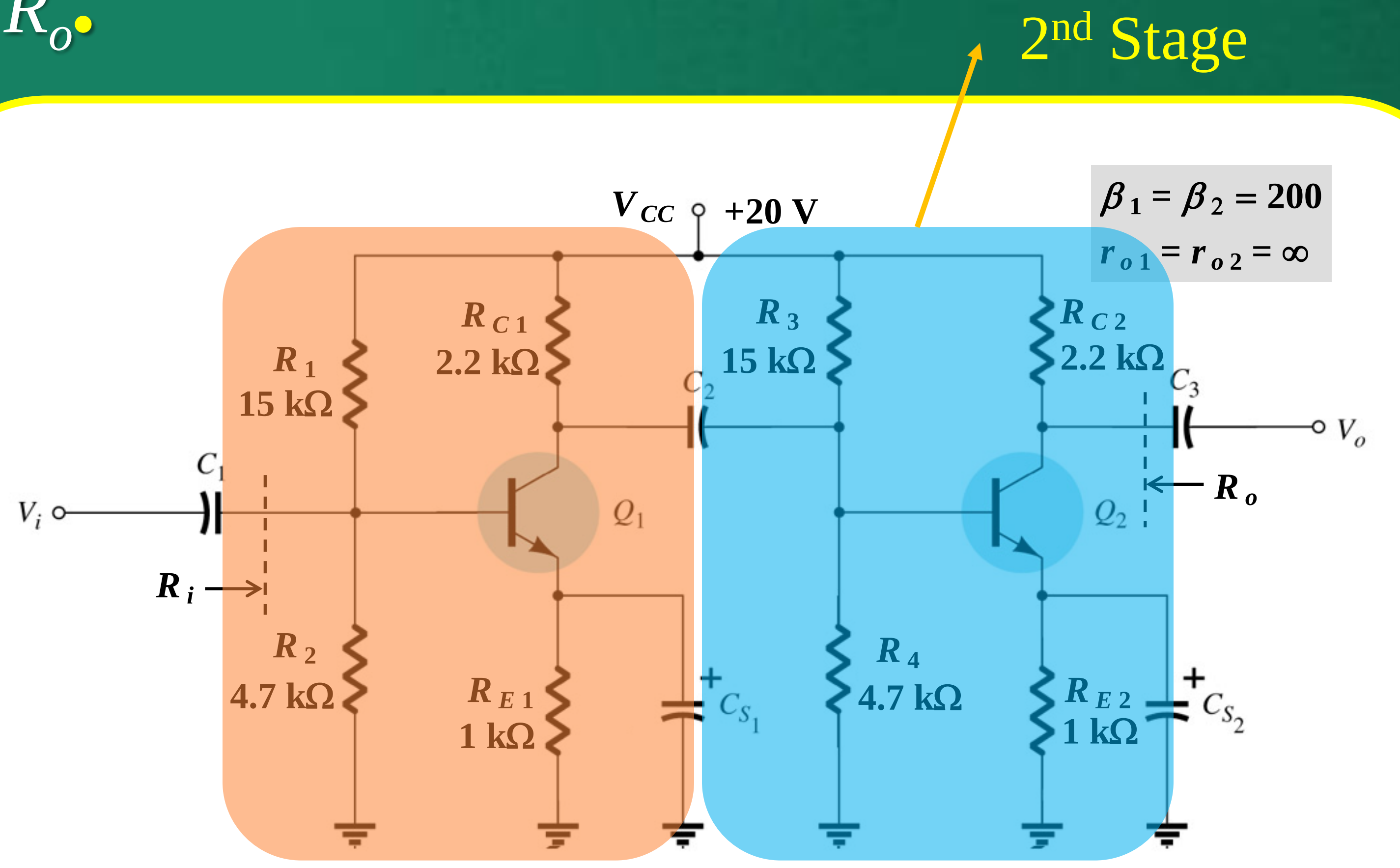
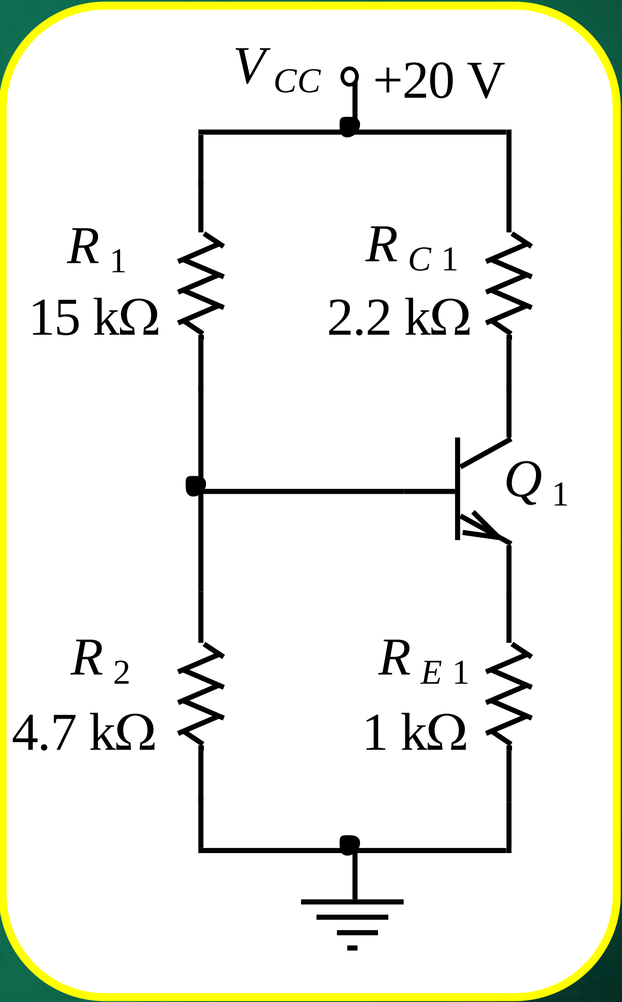


Example (continued)

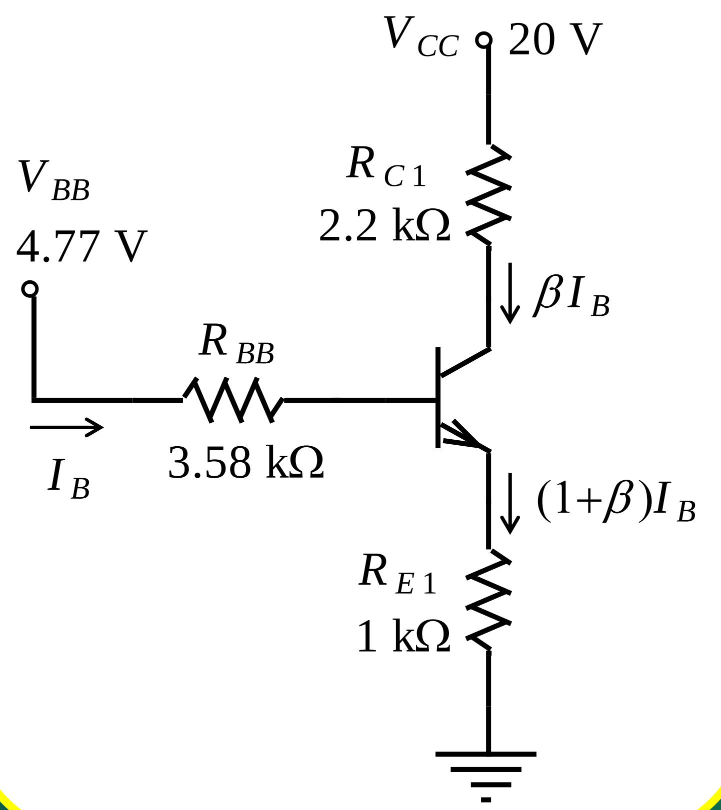
DC analysis

The circuit under DC condition (stage 1 and 2 are identical)



Example (continued)

Applying Thévenin's theorem, the circuit becomes



$$I_{BQ1} = I_{BQ2} = 19.89\ \mu\text{A}$$

$$I_{CQ1} = I_{CQ2} = 3.979\ \text{mA}$$

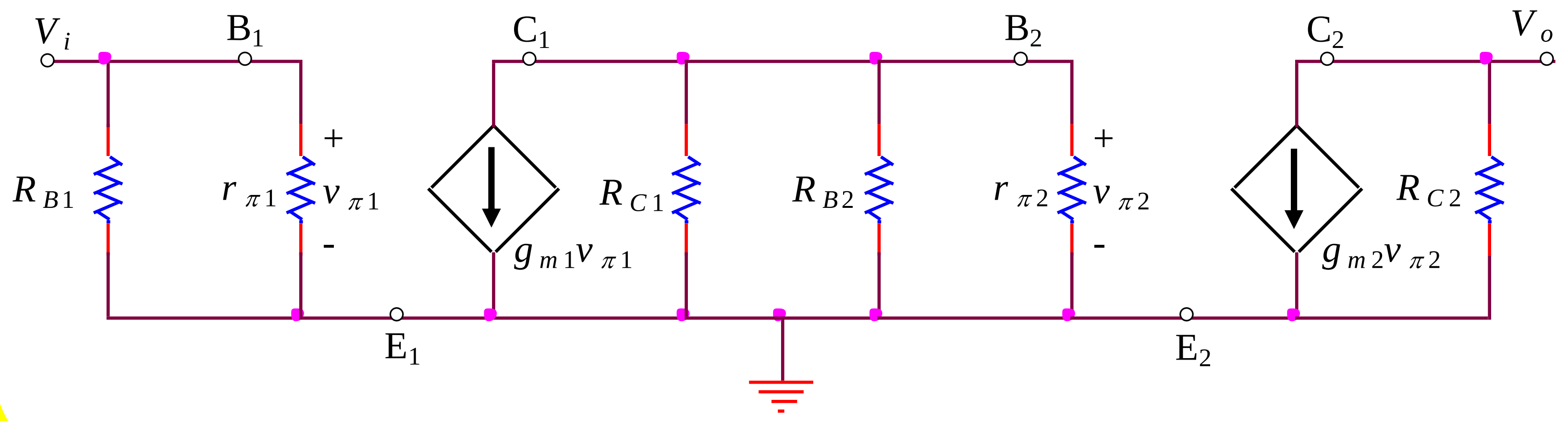
$$r_{\pi 1} = r_{\pi 2} = 1.307\ \text{k}\Omega$$

$$g_{m1} = g_{m2} = 0.153\ \text{A/V}$$

Example (continued)

AC analysis

The small-signal equivalent circuit



Example (continued)

AC analysis

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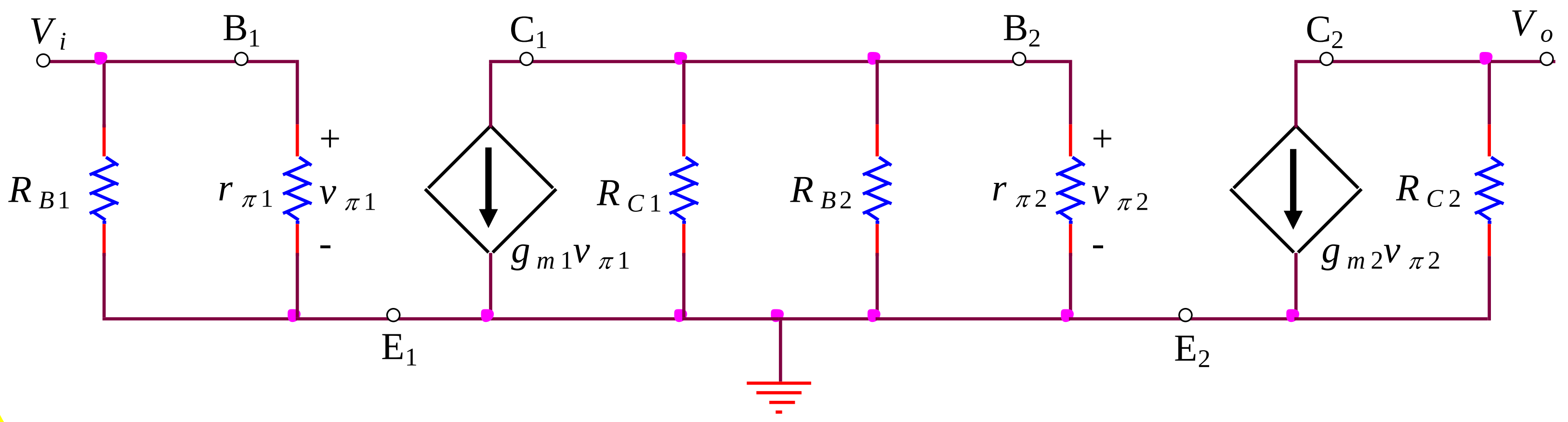
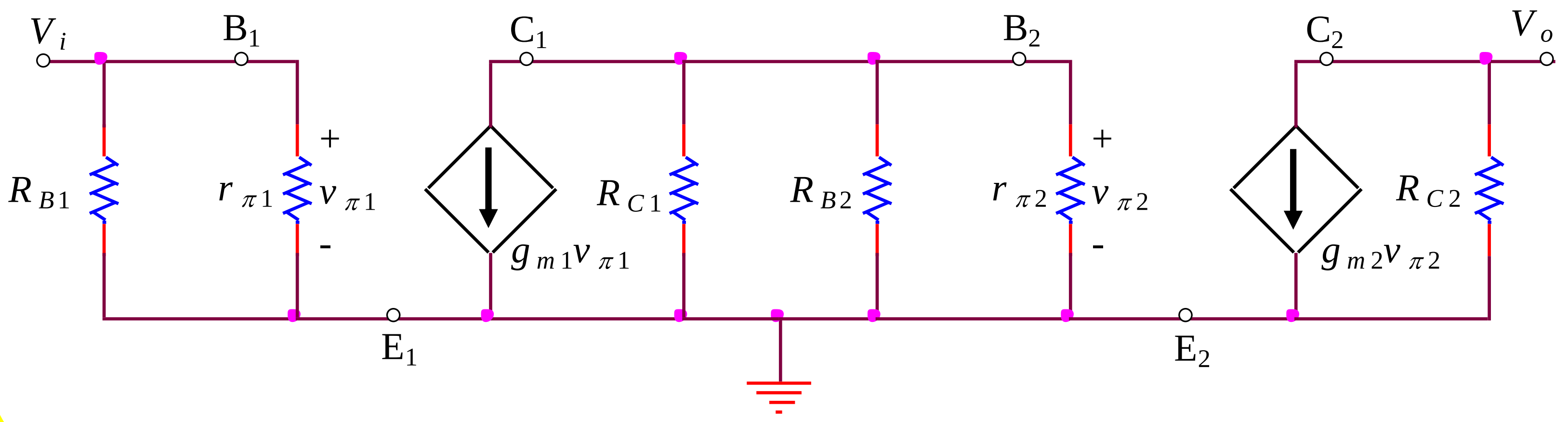


