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Using Detrended Cross-Correlation Analysis in geophysical data

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ABSTRACT

Detrended cross correlation analysis (DCCA) is used to identify and characterize correlated data obtained in drilled oil wells. The investigation is focused on different petro-physical measurements within the same well, and of the same measurement from two wells in the same oil field. The evaluation of cross correlation exponents indicates if scaling properties in two measurements are alike. The work considers also the values of cross correlated coefficients, which provide an assessment on the local correlation between measurements. The existence of several highly correlated events provides information on the continuity of geological structures, including partial and global dislocations of deposited layers.

1. Introduction

The knowledge of the earth subsurface is central to human activities, either in the construction or exploration activities or to minimize the damage by natural phenomena. Such study, however, is hampered by the high cost of a direct access to the system or by its inherent complexity. Indeed, the different structures within the system were formed and modified in a wide variety of processes over geologic time scales [1–3]. To fully understand this system, it is necessary to unravel its preceding history using the fingerprints they have left in the geological structures. The ultimate goal is to identify the agents that have driven the system, the kind of intervening materials, and to present the sequence of processes and the conditions under which they occurred. Advances in the understanding of the geosphere, in particular of the lithosphere, are obtained by analyzing geological and geophysical data. Such knowledge is of utmost importance in the exploitation of natural resources, such as oil, reducing risk levels and making it more efficient. Well data profiles (well-logs) are particularly of great importance for the oil industry. They consist in the systematic recording of several physical and chemical properties of the subsurface materials, e.g., resistivity, radioactivity, temperature, as a function of depth. These data provide information about the sequence of rock layers in the drilled wells, layer interfaces, geological structures, being used to characterize hydrocarbon reservoirs and to assess the feasibility of oil exploitation [1].

Several mathematical tools that uncover scaling fluctuation properties have been used to analyze geophysical data [4–7]. In particular, well-log data sets have been subject of a comparative analysis by several nonlinear methods [8]. One contribution for the understanding of well log data is based on the analysis of local and global fluctuations existing in the series. Such a procedure allows to identify the presence of (auto) correlation in a single signal as a function of the distance between records, or between two different signals (cross-correlation). Due to short and large scale variability, respectively related to the local inhomogeneity in rock composition and the superposed layered structures, well-logs are clear examples of irregular, non-stationary data sets. There are various methods to identify and characterize both stationary and non-stationary complex system records. They are mainly targeted to evaluating the range of fluctuations existing in the series, the magnitude of which can vary over many time scales.

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One key idea to characterize different fluctuation properties is to separate actual fluctuations from contributions due to local trends that persist over larger scales. The identification of patches within which the series can be assumed as stationary is also of great importance [9,10]. The so-called detrended fluctuation analysis (DFA) [11,12] leads to good estimates of usual Hurst exponent H [13,14], provided the series are stationary over the considered patch. The identification of correlation within the same signal (autocorrelation), or between two different signals (cross-correlation) is another strategy in the record analysis. In the case of well-logs, for example, the strong correlation between two signals of different profiles may indicate the continuity of a geological layer. In this work, we show that correlations in well-logs can be detected and characterized by Detrended Cross-Correlation Analysis (DCCA) [15–17], which makes use of the same idea that justifies the DFA strategy: to separate the contribution of local trends from that of actual fluctuations. The DCCA framework has been recently enlarged to handle series where the presence of periodic component can be identified [18]. Although this phenomenon occurs very often in real world data, it was not the case of the well-log data we analyzed. Our results are based both on the scaling exponents of cross correlations between data fluctuations, as well as on the coefficients of the correlation function between the same data. It is worth commenting that recent works have been able to identify and quantify asymmetries between the upward and downward scaling behavior in financial data [19,20]. In the current work, we have not been able to identify clear asymmetric scaling properties in the data sets.

In the next sections of this paper we discuss the following issues. The DCCA procedure is reviewed in Section 2. It is divided into two subsections, where we briefly discuss the exponent and coefficient evaluation. Results obtained from these two different approaches are presented in Sections 3 and 4. Finally, Section 5 closes the paper with our concluding remarks and discussions.

