Solutions to Tutorial Sheet - 1 IEC103

Q1 Classify the following signals using one descriptor each from A, B, C, & D from the following list

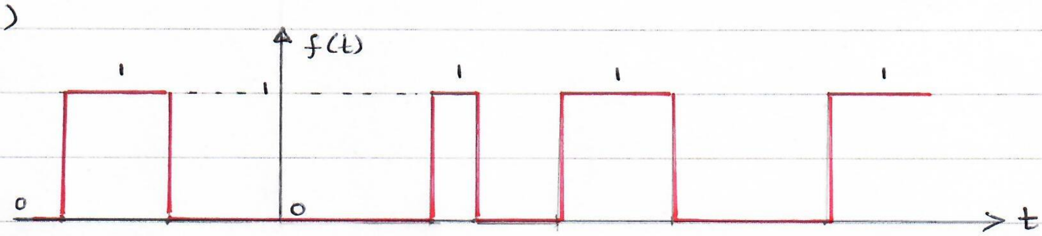
A	B	C	D
Continuous	Time continuous	Amplitude continous	Periodic
Discontinuous	Time discrete	Amplitude discrete	Aperiodic
		Binary	

i) $\cos(w_0 t)$ for $-\infty < t < \infty$

ii) $f(t) = 2$ for $0 < t \leq 8$ and '0' elsewhere

iii) $f(t) = \cos(w_0 t)$ where $w_0 = \frac{2\pi}{T}$ with $t = \frac{nT}{N}$ $n \in \mathbb{Z}$ $N \gg 1$

iv)



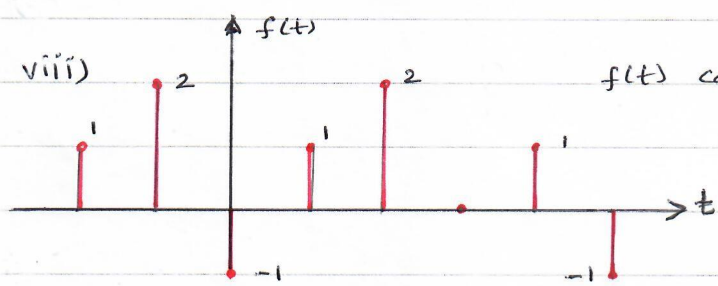
v) $\sin(1000t) + \cos(4000t)$ where $t \in \mathbb{R}$

vi) $f(t) = e^{-2t}$ for $t \geq 0$

$= 0$ elsewhere

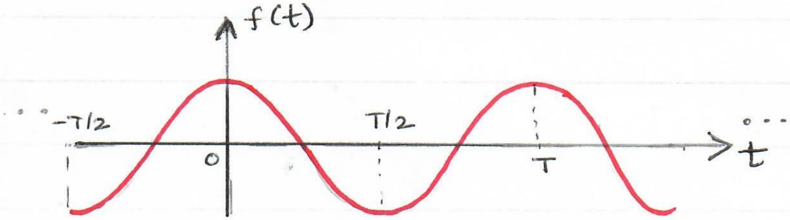
vii) $f(t) = e^{-t^2/2}$ for $-\infty < t < \infty$

viii)