Example (continued)

AC analysis

The small-signal equivalent circuit



$$R_{B1} = R_1 // R_2$$

$$R_{B2} = R_3 // R_4$$

Example (continued)

$$V_o = -g_{m2} v_{\pi 2} R_{C2}$$

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Example (continued)

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$$= -g_{m1} V_i \left(R_{C1} \,//\, R_{B2} \,//\, r_{\pi 2} \right)$$

$$\left[v_{\pi 1} = V_i \right]$$

Example (continued)

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$$\left[v_{\pi 1} = V_i \right]$$

$$A_1 = \frac{v_{\pi 2}}{V_i} = -g_{m2} \left(R_{C1} \,//\, R_{B2} \,//\, r_{\pi 2} \right)$$

Example (continued)

The small-signal voltage gain

$$A = A_1 A_2 = g_{m1} g_{m2} R_{C2} \left(R_{C1} \,//\, R_{B2} \,//\, r_{\pi 2} \right)$$

Example (continued)

The small-signal voltage gain

$$A = A_1 A_2 = g_{m1} g_{m2} R_{C2} \left(R_{C1} \,//\, R_{B2} \,//\, r_{\pi 2} \right)$$

Substituting values

$$R_{B1} = R_{B2} = R_3 \,//\, R_4 = 15 \,//\, 4.7 = 3.579\ \text{k}\Omega$$

$$R_{C1} \,//\, R_{B2} \,//\, r_{\pi 2} = 2.2 \,//\, 3.579 \,//\, 1.307 = 667\ \Omega$$

$$A = 0.153 \times 0.153 \times 2200 \times 667 = 34350\ \text{V/V}$$

Example (continued)

The input resistance

$$R_{in} = R_{B1} // r_{\pi 1} = 3.579 // 1.307 = 0.957\ \text{k}\Omega$$

Example (continued)

The input resistance

$$R_{in} = R_{B1} // r_{\pi 1} = 3.579 // 1.307 = 0.957 \text{ k}\Omega$$

The output resistance

$$R_o = R_{C2} = 2.2 \text{ k}\Omega$$

Cascode Connection

Cascode Connection

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- The i/p applied to a C-E amplifier (Q_1) whose output is used to drive a C-B amplifier. (Q_2)

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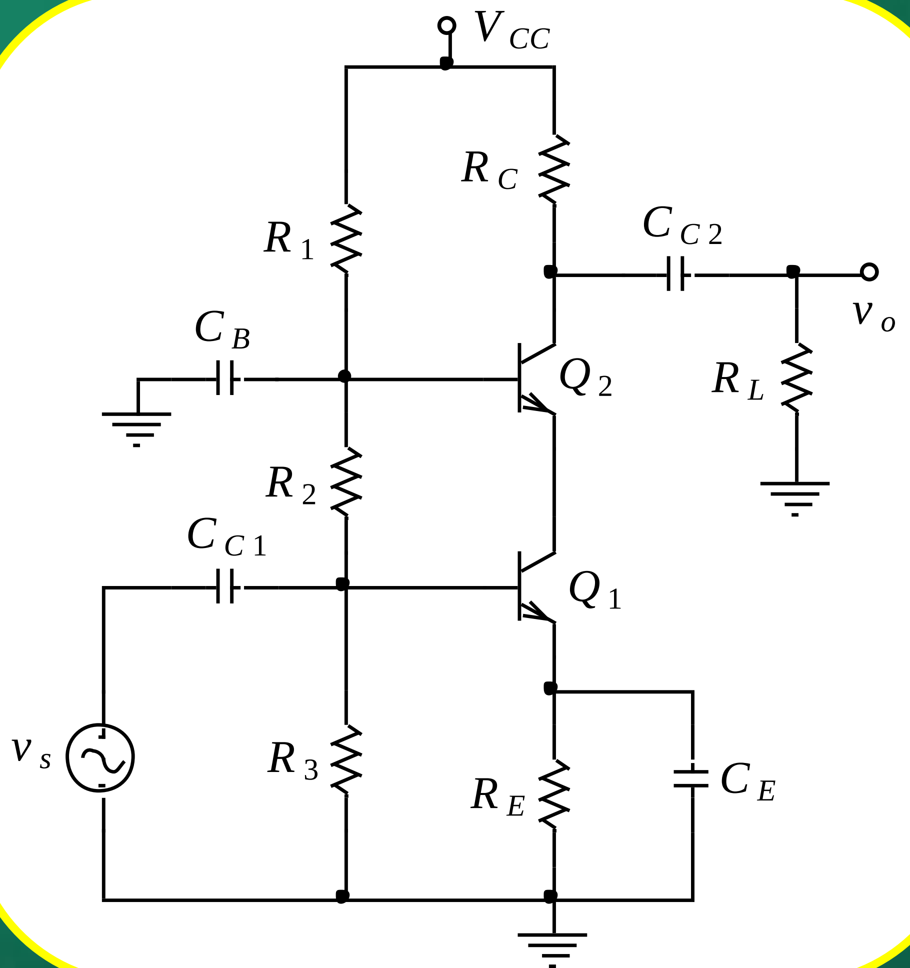
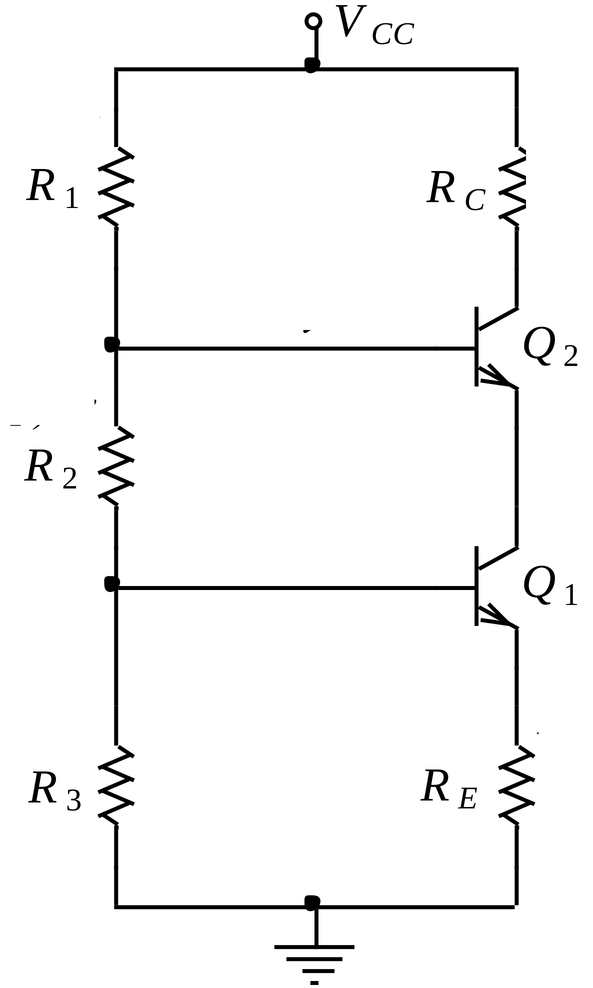
- The i/p applied to a C-E amplifier (Q_1) whose output is used to drive a C-B amplifier. (Q_2)
- The o/p signal current of Q_1 is the i/p signal of Q_2.
- The advantage: provides a high i/p impedance. The C-B stage providing good high frequency operation

Cascode Amplifier



Cascode Amplifier

DC analysis



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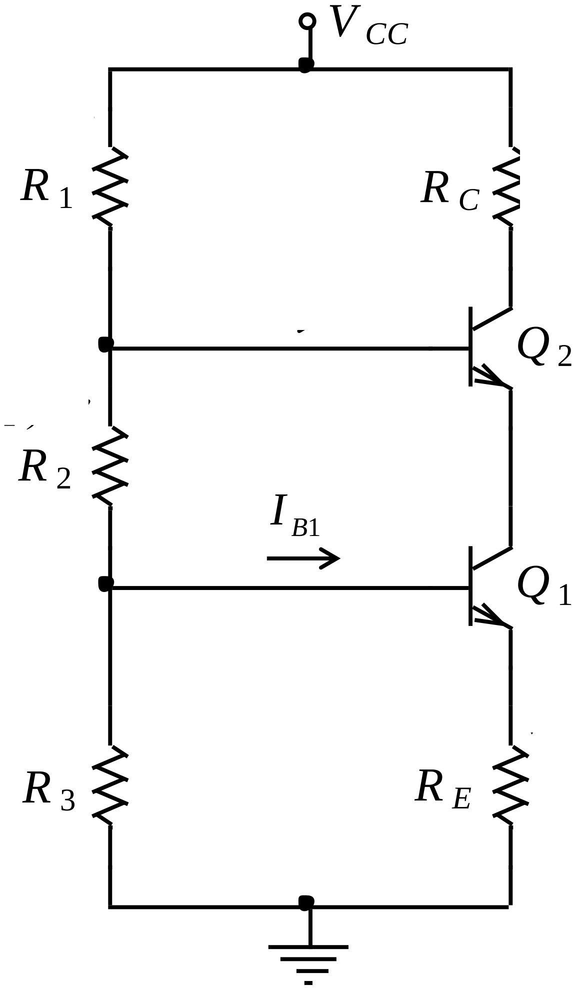

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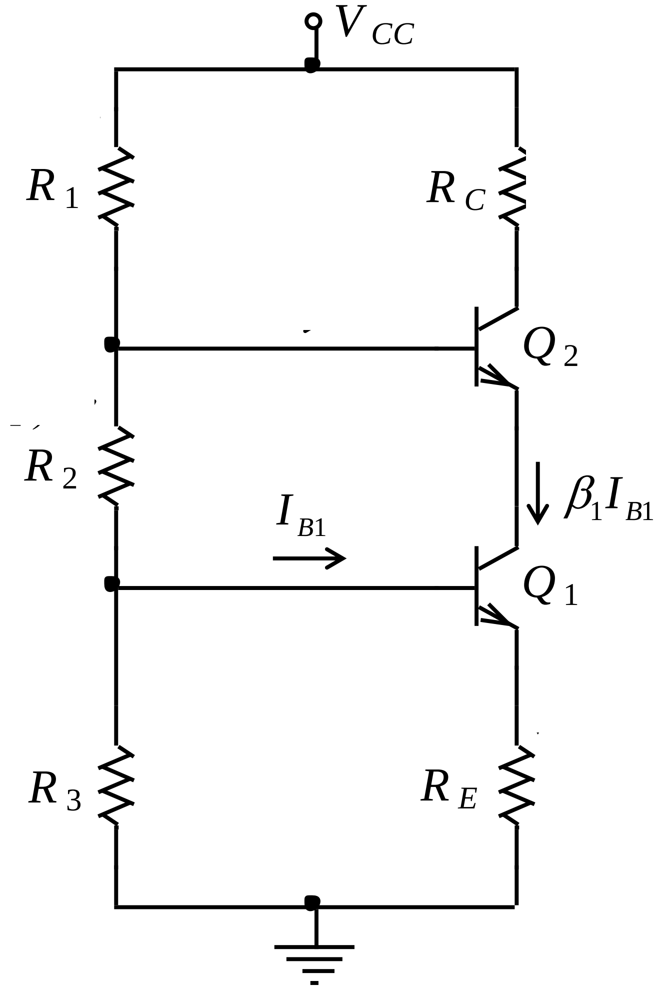

