

HW #3

3.13 LABORATORY EXERCISES AND PROJECTS

- Design a Wiener filter to remove the artifacts in the ECG signal in the file `ecg_hfn.dat`. (See also the file `ecg_hfn.m`.) The equation of the desired filter is given in Equation 3.101. The required model PSDs may be obtained as follows:
Create a piece-wise linear model of the desired version of the signal by concatenating linear segments to provide P, QRS, and T waves with amplitudes, durations, and intervals similar to those in the given noisy ECG signal. Compute the PSD of the model signal.
- Redo the above exp by using the ECG filtered by the Comb filter as the template.
- Also Compare the results with the results of the lowpass filter.
- Due date of PPT report: 11/6 2019

HW #3

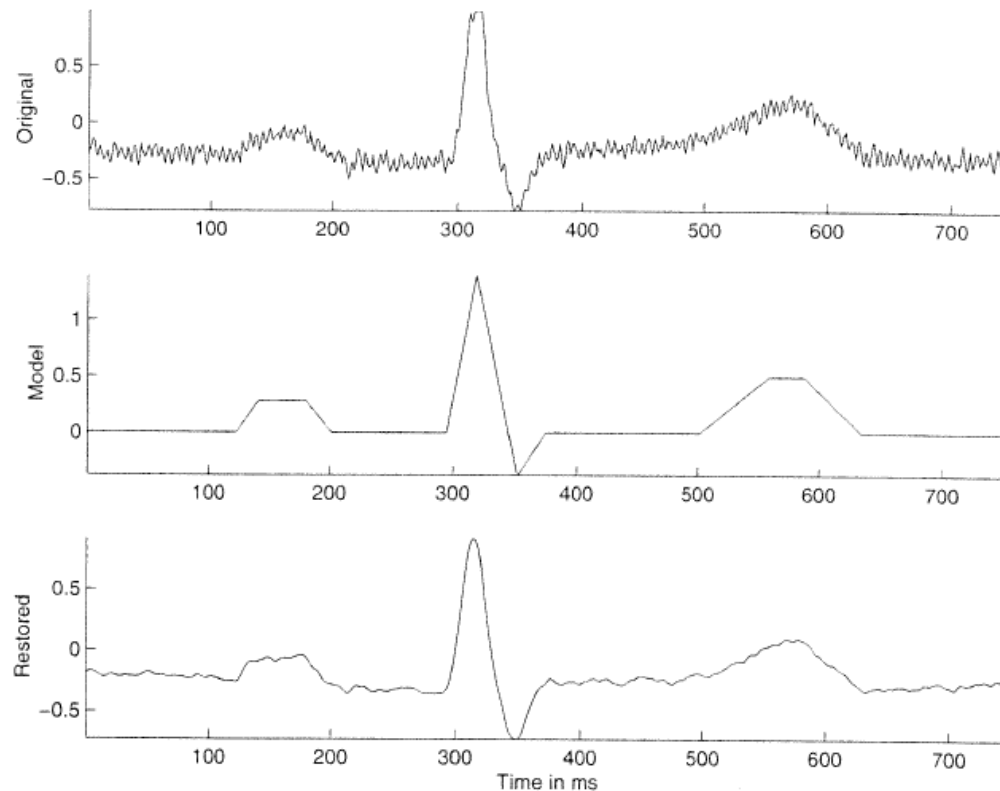


Figure 3.47 From top to bottom: one cycle of the noisy ECG signal in Figure 3.5 (labeled as Original); a piece-wise linear model of the desired noise-free signal (Model); and the output of the Wiener filter (Restored).

補充說明-I

$$\sum_{i=0}^{M-1} w_{oi} \phi(k-i) = \theta(k), \quad k = 0, 1, 2, \dots, M-1. \quad (3.88)$$

Thus we have the convolution relationship

$$w_{ok} * \phi(k) = \theta(k). \quad (3.89)$$

Applying the Fourier transform to the equation above, we get

$$W(\omega) S_{xx}(\omega) = S_{xd}(\omega), \quad (3.90)$$

which may be modified to obtain the Wiener filter frequency response $W(\omega)$ as

$$W(\omega) = \frac{S_{xd}(\omega)}{S_{xx}(\omega)}, \quad (3.91)$$

where $S_{xx}(\omega)$ is the PSD of the input signal and $S_{xd}(\omega)$ is the cross-spectral density (CSD) between the input signal and the desired signal.

補充說明

Let us now consider the problem of removing noise from a corrupted input signal. For this case, let the input $x(n)$ contain a mixture of the desired (original) signal $d(n)$ and noise $\eta(n)$, that is,

$$x(n) = d(n) + \eta(n). \quad (3.92)$$

Using the vector notation as before, we have

$$\mathbf{x}(n) = \mathbf{d}(n) + \boldsymbol{\eta}(n), \quad (3.93)$$

where $\boldsymbol{\eta}(n)$ is the vector representation of the noise function $\eta(n)$. The autocorrelation matrix of the input is given by

$$\boldsymbol{\Phi} = E[\mathbf{x}(n)\mathbf{x}^T(n)] = E[\{\mathbf{d}(n) + \boldsymbol{\eta}(n)\}\{\mathbf{d}(n) + \boldsymbol{\eta}(n)\}^T]. \quad (3.94)$$

If we now assume that the noise process is statistically independent of the signal process, we have

$$E[\mathbf{d}(n)\boldsymbol{\eta}^T(n)] = E[\boldsymbol{\eta}^T(n)\mathbf{d}(n)] = \mathbf{0}. \quad (3.95)$$

Then,

$$\boldsymbol{\Phi} = E[\mathbf{d}(n)\mathbf{d}^T(n)] + E[\boldsymbol{\eta}(n)\boldsymbol{\eta}^T(n)] = \boldsymbol{\Phi}_d + \boldsymbol{\Phi}_\eta, \quad (3.96)$$

where $\boldsymbol{\Phi}_d$ and $\boldsymbol{\Phi}_\eta$ are the $M \times M$ autocorrelation matrices of the signal and noise,

補充說明

$$\Phi = E[\mathbf{d}(n)\mathbf{d}^T(n)] + E[\boldsymbol{\eta}(n)\boldsymbol{\eta}^T(n)] = \Phi_d + \Phi_{\eta}, \quad (3.96)$$

where Φ_d and Φ_{η} are the $M \times M$ autocorrelation matrices of the signal and noise, respectively. Furthermore,

$$\Theta = E[\mathbf{x}(n)d(n)] = E[\{\mathbf{d}(n) + \boldsymbol{\eta}(n)\}d(n)] = E[\mathbf{d}(n)d(n)] = \Phi_{1d}, \quad (3.97)$$

where Φ_{1d} is an $M \times 1$ autocorrelation vector of the desired signal. The optimal Wiener filter is then given by

$$\mathbf{w}_o = (\Phi_d + \Phi_{\eta})^{-1} \Phi_{1d}. \quad (3.98)$$

The frequency response of the Wiener filter may be obtained by modifying Equation 3.91 by taking into account the spectral relationships

$$S_{xx}(\omega) = S_d(\omega) + S_{\eta}(\omega) \quad (3.99)$$

and

$$S_{xd}(\omega) = S_d(\omega), \quad (3.100)$$

which leads to

$$W(\omega) = \frac{S_d(\omega)}{S_d(\omega) + S_{\eta}(\omega)} = \frac{1}{1 + \frac{S_{\eta}(\omega)}{S_d(\omega)}}, \quad (3.101)$$