2. Detrended cross correlation analysis

2.1. DCCA exponents

In general fluctuation analysis, we consider two data sets of increments x_i and x'_i, both consisting of N records, and build the corresponding integrated series

$$y_k = \sum_{i=1}^{k} x_i, \qquad y'_k = \sum_{i=1}^{k} x'_i, \tag{1}$$

with $k = 1, \ldots, N$. Each integrated series is divided into M_ν (possibly overlapping) boxes of width ν. In each box labeled by (m, ν), with $1 \leq m \leq M_\nu$, we compute a measure of fluctuation that depends on the method. Throughout this work, we evaluate four fluctuation measures, respectively labeled by *DFA* (Detrended Fluctuation Analysis - Eq. (2)), *SCCA* (Standard Cross Correlation Analysis - Eq. (3)), *DCCA* (Detrended Cross Correlation Analysis - Eq. (4)), and $|DCCA|$ (Eq. (5)):

$$f^2_{DFA}(m, \nu) = \frac{1}{\nu} \sum_{k=I_{min}(m,\nu)}^{I_{max}(m,\nu)} [y_k - p_k(m, \nu)]^2, \tag{2}$$

$$f^2_{SCCA}(m, \nu) = \frac{1}{\nu} \sum_{k=I_{min}(m,\nu)}^{I_{max}(m,\nu)} [y_k - \bar{y}(m, \nu)][y'_k - \bar{y}'(m, \nu)], \tag{3}$$

$$f^2_{DCCA}(m, \nu) = \frac{1}{\nu} \sum_{k=I_{min}(m,\nu)}^{I_{max}(m,\nu)} [y_k - p_k(m, \nu)][y'_k - p'_k(m, \nu)], \tag{4}$$

$$f^2_{|DCCA|}(m, \nu) = \frac{1}{\nu} \sum_{k=I_{min}(m,\nu)}^{I_{max}(m,\nu)} |[y_k - p_k(m, \nu)][y'_k - p'_k(m, \nu)]|. \tag{5}$$

The label $|DCCA|$ in Eq. (5) emphasizes the use of the *absolute* value of the local fluctuation in each of the series. In Eqs. (2)–(5), $\bar{y}$ and $\bar{y}'$ are the mean values of y_k and y'_k in the box (m, ν) limited by $I_{min}(m, \nu)$ and $I_{max}(m, \nu)$; $p_k(m, \nu) = a(m, \nu)x_k + b(m, \nu)$ and $p'_k(m, \nu) = a'(m, \nu)x'_k + b'(m, \nu)$ are the first degree polynomials evaluated by the least squares method that express the local linear trends of the series in the box (m, ν). In particular, the $f^2_{DFA}(m, \nu)$ is the particular case of f_{DCCA} or $f_{|DCCA|}$ with two equal series.

In the sequence, a fluctuation function is calculated for each width ν according to

$$F^2_X(\nu) = \frac{1}{M_\nu} \sum_{m=1}^{M_\nu} f^2_X(m, \nu), \quad X = DFA, SCCA, DCCA, |DCCA|. \tag{6}$$

If the series present scale properties related to cross-correlations, it is expected that a power law $F_X(\nu) \sim \nu^\lambda$ holds. The exponent λ represents a measure of cross correlation between the two analyzed series. In the case of DFA, the exponent λ becomes equivalent to the Hurst or roughness exponent, usually denoted by H or α.