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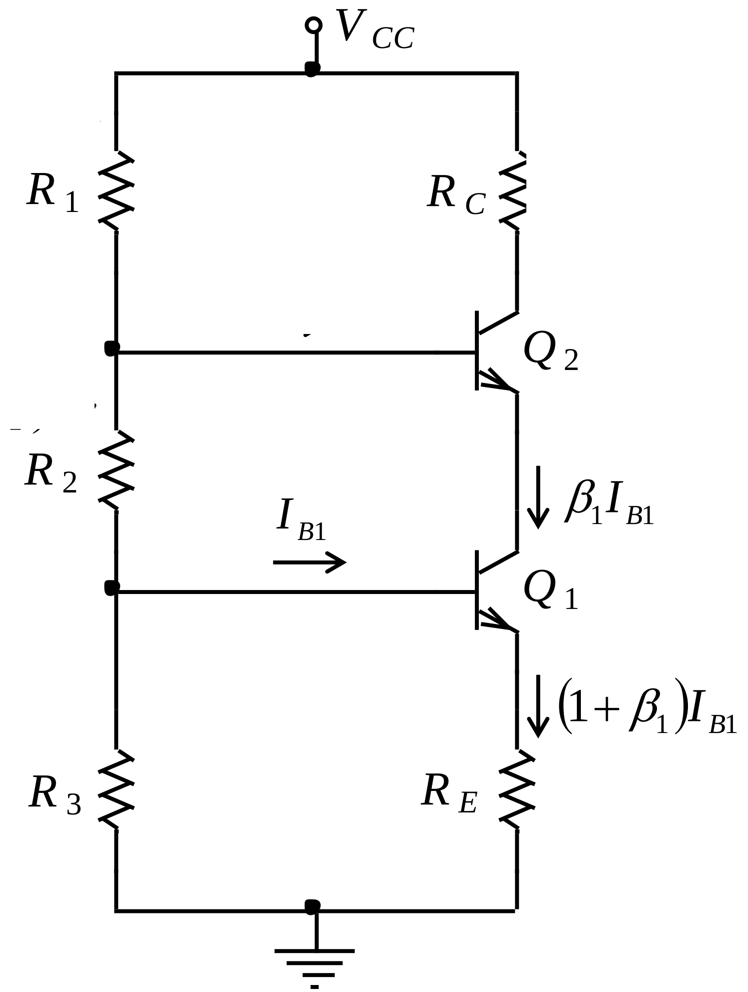

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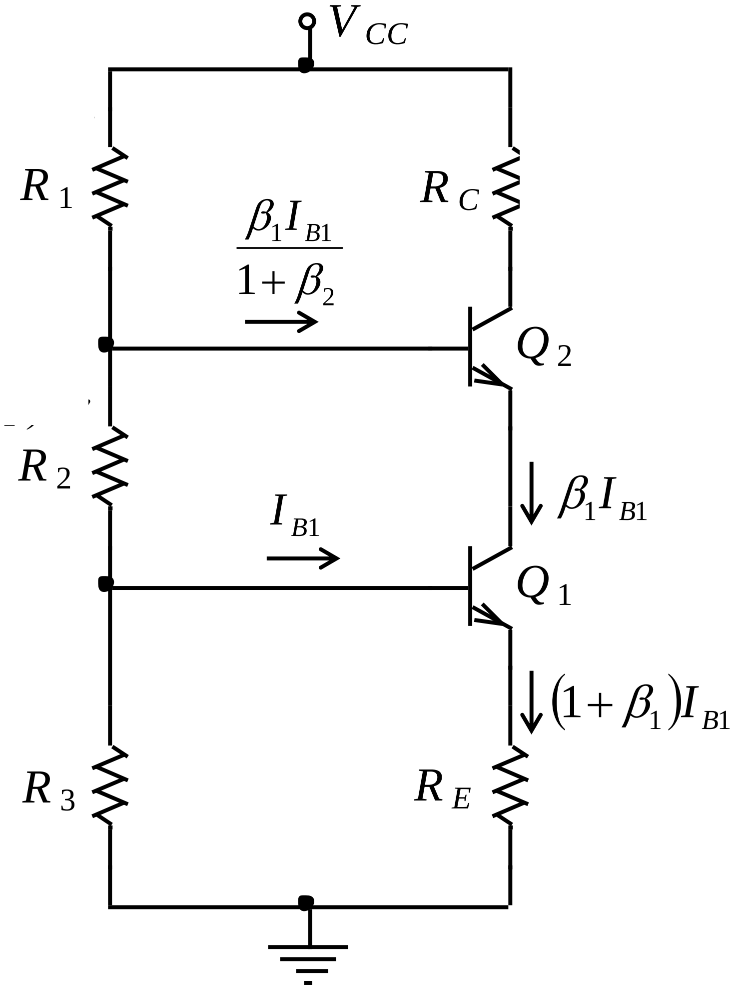

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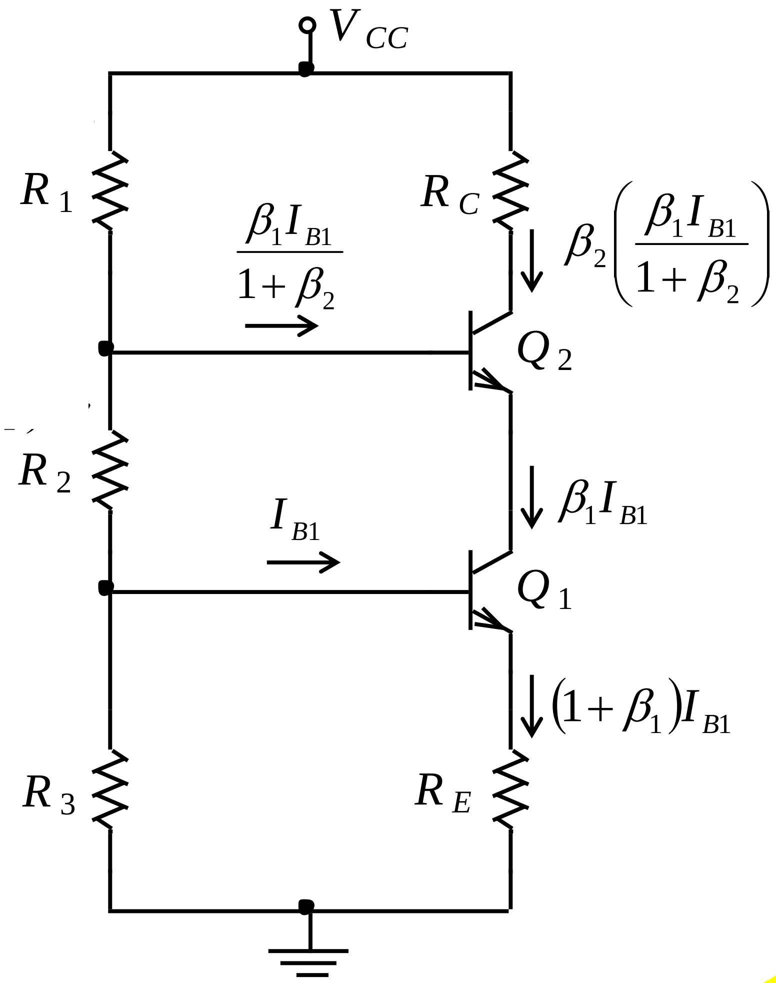

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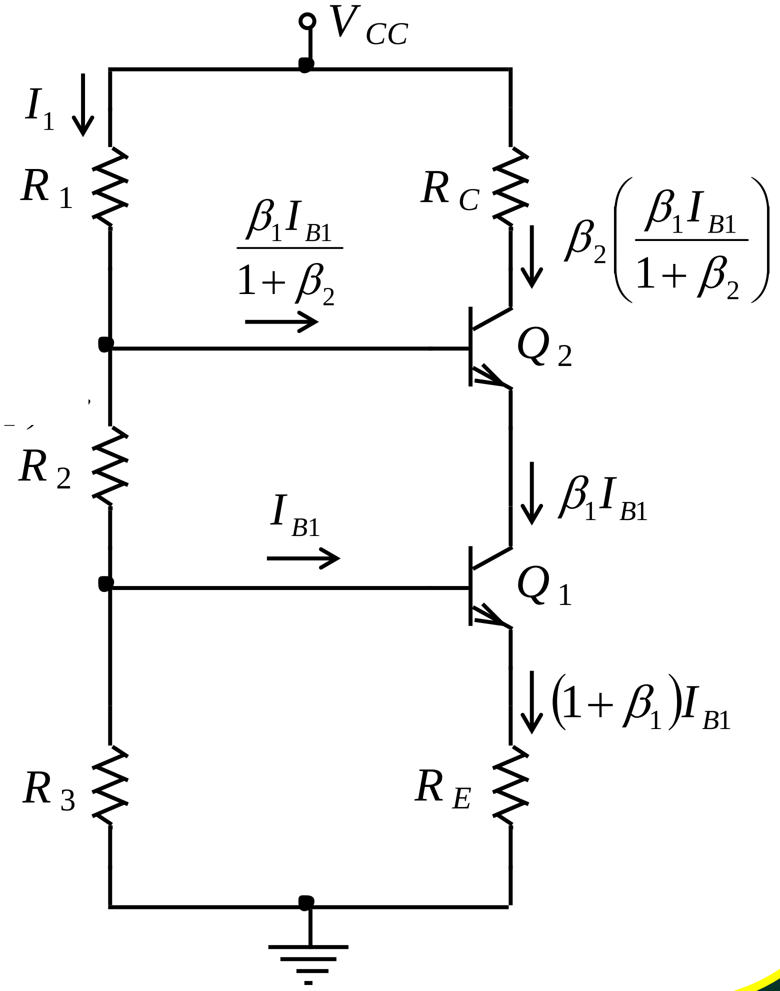

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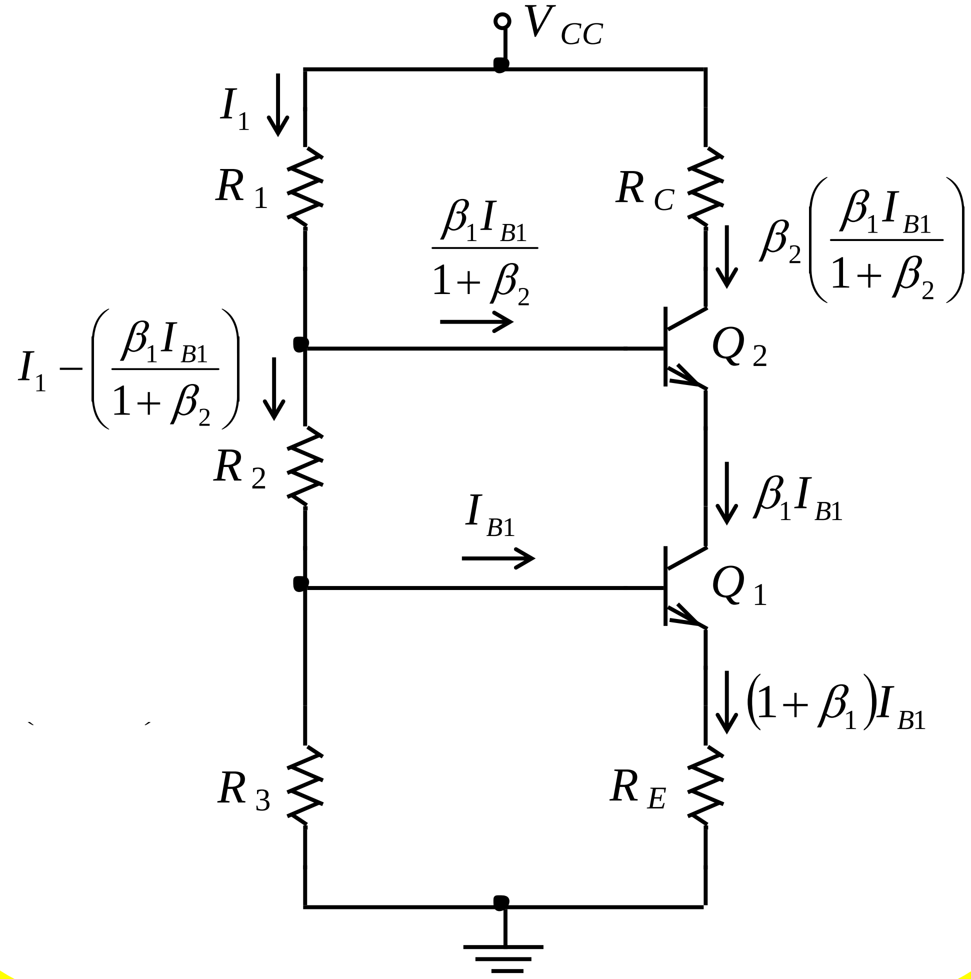