f(t) can take only -1, 0, 1, and 2 values

i) $\cos(w_0 t)$ for $-\infty < t < \infty$



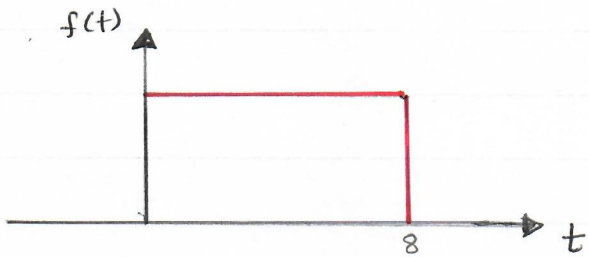
A : Continous

B : Time continuous

C : Amplitude continuous

D : Periodic

ii) $f(t) = 2$ for $0 < t \leq 8$ and '0' elsewhere



A : Discontinuous

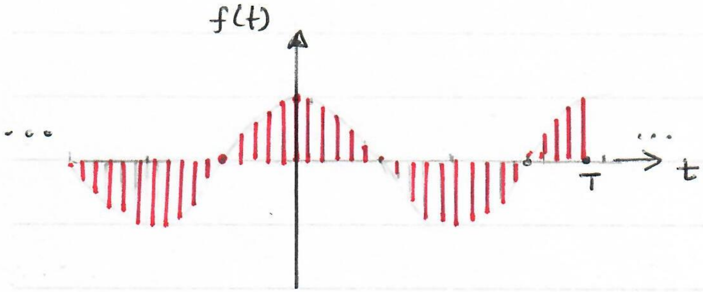
B : Time continuous

C : Amplitude discrete

D : A periodic

iii) $f(t) = \cos(w_0 t)$ where $w_0 = \frac{2\pi}{T}$ with $t = \frac{nT}{N}$ $n \in \mathbb{Z}$

$N >> 1$



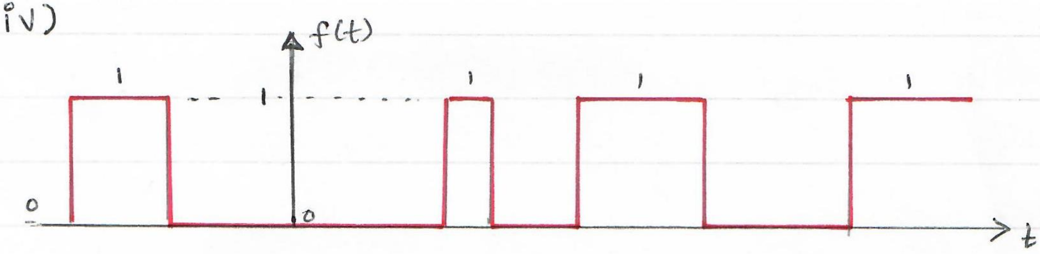
A : Discontinuous

B : Time discrete

C : Amplitude continuous

D : Periodic

iv)



A: Discontinuous

B: Time-continuous

C: Binary

D: Aperiodic

v) $\sin(1000t) + \cos(4000t)$ where $t \in \mathbb{R}$

It is the sum of two periodic continuous (sinusoids) signal

∴ The signal is

A: continuous

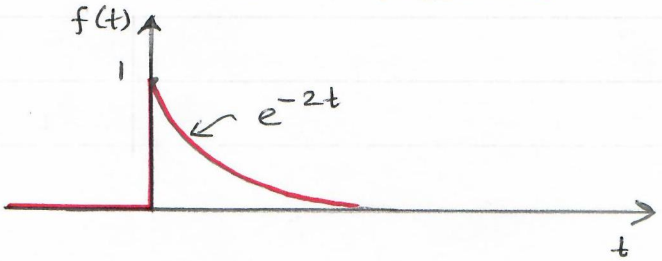
B: Time continous

C: Amplitude continuous

D: Periodic

vi) $f(t) = e^{-2t}$ for $t \geq 0$

$= 0$ elesewhere



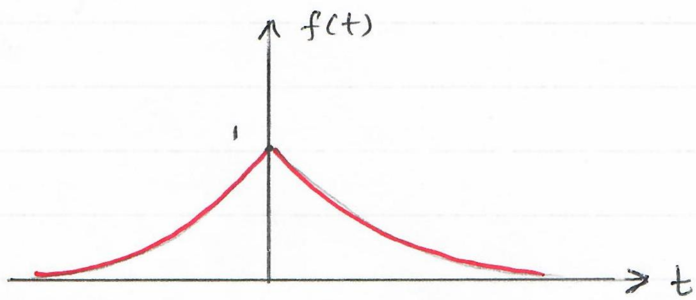
A: Discontinuous

B: Time continuous

C: Amplitude continuous

D: Aperiodic

vii) $f(t) = e^{-t^2/2}$ for $-\infty < t < \infty$



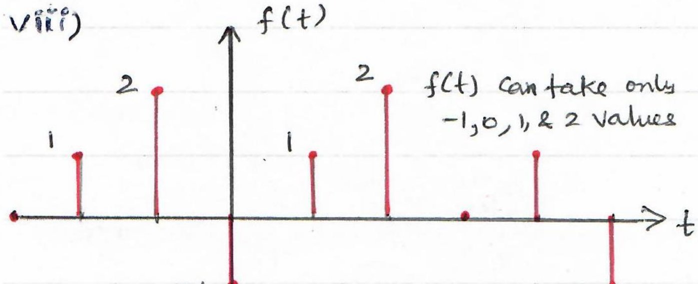
A: continuous

B: Time continuous

C: Amplitude continuous

D: Aperiodic

viii)