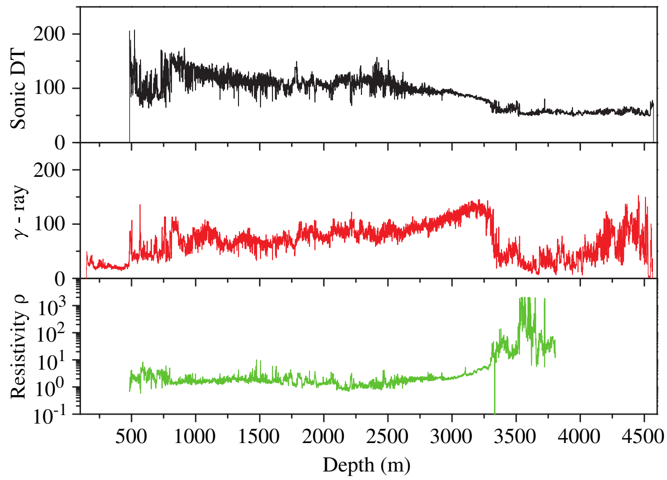

Fig. 1. Well-log data of the well 1BAS68 showing the signals of transient sonic irradiation (DT), gamma radiation (γ) and resistivity (ρ). The γ series is the most complete one. Missing data for DT and ρ series restrict our analyses to the depth interval ranging from 500 to 3000 m (∼17,000 points).

The values of λ indicate the type of correlations between the series. Much as the exponent H [21], λ is related to the concept of persistence and anti-persistence. For $\lambda > 0.5$ the series are persistent, i.e., a large (small) increase in one of the series is followed by (in average) large (small) increase in the future of the other series. Anti-persistence when $\lambda < 0.5$ represents the opposite situation, with both series reversing the direction of the recent moves. Finally, when $\lambda = 0.5$ the increments are, in average, not correlated with the previous ones.

2.2. DCCA coefficients

The DCCA cross-correlation coefficient σ_{DCCA} was proposed to quantify the cross-correlation between two non-stationary series [22–24]. This coefficient is defined for each scale of analysis ν (the width of boxes) as the ratio

$$\sigma_{DCCA}(\nu) = \frac{F^2_{DCCA}(\nu)}{F_{DFA}(\nu)F'_{DFA}(\nu)}. \tag{7}$$

The DCCA coefficient is a dimensionless quantity ranging between $-1 \leq \sigma_{DCCA} \leq 1$. Similar to the standard correlation coefficient, $\sigma_{DCCA} = 1$ indicates a perfect cross-correlation while $\sigma_{DCCA} = -1$ means a perfect anti cross-correlation. If $\sigma_{DCCA} = 0$, there is no cross-correlation between the series. Because the DCCA coefficient is a function of the scale ν, it is possible to observe how the cross-correlation between the series behaves on different scales and determine, e.g., if a high cross-correlation holds for all scales or if the intensity of cross-correlation changes in a particular scale.

It is important to notice that the expression for the DCCA coefficient Eq. (7) uses the fluctuation function $F_{DCCA}(\nu)$ and not $F_{|DCCA|}(\nu)$. For this analysis, the sign of the coefficient provides information about the type of correlation and it would be lost if the absolute values of fluctuations were considered.

3. Results: DCCA exponents

We applied the above described method to analyze well-logs of two drilled wells in the off-shore Jequitinhonha basin, Brazil, labeled as 1BAS68 and 1BAS121. Each of them comprise various petro-physical measures, including, the transient sonic irradiation (DT), gamma radiation (γ), and resistivity (ρ). These data sets have also been analyzed within the framework of wavelet cross-correlation analysis [25]. In Fig. 1 we illustrate typical structure of such data sets by drawing the DT, γ and ρ profiles of the 1BAS68 well. The measurements were recorded at intervals of 15.24 cm, (1/2 *foot*), reaching the maximal depth of 4,500 m corresponding to approximately 30,000 entries. The choice of these three petro-physical measures was based both on the completeness of the records and on their relevance.

The original well-log sets, like those shown in Fig. 1, are interpreted as the integrated series y_k. The subtraction of two contiguous values leads to step series x_i, from which two further series for step magnitude and sign can be obtained, respectively defined as $M = |x_i|$ and $S = \text{sign}(x_i)$ [26]. Next, the corresponding integrated series y_k^M and y_k^S are calculated with the help of Eq. (1).