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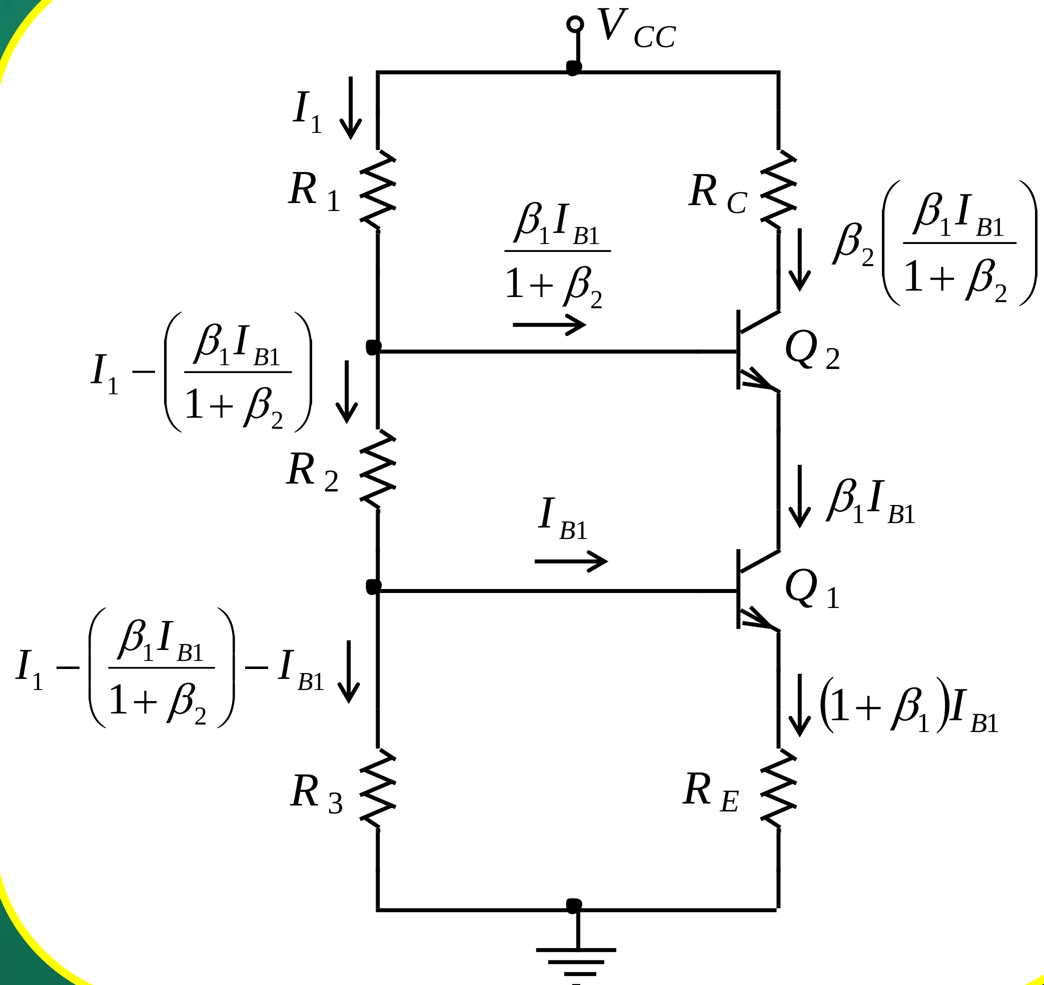

Cascode Amplifier

Assuming $V_{BE} = 0.7\ \mathrm{V}$ **for both BJT's**

Cascode Amplifier

Assuming $V_{BE} = 0.7$ V **for both BJT's**

$$R_1 I_1 + R_2\left(I_1 - \frac{\beta_1 I_{B1}}{\beta_2 + 1}\right) + R_3\left(I_1 - \frac{\beta_1 I_{B1}}{\beta_2 + 1} - I_{B1}\right) = V_{CC}$$

Cascode Amplifier

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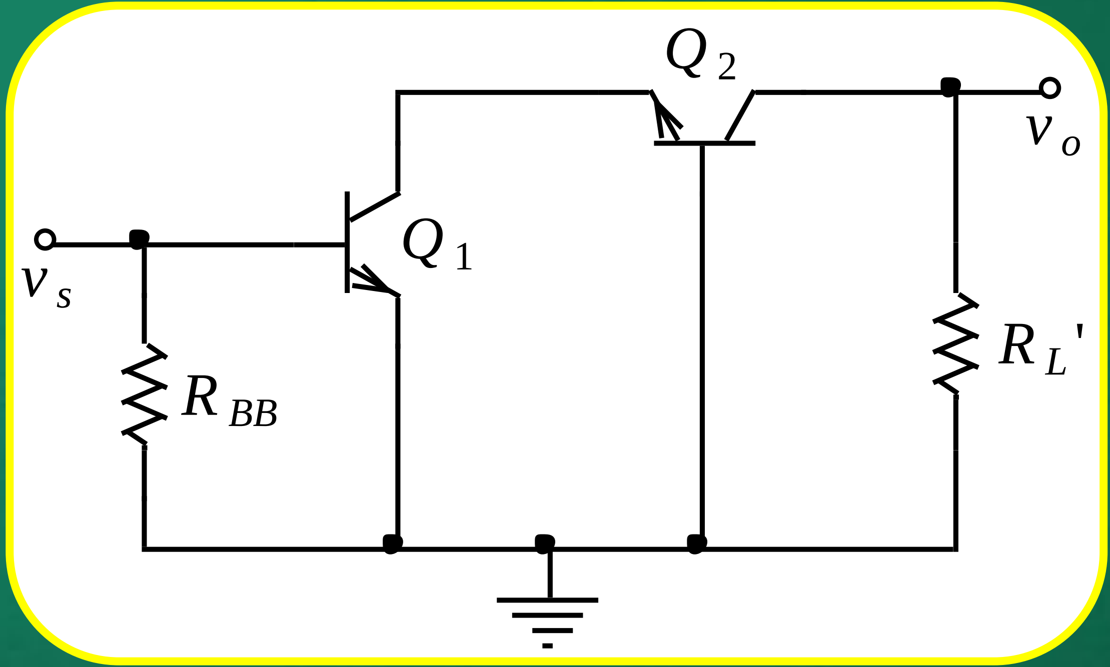
The above equations may solved for the two unknown currents namely I_1 **and** I_{B1}**.**

Cascode Amplifier

AC analysis

The equivalent circuit under AC condition



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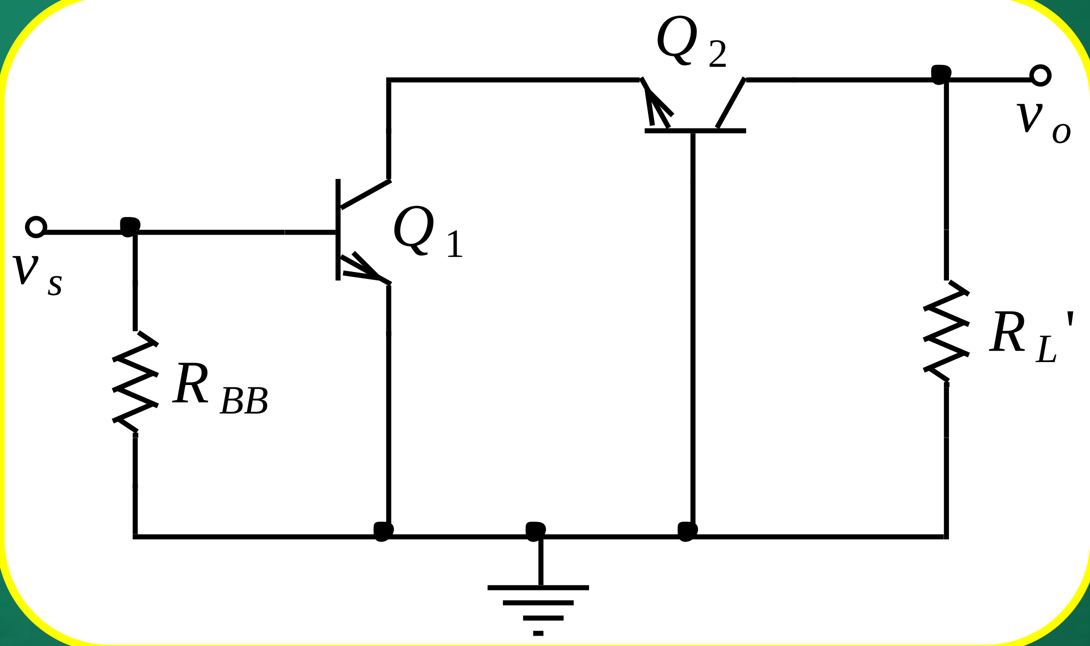
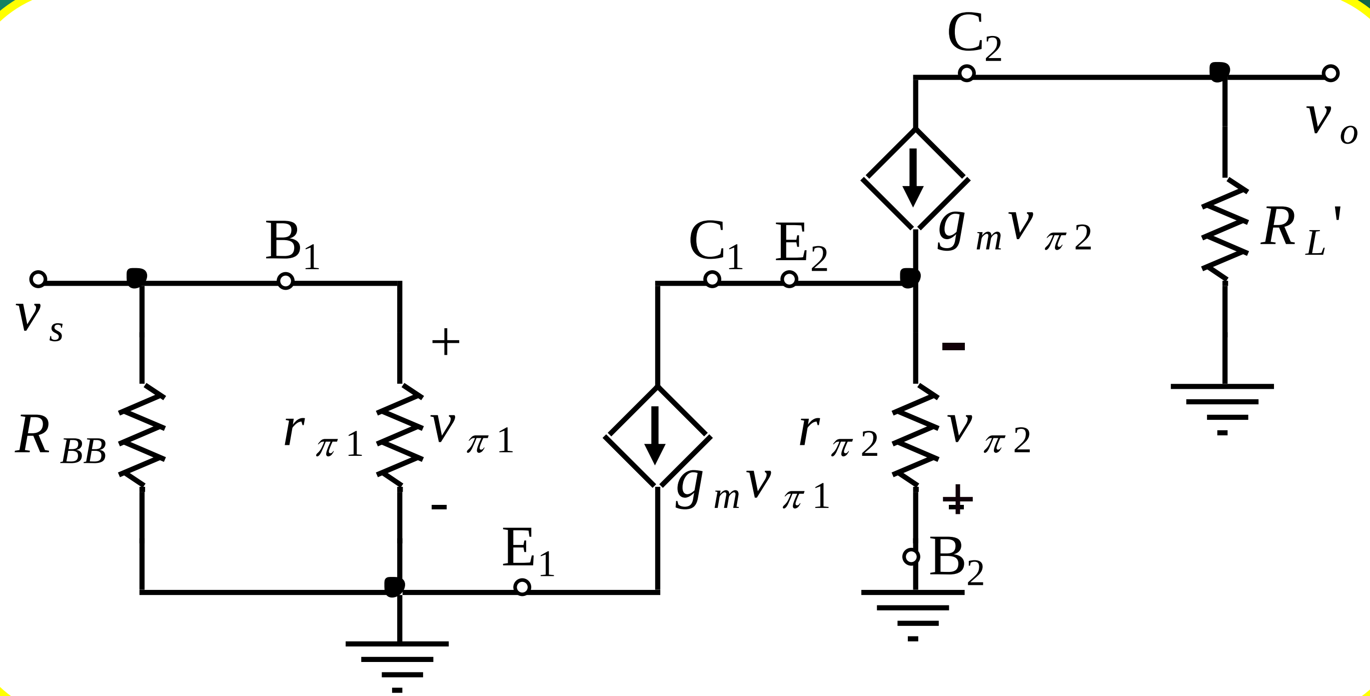