f(t) can take only -1, 0, 1, & 2 values

A: Discontinuous

B: Time discrete

C: Amplitude discrete

D: Aperiodic

Q2) Two signals $f_1(t) = A\cos(\omega t)$ and $f_2(t) = A\cos(\omega t + \theta)$ are multiplied to get $f(t) = A^2\cos(\omega t)\cos(\omega t + \theta)$. What is the average and RMS value of $f(t)$?

Sol:

$$f(t) = f_1(t) f_2(t) = A^2 \cos(\omega t)\cos(\omega t + \theta)$$

$$= \frac{A^2}{2}\left[\cos(2\omega t + \theta) + \cos(\theta)\right]$$

$$= \frac{A^2}{2}\cos(2\omega t + \theta) + \frac{A^2}{2}\cos\theta \quad ; \omega = \frac{2\pi}{T}$$

$$f_{av} = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\,dt$$

$$= \frac{1}{T}\int_{-T/2}^{T/2}\left[\frac{A^2}{2}\cos(2\omega t + \theta) + \frac{A^2}{2}\cos\theta\right]dt$$

$$= \frac{A^2}{2T}\int_{-T/2}^{T/2}\cos\left(2\times\frac{2\pi}{T}t + \theta\right)dt + \frac{A^2\cos\theta}{2T}\int_{-T/2}^{T/2} dt$$

$$= 0 + \frac{A^2}{2T}\cos\theta\,(t)\Big|_{-T/2}^{T/2}$$

$$= \frac{A^2}{2T}\cos\theta \times \left[\frac{T}{2} + \frac{T}{2}\right] = \frac{A^2}{2\cancel{T}}\cos\theta \times \cancel{T} = \frac{A^2}{2}\cos\theta$$

$$\boxed{f_{av} = \frac{A^2}{2}\cos\theta}$$

$$f_{RMS} = \sqrt{\frac{1}{T}\int_{-T/2}^{T/2}[f(t)]^2 dt}$$

$$f_{RMS}^2 = \frac{1}{T}\int_{-T/2}^{T/2}(f(t))^2 dt$$

$$= \frac{1}{T}\int_{-T/2}^{T/2}\left\{\frac{A^2}{2}\left[\cos(2\omega t+\theta)+\cos\theta\right]\right\}^2 dt$$

$$= \frac{A^4}{4T}\int_{-T/2}^{T/2}\left[\cos^2(2\omega t+\theta)+\cos^2\theta+2\cos(2\omega t+\theta)\cos\theta\right]dt$$

$$= \frac{A^4}{4T}\left[\int_{-T/2}^{T/2}\cos^2(2\omega t+\theta)dt + \int_{-T/2}^{T/2}\cos^2\theta\, dt + 2\int_{-T/2}^{T/2}\cos(2\omega t+\theta)\cos\theta\, d\theta\right]$$

$$= \frac{A^4}{4T}\left[\int_{-T/2}^{T/2}\left\{\frac{1+\cos(4\omega t+2\theta)}{2}\right\}dt + \int_{-T/2}^{T/2}\cos^2\theta\, dt + 2\int_{-T/2}^{T/2}\left\{\frac{\cos(2\omega t+2\theta)+\cos(2\omega t)}{2}\right\}dt\right]$$

$$= \frac{A^4}{4T}\left[\frac{1}{2}\int_{-T/2}^{T/2}dt + \frac{1}{2}\int_{-T/2}^{T/2}\cos(4\omega t+2\theta)dt + \cos^2\theta\int_{-T/2}^{T/2}dt + \int_{-T/2}^{T/2}\cos(2\omega t+2\theta)dt + \int_{-T/2}^{T/2}\cos(2\omega t)dt\right]$$