For the exponent analysis, we use the fluctuation function $F_{|DCCA|}(\nu)$ instead of $F_{DCCA}(\nu)$. If the last one is used, we observe a reduction of the fluctuation function caused by the negative contributions due to possible different signs of the terms in Eq. (3). In this case we obtain graphs with no clear scaling behavior, as seen in [22]. To overcome this difficulty we use the

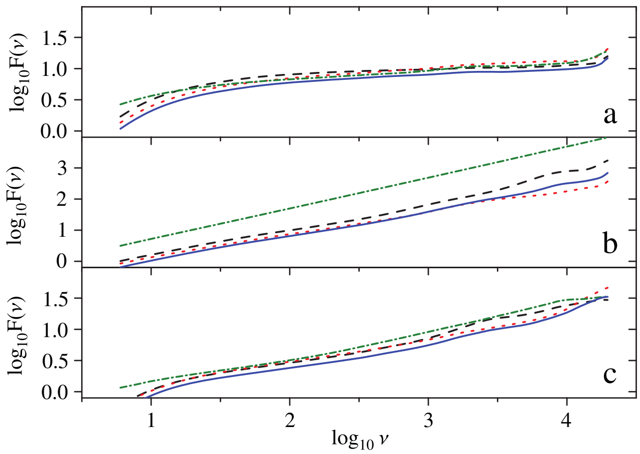

Fig. 2. Dependence of $DTF_{DFA}(\nu)$ (dashes), $\gamma F_{DFA}(\nu)$ (dots), $(DT, \gamma)F_{SCCA}(\nu)$ (dash-dots) and $(DT, \gamma)F_{|DCCA|}(\nu)$ (solid) as a function of ν. (a) Original series, with weak scaling behavior. Linear least square fits become meaningful in a small ν interval only. In such cases, the slopes are very small. (b) Magnitude series. Here, the measured slopes are robust and lead to the following values of the corresponding exponents: 0.88, 0.71, 0.99, 0.81. They indicate clear cut persistent scaling behavior for all four fluctuation measures. (c) Signal series, with anti-persistent scaling behavior for all four fluctuation measures. In this case, the obtained values for the exponents are, respectively: 0.45, 0.42, 0.44, 0.43.

absolute values of fluctuations. In Fig. 2, we show the dependence of $F(\nu)$ as a function of ν for three series y_k, y_k^M, and y_k^S obtained from the DT and γ records of the 1BAS68 well.

The results comprising a set of about 17,000 points suggest that the original, magnitude and sign series have their own scale properties, which are also manifested in the cross-correlation analyses of two different measures in the same well. Scale intervals and exponents are different for the three series types. In the case of y_k^M, the scaling interval spans more than three orders of magnitude while, for the original series, the scaling behavior over a shorter interval is much weaker. Due to the small range of scaling region, we avoid associating a growth exponent to the original series. The slope of the intermediate region, which could be a proxy for H, is very small, indicative of anti-persistent behavior. The numerical values of the exponents of the magnitude and signal series are indicated in the caption of Fig. 2. The signal series are anti-persistent, while the series of magnitudes are persistent. The cross-correlations between the series follow the same pattern. Furthermore, there is the following approximate relation between the exponents, which was already noticed for distinct data sets analyzed in [15]: $\lambda \simeq (H + H')/2$, where H and H' are the Hurst exponents of each series. With respect to the magnitude and sign series, it can be noticed that the cross correlation exponents are overestimated when obtained by SCCA. Such difference is particularly evident for the magnitude series that are formed by summing only by positive increments. They have a strong tendency to upward masking the actual fluctuations, indicating that the DCCA is most suitable to provide a more accurate measure of the scaling behavior in the cross correlation.

It can be seen that the decomposition of the original y_k set into magnitude and sign series represents a useful strategy that leads to a better characterization of the well logs. Each of these elements can present different behaviors, which become mixed up in the results for $F(\nu)$ of the original series. Effectively, the scaling properties in the graphs based on y_k are less evident than those of the other two series. We obtained results for smaller windows with width larger than 5,000 points in different regions of the profile, which were similar to those for the whole data set presented above in a corresponding reduced scaling region. They show the existence of scaling properties in all patches of the data, and validating DCCA as a trustful tool to effectively reveal the presence of cross-correlations between related data. The geological interpretations provided by these results, which amounts to compare them with the region of lithology (rock types, thickness of layers, interfaces) will be published elsewhere.