$$R_{BB} = R_2 \,//\, R_3$$

$$R_L' = R_C \,//\, R_L$$

Cascode Amplifier

The ac equivalent circuit using hybrid-π model



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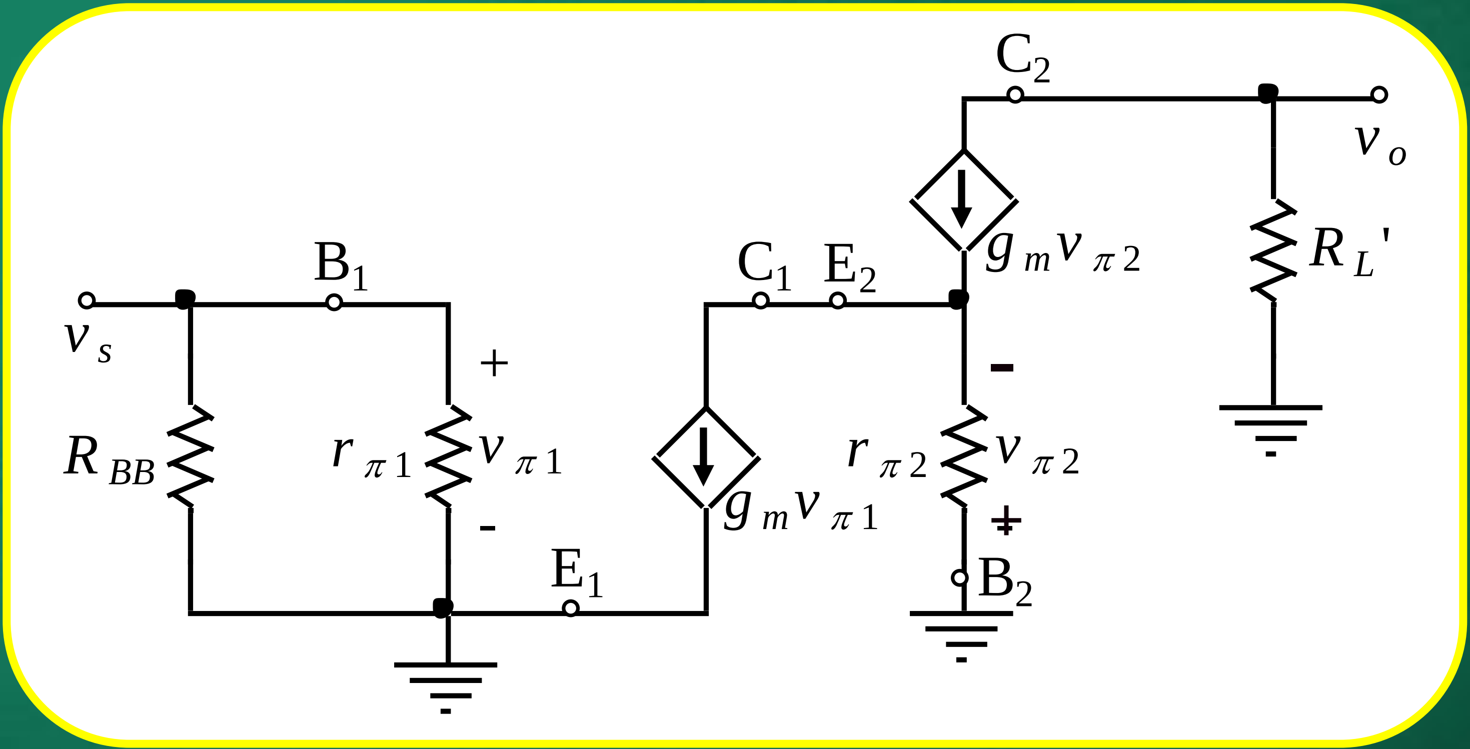

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At node E_2

$$g_{m1} v_{\pi 1} = \frac{v_{\pi 2}}{r_{\pi 2}} + g_{m2} v_{\pi 2}$$

Or

$$v_{\pi 2} = \frac{g_{m1} r_{\pi 2} v_{\pi 1}}{1 + g_{m2} r_{\pi 2}}$$

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Or

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Substituting in (1);

$$v_o = -\left(\frac{g_{m1} g_{m2} r_{\pi 2}}{1 + g_{m2} r_{\pi 2}} \right) R_L' v_{\pi 1} = -g_{m1} \left(\frac{\beta_2}{1 + \beta_2} \right) R_L' v_s$$

Cascode Amplifier

The small-signal voltage gain

$$A_v = \frac{v_o}{v_s} = -g_{m1}\left(\frac{\beta_2}{1+\beta_2}\right)R_L'$$

Cascode Amplifier

The small-signal voltage gain

$$A_v = \frac{v_o}{v_s} = -g_{m1}\left(\frac{\beta_2}{1+\beta_2}\right)R_L'$$

When $\beta_2 >> 1$

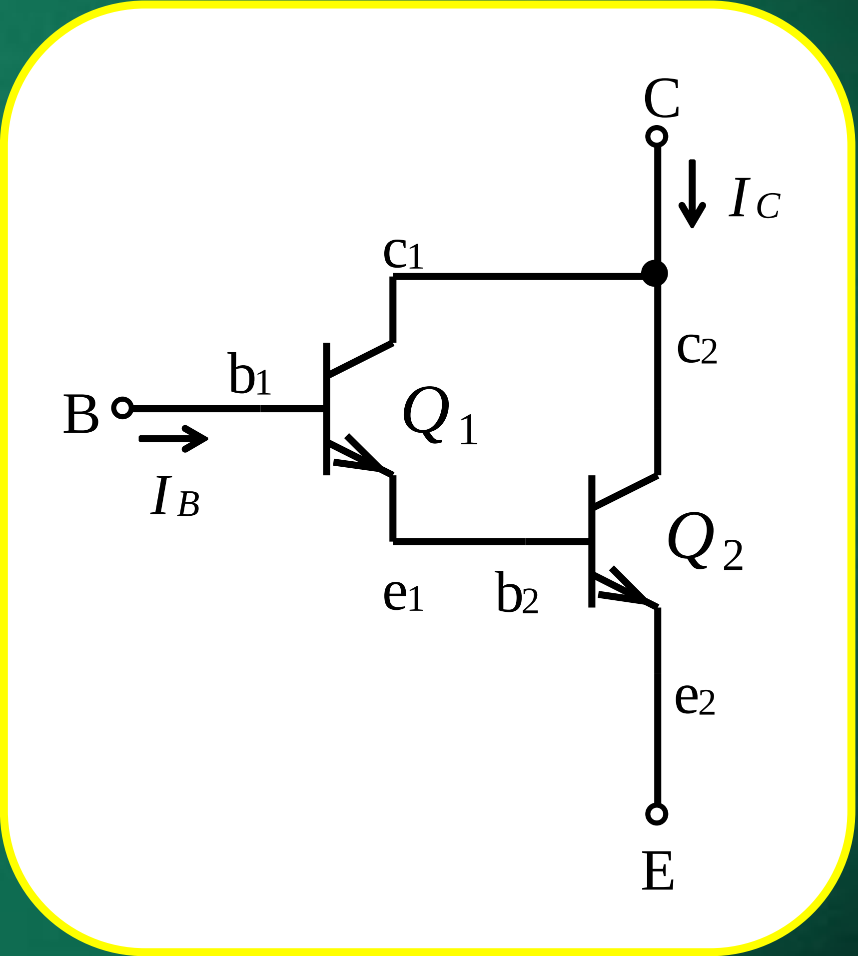
$$A_v \cong -g_{m1}R_L'$$

Darlington Connection

Darlington pair

Internal connection;

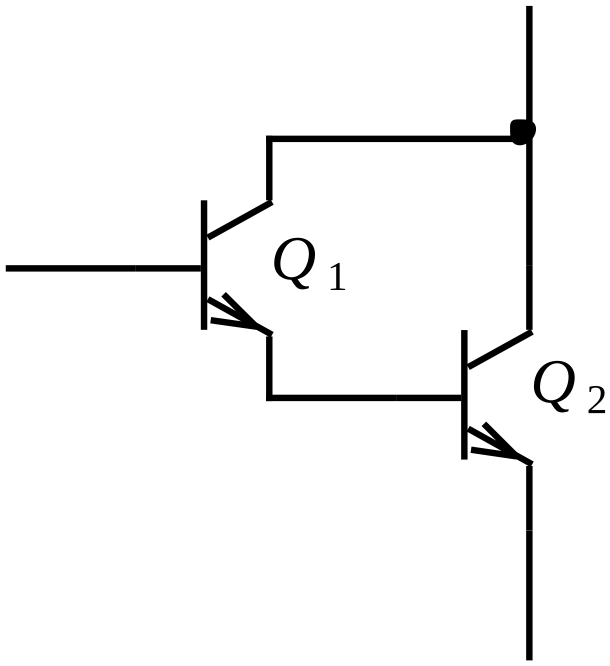
- **Collectors of Q_1 and Q_2**
- **Emitter of Q_1 and base of Q_2**



Provides high current gain : $I_C \cong \beta^2 I_B$

Darlington Connection

Currents in darlington pair



Darlington Connection

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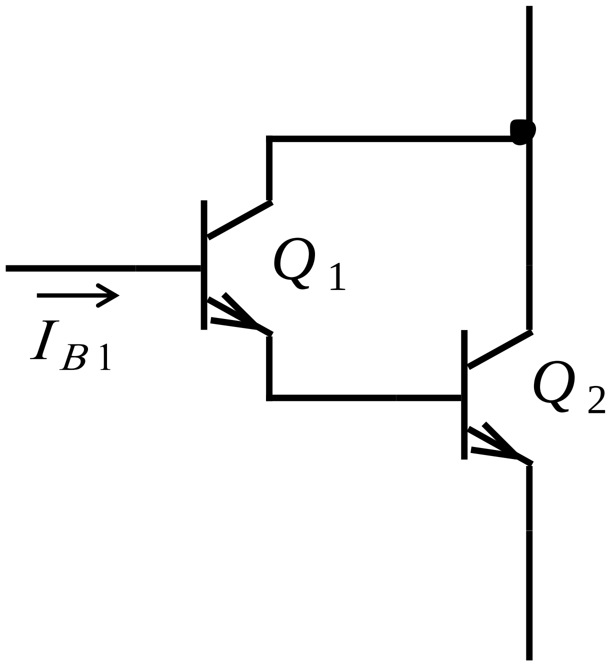

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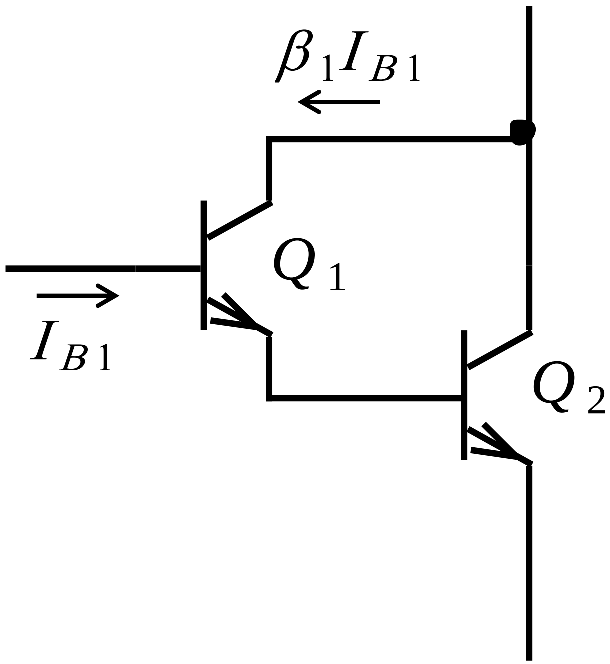

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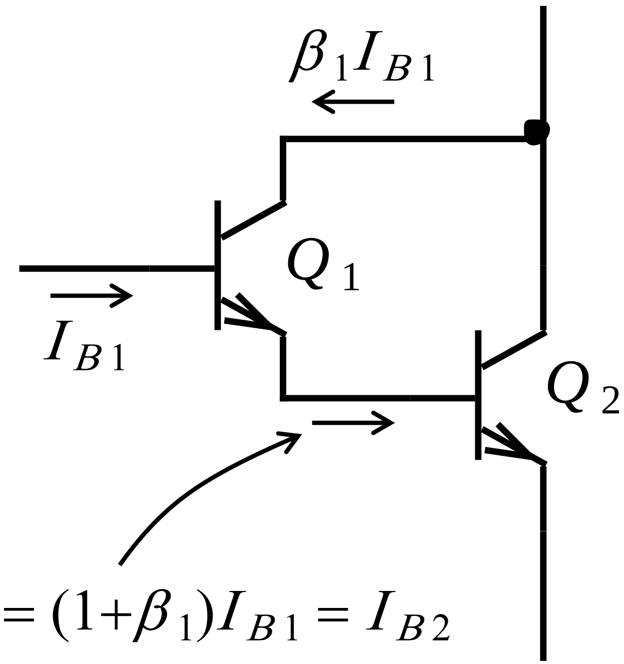
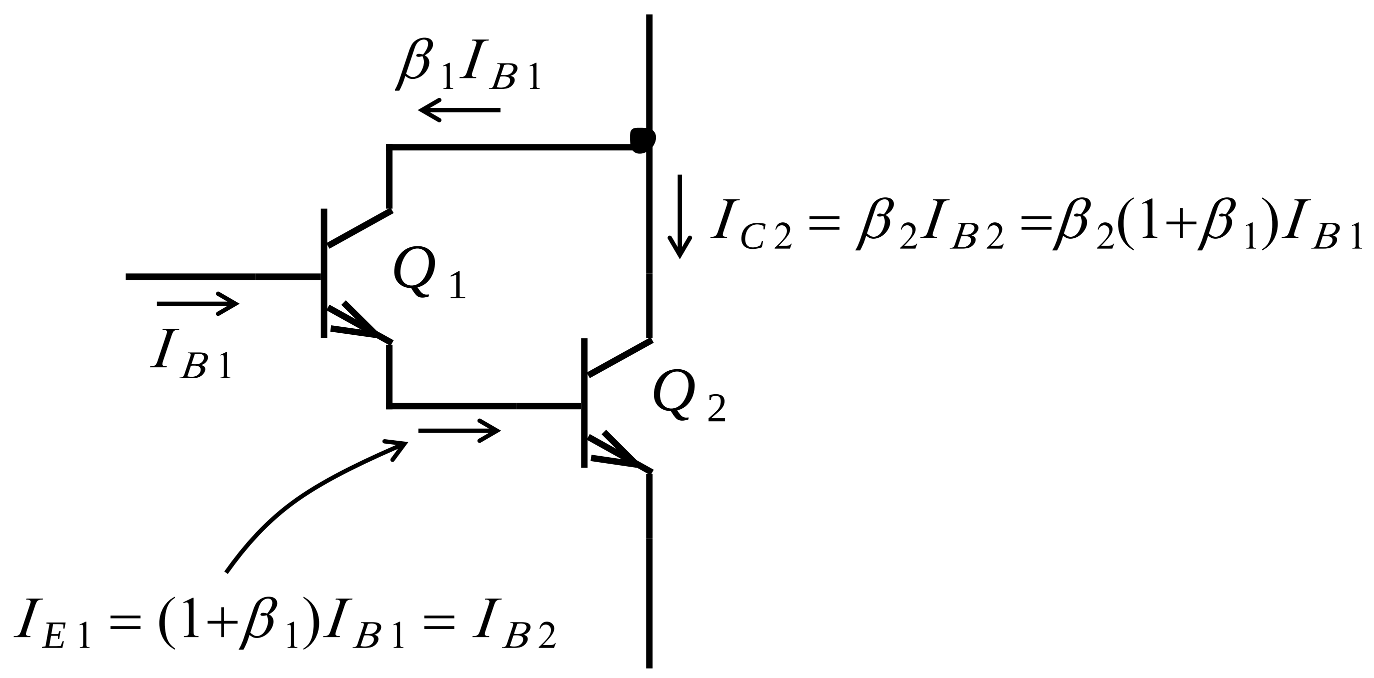