$$= \frac{A^4}{4T}\left[\frac{T}{2} + 0 + T\cos^2\theta + 0 + 0\right]$$

$$= \frac{A^4}{4T}\left(\frac{T}{2}+T\cos^2\theta\right) = \frac{A^4}{4}\left(\frac{1}{2}+\cos^2\theta\right) = \frac{A^4}{8}(1+2\cos^2\theta) = f_{RMS}^2$$

$$\therefore \boxed{f_{RMS} = \frac{A^2}{2\sqrt{2}}(1+2\cos^2\theta)^{1/2}}$$

(Q3) Let $x_1(t)$ and $x_2(t)$ be periodic with period T_1 and T_2 respectively. What is the condition to make the signal $x_1(t) + x_2(t)$ periodic?

Sol.

Given signals $x_1(t)$ and $x_2(t)$ are periodic with fundamental periods T_1 and T_2 respectively. So,

$$x_1(t) = x_1(t+T_1) = x_1(t+mT_1) \text{ where } m \text{ is +ve integer}$$

$$x_2(t) = x_2(t+T_2) = x_2(t+nT_2) \text{ where } n \text{ is +ve integer}$$

Given signal is $x(t) = x_1(t) + x_2(t)$

$$= x_1(t+mT_1) + x_2(t+nT_2)$$

For $x(t)$ to be periodic T, we require

$$x(t) = x(t+T) = x_1(t+T_1) + x_2(t+T)$$

$$= x_1(t+mT_1) + x_2(t+nT_2)$$

$$mT_1 = nT_2 = T$$

$$\Rightarrow \frac{T_1}{T_2} = \frac{n}{m} = \text{rational number}$$

Then the fundamental period T of $x(t)$ is the LCM of T_1 and T_2. Therefore $T = mT_1 = nT_2$

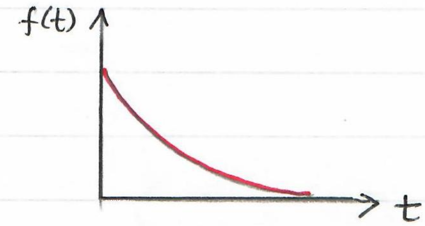
So, we can say that the sum of two periodic signals is periodic only if the ratio of there respective periods is a rational numbers. Then the fundamental period is the LCM of the respective fundamental periods.

(Q4) A signal $f(t) = \begin{cases} 0 & \text{for } -\infty < t < 0 \\ e^{-\alpha t} & \text{for } 0 \le t < \infty \end{cases}$

What is the DC value and the power of the signal.

Sol. The DC value of the signal is nothing but the average value of the signal. The RMS value of the signal indicate the power contained in the signal.

$$f_{av} = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\,dt$$



$$f_{RMS}^2 = \frac{1}{T}\int_{-T/2}^{T/2} [f(t)]^2\,dt$$

This signal is an aperiodic signal. So, the signal can be considered as a signal with period tending to ∞.

$$F_{av} \text{ for an aperiodic signal} = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} f(t)\,dt$$

$$F_{RMS} \text{ for an aperiodic signal} = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} [f(t)]^2\,dt$$

$$\therefore\ f_{av} = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} f(t)\,dt = \lim_{T\to\infty}\left[\frac{1}{T}\int_{-T/2}^{0} 0\,dt + \frac{1}{T}\int_{0}^{T/2} e^{-\alpha t}\,dt\right]$$

$$= \lim_{T\to\infty}\left[0 + \frac{-1}{T}\,\frac{e^{-\alpha t}}{\alpha}\Big|_0^{T/2}\right]$$

$$= \lim_{T\to\infty} -\frac{1}{\alpha T}\,e^{-\alpha t}\Big|_0^{T/2}$$

$$= \lim_{T\to\infty} \frac{-1}{\alpha T}\left[e^{-\alpha T/2} - 1\right]$$

$$= \lim_{T\to\infty} -\frac{e^{-\alpha T/2}}{\alpha T} + \lim_{T\to\infty} \frac{1}{\alpha T},$$