When we further reduce the window width within which $F(\nu)$ is evaluated (∼100–300 points), the scaling region becomes very short so that a linear dependence between $\log_{10} F(\nu)$ and $\log_{10} \nu$ may become questionable. This is observed even for the evaluation of the Hurst exponent, which is subject to strong variability. This restricts the possibility of developing a local analysis, in which a DCCA exponent calculated in a local small window could be associated with a given depth with respect to the earth surface. Such a measure would be very useful for the identification of the continuity of geological structures and the understanding of the properties in the context of the available geophysical/geological data.

The natural extension of the presented results is the analysis of correlations between properties of different wells. We have found that, even if the two wells are distant from each other by only 4.5 km, the detrended cross correlation for large width windows shows very weak scaling properties. In order to overcome difficulties in obtaining local information, in the next Section we switch our focus from the DCCA exponents to the DCCA coefficients. We will show that DCCA coefficients can be used to identify local correlations, even in situations where a relative vertical displacement of the geological layers between the wells can be observed.

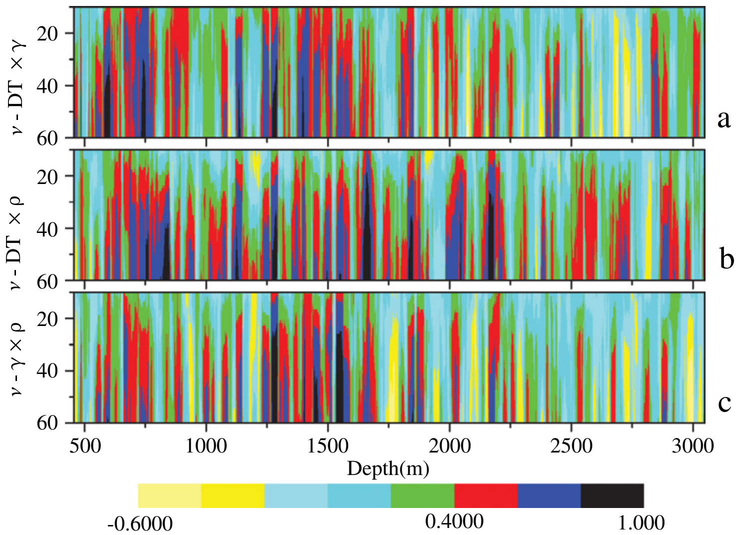

Fig. 3. Diagrams of DCCA coefficients $\sigma_{DCCA}(\nu, z)$ for different properties: (a) $DT \times \gamma$, (b) $DT \times \rho$ and (c) $\gamma \times \rho$ for the well 1BAS68. The horizontal bar indicates the color (or gray tone) associated to the value of $\sigma_{DCCA}(\nu, z)$: black/blue (dark) correspond to a high correlation, going through red/green/cian (mid gray tones) for no correlation, until yellow/light yellow (light gray and white) when the signals are anti-correlated. In these diagrams there are events of high correlation for all scales in various depths, showing that the analyzed properties are correlated.

4. Results: DCCA coefficients

In order to perform a local analysis using the DCCA coefficients, we used a sliding window procedure. In such operation, we move a window (subset of the series) of a given size along the series and compute $\sigma_{DCCA}(\nu)$ for each window. If we relate the window n to the depth z, we can obtain DCCA coefficients as functions of the scale ν and the depth z, $\sigma_{DCCA}(\nu, z)$. The size of the window is an important parameter; it must be small enough to permit a local analysis, but sufficient large to ensure the statistical significance of the fluctuation analysis. Here we evaluated the local DCCA cross-correlation coefficients for the geophysical data described in the previous section, using the magnitude series and a moving window of size 200 points.