$I_{E1} = (1+\beta_1)I_{B1} = I_{B2}$

Darlington Connection

Currents in darlington pair



Darlington Connection

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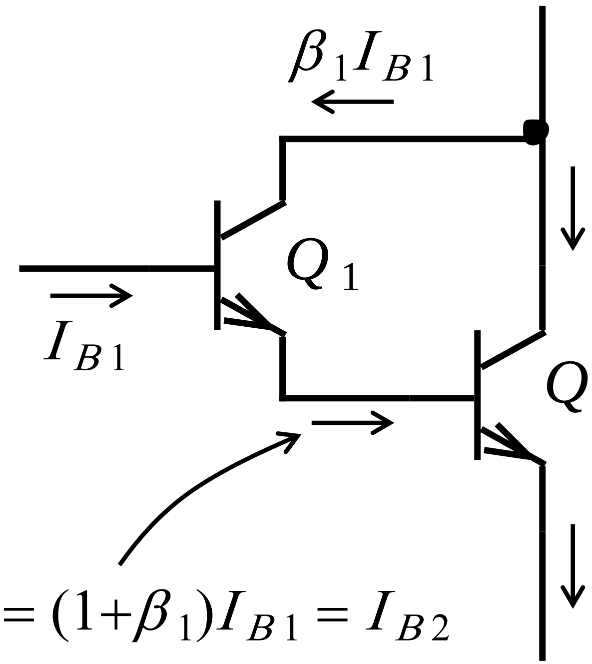


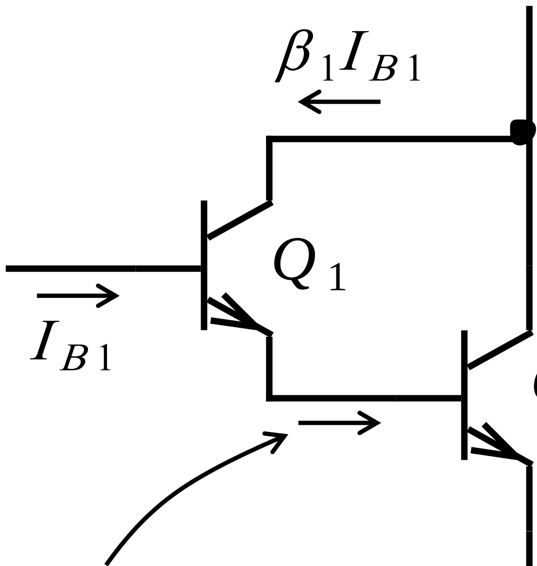
$$I_{C2} = \beta_2 I_{B2} = \beta_2 (1+\beta_1) I_{B1}$$

$$I_{E2} = (1+\beta_2) I_{B2} = (1+\beta_1)(1+\beta_2) I_{B1}$$

$$I_{E1} = (1+\beta_1) I_{B1} = I_{B2}$$

Darlington Connection

Currents in darlington pair



$$I_{C1} + I_{C2} = [\beta_1 + \beta_2(1+\beta_1)]I_{B1}$$

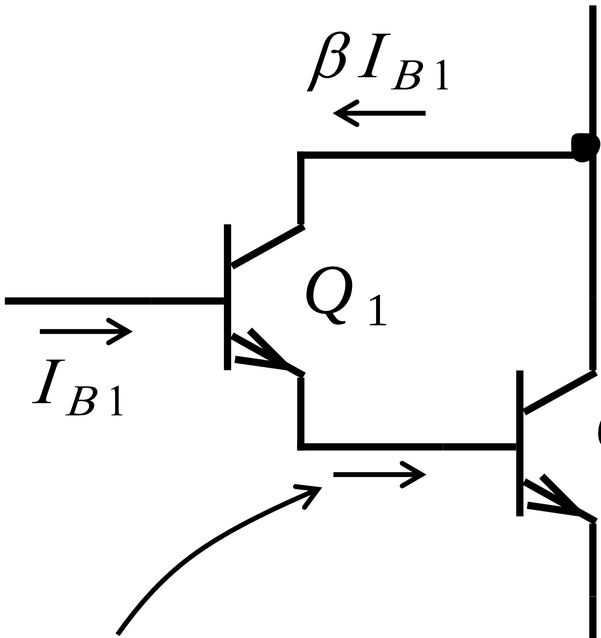
$$I_{C2} = \beta_2 I_{B2} = \beta_2(1+\beta_1)I_{B1}$$

$$I_{E2} = (1+\beta_2)I_{B2} = (1+\beta_1)(1+\beta_2)I_{B1}$$

$$I_{E1} = (1+\beta_1)I_{B1} = I_{B2}$$

Darlington Connection

If $\beta_1 = \beta_2 = \beta$ **and assuming** β **is large**



$$I_{C1} + I_{C2} \cong \beta^2 I_{B1}$$

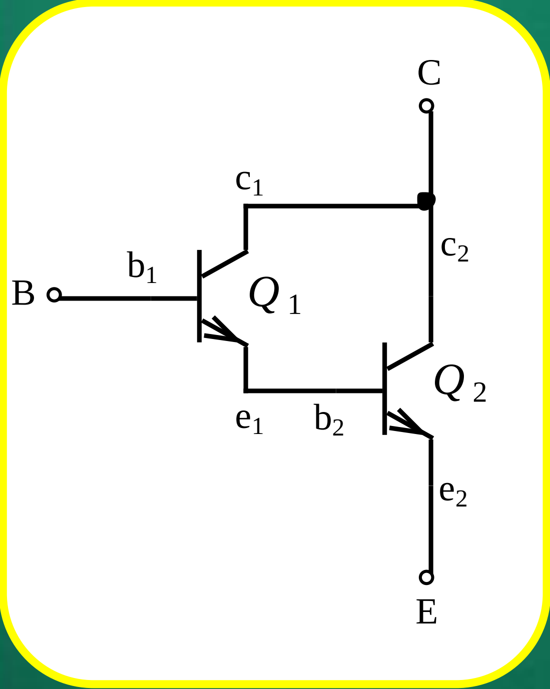
$$I_{C2} = \beta I_{B2} = \beta(1+\beta) I_{B1}$$

$$I_{E1} = (1+\beta) I_{B1} = I_{B2}$$

$$I_{E2} = (1+\beta) I_{B2} = (1+\beta)^2 I_{B1}$$

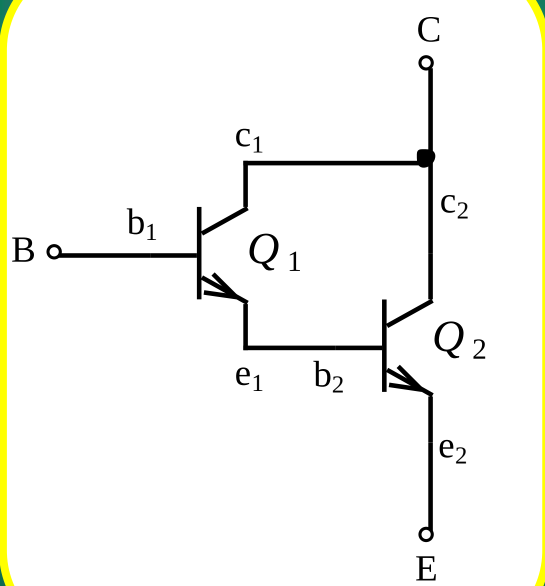
Darlington Connection

Hybrid-π model



Darlington Connection

Hybrid-π model



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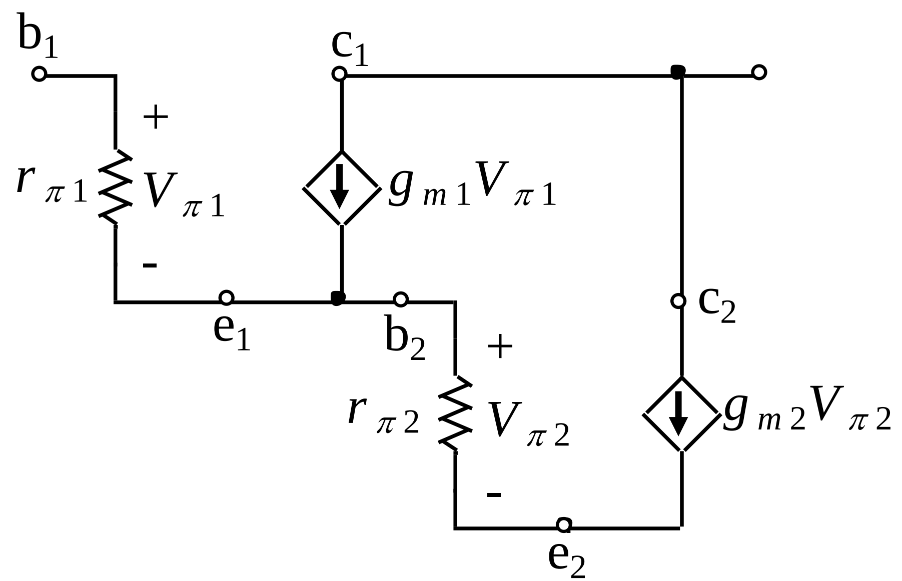
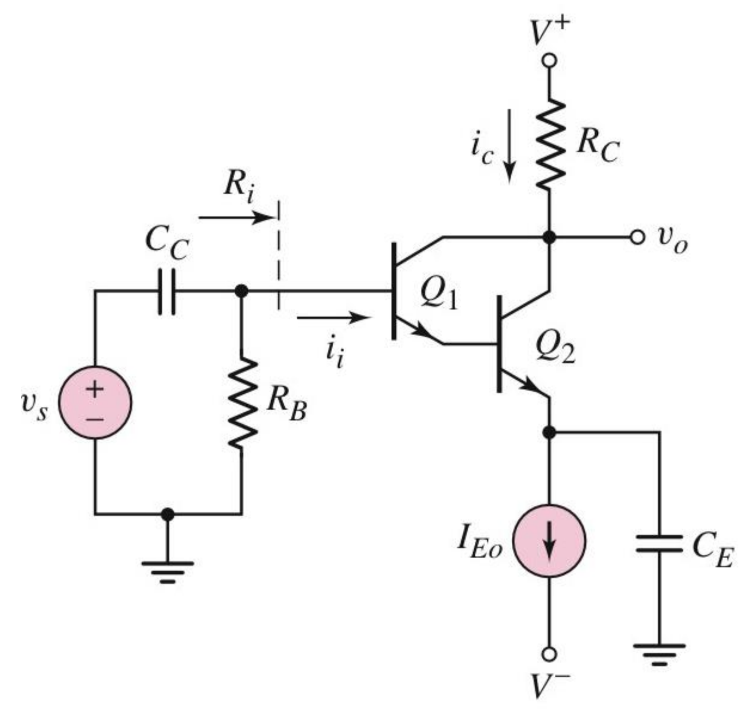


