Practice Questions

Minimum spanning Trees
Data Structures

March 31, 2018

Problem 1.

Does any subset of edges that connects all the vertices and has minimum weight necessarily forms a tree ? You may consider negative edge weights too.

Problem 2. Given a graph G and its Minimum spanning tree T, suppose we modify one of the edge of the graph G, give an efficient algorithm to find the new MST.

Problem 3. True/False :
In a graph G let e_{min} be the edge with minimum weight, then e_{min} is always part of some MST of graph G.
In a graph G let e_{max} be the edge with maximum weight, then e_{max} is never part of any MST of graph G.

Problem 4. Suppose you are to compute the minimum product spanning tree where the product of all the weights should be minimum. Would the minimum product spanning tree be the same as the minimum weight spanning tree. Justify.

Problem 5. How would you find a spanning tree that is not a MST in a graph of distinct edge weights? How would you address the same if edge weights are not distinct ?

Problem 6. Given is a graph G=(V,E) of distinct edge weights and a spanning tree T. Let adjacent spanning tree be a spanning tree derived by changing one edge(adding one edge and removing another) in the input spanning tree. Prove that every spanning tree except the MST has an adjacent spanning tree of smaller weight.

Problem 7. Given is an undirected graph G=(V,E) with a cost c(e) on each edge e in E. Also given is a subset S of vertices. You need to report the spanning tree wherein all elements of S are leaf nodes. Also among all such spanning trees, the tree you report should have the least weight. There could be other leaf nodes as well, in the output tree you report. Provide an algorithm that either outputs the spanning tree that satisfies the constraints or state that such a result is not possible.

Problem 9. Consider the modified divide and conquer algorithm to find the minimum spanning tree of a graph G. We divide the graph into two sets of vertices V1 and V2 such that both are of size atleast 1. We recursively call the MST algorithm recursively on both the partitions, and then add the edge with the minimum weight (u,v) such that u $\in$ V1 and v $\in$ V2 in MST.
Base case would be when there is only one vertex in the partition, then we return the node itself.
Does the modified algorithm give correct MST ?

Problem 10. Consider the modified algorithm to find the MST

c. MAYBE-MST-C(G, w)

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T = ∅
for each edge e, taken in arbitrary order
    T = T ∪ {e}
    if T has a cycle c
        let e' be a maximum-weight edge on c
        T = T − {e'}
return T
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Argue if the function gives correct MST, if not give a counter example.

Problem 11. Given that we are storing a graph G in the form of adjacency matrix, give an implementation of Prims Algorithm that runs in O(V^2) complexity, where V is the number of vertices in the graph G.