First we applied the method to different petro-physical properties (DT, γ, ρ), in each well (see Fig. 3). Because this analysis is performed in a single well, it does not provide relevant information about geological structures, but shows the level of cross-correlation between the properties. Such result is related to that obtained with the DCCA exponents. We can see that $\lambda > 0.5$, which represents a persistent behavior, corresponds to a diagram of DCCA coefficients with events of high correlation in all scales for various depths. The advantage of the local DCCA coefficient is that we can identify where the high correlation events occur and their intensities, while the DCCA exponent provides just a global characterization about persistence. The diagrams confirm a large degree of cross correlation between DT and ρ, which is spread out over almost all well depth. Several spots with highly correlated coefficients over all scales ν can be identified. This somewhat expected result can be understood within the framework of the empirical Archie's law as well as from the phenomenological approaches that have been proposed to derive it from microscopic structural rock properties [27–29]. Next in the rank, DT and γ signs are mainly correlated on smaller depths, while patches where γ and ρ are strongly correlated reach $\sim 1/4$ of the well depth.

Next we calculated the local DCCA coefficients for the same property in the two different wells in order to identify marks of continuity of geological structures as discussed before. As a matter of fact, diagrams $\sigma_{DCCA}(\nu, z)$ for DT, γ, and ρ are characterized by the absence of any relevant contrast. The values of $\sigma_{DCCA}(\nu, z)$ present only random small fluctuations around $\sigma_{DCCA} = 0$, indicating no cross correlation between wells. This stays in agreement with the reported result in the former Section.

We considered further possibilities, corresponding to relative vertical shifts between the wells. In such case, the identification of similar geologic structures that might have been displaced by natural agents could be taken into account. Applying the method for 1BAS68 shifted upward with respect to 1BAS121, an event of strong cross-correlation in all scales was found for shift 101 points (15.3924 m) around the depth 10000 points (1,524 m) (depth measured with respect to the unshifted well-1BAS121) as shown in Fig. 4. We observe that this event is present for all the analyzed properties, although weaker for gamma radiation. Similar results have also been confirmed within the framework of wavelet cross correlation method [25,30,31].

To find other possible events for different shifts, we developed an automatic search procedure that calculate the local DCCA coefficient for different shifts and select the shift and depth where a given rule is satisfied. Fig. 5 shows the result for the automatic search that selected points where the mean of the DCCA coefficient for all scales was greater than 0.70. This rule seems reasonable to consider only events of strong cross-correlation in all scales. As can be observed, there is only

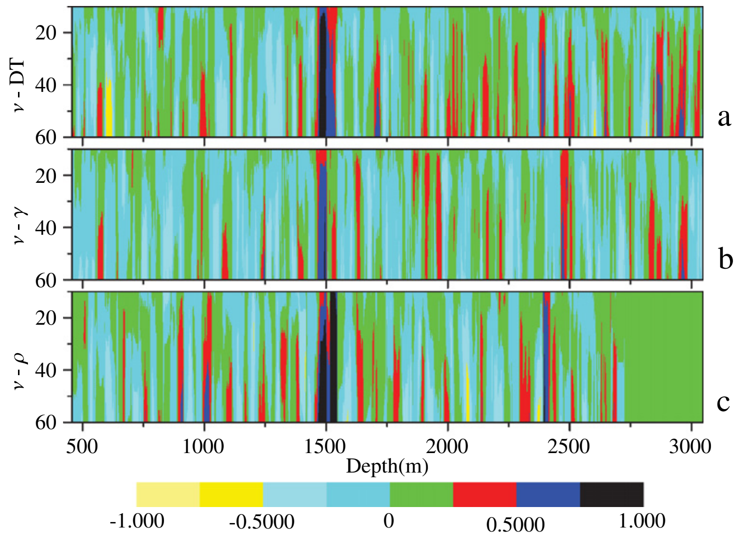

Fig. 4. Diagrams of DCCA coefficients for the same property (a) DT, (b) γ and (c) ρ for the wells 1BAS68 and 1BAS121, where the well 1BAS68 was shifted upward by an amount of 101 points (15.3924 m) with respect to 1BAS121. As in Fig. 3, the horizontal bar indicates the color (gray tone) associated to the value of $\sigma_{DCCA}(\nu, z)$. We observe a high correlation event around the depth 1,524 m for all the three properties, which may indicate a continuity of a geological structure.