Darlington Connection

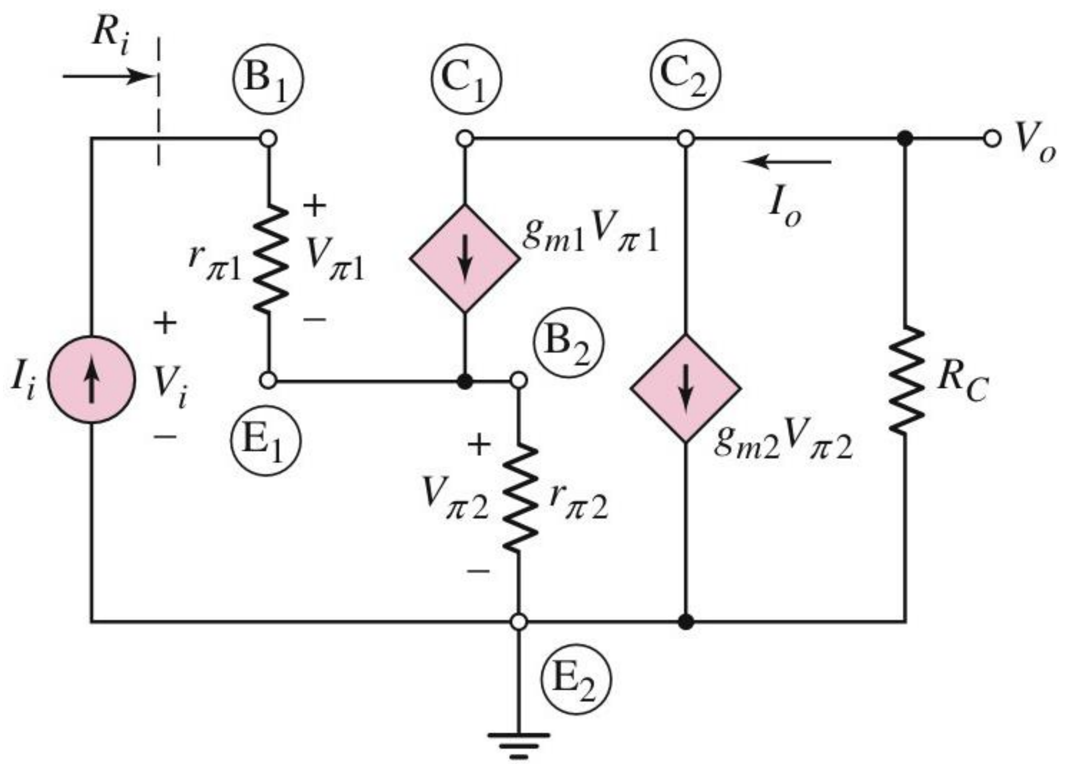
Darlington configuration provides

- Increased current
- High input resistance

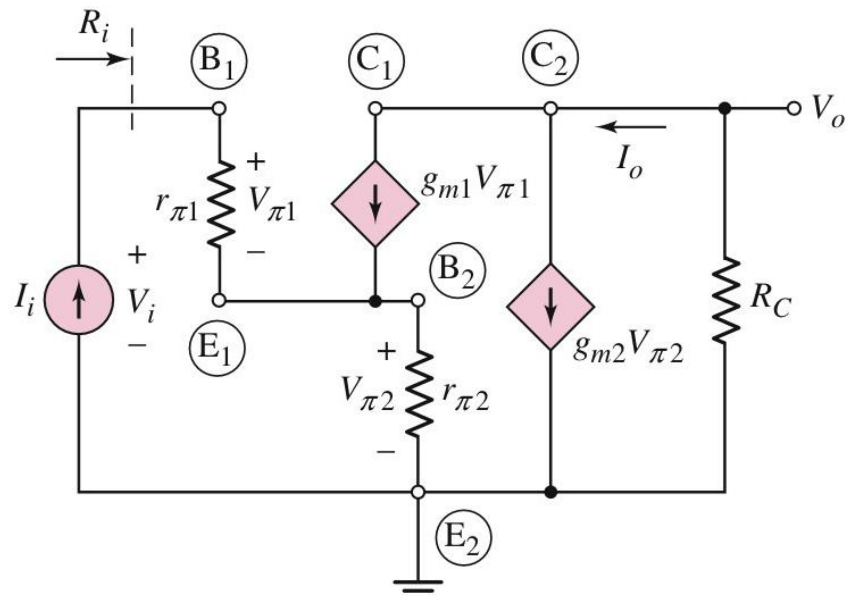


Darlington Connection

Small-signal equivalent circuit

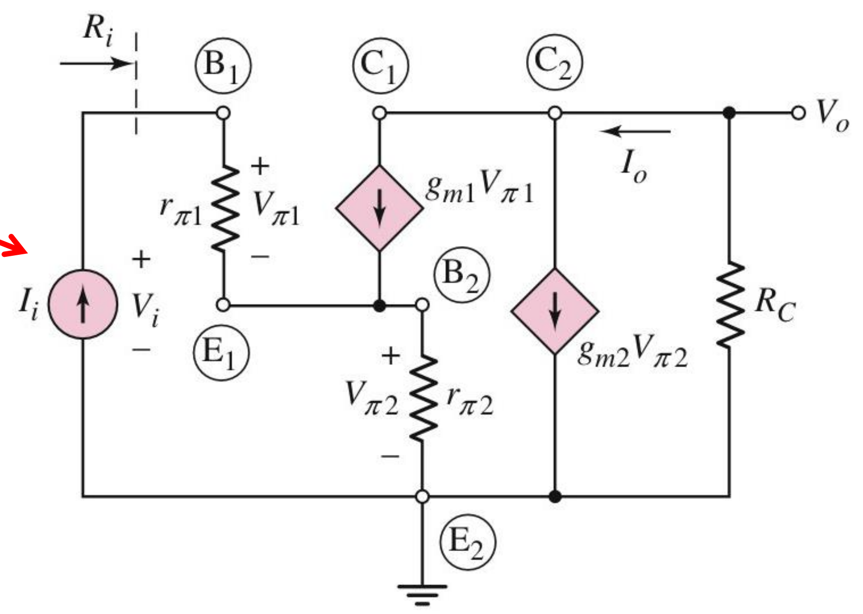


Darlington Connection



Darlington Connection

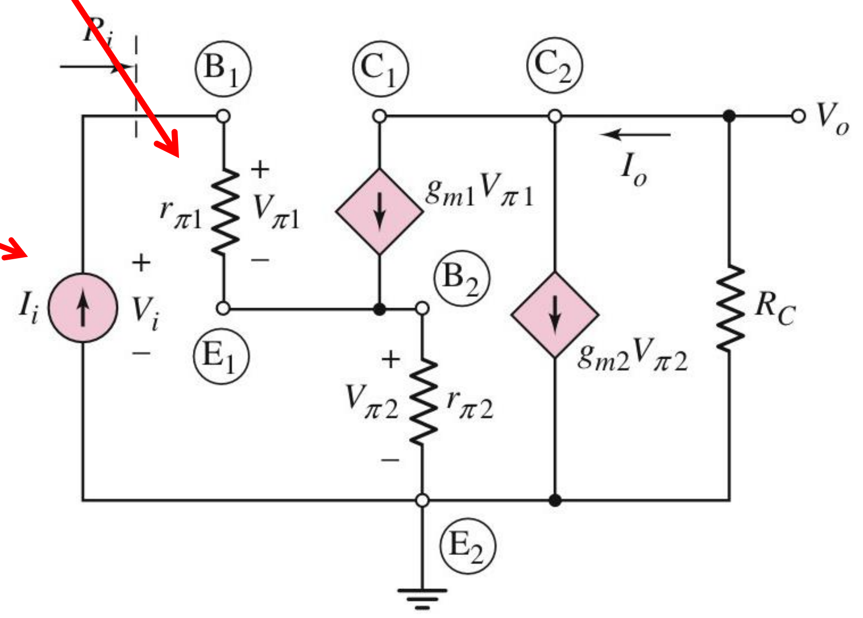
Input voltage source is transformed into current source



Darlington Connection

$$V_{\pi 1} = I_i r_{\pi 1}$$

Input voltage source is transformed into current source

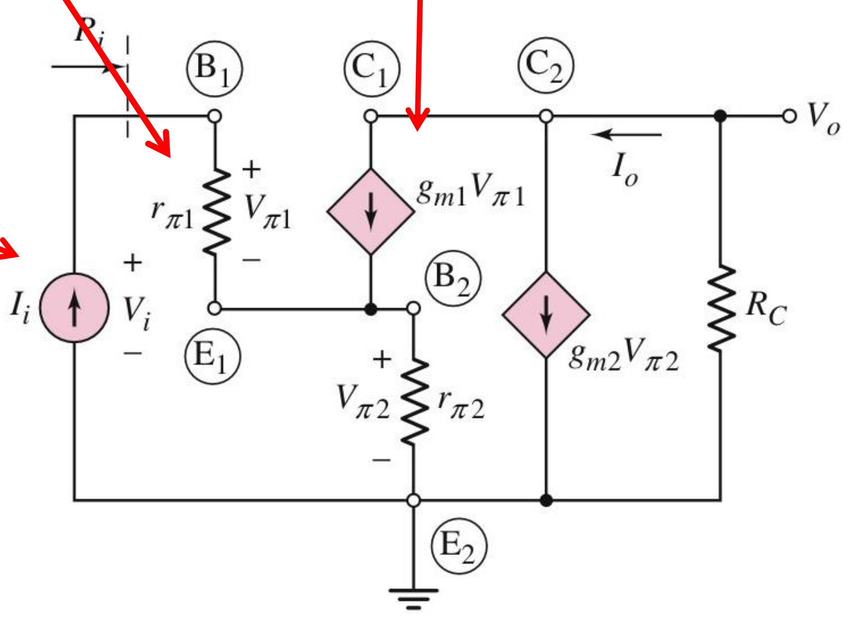


Darlington Connection

$$V_{\pi 1} = I_i r_{\pi 1}$$

$$g_{m1} V_{\pi 1} = g_{m1} r_{\pi 1} I_i = \beta_1 I_i$$

Input voltage source is transformed into current source



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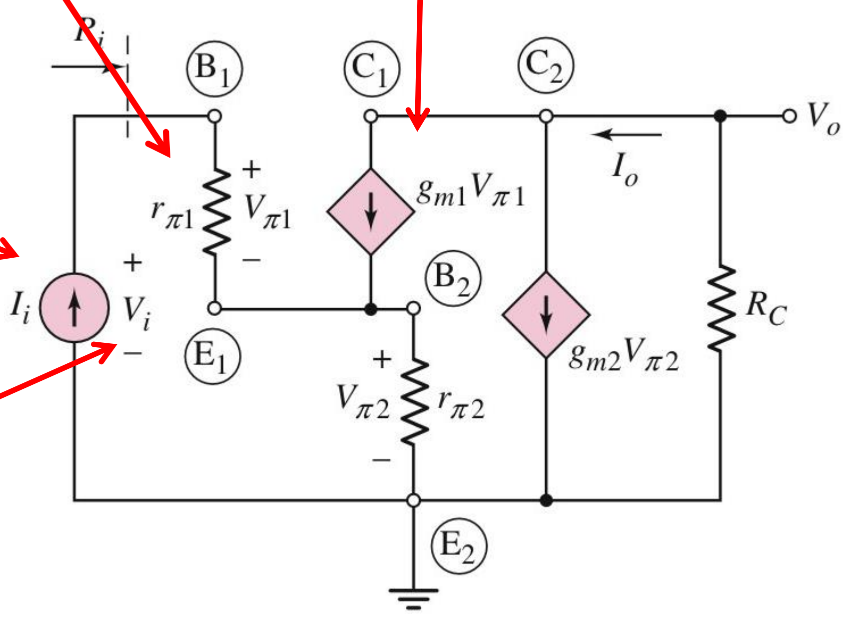