$$= 0 + 0 = 0$$

$$f^2_{RMS} = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} [f(t)]^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{0} 0\, dt + \lim_{T\to\infty} \frac{1}{T}\int_{0}^{T/2} (e^{-\alpha t})^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{0}^{T/2} e^{-2\alpha t} dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \left.\frac{e^{-2\alpha t}}{-2\alpha}\right|_0^{T/2}$$

$$= \lim_{T\to\infty} -\frac{1}{2\alpha T} \left. e^{-2\alpha t}\right|_0^{T/2}$$

$$= \lim_{T\to\infty} -\frac{1}{2\alpha T}\left[e^{-\alpha T} - 1\right]$$

$$\lim_{T\to\infty} \frac{1}{2\alpha T} + \lim_{T\to\infty} \frac{e^{-\alpha T}}{2\alpha T}$$

$$0 + 0 = 0$$

Q5. A linear time invariant system yields output $y(t)$ for an input $x(t)$ as shown in Fig. Q a). What will be the system's output for an input $x_1(t)$ shown in Fig. Q b) ?

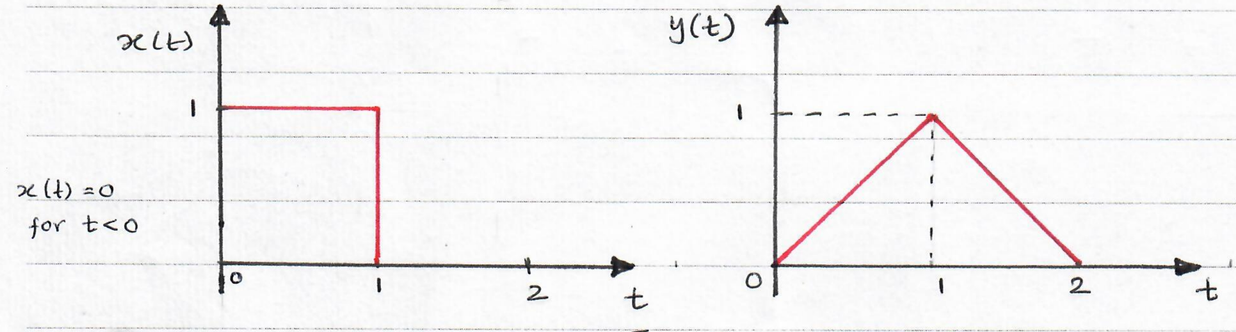


Fig. Q a

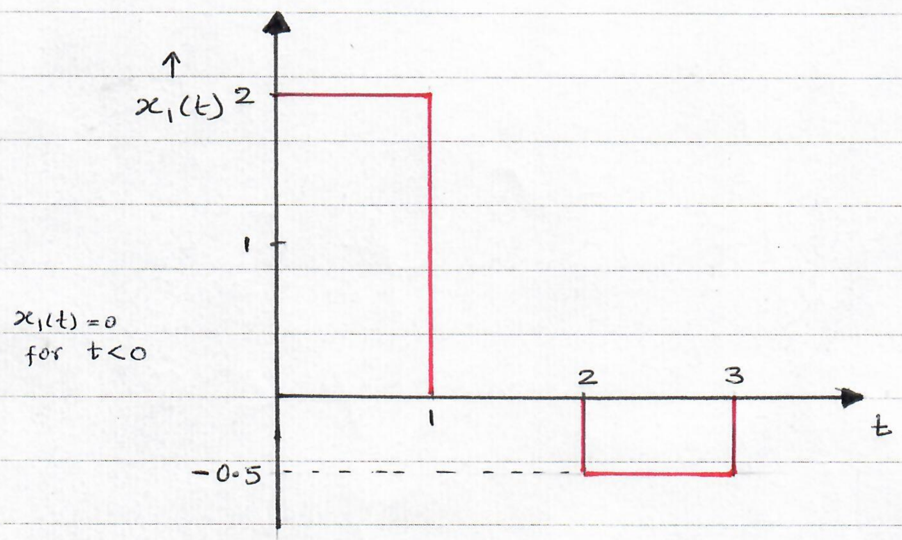


Fig. Q b

Sol. The output $y(t)$ due to input $x_1(t)$ is shown below.