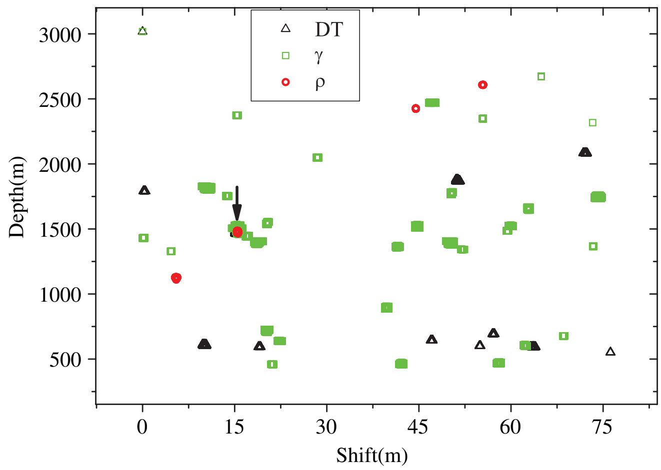

Fig. 5. Result of the automatic search procedure for high correlation events between the wells 1BAS68 and 1BAS121 for the properties: $DT(\triangle)$, $\gamma(\circ)$ and $\rho(\square)$. The points (*shift*, *depth*) were plotted when the mean value of the DCCA coefficients for all scales was greater than 0.70. The indicated point corresponds to the event of shift 15.3924 m and depth ∼1,524 m.

one point in which all the three properties present an event of high correlation that is found at the depth ∼1,524 m, after shifting upward the 1BAS68 series by 15.3924 m (see Fig. 4). Other strong correlation events occur for sonic irradiation and resistivity (shift 0 m, depth ∼3,000 m and shift ∼8 m, depth ∼1,750), but are weaker than that shown in Fig. 5. Again the analysis for gamma radiation is different, presenting fewer and weaker events then for sonic irradiation and resistivity.

The results in this Section clearly show that local analysis of the DCCA coefficients is particularly useful for well-log data. The identification of strong cross-correlation for different wells in all scales in a given depth may represent the continuity of a geological structure since this kind of event only occur if there is a high similarity between the fluctuation of the signals.

5. Conclusions

This work used the general DCCA framework to investigate actual well-logs from two neighboring wells in an off-shore oil field. Both DCCA exponents and coefficients were used to obtain information about the correlation between the recorded data. In the case of DCCA exponents, we have shown that some pairs of signals (e.g. DT and γ-ray) from the same well have large tendency of being, in the average, globally correlated. Unlike the SCCA analysis, the DCCA results do not over estimate the magnitude and sign cross exponents. However, the attempt to scale the global analysis to smaller patches faced difficulties: the exponents are subject to large fluctuations, mostly due to the lack of clear scaling behavior. The analysis carried out by DCCA coefficients was able to detect the presence of events that are correlated of two distinct nature: different measures in the same well and the same measure between wells. In the former case, correlation is related to specific properties that are concomitantly present in some layers. They may corroborate specific layer composition indicated by lithographic studies. In the latter case, we have found that correlated events are rare events. However, several correlated patches have been detected once the series are displaced with respect to each other.

The hypothesis of vertical layer displacement to explain the high correlated signals does not require large scale changes. Indeed, in the observed event illustrated in Fig. 4, the series are shifted by ∼15 m, which requires a very small tilt in a geological layer, taken into account the fact that the two wells are ∼5 km apart. Since the lithology at the two locations is not precise, the strongest support to our claim (at the current knowledge stage) comes from the fact that high correlated values occur over a broad scale interval. The likelihood of fortuitous match is strongly depressed in such case. Nevertheless, this still needs to be checked by further geological analyses. If this can be conveniently clarified, this method can be used systematically for the purpose of the characterization of the earth subsurface.

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