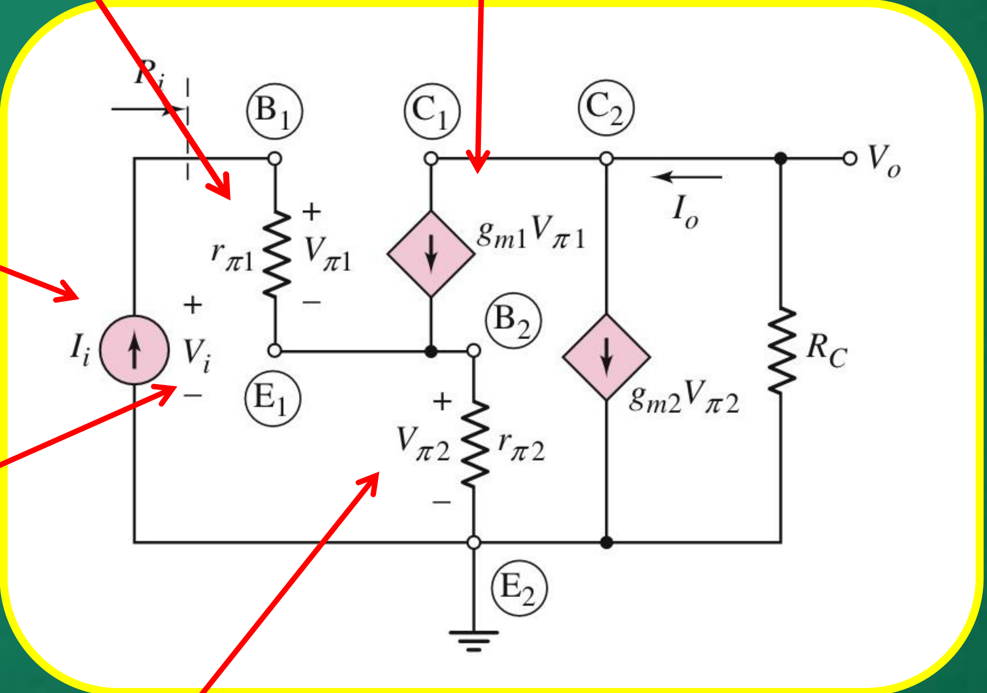
$$V_i = V_{\pi 1} + V_{\pi 2}$$

Darlington Connection

$$V_{\pi 1} = I_i r_{\pi 1}$$

$$g_{m1} V_{\pi 1} = g_{m1} r_{\pi 1} I_i = \beta_1 I_i$$

Input voltage source is transformed into current source



$$V_i = V_{\pi 1} + V_{\pi 2}$$

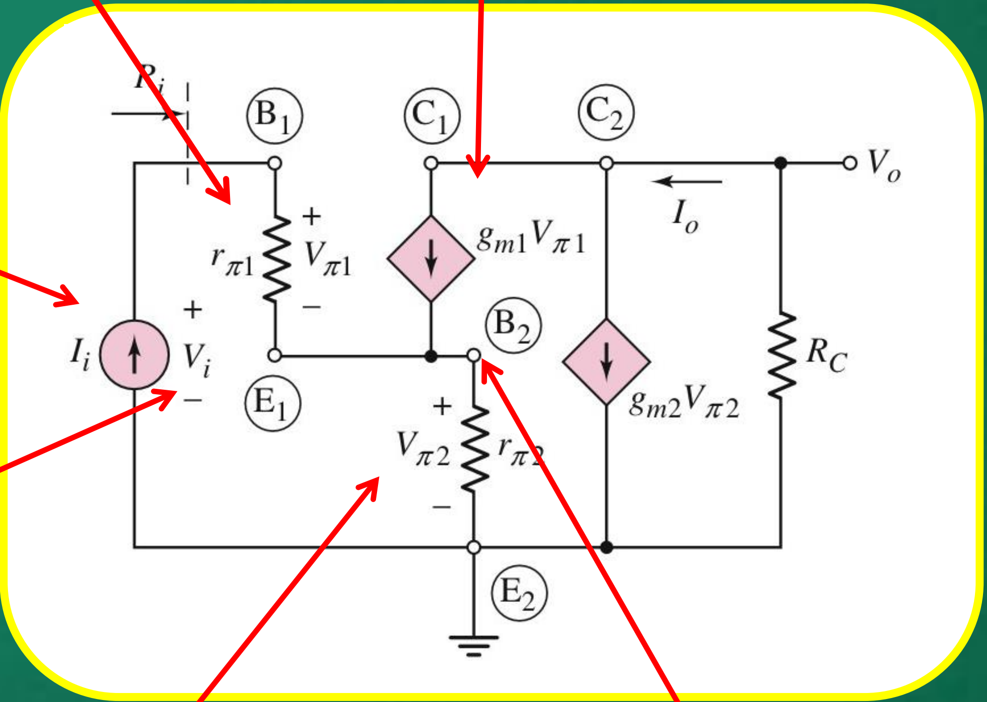
$$V_{\pi 2} = (1+\beta_1) I_i r_{\pi 2}$$

Darlington Connection

$V_{\pi 1} = I_i r_{\pi 1}$

$g_{m1} V_{\pi 1} = g_{m1} r_{\pi 1} I_i = \beta_1 I_i$

Input voltage source is transformed into current source



$V_i = V_{\pi 1} + V_{\pi 2}$

$V_{\pi 2} = (1 + \beta_1) I_i r_{\pi 2}$

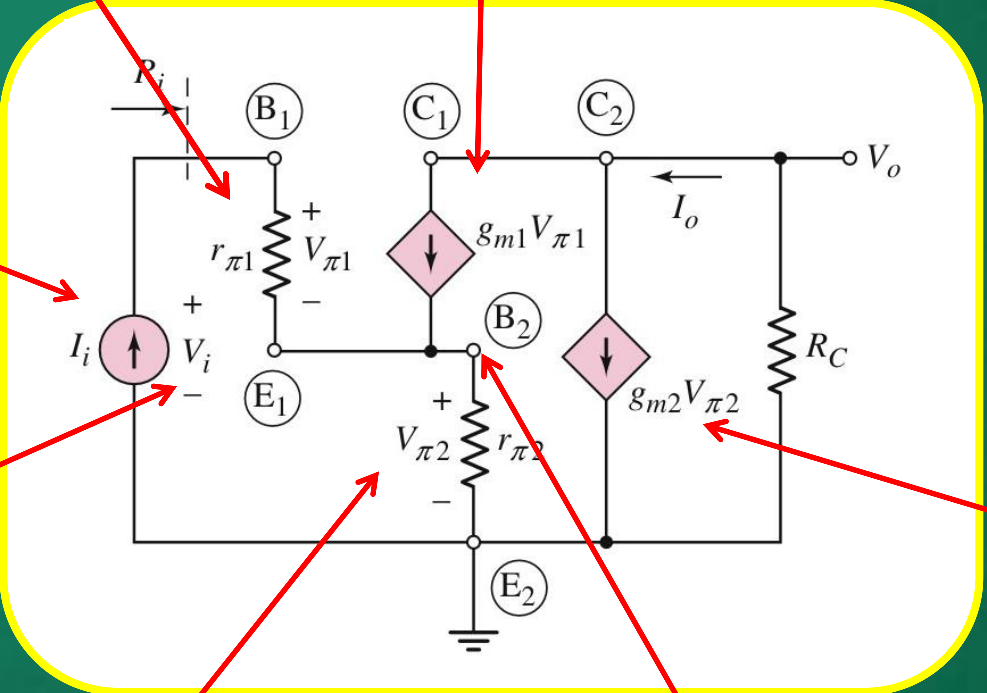
$I_{b2} = I_i + \beta_1 I_i = (1 + \beta_1) I_i$

Darlington Connection

$$V_{\pi 1} = I_i r_{\pi 1}$$

$$g_{m1} V_{\pi 1} = g_{m1} r_{\pi 1} I_i = \beta_1 I_i$$

Input voltage source is transformed into current source



$$V_i = V_{\pi 1} + V_{\pi 2}$$

$$g_{m2} V_{\pi 2} = g_{m2} (1+\beta_1) I_i r_{\pi 2} = \beta_2 (1+\beta_1) I_i$$

$$V_{\pi 2} = (1+\beta_1) I_i r_{\pi 2}$$

$$I_{b2} = I_i + \beta_1 I_i = (1+\beta_1) I_i$$

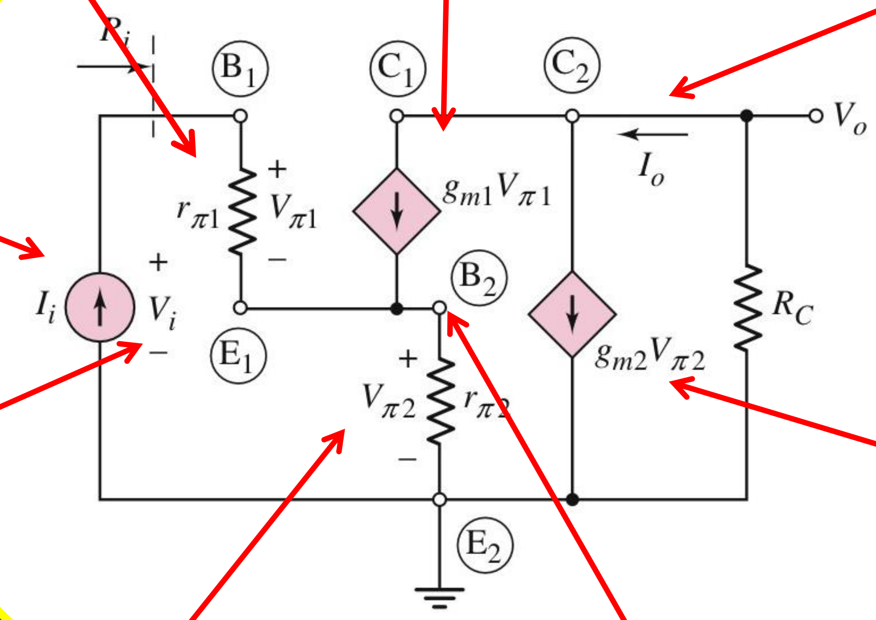
Darlington Connection

$$V_{\pi 1} = I_i r_{\pi 1}$$

$$g_{m1} V_{\pi 1} = g_{m1} r_{\pi 1} I_i = \beta_1 I_i$$

$$I_o = g_{m1} V_{\pi 1} + g_{m2} V_{\pi 2}$$
$$= \beta_1 I_i + \beta_2 (1 + \beta_1) I_i$$

Input voltage source is transformed into current source



$$g_{m2} V_{\pi 2}$$
$$= g_{m2} (1 + \beta_1) I_i r_{\pi 2}$$
$$= \beta_2 (1 + \beta_1) I_i$$

$$V_i = V_{\pi 1} + V_{\pi 2}$$

$$V_{\pi 2} = (1 + \beta_1) I_i r_{\pi 2}$$

$$I_{b2} = I_i + \beta_1 I_i = (1 + \beta_1) I_i$$

Darlington Connection

$$V_{\pi 1} = I_i r_{\pi 1}$$

$$g_{m1} V_{\pi 1} = g_{m1} r_{\pi 1} I_i = \beta_1 I_i$$

Darlington Connection

$$V_{\pi1} = I_i r_{\pi1}$$ → $$g_{m1}V_{\pi1} = g_{m1}r_{\pi1}I_i = \beta_1 I_i$$

$$I_{b2} = I_i + \beta_1 I_i = (1+\beta_1)I_i$$ → $$V_{\pi2} = I_{b2}r_{\pi2} = (1+\beta_1)I_i r_{\pi2}$$

Darlington Connection

$$V_{\pi 1} = I_i r_{\pi 1} \rightarrow g_{m1} V_{\pi 1} = g_{m1} r_{\pi 1} I_i = \beta_1 I_i$$

$$I_{b2} = I_i + \beta_1 I_i = (1+\beta_1) I_i \rightarrow V_{\pi 2} = I_{b2} r_{\pi 2} = (1+\beta_1) I_i r_{\pi 2}$$

$$g_{m2} V_{\pi 2} = g_{m2} (1+\beta_1) I_i r_{\pi 2} = \beta_2 (1+\beta_1) I_i$$

Darlington Connection

$$V_{\pi 1} = I_i r_{\pi 1} \rightarrow g_{m1} V_{\pi 1} = g_{m1} r_{\pi 1} I_i = \beta_1 I_i$$

$$I_{b2} = I_i + \beta_1 I_i = (1+\beta_1) I_i \rightarrow V_{\pi 2} = I_{b2} r_{\pi 2} = (1+\beta_1) I_i r_{\pi 2}$$

$$g_{m2} V_{\pi 2} = g_{m2} (1+\beta_1) I_i r_{\pi 2} = \beta_2 (1+\beta_1) I_i$$

$$I_o = g_{m1} V_{\pi 1} + g_{m2} V_{\pi 2} = \beta_1 I_i + \beta_2 (1+\beta_1) I_i$$

Darlington Connection

$$V_{\pi 1} = I_i r_{\pi 1} \rightarrow g_{m1} V_{\pi 1} = g_{m1} r_{\pi 1} I_i = \beta_1 I_i$$

$$I_{b2} = I_i + \beta_1 I_i = (1+\beta_1) I_i \rightarrow V_{\pi 2} = I_{b2} r_{\pi 2} = (1+\beta_1) I_i r_{\pi 2}$$

$$g_{m2} V_{\pi 2} = g_{m2} (1+\beta_1) I_i r_{\pi 2} = \beta_2 (1+\beta_1) I_i$$

$$I_o = g_{m1} V_{\pi 1} + g_{m2} V_{\pi 2} = \beta_1 I_i + \beta_2 (1+\beta_1) I_i$$

The current gain is

$$A_i = \frac{I_o}{I_i} = \beta_1 + \beta_2 (1+\beta_1) \cong \beta_1 \beta_2$$

Darlington Connection

$$V_i = V_{\pi 1} + V_{\pi 2} = I_i r_{\pi 1} + (1 + \beta_1) I_i r_{\pi 2}$$

Darlington Connection

$$V_i = V_{\pi 1} + V_{\pi 2} = I_i r_{\pi 1} + (1 + \beta_1) I_i r_{\pi 2}$$

The input resistance is

$$R_i = \frac{V_i}{I_i} = r_{\pi 1} + (1 + \beta_1) r_{\pi 2}$$

Darlington Connection

$$V_i = V_{\pi 1} + V_{\pi 2} = I_i r_{\pi 1} + (1+\beta_1) I_i r_{\pi 2}$$

The input resistance is

$$R_i = \frac{V_i}{I_i} = r_{\pi 1} + (1+\beta_1) r_{\pi 2}$$

Approximate expression for the input resistance of the darlington configuration above is

$$R_i \cong 2\beta_1 r_{\pi 2}$$

since

$$r_\pi = \frac{\beta V_T}{I_{CQ}}$$

&

$$I_{CQ1} \cong \frac{I_{CQ2}}{\beta_2}$$